Classification of numbers

Number System

1. **Natural Numbers: –** The numbers 1, 2, 3, 4, 5………are called natural numbers or positive numbers.

   Example: 1, 2, 3, 4, 5………………………..

2. **Whole Numbers: –**The numbers including "0" and all natural numbers are called the whole numbers.

   Example: 0, 1, 2, 3, 4, 5………………………..

3. **Integers –** The numbers including 0 and all the positive as well as negative of the natural numbers are called integers.

   Example: ………………………-3, -2, -1, 0, 1, 2, 3……………………………

4. **Rational Numbers: –** A number which can be expressed in the form p/q where p and q are integers and $q \neq 0$ is called a rational number.

   For example, 4 is a rational number. Since, 4 can be written as 4/1 where 4 and 1 are integers and the denominator $1 \neq 0$. Similarly, the numbers ¾, -2/5, etc. are also rational numbers.

   Between any two numbers, there can be an infinite number of other rational numbers.

5. **Irrational Numbers: –** Numbers which are not rational but which can be represented by points on the number line are called irrational numbers. Examples for irrational numbers are

   Example: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{8}$, etc.

   Numbers like π, e are also irrational numbers.

   Between any two numbers, there are infinite irrational numbers.

   Another way of looking at rational and irrational numbers is

   Any terminating or recurring decimal is a rational number.

   Any non-terminating non-recurring decimal is an irrational number.

6. **Real numbers: –** The set of natural numbers, integers, whole numbers, rational numbers, and irrational numbers constitute the set of real numbers.

7. **Even Numbers: –** The numbers that are divisible by 2 are called even numbers.

   Example: 2, 4, 6, 8, 16, 32 etc.

8. **Odd Numbers: –** The numbers that are not divisible by 2 are called odd numbers.

   Example: 3, 5, 7, 9, 15 etc.

9. **Prime Numbers: –** Those numbers which are divisible by themselves and 1 are called prime numbers or a number which has only two factors 1 and itself is called a prime number.

   Example: 2, 3, 5, 7 etc.

10. **Twin Primes: –** A pair of prime numbers when they differ by 2 are called twin prime numbers.

    Example: (3, 5), (5, 7), (11, 13), (17, 19) etc.

11. **Co-prime Numbers: –** A pair of two natural numbers are said to be co-prime if their G.C.D. or H.C.F. is 1.

    Example: H.C.F. (3, 4) = 1, H.C.F. (13, 15) = 1 then (3, 4) and (13, 15) are co-prime numbers.

12. **Composite Numbers: –** The natural numbers which are not prime are called composite numbers OR numbers that have factors other than itself and 1, are called composite numbers.

Example: 4, 6, 9, 16, 25 etc.

Note: 1 is neither a composite number nor a prime number.

13. **Perfect Numbers: –** If the addition of all the factors of a number excluding the number itself happens to be equal to the number, it is called a perfect number.

First perfect number is 6.

Factors of 6 are 1, 2, 3, 6.

Now add all the factors excluding

6. 1+2+3 = 6, hence 6 is a perfect

number.

Example: 28, 496 and 8128.

14. **Complex Numbers: –** The number which has real and imaginary components is called a complex number.
Example: 3+4*i*, 5+6*i*, where □ = $\sqrt{-1}$ = □ □□□□□□□□□ □□□□□□

15. **Face Value** of a digit in a number is its own value.

    Example: 6728, Face Value ⇒6 = 6, 7 = 7, 2 = 2 and 8 = 8

16. **Place Value** of a digit is given by multiplying it with the value of the place

    where it is placed. Example: 6729

    Place Value of 9 ⇒9 × 1 = 9

    Place Value of 2 ⇒2 × 10 = 20

    Place Value of 7 ⇒7 × 100 = 700

    Place Value of 6 ⇒6 × 1000 = 6000

17. Fractions: A fraction is a quantity which expresses a part of the whole, eg: ¼ means one fourth of the whole

✓ **Types of Fractions**:

- A **Proper Fraction** is one whose numerator is less than its denominator Example: 2/3 is proper fraction, as 2<3
- An **Improper Fraction** is one whose numerator is equal to or greater than its denominator

    Example: 3/2 is an improper fraction, as 3>2 ;
    3/3 is an improper fraction, as 3=3

Some Important Points:

1. Addition or subtraction of any two odd numbers will always result in an even number or zero. E.g. 1 + 3 = 4.
2. Addition or subtraction of any two even numbers will always result in an even number or zero. E.g. 2 + 4 = 6.
3. Addition or subtraction of an odd number from an even number will result in an odd number. E.g. 4 + 3 = 7.
4. Addition or subtraction of an even number from an odd number will result in an odd number. E.g. 3 + 4 = 7.
5. Multiplication of two odd numbers will result in an odd number. E.g. 3 × 3 = 9.
6. Multiplication of two even numbers will result in an even number. E.g. 2 × 4 = 8.
7. Multiplication of an odd number by an even number or vice versa will result in an even number. E.g. 3 × 2 = 6.
8. An odd number raised to an odd or an even power is always odd.
9. An even number raised to an odd or an even power is always even.
10. The standard form of writing a number is m × 10n where m lies between 1 and 10 and n is an integer.
    - If n is odd. $n(n^2 - 1)$ is divisible by 24. Take n = 5 ⇒$5(5^2 - 1) = 120$, which is divisible by 24.
    - If n is an odd prime number except 3, then $n^2 - 1$ is divisible by 24.
    - If n is odd. $2^n + 1$ is divisible by 3.

- If n is even. $2^n - 1$ is divisible by 3.
- If n is odd. $2^{2n} + 1$ is divisible by 5.
- If n is even. $2^{2n} - 1$ is divisible by 5.
- If n is odd. $5^{2n} + 1$ is divisible by 13.
- If n is even. $5^{2n} - 1$ is divisible by 13

11. Some properties of Prime Numbers

- The lowest prime number is 2.
- 2 is also the only even prime number.
- The lowest odd prime number is 3.
- There are 25 prime numbers between 1 to 100.
- The remainder of the division of the square of a prime number p ≥ 5 divided by 24 or 12 is 1.

Division and BODMAS

Hierarchy of Arithmetic Operations

To simplify arithmetic expressions, which involve various operations like brackets, multiplications, addition, etc. a particular sequence of the operations has to be followed. For Example, 2 + 3 X 4 has to be calculated by multiplying 3 with 4 and the result 12 added to 2 to give the final result of 14 (you should not add 2 ro 3 first to take the result 5 and multiplying this 5 by 4 to give the final result as 20). This is because in arithmetic operations, multiplication should be done first before addition is taken up.

The hierarchy of arithmetic operations is given by a rule called the BODMAS rule. The operations have to be carried out in the order, in which they appear in the word BODMAS, where different letters of the word BODMAS stand for the following operations:

B Brackets

O Of

D Division

M Multiplication

A Addition

S Subtraction

There are four types of brackets here:

(i) **Vinculum:** This is represented by a bar on the top of the numbers. For example,
$2 + 3 - \overline{4 + 3}$;
Here, the figures under the vinculum have to be calculated as 4 + 3 first and the “minus” sign before 4 is applicable to 7, Thus the given expression is equal to 2 + 3 – 7 which is equal to -2.

(ii) **Simple Brackets:** These are represented by ( )

(iii) **Curly Brackets:** These are represented by { }

(iv) **Square Brackets:** These are represented by [ ]

The brackets in an expression have to be opened in the order of vinculum, simple brackets, curly brackets and square brackets, i.e., [{ ($\overline{\ }$) }] to be opened from inside to outwards.

After brackets is O in the BODMAS rule standing for “of” which means multiplication. For example, ½ of 4 will be equal to ½ x 4 which is equal to 2.

After O, the next operation is D standing for division. This is followed by M standing for multiplication. After Multiplication, A standing for addition will be performed. Then, S standing for subtraction is performed.

Example 1: simplify: $[3+\frac{1}{6}$ of $\{29 - (20 + \overline{9-6}) + \frac{1}{2}$ of $48\}$ $-2]$

Solution: $[3 + \frac{1}{6}$ of $\{29 - (\ 20 + \overline{9-6}\ ) + \frac{1}{2}$ of $48\} - 2]$

$= [3 + \frac{1}{6}$ of $\{29 - (\ \ 20 + 3) + \frac{1}{2}$ of $48\} - 2]$

$= [3 + \frac{1}{6}$ of $\{29 - 23 + 24\} - 2]$

$= [3 + \frac{1}{6}$ of $\{30\} - 2] = [3+5-2] = 6$

Recurring Decimals

A decimal in which a digit or set of digits is repeated continually is called a recurring decimal. Recurring decimals are written in a shortened form, the digits which are repeated being marked by dots placed over the first and the last of them, thus

$8/3 = 2.666\ldots\ldots = 2.\dot{6}$ or $2.\bar{6}$;

$1/7 = 0.142857142857142857\ldots = 0.\overline{142857}$

In case of 1/7, where the set of digits 142857 is recurring, the dot is placed on top of the first and the last digits of the set or alternatively, a bar is placed over the entire set of the digits that occur.

A recurring decimal like [illegible] is called a pure recurring decimal because all the digits after the decimal point are recurring.

A recurring decimal like $0.1\bar{6}$ (which is equal to 0.16666…) is called a mixed recurring because some of the digits after the decimal are not recurring (in this case, only the digit 6 is recurring and the digit 1 is not recurring).

A recurring decimal is also called a "circulator". The digit, or set of digits, which is repeated is called the "period" of the decimal. In the decimal equivalent to 8/3, the period is 6 and in 1/7 it is 142857.

As already discussed, all recurring decimals are rational numbers as they can be expressed in the form p/q, where p and q are integers. The general rule for converting recurring decimals into fractions will be considered later. Let us first consider a few examples so that we will be able to understand the rule easily.

Example 2: Express $0.\overline{3}$ in the form of a fraction.

Solution: $0.\overline{3} = 0.3333$ $\rightarrow$ (1)

As the period is of one digit, we multiply by 10^1
i.e. 10

$\therefore$ 10 x $0.\overline{3}$ = 3.333 $\rightarrow$ (2)

(2) – (1) gives

9 x $0.\overline{3}$ = 3 $\Rightarrow$ $0.\overline{3}$ = 3/9 = 1/3

Example 3: Express $0.\overline{54}$ in the form of a fraction,

Solution: $0.\overline{54}$ = 0.545454 $\rightarrow$ (1)

As the period is containing 2 digits, we multiply by
10^2 i.e. 100

$\therefore$100 x $0.\overline{54}$ = 54

$\Rightarrow$ $0.\overline{54}$ = 54/99 = 6/11

Example 4: Express the recurring decimal $0.\overline{026}$ in the form of a fraction.

Solution: 0.026 = 0.026026026 $\Rightarrow$ (1)

As the period is containing 2 digits, we multiply by 10^3,
i.e. 1000, therefore

1000 x 0.026 = 26.026026 $\Rightarrow$ (2)

(2) – (1) gives

999 x 0.026 = 26 $\Rightarrow$ 0.026 = 26/999

We can now write down the rule for converting a pure recurring decimal into a fraction as follows:

A pure recurring decimal is equivalent to a vulgar fraction which has the number formed by the recurring digits 9 called the period of the decimal) for its numerator, and for its denominator the number which has for its digits as many nines as their digits in the period.

Thus $0.\overline{37}$ can b written as equal to 37/99

$0.\overline{225}$ can be written as equal to 225/999 which is the same as 25/111

$0.\overline{63}$ = 63/99 = 7/11

A mixed recurring decimal becomes the sum of a whole number and a pure recurring decimal, when it is multiplied by a suitable power of 10 which will bring the decimal point to the left of the first recurring figure. We can then find the equivalent vulgar fraction by the process as explained in case of a pure recurring decimal.

Examples 5: Express $0.2\overline{3}$ as a fraction.

Solution:

Let x = $0.2\overline{3}$ Then 10 x = 2.3 = 2 + 0.3

= 2+3/9 $\left(\text{since } 0.\overline{3} = 3/9\right)$ = 2 + 1/3 = 7/3

⇨ 10x = 7/3 ∴ x = 7/30

Example 6: Express $0.1\overline{36}$ in the form of a fraction.

Solution: Let x = $0.1\overline{36}$

⇨ 10x = $1.\overline{36}$ = 1 + 36/99 = 1 + 4/11 = 15/11

10x = 15/11 ⇨ x = 15/110 = 3/22

Now we can write the rule to express a mixed recurring decimal into a (vulgar) fraction as below.

In the numerator write the entire given number formed by the (recurring and non-recurring parts) and subtract from it the part of the decimal that is not recurring. In the denominator, write as many nines as the period (i.e., as many nines as the number of digits recurring) and then place next to it as many zeroes as there are digits without recurring in the given decimal.

i.e. $0.1\overline{56}$= 156 -1/990 = 155/990 = 31/198

$0.7\overline{3}$ = 73-7/90 = 66/90 = 11/15

RULES OF DIVISIBILITY

(i) Divisibility by 2^n (2, 4, 8, 16 etc)

If the number formed by the last 'n' digits of a number is divisible by 2^n, then the entire number is divisible by 2^n.

Divisibility by 2 (2^1) → Look for last 1 digit

Divisibility by 4 (2^2) → Look for last 2 digits

Divisibility by 8 (2^3) → Look for last 3 digits

Divisibility of 2^n→ Look for last n digits

Example: Check the divisibility of 24138424 by 2, 4, 8, 16, 32

Please highlight that if a number is not divisible by 2^n, then naturally it cannot be divisible by 2^{n+1}.

Solution: divisible by 2,4 and 8 only

(ii) Divisibility by 3, 9

A number is divisible by 3 if the sum of its digits is divisible by 3.

Please highlight that the same rule is also applicable for 9, but not for any higher power of 3. (The sum of digits of 27 is 9).

Use your own examples for checking divisibility with 3 and 9

(iii) Divisibility by 5^n (5, 25, 125 etc) and 10^n (10,100 etc)

The divisibility rule for 5^n is similar to the rule for 2^n. i.e. if the number formed by the last n digit is divisible by 5^n, then that number is divisible by 5^n. Explain with a few examples. A similar formula would be applicable for 10^n as well.

Use your own examples for checking divisibility with 5^n

(iv) Divisibility by composite numbers (6, 12, 15 etc)

If a number N can be written as a product of 2 coprimes m and n, then if any number is divisible by m and n, it would also be divisible by m × n = N. Here please emphasize the importance of the numbers m and n being co - prime.

So can we use the following combination for checking divisibility for 18?

18 = 18 × 1 ✓ (1 is co-prime with every other number) 9 × 2 ✓ (co-primes need not be prime numbers)

6 × 3 × (A no. divisible by 6 is naturally divisible by 3, but does not mean it is divisible by 18)

(v) Divisibility by 7

There are several methods for testing the divisibility of a number by 7. For our purpose, we use the following:

(a) If the number has 3 digits or less, we can simply divide and check - it should be easier than any method we could discuss.

(b) For a number with more than 3 digits, we separate the last 3 digits from the remaining and take their positive difference. If this difference is divisible by 7, then the entire number is divisible by 7.

For example,

$$\frac{1234}{a}\,\Big|\,\frac{562}{b}$$

a – b = 1234 – 562 = 672

As 672 is divisible by 7, the entire number is divisible by 7.

Another Example :
For 35637, the required difference is 637 – 35 i.e., 602.
As 602 ÷ 7, 35637 is also divisible by 7.

∗ Please note that the same divisibility test is applicable for 13 as well.

(vi) Divisibility by 11:

Consider a number. Let the sum of the alternate digits starting from the unit's place be 'A' and the sum of the alternate digits starting from the tens place be 'B'. If (A - B) is divisible by 11, then the number under consideration is divisible by 11. If (A - B) is not divisible by 11, the remainder obtained is the remainder when the number is divided by 11.

For example, 15258595 has the sum of the digits in odd places as 5 +5 + 5 + 5 i.e., 20
Also the sum of the digits in even places = 9 + 8 + 2 + 1 i.e., 20
As the sum of the digits in odd places is equal to the sum of the digits in even places, 15258595 is divisible by 11.

Another Example : 9172816
The sum of the digits in odd places = 9 + 7 +8 + 6 = 30
The sum of the digits in even places = 1 + 2 + 1 = 4

Required difference 30 – 4 = 26.
As, 26 is not a multiple of 11, 9172816 is not divisible by 11.

Example 1: If the number 481 * 673 is completely divisible by 9, then the smallest whole number in place of * will be:
Solution: Sum of digits = $(4 + 8 + 1 + x + 6 + 7 + 3) = (29 + x)$, which must be divisible by 9.
➔ $x = 7$.

Example 2: If the number 97215 * 6 is completely divisible by 11, then the smallest whole number in place of * will be:
Solution: Given number = $97215x6$
$(6 + 5 + 2 + 9) - (x + 1 + 7) = (14 - x)$, which must be divisible by 11.
➔ $x = 3$

Example 3: If 5668425y is divisible by 48, find the value of y.
Solution: Since 48 is a composite number, it should be expressed as a product of co-prime numbers and divisibility should be checked by these co-primes. For 48, we have to use 16 and 3. To be divisible by 3, the sum of the digits should be divisible by 3 ⇒36 + y should be divisible by 3. Therefore y can be 0 or 3 or 6 or 9.For 16; last 4 digits should be divisible by 16

- ➔ 425y should be divisible by 16.
- ➔ 4256 is divisible by 16.
- ➔ Therefore, y = 6