Fractions

Fractions play an important role in most of the competitive exams. As the questions on fractions do appear directly and do have some indirect application in few of the questions.

Following are some definitions and fundamentals one needs to know before understanding the topic of fractions.

- Numbers in the form of 3/4, 4/5, are called fractions. A fraction can be written as p/q where q should not be equal to zero.

- If the numerator and denominator of a fraction are multiplied/divided by the same number, then the value of the fraction does not change.

- For any positive proper fraction p/q (p<q), the value of the fraction increases when both the denominator and numerator are increased by the same positive number.

  Example: (3/4) = 0.75, $\frac{3+1}{4+1} = \frac{4}{5}$ = 0.8

- For any positive proper fraction p/q (p<q), the value of the fraction decreases when both the numerator and denominator are decreased by the same positive number.

  Example: $\frac{3}{4}$= 0.75, $\frac{3-1}{4-1} = \frac{2}{3}$= 0.67.

- For any positive improper fraction p/q (p>q), the value of the fraction decreases when both the numerator and the denominator are increased by the same positive. e.g. $\frac{5}{4}$= 1.25, adding 1 to the numerator and the denominator, we get $\frac{5+1}{4+1} = \frac{6}{5}$= 1.2, which is less than 1.25.

- For any positive improper fraction p/q (p>q), the value of the fraction increases when both the numerator and denominator are decreased by the same positive number. e.g. $\frac{5}{4}$ = 1.25, by subtracting 1 from both the numerator and denominator we get, $\frac{5-1}{4-1} = \frac{4}{3}$ = 1.33 > 1.25.

Types of Fractions

- **Common Fractions:** Fractions such 3/4, 32/43 etc are called common or vulgar fractions.

- **Decimal Fractions:** Fractions whose denominators are 10, 100, 1000, ... are called decimal fractions.

- **Proper Fraction:** A fraction whose numerator is less than its denominator is known as a proper fraction. e.g. 3/4

- **Improper Fraction:** A fraction whose numerator is greater than its denominator is known as an improper fraction. e.g. 4/3

- **Mixed Fractions:** Fractions which consist of an integral part and a fractional part are called mixed fractions. All improper fractions can be expressed as mixed fractions and vice versa. e.g. (3/14).

- **Recurring Decimals:** A decimal in which a set of figures is repeated continually is called a recurring or periodic or a circulating decimal. e.g. 17 = 0.142857… the dots indicate that the figure between 1 and 7 will repeat continuously.

Addition of fractions:

Addition of fractions with same denominators

If denominators of two or more fractions are the same, then we can directly add the numerators, keeping the denominator common.
Follow the below steps to add the fractions with same denominators:

- Add the numerators together, keeping the denominator common
- Write the simplified fraction

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$$

For example: Add the fractions: 2/5 and 3/5.
Since the denominators are the same values, so we can add the numerators directly.

- (2/5) + (3/5) = (2 + 3)/5 = 5/5

Simplify the fraction,

- 5/5 = 1
- Hence, the sum of 2/5 and 3/5 is 1.

Adding fractions with Different Denominators

When two or more fractions with different denominators are added together, then we cannot add the numerators directly. Follow the below steps to add fractions with different denominators:

- Check the denominators of the fractions.
- Make the denominators of the fractions same, by finding the LCM of denominators and rationalizing them
- Add the numerators of the fractions, keeping the denominator common
- Simplify the fraction to get final sum

For example: Add 2/12 + 3/2
Solution: Both the fractions 3/12 and 5/2 have different denominators.

- We can write 2/12 = 1/6, in a simplified fraction.
- Now, 1/6 and 3/2 are two fractions.
- LCM of 6 and 2 = 6
- Multiply 3/2 by 3/3.
- 3/2 x 3/3 = 9/6
- Now add 1/6 and 9/6
- 1/6 + 9/6= (1+9)/6 = 10/6= 5/3
- Hence, the sum of 2/12 and 3/2 is 5/3.

Adding fractions with whole numbers

Add the fraction and a whole number with three simple steps:

- Write the given whole number in the form of a fraction
- Make the denominators same and add the fractions
- Simplify the fraction

For example: Add 5/2 + 6

Solution: Here, 5/2 is a fraction and 6 is a whole number.

- We can write 6 as 6/1.

Now making the denominators same, we get;

- 5/2 and 6/1 x (2/2) = 12/2
- Add 5/2 and 12/2
- 5/2 + 12/2 = 17/2
- Hence, the sum of 5/2 and 6 is 17/2.

Adding Mixed Fractions

A mixed fraction is a combination of a whole number and a fraction. To add two mixed fractions, we need to convert them first into improper fractions and then add them together.

Follow the below steps to add mixed numbers:

- Convert the given mixed fraction into improper fractions
- Check if denominators are the same or different
- If different denominators, then rationalize them
- Add the fractions and simplify

Let's understand how to add mixed fractions with an example:

Example: Add : 3 ⅓ + 1 ¾

Solution:

Step1: Convert the given mixed fractions to improper fractions.

3 ⅓ = 10/3

1 ¾ = 7/4

Step 2: Make the denominators same by taking the LCM and multiplying the suitable fractions for both.

LCM of 3 and 4 is 12.

So, 10/3 = (10/3) × (4/4) = 40/12

7/4 = (7/4) × (3/3) = 21/12

Step 3: Take the denominator as common and add numerators. Then, write the final answer.

(40/12) + (21/12) = (40 + 21)/12 = 61/12

Therefore, 3 ⅓ + 1 ¾ = 61/12 = 5(1/12)

Decimals and Fractions to be remembered

You get many questions in the exams based on Percentage, Profit, Interest etc. in which you have to calculate, say 87.5 % of 800, 58.33 % of 2400 etc. Calculating these values with the help of traditional methods is time- consuming. If you have the fraction approach, you can crack these easily, i.e., if you know that 87.5 % is just 7/8th of the number and 58.33 % is 7/12th of the number, then it becomes easy to calculate

1%	=	1/100	25%	=	1/4	80%	=	4/5
2%	=	1/50	33.33%	=	1/3	83.33%	=	5/6
4%	=	1/25	37.50%	=	3/8	87.50%	=	7/8
5%	=	1/20	40%	=	2/5	100%	=	1
8.33%	=	1/12	50%	=	1/2	120%	=	6/5
10%	=	1/10	60%	=	3/5	125%	=	5/4
12.50%	=	1/8	62.50%	=	5/8	133.33%	=	4/3
16.67%	=	1/6	66.67%	=	2/3	150%	=	3/2
20%	=	1/5	75%	=	3/4	175%	=	7/4

- Anything doubles to increase by 100 % and becomes 200 %.
- Anything trebles to increase by 200 % and becomes 300 %.

Solved Examples

Example 1: One-quarter of one-seventh of land is sold for a total of Rs.30,000.
What would be the value of eight thirty-fifths of the land?

Solution:

- One-quarter of one-seventh = 1/4 x 1/7= 1/28 Now,
- 1/28 of a land cost = Rs. 30,000
- Thus, 8/35 of the land will cost = 30,000 x 28 x 8/55
  = Rs. 1,92,000.

Example 2: After taking out of a purse (1/5) of its contents, (1/12) of the remainder was found to be Rs 7.40. What sum did the purse contain at first?

Solution:

- LCM (5,12) =60
- let total money = 60
- After taking out =(1/5)×60=12
- Remaining=60-12=48
- (1/12) ×48=7.40-------(1)
- Solving equation 1
- for 4=>7.40
- for 1=> 7.40/4=1.85
- total =60×1.85 =111
- total initial contents=111

Example 3: A sum of money increased by its seventh part amounts to Rs. 40.
Find the sum.

Solution:

As per the question,

- S + (S/7) = Rs.40
- (8S/7) = Rs. 40 S = Rs. 35.

Example 4: A train starts full of passengers. At the first station, it drops one- third of passengers and takes in 96 more. At the next, it drops one – half of the new total and takes in 12 more. On reaching the next station, there are found to be 248 left. With how many passengers did the train start?

Solution:
Let the full number of passengers be x

- Number of passengers at the first station, after drops one-third of the passenger $=[x-(1/3)x]+96=2x/3+96$
- Number of passengers at the second station , after one-half get down and 12 added $=1/2\,(2x/3+96)+12=x/3+48+12=x/3+60$

Given $x/3+60=248$

- $x/3=248-60=188$
- $x=188\times3=564$

Example 5: A motorcycle, before overhauling, requires 5/6-hour service time every 90 days, while after overhauling. It requires 5/6-hour service time every 120 days. What fraction of the pre-overhauling service time is saved in the latter case?

Solution:

- Time required for service before overhauling = 5/6 h = 50 min in 90 days
- Time required for service after overhauling =5/6 h= 50 min in 120 days

Now, LCM of 90 and 120 = 360 days

- Time required for service before overhauling is 360 days = 50 x 4 = 200 min
- Time required for service before overhauling is 360 days =50 x 3 = 150 min
- Fraction of time saved = 200-150/360 =50/360 = 5/36

Square Root

In order to find the square root of a number, there are two methods available. The first method is prime factorization and the second is the conventional square root method. Factorization is suitable only when the numbers are relatively small and their factors can be easily found. Considering the kind of questions which appear in the competitive exams, firstly we are going to learn the conventional square root calculation method. In this, firstly the number is divided into pairs from the right hand side. If in the beginning there is a pair, then the starting is done with that pair, and if there is a single digit number, then that would be the starting point.

Method to find Square Root

Square root By Prime Factorisation

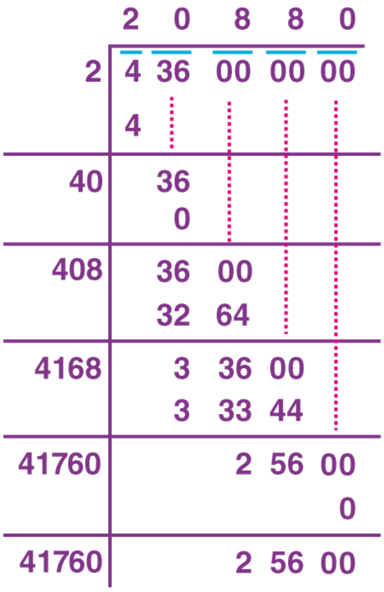
The square root of a perfect square number is easy to calculate using the prime factorisation method. Let us solve some of the examples here:

Number	Prime Factorisation	Square Root
16	2x2x2x2	$\sqrt{16} = 2\times2 = 4$
144	2x2x2x2x3x3	$\sqrt{144}$ = 2x2x3 = 12
169	13×13	$\sqrt{169} = 13$

256	256 = 2×2×2×2×2×2×2×2	√256 = (2x2x2x2) = 16
576	576 = 2x2x2x2x2x2x3x3	√576 = 2x2x2x3 = 24

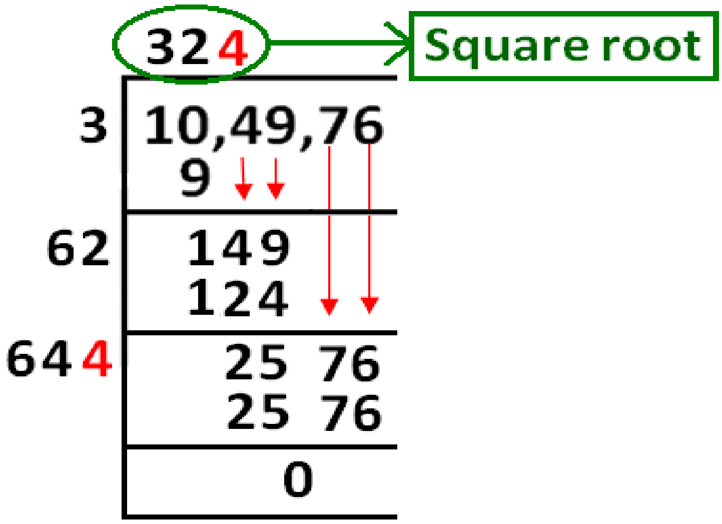
Square Root by Long Division Method

Finding square roots for the imperfect numbers is a bit difficult but we can calculate using a long division method. This can be understood with the help of the example given below. Consider an example of finding the square root of 436.



Example: Find the square root of 104976.

Solution:
Applying the square root calculation method,
In the first place after making pairs from the RHS, you are left with the number 10. Now you should take a number, which can be multiplied by the same number itself, and the result is less than equal to 10, which is 3.
After subtracting 9 from it, the new pair 49 is taken. The number now becomes 149. The previous quotient is doubled and 6 is obtained. Then,



a number ‘x’ is written with 6, in such a way that the product of ‘6x’ and ‘x’ is less than or equal to 149. So the value of x is 2.
The remainder of the next step is 25 and the last pair 76 is written with it. Then the previous quotient 32 is doubled and 64 is obtained and a number ‘y’ is written with it in such a way that the product of ‘64y’ and ‘y’ is less than or equal to 2576. When y is substituted by 4, the product is 2576. In this way, the final quotient 324 is the square root of 104976.

Solved Examples

Example 1: Find the smallest number with which 60 should be multiplied so that it becomes a perfect square.

Solution: In order to answer such questions, firstly the prime factorisation of the number is done. The factors of 60 are $2 \times 2 \times 3 \times 5$.
In this, it can be seen that ‘2’ is occurring twice, but 3 and 5 are occurring only once. In order to make a number a perfect square every prime factor should be there twice or an even number of times. So a ‘5’ and ‘3’ is required, the product of which is 15. Therefore 15 is the smallest number.

Example 2: In a class, each of the students contributed as many paisa as there is number of students. If the total collection was Rs. 144, what is the number of students in the class?

Solution: Let the number of students in the class be x. Now each of these students contributed ‘x’ paisa each. So the total collection will be x^2 paisa.
Now the total collection is given to be Rs. 144, which is 14400 paisa.
As per the statement of the question $x^2 = 14400 \Rightarrow x = 120$. Thus, there are 120 students.

Example 3: What is the value of

$$\sqrt{10+\sqrt{25+\sqrt{108\sqrt{154+15}}}} =$$

Solution:

$$= \sqrt{10+\sqrt{25+\sqrt{108\sqrt{154+15}}}}$$

$$= \sqrt{10+\sqrt{25+\sqrt{108+\sqrt{169}}}}$$

$$= \sqrt{10+\sqrt{25+\sqrt{108+13}}} = \sqrt{10+\sqrt{25+\sqrt{121}}}$$

$$\sqrt{10+\sqrt{25+11}} = \sqrt{10+\sqrt{36}} = \sqrt{10+6} = \sqrt{16} = 4$$

Example 4 What approximate value will come (?) in the following equation?
84.95% of 280 + √? = 253.001

A. 256 B. 324 C. 18 D. 225

Solution:

➢ 253.001=253,
➢ 84.95% of 280 + √? =253.
➢ Hence, answer is 225

Example 5 What is the value of √1.5625=?

A. 125 B. 12.5 C. 1.05 D. 1.25

Solution: Option D

$$\sqrt{\frac{15625}{10000}} = 1.25$$

Example 6 In a class each of the students contributed as many paise as there are a number of students. If the total collection was Rs. 64, what is the number of students in the class?

A. 90 B. 82 C. 80 D. None of these

Solution:

➢ Let the number of students in the class be x.
➢ Each of these students contributed 'x' paise.
➢ The total collection will be x^2 paise.
➢ The total collection is given to be Rs. 64, which is 6400 paise.

As per the statement of the question,

➢ $x^2 = 6400 \Rightarrow x = 80$.

Thus, there are 80 students.

Example 7 A person wants to arrange his colleagues in the form of a perfect square, but he finds there are 9 people too many. What will be the total number of persons in the front row, if the total number of persons with him is 2410?

A. 41 B. 47 C. 48 D. 49

Solution:

➢ Let persons in the first row be y,
(y × y) + 9 = 2410.

Solving this,

➢ we get the value of y as 49.

Example 8: An Army man wants to arrange his men in the form of a perfect square, but he finds there are 52 men too many. What will be the total number of men in front row, if the total number of men with him is 14693?

Solution:
Required number of men in the front row = square root (14693-52)= square root(14641)= 21

Note: No perfect square ends with 2, 3, 7, 8, and an odd number of zeroes,
i.e. any number, which has 2, 3, 7 and 8 at its unit's place and any number ending with an odd number of zeroes can never be a perfect square.