True graphical model:

$$X_1 \longrightarrow X_2 \longrightarrow X_3$$

- Here, we're just doing parameter estimation, so we'll assume we're already provided the graph structure $\mathcal{G}$

- Given the graph structure:

$$P(\vec{x}) = P(x_1) \cdot P(x_2 | x_1) \cdot P(x_3 | x_2)$$

$$P(x_1) = \frac{1}{Z_1(a_1)} \exp(-a_1 X_1^2)$$

For $i = 2, 3$:

$$P(x_i | x_{i-1}) = \frac{1}{Z_i(a_i, b_i)} \exp(-a_i X_i^2 - b_i (x_i - x_{i-1})^2)$$

Parameter $\Theta = \{a_1, a_2, a_3; b_2, b_3\}$

Given a data set D,

$$L(\Theta : D) = \prod_{i=1}^{|D|} P(\vec{x}[i] : \Theta)$$

$$\log[L(\Theta : D)] = \sum_{i=1}^{|D|} \log[P(\vec{x}[i] : \Theta)]$$

$$\log[L(\Theta : D)] = \sum_{i=1}^{|D|} \log[P(x_1[i] : \Theta) \cdot P(x_2[i] | x_1[i] : \Theta) \cdot P(x_3[i] | x_2[i] : \Theta)]$$

$$\log[L(\Theta : D)] = \left(\sum_{i=1}^{|D|} \log[P(x_1[i] : \Theta)]\right) + \left(\sum_{i=1}^{|D|} \log[P(x_2[i] | x_1[i] : \Theta)]\right) + \left(\sum_{i=1}^{|D|} \log[P(x_3[i] | x_2[i] : \Theta)]\right)$$

$$\log[L(\Theta : D)] = \underbrace{\left(\sum_{i=1}^{|D|} \log[P(x_1[i] : a_1)]\right)}_{\text{only depends on } a_1} + \underbrace{\left(\sum_{i=1}^{|D|} \log[P(x_2[i] | x_1[i] : a_2, b_2)]\right)}_{\text{only depends on } a_2, b_2} + \underbrace{\left(\sum_{i=1}^{|D|} \log[P(x_3[i] | x_2[i] : a_3, b_3)]\right)}_{\text{only depends on } a_3, b_3}$$

We can maximize each term in Θ independently!