Data Structures and Algorithms (L7 – Appendix A)
(CS F 211 / IS F 211)

Random Variables

A ***random variable*** is a function or rule that assigns a number to each outcome of an experiment.

Basically it is just a symbol that represents the outcome of an experiment.

Examples

- ✓ X = number of heads when the experiment is flipping a coin 20 times.
- ✓ C = the daily change in a stock price.
- ✓ R = the number of kilometers per litter you get on your car during a family vacation.

Random Variables

Discrete Random Variable

➢ usually count data [Number of]

➢ one that takes on a ***countable*** number of values

➢ this means you can sit down and list **all** possible outcomes without missing any

Example:

✓ X = values on the roll of two dice: X has to be either 2, 3, 4, …, or 12.

✓ Y = number of accidents in Hyderabad during a week: Y has to be 0, 1, 2, 3, 4, 5, 6, 7, 8, ……………”real big number”

Random Variables

Continuous Random Variable

➢ usually measurement data [time, weight, distance, etc]

➢ one that takes on an uncountable number of values

➢ this means you can never list all possible outcomes even if you had an infinite amount of time

Example:

✓ X = time it takes you to drive home from class: X > 0, might be 30.1 minutes measured to the nearest tenth but in reality the actual time is 30.10000001………………… minutes?)

✓ Exercise: try to list all possible numbers between 0 and 1

Random Variables & Probability Distributions

A ***probability distribution (density function)*** is a table, formula, or graph that describes the values of a random variable and the probability associated with these values.

Discrete Probability Distribution

X = outcome of rolling one die

X	1	2	3	4	5	6
P(X)	1/6	1/6	1/6	1/6	1/6	1/6

Discrete Probability Notation…

✓ An upper-case letter will represent the ***name*** of the random variable, usually **X**.

✓ Its lower-case counterpart, x, will represent the ***value*** of the random variable.

✓ The probability that the random variable **X** will equal x is:

P(**X** = x) or more simply P(x)

✓ **X** = number of heads in 10 flips of coin

P(X = 5) = P(5) = probability of 5 heads (x) in 10 flips

Mean, Variance & Standard Deviation

- ✓ The mean of a discrete random variable is the ***weighted average*** of all of its values. The weights are the probabilities.
- ✓ This parameter is also called the expected value of X and is represented by E(X).

$$E(X) = \mu = \sum_{all\ x} xP(x)$$

- ✓ The variance is

$$V(X) = \sigma^2 = \sum_{all\ x} (x - \mu)^2 P(x)$$

- ✓ The standard deviation is

$$\sigma = \sqrt{\sigma^2}$$

Computing Mean, Variance, and Std. Dev. for Discrete Random Variable

Example A mutual fund sales person knows that there is *20% chance of closing a sale* on each call she makes. What is the ***probability distribution and mean of the number of sales*** if she plans to call three customers?

Solution:

Random Variable = X = # Sales Made in 3 Attempts

Let S denote the event of closing a sale **P(S)=.20**

Thus S^C is the event of not closing a sale, and **P(S^C)=.80**

Seems reasonable to assume that sales are **independent**.

Developing Discrete Probability Distributions

Sample Space: List of all possible outcomes

SSS : P(X = 3) = (.2)*(.2)*(.2) = 0.008

P(3) = .008

SSS^C : P(X = 2) = (.2)*(.2)*(.8) = 0.032

SS^CS : P(X = 2) = (.2)*(.8)*(.2) = 0.032

S^CSS : P(X = 2) = (.8)*(.2)*(.2) = 0.032

P(2) = .032+.032+.032

(Additive Law)

SS^CS^C : P(X = 1) = (.2)*(.8)*(.8) = 0.128

S^CSS^C : P(X = 1) = (.8)*(.2)*(.8) = 0.128

S^CS^CS : P(X = 1) = (.8)*(.8)*(.2) = 0.128

P(1) = .128+.128+.128

(Additive Law)

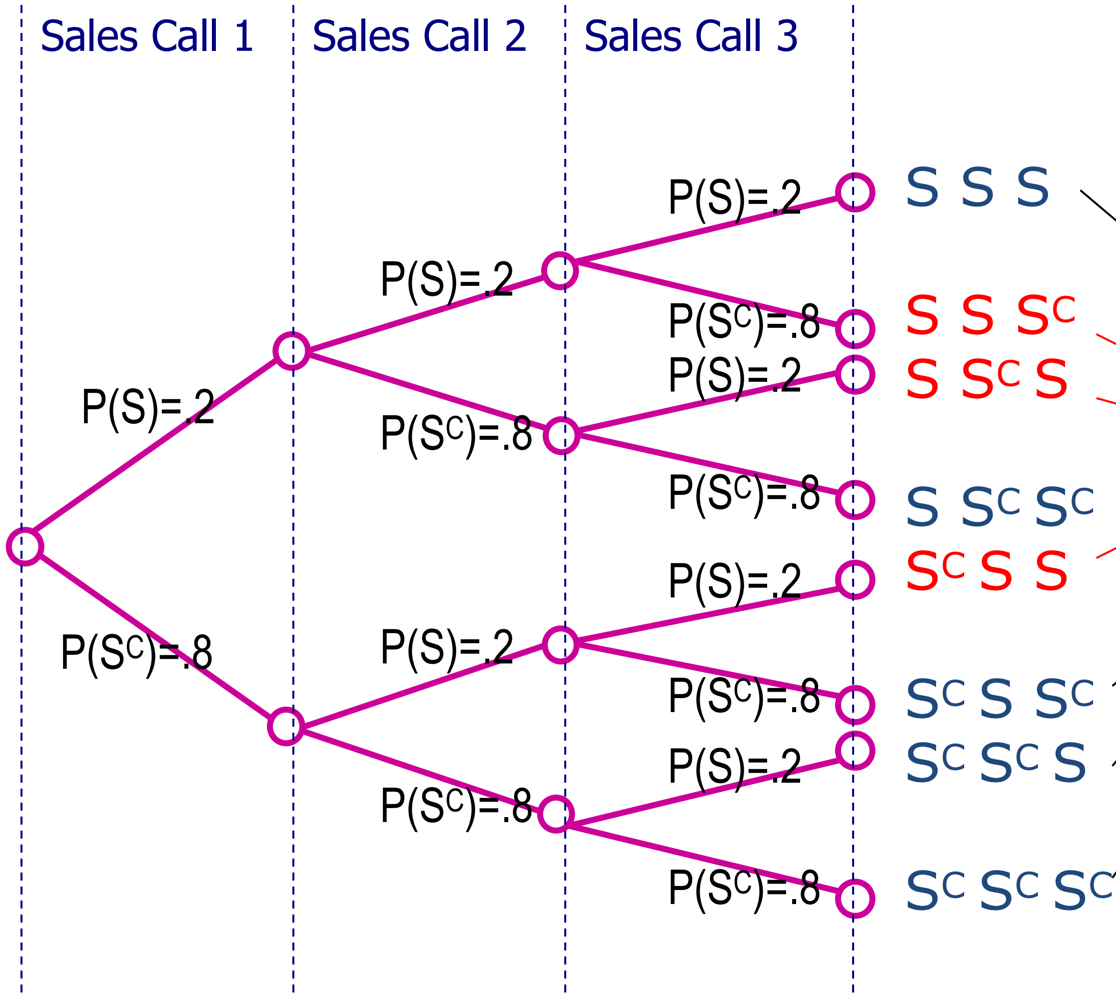
$S^CS^CS^C$: P(X = 0) = (.8)*(.8)*(.8) = 0.512

P(0) = .512

X	0	1	2	3
P(x)	0.512	0.384	0.096	0.008

Another Approach: Tree Diagram

Developing a Probability Distribution…



X	P(x)
3.	$.2^3 = .008$
2.	3(.032)=.096
1.	3(.128)=.384
0	$.8^3 = .512$

P(**X**=2) is illustrated here…

Computing Mean, Variance, and Std. Dev. for Discrete Random Variable

X	0	1	2	3
P(x)	0.512	0.384	0.096	0.008

- ✓ Mean = 0*(.512) + 1*(.384) + 2*(.096) + 3*(.008)
  = 0 + 0.384 + 0.192 + 0.024
  = 0.6
- ✓ Variance =
- ✓ Std. Dev. =