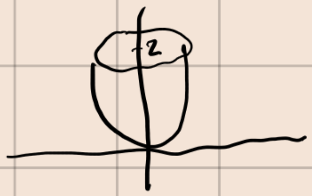
2b) 

$$\varphi(u,v) = (u\cdot\cos v,\ u\cdot\sin v,\ u^2)$$

$$u \in [0,\sqrt{2}],\ v \in [0,2\pi]$$

$$\int_S 1\, d\sigma = \int_0^{\sqrt{2}}\int_0^{2\pi} 1\cdot\|(\cos v,\ \sin v,\ 2u)\times(-u\cdot\sin v,\ u\cdot\cos v,\ 0)\|\, dv\, du$$

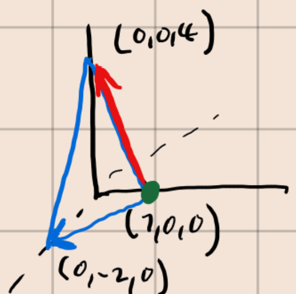
$$= \int_0^{\sqrt{2}}\int_0^{2\pi} \|(-2u^2\cdot\cos v,\ -2u^2\sin v,\ u\cdot\cos^2 v)\|\, dv\, du =$$

$$= \int_0^{\sqrt{2}}\int_0^{2\pi}\sqrt{4u^4\cdot\cos^2 v + 4u^4\sin^2 v + u^2}\, dv\, du = \int_0^{\sqrt{2}}\int_0^{2\pi}\sqrt{4u^4+u^2}\, dv\, du$$

$$= 2\pi\int_0^{2\pi} u\cdot\sqrt{4u^2+1}\, du = \left| \begin{matrix} 4u^2+1=t \\ 8u\, du = dt \end{matrix} \right| = \frac{2\pi}{8}\int_1^9 \sqrt{t}\, dt = \frac{\pi}{2}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^9 =$$

$$= \frac{\pi}{6}(27-1) = \frac{26}{6}\pi = \frac{13}{3}\pi$$

3a) $\int_S x\, d\sigma$

S: 

parametrizace:

$$\varphi(u,v) = (1,0,0) + u\big((0,-2,0)-(1,0,0)\big) + v\big((0,0,4)-(1,0,0)\big)$$

$$= (1-u-v,\ -2u,\ 4v)$$

konvexni kombinace:

$$(1-u-v) + u + v = 1$$

$$0 \le 1-u-v \le 1$$

$$v \le 1-u$$

$$u \in [0,1],\ v \in [0,1-u]$$

$$\int_S x \, d\sigma = \int_0^1 \int_0^{1-u} (1-u-v) \cdot \|(-1,-2,0) \times (-1,0,4)\| \, dv \, du =$$

$$= \int_0^1 \int_0^{1-u} (1-u-v) \cdot \|(-8,4,-2)\| \, dv \, du =$$

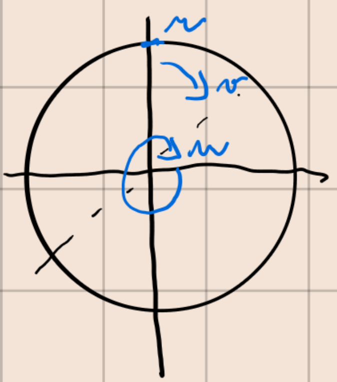
$$= \int_0^1 \int_0^{1-u} (1-u-v)\sqrt{64+16+4} \, dv \, du = \sqrt{84} \cdot \int_0^1 \left[v - uv - \frac{v^2}{2} \right]_0^{1-u} =$$

$$= 2\sqrt{21} \int_0^1 \left(1-u-u(1-u) - \frac{(1-u)^2}{2}\right) du = 2\sqrt{21} \int_0^1 \left(1-u-u+u^2 + \frac{1}{2} + u - \frac{u^2}{2}\right) du$$

$$= 2\sqrt{21} \int_0^1 \left(\frac{1}{2} - u + \frac{u^2}{2}\right) du = 2\sqrt{21} \left[\frac{1}{2}u - \frac{u^2}{2} + \frac{u^3}{6}\right]_0^1 =$$

$$= 2\sqrt{21}\left(\frac{1}{2} - \frac{1}{2} + \frac{1}{6}\right) = \frac{\sqrt{21}}{3} = \sqrt{\frac{7}{3}}$$

Sféra $S(0,r)$ - fixní poloměr



$$\phi(u,v) = (r \cdot \cos u \cdot \sin v, r \cdot \sin u \cdot \sin v, r \cdot \cos v)$$

$$u \in [0, 2\pi], \; v \in [0, \pi]$$

$$\frac{\partial \phi}{\partial u} \times \frac{\partial \phi}{\partial v} = (-r \cdot \sin u \cdot \sin v, r \cdot \cos u \cdot \sin v, 0) \times (r \cdot \cos u \cdot \cos v, r \cdot \sin u \cdot \cos v, -r \cdot \sin v)$$

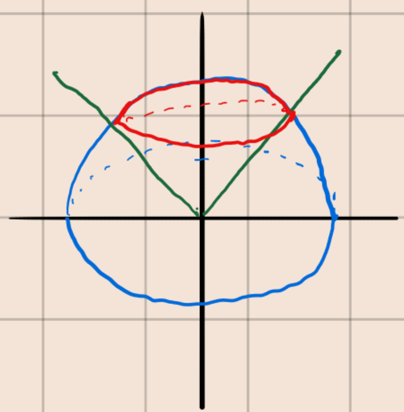
$$= (-r^2 \cos u \cdot \sin^2 v, -r^2 \sin u \cdot \sin^2 v, -r \cdot \sin^2 u \cdot \sin v \cdot \cos v - r^2 \cos^2 u \cdot \sin v \cdot \cos v)$$

$= \left(-u^2 \cos u \cdot \sin^2 v, -u^2 \sin u \cdot \sin^2 v, -u^2 \sin v \cdot \cos v\right)$

$$\left\|\frac{\partial\phi}{\partial u} \times \frac{\partial\phi}{\partial v}\right\| = \frac{\left(-u^2 \cos u \cdot \sin^2 v, -u^2 \sin u \sin^2 v, -u^2 \sin v \cdot \cos v\right)}{\underbrace{\sqrt{u^4 \cos^2 u \sin^4 v + u^4 \sin^2 u \sin^4 v + u^4 \sin^2 v \cos^2 v}}}$$

$$= \sqrt{u^4 \sin^4 v + u^4 \sin^2 v \cdot \cos^2 v} = \sqrt{u^4 \sin^2 v} = \sqrt{u^2 \sin v}$$

3b) $\displaystyle\int_S y^2 = \int_0^{2\pi}\int_0^{\frac{\pi}{4}} \sin^2 u \cdot \sin^2 v \cdot \sin v \, dv \, du = \int_0^{2\pi} \sin^2 u \, du \cdot \int_0^{\frac{\pi}{4}} \sin^3 v \, dv$



$\phi(u,v) = (\cos u \cdot \sin v, \sin u \cdot \sin v, \cos v)$

$u \in [0, 2\pi], v \in [0, \frac{\pi}{4}]$

$\left\|\frac{\partial\phi}{\partial u} \times \frac{\partial\phi}{\partial v}\right\| = \sin v$

$$= \pi \cdot \int_0^{\frac{\pi}{4}} \sin v \left(1 - \cos^2 v\right) dv = \left[\begin{array}{c} \cos v = t \\ -\sin v \, dv = dt \\ \begin{array}{c|c|c} v & 0 & \frac{\pi}{4} \\ \hline t & 1 & \frac{\sqrt{2}}{2} \end{array}\end{array}\right] = \pi \int_{\frac{\sqrt{2}}{2}}^{1} \left(1 - t^2\right) dt =$$

$$= \pi \left[t - \frac{t^3}{3}\right]_{\frac{\sqrt{2}}{2}}^{1} = \pi \left(1 - \frac{1}{3} - \frac{6\sqrt{2}}{12} + \frac{2\sqrt{2}}{24}\right) = \pi\left(\frac{2}{3} - \frac{5}{12}\sqrt{2}\right)$$

4a) $\displaystyle\int_{(S,\downarrow)} (y, -x, 2z) \cdot d\sigma = *$

$\phi(u,v) = (2\cos u \cdot \sin v, 2 \cdot \sin u \cdot \sin v, 2 \cdot \cos v)$

$$\frac{\partial\phi}{\partial u} \times \frac{\partial\phi}{\partial v} = \left(-4\cos u \cdot \sin^2 v, -4 \sin u \sin^2 v, \underbrace{-4 \sin v \cdot \cos v}_{\leq 0}\right)$$

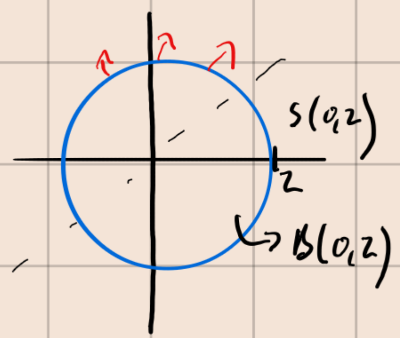
$u \in [0, 2\pi], v \in [0, \frac{\pi}{2}]$

$$* = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} (2\sin u \cdot \sin v, -2\cos u \cdot \sin v, 4\cos v) \bullet \left(\frac{\partial\phi}{\partial u} \times \frac{\partial\phi}{\partial v}\right) dv\, du =$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \left(-8\sin^3 v \cdot \sin u \cdot \cos u + 8\sin^2 v \cdot \sin u \cdot \cos u - 16 \cdot \sin v \cdot \cos^3 v\right) dv\, du$$

$$= 2\pi \int_0^{\frac{\pi}{2}} \left(-16 \cdot \sin v \cdot \cos^2 v\right) dv = \left|\text{stejná substituce}\right| = \frac{-32}{3}\pi$$

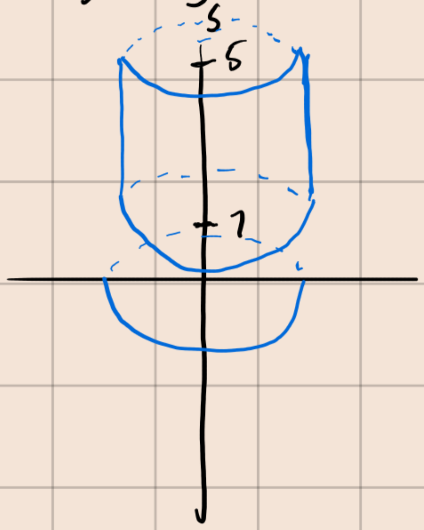
6a) $$\int_{(S,1)} (4x, y, 4z)\cdot d\sigma \overset{\text{Gauss}}{=} \int_{B(0,2)} \operatorname{div}(4x, y, 4z)\, d\lambda^3 =$$



$$= \int_{B(0,2)} (4+1+4)\, d\lambda^3 = 9 \int_{B(0,2)} 1\, d\lambda^3 = 9 \cdot \frac{4}{3}\pi \cdot 8 =$$

$$= 3 \cdot 32\pi = 96\pi$$

6c) $$\int_S (y^2, xz^3, (z-1)^2)\cdot d\sigma \overset{\text{Gauss}}{=} \int \operatorname{div}(y^2, xz^3, z^2-2z-1)\, d\lambda^3 =$$



$$= \int_1^5 \int_{\odot} (2z-2)\, d\lambda^3\, dz = \int_1^5 (2z-2)\cdot \pi 4^2\, dz =$$

$$= 32\pi \int_1^5 (z-1)\, dz = 32\pi \left[\frac{z^2}{2} - z\right]_1^5 =$$

$$= 32\pi \left[\frac{25}{2} - 5 - \frac{1}{2} + 1\right] = 8 \cdot 32\pi$$