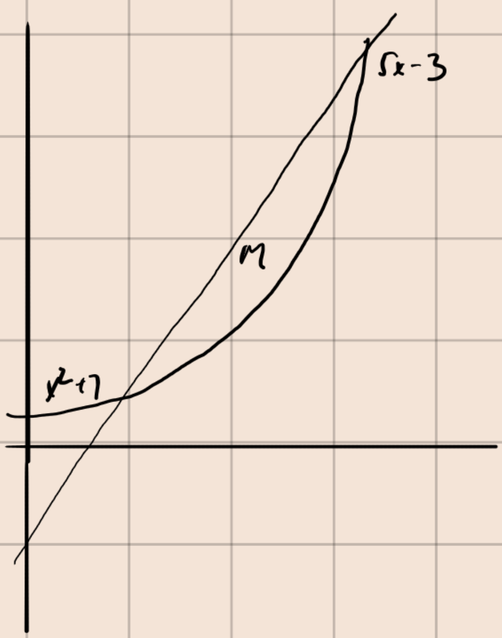
12a)

$$\int_{(c,+)} (ye^x, 2e^x)\, ds \overset{\text{Green}}{=} \int_\Omega \left( \frac{\partial(2e^x)}{\partial x} - \frac{\partial(ye^x)}{\partial y} \right) d\lambda^2 = \int_0^3 \int_0^4 \left(2e^x - e^x\right) dy\, dx$$


13a) $$\int_M 1\, d\lambda^2 = \int_M \left( \frac{\partial(x)}{\partial x} - \frac{\partial 0}{\partial y} \right) d\lambda^2 \overset{\text{green}}{=} \int_{(c,+)} (0, x)\, ds = *$$




$x^2+1 = 5x-3$

$(x-1)(x-4) = 0$

$c_1 \dots \varphi_1(t) = (t, t^2+1),\ t \in [1,4]$

(správný směr)

$c_2 \dots \varphi_2(t) = (t, 5t-3),\ t \in [1,4]$

(opačný směr)

$$* = \int_{(c_1,+)} (0,x)\, ds - \int_{(c_2,+)} (0,x)\, ds = \int_1^4 (0,t)(1,2t)\, dt - \int_1^4 (0,t)(1,5)\, dt =$$

$$= \int_1^4 2t^2\, dt - \int_1^4 5t\, dt = \left[ \frac{2t^3}{3} - \frac{5t^2}{2} \right]_1^4 = \frac{9}{2}$$

Potenciál vektorového pole

7) $F(x,y) = (\alpha xy + y^2, x^2 + 2xy)$

$$\left.\begin{aligned} \frac{\partial(x^2+2xy)}{\partial x} &= 2x+2y \\ \frac{\partial(\alpha xy + y^2)}{\partial y} &= \alpha x + 2y \end{aligned}\right\} \alpha = 2$$

$F \in C^1(\mathbb{R})$ ... nutná i postačující

9) $F(x,y) = (3+2xy^2, 2x^2y)$

a) $$\left.\begin{aligned} \frac{\partial(2x^2y)}{\partial x} &= 4xy \\ \frac{\partial(3+2xy^2)}{\partial y} &= 4xy \end{aligned}\right\} \begin{aligned} &F \in C^1(\mathbb{R}) \\ &\Rightarrow \text{je potenciální} \end{aligned}$$

Hledáme $f$: $\nabla f = F$ : $\frac{\partial f}{\partial x} = F_1$ , $\frac{\partial f}{\partial y} = F_2$

$$f_{C}(y) = \int F_1 \, dx = \int (3+2y^2)\, dx = 3x + \frac{2x^2}{2} y + C(y)$$

$\frac{\partial f_C(y)}{\partial y} = 2x^2y + C'(y)$, chceme: $F_2 = 2x^2y \Rightarrow C'(y) = 0$, $C(y) = C$, $C \in \mathbb{R}$

potenciál:

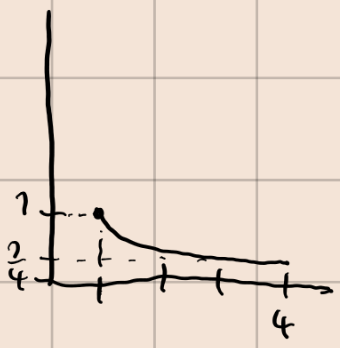
$f_C(x,y) = 3x + x^2y^2 + C$, $C \in \mathbb{R}$

$f(0,0) = 0$

$0 = f_C(0,0) = 0 + 0 + C \Rightarrow C = 0$

$f(x,y) = 3x + x^2y^2$

6) $\int_C F\, ds = f(4, \frac{1}{4}) - f(1,1) = 3 \cdot 4 + 16 \cdot \frac{1}{16} - 3 - 1 = 9$




1c) $y = \frac{1}{1+x^2}$


$x = u$

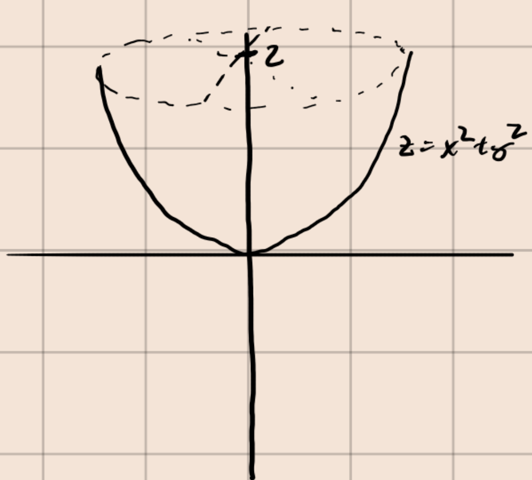
$y = \frac{1}{1+u^2} \cos v$

$z = \frac{1}{1+u^2} \sin v$

$u \in [-1, 1]$

$v \in [-\pi, \pi]$

2b) $\int_S 1$




$$\left.\begin{aligned} x &= \sqrt{z} \cdot \cos v \\ y &= \sqrt{z} \cdot \sin v \\ z &= z \end{aligned}\right\} \quad \phi(u,v) = (u \cdot \cos v, u \cdot \sin v, u^2)$$

$u \in [0, \sqrt{z}]$

$v \in [0, 2\pi]$

$\frac{\partial \phi}{\partial u} = (\cos v, \sin v, 2u)$

$\frac{\partial \phi}{\partial v} = (-u \cdot \sin v, u \cdot \cos v, 2)$

$$\frac{\quad}{\partial v} = ( \quad , \quad , \quad )$$

$$\frac{\partial \phi}{\partial u} \times \frac{\partial \phi}{\partial v} = \left(2u^2 \cos v, -2u^2 \sin v, u \cdot \cos^2 v + u \cdot \sin^2 v\right)$$