Code for deriving parameters of NIF target during its x-ray burst

The code runs along scenario **prescribed by user** for three zones of a target and computes signals of power detectors, sequence of spectrograms, images of the target. Spectral response of each detector is **prescribed by user**. Comparing simulated data to that in experiment, **users may improve scenarios** until satisfied with results of comparison, thus deriving time-dependent radius of each zone and thermodynamic parameters in it.

I. Advantages of the code

A. Plasma may be composed of 1 - 3 light chemical elements (CEs) and one much-heavier CE admixed for x-ray spectroscopy. For each spatial zone **user may choose** time-dependent ion number density of each CE. This option enables simulations of zone-to-zone mass transfer and time dependence of total radiating mass.

B. **Users may prescribe** a distribution of free electrons over energy, $f(\varepsilon_e,t)$. This option enables (i) analysis of plasma response to irradiation by external sources and/or generation of electron beam, (ii) comparison of spectrograms computed using various $f(\varepsilon_e,t)$, e.g. comparison to spectrograms of maxwellian plasma.

C. We derived and included expression for continuum lowering in plasmas of arbitrary chemical composition.

D. Code does not assume any type of equilibrium, therefore, is correct for arbitrarily quick changes in parameters and $f(\varepsilon_e,t)$. Moreover, the code may be used for determination of time needed for plasma relaxation to collisional-radiative equilibrium (CRE) at any parameters and $f(\varepsilon_e)$.

E. Code may be converted in post-processor of hydrodynamic simulations, that enables computation of abovementioned spectroscopic data, thus comparison to spectroscopic data obtained during the x-ray burst.

II. Model and the code

We simulate NIF target during its x-ray burst. The model consists of N_{BS} identical spheric bright sources (**BS**s) **randomly** located in spheric **core** surrounded by plasma **capsule**. Radius of BS is $\mathbf{R_1}$, radius of core is $\mathbf{R_2 \gg R_1}$, capsule is spheric layer of inner radius R_2, outer radius $\mathbf{R_3}$. For brevity, capsule, core, BSs are mentioned as three zones. Zones affect each other by **radiation**. **Within each zone plasma is uniform.** Number of BSs, radii, thermodynamic parameters of zones change in time. Full set of parameters is mentioned as scenario, **to be prescribed** in file **Params2.inp**.

In computation of radiated power, code treats a target as a sequence of coaxial cylindrical tubes, **Fig. 1**, and takes central line of sight (LoS) of each tube as mean for this tube. Total number of cylindrical tubes, N_T, **to be prescribed** in file **Params1.inp**. Width of each tube is R_3/N_T. Computation of images requires at least two tubes outside the core, it limits R_3 to

$$\mathbf{R_3 > R_2/(1 - 2/N_T)}.$$

Each detector sees a target as plane circle of radius R_3. In this circle, BSs are behind area $A_\Sigma \le N_{BS}\, \pi R_1^2$ within central πR_2^2, thus mean LoS has probability $W_1 \le N_{BS}\,(R_1/R_2)^2$ to cross a BS and $W_{11} \le (N_{BS}\,-1)\,(R_1/R_2)^2$ to cross one more BS; sign < is due to possible partial overlap of 2D images of BSs. Two crosses are independent events, therefore the probability of two $W_2 = W_1 W_{11}$. Closer-to-detector BS attenuates radiation of behind-it BS. Optically thick lines are attenuated more than thin lines and continuum; therefore, the attenuation changes their ratio (*vs that in radiation of single BS*). Coordinates of BSs are unknown; therefore, A_Σ and spectrum of attenuated radiation cannot be computed; therefore, we restricted the model to negligible W_2, namely, $W_2 < W_2^{max} \ll 1$ that limits N_{BS} to

$$N_{BS}\,(N_{BS}\,-1) < W_2^{max}\,(R_2/R_1)^4. \qquad \text{(II.1)}$$

Users **prescribe** W_2^{max} according to their understanding of negligibility, see parameter "W2*max*" in file **Params2.inp**.

Mean volume of core per BS is $V_{p1} = 4\pi R_2^3/3N_{BS}$, thus mean distance between centers of BSs

$$d \approx V_{p1}^{1/3} = R_2 (4\pi/3N_{BS})^{1/3}.$$

In 2D images distance between BSs **may look shorter** than d that is 3D.

Computation of probabilities of photoionization and other photoinduced transitions requires specific intensity of radiation in each zone. Due to uncertain location of BSs, backlighting of BS by other BSs cannot be computed, therefore we restrict consideration to $d \gg R_1$ that justifies negligibility of backlighting by BSs *in comparison to radiation of BS itself plus backlighting of BS by core and capsule*. Inequality $d \gg R_1$ limits N_{BS} to

$$N_{BS} \ll 4 (R_2/R_1)^3$$

that is satisfied if (II.1) is satisfied.

Code is applicable to plasmas of up to four chemical elements (CEs). Three of them are light CEs. One is much-heavier CE for spectroscopy at photon energy $h\nu \gg 1$ keV. This CE is **denoted X**. In present database X is Kr, light isotopes are ^{2}H, ^{4}He, ^{12}C. **User** can change chemical composition of plasma folloving explanation in Section IV. One or two of light CEs may be excluded from computation by prescribing negligible ion number density in file **Params2.inp**.

In present database, ions of Kr are N-like to nuclei, that limits the electron temperature to

$$\mathbf{T_{e,b}(t) \gg 100\ eV}\ , \qquad \text{(II. 2)}$$

subscript b = 1, 2, 3 denotes zone (i.e. BSs, core, capsule, respectively). Each zone is characterized with **time-dependent**

(i) **number density of ions** $n_{a,b}$, subscript a denotes CE, a = 1, 2, 3, 4 denote Kr, C, He, H, respectively.

(ii) **distribution** $f_b(\varepsilon_e)$ **of free electrons over their energy** ε_e. In the code function $f_b(\varepsilon_e)$ is normalized to the under-shape area = 1. Units of ε_e, f_b are [eV] and [eV^{-1}], respectively. Function $f_b(\varepsilon_e)$ **must be prescribed** in "*function EED*" in the source file **Co2nov25.for**. Possibility of choosing

arbitrary $f_b(\varepsilon_e, t)$ is unique advantage of present code. This option enables (i) analysis of plasma response to irradiation by external sources and/or generation of electron beam, (ii) comparison of spectrograms computed using different $f(\varepsilon_e, t)$, e.g. comparison to spectrograms of plasma with Maxwell $f(\varepsilon_e, t)$. This possibility is due to characterization of inelastic electron-ion collisions **by cross sections** (*not by widely used maxwellian rates*). The cross sections are computed using Flexible Atomic Code (FAC) **[1]**. In demo version of the code, function $f_b(\varepsilon_e, t)$ consists of maxwellian and beam parts:

$$f_b(\varepsilon_e) = (1 - \xi_b)\, f_{\mathrm{Maxw}}(\varepsilon_e, T_{e,b}) + \xi_b\, f_{\mathrm{beam}}(\varepsilon_e, \varepsilon_{e,c,b}, \varepsilon_{e,w,b}) \qquad \text{(II. 3)}$$

where $f_{\mathrm{Maxw}}(\varepsilon_e, T_{e,b})$ is Maxwell distribution at $T_{e,b}$; parameter ξ_b **is the beam part** of local number density of free electrons; $f_{\mathrm{beam}}(\varepsilon_e, \ldots)$ is the shape of electron beam. In demo packet of files, the shape is rectangular; $\varepsilon_{e,c,b}$, $\varepsilon_{e,w,b}$ are central energy and width of the e-beam; f_{Maxw} and f_{beam} are each normalized to the under-shape area = 1. Let v_{ee} denote velocity of relative motion of e-beam electrons vs electrons of Maxwellian.
Cross section of elastic e-e collisions is proportional to v_{ee}^{-4} **[2]** therefore collisions of high-energy electron beam (that with $(\varepsilon_{e,c,b} - \varepsilon_{e,w,b}/2) \gg T_{e,b}$) with Maxwell electrons don't distort f_{Maxw} or f_{beam}.

(iii) **mean absolute velocity of ions in collective** (i.e. non-thermal**) motion**, denoted $u_{3D,b}$. Velocity $u_{3D,1}$ includes motion of BSs in the core.

Time-dependent parameters R_b, $n_{a,b}$, $T_{e,b}$, $u_{3D,.b}$, ξ_b, $\varepsilon_{e,c,b}$, $\varepsilon_{e,w,b}$ are prescribed in file **Params2.inp**.

High density in compressed target results in sub-ps equilibration between $T_{e,b}$ and **ion temperature** $T_{i,b}$, therefore in the code

$$\mathbf{T_{i,b} = T_{e,b}}. \qquad \text{(II. 4)}$$

Density of free electrons $\boldsymbol{n_{e,b}}$ is given by equation of plasma neutrality

$$\boldsymbol{n_{e,b}} = \sum_a Z_{a,b}\, n_{a,b}, \qquad \text{(II. 5)}$$

$Z_{a,b}$ is the mean ion charge of CE a in zone b. Inequality (II.2) results in complete ionization of light CEs; therefore their $Z_{a,b}$ equals to the charge of nuclei. Ionization energy of heavier ions is lowered in the Ion Sphere

approach that we advanced to plasmas of **arbitrary chemical composition**, see **Appendix II**.

The database of X **must** include nucleus, H-like, He-like, Li-like ions and **can** include ionization stages below Li-like; these stages must follow in one sequence (no missing stages) down to the stage that has ground state ionization energy < 1 keV. Lower stages are omitted because at $T_{e,b} \gg 100$ eV (II.2) their population is negligible. Detailed info on database and instruction for replacement of database are given in Section IV.

Let $n_{b,l}$ denote the ion number density of X on energy level number l in zone b. Compiler restricts total number of energy levels in database of X to

$$l_{\max} < 2300. \qquad \text{(II. 6)}$$

Sum over energy levels

$$\sum_l n_{b,l} = n_{a,b} . \qquad \text{(II. 7)}$$

Functions $n_{b,l}(t)$ are computed by integration of system of $l_{\max}$ rate equations of level-population kinetics **[5 - 7]**

$$dn_{b,l}/dt = \sum_{l' \neq l} n_{b,l'} W_{l',l} - n_{b,l} \sum_{l' \neq l} W_{l,l'} \qquad \text{(II. 8)}$$

where $W_{l',l}$ is the probability of $l' \rightarrow l$ transition due to all channels possible for this pair, namely, spontaneous radiative transition, spontaneous radiative recombination, autoionization, dielectronic capture, photoionization, photoinduced recombination, photo-excitation, photoinduced deexcitation, ionization by electron impact, three-body recombination, excitation and deexcitation by electron impact.

Computation of probabilities of photoinduced transitions in zone b requires **mean specific intensity of radiation** $J_{\nu,b}(t)$ **[3, 5, 14]**. Here "mean" is average over volume V_b of zone b and over full solid angle (4π sr) towards small volume dV around each point, **r**, inside V_b, thus **<u>by definition</u>**

$$J_{\nu,b}(t) = (4\pi V_b)^{-1} \int_{(Vb)} dV \int_{(4\pi)} I_\nu(t, \mathbf{r}, \mathbf{\Omega})\, d\mathbf{\Omega}. \qquad \text{(II.9)}$$

Specific intensity $I_\nu(t, \mathbf{r}, \mathbf{\Omega})$ is solution of the radiative transfer equation (RTE) **[4, 5]** along a ray to point **r** from sector $d\mathbf{\Omega}$ around direction $\mathbf{\Omega}$.

Functions $J_{\nu,b}$ and $I_{\nu,b,j}$ are expressed in [W/eV/sr/cm^2]. For solution of RTE see **Appendix I**.

Code integrates rate equations (II.8) by 4th order Runge-Kutta method using D02EAF subroutine of NAG library (included in the packet). At start time of computation (denoted t_0) the code loads all ions of X on one energy level **to be chosen** in file **Params1.inp**. Other levels are given $n_{b,l}(t_0) = 0$. With time, inelastic collisions and radiation distribute ions over all energy levels according to scenario **prescribed** in file **Params2.inp**.

Let Δt be the time step of computation, let t_i be the first instant of Δt, let $t_f = t_i + \Delta t$ be the final instant of this Δt. Integration of eqs, (II.8) from t_i to t_f gives $n_{b,l}(t_f)$. For next Δt these densities are initial, i.e. $n_{b,l}(t_i)$. Parameters $T_{e,b}(t_i)$ and $u_{3D,b}(t_i)$ are given in scenario. We use $n_{b,l}(t_i)$, $T_{e,b}(t_i)$, $u_{3D,b}(t_i)$ for computation of $J_{\nu,b}(t_i)$ of this "next" Δt; Knowledge of $n_{b,l}(t_i)$, $T_{e,b}(t_i)$, $u_{3D,b}(t_i)$, $J_{\nu,b}(t_i)$ is sufficient for computation of probabilities of all inelastic and photoinduced transitions, thus for integration of eqs. (II.8) until next t_f, and so on along entire scenario.

Computation of $J_{\nu,b}(t_i)$ requires emissivity and effective absorption coefficient in all zones. Let $\varepsilon_{\nu,b}$ denote the emissivity **[5]** (*also known as emission coefficient* **[6]**) and let $\kappa'_{\nu,b}$ denote effective absorption coefficient (*difference of absorption and induced emission coefficients*) **[6]**. In the code, $\varepsilon_{\nu,b}$ is expressed in [W/eV/cm^3/sr], $\kappa'_{\nu,b}$ is expressed in [cm^{-1}]. In the bound-bound part of $\varepsilon_{\nu,b}$, $\kappa'_{\nu,b}$ each spectral line is given **Voigt shape** with **Gaussian part of Voigt FWHM** defined by Doppler broadening, thus by $T_{i,b}$ and $u_{3D,b}$. **Lorentzian part of Voigt FWHM** is defined by bigger of two: Stark effect or the uncertainty principle. In computations of Stark width we follow Refs. **[8,9]**. For the uncertainty principle, life time of each energy level is taken in Baranger' approach **[6]**. **Shape of radiated lines differs from Voigt** because the lines are broadened by plasma opacity along LoSs and distorted/broadened by merging. In spectrograms, line shapes are further broadened by instruments. Effect of each mechanism on the width of each

line at t = t_{inf} is shown in files "….**LineInfo.dat**", *see* ***Section VI*** *for details*. Time t_{inf} **should be given by user** as parameter "tiInf" in file **Params2.inp**.

Note: in the code, no spectral lines except those satisfying **each** of three conditions: (i) have center in interval [$h\nu_{min}$, $h\nu_{max}$],

(ii) have absorption oscillator strength f_{lu} > "f_{lu}Min"

(iii) have Einstein coefficient A_{ul} > "A_{ul}Min".

Parameters $h\nu_{min}$, $h\nu_{max}$, "f_{lu}Min", "A_{ul}Min" **should be prescribed** in file **Params1.inp**.

Due to **differential** eqs. (II.8), present code has substantial advantage over widely used **CRE codes** because those codes assume $dn_{b,l}/dt = 0$ thus $n_{b,l}(t)$ are defined by instant values of $R_b(t)$, $T_{e,b}(t)$, $u_{3D,b}(t)$, $J_{\nu,b}(t)$. Non-CRE computations show that **assumption of CRE is often wrong** because population of energy levels depends on entire scenario from t_0 to t. Comparing $n_{b,l}(t)$ given by eqs. (II.8) to those in CRE, **user may analyze** errors caused by assumption of CRE for his scenario. Present code enables an estimate of time, $\boldsymbol{t_{CRE}}$, required for relaxation of $n_{b,l}(t)$ to their CRE values at parameters of interest. If user wants this estimate, he should keep parameters constant during a few frames. Then a sequence of frames converges with t. The computation should be repeated with 3x, 10x, 30x, ... shorter Δt, until the time requied for convergence becomes independent of Δt. **Thus found time is $\boldsymbol{t_{CRE}}$**. Note: t_{CRE} depends on (i) database of energy levels, (ii) database of cross sections, (ii) energy level chosen for loading initial population.

III. Target radiation. Simulation of spectrograms, images of the target, signals of power detectors.

Instruments see a target as **plane** circle of radius $R_3(t)$. Let r be radial coordinate in this circle and let $I_\nu(r,t)$ be specific intensity of radiation from target towards an instrument along LoS that crossed the plane circle at distance r from its center. In other words, $I_\nu(r,t)$ is solution of RTE along normal-to-the-circle LoS that is at distance r from central LoS, **Section VI**.

Specific power of radiation from target towards an instrument is integral over plane circle seen to this instrument

$$P_\nu(t) = 2\pi \int_0^{R3(t)} I_\nu(r,t)\, r dr\,. \qquad \text{(III.1)}$$

In the code, I_ν is expressed in [W/eV/sr/cm^2], P_ν is expressed in [W/eV/sr], integral (III.1) is computed as a sum of N_T radial belts of width $\Delta r = R_3(t)/N_T$ within the circle mentioned above. For parameters derived from analysis of NIF experiments, spectrograms simulated with $N_T = 3$ differ from those with $N_T = 12$ by less than 10% at any hν; spectrograms simulated with $N_T = 48$ differ from those with $N_T = 12$ by less than 1.2% at any hν; thus $N_T = 12$ is sufficient for computation of spectrograms accurate to ~1%. However, if users want to compute **images** of radiating plasma, **they should choose** N_T » 12 for sufficient spacial resolution.

Spectral distribution of energy radiated during k^{th} frame is

$$Y_\nu(t_k) = \int_{t1}^{t2} P_\nu(t)\, dt. \qquad \text{(III.2)}$$

Here t_k is central time of k^{th} frame, $t_1 = t_k - \delta/2$, $t_2 = t_k + \delta/2$, δ is duration of a frame. Eight functions $Y_\nu(t_k)$ are displayed in columns 2-9 of file **Frames.dat** where $Y_\nu(t_k)$ is expressed in [J/eV/sr], 10^{th} column displays integral (III.2) in limits from t_0 to t^ that is the end of computation. Parameters t_k, δ, t^ **should be prescribed** in file **Params2.inp**. Besides integrals (III.2) file **Frames.dat** displays spectral response of spectrometer, $\Psi_{\nu,1}$ (column 11) and eight integrals

$$Y^*_\nu(t_k) = \Psi_{\nu,1} \int_{t1}^{t2} P_\nu(t_k)\, dt \qquad \text{(III.3)}$$

that show specific energy recorded by spectrometer. Spectral response is the product of crystal reflectivity, filter transparency, geometric factor and MCP sensitivity. Function $\Psi_{\nu,1}$ **should be loaded** in input file **InpResp1.inp** in arbitrary units, index **1** is serial number of instrument in the code. The last column displays integral (III.3) in limits [t_0, t^]. Due to arbitrary units of $\Psi_{\nu,1}$ columns 12-20 are expressed in arbitrary units as well.

Users may compare specific intensity of radiation along four towards-spectrometer LoSs at $t = t_{inf}$. First of these four is "central" LoS, that through center of target ($r = 0$). Another three LoSs are at distance $r_\#$ from $r = 0$. Distances $r_\#$ **should be given** in file **Params2.inp**; # = 1,2,3 is serial number of non-central LoS. Spectral intensity of target radiation along these three LoSs, $I_\nu(r_\#)$, is displayed in files **SpIn1.dat**, **SpIn2.dat**, **SpIn3.dat**. Details of computation are given in comments to these files in **Section VI**. Besides $I_\nu(r_\#)$ files **SpIn#.dat** display spectral response of spectrometer, $\Psi_{\nu,n}$ and spectrograms

$$S_{\nu,\#} = I_\nu(r_\#) \cdot \Psi_{\nu,n}. \qquad \text{(III.4)}$$

n = 1+ # = 2,3,4 is serial number of instrument in the code. Functions $\Psi_{\nu,n}$ **should be loaded by user** in files **InpRespn.inp**. $I_\nu(r_\#)$ is expressed in [W/eV/cm^2/sr], $\Psi_{\nu,n}$ is prescribed in arbitrary units, therefore spectrograms (III.4) are expressed in arbitrary units as well.

Spectral intensity of radiation on the central LoS exit from target is displayed in 4th column of file "**AlongCeLOS.dat**". 2nd column of this file displays spectral intensity of radiation on central LoS exit from capsule into core. 3rd column displays spectral intensity of radiation on central LoS exit from core into near-spectrometer segment of capsule. Spectral intensity (column 4) includes possible contribution of BS. For details of computation and explanation see **Section VI**.

Code simulates images of radiating plasma on two cameras at $t = t_{inf}$

$$Q_\#(r, t_{inf}) = \int_0^\infty I_\nu(r, t_{inf}) \cdot \Psi_{\nu,n}\, d\nu\,. \qquad \text{(III.5)}$$

Here # = 1,2 is serial number of camera, n = 4+# = 5,6 is serial numbers of these instruments in the code. Function $\Psi_{\nu,n}$ is spectral response of #th camera; two cameras can display the target in different parts of spectrum. Files **Image#.dat** display images normalized to their values at $r = 0$, i.e. ratios

$$Q_{\#}(r, t_{inf}) \,/\, Q_{\#}(0, t_{inf}) \qquad \text{(III.6)}$$

Files **PowDet#.dat** display signals of three power detectors

$$\Pi_{\#}(t) = \int_0^{\infty} P_{\nu}(t)\, \Psi_{\nu,n}\, d\nu. \qquad \text{(III.7)}$$

Here # = 1,2,3 is serial number of power detector, n = #+6 = 7,8,9 is serial number of instrument in the code. Due to $\Psi_{\nu,n}$ the signals are expressed in arbitrary units.

Note: **spectral response** of each instrument **should be given** in input file **InpResp<u>n</u>.inp** in any units; <u>n</u> = 1, 2, …, 9 is serial number of instrument in the code. If in some experiment the number of frames > 8 or the number of spectrometers > 4 or the number of imaging cameras > 2 or the number of power detectors > 3, **users should** repeat the computation using $\Psi_{\nu,n}$ of new instruments. Further explanations to input and output files are given in **Sections IV** and **VI**, respectively.

IV. Source file, module, input files, library

Source file of present code is **Co2nov25.for**. This file is written in **Microsoft Fortran for Power Station**. File **Co2nov25.for** and module **mo1co2nov.for** should be compiled using **Microsoft Developer Studio**. Note: statement "*warning LNK4084: total image size 940171264 exceeds max (268435456); image may not run"* generated during compilation of the code **should be ignored**. It doesn't prevent computations.

Atomic database of high-Z dopant consists of files **QSs.inp**, **AIw.inp**, **Exc.inp**, **Inz.inp**. In present packet of files, the dopant is Krypton. To change the dopant, **user should replace** the database of Kr by similar files **printed in the format of the above four files**. Database of dopant **must** include nuclei, H-like, He-like, Li-like ions and **can** include ions of spectroscopic symbols (SpSs) below Li-like; all SpSs must follow in one sequence (no missing SpSs). Computations showed that number of energy levels (ELs) in database of dopant must be smaller than 2300. Changes in a database require changes in parameters nXE, FSS, HSS, Nnu, NST, NSTm, HSSm, nFAI introduced and explained in module **mo1co2nov.for**.

File **QSs.inp** contains atomic data of ions and ELs. In present packet of files, this is atomic data of Kr in the last eight charge states (N-like to nucleus). For each spectroscopic symbol, lines 3-9 give the number of non-autoionizing ELs, the number of autoionizing (AI) ELs, ionization energies (computed with FAC **[1]**). Lines from 12 until the end of the file give info on each EL. In these lines:

first group of columns gives configuration of electron shells,
column "g0" gives the degeneracy of EL,
column "E(eV)" gives the energy of EL relative to the ground state,
column "#" gives serial number of EL among ELs of same-charge ions;
AI ELs are placed below nucleus; for convenience of programing they are given negative serial numbers,
column "##" gives serial number of EL in full list of ELs of Kr.

AIw.inp is the file of autoionization (AI) probabilities of Els. In present packet of files, these probabilities are computed for Kr (using FAC).

Column 1, 2 give spectroscopic symbol (SpS) and serial number of AI EL.
Columns 3, 4 give SpS and serial number of final EL (that after AI).
Column "WAI" gives the probability of AI in units [s^{-1}].
Column DE(eV) gives the difference [eV] between energies of initial EL and final ELs prior to the lowering of continuum.

File **Exc.inp** contains coefficients for expression that fits electron impact excitation cross sections of ELs. In present packet of files the cross sections are computed for ELs of Kr. FAC gives each cross section by points for 6-8 energies of projectile electron (these energies are chosen by FAC), we interpolate these points with smooth function that has six fitting coefficients, then tabulate these coefficients in columns 5-10. Column 1 gives SpS of an ion. Column 2 gives serial number of lower EL in the list of ELs of this SpS. Column 3 gives serial number of upper EL in this list. Column 4 gives the number of interpolation formula, see *function "SigExc(eeV)"* in file **Co2nov25.for**. In column 11 minus marks computation with FAC. Absolute value is absorption oscillator strength for this transition.

File **Inz.inp** contains coefficients for expression that fits ionization cross sections of each EL (ionization by electron impact and photoionization). In present packet of files the cross sections are computed for ELs of Kr. For each EL⟶ EL′ ionization FAC gives cross section for 6-8 energies of projectile electron or photon. The energies are chosen by FAC. We interpolate these points with smooth function that has four fitting coefficients (see *functions "SigInz"* and *"SigPhi"* in file **Co2nov25.for**), then tabulate the fitting coefficients in columns 5-8 for ionization by electron impact and columns 10-13 for photoionization. Columns 1,2 give info on EL to be ionized (namely, SpS of an ion and serial number of EL in the list of ELs of this SpS. Columns 3, 4 give similar info on resulting (post-ionization) EL. Last column gives the difference between energies [eV] of EL and EL′ prior to the lowering of continuum.

Files **QSsC.inp**, **QSsHe.inp**, **QSsH.inp** show atomic data of C, He, H. **User may replace** ^{2}H, ^{4}He, ^{12}C with any isotopes of CE up to neon. If any zone contains only one or two light chemical elements (CEs), then missing CEs should be given much-lower-than-others ion number density in this zone. Present database of dopant is composed for hot (x-emiting) plasma, therefore it includes only the last eight ionization stages. In such plasma, ions of light CEs are stripped to nuclei, therefore for light CEs we use two-state databases that consist of nuclei and ground state of H-like ion. The ground state is required by structure of the code but remains empty. No ionization of this state, no recombination on it.

Array of photon energies hν(i) consists of N_ν domains that differ by spectral resolution $d\nu/\nu$ that is relative interval between neibouring hν point, namely $[h\nu(i+1)-h\nu(i)]/h\nu(i)$. For new set of parameters **user should perform** test computation with N_ν =1, $d\nu/\nu < 10^{-4}$ for determination of domains with spectral lines. These domains require high spectral resolution (usually $d\nu/\nu \sim 10^{-5}$ to 10^{-4}) while for domains of continuum $d\nu/\nu \sim 10^{-3}$ to 10^{-2} is sufficient. N_ν and hard edge of each domain should be shown in file **Params1.inp** that adds comments and example.

Files **Params0.inp**, **Params1.inp**, **Params2.inp** contain parameters of simulation. **These files should be filled in by user**. Each parameter is given and explaned in same line. Sample scenario given in file **Params2.inp** of present packet isn't derived from experimental data.

Nine input files **InRespn.inp** give spectral response $\Psi_{\nu,n}$ of nine diagnostic instruments; here n = 1, 2, …, 9 is serial number of instrument in the code. These input files **should be loaded by user** (in arbitrary units). In present version of the code

n = 1 relates to spectrometer that shows spectrograms of specific power radiated by entire target (III.3) at eight times prescribed in file **Params2.inp**. Also, see comments to file **Frames.dat** in **Sections III, VI**.

n = 2, 3, 4 relate to three spectrometers that shows specific intensity of target radiation along three non-central LoSs at $t = t_{inf}$, see (III.4) and comments to files **SpIn1.dat**, **SpIn2.dat**, **SpIn3.dat** in **Sections III, VI**.

n = 5, 6 relate to two imaging cameras, see (III.5), (III.6) and comments to files **Image1.dat** and **Image2.dat** in **Sections III, VI**.

n = 7, 8, 9 relate to three power detectors, see (III.7) and comments to files **PowDet1.dat**, **PowDet2.dat**, **PowDet3.dat** in **Sections III, VI**.

If simulation of some experiment requires more detectors of any type, **users should** repeat the computation with no changes in any parameter but using spectral response of new instruments. If this response is given by less (or more) hν points, change corresponding number in line "*DATA npInpRes*" in file **mo1co2nov.for**.

File **Naglib.lib** is part of NAG library, namely, (i) subroutine **D02EAF** that integrates the system of ordinary differential equations by the 4th order Runge-Kutta method, (ii) definite integral **D01AH**.

Module **mo1co2nov.for** gives parameters of atomic database, parameters of computation, maximal size of arrays, explains the meaning of main parameters.

V. Info on display

Code runs along scenario by time steps Δt. On each Δt the code displays initial instant, t_i, of this Δt, parameters of each zone at $t = t_i$, namely, serial nunber of zone (1, 2, 3 for BS, core, capsule, respectively), outer radius of zone [cm], electron temperature [eV], mean absolute velocity of ions in their collective (i.e., non-thermal) motion [cm/s], electron density [cm^{-3}], mean charge of ions of dopant (here Kr).

VI. Output files

File **EffSpIns.dat** displays $J_{\nu,b}$ in each zone at $t = t_{inf}$. Time t_{inf} **should be prescribed** in file **Params2.inp**. Function $J_{\nu,b}$ is given by its definition (II.9) and is approximated by sums (AI.0); for details see **Appendix I**. Columns of **EffSpIns.dat** display as follows:

Column "hvKeV" displays the photon energy in [keV].

Columns "BS", "Core", "Capsule" display $J_{\nu,b}$ for b = 1, 2, 3 respectively. For correct integral over hν [keV], $J_{\nu,b}(t_{inf})$ is expressed in [kW/keV/sr/cm^2].

Columns 5-7 display typical opacity of zones, namely, $\kappa'_{\nu,1} R_1$, $\kappa'_{\nu,2} R_2$, $\kappa'_{\nu,3} (R_3 - R_2)$.

Files **"BSinfo.dat", "CoreInfo.dat", "CapInfo.dat"** display (i) parameters of plasma in each zone on each time step Δt, according to scenario prescribed in file **Params2.inp**; (ii) a few of computed parameters of plasmin this zone.

Column 1 displays time [ps] ,
column 2 displays outer radius of zone [μm],
column 3 displays T_e [keV] in zone,
column 4 displays u_{3D} [cm/s] **/10^7** in zone,
column 5 displays $n_{e,b}$ [cm^{-3}] in zone,
column 6 displays mean (over CEs) ion charge in zone,
columns 7-10 display mean ion charge of each CE in zone,
columns 11-14 display number density of ions [cm^{-3}] of each CE in zone,
column "bp" displays parameter ξ_b that is the part ($0 \le \xi_b \le 1$) of beam electrons in $n_{e,b}$, see (II.3),
columns "bcKeV" , "bwKeV" display central energy [keV] and width [keV] of electron beam in zone, see (II.3).

File **"LineList.dat"** is the list of spectral lines that satisfy three inequalities:

(i) line center, $h\nu_c$, is in the interval $[h\nu_{min}, h\nu_{max}]$,

(ii) absorption oscillator strength f_{lu} > "f_{lu}Min",

(iii) Einstein coefficient A_{ul} > "A_{ul}Min".

These lines are included in computations. Weaker lines and out-of-interval lines are ignored.

Parameters $h\nu_{min}$, $h\nu_{max}$, "f_{lu}Min", "A_{ul}Min" are given in file **Params1.inp** .

Column "ChE" displays chemical element (1 means dopant, here Kr),
Column "SpS" displays spectroscopic symbol of the ion,
3rd column displays the energy of photons, $h\nu_c$ [eV] in line center,
4th column displays the wavelength of the line, λ_c [A],
5th column displays Einstein coefficient A_{ul} [s^{-1}],
6th column displays serial number of upper level in file "**QSs.inp**",
7th column displays electron configuration of this level,
8th column displays serial number of lower level in file "**QSs.inp**",
9th column displays electron configuration of this level,
10th column displays the absorption oscillator strength, f_{lu}.

Files "**BSsLineInfo.dat**", "**CorLineInfo.dat**", "**CapLneInfo.dat**' display info on contribution of each broadening mechanism to the widths of spectral lines in BSs, core, capsule, respectively. We remind that computations include only lines that satisfy all three of the following:

(i) line center, $h\nu_c$, is in the interval $[h\nu_{min}, h\nu_{max}]$,

(ii) absorption oscillator strength f_{lu} > "f_{lu}Min",

(iii) Einstein coefficient A_{ul} > "A_{ul}Min".

Parameters $h\nu_{min}$, $h\nu_{max}$, "f_{lu}Min", "A_{ul}Min" are given in file **Params1.inp**.

Column "hvCeV" displays $h\nu_c$ [keV],
column "XE" displays chemical element that radiated the line (now it shows "1" that means "the dopant"),
column "SS" displays spectroscopic symbol of ion that radiated this line,
column "Lambda" displays the line-center wavelength (in Angstroms),

two columns “Upper” display serial number and electron configuration
of the upper level of this sppectral line,
two columns “Lower” display serial number and the electron configuration
of the lower level of this spectral line,
column “viaTi” displays contribution of thermal broadening to the
FWHM [eV] of Voigt shape of the line in k_ν', ε_ν; i.e.,
FWHM [eV] of thermal part of Gaussian part of Voigt FWHM;
column “via3D” displays contribution of hydro (collective) broadening to
FWHM [eV] of Voigt shape of the line in k_ν', ε_ν; i.e.,
FWHM [eV] of hydro part of Gaussian part of Voigt FWHM;
column “Baranger” displays the lower limit for Lorentzian part of
FWHM [eV] of Voigt shape of the line in k_ν', ε_ν;
this limit is caused by the uncertainty principle;
column “Lorentz” displays FWHM [eV] of Lorentzian caused by either the
uncertainty principle or Stark effect (bigger one of these two);
“Baranger” = “Lorentz” usually means that Stark width is
not computed for this spectral line.
column “Voigt” displays FWHM [eV] of Voigh shape of the line in k_ν', ε_ν;
column “PopUpper” displays population of the upper level: $n_{b,Up}/n_{A,b}$,
column “flu” displays the absorption oscillator strength f_{lu}.

File **AlongCeLOS.dat** displays specific intensity of radiation on three interfaces along central towards-detector LoS (that through $r = 0$).

Column 1 displays the photon energy [keV].

Column 2 displays specific intensity of radiation on the exit from capsule into core: $I_{\nu,3/2} = (\varepsilon_{\nu,3}/\kappa'_{\nu,3})[1 - \exp(-\tau_3)]$,
the units are [kW/keV/sr/cm^2], $\tau_3 = \kappa'_{\nu,3}\cdot(R_3 - R_2)$.

Column 3 displays specific intensity on exit from core into capsule:

$$I_{\nu,2/3} = I_{\nu,3/2}\cdot\exp(-\tau_2) + (\varepsilon_{\nu,2}/\kappa'_{\nu,2})[1 - \exp(-\tau_2)] + (\varepsilon_{\nu,1}/\kappa'_{\nu,1})[1 - \exp(-\tau_1)]\cdot W_{BS}.$$

Here $\tau_2 = 2R_2\cdot\kappa'_{\nu,2}$, $\tau_1 = (4/3)R_1\cdot\kappa'_{\nu,1}$, $4R_1/3$ is mean path through BS. W_{BS} is probability to cross a BS by central LoS, see explanation to (AI.4).

Column 4 displays the radiation intensity on exit from the target:

$$I_{\nu,out} = I_{\nu,/2/3} \cdot \exp(-\tau_3) + (\varepsilon_{\nu,3} / \kappa'_{\nu,3})[1 - \exp(-\tau_3)].$$

Columns 5,6 display plasma opacity along central LoS, namely, τ_2 and τ_3 . In file **"Frames.dat"** the 1st column displays the energy of photons [keV], columns 2-9 display specific energy [kJ/keV/sr] radiated by the target during each of eight frames, see (III.2). All frames are of equal duration δ. This duration and central time of each frame, "FrP(k)" **should be given** in file "**Params2.inp**". The 10th column displays the energy [kJ/keV/sr] radiated by the target during entire computation. End-of-computation time is denoted t^ and is given in file "**Params2.inp**". Time t^ must be > [FrP(k_{max}) + $\delta/2 + \Delta t$] where k_{max} is requied number of frames. Column 11 displays spectral response of spectrometer #1, namely $\Psi_{\nu,1}$. This function [arb. u.] is the product of crystal reflectivity, filter transparency, geometric factor, sensitivity of MCP. Columns 12-19 display signal [arb.u.] on spectrometer #1, namely functions (III.3) at eight times "FrP(k)". The last column displays integral type (III.3) in limits [t_0 , t^].

In file "**FrameGauLor.dat**" the first column displays photon energy [keV], the second column reproduces one of columns 12-19 of file "**Frames.dat**", i.e. displays signal (III.3) recorded by spectrometer #1 during one frame prescribed in file "**Params1.inp**". Columns 3 and 4 display results of convolution of column 2 with Gaussian and Lorentzian instrumental broadening broadenin ($d\nu/\nu$) **prescribed by user** in input file "**Params1.inp**" together with edges of spectral interval where instrumental broadening is known and approximated using quadratic fit

$$d\nu/\nu = A_{ins} + B_{ins}\,(h\nu[keV]) + C_{ins}\,(h\nu[keV])^2 .$$

Constants A_{ins} , B_{ins} , C_{ins} are **prescribed by user** in file "**Params1.inp**".

Files "**BSemiAbso.dat**", "**CoreEmiAbso.dat.dat**", "**CapEmiAbso.dat**" display plasma emissivity ε_ν [W/eV/cm^3/sr] and plasma absorption coefficient corrected for induced emission k_ν' [cm^{-1}] in BSs, core, capsule, respectively, all at $t = t_{inf}$.

Column "hvKeV" displays the photon energy [keV].

Columns 2 - 4 display three terms of plasma emissivity [W/eV/cm^3/sr]:
free-bound (i.e. due to the radiative recombination),
bound-bound (i.e. due to radiation in spectral lines),
free-free (i.e. due to radiation of free electrons);
Column 5 displays full ε_ν [W/eV/cm^3/sr] that is the sum of three abovementioned terms.
Columns 6 - 8 display three terms of k_ν' [cm^{-1}]:
bound-free (i.e. due to photoionization),
bound-bound (i.e. absorption in spectral lines),
free-free (due to absorption by free electrons),
Column 9 displays full k_ν' [cm^{-1}] that is sum of he tabove three terms.

Files "**PIR_BS.dat**", "**PIR_Core.dat**", "**PIR_Cap.dat**" display info on the lowering of continuum in BSs, core, capsule, respectively, all in the Ion Sphere approach,see **Appendix II**.

Columns 1-4 display time [ps], $T_{e,b}$ [keV], $n_{e,b}$ [cm^{-3}], $n_{A,b}$ [cm^{-3}], columns 5-11 relate to ions of Kr in N-like to H-like ionization stages. These columns display ratio of the **ground state** ionization energy in plasma to this parameter for in vacuum. For ions in vacuum, the ionization energy from the ground state is shown in the last column of lines 3-9 of input file "**QSs.inp**".

Three files "**SpInDi#.dat**" display $I_\nu(r_\#)$ that is specific intensity of plasma radiation on exit from the target at $t = t_{inf}$ along three LoSs that cross the target at distanse $r_\#$ from its center, see Section III for details. # = 1,2,3 is serial number of LoS. First column of each "**SpInDi#.dat**" displays photon energy [keV], second columns display $I_\nu(r_\#)$ in units [kW/keV/sr/cm^2] Column 3 displays spectral response of spectrometer $\Psi_{\nu,\ \#+1}$, column 4 displays spectrograms (III.4) in arbitrary units. Computation is similar to that described above in explanation to file "**AlongCeLOS.dat**".

Two files "**Images#.dat**" display images of radiating plasma on two imaging cameras at t = t_{inf} , see (III.6). Here # = 1,2 is serial number of camera. In each of these two files 1st column displays radial coordinate [μm], 2nd column display the images normalized to unity at $r = 0$.

Three files "**PowDet#.dat**" display signal of three power detectors (III.7). Here # = 1,2,3 is serial number of power detector. Columns display time [ps], power [TW] radiated by the target in full spectrum (independent of detector), signal of the detector [arb.u] with account of its spectral response $\Psi_{\nu,\,\#+6}$.

In files "**ZpopsBS.dat**", "**ZpopsCore.dat**", "**ZpopsCap.dat**" :
column 1 displays time [ps] along the scenario.
columns 2-4 display $T_{e,b}$ [keV], $n_{e,b}$ [cm^{-3}], $n_{Kr,b}$ [cm^{-3}] in BSs, core, capsule, respectively;
columns 5-12 display population of the last 8 ionization stages of Kr, namely N-like to nucleus. Sum over the stages = 1.

File "**Contacts.dat**" includes all explanations except notation of the columns. Each column displays probability [s^{-1}] of certain type of transition.

Column **A** displays probability of spontaneous radiative transition (emission in spectral line).
Column **WInd** displays probability of photoinduced radiative transition (photoinduced emission in spectral line).
Column **Wab** displays probability of photoinduced excitation.
Column **Wex** displays probability of excitation by electron impact.
Column **Wdx** displays probability of de-excitation by electron impact.
Column **Wi** displays probability of ionization by electron impact.
Column **Wphi** displays probability of photo-ionization.
Column **Waiz** displays probability of auto-ionization.
Column **WDC** displays probability of dielectronic capture.
Column **WTB** displays probability of three-body recombination.
Columns **WRR** and **WiRR** displays probabilities of spontaneous and photoinduced radiative recombination, respectively.

Appendix I. Computation of $J_{\nu,b}$

As we mentioned in **Section II**, mean specific intensity of radiation in zone b is defined by expression

$$J_{\nu,b}(t) = (4\pi V_b)^{-1} \int_{(Vb)} dV \int_{(4\pi)} I_\nu(t, \mathbf{r}, \mathbf{\Omega})\, \mathbf{d\Omega} \qquad (II.9)$$

where $I_\nu(t, \mathbf{r}, \mathbf{\Omega})$ is solution of RTE along a ray directed to point **r**. Vector **Ω** gives direction of the ray in spheric coordinate system built in point **r**. $\mathbf{d\Omega} = \sin(\theta)\, d\theta\, d\varphi$ is small solid angle around direction **Ω**. Functions $J_{\nu,b}$ and $I_{\nu,}(\mathbf{r}, \mathbf{\Omega})$ are expressed in [W/eV/sr/cm^2].

In the code, integral over V_b is approximated by sum of N_r spherical layers around center of zone *b*. In BS and in core thickness of a layer $\Delta r_b = R_b/N_r$, outer radius of jth layers is $r_{b,j} = j\,\Delta r_b$. j = 1, 2, …, N_r . In capsule, thickness of a layer $\Delta r_3 = (R_3 - R_2)/N_r$, outer radii of layers $r_{3,j} = R_2 + j\,\Delta r_3$. Volume of jth spherical layer of zone *b*

$$V_{b,j} = 4\pi(r^3_{b,j} - r^3_{b,j-1})/3 \ ,$$

in core and in BSs $r_{b,0} = 0$, in capsule $r_{b,0} = R_2$.

We treat volume $V_{b,j}$ as a sum of $N_{\Delta,j}$ identical volumes $\Delta V_{b,j} \approx (\Delta r_b)^3$, each almost cubical, except a bit curved by sphericity of layers, see **Fig. 1.** Due to central symmetry of each zone, integral over 4π sr in (II.9) is calculated for one $\Delta V_{b,j}$ of each j, then multiplied by $N_{\Delta,j}$. Let point $C_{b,j}$ be central point of $\Delta V_{b,j}$. Integral over full solid angle around $C_{b,j}$ is approximated with the sum of sectors **ΔΩ** of spherical coordinates that have polar axis towards center of the target. We divide polar angle θ into N_θ sectors of width $\Delta\theta = \pi/N_\theta$. Sector $\mathbf{\Delta\Omega} = \sin(\theta)\Delta\theta\Delta\varphi$ where φ is athimutal angle. Due to symmetry of each zone relative to the polar axis, integral over φ gives factor 2π, thus for each $\Delta V_{b,j}$

$$\int_{(4\pi)} I_\nu(\mathbf{r}, \mathbf{\Omega})\, d\mathbf{\Omega} \approx 2\pi \mathbf{\Sigma_i}\, I_\nu(C_{b,j}, \theta_i) \sin(\theta_i)\Delta\theta \ .$$

Here $\theta_i = \Delta\theta\ (i - 1/2)$ is middle polar angle of ith sector, i = 1, 2, …, N_θ is serial number of sector. Let us denote $I_\nu(C_{b,j}, \theta_i)$ with $I_{\nu,b,j,i}$, then $J_{\nu,b}$ may be presented as

$$J_{\nu,b} \approx [2\pi^2/(3N_\theta V_b)]\, \mathbf{\Sigma_j}\, (r^3_{b,j} - r^3_{b,j-1})\, \mathbf{\Sigma_i} \sin(\theta_i)\, I_{\nu,b,j,i} \ . \qquad (AI.0)$$

Specific intensity $I_{\nu,b,j,i}$ is solution of RTE along a ray to point $C_{b,\,j}$ under polar angle θ_i , see **Figs. 3 - 6**. For BS, i.e. for $C_{1,j}$ the solution is

$$I_{\nu,\mathbf{1},j,i} = (\varepsilon_{\nu,3}/\kappa'_{\nu,3})\left\{1 - \exp[-\kappa'_{\nu,3} L_{3,j}(\theta_i)]\right\} \exp[-\kappa'_{\nu,2} L_{2,j}(\theta_i) - \kappa'_{\nu,1} L_{1,j}(\theta_i)] +$$
$$(\varepsilon_{\nu,2}/\kappa'_{\nu,2})\left\{1 - \exp[-\kappa'_{\nu,2} L_{2,j}(\theta_i)]\right\}\exp[- \kappa'_{\nu,1} L_{1,j}(\theta_i)] +$$
$$W_{BS,1}\,(\varepsilon_{\nu,1}/\kappa'_{\nu,1})\left[1 - \exp(-\kappa'_{\nu,1} L_4)\right] \exp[-\kappa'_{\nu,2} L_2/2 - \kappa'_{\nu,1} L_{1,j}(\theta_i)] +$$
$$(\varepsilon_{\nu,1}/\kappa'_{\nu,1})\left\{1 - \exp[-\kappa'_{\nu,1} L_{1,j}(\theta_i)]\right\} \qquad \text{(AI.1)}$$

where $L_{3,j}(\theta_i)$ is the ray path in capsule, $L_{2,j}(\theta_i)$ is the ray path in core, $L_{1,j}(\theta_i)$ is the ray path in BS from its surface to point $C_{1,j}$, see **Fig. 3**. $W_{BS,1}$ is the probability that path $L_{2,j}(\theta_i)$ crosses one of other $(N_{BS}-1)$ BSs. Mean path of the ray in BS is $L_4 = 4R_1/3$ *that is mean chord in sphere of radius* R_1.

First line of (AI.1) is capsule radiation attenuated in core and in test BS on the ray path to $C_{\mathbf{1},j}$. Second line adds core radiation attenuated in test BS on the same path. Third line adds possible radiation of crossed BS attenuated in core on path $L_2/2$ (*that is the estimate of mean distance between surface of crossed BS and surface of test BS*) and in test BS on path $L_{1,j}(\theta_i)$. Fourth line adds radiation of test BS itself on the ray path $L_{1,j}(\theta_i)$.

On path $L_{2,j}(\theta_i)$ a ray has probability $W_{BS,1}$ to cross one of $(N_{BS}-1)$ BSs. It can be only a BS that has <u>center</u> closer than R_1 to the ray, i.e. entire BS is inside cylindrical volume

$$V_{ray} = L_{2,j}(\theta_i)\,\pi\,(2R_1)^2. \qquad \text{(AI.1a)}$$

All BSs are in core of volume $V_2 = 4\pi R_2^3/3$, then the probability to cross one of $(N_{BS} - 1)$ BSs is

$$W_{BS,1} = (N_{BS} - 1)\,V_{ray}/V_2. \qquad \text{(AI.2)}$$

Solution of RTE along a ray **to** $\mathbf{C_{2,j}}$ (*that is point* $C_{b,\,j}$ *in core*, **Fig. 4**) is

$$I_{\nu,2,j}(\theta_i) = (\varepsilon_{\nu,3}/\kappa'_{\nu,3})\left\{1 - \exp[-\kappa'_{\nu,3} L_{3,j}(\theta_i)]\right\} \exp[-\kappa'_{\nu,2} L_{2,j}(\theta_i) +$$
$$(\varepsilon_{\nu,2}/\kappa'_{\nu,2})\left\{1 - \exp[-\kappa'_{\nu,2} L_{2,j}(\theta_i)]\right\} +$$
$$W_{BS,2}\,(\varepsilon_{\nu,1}/\kappa'_{\nu,1})\left[1 - \exp(-\kappa'_{\nu,1} L_4)\right]\exp[-\kappa'_{\nu,2} L_2/2] \qquad \text{(AI.3)}$$

Here $L_{3,j}(\theta_i)$ is the ray path in capsule, $L_{2,j}(\theta_i)$ is the ray path in core (*from its surface to $C_{2,j}$*). $W_{BS,2}$ is the probability to cross a BSs on path $L_{2,j}(\theta_i)$. It can be only a BS that is inside cylindrical volume V_{ray} (AI.1a), then

$$W_{BS,2} = N_{BS} V_{ray} / V_2 \qquad \text{(AI.4)}$$

In (AI.3) first line gives capsule radiation attenuated in core on the ray path from core surface to $C_{2,j}$. Second line adds radiation of core. Third line adds radiation of possible crossed BS attenuated in core on path $L_2/2$ *that is an estimate of mean distance from surface of crossed BS to $C_{2,j}$.*

Solution of RTE along a **ray to $C_{3,j}$** (*that is point $C_{b,j}$ of capsule, see* **Figs. 5, 6**) depends on yes/no path through core, namely:

Rays with $C_{3,j} \sin(\theta_i) > R_2$ pass outside core, **Fig. 5**. In this case, solution of RTE is

$$I_{\nu,3,j}(\theta_i) = (\varepsilon_{\nu,3}/\kappa'_{\nu,3}) \left\{1 - \exp[-\kappa'_{\nu,3} L_{3,j}(\theta_i)]\right\} \qquad \text{(AI.5)}$$

where $L_{3,j}(\theta_i)$ is the ray path through capsule.

Rays with $C_{3,j} \sin(\theta_i) < R_2$ cross far-from-$C_{3,j}$ side of capsule, then core, then some part of near-$C_{3,j}$ capsule, **Fig. 6**. In this case, solution of RTE is

$$\begin{aligned} I_{\nu,3,j}(\theta_i) = \Big\{ & (\varepsilon_{\nu,3}/\kappa'_{\nu,3}) \left[1 - \exp(-\kappa'_{\nu,3} L_{5,j}(\theta_i))\right] \exp[-\kappa'_{\nu,2} L_{2,j}(\theta_i)] + \\ & (\varepsilon_{\nu,2}/\kappa'_{\nu,2}) \left[1 - \exp(-\kappa'_{\nu,2} L_{2,j}(\theta_i))\right] + \\ & W_{BS,3} (\varepsilon_{\nu,1}/\kappa'_{\nu,1}) \left[1 - \exp(-\kappa'_{\nu,1} L_4)\right] \exp(-\kappa'_{\nu,2} L_2/2) \Big\} \times \\ & \exp[-\kappa'_{\nu,3} L_{6,j}(\theta_i)] + \\ & (\varepsilon_{\nu,3}/\kappa'_{\nu,3}) \left[1 - \exp(-\kappa'_{\nu,3} L_{6,j}(\theta_i))\right]. \end{aligned} \qquad \text{(AI.6)}$$

$L_{5,j}(\theta_i)$ is the ray path through far-from-$C_{3,j}$ side of capsule. $L_{2,j}(\theta_i)$ is the ray path through core, see **Fig. 6**. $W_{BS,3}$ is the probability to cross a BS. $W_{BS,3}$ is given by (AI.4) derived for $W_{BS,2}$. $L_{6,j}(\theta_i)$ is the ray path from core to $C_{3,j}$. First line of (AI.6) gives radiation of far-from-$C_{3,j}$ side of capsule (attenuated in core). Second line adds radiation of core. Third line adds radiation of possible crossed BS attenuated in core on path $L_2/2$ *that is an estimate of mean distance from surface of crossed BS to near-$C_{3,j}$ surface of the core*. Forth line gives attenuation of terms 1–3 on the ray path $L_{6,j}(\theta_i)$ that is through capsule to $C_{3,j}$. Fifth line adds radiation of capsule on this path.

Appendix II. Ionization energy of ion X^{q+} in plasma of arbitrary chemical composition

Widely used simple models of continuum lowering in plasma, $\Delta I_{q,b}$, such as Debye-Hűckel, Ion Sphere, Stewart-Pyatt **[4, 6, 10]** ignore a few fundamental effects **[11]**. On the other hand, no closed-form expression that accounts these effects; therefore we have chosen the Ion Sphere model but modified it for plasmas of complex chemical composition.

In plasmas, each free electron moves in collective electric field of other free electrons and ions of different charge and mass. This rapidly-changing collective field complicates trajectories of free electrons. In neutral uniform plasma it results in uniform distribution of free electrons; then **the mean volume per free electron**

$$V_{e,b} \approx 1/n_{e,b}.$$

Due to neutrality of plasma, mean number of free electrons traversing the ion sphere of ion X^{q+} at any time is approximately q, thus the volume of ion sphere of X^{q+} in zone b

$$V_{q,b} \approx qV_{e,b} \approx q/n_{e,b}\,, \qquad \text{(AII.1)}$$

then radius of ion sphere

$$\alpha_{q,b} \approx [3V_{q,b}/4\pi]^{1/3} \approx \mathbf{[3}q\mathbf{/(4}\pi n_{e,b}\mathbf{)]}^{1/3}. \qquad \text{(AII.2)}$$

For example, radius of ion sphere of H-like ion Kr^{+35} is 3.3x bigger than radius of ion sphere of D^+.

If ion X^{q+} is **solitary**, then it can be treated as a point-like $X^{(q+1)+}$ with one electron bound in potential

$$U(r) = -\,e\,(q+1)/r\,. \qquad \text{(AII.3)}$$

Removal of this electron from r to infinity (i.e. ionization of X^{q+}) requires energy

$$I_{q,b} = eU(\infty) - eU(r) = 0 + e^2(q+1)/r\,. \qquad \text{(AII.4)}$$

In neutral plasma, q free electrons traversing an ion sphere, weaken potential (AII.3) so that on interface between two ion spheres bound electron does not belong to any of them, thus is free; its ionization from $r = \alpha_{q,b}$ does

not require energy. It means that in comparison to a solitary ion [*see* (AII.4) *at* $r = \alpha_{q.b}$] the ionization energy in plasma is lower by $e^2(q+1)/\alpha_{q.b}$; in other words, lowering of ionization energy in plasma (also known as lowering of continuum)

$$\mathbf{\Delta I_{q,b} \approx} e^2(q+1)/\alpha_{q,b} = \mathbf{2\ (}q\mathbf{+1)\ I_H\ \alpha_0 /\alpha_{q,b}}\ . \qquad \text{(AII.5)}$$

Here we used known relation $2I_H = e^2/\alpha_0$ where e is electron charge, α_0 is Bohr radius.

Derived $\Delta I_{q,b}$ depends on the ion charge and electron density but does not depend on chemical and ionization composition of plasma, therefore expression (AII.5) does not differ (*except factor* $^4/_3$) from result known for identical ions*; numeric factor ~1 that depends on a model, *see discussion in Ref.* **[6]**. Comparison of predicted $\Delta I_{q,b}$ to experimental data does not prove any of the models because their results differ by less than factor 2, *see Figs. 1, 2 in Ref.* **[12]**, while experimental results are average along a LoS through non-uniform plasma.

---

* See expressions (7.42), (7.43) in Ref. **[6]** and (10.12) in Ref. **[13]**.

References

[1] M. F. Gu, Flexible Atomic Code, *Can. J. Phys.* ***8****, 5, (2008)*.

[2] L. D. Landau and E. M. Lifshitz, Mechanics, *Elsevier, 2005.*

[3] G. B. Rybicki, Escape probability methods;
in W. Kalkofen, ed. Methods in Radiative transfer,
Cambridge Univ. Press, Cambridge, U.K. 1984, pp. 21 – 64.

[4] Ya. B. Zel'dovich, Yu. P. Raizer, Physics of Shock Waves
and High-Temperature Hydrodynamics Phenomena,
Academic Press, New York, 1967.

[5] D. Mihalas, B. Weibel-Mihalas, Foundations of Radiation Hydrodynamics, *Dover Publications, Inc, New York, 1999.*

[6] H.R. Griem, Principles of Plasma Spectroscopy, *Cambridge University Press, Cambridge, 1997.*

[7] V. I. Fisher, D. V. Fisher, and Y. Maron. Radiation transport and density effects in non-equilibrium plasmas. *High Energy Density Phys.* ***3****, 283 (2007).*

[8] E. Stambulchik and Y. Maron, Quasicontiguous frequency-fluctuation model for calculation of hydrogen and hydrogen-like Stark-broadened line shapes in plasmas. *Phys. Rev. E* ***87****, 053108 (2013).*

[9] E. Stambulchik, private communication.

[10] R. P. Drake, High-Energy-Density Physics, *Springer-Verlag, Berlin, Heidelberg, 2006.*

[11] G. Röpke et al., Ionization potential depression and Pauli blocking in degenerate plasmas at extreme densities. *Phys. R*ev. E **99**, 033201 (2019).

[12] B. J. B. Crowley, Continuum lowering – A new perspective. *High Energy Density Phys.* ***13****, 84 (2014).*

[13] S. Atzeni and J. Meyer-ter-Vehn, The Physics of Inertial Fusion, *Oxford University Press, 2009.*

[14] V. I. Fisher, I. R. Almiev, L. L. Weingarten, Y. Maron; Mean specific intensity of radiation in cylindrical and spherical plasmas. *Laser and Particle Beams* ***27****, 91 (2009).*