$y' + y = 0$

$y_h = e^{-x} \cdot c_1$

$\lambda_1 = 1$

$y' + y = e^x$

$y_p = C \cdot e^x$

$y_p' = c \cdot e^x$

$c \cdot e^x + c \cdot e^x = e^x$

$c = \frac{1}{2}$

$y_p = \frac{1}{2} e^x$

$y' + y = x$

$y = Ax + B$

$y' = A$

$A + Ax + B = y$

$A = 1$

$B = -1$

$y_p = x - 1$

$y' + y = \sin x$

$y_p = A \cdot \sin x + B \cdot \cos x = \frac{1}{2} \sin x - \frac{1}{2} \cos x$

$y_p' = A \cdot \cos x - B \cdot \sin x$

$A \cdot \cos x - B \cdot \sin x + A \cdot \sin x + B \cdot \cos x = \sin x + 0$

$(-B + A) \sin x = \sin x \Rightarrow A - B = 1$

$\cos x \, (A + B) = 0$

$A = \frac{1}{2}$

$B = -\frac{1}{2}$

$y' + y = e^{-x}$

$y_p = C \cdot e^{-x} \cdot x$

$y_p' = e^{-x} \cdot C \cdot (1 - x)$

$e^{-x} \cdot C \cdot [(1 - x) + x] = e^{-x}$

$C \cdot e^{-x} = e^{-x} \Rightarrow C = 1$

$C \cdot e = e \Rightarrow \underline{C = 1}$

$y_p = x \cdot e^{-x}$

$y''' - 9y' =$ $\lambda = 0, -3, 3$ $\{1, e^{-3x}, e^{3x}\}$	$y'' - 4y' + 4y =$ $\lambda = 2\,(2x)$ $\{e^{2x}, x \cdot e^{2x}\}$	$/ = b(x)$ $/ \alpha + \beta i$
$e^{-3x}(Ax+B) \cdot x$	$e^{-3x}(Ax+B)$	$= x \cdot e^{-3x}$ $/ -3$
$e^{2x} \cdot (A \cdot \sin(x) + B \cdot \cos(x))$	$e^{2x}(A \cdot \sin(x) + B \cdot \cos(x))$	$= 3e^{2x} \cdot \cos(x)$ $/ 2 \pm i$
$x \cdot (Ax^2 + Bx + C) + D \cdot e^{2x}$	$(Ax^2 + Bx + C) \cdot D \cdot e^{2x} \cdot x^2$	$= 5x^2 - 7 + e^{2x}$
$(Ax+B) \cdot \sin(\pi x) + (Cx+D) \cdot \cos(\pi x)$	$(Ax+B) \cdot \sin(\pi x) + (Cx+D) \cdot \cos(\pi x)$	$= 13x + \cos(\pi x)$ $/ 0 \pm \pi \cdot i$
$A \cdot e^{x} + B \cdot e^{3x} \cdot x$	$A \cdot e^{x} + B \cdot e^{3x}$	$= e^{x} - 3e^{3x}$
$x \cdot (Ax+B) + C \cdot \sin(3x) + D \cdot \cos(3x)$	$(Ax+B) + C \cdot \sin(3x) + D \cdot \cos(3x)$	$= 2x - \sin(3x)$

podobné by mohlo být na zkoušce ↓

$y'' - y' = 4 \cdot \sin(x) - 2x$

$y(0) = 3, \quad y'(0) = 0$

1. $y'' - y' = 0$

$\lambda^2 - \lambda = \lambda(\lambda - 1) = 0$

$\lambda_1 = 0$

$\lambda_2 = 1$

$y_h = C_1 \cdot 1 + C_2 \cdot e^x = C_1 + C_2 \cdot e^x$

2. $y_p = A \cdot \sin(x) + B \cdot \cos(x) + (Cx + D)x$

$y_p = A \cdot \sin(x) + B \cdot \cos(x) + Cx^2 + Dx$

$y_p' = A \cdot \cos(x) - B \cdot \sin(x) + 2Cx + D$

$y_p'' = -A \cdot \sin(x) - B \cdot \cos(x) + 2C$

$-A.\sin(x) - B.\cos(x) + 2C - A.\cos(x) + B.\sin(x) - 2Cx - D$

$= 4.\sin(x) - 2x$

$$\left.\begin{array}{r} -A+B=4 \\ -B-A=0 \\ C-D=0 \\ -2C=-2 \end{array}\right\} \begin{array}{l} B=2 \\ A=-2 \\ C=-1 \\ D=2 \end{array}$$

$y_p = -2\sin(x) + 2.\cos(x) + x^2 + 2x$

3 $\quad y = y_p + y_h$

$y = -2\sin(x) + 2.\cos(x) + x^2 + 2x + C_1 + C_2.e^x$

$y' = -2.\cos(x) - 2.\sin(x) + 2x + 2 + 2.e^x$

$y_0 = -2.\sin(0) + 2.\cos(0) + C_1 + C_2.e^0$

$= 2 + C_1 + C_2 = 3 \quad \rightarrow C_1 = 1$

$y'(0) = -2 + 2 + C_2 = 0 \quad \rightarrow C_2 = 0$

$$y = -2 \cdot \sin(x) + 2 \cdot \cos(x) + x^2 + 2x + 1$$

Samostatně:

- $y''' - y'' + y' - y = 10e^{2x} - x$

↳ obecné řešení + chování v ∞

- $y'' - y = 9e^{2x}$
- $y'' - 4y' + 4y = -e^{2x}$
- $y'' + 3y' + 2y = \sin(x) + 3 \cdot \cos(x)$
- $y'' - 4y' = -8x$

hom. rovnice

+ návrh part. řešení

+ chování v ∞

$$y''' - y'' + y' - y = 10e^{2x} - x$$

$$\lambda^3 - \lambda^2 + \lambda - 1 = 0$$

$$\lambda^2(\lambda - 1) + (\lambda - 1)$$

$$(\lambda - 1)(\lambda^2 + 1) \qquad \lambda^2 + 1 = 0$$

$\lambda_1 = 1$ $\qquad$ $\lambda^2 = -1$

$\lambda_{2,3} = \pm i$

$y_h = C_1 + C_2 i - C_3 i$

$y_p = (Ax+B) + C \cdot e^{2x}$

$y_p = Ax \cdot C \cdot e^{2x} + B \cdot C \cdot e^{2x}$

$y_p' = A \cdot e^{2x} + e^{2x}$

$y_p' = e^{2x} + e^{2x}$

$y_p'' = e^{2x} + e^{2x}$

$y_p'' = e^{2x} + e^{2x}$

$\cancel{e^{2x} + e^{2x}} \cancel{- e^{2x} - e^{2x}} + e^{2x} + e^{2x} - A \cdot e^{2x} - e^{2x} =$

$10e^{2x} - x$

$e^{2x} - A \cdot e^{2x} = 10e^{2x} - x$

$e^{2x} = (10+A)e^{2x} - x$

$e^{2x} + x = (10+A)e^{2x}$

$x = (A+9)e^{2x}$