

- ① Given an array of $[4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9]$ integers find the maximum and minimum Product that can be obtained by multiplying two integers from the array.

Sol:-

array is $[4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9]$

we need to consider the largest and smallest Products that can be formed by selecting two numbers from the array.

1) sort the array.

sorted array

$[-9, -8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]$

2) Identify Possible candidates for maximum Product.

3) Identify Possible candidates for minimum Product.

calculating maximum Product:-

* The two largest Positive numbers are 10 and 11

$$10 \times 11 = 110$$

* The two smallest negative numbers are -9 and -8.

$$-9 \times -8 = 72.$$

The maximum Product is 110.

calculating minimum Product:-

The largest Positive and negative number is 11 and -9

$$11 \times -9 = -99$$

The smaller negative numbers are

$$-9 \times -8 = 72$$

-99 is smaller than 72 so

maximum Product = 110, and minimum Product = -99

② Demonstrate the Binary search method to search for the Key = 23 from the array = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}.

Sol: given Key = 23 and array = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}

1. Initialize pointers

low = 0 and high = 9

Calculate $mid = \left\lfloor \frac{low + high}{2} \right\rfloor = \left\lfloor \frac{0 + 9}{2} \right\rfloor = 4$

Compare arr[mid] with key:

arr[4] = 16

since $16 < 23$ update $low = mid + 1 = 5$

Calculate $mid = \left\lfloor \frac{low + high}{2} \right\rfloor = \left\lfloor \frac{5 + 9}{2} \right\rfloor = 7$

Compare arr[mid] with key:

arr[7] = 56

since $56 > 23$ update $high = mid - 1 = 6$

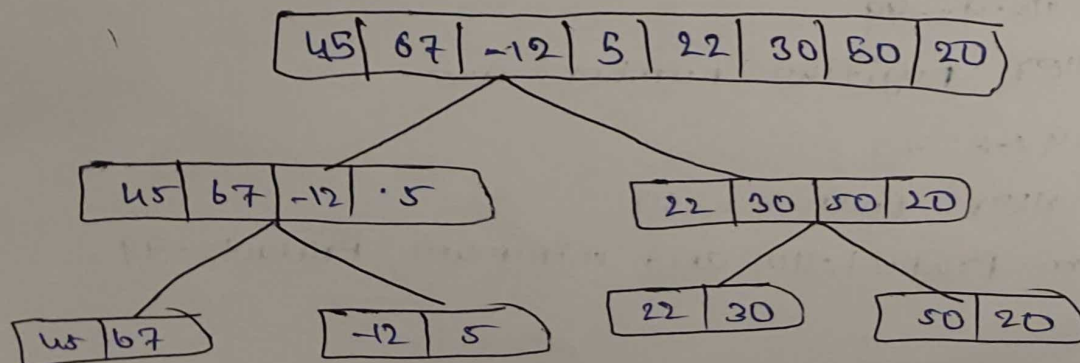
$mid = \left\lfloor \frac{5 + 6}{2} \right\rfloor = 5$

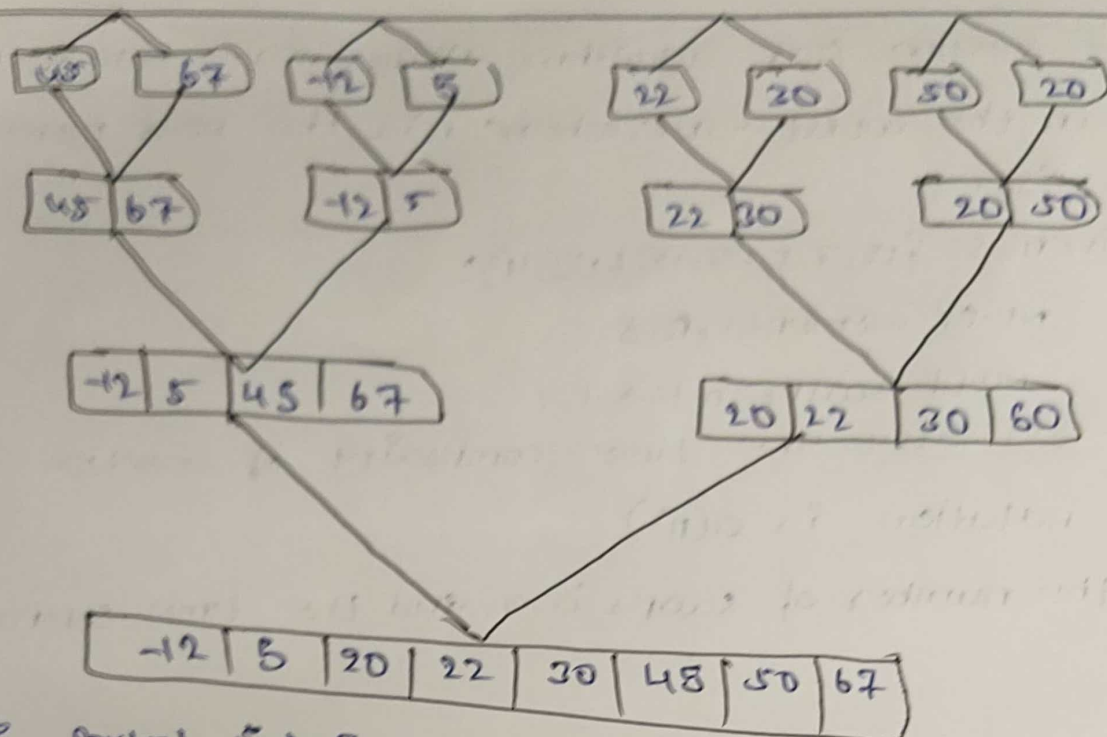
arr[mid] = arr[5] = 23

$23 = 23$ the key is found at index 5.

∴ The key = 23 is found at index 5.

③ Apply merge sort and other List of 8 elements, Pat a d = {45, 67, -12, 5, 22, 30, 50, 20}. Set up recurrence relation for the number of key comparisons made by mergesort.





∴ The sorted list = $[-12, 5, 20, 22, 30, 45, 50, 67]$

Recurrence Relation for comparisons:

$$T(n) = 2 + (n/2) + O(n)$$

if $n=1$, $T(1) = 0$. Base case

→ At each level of recursion we make at most $n-1$ comparisons to merge two halves of size $n/2$ so it becomes.

$$T(n) = 2T(n/2) + (n-1)$$

solving recurrence relation we get

$$T(n) = n \log_2(n) - n + 1$$

$$T(n) = O(n \log n)$$

∴ The recurrence relation is $T(n) = 2T(n/2) + O(n)$ or more precisely.

$$T(n) = n \log_2(n) - n + 1$$

- ④ Find the no. of times to perform swapping for selection sort also estimate the time complexity for the order of notation set $S(12, 7, 5, -2, 18, 6, 13, 4)$

The selection sort algorithm always makes exactly $n-1$ swaps in the worst case, where n is the no. of elements in the list.

Given $S = \{12, 7, 5, -2, 18, 6, 13, 4\}$:

No. of elements, $n = 8$

No. of swaps $= n - 1 = 8 - 1 = 7$

Time complexity - The time complexity of selection sort in Big O notation is $O(n^2)$

So, the number of swaps is 7, and the time complexity is $O(n^2)$

⑤ Find the index of the target value 10 using binary search from the following list of elements $[2, 4, 6, 8, 10, 2, 14, 16, 18, 20]$.

Given list $= \{2, 4, 6, 8, 10, 2, 14, 16, 18, 20\}$ and value $= 10$

low $= 0$ and high $= 9$

$$mid = \frac{low + high}{2} = \frac{0 + 9}{2} = 4$$

$list[mid] = mid = 10$ $mid == value$

Since $10 == 10$ the target is found at index

\therefore The target value $= 10$ is found at index.