Def. obor, limity v hraničních bodech def. oboru

1. $f(x) = \dfrac{\ln(2-x)}{x^2-2x-3}$

def. obor:

$2-x>0 \Rightarrow x<2$

$x^2-2x-3 = (x-3)(x+1) \neq 0$

$\rightarrow x \neq 3$

$\rightarrow x \neq -1$

$$\boxed{D_f = (-\infty; -1) \cup (-1; 2)}$$

f

a) $\displaystyle\lim_{x\to(-1)^-} \frac{\ln(2-x)}{(x+1)(x-3)} = \frac{\ln(2-x)}{0^+} = \infty$

vsf $x \to -1^-$ $\to \ln(3) > 0$; $(x+1) \to 0^-$; $(x-3) \to -4$

b) $\displaystyle\lim_{x\to(-1)^+} \frac{\ln(2-x)}{(x+1)(x-3)} = \frac{\ln(2-x)}{0^-} = -\infty$

$\ln(2-x) \to >0$; $(x+1) \to 0^+$; $(x-3) \to -4$

c) $\displaystyle\lim_{x\to 2^-} \frac{\ln(2-x)}{(x+1)(x-3)} = \frac{-\infty}{-3} = \infty$

0^+; 0^+; $(x+1) \to 3$; $(x-3) \to -1$

d) $\displaystyle\lim_{x\to -\infty} \frac{\ln(2-x)}{(x+1)(x-3)} = \left[\text{neurč. tvar } \frac{\infty}{\infty}\right] \overset{?}{\underset{\text{LH}}{=}} \frac{\frac{-1}{2-x}}{2x-2}$

$\ln(2-x) \to \infty$

$$\stackrel{?}{=} \frac{-1}{(2-x)(2x-2)} = \frac{-1}{-\infty} \underset{\text{omez. miz}}{=} 0$$

∞ $\quad -\infty$

2. $f(x) = \dfrac{\cos 2x - 1}{2^x - 1}$

$2^x - 1 = 0$

$2^x = 1$

$2^x = 2^0$

$\rightarrow x \neq 0$

$D_f = (-\infty; 0) \cup (0; \infty)$

a) $\displaystyle\lim_{x \to -\infty} \frac{\cos(2x) - 1}{2^x - 1}$ = lim. neexistuje

$\rightarrow$ osciluje

0 $\quad \rightarrow -1$

omez.

b) $\displaystyle\lim_{x \to \infty} \frac{\cos(2x) - 1}{2^x - 1} = 0$

omez. miz.

1^+ $\quad \infty$ $\quad \rightarrow 0$

c) $\displaystyle\lim_{x \to 0+} \frac{\cos(2x) - 1}{2^x - 1}$ = [neurč. výraz $\frac{0}{0}$] $\stackrel{?}{=}_{L'H}$ $\displaystyle\lim \frac{-2\sin(2x)}{2^x \cdot \ln(2)}$

1^+ $\quad \rightarrow 0$ $\quad 1$ $\quad ? \; 0$

$\rightarrow 0$ $\quad ? \; 0$

$= 0$

d) $$\lim_{x \to 0^-} \frac{\cos(2x) - 1}{\underbrace{2^x - 1}_{\to 0}} = \left[\text{neurč. výraz } \frac{0}{0}\right] \overset{l'H}{=} \lim \frac{-2\sin(2x)}{2^x \cdot \ln(2)}$$

$= 0$