CS5200: Assignment 1

1 Proof of Correctness

1. Prove the correctness of mergesort. (2 pts)

2 Asymptotic Analysis

1. Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Use the definition to show $\max(f(n), g(n)) = \Theta(f(n) + g(n))$. (1 pt)
2. Prove or disprove the statement: $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$. (1 pt)
3. Compare functions using the asymptotic notations (need to show how you get the conclusion). (2 pts)
3.1) n^c and c^n for $0 < c < 1$.
3.2) $n^{\lg c}$ and $c^{\lg n}$ for $c > 1$.

3 Recurrence

1. Consider the recursive version of Binary-Search that finds x in an array $A[1..n]$ which is sorted in ascending order. If x exists in A, its index is returned; Otherwise, the program returns 0. For simplicity, assume n is a power of 2. (2 pts)

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Binary-Search($A, left, right, x$)
if $left > right$
    return 0
$mid = \lfloor (left + right)/2 \rfloor$
if $A[mid] == x$
    return mid
elseif $A[mid] < x$
    return Binary-Search($A, mid + 1, right, x$)
else
    return Binary-Search($A, left, mid - 1, x$)
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1.1) Develop the recurrence relationship for $T(n)$.
1.2) Solve the recurrence using master formula.
1.3) Verify your solution using substitution method.

2. Solve the recurrence (2 pts)
2.1) Solve $T(n) = 7T(\frac{n}{2}) + n^2$ using recursion-tree.
2.2) Solve $T(n) = 2T(\frac{n}{3}) + 3n^2$ using recursion-tree.
2.3) Show $F_n = \frac{(\frac{\sqrt{5}+1}{2})^n - (\frac{\sqrt{5}-1}{2})^n}{\sqrt{5}}$ using substitution method where $F_n = F_{n-1} + F_{n-2}$ is the Fibonacci sequence.
2.4) Solve $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$.

4 Backtracking

1. Solve the problem then analyze the complexity of your algorithm: Given an array A with n distinct integers, return all the possible permutations. You can return the answer in any order. (2 pts)