Geometric Progressions Questions

MCQ Question 1

The third term of a G.P. is 9. The product of its first five terms is

1. 3^5

2. 3^9

3. 3^{10}

4. 3^{12}

Answer (Detailed Solution Below)

Option 3 : 3^{10}

Let's discuss the concepts related to Sequences and Series and Geometric Progressions. Explore more from Mathematics here. Learn now!

Geometric Progressions MCQ Question 1 Detailed Solution

Concept:

Five terms in a geometric progression:

If a G.P has first term a and common ratio r then the five consecutive terms in the GP are of the form $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$.

Calculation:

Let us consider a general geometric progression with common ratio r.

Assume that the five terms in the GP are $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$.

It is given that third term is 9.

Therefore, a = 9.

Now the product of the five terms is given as follows:

$$\frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 = a^5$$

But we know that a = 9.

Thus, the product is $9^5 = 3^{10}$.

MCQ Question 2

For what possible value of x are the numbers - 2/7, x, - 7/2 are in a GP ?

1. - 1

2. 1

3. Both 1 and 2

4. None of these

Answer (Detailed Solution Below)

Option 3 : Both 1 and 2

Geometric Progressions MCQ Question 2 Detailed Solution

CONCEPT:

If a, b and c are in a GP then $b^2 = ac$

CALCULATION:

Given: The numbers - 2/7, x, - 7/2 are in GP

As we know that, if a, b and c are in GP then $b^2 = ac$

Here, a = - 2/7, b = x and c = - 7/2

$\Rightarrow x^2 = (-2/7) \times (-7/2) = 1$

$\Rightarrow x = \pm 1$

Hence, correct option is 3.

MCQ Question 3

In a G.P. of positive terms , if every term is equal to the sum of next two terms. Then find the common ratio of the G.P.

1. $2 \sin 18°$

2. $2 \sin 72°$

3. $2 \cos 18°$

4. $\cos 72°$

Answer (Detailed Solution Below)

Option 1 : $2 \sin 18°$

Geometric Progressions MCQ Question 3 Detailed Solution

Concept:

Let us consider sequence a_1, a_2, a_3 .... a_n is a G.P.

- Common ratio $= r = \dfrac{a_2}{a_1} = \dfrac{a_3}{a_2} = \ldots = \dfrac{a_n}{a_{n-1}}$
- n^{th} term of the G.P. is $a_n = ar^{n-1}$

$\sin 18° = \dfrac{\sqrt{5}-1}{4}$

Calculation:

We know that if the first term of a G.P is 'a' and the common ratio is 'r' then in this case then G.P = a,

ar, ar^2 ....

Since we have given $a = ar + ar^2$

Now, $1 = r + r^2$

$\Rightarrow r^2 + r - 1 = 0$

After solving we get $r = \dfrac{\sqrt{5}-1}{2} = 2 \times \dfrac{\sqrt{5}-1}{4} = 2\,\text{Sin}18°$

MCQ Question 4

A man has 2 parents, 4 grandparents, 8 great- grand parents, and so on . Find the number of ancestors during the 8 generations preceding his own .

1. 255

2. 450

3. 505

4. 510

Answer (Detailed Solution Below)

Option 4 : 510

Geometric Progressions MCQ Question 4 Detailed Solution

Concept:

Let us consider sequence $a_1, a_2, a_3 \ldots a_n$ is an G.P.

- Sum of n terms = s = $\frac{a(r^n-1)}{r-1}$; where r >1
- Sum of n terms = s = $\frac{a(1-r^n)}{1-r}$; where r <1

Calculation:

The required no. of ancestors

= 2 + 4 + 8 +... upto 8 terms

As we know that sum of G.P ,

$S = \frac{a(r^n-1)}{r-1}$

Where , a = 2, r = 2 and n = 8

$\Rightarrow$ No. of ancestors required = $\frac{2 \times (2^8-1)}{2-1}$ = $2 \times (2^8 - 1)$ = 510

$\therefore$ No. of ancestors required is **510.**

The correct option is 4.

MCQ Question 5

If the sum of n numbers in the GP 5, 10, 20, ... is 1275 then n is ?

1. 6

2. 7

3. 8

4. 9

Answer (Detailed Solution Below)

Option 3 : 8

Geometric Progressions MCQ Question 5 Detailed Solution

Concept:

Let us consider sequence $a_1, a_2, a_3 \ldots a_n$ is a G.P.

- Common ratio = $r = \dfrac{a_2}{a_1} = \dfrac{a_3}{a_2} = \ldots = \dfrac{a_n}{a_{n-1}}$
- n^{th} term of the G.P. is $a_n = ar^{n-1}$
- Sum of n terms of GP = $s_n = \dfrac{a\,(r^n - 1)}{r - 1}$; where r >1
- Sum of n terms of GP = $s_n = \dfrac{a\,(1 - r^n)}{1 - r}$; where r <1
- Sum of infinite GP = $S_\infty = \dfrac{a}{1 - r}$; |r| < 1

Calculation:

Given series is 5, 10, 20, ...

Here, a = 5, r = 2

Sum of n numbers = s_n = 1275

To Find: nAs we know that, Sum of n terms of GP = $s_n = \dfrac{a\,(r^n - 1)}{r - 1}$; where r >1

$\therefore s_n = \dfrac{5\,(2^n - 1)}{2 - 1}$

$1275 = 5 \times (2^n - 1)$

$\Rightarrow 255 = (2^n - 1)$

$\Rightarrow 2^n = 256$

$\Rightarrow 2^n = 2^8$

$\therefore n = 8$

MCQ Question 6

The third term of a GP is 3. What is the product of its first five terms?

1. 81

2. 243

3. 729

4. Cannot be determined due to insufficient data

Answer (Detailed Solution Below)

Option 2 : 243

Geometric Progressions MCQ Question 6 Detailed Solution

Concept:

Let us consider sequence a_1, a_2, a_3 .... a_n is a G.P.

Common ratio = $r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_n}{a_{n-1}}$

Calculation:

Consider,

(a = 3) be the 3rd term of the G.P series,

So, we can write the five terms as,

$\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar^2

So, the product of the five terms (P) will be,

P = $\frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 = a^5$

Since,

a = 3,

∴ The product of the first five terms (P) = 3^5 = 243

The value of $9^{\frac{1}{3}} 9^{\frac{1}{9}} 9^{\frac{1}{27}} \ldots \infty$ **is:**

1. 3

2. 6

3. 9

4. None of these

Answer (Detailed Solution Below)

Option 1 : 3

Geometric Progressions MCQ Question 7 Detailed Solution

Concept:

Geometric Progression (GP):

- The series of numbers where the ratio of any two consecutive terms is the same is called a Geometric Progression.
- A Geometric Progression of n terms with **first term a** and **common ratio r** is represented as:

  a, ar, ar^2, ar^3, ..., ar^{n-2}, ar^{n-1}.

- The **sum of the first n terms** of a GP is: $S_n = a\left(\dfrac{r^n - 1}{r - 1}\right)$.
- The **sum to ∞** of a GP, when $|r| < 1$, is: $S_\infty = \dfrac{a}{1 - r}$.

Calculation:

Let us consider the infinite series $\dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \ldots \infty$.

Here, $a = \dfrac{1}{3}$ and $r = \dfrac{\frac{1}{9}}{\frac{1}{3}} = \dfrac{1}{3}$.

$\therefore S_\infty = \dfrac{a}{1 - r} = \dfrac{\frac{1}{3}}{1 - \frac{1}{3}} = \dfrac{\frac{1}{3}}{\frac{2}{3}} = \dfrac{1}{2}$.

Now, let $P = 9^{\frac{1}{3}} \cdot 9^{\frac{1}{9}} \cdot 9^{\frac{1}{27}} \ldots \infty$.

$\therefore P = 9^{\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots \infty} = 9^{\frac{1}{2}} = \sqrt{9} = 3$.

MCQ Question 8

For the series $1 + 3 + 3^2 + \ldots$, the sum to n terms is 3280. Find the value of n.

1. 6

2. 8

3. 10

4. 12

Answer (Detailed Solution Below)

Option 2 : 8

Geometric Progressions MCQ Question 8 Detailed Solution

Concept:

Geometric Progression (GP): The series of numbers where the ratio of any two consecutive terms is the same, is called a Geometric Progression.

- A Geometric Progression of n terms with first term a and common ratio r is represented as:

  $a, ar, ar^2, ar^3, ..., ar^{n-2}, ar^{n-1}$.

- The sum of the first n terms of a GP is:

  $S_n = a\left(\frac{r^n - 1}{r - 1}\right)$.

Calculation:

For the given geometric series $1 + 3 + 3^2 + ...$, we have a = 1 and r = 3.

Let the sum of first n terms be equal to 3280.

$\therefore S_n = 1\left(\frac{3^n - 1}{3 - 1}\right) = 3280$

$\Rightarrow 3^n - 1 = 6560$

$\Rightarrow 3^n = 6561 = 3^8$

$\Rightarrow n = \mathbf{8}$.

If the sum of n numbers in the GP 4, 8, 16, ... is 2044 then n is ?

1. 6

2. 7

3. 8

4. 9

Answer (Detailed Solution Below)

Option 4 : 9

Geometric Progressions MCQ Question 9 Detailed Solution

Concept:

Let us consider sequence a_1, a_2, a_3 .... a_n is a G.P.

- Common ratio = r = $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots = \frac{a_n}{a_{n-1}}$
- n^{th} term of the G.P. is $a_n = ar^{n-1}$
- Sum of n terms of GP = $s_n = \frac{a(r^n-1)}{r-1}$; where r >1
- Sum of n terms of GP = $s_n = \frac{a(1-r^n)}{1-r}$; where r <1
- Sum of infinite GP = $S_\infty = \frac{a}{1-r}$; |r| < 1

Calculation:

Given series is 4, 8, 16, ...

Here, a = 4, r = 2

Sum of n numbers = s_n = 2044

To Find: nAs we know that, Sum of n terms of GP = $s_n = \frac{a(r^n-1)}{r-1}$; where r >1

$\frac{4(2^n-1)}{}$

$$\therefore s_n = \frac{}{2-1}$$

$$2044 = 4 \times (2^n - 1)$$

$$\Rightarrow 511 = (2^n - 1)$$

$$\Rightarrow 2^n = 512$$

$$\Rightarrow 2^n = 2^9$$

$$\therefore n = 9$$

MCQ Question 10

What is the nth term of the sequence 25, -125, 625, -3125, ...?

1. $(-5)^{2n-1}$

2. $(-1)^{2n} 5^{n+1}$

3. $(-1)^{2n-1} 5^{n+1}$

4. $(-1)^{n-1} 5^{n+1}$

Answer (Detailed Solution Below)

Option 4 : $(-1)^{n-1} 5^{n+1}$

Geometric Progressions MCQ Question 10 Detailed Solution

Concept:

If $a_1, a_2, ..., a_n$ is a GP then the general term is given by: $a_n = a \times r^{n-1}$ where a is the 1st term and r is

the common ratio.

Calculation:

Given: 25, -125, 625, -3125, ...

Here, first term a = 25 and common ratio r = -5.

As we know that, if $a_1, a_2, ..., a_n$ is a GP then the general term is given by: $a_n = a \times r^{n-1}$ where a is the first term and r is the common ratio.

$\Rightarrow$ The general term is : $a_n = 25 \times (-5)^{n-1} = (-1)^{n-1} \times 5^{n+1}$