# Processamento e Recuperação de Informação

## Classification

Departamento de Engenharia Informática
Instituto Superior Técnico

$1^{\text{o}}$ Semestre
2018/2019

# Outline

1. Introduction
2. Supervised Learning
3. Text Classifiers
   - Nearest Neighbor Classifiers
   - Generative Bayesian Classifiers
   - Linear Discriminative Classifiers
   - Neural Networks
4. Evaluation of Classifiers
5. Other Issues

# Bibliography

- Bing Liu, Web Data Mining - Exploring Hyperlinks, Contents, and Usage Data. Chapter 3.
- Ricardo Baeza-Yates and Berthier Ribeiro-Neto, Modern Information Retrieval. Chapter 8.
- Christopher D. Manning, Prabhakar Raghavan and Hinrich Schütze, Introduction to Information Retrieval. Chapters 13, 14 and 15.
- Jure Leskovec, Anand Rajaraman, and Jeff Ullman, Mining of Massive Datasets, Chapter 12

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5. Other Issues

# Organizing Knowledge

- Organize into systematic knowledge structures
- Ontologies
  - Dewey Decimal System
  - ACM Computing Classification System
  - Patent Subject Classification
  - International Classification of Diseases
- Web catalogs
  - Yahoo Directory (RIP 2002–2014)
  - DMOZ Directory (RIP 1998–2017)
  - World Wide Web Virtual Library
  - Jasmine Directory

# Organizing Knowledge

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Problem: Manual maintenance

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# Supervised Learning

Given a set of training data as input, use learning algorithm $A$ to discover the function $\hat{h}$ that minimizes the loss (e.g. the error over the set of training instances)

Input: $\{(x_i, y_i)\}_{i=1}^{N}, \; x_i \in \mathcal{R}^M, y_i \in \mathcal{R}$

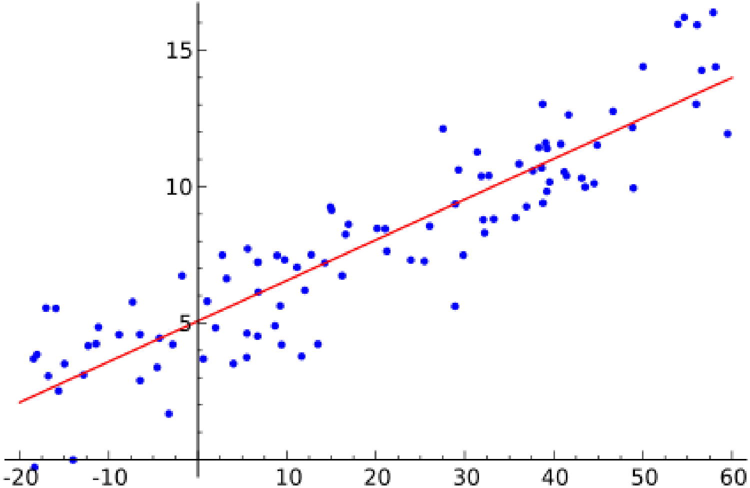
Hypothesis space: $h^* \in H$

Loss function: $L(h(x), y)$

Learning Algorithm: $\hat{h} = A(\{(x_i, y_i)\}_{i=1}^{N})$, such that

$$\hat{h} = \text{argmin}_h \sum_{i=1}^{N} L(h(x_i), y_i)$$

# An Example: Linear Regression



(source: wikipedia)

# Linear Regression (cont.)

- The hypothesis space:

$$h_{\vec{w}}(x) = w_0 + w_1 x$$

  where $\vec{w} = [w_0, w_1]$

- The loss function:

$$L(h_{\vec{w}}, y) = \frac{1}{N} \sum_{i=1}^{N} (y_i - h_{\vec{w}}(x_i))^2$$

  i.e. the sum of the squared error

- We want to find

$$w^* = \operatorname*{argmin}_{w} L(h_{\vec{w}}, y)$$

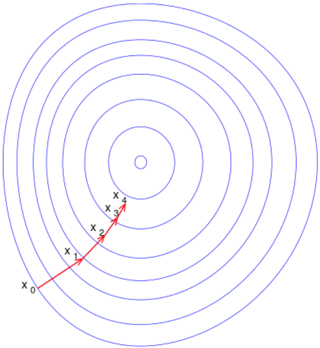
# Minimizing the Loss

- In simple cases, we can easily find one (or more) solution(s) to the problem of learning $\hat{h}$
  - For linear regression, take the derivatives and equal to 0
- In many cases this is not possible (or we may want to enforce some constraints on the parameters)
- In practice, there are many ways to estimate $w^*$

# An Example: Gradient Descent

```
$w \leftarrow$ any point in the parameter space
loop until convergence do
    for each $w_i$ in $\vec{w}$ do
        $w_i \leftarrow w_i - \alpha \frac{\partial}{\partial w_i} L(h_{\vec{w}}, y)$
```

$\alpha$ = learning rate



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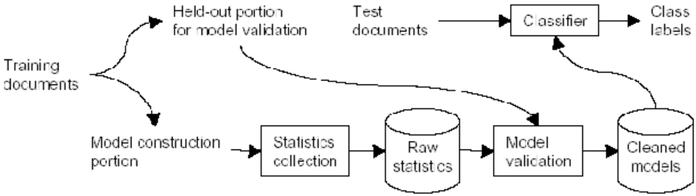
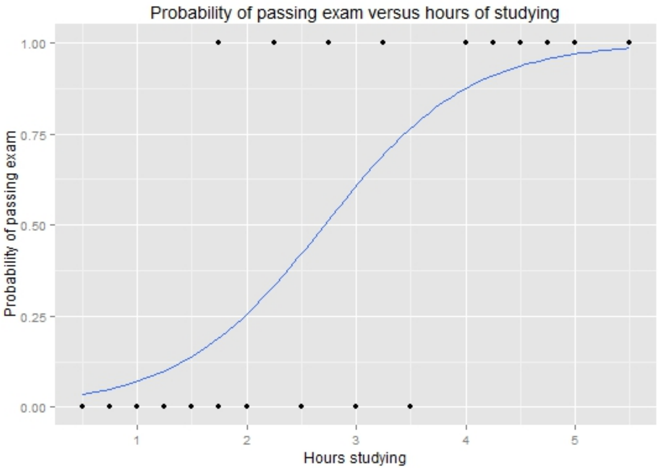
# Classification

- Learning to assign objects to classes given examples
- Learn a classifier (i.e., map the problem into supervised learning task)



# An Example: Logistic Regression



(source: wikipedia)

# Logistic Regression (cont.)

- The hypothesis space:

$$h_{\vec{w}}(x) = \frac{1}{1 + e^{-(w_0 + w_1 x)}}$$

where $\vec{w} = [w_0, w_1]$

- The loss function:

$$L(h_{\vec{w}}, y) = \frac{1}{N} \sum_{i=1}^{N} C(h_{\vec{w}}(x_i), y)$$

where

$$C(h_{\vec{w}}(x), y) = \begin{cases} -\log(h_{\vec{w}}(x)) & \text{if } y = 1 \\ -\log(1 - h_{\vec{w}}(x)) & \text{if } y = 0 \end{cases}$$

- We want to find

$$w^* = \operatorname*{argmin}_{w} L(h_{\vec{w}}, y)$$

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# Text Classification vs. Data Mining

Leverage supervised learning together with method for representing textual information (e.g., VSM with TF-IDF)

- Lots of features and a lot of noise
- No fixed number of columns
- No categorical attribute values
- Data scarcity
- Larger number of class labels
- Hierarchical relationships between classes less systematic

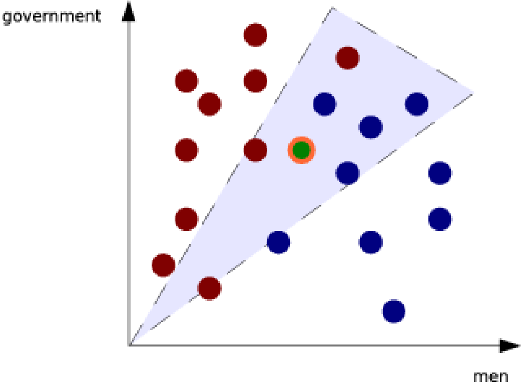
# Text Classifiers

- Nearest Neighbor Classifiers
  - Classify documents according to the class distribution of their neighbors
- Generative Bayesian classifiers (e.g., naïve Bayes)
  - Discover the class distribution most likely to have generated a test document
- Linear discriminative classifiers (e.g., the perceptron, logistic regression, or support vector machines):
  - Discover an hyperplane that separates classes
- Neural networks
  - Discover a non-linear function, often resulting from a composition of many functions, that separates classes

# Nearest Neighbor Classifiers

- Intuition: similar documents are expected to be assigned the same class label
  - Similarity: vector space model + cosine similarity
- Training:
  - Index each document and remember class label
- Testing:
  - Fetch $k$ most similar documents to the given document
  - Majority class wins
  - Alternatives:
    - Weighted counts: counts of classes weighted by the corresponding similarity measure
    - Per-class offset: tuned by testing the classifier on a portion of training data held out for this purpose

# *k*NN Classifier



$$score(c, d_q) = b_c + \sum_{d \in kNN(d_q)} sim(d_q, d)$$

# Properties of *k*NN

- Advantages:
  - Reuse of standard vector space model and availability of associated technology (e.g., inverted indexes)
  - Collection updates are trivial
  - Accuracy comparable to best known classifiers
- Problems:
  - Classification efficiency
    - many lookups over the document collection/index
    - sorting by overall similarity
    - picking the best $k$ documents
  - Space overhead and redundancy
    - Data stored at level of individual documents
    - Poor generalization
  - Choosing a value for $k$

# Improvements for *k*NN

- To reduce space requirements and speed up classification
  - Find clusters in the data and start by comparing instances against clusters (clustering covered in the next lecture)
  - Store only a few statistical parameters per cluster
  - In second step, compare with documents in only the most promising clusters
- However...
  - Ad-hoc choices for number and size of clusters and parameters
  - Number of clusters depends on the data

# Bayesian Classifiers

- Probabilistic document classifier
- Assumptions:
  1. A document can belong to exactly one class
  2. Each class $c$ has an associated prior probability $P(c)$
  3. There is a class-conditional document distribution $P(d|c)$ for each class (i.e., the likelihood)
- Given a document $d$, the probability of it being generated by class $c$ is:

$$P(c|d) = \frac{P(d|c)P(c)}{\sum_{\gamma} P(d|\gamma)P(\gamma)}$$

- The class with the highest probability is assigned to $d_q$ (i.e., we use a *maximum a-posteriory* rule)

# Learning the Document Distribution

- $P(d|c)$ is estimated based on parameters $\Theta$
- $\Theta$ are estimated based on two factors:
  1. Prior knowledge before seeing any documents
  2. Terms in the training documents
- Bayes Optimal Classifier

$$P(c|d) = \int_{\Theta} \frac{P(d|c,\Theta)P(c|\Theta)}{\sum_{\gamma} P(d|\gamma,\Theta)P(\gamma|\Theta)} P(\Theta|D)$$

  - This can be hard to compute
- Maximum Likelihood Estimate: $P(d|c,\hat{\Theta})$

$$\hat{\Theta} = argmax_{\Theta} P(d|c,\Theta)$$

# Naïve Bayes Classifier

- Naïve assumption
  - assumption of independence between terms
  - joint term distribution is the product of the marginals
- Widely used owing to
  - simplicity and speed of training, applying, and updating
- Two kinds of widely used marginals for text
  - Binary model (Bernoulli)
  - Multinomial model

# Naïve Bayes Models

Binary Model: Each parameter $\theta_{c,t}$ indicates the probability that a document in class $c$ will mention term $t$ at least once

$$P(d|c,\Theta) = \Pi_{t \in d} \theta_{c,t} \Pi_{t \notin d} (1 - \theta_{c,t})$$

$$\theta_{c,t} = \frac{N_{c,t}}{N_c}$$

$N_{c,t}$ = n. of docs in class $c$ containing term $t$
$N_c$ = n. of docs in class $c$

# Naïve Bayes Models (cont.)

## Multinomial Model:

- each class has an associated die with $|W|$ faces
- each parameter $\theta_{c,t}$ denotes probability of the face turning up on tossing the die, i.e. $\sum_{d \in c} n(d,t) / \sum_{d \in c} \ell_d$
- term $t$ occurs $n(d,t)$ times in document $d$
- document length is a random variable denoted $L$

$$\begin{aligned} P(d|c,\Theta) &= P(L=\ell_d|c)P(d|\ell_d,c) \\ &= P(L=\ell_d|c)\frac{\ell_d!}{\prod_{t \in d} n(d,t)!}\prod_{t \in d}\theta_{c,t}^{n(d,t)} \\ &\sim P(L=\ell_d|c)\prod_{t \in d}\theta_{c,t}^{n(d,t)} \end{aligned}$$

# Parameter Smoothing

- What if a test document $d_q$ contains a term $t$ that never occurred in any training document in class $c$?

# Parameter Smoothing

- What if a test document $d_q$ contains a term $t$ that never occurred in any training document in class $c$?
    - $P(c|d_q) = 0$
    - Even if many other terms clearly hint at a high likelihood of class $c$ generating the document
- Thus, MLE cannot be used directly
- We can use Laplace smoothing
    - Simply adds 1 to each count

$$\theta_{c,t} = \frac{\sum_{d \in c} n(d,t) + 1}{\sum_{d \in c} \ell_d + |W|}$$

# Performance Analysis

- Multinomial naïve Bayes classifier generally outperforms the binary variant
- *k*NN may outperform Naïve Bayes
- Naïve Bayes is faster and more compact
- Determines decision boundaries
  - Regions of the term-space where different classes have similar probabilities
  - Documents in these regions are hard to classify
  - Strongly biased

# Discriminative Classification

- Naïve Bayes classifiers are generative
- Differently, discriminative classifiers:
  - Directly map the feature space to class labels
  - Class labels are encoded as numbers
    - e.g: +1 and -1 for two a class problem
- For instance, we can try to find a vector $\alpha$ such that the sign of $\alpha \cdot d + b$ directly predicts the class of a document $d$
- Possible solutions:
  - Linear least-square regression
  - The Perceptron
  - Logistic Regression
  - Support Vector Machines

# What is a Linear Discriminative Classifier?

- Essentially:
  - Classification decision is based on the value of a linear combination of the features
  - Can be seen as the splitting of a high-dimensional input space with a hyperplane

$$y(d_1, \ldots, d_n) = f(\alpha_1 d_1 + \alpha_2 d_2 + \ldots + \alpha_n d_n)$$

- $\alpha_i$ are parameters (i.e., the weight of each feature $d_i$)
- $f$ is the activation function (e.g., $f(d) = 1_{x \geq 0}(d)$)
- The result of $y(d_1, \ldots, d_n)$ corresponds to the estimated class

# The Bias Term

- Notice that, according to the previous definition, the decision hyperplane must go through the origin
- Could be achieved by preprocessing the input, but this is not always desirable or possible
- Solution : Add a bias input:

$$y(d_1, \ldots, d_n) = f(b + \alpha_1 d_1 + \ldots + \alpha_n d_n)$$

- Same as an input connected to the constant 1
- We consider this *ghost* input implicit henceforth

# Training : The Perceptron Algorithm

- Switching to vector notation:

$$y(\mathbf{d}) = f(\alpha\mathbf{d}) = f_\alpha(d) \tag{1}$$

- Assume we need to separate sets of points $A$ (i.e., the positive examples) and $B$ (i.e., the negative examples)

$$E(\alpha) = \sum_{\mathbf{d}\in A}(1 - f_\alpha(\mathbf{d})) + \sum_{\mathbf{d}\in B} f_\alpha(\mathbf{d}) \tag{2}$$

- Goal: $E(\alpha) = 0$
- Start from a random $\alpha$ and improve it iteratively

# Algorithm Pseudo-Code

1. Start with random $\alpha$, set $t = 0$
2. Select a vector $\mathbf{d} \in A \cup B$
3. If $\mathbf{d} \in A$ and $\alpha\mathbf{d} \leq 0$, then $\alpha_{t+1} = \alpha_t + \mathbf{d}$
4. Else if $\mathbf{d} \in B$ and $\alpha\mathbf{d} \geq 0$, then $\alpha_{t+1} = \alpha_t - \mathbf{d}$
5. Conditionally go to step 2

- Guaranteed to converge iff $A$ and $B$ are linearly separable!

# Problems of Simple Perceptrons (1)

## Overfitting

- The standard Perceptron returns the most recent version of the weight vector
- Intuitively, this version is over-adapted to the last few instances, and may work less well for other instances

- The Averaged Perceptron returns the average of all versions (or the last few versions) of the weight vector
- An implementation trick involves setting a learning step that takes the averaging effect into account

# Problems of Simple Perceptrons (2)

## Multi-class classification

- Several problems involve multi-class classification
- Multi-class classification can be made through one weight vector for each category, assigning instances to the class for which the model predicts a higher value

- In practice, we can represent this with one giant weight vector and repeated features for each category
- Update rule involves changing the weights for the true class and the class that was predicted
- Other options for update rule can be considered, e.g. updating classes with higher score than correct one

# Summary of Simple/Averaged Perceptrons

- Simple and reasonably efficient online training
- Easy to extend in order to consider multi-class classification (and also structured prediction)
- Works well for document classification, and more generally for problems with many features
- Limited capabilities (e.g., does not try to optimize the separation "distance" between classes)
  - Just looks for a hyperplane that separates the two sets
  - Methods such as Support Vector Machines, on the other hand, try to maximize the distance between two closest opposite sample points (i.e., the margin of the separating hyperplane between the classes)

# Linear Discriminative Classifiers and SVMs

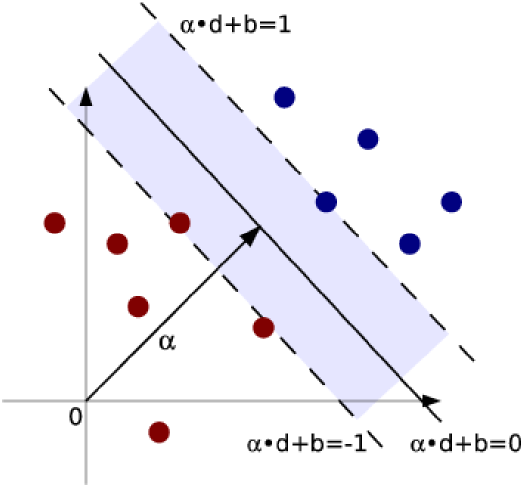
- Hypothesis:
  - The classes can be separated by an hyperplane
  - The hyperplane that is close to many training data points has a greater chance of misclassifying test instances
  - An hyperplane that passes through a "no-man's land", has lower chances of misclassifications
- Make a decision by thresholding
  - Seek an hyperplane that maximizes the distance to any training point
  - Choose the class on the same side of the hyperplane as the test document (i.e., same as in the Perceptron)

# Discovering the Hyperplane

- Assume the training documents are separable by an hyperplane perpendicular to a vector $\alpha$
- Seek a vector $\alpha$ which maximizes the distance of any training point to the hyperplane
- This corresponds to solving the following quadratic programming problem:

$$\begin{array}{rl} \text{Minimize} & \frac{1}{2}\alpha \cdot \alpha \\ \text{subject to} & c_i(\alpha \cdot d_i + b) \geq 1, \forall i = 1, \ldots, n \end{array}$$

# SVM Classifier



# Non Separable Classes

- Classes in the training data not always separable
- We introduce slack variables

$$\begin{array}{rl} \text{Minimize} & \frac{1}{2}\alpha \cdot \alpha + C\sum_i \xi_i \\ \text{subject to} & c_i(\alpha \cdot d_i + b) \geq 1 - \xi_i, \forall i = 1, \ldots, n \\ \text{and} & \xi_i \geq 0, \forall i = 1, \ldots, n \end{array}$$

- Implementations often solve the equivalent dual problem

$$\begin{array}{rl} \text{Maximize} & \sum_i \lambda_i - \frac{1}{2}\sum_{i,j} \lambda_i \lambda_j c_i c_j (d_i \cdot d_j) \\ \text{subject to} & \sum_i c_i \lambda_i = 0 \\ \text{and} & 0 \leq \lambda_i \leq C, \forall i = 1, \ldots, n \end{array}$$

# Analysis of SVMs

- Complexity:
    - Quadratic optimization problem
    - Requires on-demand computation of inner-products
    - Recent SVM packages work in linear time
- Performance:
    - Amongst most accurate classifier for text
    - Better accuracy than Naïve Bayes and most classifiers
    - Linear SVMs suffice
        - Standard text classification tasks have classes almost separable using a hyperplane in feature space
    - Non-linear SVMs can be achieved through kernel functions

# Logistic Regression as a Linear Classifier

Recall that for logistic regression, we have that:

$$P(y=1|x) = \frac{1}{1+e^{-(w_0+\sum_i w_i x_i)}}$$

$$P(y=0|x) = 1 - P(y=1|x) = \frac{e^{-(w_0+\sum_i w_i x_i)}}{1+e^{-(w_0+\sum_i w_i x_i)}}$$

We predict $+1$ if $P(y=1|x) > P(y=0|x)$, equivalent to:

$\frac{P(y=1|x)}{P(y=0|x)} > 1$ or, taking logs on both sides, $\log\left(\frac{P(y=1|x)}{P(y=0|x)}\right) > 0$

Simplifying the expression:

$$\log(1+e^{-(w_0+\sum_i w_i x_i)}) - \log(1+e^{-(w_0+\sum_i w_i x_i)}) - \log(1) + \log(e^{-(w_0+\sum_i w_i x_i)}) > 0$$

Thus, the decision boundary is given by the plane $w_0 + \sum_i w_i \dot{x}_i$ (similarly to the Perceptron or SVM classifiers).

# Neural Networks

- Multi-Layered Perceptrons leveraging VSM representations
- Other approaches leveraging sequential information and different representations (e.g., word embeddings)
  - Convolutional neural networks
  - Recurrent neural networks
- To be addressed latter in the course

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# Quality Measures

Two cases:

1. Each document is associated with exactly one class from a given set of possibilities (in either binary or multi-class scenarios), or
2. Each document is associated with a subset of classes (also refered to as multi-label classification)

# Single-Class Scenarios (Binary or Multi-Class Classification)

- For the first case, we can use a confusion matrix $M$
    - $M[i,j]$ is the number of test documents belonging to class $i$ which were assigned to class $j$
    - Perfect classifier: diagonal elements $M[i,i]$ would be nonzero
    - Example:

$$M = \left\{ \begin{array}{c|c|c} 5 & 0 & 0 \\ \hline 1 & 3 & 0 \\ \hline 1 & 2 & 4 \end{array} \right\}$$

- If $M$ is large, we use

$$\text{accuracy} = \sum_i M[i,i] / \sum_{i,j} M[i,j]$$

# Micro-Averaged Precision (Multi-Class Classification)

In a problem with $n$ classes, let $C_i$ be the number of documents in class $i$ and let $C_i'$ be the number of documents estimated to be of class $i$ by the classifier

- Micro-averaged precision is defined as

$$\frac{\sum_{i=1}^{n} C_i' \cap C_i}{\sum_{i=1}^{n} C_i'}$$

- Micro-averaged recall is defined as

$$\frac{\sum_{i=1}^{n} C_i' \cap C_i}{\sum_{i=1}^{n} C_i}$$

- Micro-averaged precision/recall measures correctly classified documents, thus favoring large classes

# Macro-Averaged Precision (Multi-Class Classification)

In a problem with $n$ classes, let $P_i$ and $R_i$ be the precision and recall, respectively, achieved by a classifier for class $i$

- Macro-averaged precision is defined as

$$\frac{1}{n}\sum_{i=1}^{n} P_n$$

- Macro-averaged recall is defined as

$$\frac{1}{n}\sum_{i=1}^{n} R_n$$

- Macro-averaged precision/recall measures performance per class, giving all classes equal importance

# Multi-Label Scenario

- One-vs-rest formulation
  - Create a two-class problem for every class
    - E.g. “sports” and “not-sports”, “science” and “not-science”, etc.
  - We have a classifier for each case
- Quality is measured by per-instance recall and precision
  - Let $C_d$ be the correct classes for document $d$ and $C'_d$ be the set of classes estimated by the classifier

$$precision = \frac{C'_d \cap C_d}{C'_d}$$

$$recall = \frac{C'_d \cap C_d}{C_d}$$

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# Other Issues (1)

- Tokenization and feature extraction
  - E.g., replacing monetary amounts and other numeric quantities by a special token, part-of-speech tagging, representations based on *n*-grams, etc.
- Evaluating text classifiers in practice
  - Accuracy
  - Training speed and scalability
  - Simplicity, speed, and scalability for document modifications
  - Ease of diagnosis, interpretation of results, and adding human judgment and feedback
- Many other practical issues...

Questions?