A Deep Reinforcement Learning Approach to Stock Portfolio Optimization

Undergraduate Research Project

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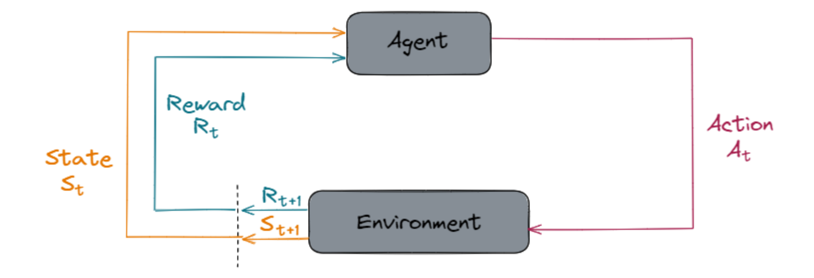
What is Reinforcement Learning?

- Reinforcement learning is a **framework for learning** how to interact with a complex **environment** from **experience**.
- Reinforcement learning is a data science problem, also known as **self-supervised learning**.
- Reinforcement learning can be used to solve **sequential decision-making** problems.

Markov Decision Processes



Example Reward Sequence

$$s_0 \rightarrow a_1 \rightarrow s_1 \rightarrow a_2 \rightarrow s_2 \rightarrow a_3 \rightarrow s_3 \rightarrow r_3 \rightarrow a_4 \rightarrow s_4 \ldots \tag{1}$$

Policy/Value/Quality Functions

Policy functions map state-action pairs to probabilities.

$$\pi(s, a) = \Pr(\text{action} = a \mid \text{state} = s) \tag{2}$$

Value functions map states to expected reward values.

$$V_\pi(s) = \mathbb{E}\left(\sum_t \gamma^t r_t \mid s_0 = s\right) \tag{3}$$

Quality functions map state-action pairs to expected reward values.

$$Q_\pi(s, a) = \mathbb{E}\left(\sum_t \gamma^t r_t \mid s_0 = s, a_0 = a\right) \tag{4}$$

Taxonomy of Reinforcement Learning

- Reinforcement Learning
  - Model Based
  - Model Free
    - On Policy
    - Off Policy

Value Iteration

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Value iteration was the **first ever reinforcement learning algorithm** [1].

Start by assuming we know the state transition probabilities

$$P(s' \mid s, a) = \Pr(s_{t+1} = s' \mid s_t = s, a_t = a) \tag{5}$$

and the reward structure

$$R(s', s, a) = \Pr(r_{t+1} \mid s_{t+1} = s', s_t = s, a_t = a) \tag{6}$$

These become our models for the reward and next state - hence value iteration is **model based**.

Value Iteration

Define the optimal value function as

$$V_\star(s) = \max_\pi V_\pi(s) = \max_\pi \mathbb{E}\left(\sum_{t=0}^{\infty} \gamma^t r_t \mid s_0 = s\right), \forall s \tag{7}$$

Bellman showed that we could write this recursively as

$$V_\star(s) = \max_\pi \mathbb{E}\left(r + \sum_{t=1}^{\infty} \gamma^k r_t \mid s_1 = s'\right) = \max_\pi \mathbb{E}\left(r + \gamma V_\star(s')\right) \tag{8}$$

This is known as the **Bellman optimality equation**.

Value Iteration

Just change the **equality to an assignment**. Bellman [1] showed that this would eventually converge to the optimal value function.

$$V(s) \leftarrow \max_a \sum_{s'} P(s' \mid s, a) \left(R(s', s, a) + \gamma V(s')\right) \tag{9}$$

We can extract the optimal policy from the optimal value function by taking the action that results in the most *valuble* state.

Q-Learning

Q-learning was the **first ever model-free algorithm** [14].

In a similar manner to 7, write the optimal quality function recursively as

$$Q_\star(s,a) = \max_{\pi} \mathbb{E}\left(r + \gamma \max_{a'} Q_\star(s',a')\right). \tag{10}$$

Define the **TD-target estimate** as

$$R_\Sigma = r + \gamma \max_{a'} Q(s',a') \tag{11}$$

This is what we expect $Q(s,a)$ to be given some r, s'.

Q-Learning

The Q-learning algorithm just iterates through the MDP, collects **experience tuples** (s, a, r, s'), and updates the Q-values based on the following update equation

$$Q(s, a) \leftarrow Q(s, a) + \alpha(R_\Sigma - Q(s, a)). \tag{12}$$

where α is some learning rate.

Watkins et al. [14] showed this process will eventually converge to the optimal Q-function.

Q-Learning

But how do we select the action when collecting experience tuples (s, a, r, s')?

- **SARSA** [10]: Select the best action always $\leftarrow$ On-policy
- **Q-Learning** [14]: Occasionally explore $\leftarrow$ Off-policy

Deep Q-Learning

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Problem: Sometimes it is impossible to enumerate the Q-values for all the state-action pairs.

Solution: Use a neural network as a functional approximator of the Q-function.

$$Q(s, a) \approx Q(s, a; \theta) \tag{13}$$

where θ is a vector representing the parameters of the Q-network.

Deep Q-Learning

Overleaf

The loss function is derived from Q-learning12

$$\mathcal{L} = \left[Q(s, a; \theta) - (r + \gamma \max_{a'} Q(s', a'; \theta)\right]^2 \tag{14}$$

and the network parameters are adjusted using **backpropagation** and **experience replay**[1].

[1] In practice since deep Q-learning is off-policy, the training data set can be created on the go by playing the game and collecting experience tuples of the form (s, a, r, s'). We then randomly sample mini-batches of experience tuples for training and perform back-propagation on entire batches at a time. Simultaneously, we add new experience tuples based on our current Q-network and exploration strategy. This mechanism is called experience replay.

Finance Terminology

- A **security** is something that can be traded e.g stocks, bonds, options, futures, crypto, index funds, etc.
- A **portfolio** is a basket of tradeable securities. Investors hold portfolios with the hope that they will grow in value over time.
- The **rate of return** on an investment is the percentage change in price for the given unit of time, for example the daily rate of return.

$$r_t = \left(\frac{p_1 - p_0}{p_0}\right) \tag{15}$$

Risk and Return of a Security

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Investing in a security for n days results in a sequence of daily returns.

$$\{r_1, r_2, r_3, \ldots, r_n\} \tag{16}$$

The **annualized mean historical return** is

$$\mu = \frac{252}{n} \sum_{i=1}^{n} r_i \tag{17}$$

and the **annualized historical risk** *(or standard deviation)* is

$$\sigma = \sqrt{\frac{252}{n} \sum_{i=1}^{n} (\mu - r_i)^2} \tag{18}$$

where there are 252 trading days in a year.

Risk and Return of a Portfolio

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Suppose an investor holds a portfolio of k securities for n days. Each security will have its own sequence of daily returns.

The portfolio can be characterized by a sequence of weights.

$$\{w_1, w_2, \ldots, w_k\} \tag{19}$$

The **annualized mean historical return** of the portfolio is

$$\mu_{\text{port}} = \sum_{i=1}^{k} w_i \cdot \mu_i \tag{20}$$

and the **annualized historical risk** of the portfolio is

$$\sigma_{\text{port}} = \sqrt{\sum_{i=1}^{k} \sum_{j=1}^{k} w_i \cdot w_j \cdot \sigma_{ij}} \tag{21}$$

where σ_{ij} is the covariance between the returns of securities i and j.

Mean-Variance Optimization

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In 1952, Harry Markowitz proposed a systematic approach to constructing portfolios using **mean-variance analysis** [7], which involves solving the following **nonlinear bi-objective optimization problem**.

$$\begin{gathered}
\max \sum_{i=1}^{k} w_i \cdot \mu_i \\
\min \sqrt{\sum_{i=1}^{k} \sum_{j=1}^{k} w_i \cdot w_j \cdot \sigma_{ij}} \\
\text{subject to} \\
\sum_{i=1}^{k} w_i = 1 \\
0 \leq w_1 \leq 1
\end{gathered} \tag{22}$$

Sharpe Ratio Optimization

The **Sharpe ratio** is a metric introduced by William F Sharpe to evaluate the performance of mutual funds according to their expected risk and return [11].

$$\max \left(\frac{\sum_{i=1}^{k} w_i \cdot \mu_i - R_f}{\sqrt{\sum_{i=1}^{k} \sum_{j=1}^{k} w_i \cdot w_j \cdot \sigma_{ij}}} \right)$$

subject to

$$\sum_{i=1}^{k} w_i = 1$$

$$0 \leq w_1 \leq 1 \tag{23}$$

Maximum Sharpe Portfolio Visualized

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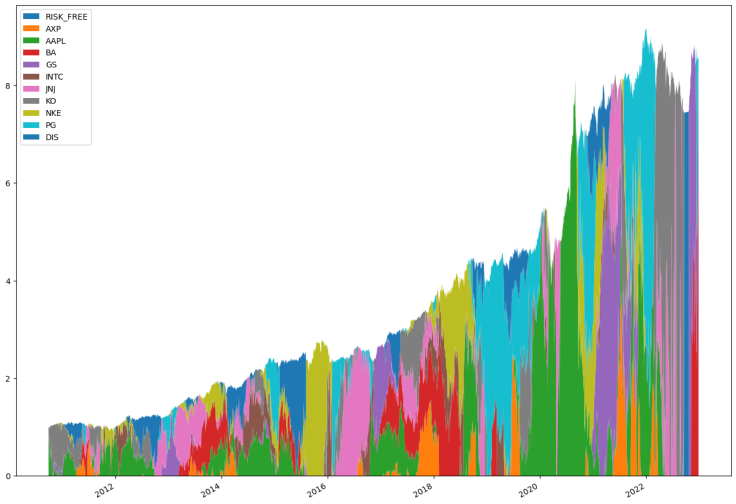

Data

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- Ten stocks[2] from the **Dow Jones Industrial Average** were arbitrarily selected.
- Price data was downloaded from *Yahoo Finance* and technical analysis-inspired features were generated e.g daily returns, rolling returns, rolling standard deviations, volume percent changes, etc.
- Data was split into training, validation, and test sets.

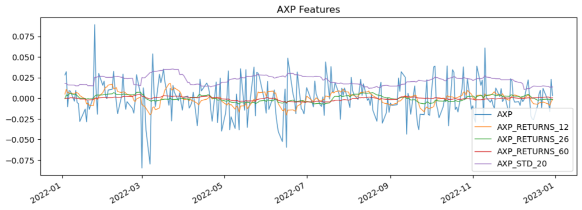

Figure: Visualization of features for one stock over one year

[2] American Express Company, Apple Inc, Boeing Co, Goldman Sachs Group Inc, Intel Corporation, Johnson & Johnson, Coca-Cola Co, Nike Inc, Procter & Gamble Co, and Walt Disney Co.

Environment Details

State and Observation

- The **state** is the current market situation.
- The **observation** is a 10 day history of the features.

Action

Actions change portfolio weights. There are 2 actions per stock - **buy or sell** - and a hold action which does nothing. Buying and selling occur in **10% increments**.

Reward Structure

A good **reward structure** is essential for success since we want to *reinforce* the right behavior.

1. Return of the portfolio ✗
2. Cumulative return of the portfolio ✗
3. Weighted cumulative return of the portfolio ✓

Model

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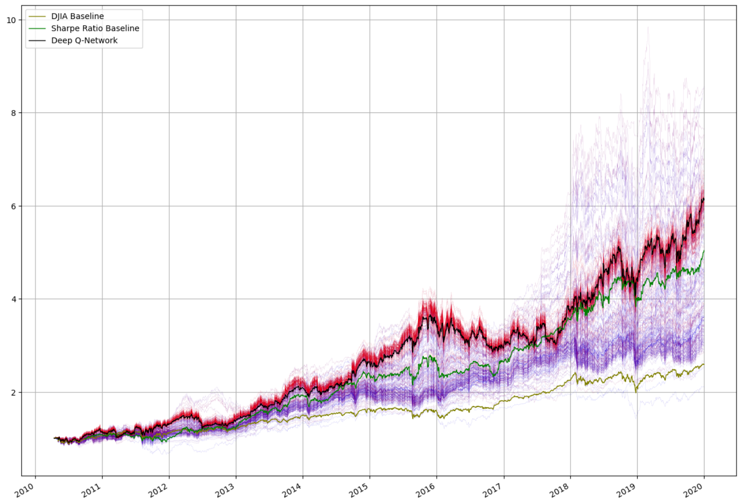
- **Deep Q-learning model** with **experience replay**.
- Default hyper-parameters[3].
- Default network architecture (2 hidden layers, each with 64 nodes).
- Trained over $3,000,000$ time-steps.

[3] except for *learning rate* being set to 0.0003 and the *batch size* being set to 64

Performance on Training Data



Performance on Validation Data

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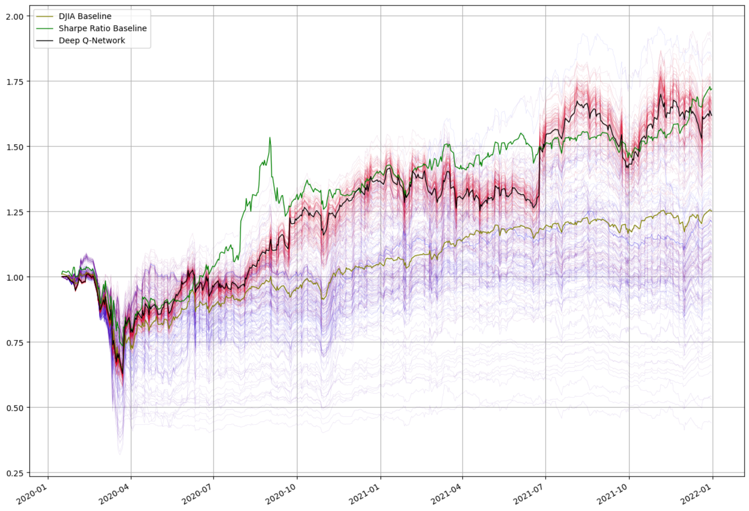
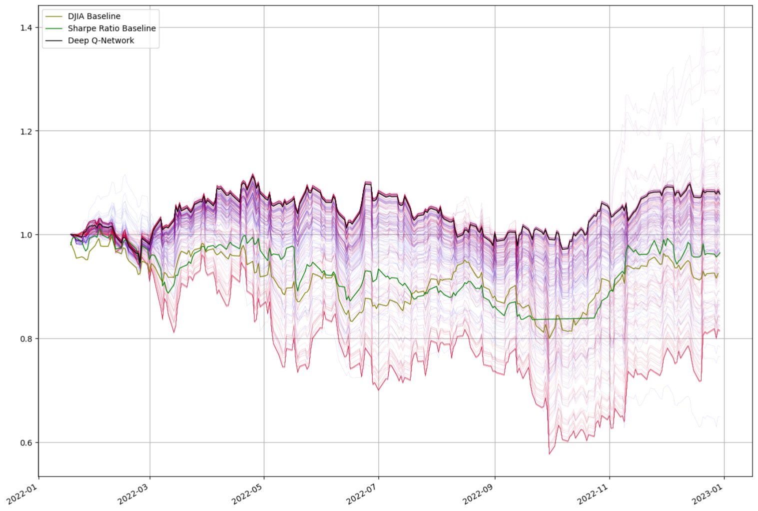


Performance on Test Data



Performance

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Training Data	Mean Return	Risk	Sharpe Ratio
DJIA	0.103	0.141	0.730
Max Sharpe	0.191	0.170	1.129
Deep Q-Network	0.206	0.216	0.951

Validation Data	Mean Return	Risk	Sharpe Ratio
DJIA	0.121	0.277	0.438
Max Sharpe	0.318	0.319	0.996
Deep Q-Network	0.278	0.333	0.835

Test Data	Mean Return	Risk	Sharpe Ratio
DJIA	-0.079	0.202	-0.389
Max Sharpe	-0.037	0.196	-0.189
Deep Q-Network	0.082	0.172	0.474

Limitations and Future Work

- Hyper-parameter tuning
- Verify with different underlying securities
- More informative features (e.g market sentiment, news headlines)
- Different model (e.g Actor-Critic, DDPG)
- Different network architecture (e.g LSTM)

References I

[1] R. Bellman.
A markovian decision process.
Journal of mathematics and mechanics, pages 679–684, 1957.

[2] S. L. Brunton and J. N. Kutz.
Data-driven science and engineering: Machine learning, dynamical systems, and control.
Cambridge University Press, 2022.

[3] H. Hasselt.
Double q-learning.
Advances in neural information processing systems, 23, 2010.

[4] R. A. Howard.
Dynamic programming and markov processes.
1960.

References II

[5] V. Konda and J. Tsitsiklis.
Actor-critic algorithms.
Advances in neural information processing systems, 12, 1999.

[6] D. Lee, H. Seo, and M. W. Jung.
Neural basis of reinforcement learning and decision making.
Annual review of neuroscience, 35:287–308, 2012.

[7] H. Markowitz.
Portfolio selection the journal of finance, vol. 7, no. 1, 1952.

[8] V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, and M. Riedmiller.
Playing atari with deep reinforcement learning.
arXiv preprint arXiv:1312.5602, 2013.

References III

[9] G. V. Pai.
Metaheuristics for portfolio optimization: an introduction using MATLAB.
John Wiley & Sons, 2017.

[10] G. A. Rummery and M. Niranjan.
***On-line Q-learning using connectionist systems*, volume 37.**
University of Cambridge, Department of Engineering Cambridge, UK, 1994.

[11] W. F. Sharpe.
The sharpe ratio.
Streetwise–the Best of the Journal of Portfolio Management, 3:169–185, 1998.

[12] R. S. Sutton and A. G. Barto.
Reinforcement learning: An introduction.
MIT press, 2018.

References IV

[13] R. S. Sutton, D. McAllester, S. Singh, and Y. Mansour.
Policy gradient methods for reinforcement learning with function approximation.
Advances in neural information processing systems, 12, 1999.

[14] C. J. Watkins and P. Dayan.
Q-learning.
Machine learning, 8:279–292, 1992.