CS5200: Assignment 2

1 Randomized Algorithms

1. Design a deterministic algorithm, a Monte Carlo algorithm, and a Las Vegas algorithm to solve the following problem. Analyze the expected time complexity and accuracy of your algorithm. (4 pts)

Given an array A with distinct numbers from 1 to n. Return any number that is both smaller and larger than $\frac{1}{3}$ of the elements simultaneously. (E.g., if A={1, 2, 3, 4, 5, 6, 7, 8, 9}, valid answers would be 4, 5, or 6.)

2 Sorting

1. Consider a sequence $A = \{27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0\}$. (6 pts)

1) Show the heap structure of A. Is A a max-heap? Build a max-heap on A using the BUILD-MAX-HEAP algorithm introduced in the class. Show both the heap structure and array representation of the produced output. (1 pt)

2) A d-ary heap is like a binary heap, but non-leaf nodes have d children instead of 2. Let's consider 3-ary heap in this exercise.

1.2.1) Show the 3-ary heap structure of A and the indexing method for the left, middle, and right child in the 3-ary heap? (1 pt)

1.2.2) Develop an algorithm to build the max 3-ary heap and analyze the complexity of your algorithm. (1 pt)

1.2.3) Develop an algorithm for 3-ary heapsort and analyze the complexity of your algorithm. (1 pt)

1.2.4) Develop an algorithm to generalize the heapsort algorithm to d-ary heaps. Analyze the complexity of your algorithm in terms of d and n. (2 pts)

1.2.5) What are the advantages and disadvantages of d-ary heap compared with the binary one (2 pts)?

2. Design an efficient algorithm to solve the problem and then analyze the complexity of your algorithm: Merge k sorted arrays into one sorted array containing all the elements. (4 pts)