Q-2)

1) Sum of all terms in while loop after running x times is:-

$$\frac{x^2+x}{2}.$$

This is $\frac{x(x+1)}{2} <= n$.

$$\Rightarrow \frac{x^2+x}{2} <= n$$

$$\Rightarrow O(\sqrt{n})$$ #.

2). i runs for $n+1-\frac{n}{2}$ times

$$\Rightarrow i = \frac{2n+2-n}{2} = \frac{n+2}{2}$$

j runs for $n - \frac{n}{2} + 1 - 1$

$$\Rightarrow \frac{2n-n+2-2}{2} \Rightarrow \frac{n}{2} \text{ times}$$

$\Rightarrow$ and k is $k, k^2, k^4, k^8, k^8 \ldots$

$\therefore \dfrac{k(1-k^x)}{1-k}$ times $= n$.

$\therefore k = \log(n)$

k^2

$k^{2 \cdot 2}$

$k^{4 \cdot 2}$ =

$\Rightarrow$ So, $\left(\dfrac{n+2}{2}\right) \times \left(\dfrac{n}{2}\right) \times \left(\dfrac{1}{\log(n)}\right)$

$\Rightarrow \cancel{O\left(n^2 k - \frac{n}{2}\right)}$

$\Rightarrow O(n^2 \log n)$ #.

3) $O(\sqrt{n})$.

4) $O(\log(nm))$.

1

Q5]

1) <u>To prove:</u>

$f(n) = 5n^3 + 2n^2 + 7n + 1$ is $O(n^3)$, $\theta(n^3)$.

⇒ According to Big-Oh Notation,

$O(g(n)) = f(n)$ ranges from

$\{ 0 \leq f(n) \leq c \cdot g(n).$
$\forall n \geq n_0, \exists c > 0,$
$\exists n_0 > 0 \}$

$\Rightarrow 5n^3 + 2n^2 + 7n + 1 \leq c \cdot n^3.$

$\Rightarrow 5 + \frac{2}{n} + \frac{7}{n^2} + \frac{1}{n^3} \leq c$

Here we can see that for $n_0 = 1$, $c \geq 15$, and for every increase in n_0, the ~~sav~~ left side value will always be less than 15.

So, for $n_0 = 1$, $c = 15$ and for values $n_0 > 1$, c will be less than 15. So, the equation on left side $5n^3 + 2n^2 + 7n + 1$ is upper bounded by $c \cdot n^3$, $O(n^3)$.

For $\Theta(n^3)$:-

According to Big theta notation

$$\{(0 \le c_2 \cdot g(n) \le f(n) \le c_1 \cdot g(n)) \; \forall n \ge n_0 \; \exists (c_1 > 0, c_2 > 0, n_0 > 0)\}$$

We have already proved the RHS $\because$ ~~Eq~~ $f(n) \le c_1 \cdot g(n)$ in $O(n^3)$.

So, for LHS:-

$\Rightarrow \quad c_2 \cdot n^3 \le 5n^3 + 2n^2 + 7n + 1$.

$\Rightarrow \quad c_2 \le 5 + \frac{2}{n} + \frac{7}{n^2} + \frac{1}{n^3}$.

For

$$C_2 \cdot n^3 \leq 5n^3 + 2n^2 + 7n + 1 \leq C_1 \cdot n^3.$$

$$C_2 \leq 5 + \frac{2}{n} + \frac{7}{n^2} + \frac{1}{n^3} \leq C_1$$

RHS was already proven,

For LHS:-

$$C_2 \cdot n^3 \leq 5n^3 + 2n^2 + 7n + 1,$$

→ For large values of n, the n^3 will dominate, so,

$$\cancel{\Rightarrow c \cdot n^3 \leq 5n^3 + \boxed{} \Rightarrow \text{constants.}}$$

$$\cancel{\Rightarrow c \leq 5.}$$

~~For $c \leq 5$ $c \leq 4$,~~

$$\Rightarrow (C_2 - 5)n^3 - 2n^2 - 7n - 1 \leq 0$$

For large n values, this will be dominated by n^3 term.

4

So, $(5 - c_2')n^3 \geq 0$

For this term to be positive,
c_2 should be less than or equal to
5., like $c_2 = 4$.

$\Rightarrow 4 \cdot n^3 \leq 5n^3 + 2n^2 + 7n + 1 \leq 15 \cdot n^3$

So, $\Theta(n^3)$ for this $f(n)$ is true.

2]. $f(n) = 5n^3 + 2n^2 + 7n + 1$.

To prove :-

$\omega(n^2)$, $\Omega(n^2)$.

$\Rightarrow$ For $\omega(n^2)$:-

$\cancel{\{0 \leq c \cdot g(n) \leq}$

$\{ 0 \leq c \cdot g(n) < f(n) \ \forall n \geq n_0,$
$\forall c > 0, \exists n_0 > 0 \}$

$\Rightarrow$ Also, $\left\{ f(n) \mid \lim_{n \to \infty} \frac{g(n)}{f(n)} = 0 \right\}$

$$\Rightarrow \lim_{n \to \infty} \frac{n^2}{5n^3 + 2n^2 + 7n + 1} = 0$$

$$\Rightarrow \lim_{n \to \infty} \frac{1}{5n + 2 + \frac{7}{n} + \frac{1}{n^2}} = 0$$

This equation can be achieved if $n_0 \to \infty$. $(n \geq n_0)$.

And, $c \cdot n^2 < 5n^3 + 2n^2 + 7n + 1$.

For $c = 1$ to any number, $c \cdot n^2$ will always be less than the RHS because the right side is n^3, and any value of c for $c \cdot n^2$ will never exceed or equal n^3.

For $c = 1000$, there is some root for $5n^3 - 998n^2 + 7n + 1 > 0$.

6 135+

For $\Omega(n^2)$

$\{ 0 \le c \cdot g(n) \le f(n), \forall n \ge n_0,$
$\exists c > 0, \exists n_0 > 0 \}$

$\Rightarrow$ Similar to previous $\omega(n^2)$,

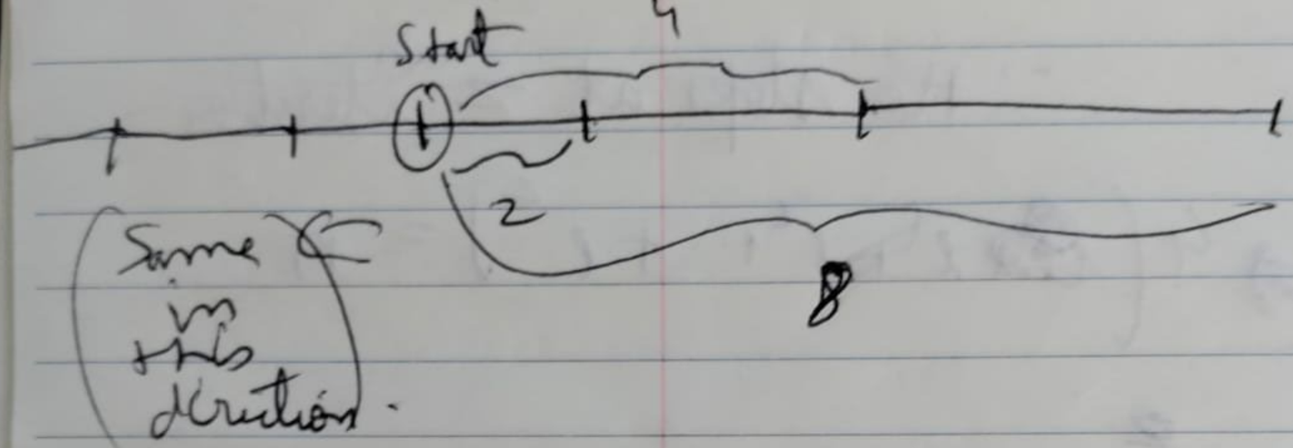
$c \cdot n^2 \le 5n^3 + 2n^2 + 7n + 1$.

For ~~no~~ example, when $n_0 = 3$,
$c \le 19$; so, there exits some values of c where $n_0 > 0$ and
$c \cdot n^2 \le 5n^3 + 2n^2 + 7n + 1$.

$\therefore$ $\Omega(n^2)$ is true for ~~sen~~
$f(n) = 5n^3 + 2n^2 + 7n + 1$.

Q 5.

Assuming Erlich increases his steps exponentially farther from the initial steps.



First iteration is Erlich takes l steps to left, then after arriving at initial position, he goes l^2 steps in next iteration, Same for both directions.

→ For each iteration, he takes 4 functions (left, back to centre, right, back to center).

→ Number of steps:

$l^1(4), l^2(4), l^3(4)$

$\dots l^x(4)$

Assuming he walks z number of iterations,

So, $l^1(4) + l^2(4) + l^3(4) + \dots + l^z(4) = n$

$\because$ He stops at z^{th} iteration.

$\Rightarrow 4\left(l^1 + l^2 + \dots + l^z\right) = n$

$\Rightarrow l^1 + l^2 + \dots + l^z = \frac{n}{4}$

This is a Geometric Progression,

and $\sum_{i=1}^{n} ar^{i-1} = \frac{a(1-r^n)}{1-r}$.

~~Using this, $r = l$, $a = 1$~~

$\Rightarrow \cancel{\frac{1(1-l^z)}{1-l} = \frac{n}{4}}$

Using this,

$a = 4l$

$r = l$

$\Rightarrow \dfrac{4l(1-l^{z})}{1-l} = n.$

$\Rightarrow \cancel{l}(1-l)n = 4l - 4l^{z+1}$

$\Rightarrow n - nl = 4l - 4l^{z+1}$

$\Rightarrow 4l - 4l^{z+1} + nl = n.$

$\cancel{4l(1-}$

$\therefore$ Total steps $\Rightarrow \dfrac{4l(1-l^{z})}{1-l}$ #.

$\Rightarrow$ Here the dominant term is

l^{z},

so, $f(\cancel{z}) = l^{z}$.

$\Rightarrow \cancel{g}\ f(\cancel{n}) \leq c \cdot g(\cancel{n})$.

$\Rightarrow O(l^{\cancel{z}})$ #.