



Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

Processamento e Recuperação de Informação

Evaluation of IR and IE Systems

Departamento de Engenharia Informática
Instituto Superior Técnico

1º Semestre
2018/2019



Bibliography

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

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Outline

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

1 Evaluation and Relevance

2 Precision vs. Recall

3 Other Measures

4 Ranking Comparison

5 Obtaining the Ground Truth

6 Evaluation of Classifiers

7 Evaluation of Clustering



IR System Evaluation

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

Why evaluate?

- Measure the benefit of using an IR system
- Measure how well an IR system fulfills its goal
- Compare IR systems

What to evaluate?

- Collection coverage
- Processing time
- Output presentation
- User effort
- Recall and Precision



Elements of an information retrieval performance evaluation experiment

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

The Cranfield Paradigm

An IR experiment, as devised by Cyril Cleverdon (1950s), must include:

- 1 A reference collection
- 2 Relevance judgments
- 3 An evaluation metric



Relevant Documents

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

Recall and Precision

Measure the ability of a system to return **relevant** documents.

Relevance

- Subjective notion
- Usually **evaluated by a set of experts**



Outline

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

1 Evaluation and Relevance

2 Precision vs. Recall

3 Other Measures

4 Ranking Comparison

5 Obtaining the Ground Truth

6 Evaluation of Classifiers

7 Evaluation of Clustering



Evaluating Prediction

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

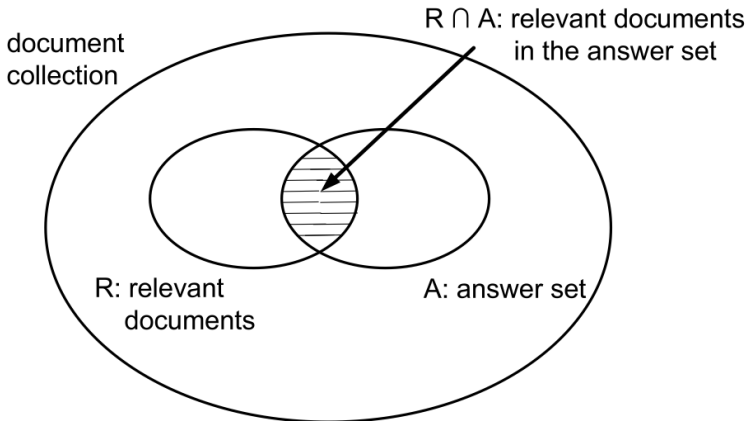
Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering





Measuring Precision and Recall

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

Definition

Let A be the set of documents retrieved for query Q .

Let R be the set of documents that are relevant to query Q .

Precision is the proportion of retrieved documents that are relevant, i.e.:

$$Pr = \frac{|R \cap A|}{|A|}$$

Recall is the proportion of relevant documents retrieved, i.e.:

$$Re = \frac{|R \cap A|}{|R|}$$



Precision-Recall Curves

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- Retrieved documents are ordered \Rightarrow we are interested in measuring how precision changes as recall increases

Example

Let $A = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{10}\}$ be an ordered set of retrieved documents, for a query Q .

Let $R = \{d_2, d_5, d_8, d_{15}\}$ be the set of relevant documents for query Q .

Re	Pr
0.25	0.50
0.50	0.40
0.75	0.38



Interpolated Precision-Recall

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- Precision is usually measured at 10 standard recall points: 0%, 10%, 20%, ..., 90%, 100%
- Precision at $r^0\%$ recall is defined as

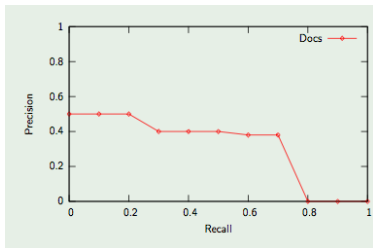
$$P(r) = \max_{i \geq r} P(i)$$

- Precision is zero after no more relevant documents are found

Interpolated Precision-Recall (cont.)

Let $A = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{10}\}$ be an ordered set of retrieved documents, for a query Q . Let $R = \{d_2, d_5, d_8, d_{15}\}$ be the set of relevant documents for query Q .

	Re	Pr
	0.00	0.50
	0.10	0.50
	0.20	0.50
	0.30	0.40
	0.40	0.40
	0.50	0.40
	0.60	0.38
	0.70	0.38
	0.80	0.00
	0.90	0.00
	1.00	0.00





Interpolated Precision-Recall (cont.)

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

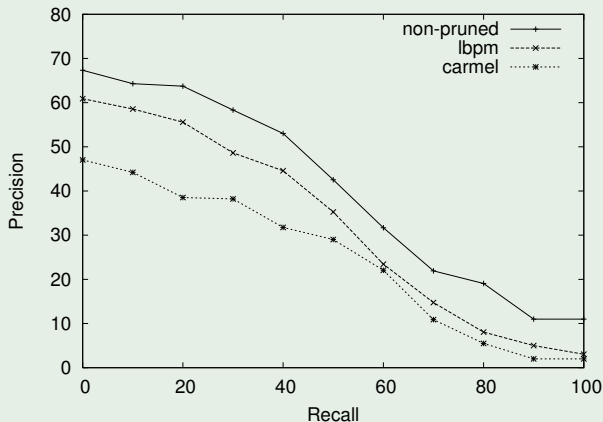
Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

Example





Outline

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

1 Evaluation and Relevance

2 Precision vs. Recall

3 Other Measures

4 Ranking Comparison

5 Obtaining the Ground Truth

6 Evaluation of Classifiers

7 Evaluation of Clustering



P@N, R-precision

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

P@N – Precision at the N -th retrieved document

Most commonly used

- $P@5$,
- $P@10$
- $P@20$

Usefull for Web retrieval

R-precision - Precision at the R -th document, where R is the number of relevant documents



F-measure

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

Harmonic mean of precision and recall:

$$F_{\beta} = \frac{(1 + \beta^2) \times Pr \times Re}{(\beta^2 \times Pr) + Re}$$

Usually we adopt F_1 :

$$F_1 = \frac{2 \times Pr \times Re}{Pr + Re}$$



AP, MAP

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- **AP** - Average of the values for the precision at each recall point

$$AP = \frac{\sum_{i=1}^N Pr@i \times R_i}{|R|}$$

where $R_i = 1$ if document at rank i is relevant and $R_i = 0$ otherwise.

- **MAP** - Mean Average Precision

$$MAP = \frac{\sum_{q=1}^Q AP_q}{Q}$$

- AP can also be interpolated



Discounted Cumulative Gain

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

Cumulative gain: sum the relevance weights

- **DCG** - Discounted cumulative gain

$$\text{DCG}_p = R_1 + \sum_{i=2}^p \frac{R_i}{\log_2 i}$$

where $R_i = 1$ if document at rank i is relevant and $R_i = 0$ otherwise.

- **nDCG** - Normalized discounted cumulative gain

$$\text{nDCG}_p = \frac{\text{DCG}_p}{\text{Ideal DCG}_p}$$



MRR

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

MRR - Mean Reciprocal Rank

$$\text{MRR} = \frac{1}{N} \sum_{i=1}^N \frac{1}{\text{rank}_i}$$

where rank_1 is the rank of the first relevant document.



User Models

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- **Position models:** assume independence among documents in different positions and model the examination probability as a function of the position
- **Cascade Models:** consider the dependency among URLs on a search results page — at each position, the user has a certain probability of being satisfied depending on the relevance of the previous documents

Previous measures assumed a position model; following we show *ERR*, which assumes a cascade model.



ERR - Expected Reciprocal Rank

$$\begin{aligned}\text{ERR} &= \sum_{i=1}^N \frac{1}{i} P(\text{user stops at position } i) \\ &= \sum_{i=1}^N \frac{1}{i} \prod_{j=1}^{i-1} (1 - R_j) R_i\end{aligned}$$

where $R_i = 1$ if document at rank i is relevant and $R_i = 0$ otherwise.

R_i can also be the result of mapping from relevance grades to probability of relevance $R_i := \mathcal{R}(g_i)$, where:

$$\mathcal{R}(g) = \frac{2^g - 1}{2^{g_{\max}}}, \quad g \in \{0, \dots, g_{\max}\}$$



Outline

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

1 Evaluation and Relevance

2 Precision vs. Recall

3 Other Measures

4 Ranking Comparison

5 Obtaining the Ground Truth

6 Evaluation of Classifiers

7 Evaluation of Clustering



Spearman Coefficient

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

Computes the difference between the positions of a same document in two rankings

$$\rho(X, Y) = 1 - \frac{6 \sum_{i=1}^N d_i^2}{N(N^2 - 1)}$$

where $d_i = \text{rank}(X)_i - \text{rank}(Y)_i$ is the difference in rankings of document i .



Kendall's Tau

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

Let $(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)$, where each x_i is the rank of document i in ranking X , and y_i is the rank of document i in ranking Y .

$$\tau = \frac{|\text{concordant pairs}| - |\text{discordant pairs}|}{N(N-1)/2}$$

where a pair (x_i, y_i) is concordant with (x_j, y_j) if either:

$$\begin{cases} x_i > x_j \wedge y_i > y_j \\ x_i < x_j \wedge y_i < y_j \end{cases}$$

and discordant if either:

$$\begin{cases} x_i > x_j \wedge y_i < y_j \\ x_i < x_j \wedge y_i > y_j \end{cases}$$



Outline

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- 1 Evaluation and Relevance
- 2 Precision vs. Recall
- 3 Other Measures
- 4 Ranking Comparison
- 5 Obtaining the Ground Truth**
- 6 Evaluation of Classifiers
- 7 Evaluation of Clustering



Reference Collections

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

TREC Various collections of documents (Ad hoc, Web, Blog, Clinical Decision Support, ...)

CACM Articles from Communications of the ACM

ISI Information science papers

CFC Cystic Fibrosis Collection

...

- Standards for research in IR
- Provide sets queries + evaluated documents



Human Experimentation in the Lab

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- User preferences are affected by the characteristics of the user interface (UI)
 - For instance, the users of search engines look first at the upper left corner of the results page.
 - Changing the layout is likely to affect the assessment made by the users and their behavior.
- Proper evaluation of the user interface requires going beyond the framework of the Cranfield experiments



A/B Testing

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- A/B testing consists of displaying to selected users a modification in the layout of a page
 - The group of selected users constitute a fraction of all users such as, for instance, 1%
 - The method works well for sites with large audiences
- By analysing how the users react to the change, it is possible to analyse if the modification proposed is positive or not

A/B testing provides a form of human experimentation, even if the setting is not that of a lab



Crowdsourcing

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

Amazon Mechanical Turk

Make Money by working on HITs

HITs - *Human Intelligence Tasks* - are individual tasks that you work on. [Find HITs now.](#)

As a Mechanical Turk Worker you:

- Can work from home
- Choose your own work hours
- Get paid for doing good work



Get Results from Mechanical Turk Workers

Ask workers to complete HITs - *Human Intelligence Tasks* - and get results using Mechanical Turk. [Get started.](#)

As a Mechanical Turk Requester you:

- Have access to a global, on-demand, 24 x 7 workforce
- Get thousands of HITs completed in minutes
- Pay only when you're satisfied with the results



<https://www.mturk.com>

- The participants execute human intelligence tasks, called HITs, in exchange for small sums of money
- The tasks are filed by requesters who have an evaluation need
- While the identity of participants is not known to requesters, the service produces evaluation results of high quality (except for **free-loaders**, etc)



Evaluation using Clickthrough Data

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

A promising alternative...

The data can be obtained by observing how frequently the users click on a given document, when it is shown in the answer set for a given query

Attractive, because...

The data can be collected at a low cost without overhead for the users



Evaluation using Clickthrough Data (2)

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

Click models

An accurate user model, which closely reflects users' interactions with the retrieval system, is essential for developing a good relevance metric from clickthrough data.

Example: Cascade model used in ERR metric, corresponding to

$$\prod_{j=1}^{i-1} (1 - R_j) R_i$$

where the values R_i (i.e., document satisfies the user with probability R_i) can be estimated by maximum likelihood on the click logs.



Outline

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- 1 Evaluation and Relevance
- 2 Precision vs. Recall
- 3 Other Measures
- 4 Ranking Comparison
- 5 Obtaining the Ground Truth
- 6 Evaluation of Classifiers**
- 7 Evaluation of Clustering



Classifier Evaluation

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- Previous lectures have shown that tasks such as document classification or information extraction from text can be modeled as classification problems
 - I.e., techniques in this section also apply to IE systems
- Goal in supervised classification is the minimization of classification error on test data
- We can evaluate through measures like recall, precision, and accuracy (i.e., one minus error)
 - But classification tasks can involve more than two classes (i.e., more than distinguishing relevant from non-relevant)



Confusion Matrix

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- $M[i, j]$ is the number of test documents belonging to class i which were assigned to class j
- Perfect classifier: diagonal elements $M[i, i]$ would be nonzero
- Example:

$$M = \left\{ \begin{array}{c|c|c} 5 & 0 & 0 \\ \hline 1 & 3 & 0 \\ \hline 1 & 2 & 4 \end{array} \right\}$$

- If M is large, we use

$$\text{accuracy} = \sum_i M[i, i] / \sum_{i,j} M[i, j]$$

- Notice that accuracy is not a good measure for *small* classes



Micro-Averaged Precision

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

In a problem with n classes, let C_i be the number of documents in class i and let C'_i be the number of documents estimated to be of class i by the classifier

- **Micro-averaged precision** is defined as

$$\frac{\sum_{i=1}^n C'_i \cap C_i}{\sum_{i=1}^n C'_i}$$

- **Micro-averaged recall** is defined as

$$\frac{\sum_{i=1}^n C'_i \cap C_i}{\sum_{i=1}^n C_i}$$

- Micro-averaged precision/recall measures correctly classified documents, thus favoring large classes



Macro-Averaged Precision

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

In a problem with n classes, let P_i and R_i be the precision and recall, respectively, achieved by a classifier for class i

- **Macro-averaged precision** is defined as

$$\frac{1}{n} \sum_{i=1}^n P_n$$

- **Macro-averaged recall** is defined as

$$\frac{1}{n} \sum_{i=1}^n R_n$$

- Macro-averaged precision/recall measures performance per class, giving all classes equal importance
- The **F_1 measure** is also commonly used



Multi-Label Scenario

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- Quality can be measured by per-instance **recall** and **precision**
 - Let C_d be the correct classes for document d and C'_d be the set of classes estimated by the classifier

$$precision = \frac{C'_d \cap C_d}{C'_d}$$

$$recall = \frac{C'_d \cap C_d}{C_d}$$



Train-Test Split

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- When evaluating a classifier, you cannot rely on the data used for training
 - Your estimate is likely to be overly optimistic
 - Your model will tend to **overfit**
- Data must be split into a test and training sets
 - Common train/test splits: 80%/20% or 70%/30%



Cross-Validation

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- Splitting the dataset and testing once may lead to a biased evaluation
- One way to avoid this is to use **cross-validation**
 - Leave-p-out
 - Leave-one-out
 - **k-fold**
 - ...

K-Fold Cross-Validation

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

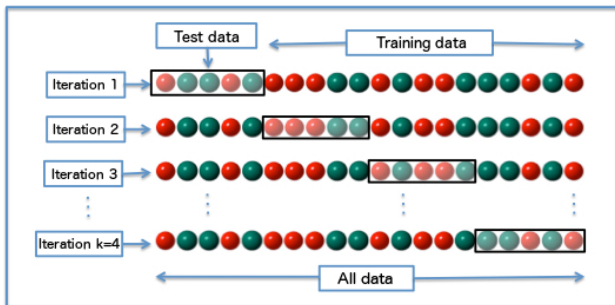
Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- 1 Split the data into k partitions
- 2 For each fold $i \in [1, k]$
 - 1 Train your model using all partitions $P_j, j \neq i$
 - 2 Evaluate your model in partition P_i
- 3 Average your evaluation metrics overall all folds





Outline

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- 1 Evaluation and Relevance
- 2 Precision vs. Recall
- 3 Other Measures
- 4 Ranking Comparison
- 5 Obtaining the Ground Truth
- 6 Evaluation of Classifiers
- 7 Evaluation of Clustering



What to evaluate

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- Goal of clustering: attain **high intra-cluster similarity** (documents within a cluster are similar) and **low inter-cluster similarity** (documents from different clusters are dissimilar)
- These are **internal criteria**
- Alternatively, we can evaluate the results of the application of interest
 - E.g. for search result clustering, evaluate search results
- Or we can use **external criteria**, comparing the clusters to a gold standard



External Criteria

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- Clusters are evaluated using a gold standard (as in classification problems)
 - Each document will be assigned a class
- Unlike classification problems, we don't know to which class each cluster corresponds
 - Thus we cannot directly computed false positives, false negatives, etc.
- There are many proposals to address this issue



Purity

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

For a set $\Omega = \{w_1, w_2, \dots, w_K\}$ of clusters and a set $\mathcal{C} = \{c_1, c_2, \dots, c_K\}$ of classes, **purity** is defined as:

$$\text{purity}(\Omega, \mathcal{C}) = \frac{1}{N} \sum_k \max_j |w_k \cap c_j|$$

where N is the number of documents, w_k is the set of documents in cluster w_k , and c_k is the set of documents in class c_k .

Each cluster is assigned to the class most frequent in the cluster and accuracy is measured.



Normalized Mutual Information

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- Purity does not show the tradeoff between quality and the number f clusters
 - E.g., if each document gets a cluster, $\text{purity} = 1$
- **NMI** takes this tradeoff into account
 - Measures the increase in the amount of information when we know what the clusters are,
 - But normalizes it by the entropy of the clusters



Normalized Mutual Information (cont.)

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

$$\text{NMI}(\Omega, \mathcal{C}) = \frac{I(\Omega, \mathcal{C})}{(H(\Omega) + H(\mathcal{C})) / 2}$$

where

$$\begin{aligned} I(\Omega, \mathcal{C}) &= \sum_k \sum_j P(w_k \cap c_j) \log \frac{P(w_k \cap c_j)}{P(w_k)P(c_j)} \\ &= \sum_k \sum_j \frac{|w_k \cap c_j|}{N} \log \frac{N |w_k \cap c_j|}{|w_k| |c_j|} \end{aligned}$$

is the mutual information between clusters and classes and

$$H(\Omega) = - \sum_k P(w_k) \log P(w_k) = - \sum_k \frac{|w_k|}{N} \log \frac{|w_k|}{N}$$

is the entropy (equally for $H(\mathcal{C})$)



Rand Index

Processamento
e Recuperação
de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

Ranking
Comparison

Obtaining the
Ground Truth

Evaluation of
Classifiers

Evaluation of
Clustering

- The **Rand index** views clustering as a series of decisions for each of the $N(N - 1)/2$ pairs of documents
- Decisions can be:
 - **True Positive**: the documents are similar and in the same cluster
 - **True Negative**: the documents are *not* similar and in different clusters
 - **False Positive**: the documents are *not* similar but in the same cluster
 - **False Negative**: the documents are similar but in different clusters
- We can consider documents as **similar if they are in the same class**



Rand Index (cont.)

Processamento
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The Rand index simply measures the percentage of correct decisions (i.e. the accuracy):

$$RI = \frac{TP + TN}{TP + FP + TN + FN}$$



The F_β Measure (Again)

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- The Rand index gives equal weight to false positives and false negatives
- Separating similar documents is sometimes worse than putting pairs of dissimilar documents in the same cluster.
- We can use the F measure to penalize false negatives more strongly than false positives, giving more weight to recall
 - selecting a value $\beta > 1$



The F_β Measure (cont.)

Processamento
e Recuperação
de Informação

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$$P = \frac{TP}{TP + FP} \quad R = \frac{TP}{TP + FN}$$

$$F_\beta = \frac{(1 + \beta^2) \cdot P \cdot R}{(\beta^2 \cdot P) + R}$$

or, using the decision framework:

$$F_\beta = \frac{(1 + \beta^2) \cdot TP}{(1 + \beta^2) \cdot TP + \beta^2 \cdot FN + FP}$$



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de Informação

Evaluation
and Relevance

Precision vs.
Recall

Other
Measures

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Comparison

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Ground Truth

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Classifiers

Evaluation of
Clustering

Questions?