

**Final project**  
**Estimation of a site where motion of a moving vehicle started using radar data.**  
**Extended Kalman filter**

The objectives of this assignment are to encourage you to think creatively and critically to extract a useful signal from noisy experimental data, find best estimation method of a dynamical process and make forecast of its future development.

This assignment is to be done in groups of 3-4 students, and only one document is submitted for the group. You may also freely talk with students in other groups, but the final documents that you submit must be done only by your group.

**Formulation**

1. It is of prime importance for many practical problems to estimate the site from which the vehicles arrived to the destination point.
2. There are radar measurements of range  $D$  and azimuth  $\beta$  for your availability.  
files/folder/Final projects/Project 9/data/trajectory\_data.mat  
The file contains three variables  $z1$ ,  $z2$  and  $z3$  corresponding to the radar measurements of three vehicles. Measurements are available every second.

The format of  $z1$ ,  $z2$ , and  $z3$  in Matlab:

First column – measurements of the range  $D$  in meters

Second column – measurements of the azimuth  $\beta$  in radians

The format of txt data

Element in the first column - measurements of the range  $D$  in meters

Element in the second column - measurements of the azimuth  $\beta$  in radians

3. Could you tell which of the vehicles arrived from one site and which vehicle arrive from another site?  
How to solve this problem?  
(a) Construct the Extended Kalman filter for all datasets.  
(b) Develop optimal smoothing to Kalman filter estimates. Thus you can reconstruct precisely the initial point or the site where the vehicle started its motion.

**Useful hints:**

$$\text{State vector } X_i = \begin{bmatrix} x_i \\ V_i^x \\ y_i \\ V_i^y \end{bmatrix}$$

- (a) Let's assume that the true trajectory  $X_i$  is described by a motion with normally distributed unbiased random acceleration  $a_i$  with variance  $\sigma_a^2 = 0.01^2$  for both  $a_i^x, a_i^y$ .

$$x_i = x_{i-1} + V_{i-1}^x T + \frac{a_{i-1}^x T^2}{2}$$

$$V_i^x = V_{i-1}^x + a_{i-1}^x T$$

$$y_i = y_{i-1} + V_{i-1}^y T + \frac{a_{i-1}^y T^2}{2}$$

$$V_i^y = V_{i-1}^y + a_{i-1}^y T$$

$$T = 1$$

(b) Initial filtration error covariance matrix  $P_{0,0}$

$$P_{0,0} = \begin{bmatrix} 10^4 & 0 & 0 & 0 \\ 0 & 10^4 & 0 & 0 \\ 0 & 0 & 10^4 & 0 \\ 0 & 0 & 0 & 10^4 \end{bmatrix}$$

(c) The measurement vector consists of range  $D$  and azimuth  $\beta$  and thus the measurement equation is nonlinear.

(d) From a prior information you know that the variance of measurement noise of range  $D$  is  $\sigma_D^2 = 200^2$  (in meters), and the variance of measurement noise of azimuth  $\beta$  is  $\sigma_\beta^2 = 0.01^2$  (in radians).

(e) To answer the question of this project plot the smoothed results as a function of  $y(x)$ .

4. Contact the group of students working on project 10. They are doing the same, but doing coordinate transformation of measurements instead of Extended Kalman filter. Compare the accuracy of predictions with your and their approach.

### October 19 and October 21

1. Present your results with charts in 15 minutes.

The presentation should include the problem formulation, why it is important, nice figures, grounds why the chosen method is the best method (visual analysis, quantitative criteria, simplicity of implementation, and any other arguments). Which regularities are found. Discuss what are the risks of obtained estimations and conclusions about the process. Make general conclusions about the efficiency of method.

Try to **share** with the audience a practical and useful idea behind the project, **to make the overall exchange of practical and efficient tools and approaches and for what they can be applied.**

2. Submit the final version of your project to canvas.