# Processamento e Recuperação de Informação

## Classification

Departamento de Engenharia Informática
Instituto Superior Técnico

1º Semestre
2018/2019

# Outline

# Bibliography




- Bing Liu, Web Data Mining - Exploring Hyperlinks, Contents, and Usage Data. Chapter 3.
- Ricardo Baeza-Yates and Berthier Ribeiro-Neto, Modern Information Retrieval. Chapter 8.
- Christopher D. Manning, Prabhakar Raghavan and Hinrich Schütze, Introduction to Information Retrieval. Chapters 13, 14 and 15.
- Jure Leskovec, Anand Rajaraman, and Jeff Ullman, Mining of Massive Datasets, Chapter 12

# Outline

# Organizing Knowledge



- Organize into systematic knowledge structures
- Ontologies
  - Dewey decimal system
  - ACM Computing Classification System
  - Patent subject classification
- Web catalogs
  - Yahoo Directory (RIP 2002–2014)
  - Dmoz Directory

# Organizing Knowledge



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Problem: Manual maintenance

# Outline

# Supervised Learning



Given a set of training data as input, use learning algorithm $A$ to discover the function $\hat{h}$ that minimizes the loss (e.g. the error)

Input: $\{(x_i, y_i)\}_{i=1}^{N}, x_i \in \mathcal{R}^M, y_i \in \mathcal{R}$

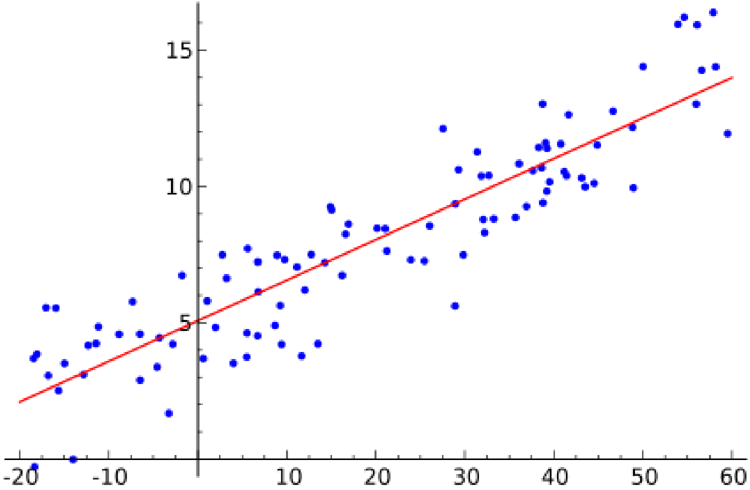
Hypothesis space: $h^* \in H$

Loss function: $L(h(x), y)$

Learning Algorithm: $\hat{h} = A(\{(x_i, y_i)\}_{i=1}^{N})$, such that

$$\hat{h} = \text{argmin}_h \sum_{i=1}^{N} L(h(x_i), y_i)$$

# An Example: Linear Regression




(source: wikipedia)

# Linear Regression (cont.)



- The hypothesis space:

$$h_{\vec{w}}(x) = w_0 + w_1 x$$

where $\vec{w} = [w_0, w_1]$

- The loss function:

$$L(h_{\vec{w}}, y) = \frac{1}{N} \sum_{i=1}^{N} (y_i - h_{\vec{w}}(x_i))^2$$

i.e. the sum of the squared error

- We want to find

$$w^* = \underset{w}{\operatorname{argmin}}\, L(h_{\vec{w}}, y)$$

# Minimizing the Loss

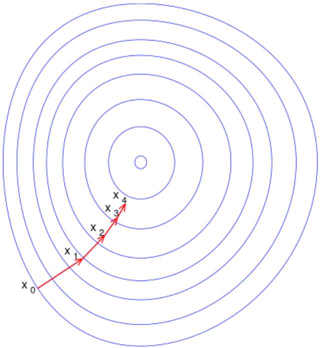


- In the most simple case, we can easily find one (or more) solution(s)
  - Just take the derivatives and equal to 0
- In many cases this is not possible (or we may want to enforce some constraints on the parameters)
- In practice, there are many ways to estimate $w^*$

# An Example: Gradient Descent

$w \leftarrow$ any point in the parameter space
**loop** until convergence **do**
  **for each** $w_i$ **in** $\vec{w}$ **do**
    $w_i \leftarrow w_i - \alpha \frac{\partial}{\partial w_i} L(h_{\vec{w}}, y)$

$\alpha$ = learning rate

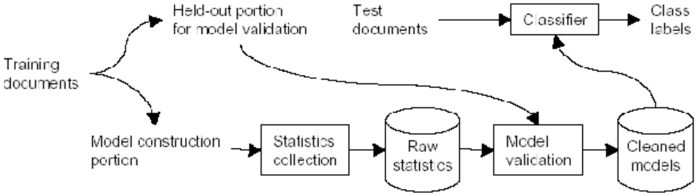



(source: wikipedia)

# Classification

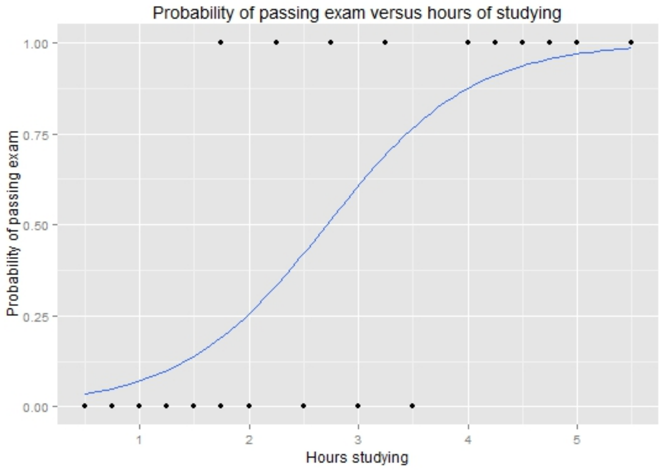


- Learning to assign objects to classes given examples
- Learn a classifier



# An Example: Logistic Regression






(source: wikipedia)

# Logistic Regression (cont.)



- The hypothesis space:

$$h_{\vec{w}}(x) = \frac{1}{1 + e^{-(w_0 + w_1 x)}}$$

where $\vec{w} = [w_0, w_1]$

- The loss function:

$$L(h_{\vec{w}}, y) = \frac{1}{N} \sum_{i=1}^{N} C(h_{\vec{w}}(x_i), y)$$

where

$$C(h_{\vec{w}}(x), y) = \begin{cases} -\log(h_{\vec{w}}(x)) & \text{if } y = 1 \\ -\log(1 - h_{\vec{w}}(x)) & \text{if } y = 0 \end{cases}$$

- We want to find

$$w^* = \operatorname*{argmin}_{w} L(h_{\vec{w}}, y)$$

# Outline

# Text Classification vs. Data Mining

- Lots of features and a lot of noise
- No fixed number of columns
- No categorical attribute values
- Data scarcity
- Larger number of class labels
- Hierarchical relationships between classes less systematic

# Text Classifiers

- Nearest Neighbor Classifiers
  - Classify documents according to the class distribution of their neighbors
- Generative Bayesian classifiers (e.g., naïve Bayes)
  - Discover the class distribution most likely to have generated a test document
- Linear discriminative classifiers (e.g., the perceptron, or support vector machines):
  - Discover an hyperplane that separates classes

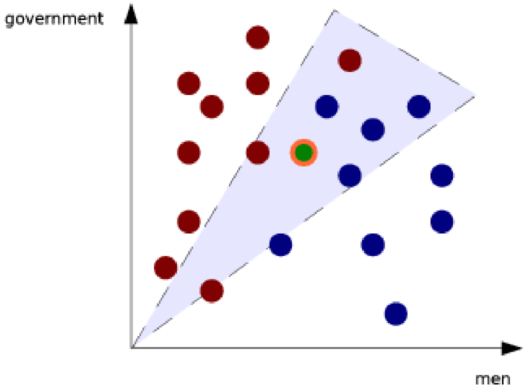
# Nearest Neighbor Classifiers



- Similar documents are expected to be assigned the same class label
    - Similarity: vector space model + cosine similarity
- Training:
    - Index each document and remember class label
- Testing:
    - Fetch *k* most similar documents to the given document
    - Majority class wins
    - Alternatives:
        - Weighted counts: counts of classes weighted by the corresponding similarity measure
        - Per-class offset: tuned by testing the classifier on a portion of training data held out for this purpose

# $k$NN Classifier






$$score(c, d_q) = b_c + \sum_{d \in kNN(d_q)} sim(d_q, d)$$

# Properties of *k*NN



- Advantages:
  - Reuse of standard vector space model and availability of associated technology (e.g., inverted indexes)
  - Collection updates trivial
  - Accuracy comparable to best known classifiers
- Problems:
  - Classification efficiency
    - many lookups over the document collection/index
    - sorting by overall similarity
    - picking the best $k$ documents
  - Space overhead and redundancy
    - Data stored at level of individual documents
    - Poor generalization
  - Choosing a value for $k$

# Improvements for $k$NN



- To reduce space requirements and speed up classification
  - Find clusters in the data (clustering will be covered in the next lecture)
  - Store only a few statistical parameters per cluster
  - Compare with documents in only the most promising clusters
- However...
  - Ad-hoc choices for number and size of clusters and parameters
  - Number of clusters depends on the data

# Bayesian Classifiers

- Probabilistic document classifier
- Assumptions:
  1. A document can belong to exactly one class
  2. Each class $c$ has an associated prior probability $P(c)$
  3. There is a class-conditional document distribution $P(d|c)$ for each class (i.e., the likelihood)
- Given a document $d$, the probability of it being generated by class $c$ is:

$$P(c|d) = \frac{P(d|c)P(c)}{\sum_\gamma P(d|\gamma)P(\gamma)}$$

- The class with the highest probability is assigned to $d_q$ (i.e., we use a *maximum a-posteriory* rule)

## Learning the Document Distribution

- $P(d|c)$ is estimated based on parameters $\Theta$
- $\Theta$ is estimated based on two factors:
  1. Prior knowledge before seeing any documents
  2. Terms in the training documents
- Bayes Optimal Classifier

$$P(c|d) = \int_{\Theta} \frac{P(d|c,\Theta)P(c|\Theta)}{\sum_{\gamma} P(d|\gamma,\Theta)P(\gamma|\Theta)} P(\Theta|D)$$

  - This can be hard to compute
- Maximum Likelihood Estimate: $P(d|c,\hat{\Theta})$

$$\hat{\Theta} = argmax_{\Theta} P(d|c,\Theta)$$

# Naïve Bayes Classifier



- Naïve assumption
  - assumption of independence between terms
  - joint term distribution is the product of the marginals
- Widely used owing to
  - simplicity and speed of training, applying, and updating
- Two kinds of widely used marginals for text
  - Binary model
  - Multinomial model

# Naïve Bayes Models

Binary Model: Each parameter $\theta_{c,t}$ indicates the probability that a document in class $c$ will mention term $t$ at least once

$$P(d|c,\Theta) = \Pi_{t \in d} \theta_{c,t} \Pi_{t \notin d} (1 - \theta_{c,t})$$

$$\theta_{c,t} = \frac{N_{c,t}}{N_c}$$

$N_{c,t}$ = n. of docs in class $c$ containing term $t$
$N_c$ = n. of docs in class $c$

# Naïve Bayes Models (cont.)



## Multinomial Model:

- each class has an associated die with $|W|$ faces
- each parameter $\theta_{c,t}$ denotes probability of the face turning up on tossing the die, i.e. $\sum_{d \in c} n(d,t) / \sum_{d \in c} \ell_d$
- term $t$ occurs $n(d,t)$ times in document $d$
- document length is a random variable denoted $L$

$$\begin{aligned} P(d|c,\Theta) &= P(L = \ell_d|c)P(d|\ell_d, c) \\ &= P(L = \ell_d|c)\frac{\ell_d!}{\prod_{t \in d} n(d,t)!}\prod_{t \in d}\theta_{c,t}^{n(d,t)} \\ &\sim P(L = \ell_d|c)\prod_{t \in d}\theta_{c,t}^{n(d,t)} \end{aligned}$$

# Parameter Smoothing

- What if a test document $d_q$ contains a term $t$ that never occurred in any training document in class $c$?

# Parameter Smoothing

- What if a test document $d_q$ contains a term $t$ that never occurred in any training document in class $c$?
    - $P(c|d_q) = 0$
    - Even if many other terms clearly hint at a high likelihood of class $c$ generating the document
- Thus, MLE cannot be used directly
- We can use Laplace smoothing
    - Simply adds 1 to each count

$$\theta_{c,t} = \frac{\sum_{d \in c} n(d,t) + 1}{\sum_{d \in c} \ell_d + |W|}$$

# Performance Analysis



- Multinomial naïve Bayes classifier generally outperforms the binary variant
- *k*NN may outperform Naïve Bayes
- Naïve Bayes is faster and more compact
- Determines decision boundaries
  - Regions of the term-space where different classes have similar probabilities
  - Documents in these regions are hard to classify
  - Strongly biased

# Discriminative Classification

- Naïve Bayes classifiers are generative
- Differently, discriminative classifiers:
  - Directly map the feature space to class labels
  - Class labels are encoded as numbers
    - e.g: +1 and -1 for two a class problem
- For instance, we can try to find a vector $\alpha$ such that the sign of $\alpha \cdot d + b$ directly predicts the class of a document $d$
- Possible solutions:
  - Linear least-square regression
  - The Perceptron
  - Support Vector Machines

# What is a Linear Discriminative Classifier?



- Essentially:
  - Classification decision is based on the value of a linear combination of the features
  - Can be seen as the splitting of a high-dimensional input space with a hyperplane

$$y(d_1, \ldots, d_n) = f(\alpha_1 d_1 + \alpha_2 d_2 + \ldots + \alpha_n d_n)$$

- $\alpha_i$ are parameters (i.e., the weight of each feature $d_i$)
- $f$ is the activation function (e.g., $f(d) = 1_{x \geq 0}(d)$)
- The result of $y(d_1, \ldots, d_n)$ corresponds to the estimated class

# The Bias Term



- Notice that the decision hyperplane must go through the origin
- Could be achieved by preprocessing the input, but this is not always desirable or possible
- Solution : Add a bias input:

$$y(d_1, \ldots, d_n) = f(b + \alpha_1 d_1 + \ldots + \alpha_n d_n)$$

- Same as an input connected to the constant 1
- We consider this *ghost* input implicit henceforth

# Training : The Perceptron Algorithm



- Switching to vector notation:

$$y(\mathbf{d}) = f(\alpha\mathbf{d}) = f_\alpha(d) \tag{1}$$

- Assume we need to separate sets of points $A$ (i.e., the positive examples) and $B$ (i.e., the negative examples)

$$E(\alpha) = \sum_{\mathbf{d}\in A}(1 - f_\alpha(\mathbf{d})) + \sum_{\mathbf{d}\in B} f_\alpha(\mathbf{d}) \tag{2}$$

- Goal: $E(\alpha) = 0$
- Start from a random $\alpha$ and improve it iteratively

# Algorithm Pseudo-Code

1. Start with random $\alpha$, set $t = 0$
2. Select a vector $\mathbf{d} \in A \cup B$
3. If $\mathbf{d} \in A$ and $\alpha\mathbf{d} \leq 0$, then $\alpha_{t+1} = \alpha_t + \mathbf{d}$
4. Else if $\mathbf{d} \in B$ and $\alpha\mathbf{d} \geq 0$, then $\alpha_{t+1} = \alpha_t - \mathbf{d}$
5. Conditionally go to step 2

- Guaranteed to converge iff $A$ and $B$ are linearly separable!

# Summary of Simple Perceptrons

- Simple and reasonably efficient online training
- Easy to extend in order to consider multi-class classification
- Works well for document classification, and more generally for problems with many features
- Limited capabilities (e.g., does not try to optimize the separation "distance" between classes)
  - Just looks for a hyperplane that separates the two sets
  - Methods such as Support Vector Machines, on the other hand, try to maximize the distance between two closest opposite sample points (i.e., the margin of the separating hyperplane)

# Linear Discriminative Classifiers and SVMs

- Hypothesis:
  - The classes can be separated by an hyperplane
  - The hyperplane that is close to many training data points has a greater chance of misclassifying test instances
  - An hyperplane that passes through a "no-man's land", has lower chances of misclassifications
- Make a decision by thresholding
  - Seek an hyperplane that maximizes the distance to any training point
  - Choose the class on the same side of the hyperplane as the test document (i.e., same as in the Perceptron)

# Discovering the Hyperplane

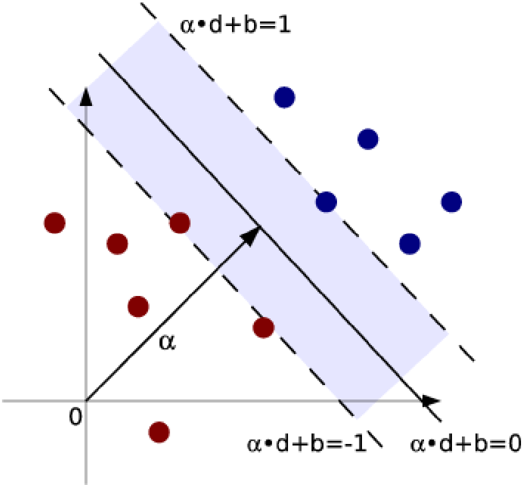


- Assume the training documents are separable by an hyperplane perpendicular to a vector $\alpha$
- Seek a vector $\alpha$ which maximizes the distance of any training point to the hyperplane
- This corresponds to solving the following quadratic programming problem:

$$\begin{array}{rl} \text{Minimize} & \frac{1}{2}\alpha \cdot \alpha \\ \text{subject to} & c_i(\alpha \cdot d_i + b) \geq 1, \forall i = 1, \ldots, n \end{array}$$

# SVM Classifier





# Non Separable Classes



- Classes in the training data not always separable
- We introduce slack variables

$$\begin{aligned} \text{Minimize} \quad & \tfrac{1}{2}\alpha \cdot \alpha + C \textstyle\sum_i \xi_i \\ \text{subject to} \quad & c_i(\alpha \cdot d_i + b) \geq 1 - \xi_i, \forall i = 1, \ldots, n \\ \text{and} \quad & \xi_i \geq 0, \forall i = 1, \ldots, n \end{aligned}$$

- Implementations often solve the equivalent dual problem

$$\begin{aligned} \text{Maximize} \quad & \textstyle\sum_i \lambda_i - \tfrac{1}{2} \sum_{i,j} \lambda_i \lambda_j c_i c_j (d_i \cdot d_j) \\ \text{subject to} \quad & \textstyle\sum_i c_i \lambda_i = 0 \\ \text{and} \quad & 0 \leq \lambda_i \leq C, \forall i = 1, \ldots, n \end{aligned}$$

# Analysis of SVMs

- Complexity:
  - Quadratic optimization problem
  - Requires on-demand computation of inner-products
  - Recent SVM packages work in linear time
- Performance:
  - Amongst most accurate classifier for text
  - Better accuracy than Naïve Bayes and most classifiers
  - Linear SVMs suffice
    - Standard text classification tasks have classes almost separable using a hyperplane in feature space
  - Non-linear SVMs can be achieved through kernel functions

# Outline

# Other Issues (1)



- Tokenization and feature extraction
  - E.g.: replacing monetary amounts by a special token, part-of-speech tagging, representations based on *n*-grams, etc.
- Handling scenarios with multiple classes, or with multiple labels per test document, with binary classifyers like SVMs
  - E.g., one-vs.-rest heuristic
    - e.g. "sports" vs. "not-sports", "science" vs. "not-science", etc.
    - Create a classifier for each case
    - Assign class(es) with the highest confidence

# Other Issues (2)



- Evaluating text classifiers
  - Accuracy
  - Training speed and scalability
  - Simplicity, speed, and scalability for document modifications
  - Ease of diagnosis, interpretation of results, and adding human judgment and feedback
- Many other practical issues...

Questions?