Eccentric anomaly

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ABSTRACT

In orbital mechanics, the eccentric anomaly is an angular parameter that defines the position of a body that is moving along an elliptic Kepler orbit. The eccentric anomaly is one of three angular parameters ("anomalies") that define a position along an orbit, the other two being the true anomaly and the mean anomaly.

Key words: miscellaneous

1 INTRODUCTION

This is a copy-paste of the Wikipedia article of the same name, which is based on Murray (1999); Plummer (1960). The point of this exercise is to get familiarized with the style conventions of scientific journals (here, MNRAS).

2 GRAPHICAL REPRESENTATION

Consider the ellipse with equation given by:

$$x = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \tag{1}$$

where a is the semi-major axis and b is the semi-minor axis.

For a point on the ellipse, $P = P(x, y)$, representing the position of an orbiting body in an elliptical orbit, the eccentric anomaly is the angle E in the figure. The eccentric anomaly E is one of the angles of a right triangle with one vertex at the center of the ellipse, its adjacent side lying on the major axis, having hypotenuse a (equal to the semi-major axis of the ellipse), and opposite side (perpendicular to the major axis and touching the point P' on the auxiliary circle of radius a) that passes through the point P. The eccentric anomaly is measured in the same direction as the true anomaly, shown in the Fig. 1 as f. The eccentric anomaly E in terms of these coordinates is given by:

$$\cos E = \frac{x}{a}, \tag{2}$$

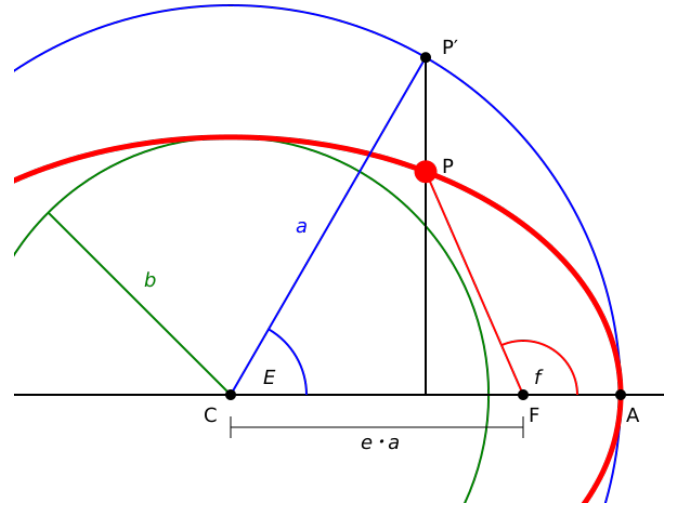and

$$\sin E = \frac{y}{b}. \tag{3}$$

The second equation is established using the relationship

$$\left(\frac{y}{b}\right)^2 = 1 - \cos^2 E = \sin^2 E, \tag{4}$$

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Figure 1. The eccentric anomaly of point P is the angle E. The center of the ellipse is point C, and the focus is point F.

which implies that $\sin E = \pm\frac{y}{b}$. The equation $\sin E = \frac{-y}{b}$ is immediately able to be ruled out since it traverses the ellipse in the wrong direction. It can also be noted that the second equation can be viewed as coming from a similar triangle with its opposite side having the same length y as the distance from P to the major axis, and its hypotenuse b equal to the semi-minor axis of the ellipse.

3 CONCLUSIONS

Hence, the eccentric anomaly is one of the parameters that determines the position of a body on an elliptic Kepler orbit.

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DATA AVAILABILITY

The official MNRAS template and guide can be ontained at the Comprehensive TeX Archive Network (CTAN) site in their directory (https://www.ctan.org/tex-archive/macros/latex/contrib/mnras). Sources of this file can be obtained from the Git repository (https://github.com/paveloom-university/Graphics-in-Scientific-Publications-S10-2022).

REFERENCES

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