| Parameter | Description | Category | Amplitude | Steady state concentration | Frequency | Speed |
|---|---|---|---|---|---|---|
| $k_1$ | Rate constant of calcium leak from ER[1] | $J_{flux}$ | X | X | - | X |
| $k_2$ | Rate constant of calcium release through IP3R[1] | | X | X | - | X |
| $K_{Ca^{2+}}$ | Half-saturation constant for calcium activation of IP3R[1] | | X | X | X | - |
| $K_{IP_3}$ | Half-saturation constant for IP3 activation of IP3R[1] | | X | X | X | - |
| $v_{SERCA}$ | Maximum rate of calcium pumping into the ER[2] | $J_{SERCA}$ | X | X | - | X |
| $k_{SERCA}$ | Half-saturation constant for calcium pumping into the ER[2] | | X | X | - | X |
| $v_{leak}$ | Rate of calcium leak from medium[1] | $J_{media}$ | X | X | - | X |
| $k_{leak}$ | Rate constant of calcium flux to the medium[1] | | - | - | X | X |
| $V_{ER}/V$ | Volumetric fraction of ER[2] | $[Ca_{ER}^{2+}]$ | - | - | - | - |
| $k_6$ | Rate constant of IP3R inactivation[1,2] | $[IP_3R]$ | - | - | - | - |
| $K_i$ | Half-saturation constant for calcium inhibition of IP3R[1,2] | | X | X | - | - |
| $k_{deg}$ | Rate constant of IP3 degradation[1] | $[IP_3]$ | X | X | X | X |
| $\mu_{PLC}$ | Mean rate of IP3 generation | | - | - | - | X |
| $D_{IP_3}$ | Effective diffusivity of IP3[1,2,3] | Diffusion | - | - | X | X |
| $D_{Ca^{2+}}$ | Effective diffusivity of calcium[1,2,3] | | - | - | - | - |


1. Höfer, Thomas, Laurent Venance, and Christian Giaume. "Control and plasticity of intercellular calcium waves in astrocytes: a modeling approach." *The Journal of neuroscience* 22.12 (2002): 4850-4859.
2. Intercellular calcium waves mediated by diffusion of inositol trisphosphate: a two-dimensional model
3. Keener, James P., and James Sneyd. *Mathematical physiology*. Vol. 1. New York: Springer, 1998.

# Boundary and initial conditions

No-flux boundary conditions:

$$D_{IP_3}\nabla^2[IP_3] = 0 \; at \; x = \{0, L\}$$

$$D_{Ca^{2+}}\nabla^2[Ca^{2+}] = 0 \; at \; x = \{0, L\}$$

Steady-state initial conditions:

$at \; t = 0$:

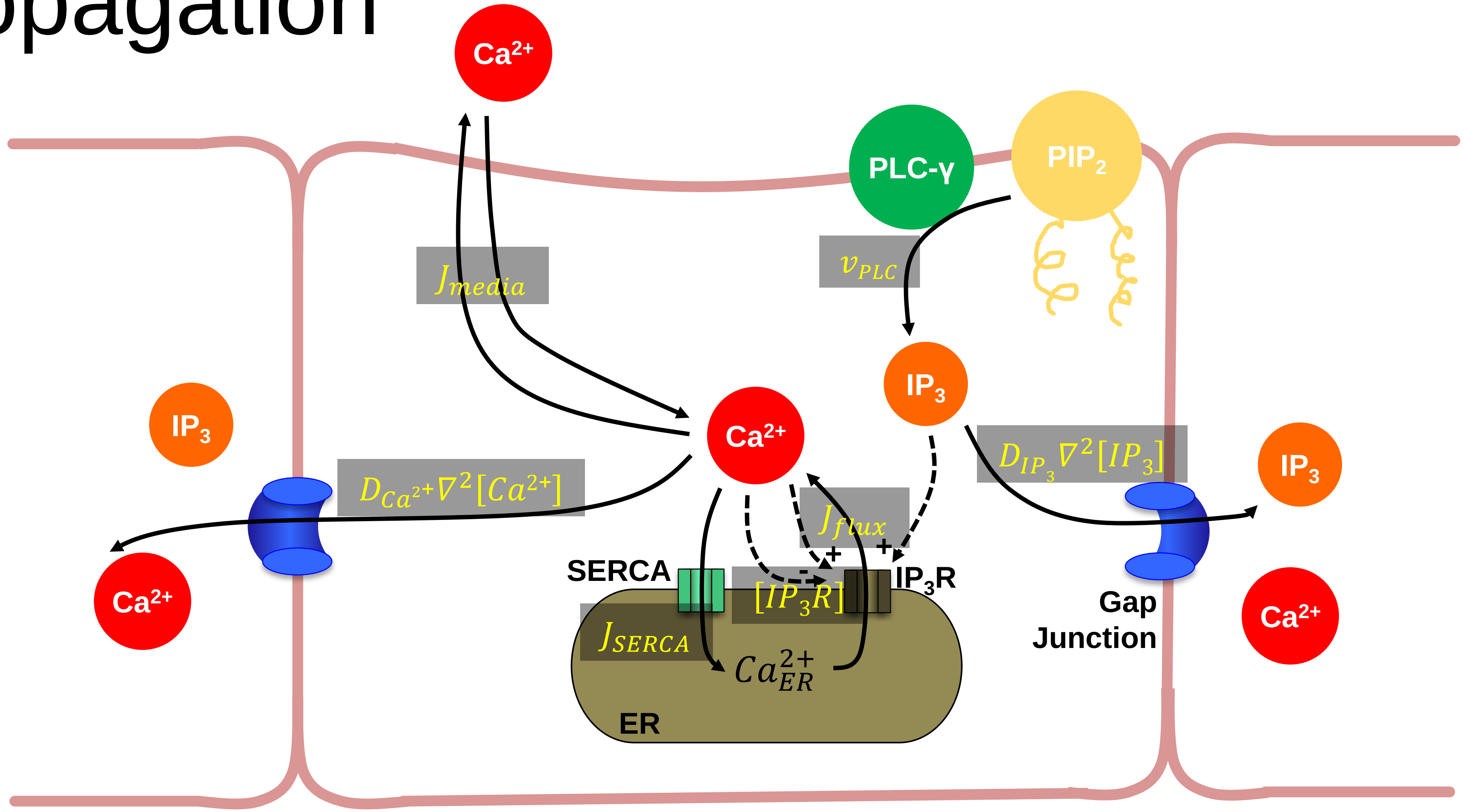
$$[Ca^{2+}] = 0.05 \; \mu M$$

$$[IP_3] = 0.15 \; \mu M$$

$$[Ca_{ER}^{2+}] = 80 \; \mu M$$

$$[IP_3R] = 1$$

# Molecular mechanism of calcium propagation



# PDE model (Version 1)

$$\frac{\partial[Ca^{2+}]}{\partial t} = \frac{(k_1 + k_2[IP_3R][Ca^{2+}]^2[IP_3]^2)([Ca^{2+}_{ER}] - [Ca^{2+}])}{(K^2_{Ca^{2+}} + [Ca^{2+}]^2)(K^2_{IP_3} + [IP_3]^2)} - \frac{v_{SERCA}[Ca^{2+}]^2}{k^2_{SERCA} + [Ca^{2+}]^2} + (v_{leak} - k_{leak}[Ca^{2+}]) + D_{Ca^{2+}}\nabla^2[Ca^{2+}]$$

$$\frac{d[Ca^{2+}_{ER}]}{dt} = -\frac{V}{V_{ER}}\left(\frac{(k_1 + k_2[IP_3R][Ca^{2+}]^2[IP_3]^2)([Ca^{2+}_{ER}] - [Ca^{2+}])}{(K^2_{Ca^{2+}} + [Ca^{2+}]^2)(K^2_{IP_3} + [IP_3]^2)} - \frac{v_{SERCA}[Ca^{2+}]^2}{k^2_{SERCA} + [Ca^{2+}]^2}\right)$$

$$\frac{\partial[IP_3]}{\partial t} = \Gamma(1, \mu_{PLC}) - k_{deg}[IP_3] + D_{IP_3}\nabla^2[IP_3]$$

$$\frac{d[IP_3R]}{dt} = k_6\left(\frac{K_i^2}{K_i^2 + [Ca^{2+}]^2} - [IP_3R]\right)$$

# PDE model (Version 2)

$$\frac{\partial[Ca^{2+}]}{\partial t} = J_{flux} - J_{SERCA} + J_{media} + D_{Ca^{2+}}\nabla^2[Ca^{2+}]$$

$$\frac{d[IP_3R]}{dt} = k_6\left(\frac{K_i^2}{K_i^2 + [Ca^{2+}]^2} - [IP_3R]\right)$$

$$\frac{d[Ca_{ER}^{2+}]}{dt} = -\frac{V}{V_{ER}}(J_{flux} - J_{SERCA})$$

$$\frac{\partial[IP_3]}{\partial t} = v_{PLC} - k_{deg}[IP_3] + D_{IP_3}\nabla^2[IP_3]$$

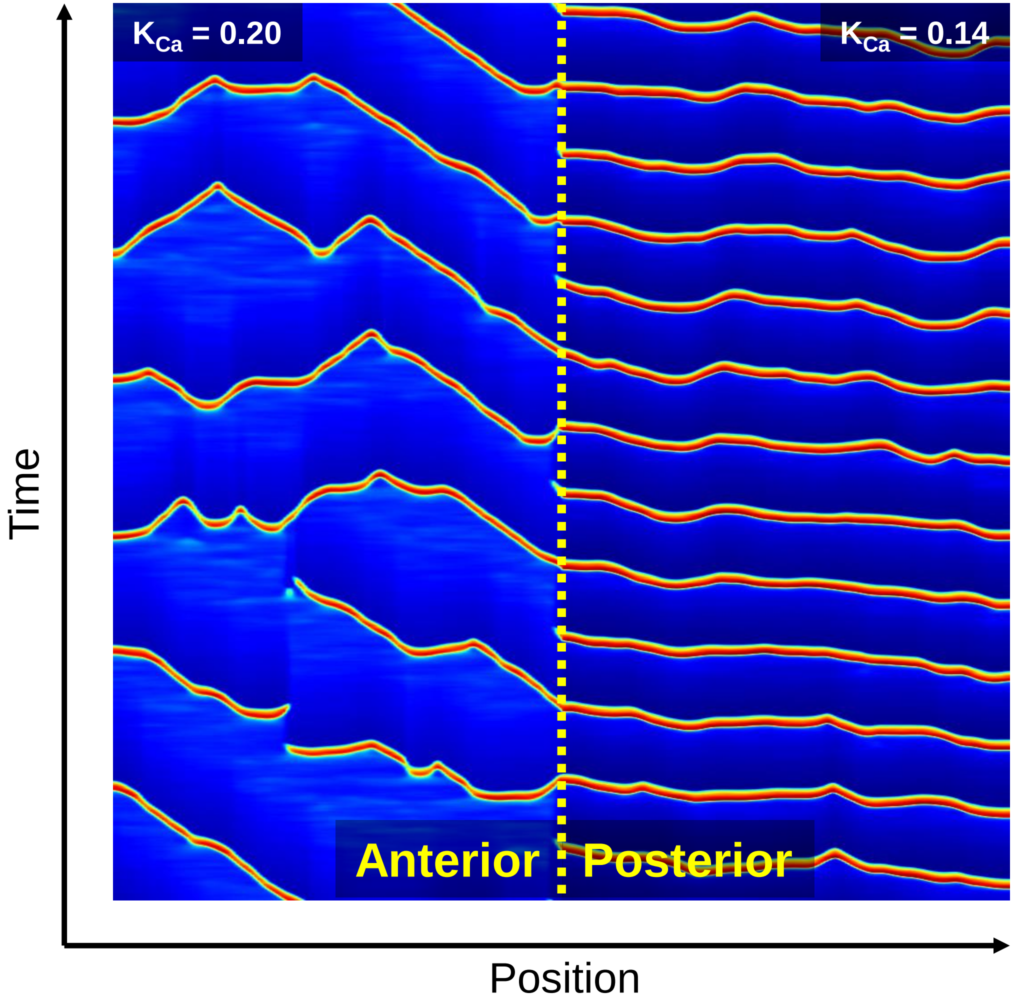
$$J_{flux} = \frac{(k_1 + k_2[IP_3R][Ca^{2+}]^2[IP_3]^2)([Ca_{ER}^{2+}] - [Ca^{2+}])}{(K_{Ca^{2+}}^2 + [Ca^{2+}]^2)(K_{IP_3}^2 + [IP_3]^2)}$$

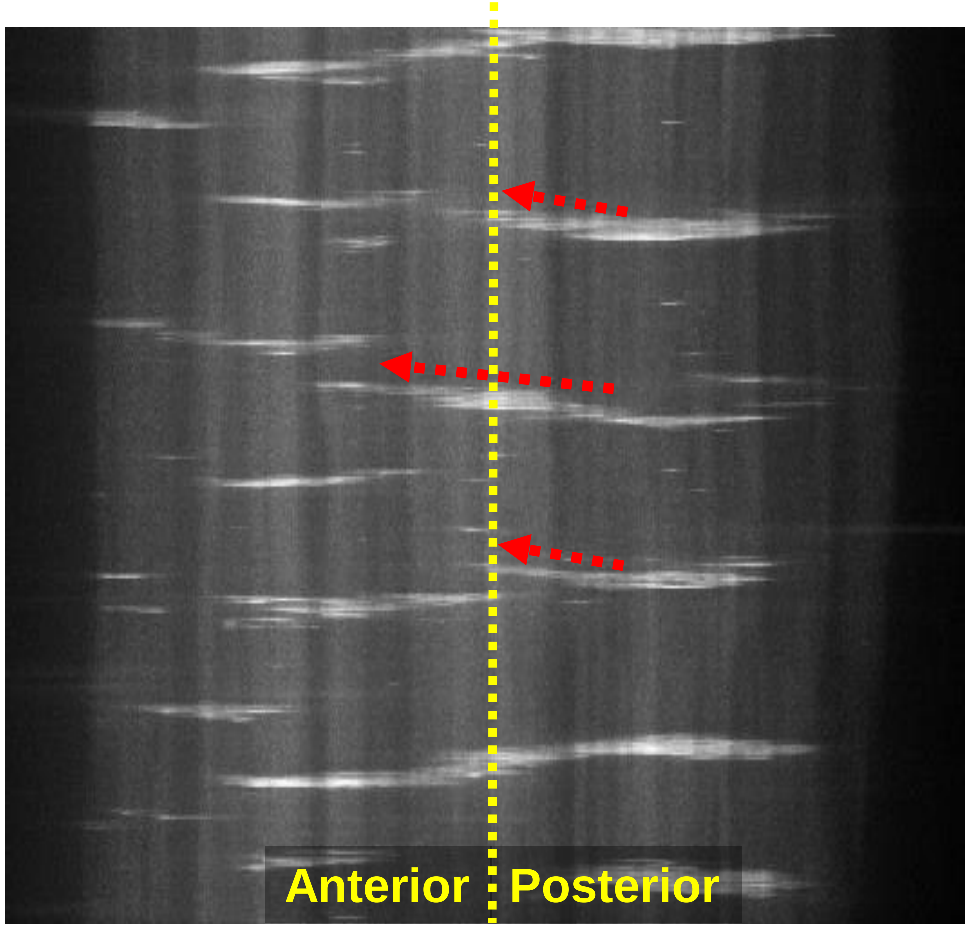
$$J_{SERCA} = \frac{v_{SERCA}[Ca^{2+}]^2}{k_{SERCA}^2 + [Ca^{2+}]^2}$$

$$J_{media} = v_{leak} - k_{leak}[Ca^{2+}]$$

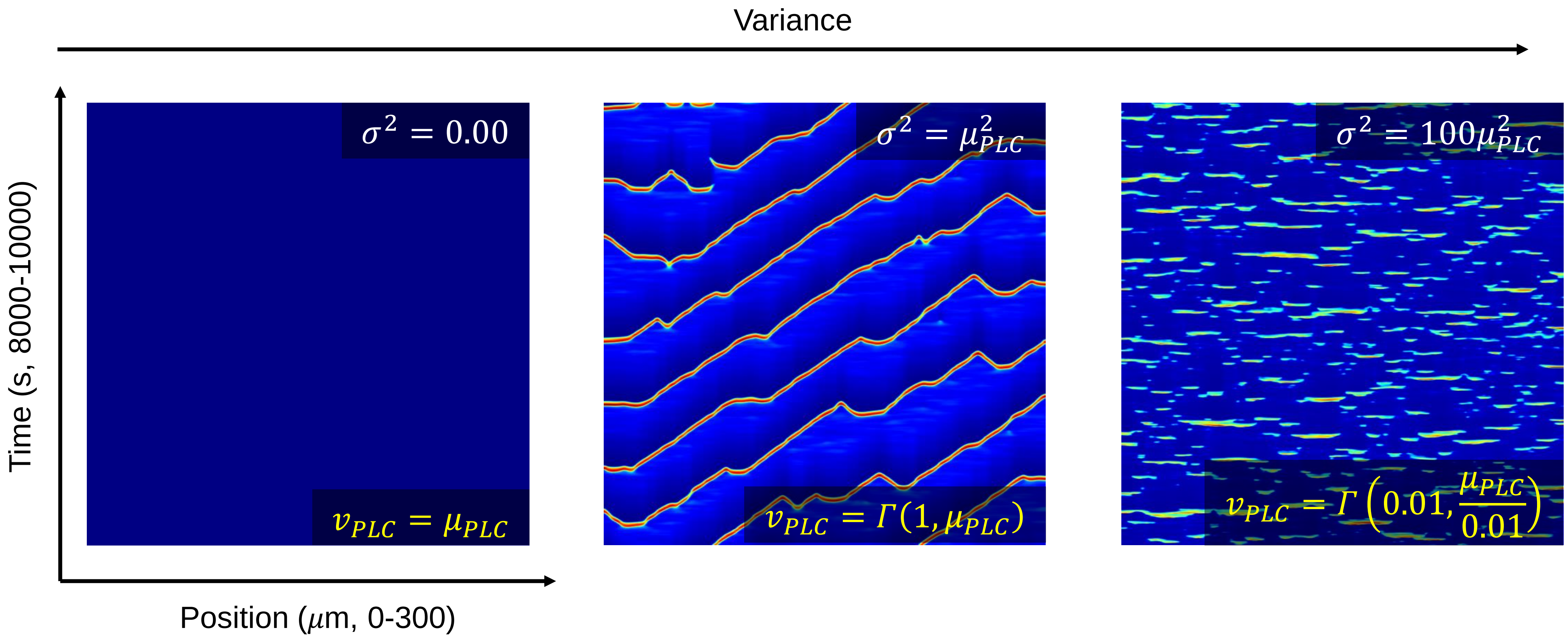
$$v_{PLC} = \Gamma(1, \mu_{PLC})$$

# Boundary penetrance may be explained by differential cell properties in compartments



# Signal variance may explain pulse to wave transition

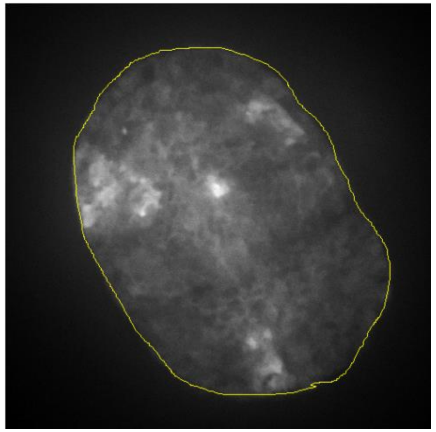



t = 8000s to 10000s

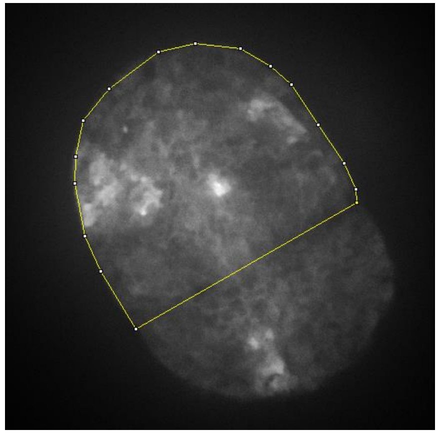
# OTHER SLIDES:

# Domain specificity of $Ca^{2+}$ transients

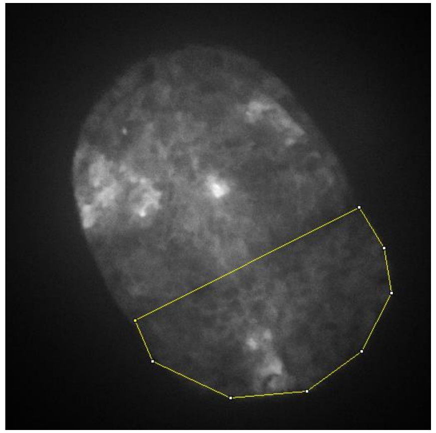
Whole disc

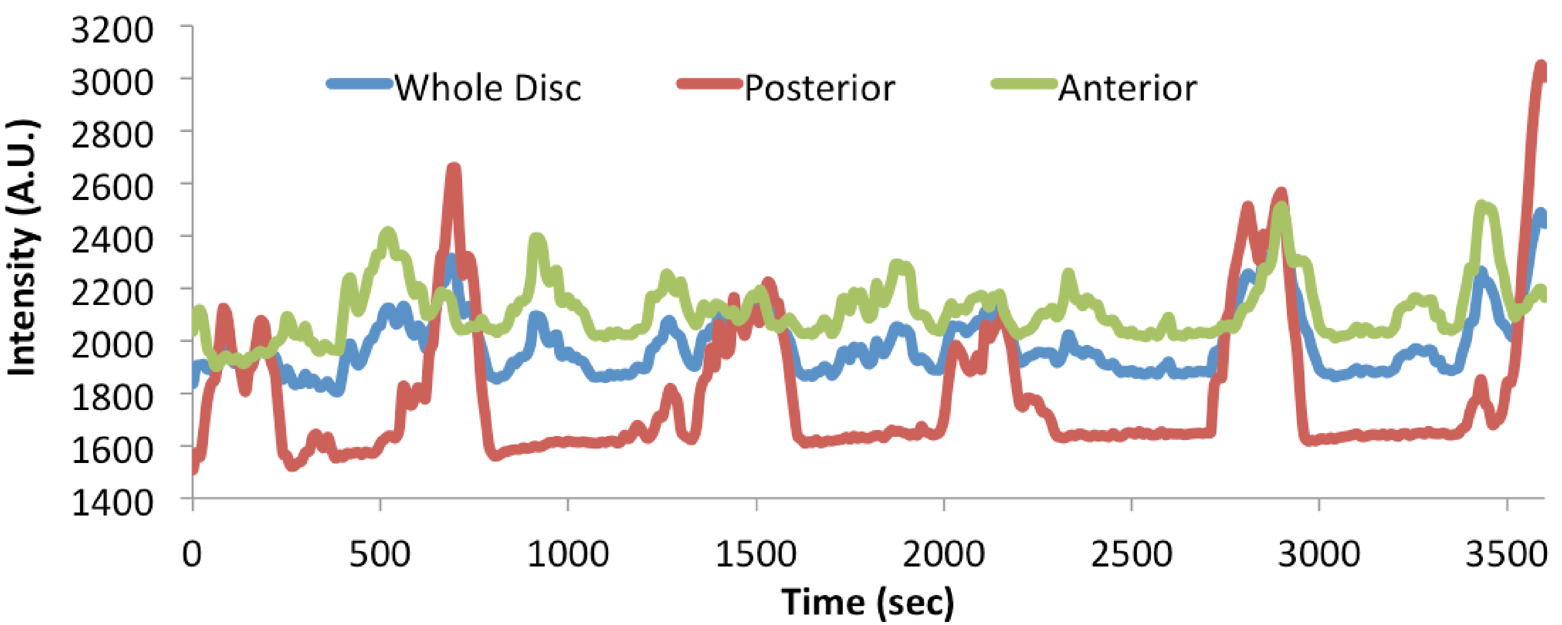


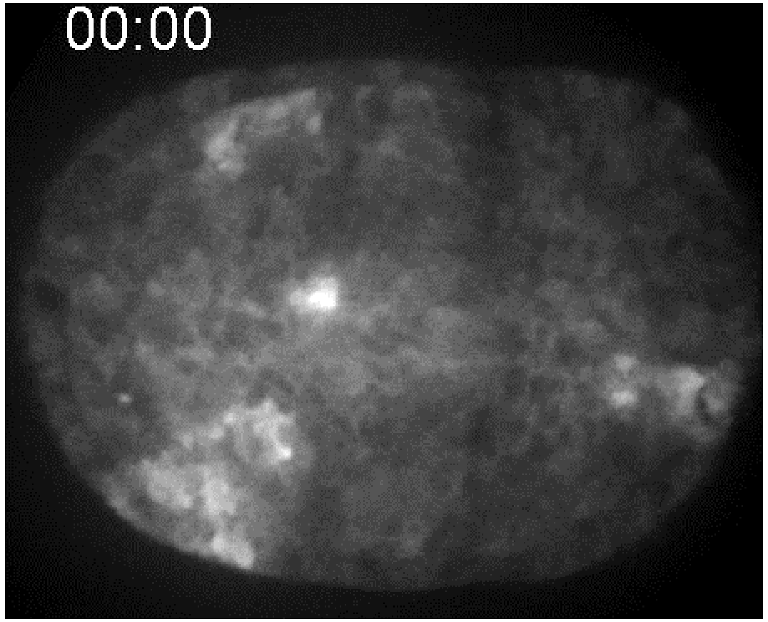
Anterior



Posterior






00:00

# Molecular mechanism of calcium propagation