$\max(f(n), g(n)) = \Theta(f(n) + g(n))$

$f(n) \geq 0$; $g(n) \geq 0$ (non negative fun).

$f(n) \leq f(n) + g(n)$

$g(n) \leq f(n) + g(n)$

$\max(f(n) + g(n)) \leq f(n) + g(n)$

By formal definition,

$\Theta(g(n)) = \{f(n) \mid$ there exists positive real constants c_1, c_2 and a positive integer n_0 Such that for all $n \geq n_0$, $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)\}$

$\exists$ $c = 1$, $n_0 = 1 \Rightarrow n \geq n_0$

$0 \leq \max(f(n), g(n)) \leq f(n) + g(n)$

$\max(f(n) + g(n)) = \Theta(f(n) + g(n))$

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$$f(n) = O(g(n)) \Rightarrow 2^{f(n)} = O(2^{g(n)})$$

$$f(n) = O(g(n))$$

$$\Rightarrow 0 \le f(n) \le c \cdot g(n)$$

Let us assume

$f(n) = 2\log n \qquad g(n) = \log n$

$$2^{f(n)} = 2^{2\log n} \qquad 2^{g(n)} = n$$

$$= n^2$$

$$n^2 > n$$

$$\therefore 2^{f(n)} \neq O(2^{g(n)})$$

$f(n) = n^c \qquad g(n) = c^n \qquad 0 < c < 1$

3.2 $$\lim_{n\to\infty} \frac{n^{\log c}}{c^{\log n}} =$$

$$= \lim_{n\to\infty} \frac{\log c \cdot \log n}{\log n \ \log c} = 1$$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1 \Rightarrow f(n) = \Theta(g(n))$$

$$\Rightarrow n^{\log c} = \Theta(c^{\log n})$$

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3. Recurrence

a) $T(n) = T\left(\frac{n}{2}\right) + 1$

b) Master theorem

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

$a = 1$, $b = 2$, $f(n) = 1$ $n^{\log_b a} = n^{\log_2 1} = n^0 =$

$$T(n) = O(n^{\log_b a} \log n)$$

$$= O(n^{\log_2 1} \log n) =$$

$$= O(\log n).$$

② $T(n) = 7T\left(\frac{n}{2}\right) + n^2$

$n \to n^2 \qquad 7\left(\frac{n}{2}\right)^2$

$\left.\begin{matrix} n/2 \\ n/2 \\ \vdots \\ n/2 \end{matrix}\right\}$ 7 times

$\rightarrow$ level $i \qquad 7^i \left(\frac{n}{2^i}\right)^2$

$\rightarrow$ level $h, \qquad 7^h$

$$T(n) = \sum_{i=0}^{h} 7^i \left(\frac{n}{2^i}\right)^2 + 7^h (T(1))$$

$$= n^2 \sum_{i=0}^{h} (7/4)^i + 7^h T(1)$$

$$= n^2 \frac{(7/4)^h - 1}{7/4 - 1} + 7^h T(1)$$

$$= 4/3\, n^2 \left((7/4)^h - 1\right) + 7^h T(1)$$

height: $\frac{n}{2^h} = 1 \Rightarrow h = \log_2 n$

$$= 4/3\, n^2 \, (7/4)^{\log_2 n} + 7^{\log_2 n} T(1)$$

$$= \frac{4n^2}{3}\left(n^{\log_2 7} - 1\right) + n^{\log 7} T(1)$$

$$T(n) = O\left(n^{\log_2 7}\right)$$

2.2 $T(n) = 2T\left(\frac{n}{3}\right) + 3n^2$

$$3n^2$$

$$3\left(\frac{n}{3}\right)^2 \qquad 3\left(n/3\right)^2 \qquad\qquad 3n^2$$

$$\vdots$$

$$T(1) \;\cdots\; T(1) \qquad\qquad 2^h\, T(1)$$

$h = \log_3 n$

$$\sum_{i=0}^{h-1} 2^i \cdot 3\left(\frac{n}{3^i}\right)^2 + 2^h\, T(1)$$

$$= 3n^2\left(\frac{1-(2/9)^h}{7/9}\right) + 2^h\, T(1)$$

$$= 3n^2\left(\frac{1-(2/9)^{\log_3 n}}{7/9}\right) + 2^{\log_3 n}\, T(1)$$

$$= O(n^2)$$

2.4

$T(n) = 2 \cdot T\left(\frac{n}{4}\right) + \sqrt{n}$

$a = 2, \quad b = 4$

$n^{\log_b a} = n^{\log_4 2} = \sqrt{n} = f(n)$

$T(n) = \Theta(n^{\log_b a} \lg n)$

$= \Theta(\sqrt{n} \lg n)$

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