

Data Structures and Algorithms (4)

CS C363 / IS C 363

Sorting

- Input: n **distinct** numbers x_1, x_2, \dots, x_n .
- Output: a sorted list of x_1, x_2, \dots, x_n .



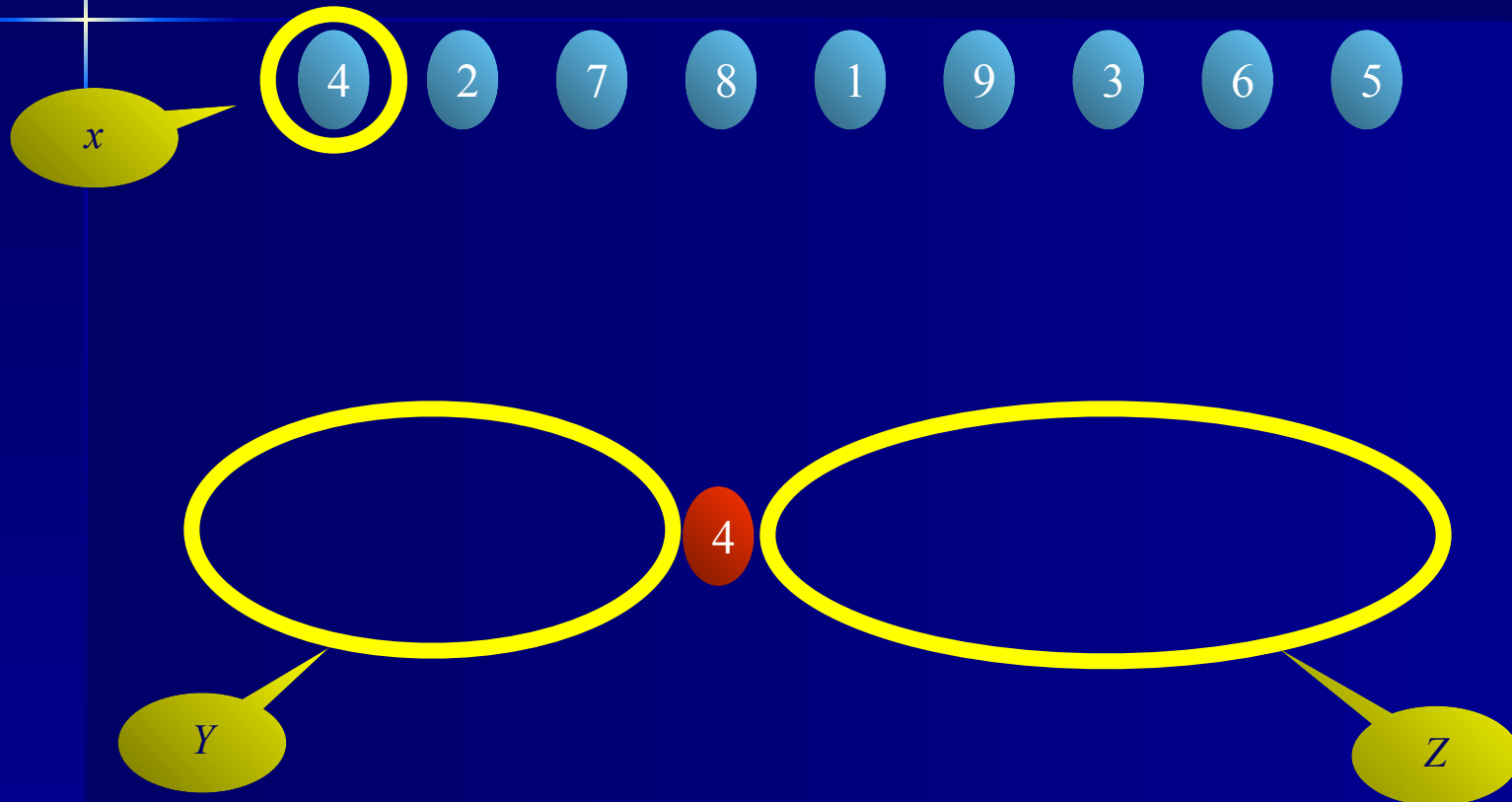
Quick sort: divide & conquer

```

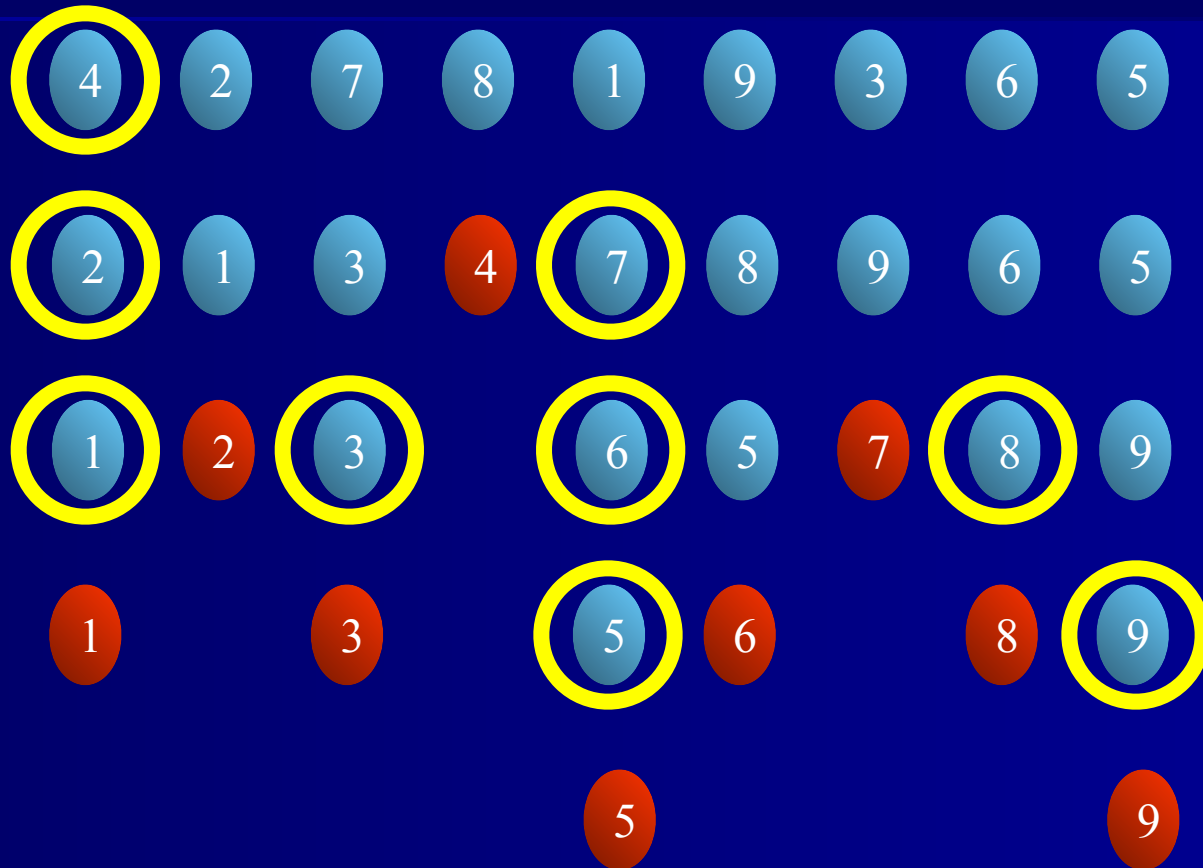
    if (size of X) < 2
    then
        return;
    select a number x;
    let Y = { y in X | y < x };
    let Z = { z in X | z > x };
    call quicksort (Y);
    print x;
    call quicksort (Z);

```

x divides X into Y and Z.



The whole process

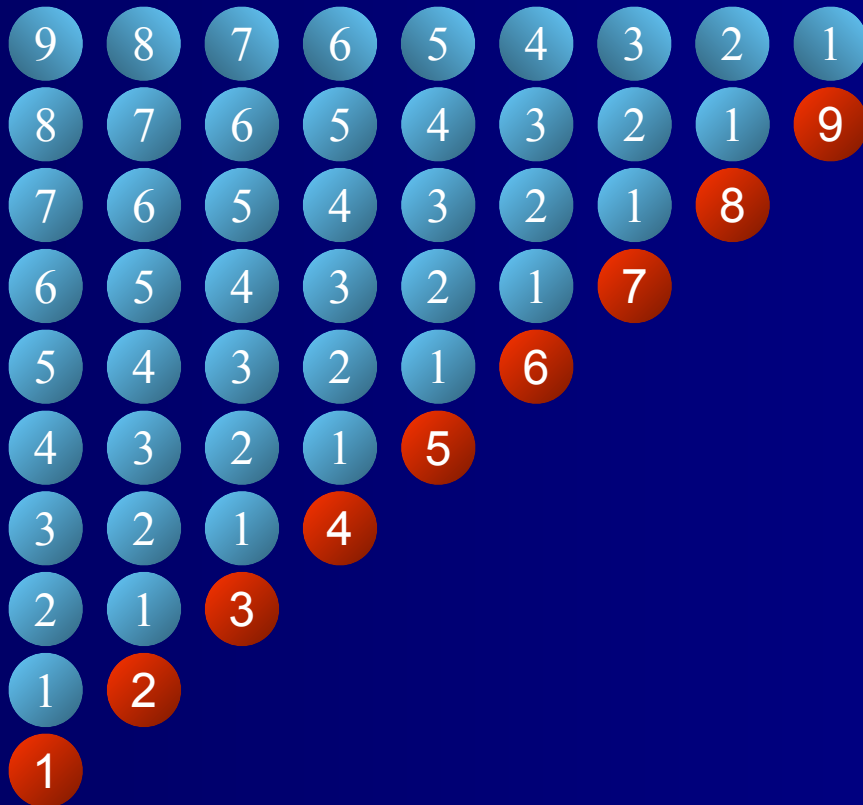


Efficiency depends on ...

```
if ! f
  algor hm quicksort (X) f
    f X f s emp y hen
    return;
  selec a number x;
  let Y = { y ∈ X | y < x };
  let Z = { z ∈ X | z > x };
  call quicksort (Y);
  print x;
  call quicksort (Z);
```

Critical step

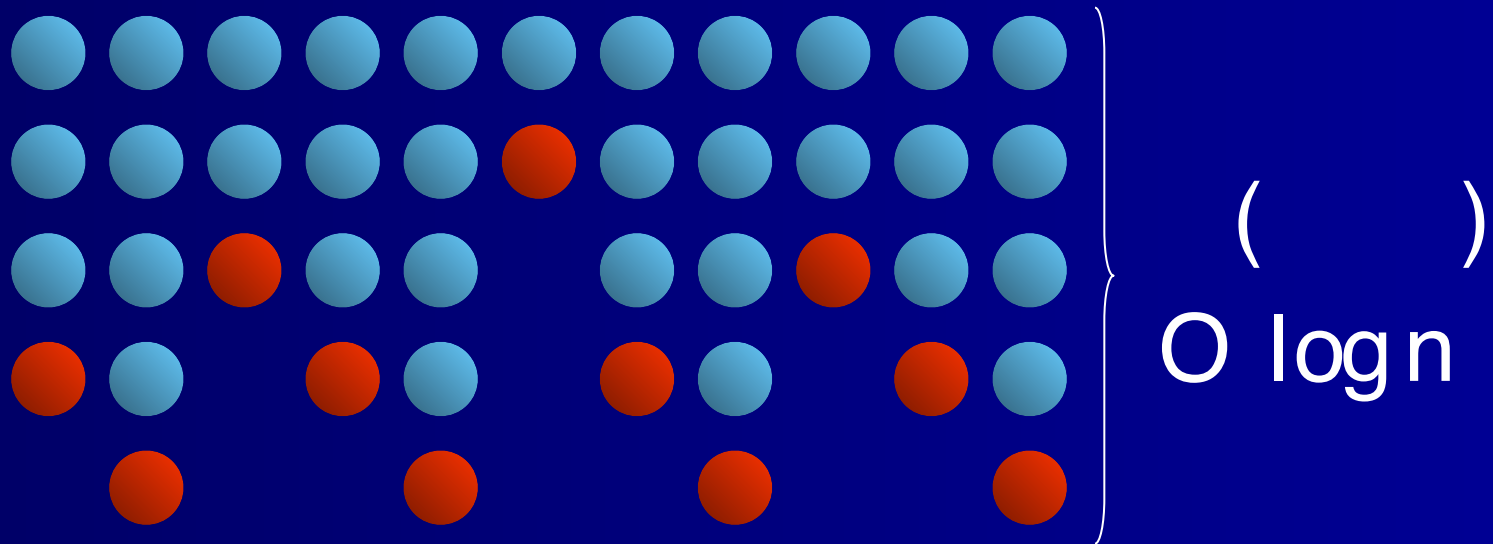
A bad case



()
- n

A good case

- Each x divides X evenly into Y and Z , i.e., when $-1 \cdot |Y| - |Z| \cdot 1$.
- The running time is $O(n \log n)$.



Quick Sort, an implementation

```
quicksort( void *a, int low, int high )  
{  
    int pivot;  
    /* Termination condition! */  
    if ( high > low )  
    {  
        pivot = partition( a, low, high );  
        quicksort( a, low, pivot-1 );  
        quicksort( a, pivot+1, high );  
    }  
}
```

Divide

Conquer

Quick Sort - Partition

```
int partition( int *a, int low, int high ) {
    int left, right;
    int pivot_item;
    pivot_item = a[low];
    pivot = left = low;
    right = high;
    while ( left < right ) {
        /* Move left while item < pivot */
        while( a[left] <= pivot_item ) left++;
        /* Move right while item > pivot */
        while( a[right] >= pivot_item ) right--;
        if ( left < right ) SWAP(a,left,right);
    }
    /* right is final position for the pivot */
    a[low] = a[right];
    a[right] = pivot_item;
    return right;
}
```

Quicksort - Partition

This example
uses `int`'s
to keep things
simple!

```
int partition( int *a, int low, int high ) {  
    int left, right;  
    int pivot_item;  
    pivot_item = a[low];  
    pivot = left = low;  
    right = high;  
    while ( left < right ) {  
        /* Move left while item < pivot */  
        while( a[left] <= pivot_item ) left++;  
        /* Move right while item > pivot */  
        while( a[right] >= pivot_item ) right--;  
        if ( left < right )  
            swap( a[left], a[right] );  
    }  
    /* right is final position for the pivot */  
    a[low] = a[right];  
    a[right] = pivot_item;  
    return right;  
}
```

Any item will do as the pivot,
choose the leftmost one!

23 12 15 38 42 18 36 29 27

high

Quick Sort - Partition

```
int partition( int *a, int low, int high ) {  
    int left, right;  
    int pivot_item;  
    pivot_item = a[low];
```

Set left and right markers

```
    pivot = left = low;
```

```
    right = high;
```

```
    while ( left < right ) {
```

```
        /* Move left while item < pivot */
```

```
        while( a[left] <= pivot_item ) left++;
```

```
        /* Move right while item > pivot */  
        while( a[right] > pivot_item ) right--;
```

```
        if ( left < right ) SWAP(a, left, right);
```

```
    }
```

```
    /* Place pivot in final position  
    a[low] = a[right];  
    a[right] = pivot_item;  
    return right;  
}
```

left



23



low

12

15

38

42

18

36

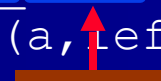
29

27

right



27



high

pivot: 23

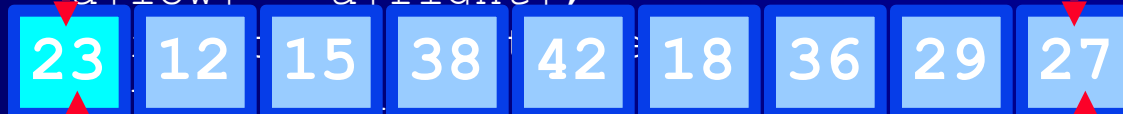
Quick Sort - Partition

```
int partition( int *a, int low, int high ) {  
    int left, right;  
    int pivot_item;  
    pivot_item = a[low];  
    pivot = left = low;  
    right = high;
```

Move the markers
until they cross over

```
    while ( left < right ) {  
        /* Move left while item < pivot */  
        while( a[left] <= pivot_item ) left++;  
        /* Move right while item > pivot */  
        while( a[right] >= pivot_item ) right--;  
        if ( left < right ) SWAP(a,left,right);  
    }
```

left → right is final position for the pivot */
a[low] = a[right];



low

pivot: 23

high

Quick Sort - Partition

```
int partition( int *a, int low, int high ) {
```

```
    int left, right;  
    int pivot_item;  
    pivot_item = a[low];  
    pivot = left = low;  
    right = high;
```

```
    while ( left < right ) {
```

```
        /* Move left while item < pivot */
```

```
        while( a[left] <= pivot_item ) left++;
```

```
        /* Move right while item > pivot */
```

```
        while( a[right] >= pivot_item ) right--;
```

```
        if ( left < right ) SWAP(a, left, right);
```

```
    }
```

```
    /* right is final position the pivot */
```

```
    a[low] = a[right];
```

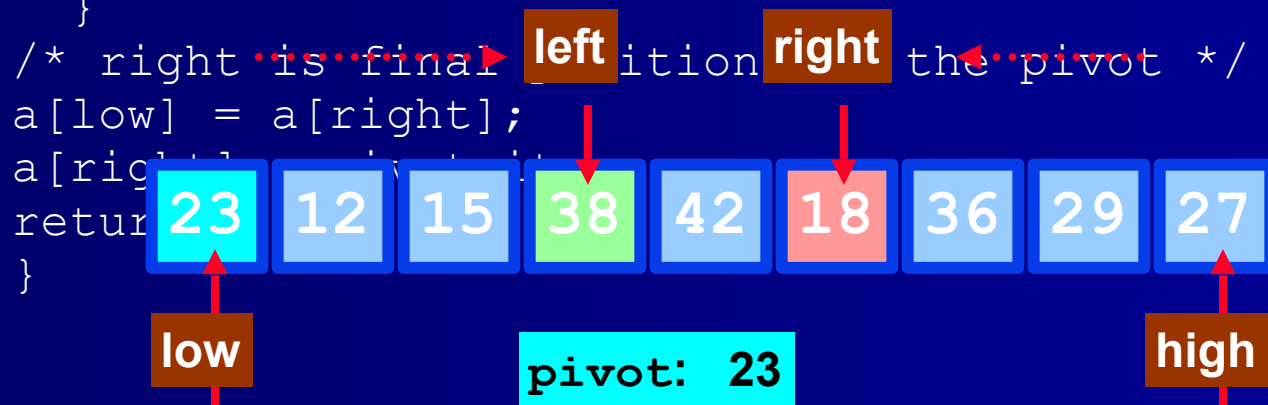
```
    a[right] = pivot_item;
```

```
    return right;
```

```
}
```

Move the left pointer while
it points to items \leq pivot

Move right
similarly



Quick Sort - Partition

```
int partition( int *a, int low, int high ) {
```

```
    int left, right;
```

```
    int pivot_item;
```

```
    pivot_item = a[low];
```

```
    pivot = left = low;
```

```
    right = high;
```

```
    while ( left < right ) {
```

```
        /* Move left while item < pivot */
```

```
        while( a[left] <= pivot_item ) left++;
```

```
        /* Move right while item > pivot */
```

```
        while( a[right] >= pivot_item ) right--;
```

```
        if ( left < right ) SWAP(a,left,right);
```

```
    }
```

```
    /* right is final position for the pivot */
```

```
    a[low] = a[right];
```

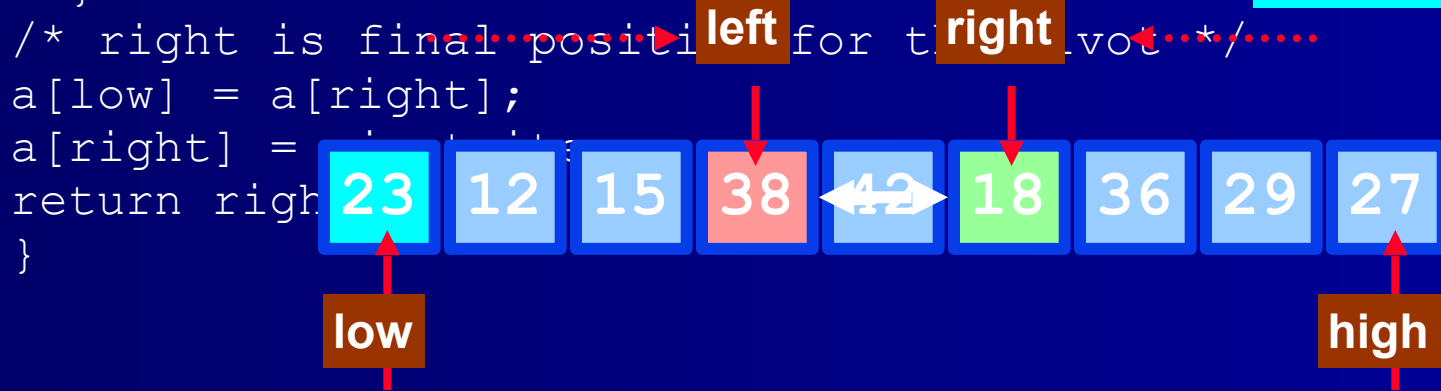
```
    a[right] = pivot_item;
```

```
    return right;
```

```
}
```

Swap the two items
on the wrong side of the pivot

pivot: 23



Quick Sort - Partition

```
int partition( int *a, int low, int high ) {
```

```
    int left, right;  
    int pivot_item;  
    pivot_item = a[low];  
    pivot = left = low;  
    right = high;
```

left and right
have swapped over,
so stop

```
    while ( left < right ) {
```

```
        /* Move left while item < pivot */
```

```
        while( a[left] <= pivot_item ) left++;
```

```
        /* Move right while item > pivot */
```

```
        while( a[right] >= pivot_item ) right--;
```

```
        if ( left < right ) SWAP(a,left,right);
```

```
    }
```

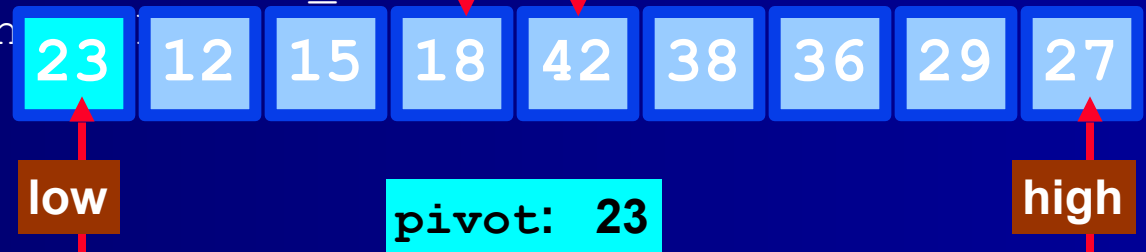
```
    /* right is final position for the pivot */
```

```
    a[low] = a[right];
```

```
    a[right] = pivot_item;
```

```
    return
```

```
}
```



Quick Sort - Partition

```
int partition( int *a, int low, int high ) {
```

```
    int left, right;
```

```
    int pivot_item;
```

```
    pivot_item = a[low];
```

```
    pivot = left = low;
```

```
    right = high;
```

```
    while (right < left) {
```

```
        /* Move left while item < pivot */
```

```
        while (a[left] < pivot_item) left++;
```



```
        if ( left < right ) SWAP(a, left, right);
```

```
    }
```

```
    /* right is final position for the pivot */
```

```
    a[low] = a[right];
```

```
    a[right] = pivot_item;
```

```
    return right;
```

```
}
```

Finally, swap the pivot
and right

Quick Sort - Partition

```
int partition( int *a, int low, int high ) {
```

```
    int left, right;
```

```
    int pivot_item;
```

```
    pivot_item = a[low];
```

```
    pivot = left = low;
```

```
    right = hi
```

```
    while ( right < right ) {
```

```
        /* Move left while item < pivot */
```

```
        while ( a[left] < pivot_item ) left++;
```

```
        while ( a[right] > pivot_item ) right--;
```

```
        if ( left < right ) SWAP(a, left, right);
```

```
    }
```

```
    /* right is final position for the pivot */
```

```
    a[low] = a[right];
```

```
    a[right] = pivot_item;
```

```
    return right;
```

```
}
```



pivot: 23

Return the position
of the pivot

Quick Sort - Conquer

