

Machine Learning Spring 2014

Study Guide

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Mean

[Khan's Academy](#)

$$\mu \triangleq \sum xP(x) = \frac{1}{N} \sum_{i=1}^N x_i \quad (1)$$

Variance and Standard Deviation

Khan's Academy

$$\begin{aligned}\text{Var}(X) &= E[(X - \mu)^2] \triangleq \frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2 \\ \text{Var}(X) &= \sigma^2 \iff \sigma = \sqrt{\text{Var}(X)} \\ \iff \int (x - \mu)^2 f(x) dx &= \int x^2 f(x) dx - \mu^2\end{aligned}\tag{2}$$

Bernoulli/Binomial Distribution

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$$\text{Bin}(k \mid n, \theta) \triangleq \binom{n}{k} \theta^k (1 - \theta)^{n-k}\tag{3}$$

Models outcomes of coin tosses. Coin toss n times with $X \in \{0, \dots, n\}$. Probability θ is the parameter of, for example, fairness of the coin.

- Expected number of “heads” outcomes in 10 flips.
- Given 10 flips what is the probability of at least 8 being “heads”?
- Given 3 heads, 2 tails, what is the estimate of θ ?
- How far will this estimate be from true θ^* ?

Likelihood Per Book

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$$P(\mathcal{D} \mid h) = \left[\frac{1}{|h|} \right]^N\tag{4}$$

Size principle/Occam's razor: model favors simplest hypothesis consistent with data.

Likelihood Function

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$$P(\mathcal{D} \mid \theta)\tag{5}$$

Probability of data given this model hypothesis.

Maximum Likelihood Estimation (MLE)

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$$\hat{\theta}_{\text{MLE}} = \underset{\theta}{\operatorname{argmax}} P(\mathcal{D} \mid \theta) \quad (6)$$

Choose parameters that make the data most probable.

Maximum a Posteriori Estimate (MAP)

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$$\hat{\theta}_{\text{map}} = \underset{\theta}{\operatorname{argmax}} P(\mathcal{D} \mid \theta)P(\theta) \quad (7)$$

Somewhat Bayesian (“point + prior”).

- Choose a likelihood function $P(\mathcal{D} \mid \theta)$
- Choose a prior parameter dist $P(\theta)$
- Use MAP.
- Make predictions using MAP: $P(\mathcal{D}' \mid \hat{\theta}_{\text{map}})$

Likelihood and MLE Example

Office Hours

$$P(\theta) = \begin{cases} \frac{1}{3} & \theta = \theta_1 \\ \frac{1}{3} & \theta = \theta_2 \\ \frac{1}{3} & \theta = \theta_3 \end{cases}$$

$\mathcal{D} = \{H, H\}$ (two heads)

$$\theta_1 = .5$$

$$\theta_2 = .3$$

$$\theta_3 = .8$$

$$\mathcal{L}_1 = P(\mathcal{D} \mid \theta) = .5 \cdot .5 = .25$$

Probability of two heads (H H) using likelihood and MAP estimates:

N	\mathcal{L}_N	$P(\mathcal{D} \mid \theta)P(\theta)$ (MAP)
1	$.5 \cdot .5 = .25$	$.25 \cdot \frac{1}{3} = .083$
2	$.3 \cdot .3 = .09$	$.09 \cdot \frac{1}{3} = .03$
3	$.8 \cdot .8 = .64$	$.64 \cdot \frac{1}{3} = .213$

Parameter θ_3 wins with an MLE $\hat{\theta}_{\text{MLE}} = .64$.

Parameter θ_3 wins with an MAP $\hat{\theta}_{\text{map}} = .213$.

Bayes:

$$P(\theta | \mathcal{D}) = \frac{P(\mathcal{D} | \theta)P(\theta)}{P(\mathcal{D})} = \frac{P(\mathcal{D} | \theta)P(\theta)}{\sum_{\theta'} P(\mathcal{D} | \theta')P(\theta')}$$

$$P(H | \mathcal{D}) = \sum_{\theta' \in \{\theta_1, \theta_2, \theta_3\}} P(\theta' | \mathcal{D})P(H | \mathcal{D}) =$$

$$\frac{.083 \cdot .5 + .03 \cdot .3 + .213 \cdot .8}{.083 + .03 + .213} = .67$$

Binomial Coefficient

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$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad (8)$$

Binomial coefficients are a family of positive integers that occur as coefficients in the binomial theorem.

Multinomial Distribution

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$$\text{Mu}(\mathbf{x} | n, \theta) \triangleq \binom{n}{x_1 \dots x_K} \prod_{j=1}^K \theta_j^{x_j}$$

θ_j is the probability that side j shows up, and

$$\binom{n}{x_1 \dots x_K} \triangleq \frac{n!}{x_1! x_2! \dots x_K!} \quad (9)$$

Models the outcomes of tossing a K -sided die. Let $\mathbf{x} = (x_1 \dots x_K)$ be a random vector, where x_j is the number of times side j of the die occurs.

Poisson/Empirical Distribution

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Gaussian (Normal) Distribution

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$$P(x) \triangleq N(x | \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \quad (10)$$

Beta Distribution

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$$\text{Beta}(x \mid a, b) = \frac{1}{B(a, b) x^{a-1} (1-x)^{b-1}}$$
$$B \triangleq \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$
(11)

Where Γ is the conjugate prior.

Product Rule

ES Class

$$P(a \wedge b) = P(a \mid b)P(b) \Leftrightarrow$$
$$P(X = x, Y = y) = P(X = x \mid Y = y) \cdot P(Y = y)$$
(12)

Bayes Rule

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$$P(X = x \mid Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)} = \frac{P(X = x)P(Y = y \mid X = x)}{\sum_{x'} P(X = x')P(Y = y \mid X = x')}$$
(13)