Processamento e Recuperação de Informação

Efficient Similarity Search

Departamento de Engenharia Informática
Instituto Superior Técnico

1º Semestre
2018/2019

Outline

Bibliography

Jure Leskovec, Anand Rajaraman, and Jeff Ullman, Mining of Massive Datasets, Chapter 3

High Dimensional Data

Many real-world problems

- Web Search and Text Mining
  - Billions of documents, millions of terms
- Product Recommendations
  - Millions of customers, millions of products
- Scene Completion, other graphics problems
  - Image features
- Online Advertising, Behavioral Analysis
  - Customer actions (e.g., websites visited, searches)

A Common Metaphor

Many problems can be expressed as finding similar sets.

Find near-neighbors in high-dimensional space.

Examples:

- Pages with similar words
  - For duplicate detection, classification by topic
- Customers who purchased similar products
  - NetFlix users with similar tastes in movies
- Products with similar customer sets
- Images with similar features
- Users who visited the similar websites

Distance Measures

We formally define near neighbors as points that are a small distance apart.

For each use case, we need to define what distance means.

Two major classes of distance measures:

- A Euclidean distance is based on the locations of points in such a space
- A Non-Euclidean distance is based on properties of points, but not their location in a space
  - Cosine similarity, Jaccard similarity coefficient, ...

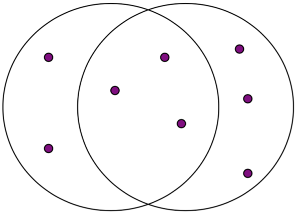
Jaccard Similarity

The Jaccard Similarity of two sets is the size of their intersection over the size of their union.

$$Sim(C_1, C_2) = \frac{|C_1 \cap C_2|}{|C_1 \cup C_2|}$$

The Jaccard Distance between sets is 1 minus their Jaccard similarity.

$$d(C_1, C_2) = 1 - \frac{|C_1 \cap C_2|}{|C_1 \cup C_2|}$$



3 in intersection
8 in union
Jaccard similarity= 3/8
Jaccard distance = 5/8

Outline

Finding Similar Documents

Goal:

Given a large number (N in the millions or billions) of text documents, find pairs that are near duplicates

Applications:

- Mirror websites, or approximate mirrors
  - Don't want to show both in a search
- Similar news articles at many news sites
  - Cluster articles by same story

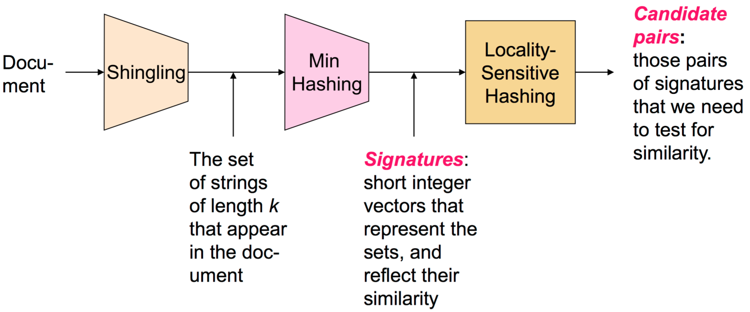
Problems:

- Many small pieces of one doc can appear out of order in another
- Too many docs to compare all pairs
- Docs are so large or so many that they cannot fit in main memory

Three Essencial Steps

1. Shingling: Convert documents, emails, etc., to sets;
2. Minhashing: Convert large sets to short signatures, while preserving similarity;
   - Depends on the distance metric;
3. Locality-sensitive hashing: Focus on pairs of signatures likely to be from similar documents.

The Big Picture



Outline

Documents as High Dimensional Data

Step 1:

Shingling: Convert documents, emails, etc., to sets

- Simple approaches...
  - Document = set of words appearing in document
  - Document = set of *important* words
- ...don't work well for this application!
  - Need to account for ordering of words
- A different way: Shingles

Shingles

- A k-shingle (or k-gram) for a document is a sequence of k tokens that appears in the document
  - Tokens can be characters, words or something else, depending on application
  - Assume tokens = characters for next examples

Example: $k = 2$; $D_1 = abcab$

Set of 2-shingles: $S(D_1) = \{ab, bc, ca\}$

Option: Shingles as a bag (i.e., multi-set), counting ab twice

- Represent a doc by the set of hash values of its k-shingles

Compressing Shingles

- To compress long shingles, we can hash them into a convenient/efficient representation (e.g. 4 bytes)
- Represent a doc by the set of hash values of its k-shingles
- Idea: Two documents could (rarely) appear to have shingles in common, when in fact only the hash-values were shared

Example: $k = 2$; $D_1 = abcab$

Set of 2-shingles: $S(D_1) = \{ab, bc, ca\}$

Hash the shingles: $h(D_1) = \{1, 5, 7\}$

Similarity Metric for Shingles

- Document D_1 = set of k-shingles $C_1 = S(D_1)$
- Equivalently, each document is a $0/1$ vector in the space of k-shingles
  - Each unique shingle is a dimension
  - Vectors are very sparse
- A natural similarity measure is the Jaccard similarity:

$$Sim(C_1, C_2) = \frac{|C_1 \cap C_2|}{|C_1 \cup C_2|}$$

Working Assumption

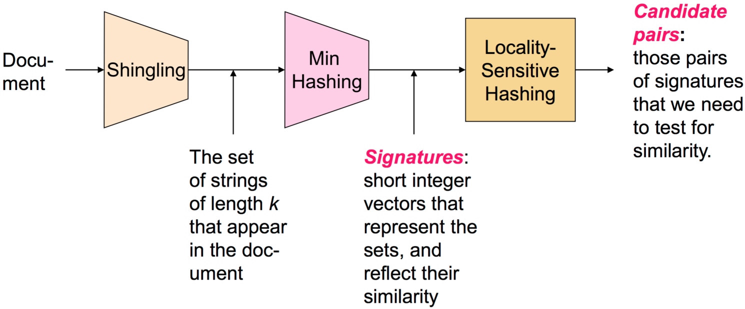
- Documents that have lots of shingles in common have similar text, even if the text appears in different order
- Careful: You must pick k large enough, or most documents will have most shingles
  - $k = 5$ is OK for short documents
  - $k = 10$ is better for long documents

Motivation for Minhash/LSH

- Suppose we need to find near-duplicate documents among $N = 1$ million documents
- Naively, we'd have to compute pairwaise Jaccard similarites for every pair of docs
  - i.e, $\frac{N \times (N-1)}{2} \approx 5 \times 10^{11}$ comparisons
  - At 10^5 secs/day and 10^6 comparisons/sec, it would take 5 days to compute all pairwaise Jaccard similarites
- For $N = 10$ million, it takes more than a year...

Outline

The Big Picture



Encoding Sets as Bit Vectors

Many similarity problems can be formalized as finding subsets hat have significant intersection

- Encode sets using 0/1 (bit, Boolean) vectors
- One dimension per element in the universal set
- Interpret set intersection as bitwise AND, and set union as bitwise OR

Example: $C_1 = 10111$; $C_2 = 10011$

- Size of intersection $= 3$;
- Size of union $= 4$
- Jaccard similarity (not distance) $= 3/4$
- $d(C_1, C_2) = 1 -$ (Jaccard similarity) $= 1/4$

From Sets to Boolean Matrices

- Rows = elements of the universal set
- Columns = sets
- 1 in row *e* and column *s* if and only if *e* is a member of *s*

- Column similarity is the Jaccard similarity of the sets of their rows with 1

- Typical matrix is sparse

shingles				
	1	0	1	0
	1	1	0	1
	0	1	0	1
	0	0	0	1
	0	0	0	1
	1	1	1	0
	1	0	1	0

documents

Jaccard of Columns

Each document is a column:

Example: $C1 = 1100011$; $C2 = 0110010$

- Size of intersection $= 2$;
- size of union $= 5$,
- Jaccard similarity (not distance) $= 2/5$
- $d(C_1, C_2) = 1$˘(Jaccard similarity) $= 3/5$

We might not really represent the data by a Boolean matrix

- Sparse matrices can be represented by the list of places with non-zero values

shingles

1	0	1	0
1	1	0	1
0	1	0	1
0	0	0	1
0	0	0	1
1	1	1	0
1	0	1	0

documents

Finding Similar Columns

So far:

- Documents represented as sets of shingles
- Represent sets as boolean vectors in a matrix

Next Goal: Find similar columns

1. Signatures of columns: small summaries of columns
2. Examine pairs of signatures to find similar columns
   - Essential that similarities of signatures and columns are related
3. Optional: check that columns with similar signatures are really similar

Warnings:

- Comparing all pairs may take too much time: job for LSH
- These methods can produce false negatives, and even false

Signatures of Columns

Key idea:

Hash each column C to a small signature $h(C)$, such that:

1. $h(C)$ is small enough that the signature fits in RAM
2. $sim(C_1, C_2)$ is the same as the similarity of signatures $h(C_1)$ and $h(C_2)$

Goal:

Find a hash function $h()$ such that:

- if $sim(C_1, C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
- if $sim(C_1, C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$

Hash docs into buckets, and expect that most pairs of near duplicate docs hash into the same bucket

Min-Hashing (1)

Goal:

Find a hash function $h()$ such that:

- if $sim(C_1, C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
- if $sim(C_1, C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$

Clearly, the hash function depends on the similarity metric:

- Not all similarity metrics have a suitable hash function
- There is a suitable hash function for Jaccard similarity: Min-hashing

Min-Hashing (2)

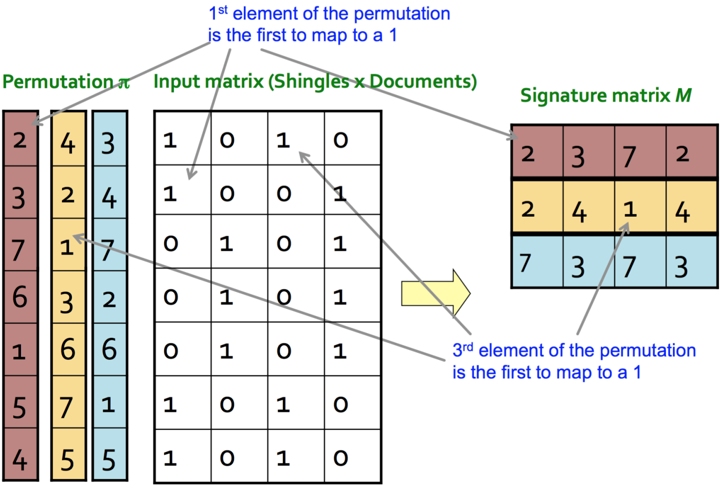
- Imagine the rows of the boolean matrix permuted under random permutation π

- Define a hash function $h_\pi(C)$ = the number of the first (in the permuted order π) row in which column C has value 1:

$$h_\pi(C) = \min_\pi(C)$$

- Use several (e.g., 100) independent hash functions (i.e., permutations) to create a signature of a column

Min-Hashing (3)



Surprising Property

Under a random permutation π,
$Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$

Sketch of proof:

- Let X be a set of shingles, and let $x \in X$
- Then: $Pr[\pi(y) = \min(\pi(X))] = \frac{1}{|X|}$
  - It is equally likely that any $y \in X$ is mapped to the min element
- Let x be s.t. $\pi(x) = \min(\pi(C_1 \cup C_2))$
- Then either $\pi(x) = \min(\pi(C_1))$ if $x \in C_1$, or $\pi(x) = \min(\pi(C_2))$ if $x \in C2$
- So the probability that both are true is the probability $x \in C1 \cap C2$
- $Pr[\min(\pi(C_1)) = \min(\pi(C_2))] = \frac{|C1 \cap C2|}{|C1 \cup C2|} = sim(C_1, C_2)$

Similarity for Signatures

- We know $Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- Now generalize to multiple hash functions
- The similarity of two signatures is the fraction of the hash functions in which they agree
- Note: Because of the minhash property, the similarity of columns is the same as the expected similarity of their signatures

Example

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	3	7	2
2	4	1	4
7	3	7	3

Similarities:

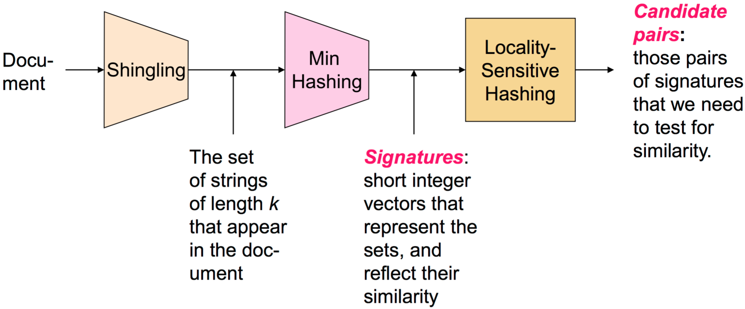
	1-3	2-4	1-2	3-4
Col/Col	0.75	0.75	0	0
Sig/Sig	0.33	0.67	0	0

Minhash Signatures

Pick 100 random permutations of the rows Think of $sig(C)$ as a column vector Let $sig(C)[i] =$ according to the i-th permutation, the index of the first row that has a 1 in column C $sig(C)[i] = min(\pi_i(C))$ Note: The sketch (signature) of document C is small – $\tilde{1}00$ bytes! We achieved the goal of compressing long bit vectors into short signatures

Outline

The Big Picture



LSH : General Intuition

Goal: Find documents with Jaccard similarity at least s (for some similarity threshold, e.g., s=0.8) LSH – Use a function f(x,y) that tells whether x and y is a candidate pair, i.e. a pair of elements whose similarity must be evaluated For minhash matrices: Hash columns of signature matrix M to many buckets Each pair of documents that hashes into the same bucket is a candidate pair

Candidates from Minhash

Pick a similarity threshold s, a fraction < 1 Columns x and y of M are a candidate pair if their signatures agree on at least fraction s of their rows:
M (i, x) $=$ M (i, y) for at least fraction s values of i
We expect documents x and y to have the same similarity as their signatures

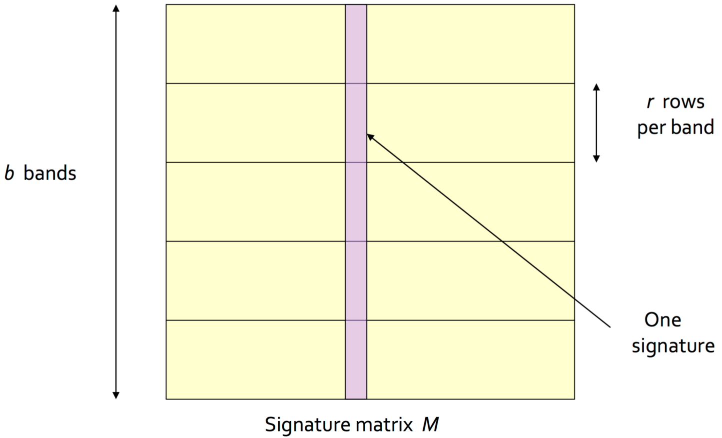
LSH for Minhash Signatures

Big idea: Hash columns of signature matrix M several times Arrange that (only) similar columns are likely to hash to the same bucket, with high probability Candidate pairs are those that hash to the same bucket

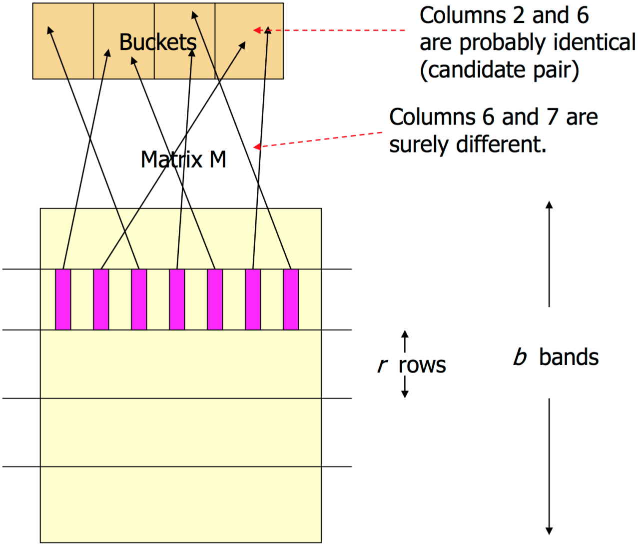
Partition M into Bands (1)

Divide matrix M into b bands of r rows For each band, hash its portion of each column to a hash table with k buckets Make k as large as possible Candidate column pairs are those that hash to the same bucket for 1 band or more Tune b and r to catch most similar pairs, but few non-similar pairs

Partition M into Bands (2)



Hashing Bands



Simplifying Assumption

There are enough buckets that columns are unlikely to hash to the same bucket unless they are identical in a particular band Hereafter, we assume that same bucket means identical in that band Assumption needed only to simplify analysis, not for correctness of algorithm

Example of Bands (1)

Assume the following case: Suppose 100,000 columns of M (100k docs) Signatures of 100 integers (rows) Therefore, signatures take 40Mb Choose 20 bands of 5 integers/band Goal: Find pairs of documents that are at least s = 80% similar

Example of Bands (2)

Assume: C1, C2 are 80% similar Since s=80Probability C1, C2 identical in one particular band: $(0.8)^5 = 0.328$ Probability C1, C2 are not similar in all of the 20 bands: $(1 - 0.328)^{20} = 0.00035$ i.e., about 1/3000th of the 80%-similar column pairs are false negatives We would find 99.965% pairs of truly similar documents

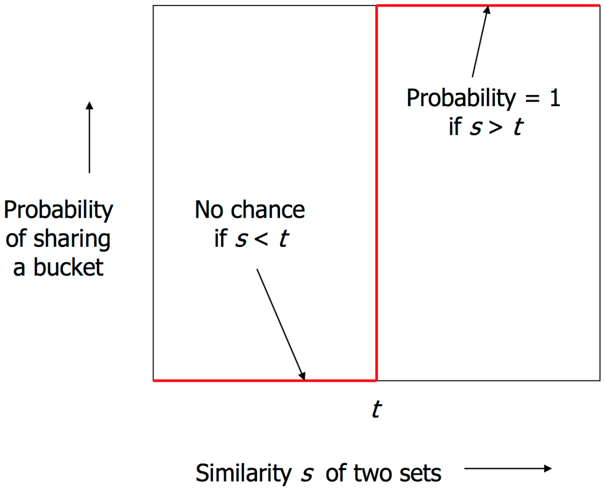
Example of Bands (3)

Assume: C1, C2 are 30% similar Since s=80Probability C1, C2 identical in one particular band: $(0.3)^5 = 0.00243$ Probability C1, C2 identical in at least 1 of 20 bands: $1 - (1 - 0.00243)^{20} = 0.0474$ In other words, approximately 4.74% pairs of docs with similarity 30% end up becoming candidate pairs – false positives

LSH Involves a Tradeoff

Pick: the number of minhashes (rows of M) the number of bands b, and the number of rows r per band to balance false positives/negatives Example: if we had only 15 bands of 5 rows, the number of false positives would go down, but the number of false negatives would go up

Analysis of LSH - What We Want



Analysis of LSH - What One Band With One Row Gives

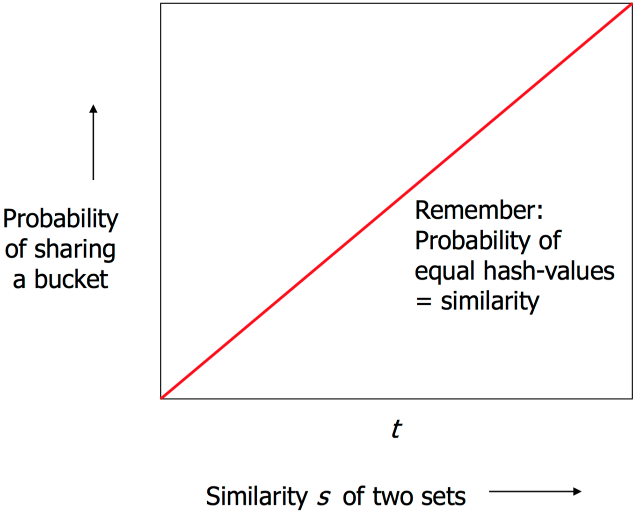
Processamento e Recuperação de Informação

Finding Similar Items

Shingles

Minhashing

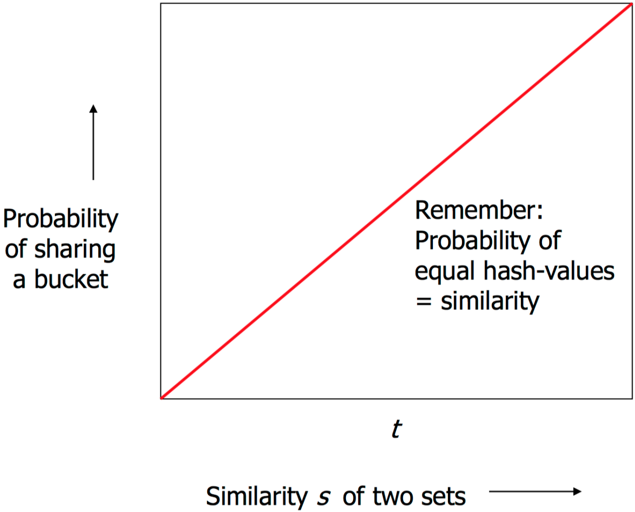
Locality-sensitive hashing



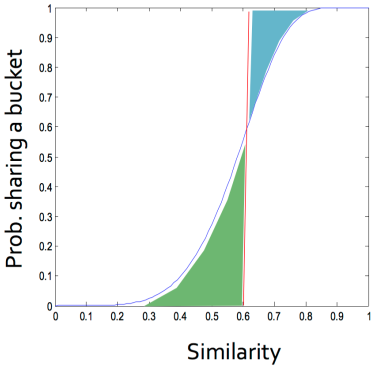
Analysis of LSH - b bands with b rows/band

Columns C_1 and C_2 have similarity s Pick any band (r rows) Prob. that all rows in band equal $= s^r$ Prob. that some row in band unequal $= 1 - s^r$ Prob. that no band identical $=$ $(1 - s^r)^b$ Prob. that at least 1 band identical $= 1 - (1 - s^r)^b$

Analysis of LSH - What b Bands With r Rows Gives



False Positives vs. False Negatives

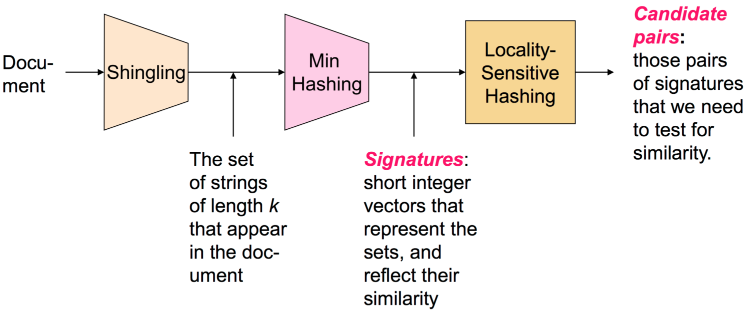


Blue area: False Negative rate
Green area: False Positive rate

LSH Summary

Tune to get almost all pairs with similar signatures, but eliminate most pairs that do not have similar signatures Check in main memory that candidate pairs really do have similar signatures Optional: In another pass through data, check that the remaining candidate pairs really represent similar documents

The Big Picture



Questions?