



Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Processamento e Recuperação de Informação

Information Retrieval Models

Departamento de Engenharia Informática
Instituto Superior Técnico

1º Semestre
2018/2019



Outline

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- 1 Generic Document Model
- 2 The Boolean Model
- 3 The Vector Space Model
- 4 Probabilistic Models
- 5 Comparison of the Classic Models



Bibliography

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Ricardo Baeza-Yates, Berthier Ribeiro-Neto, Modern Information Retrieval, 2nd edition. Chapter 3

Christopher D. Manning, Prabhakar Raghavan and Hinrich Schütze, Introduction to Information Retrieval. Chapters 1, 6, 11 and 12

Bing Liu, Web Data Mining - Exploring Hyperlinks, Contents, and Usage Data. Chapter 6.



Outline

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- 1 Generic Document Model
- 2 The Boolean Model
- 3 The Vector Space Model
- 4 Probabilistic Models
- 5 Comparison of the Classic Models



Retrieval Models

Processamento
e Recuperação
de Informação

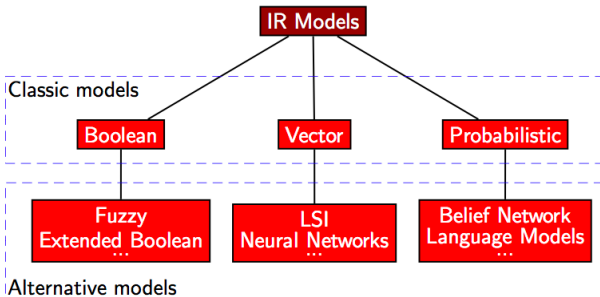
Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models





Index Terms

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

In the classic IR models, documents are represented by **index terms**

- full text/selected keywords
- structure/no structure

Not all terms are equally useful

- index terms can be **weighted**

We assume that terms are **mutually independent**

- this is, of course, a simplification



Definition of a Document Model

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Definition

Let t be the number of index terms in the collection of documents, and let k_i be a generic index term.

- $K = \{k_1, \dots, k_t\}$ is the set of all index terms.
- A weight $w_{i,j} \geq 0$ is associated with each index term k_i of a document d_j .
- For an index term which does not appear in the document text, $w_{i,j} = 0$.
- Each document d_j is associated a term vector \vec{d}_j , represented by $\vec{d}_j = (w_{1,j}, w_{2,j}, \dots, w_{t,j})$.
- Function $g_i(\vec{d}_j)$ returns the weight of index term k_i in vector \vec{d}_j .



Outline

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- 1 Generic Document Model
- 2 The Boolean Model
- 3 The Vector Space Model
- 4 Probabilistic Models
- 5 Comparison of the Classic Models



Boolean Model Queries

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- Follows Boolean algebra syntax and semantics
- Term weights are binary
 - $w_{i,j} \in \{0, 1\}$
 - $w_{i,j} = 1$ — term present,
 - $w_{i,j} = 0$ — term not present
- Queries are Boolean expressions
 - E.g., $q = k_a \wedge (k_b \vee \neg k_c)$
- Documents are considered **relevant** if the query evaluates to 1 (true)



An Example

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

d_1

That government is best which
governs least

d_2

That government is best which
governs not at all

d_3

When men are prepared for it,
that will be the kind of
government which they will have



An Example

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

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That government is best which
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When men are prepared for it,
that will be the kind of
government which they will have

$q = \text{government} \wedge \text{best}$

answer: d_1, d_2



An Example

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

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$q = \text{government} \wedge \text{best}$

answer: d_1, d_2

d_2

That government is best which
governs not at all

$q = \text{government} \wedge \text{best} \wedge \neg \text{all}$

answer: d_1

d_3

When men are prepared for it,
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An Example

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

d_1

That government is best which
governs least

$q = \text{government} \wedge \text{best}$

answer: d_1, d_2

d_2

That government is best which
governs not at all

$q = \text{government} \wedge \text{best} \wedge \neg \text{all}$

answer: d_1

d_3

When men are prepared for it,
that will be the kind of
government which they will have

$q = \text{government} \vee \text{best} \wedge \neg \text{all}$

answer: d_1, d_2, d_3



Document-Query Similarity

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- Queries can be translated to a disjunction of conjunctive vectors

$$\vec{q} = k_a \wedge (k_b \vee \neg k_c) \Leftrightarrow (1, 1, 1) \vee (1, 1, 0) \vee (1, 0, 0)$$

each tuple corresponds to a vector (k_a, k_b, k_c)

- Similarity of a document to a query is defined as:

$$\text{sim}(d_j, q) = \begin{cases} 1 & \text{if } \exists \vec{q}_c \in \vec{q} | \forall_i, g_i(\vec{d}_j) = g_i(\vec{q}_c) \\ 0 & \text{otherwise} \end{cases}$$



The Boolean Model

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Why is it good?

- Simple model based on Boolean algebra
- Intuitive concept
- Precise semantics
- Clear formal basis
- Widely adopted by early information systems



Boolean Model

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Limitations:

- Retrieval based only on binary decisions
 - More similar to *data retrieval* than *information retrieval*
 - Can retrieve too many, or too little documents
 - Some documents may be more relevant than others
- How do you translate a query to a Boolean expression?
 - Non-expert users may not be able to represent their information needs using Boolean expressions
- Terms are all equally important
 - Index term weighting can bring great improvements in performance



Outline

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

1 Generic Document Model

2 The Boolean Model

3 The Vector Space Model

4 Probabilistic Models

5 Comparison of the Classic Models



Documents as Vectors

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- Documents are represented as vectors
 - $\vec{d}_j = (w_{1,j}, w_{2,j}, \dots, w_{t,j})$
 - $w_{i,j}$ is the weight of term i in document j
- Queries are also vectors
 - $\vec{q} = (w_{1,q}, w_{2,q}, \dots, w_{t,q})$
- Vector operations can be used to compare queries×documents (or documents×documents)



An Example

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models





Defining Document Vectors

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Two questions are still unanswered:

- 1 How do we define term weights?
- 2 How do we compare documents to queries?



Defining Term Weights — TF

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Term frequency

Term frequency is a measure of term importance **within a document**

Definition

Let N be the total number of documents in the system and n_i be the number of documents in which term k_i appears. The **normalized frequency** of a term k_i in document d_j is given by:

$$f_{i,j} = \frac{freq_{i,j}}{\max_l freq_{l,j}}$$

where $freq_{i,j}$ is the number of occurrences of term k_i in document d_j .



Defining Term Weights — IDF

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

(Inverse) Document frequency

Document frequency is a measure of term importance **within a collection**

Definition

The **inverse document frequency** of a term k_i is given by:

$$idf_i = \log \left(\frac{N}{n_i} \right)$$



Defining Term Weights — TF-IDF

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Definition

The weight of a term k_i in document d_j for the vector space model is given by the **tf-idf** formula:

$$w_{i,j} = f_{i,j} \times \log \left(\frac{N}{n_i} \right)$$



Components of TF-IDF

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Different TF-IDF formulations consider alternative approaches for the TF and IDF components, and also for normalizing the resulting vectors.

Term frequency		Document frequency		Normalization	
n (natural)	$tf_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(tf_{t,d})$	t (idf)	$\log \frac{N}{df_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2 + w_2^2 + \dots + w_M^2}}$
a (augmented)	$0.5 + \frac{0.5 \times tf_{t,d}}{\max_t(tf_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N - df_t}{df_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases} 1 & \text{if } tf_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/CharLength^\alpha$, $\alpha < 1$
L (log ave)	$\frac{1 + \log(tf_{t,d})}{1 + \log(\text{ave}_{t \in d}(tf_{t,d}))}$				



Document Similarity

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- Similarity between documents and queries is a measure of the correlation between their vectors
- Documents/queries that share the same terms, with similar weights, should be more similar
- Thus, as similarity measure, we use the **cosine of the angle between the vectors**

$$\text{sim}(d_j, q) = \frac{\vec{d}_j \cdot \vec{q}}{|\vec{d}_j| \times |\vec{q}|} = \frac{\sum_{i=1}^t w_{i,j} \times w_{i,q}}{\sqrt{\sum_{i=1}^t w_{i,j}^2} \times \sqrt{\sum_{i=1}^t w_{i,q}^2}}$$



An Example

Processamento
e Recuperação
de Informação

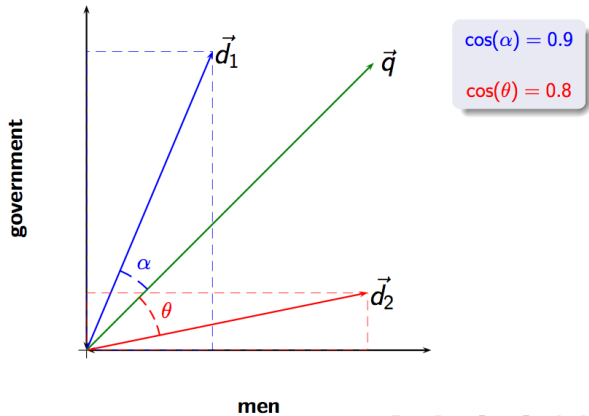
Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models





The Vector Space Model

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Why is it so good?

- Simple model, based on linear algebra
- Term weights are not binary
- Allows computing a continuous **degree of similarity** between queries and documents
- Thus, allows **ranking** documents according to their possible relevance



Improving the VSM (1)

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

The BM25 Model

Consider not only the term frequency and inverse document frequency heuristics, but also the **document length as a normalization factor** for the term frequency

$$TF_{i,j} = \frac{f_{i,j} \times (k_1 + 1)}{f_{i,j} + k_1 \times \left(1 - b + b \frac{|d_j|}{avgdl}\right)}$$

$$IDF_i = \log \frac{N - n_i + 0.5}{n_i + 0.5}$$

$$sim(d_j, q) = \sum_{i \in q} IDF_i \times TF_{i,j}$$

To be detailed in the next lecture



Improving the VSM (2)

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Latent Semantic Indexing

- Find a low-rank approximation of the matrix which describes the occurrences of terms in documents
 - Singular Value Decomposition
 - Compare the documents in the low-dimensional space
- The consequence of the rank lowering is that some dimensions are combined (e.g., **mitigates the problem of identifying synonymy**)
- To be detailed latter in the course



Outline

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- 1 Generic Document Model
- 2 The Boolean Model
- 3 The Vector Space Model
- 4 Probabilistic Models**
- 5 Comparison of the Classic Models



Probabilistic Models for IR

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

TF-IDF and VSM produces sufficiently good results in practice, but often criticized for being **too ad-hoc** or **not principled**

- Typically outperformed by probabilistic retrieval models and statistical language models in IR benchmarks
- Probabilistic retrieval models
 - use generative models of documents as bags-of-words
 - explicitly model probability of relevance $P(R|d_j, q)$
 - probabilistic justification for TF-IDF-like approaches
- Statistical language models
 - use generative models of documents and queries as sequences-of-words
 - consider likelihood of generating query from document model or divergence of document model and query model



Probabilistic Retrieval Models

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- Model the IR problem in a probabilistic framework
- Estimate the probability of document d_j being relevant to the user that submitted the query q
- When considering ranked retrieval, present documents in decreasing order of their estimated probability of relevance

Bayes optimal decision rule for set retrieval

$$d_j \text{ is relevant iff } P(R|d_j, q) > P(\bar{R}|d_j, q)$$



Binary Independence Model (BIM)

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Simplifying assumptions for $P(R|d_j, q)$

- A simple probabilistic model can assume that:
 - ① probability depends only on query and document
 - ② there is a subset R of relevant documents
 - ③ index terms are independent
 - ④ non-query terms are equally likely to appear in relevant and non-relevant documents
- Use **binary term weights**
 - documents and queries as binary term incidence vectors
 - terms not appearing in the query do not affect the ranking



Document Query Similarity

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- As a similarity measure, we can use the ratio between the probability of finding the relevant documents and the probability of finding the non-relevant documents

$$\text{sim}(d_j, q) = \frac{P(R|\vec{d}_j, \vec{q})}{P(\bar{R}|\vec{d}_j, \vec{q})}$$

- Often referred to as the **Retrieval Status Value (RSV)**



Similarity Probabilities (1)

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Initial Equation

$$\text{sim}(d_j, q) = \frac{P(R|\vec{d}_j, \vec{q})}{P(\bar{R}|\vec{d}_j, \vec{q})} = \frac{P(\vec{d}_j, \vec{q}|R) \times P(R)}{P(\vec{d}_j, \vec{q}|\bar{R}) \times P(\bar{R})} \sim \frac{P(\vec{d}_j, \vec{q}|R)}{P(\vec{d}_j, \vec{q}|\bar{R})}$$

Assuming term independence...

$$\text{sim}(d_j, q) \sim \frac{(\prod_{g_i(\vec{d}_j)=g_i(\vec{q})=1} P(k_i|R)) \times (\prod_{g_i(\vec{d}_j)=0} P(\bar{k}_i|R))}{(\prod_{g_i(\vec{d}_j)=g_i(\vec{q})=1} P(k_i|\bar{R})) \times (\prod_{g_i(\vec{d}_j)=0} P(\bar{k}_i|\bar{R}))}$$



Similarity Probabilities (2)

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Taking logs and removing constant factors...

$$\text{sim}(d_j, q) = \sum_{i=1}^t w_{i,q} \times w_{i,j} \times \left(\log \frac{P(k_i|R)}{1 - P(k_i|R)} + \log \frac{1 - P(k_i|\bar{R})}{P(k_i|\bar{R})} \right)$$

Blind assumptions

$$\begin{aligned} P(k_i|R) &= 0.5 \\ P(k_i|\bar{R}) &= \frac{n_i}{N} \end{aligned}$$



Similarity Probabilities (3)

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

After document retrieval or leveraging training data...

Let V be the number of returned documents (i.e., estimated to be relevant); let V_i be the number of returned documents containing term k_i .

$$\begin{aligned}P(k_i|R) &= \frac{V_i}{V} \\P(k_i|\bar{R}) &= \frac{n_i - V_i}{N - V}\end{aligned}$$

Avoiding small values (after retrieval)...

$$\begin{aligned}P(k_i|R) &= \frac{V_i + \frac{n_i}{N}}{V + 1} \\P(k_i|\bar{R}) &= \frac{n_i - V_i + \frac{n_i}{N}}{N - V + 1}\end{aligned}$$



Problems of this Simple Probabilistic Model

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- There is no accurate estimate for the first run probabilities
 - Index terms are not weighted
 - Terms are assumed mutually independent
-
- In fact, many different probabilistic retrieval models have been proposed, some addressing the aforementioned limitations!



The Okapi BM25 Model (1)

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- Inspired by the BIM probabilistic formulation
- Considering an alternative for term weighting
- Captures various aspects in a simple formula, tuning each component
 - Inverse Document Frequency (IDF)
 - Term Frequency (TF)
 - Document length
 - Query term frequency
- Effective and widely used model



The Okapi BM25 Model (2)

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

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$$IDF_i = \log \frac{N - n_i + 0.5}{n_i + 0.5}$$

$$sim(d_j, q) = \sum_{i \in q} IDF_i \times TF_{i,j}$$



Probabilistic Language Models

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- Another simple probabilistic retrieval model
- Each document d is treated as (the basis for) a **probabilistic language model**
- Given a query q rank documents based on $P(d|q)$

$$P(d|q) = \frac{P(d) \times P(q|d)}{P(q)}$$

- The evidence $P(q)$ is the same for all documents, so ignore
- $P(d)$ is the prior
 - often treated as the same for all d
 - we can give a higher prior to “high-quality” documents (e.g., those with high PageRank – to be seen latter)
- $P(q|d)$ is likelihood, i.e. the probability of q given d



How to compute $P(q|d)$

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- Conditional independence assumption

$$P(q|d) = P(\{t_1, \dots, t_{|q|}\}|d) = \prod_{1 \leq i \leq |q|} P(t_i|d)$$

- $|q|$ is length of q
- t_k is the token occurring at position k in q

- The above multinomial model is equivalent to:

$$P(q|d) = \prod_{\text{distinct term } t \in q} P(t_k|d)^{TF_{t,q}}$$

- Component $TF_{t,q}$ is the term frequency of t in q
- Parameters $P(t_k|d)$ computed through maximum likelihood estimates

$$P(t_k|d) = \frac{TF_{t_k,d}}{|d|}$$



LM Retrieval and Naïve Bayes

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

The next class will introduce a simple probabilistic document classifier, known as the **Naïve Bayes** approach

- We want to classify document d .
We want to classify a query q
- Human-defined classes: e.g., politics, economics, sports.
Each document in the collection is a different class
- Assume that d was produced by the generative model.
Assume that q was generated by a generative model
- Which of the classes (= class models) is most likely to have generated the document d ?
Which document (=class) is most likely to have generated the query q ?
- For which class do we have the most evidence?
For which document (as source for query) do we have the most evidence?



Outline

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- 1 Generic Document Model
- 2 The Boolean Model
- 3 The Vector Space Model
- 4 Probabilistic Models
- 5 Comparison of the Classic Models



What makes these Models Work?

Processamento
e Recuperação
de Informação

Generic
Document
Model

The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

Three main term weighting normalization driving features:

- TF - Term Frequency
- IDF - Inverse Document Frequency
- DL - Document Length



Comparison of the Classic Models

Processamento
e Recuperação
de Informação

Generic
Document
Model

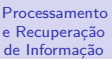
The Boolean
Model

The Vector
Space Model

Probabilistic
Models

Comparison of
the Classic
Models

- Boolean model is considered the weakest
- There is some controversy over which shows better performance: vector space or probabilistic
- However, nowadays, the vector space model is the most widely used



Generic Document Model

The Boolean Model

The Vector Space Model

Probabilistic Models

Comparison of the Classic Models

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