2.12)

Hmotný bod

a se rovnoměrně zmenšuje

$a_0 = a(t=0) = 10\,m.s^{-2}$

$a(t=\tau=20s)=0$

$a(t)=?$

$\int dt$ $v(t)=?$

$\int dt$ $s(t)=?$

$\vec{v}_0=(0,0,0)$

$\vec{r}_0=(0,0,0)$ -- polohový vektor

$a(t)=a_0-k.t$

$a_0-k\tau=0 \rightarrow k=\frac{a_0}{\tau}$

$a(t)=a_0-\frac{a_0}{\tau}.t = a_0.\left(1-\frac{t}{\tau}\right)$

$a(t)=\frac{dv(t)}{dt} \rightarrow dv=a.dt$

$= a.dt \;\Big|\; \int_{v(0)}^{v(t)} \int_0^t$

$v(t)-v_0=\int_0^t a(t')\,dt' = a_0t - a_0\frac{t^2}{2\tau}$

$v(t)=\frac{ds}{dt} \rightarrow ds=v(t)\,dt$

$s(t)-s_0=\int_0^t v(t')\,dt' = \frac{a_0t^2}{2}-a_0\frac{t^3}{6\tau}+v_0t$

---

nerovnoměrný zrychlený pohyb

$\vec{\tau}=\frac{\vec{v}}{\|\vec{v}\|}$

$\vec{a}_N=\frac{v^2}{R}\cdot\vec{n}$

$\vec{a}=\frac{d\vec{v}}{dt}=\frac{d(v.\vec{\tau})}{dt}=\frac{dv}{dt}\vec{\tau}+v\frac{d\vec{\tau}}{dt}$

tečné zrychlení

normálové zrychlení

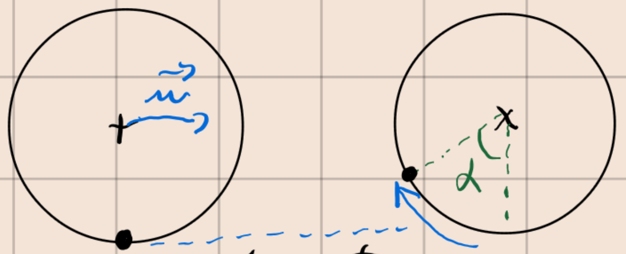
$\vec{a}_t=(\vec{a}\cdot\vec{\tau})\vec{\tau}=\left(\vec{a}\cdot\frac{\vec{v}}{v}\right)\frac{\vec{v}}{v}$

skal. součin

2.15 kamínek

auto se pohybuje konst. $\vec{u} = (u, 0, 0)$

poloměr kola R



$s = u \cdot t$

$\vec{v}(t) = ?$

$\vec{r}(t) = ?$ ... poloha kamínku

$\vec{a}(t) = ?$ ~ $a_t = ?$, $a_n = ?$

$$s = u \cdot t \to \alpha = \frac{s}{R} = \frac{u \cdot t}{R} = \omega t \quad \Big| \quad \omega = \frac{u}{R} \to u = R \cdot \omega$$

$$x(t) = u \cdot t - R \cdot \sin(\omega \cdot t) \qquad \xrightarrow{dt} \qquad \dot{x} = v_x = u - \underbrace{R\omega}_{u} \cos(\omega \cdot t) = u\,(1 - \cos(\omega \cdot t))$$

$$z(t) = R - R \cdot \cos(\omega \cdot t) \qquad \dot{z} = v_z = 0 + R\omega \cdot \sin(\omega \cdot t) = u \cdot \sin(\omega \cdot t)$$

$$v = \sqrt{v_x^2 + v_z^2} = \sqrt{u^2 (1 - \cos(\omega \cdot t))^2 + u^2 \cdot \sin^2(\omega \cdot t)}$$

$$= \sqrt{u^2 - 2u^2 \cos(\omega \cdot t) + u^2 \cdot \cos^2(\omega \cdot t) + u^2 \cdot \sin^2(\omega \cdot t)}$$

$$v = \sqrt{2u^2 - 2u^2 \cdot \cos(w \cdot t)} = u\sqrt{2(1-\cos(\underbrace{w \cdot t}_{\alpha}))}$$

$$v_{min} = 0 \quad (\alpha = 0)$$

$$v_{max} = 2u \quad (\alpha = \pi)$$

$$\left.\begin{aligned} x_{STŘED} &= u \cdot t \\ z_{STŘED} &= R \end{aligned}\right\} \to \vec{r} - \vec{r}_{STŘED} =$$

$$(-R\sin(w \cdot t), -R\cos(w \cdot t))$$

$$\ddot{x} = \dot{v}_x = a_x = 0 + Rw^2 \cdot \sin(w \cdot t)$$

$$\ddot{z} = \dot{v}_z = a_z = +Rw^2 \cdot \cos(w \cdot t)$$

$$\vec{a} = -w^2(\vec{r} - \vec{r}_{STŘED}) = \vec{a}_n$$

$$\vec{a} = -w^2 \cdot R$$

$$\vec{a} = (0, 0, -g)$$

$$\vec{a}_z = -g$$

$$g = 9{,}81 \ m \cdot s^{-2}$$

$$r_0 = (0, 0, h)$$

a) volný pád

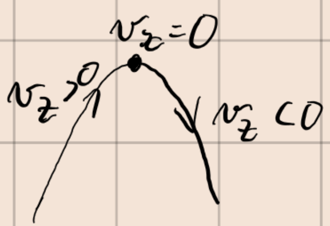
$$\vec{v}_0 = (0,0,0)$$
$$\vec{r}_0 = (0,0,h)$$
$$\vec{a} = (0,0,-g)$$

$$v(t) = v_{0z} - gt$$
$$z(t) = h + v_{0z} t - \frac{1}{2} g t^2 = h - \frac{1}{2} g t^2$$

$$t_{dopadu} = \sqrt{\frac{2h}{g}}$$

b) svislý vrh = rovn. pohyb nahoru + volný pád

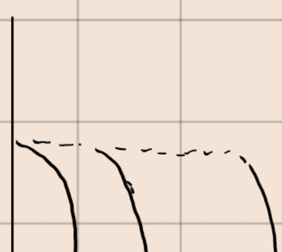
$$\vec{v}_0 = (0,0,v_{0z})$$
$$\vec{r} = (0,0,h)$$
$$\vec{a} = (0,0,-g)$$



c) vodorovný vrh

$$\vec{v}_0 = (v_{0x},0,0)$$
$$\vec{a} = (0,0,-g)$$
$$\vec{r}_0 = (0,0,h)$$

$$v_x(t) = v_{0x}$$
$$x(t) = v_{0x} \cdot t$$



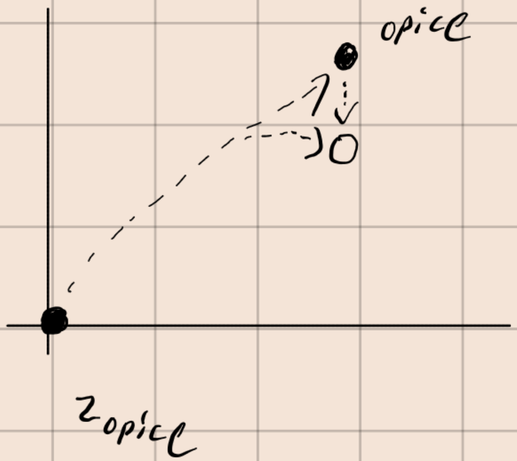
d) šikmý vrh = svislý vrh + rovn. pohyb v x

$$\vec{v_0} = (v_0 \cdot \cos\alpha, v_0 \cdot \sin\alpha)$$

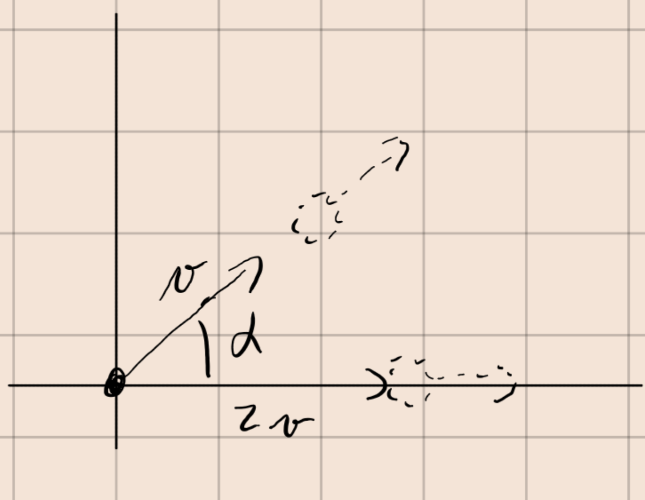
$$v_{0z} = v_0 \cdot \sin\alpha \to z(t) = h + v_0 \cdot t \cdot \sin\alpha - \frac{1}{2} g t^2$$

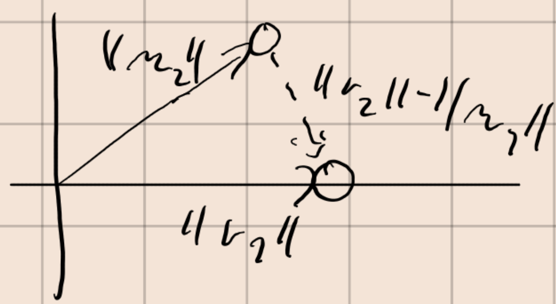
$$v_{0x} = v \cdot \cos\alpha \to x(t) = v_0 \cdot t \cdot \cos\alpha$$

2.22



působí stejné volné pády (stejné zrychlení g)





$$\vec F = G\frac{m_1 m_2}{r^2} = G\cdot\frac{m_1 m_2}{\|r_1 - r_2\|^2}\cdot\frac{-(r_1 - r_2)}{\|r_1 - r_2\|}$$

$$\vec r_1 = (2vt, 0)$$
$$\vec r_2 = (v\cdot t\cdot\cos\alpha, v\cdot t\cdot\sin\alpha) \qquad l(t) = \|\vec r_1 - \vec r_2\|$$
$$\vec r_1 - \vec r_2 = (2vt - vt\cos\alpha, -v t\cdot\sin\alpha)$$

<u>vrh střely:</u>

$$v_x(t) = v_{0x} = v_0\cdot\cos\alpha$$
$$x_s(t) = v_{0x}\cdot t = v_0 t\cdot\cos\alpha$$

$$z_s(t) = v_0 t\cdot\sin\alpha - \frac{1}{2}gt^2 + z = 0$$

<u>volný pád opice</u>

$$\vec v_0 = (0,0,0)$$
$$\vec r_0 = (d, 0, h) \longrightarrow x_{op} = d = x_s = v_0\cdot t_z\cdot\cos\alpha$$
$$\vec a = (0, 0, -g) \longrightarrow v_z(t) = -gt$$
$$z_{op}(t) = h - \frac{1}{2}gt^2$$
$$t_z = \frac{d}{v_0\cdot\cos\alpha}$$

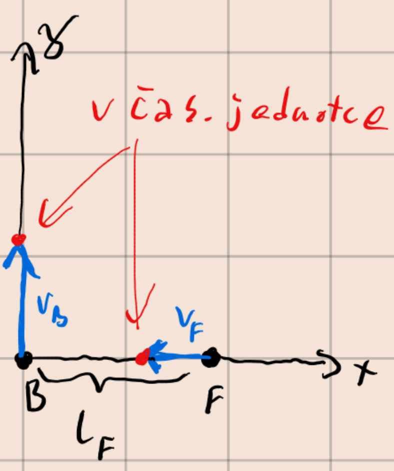
$$v_0 t\cdot\sin\alpha - \cancel{\frac{1}{2}gt^2} = h - \cancel{\frac{1}{2}gt^2}$$

$v_0 t \ldots = h$

$v_0 \cdot t_z \cdot \sin\alpha = h$

$$v_0 \cdot \frac{d}{v_0 \cos\alpha} \cdot \sin\alpha = h \rightarrow \operatorname{tg}\alpha = \frac{h}{d}$$

$\boxed{\text{úkol: 2.7}}$



- vzdálenost L_{FB} jako fce času
- t_n, L_n ... čas, vzdálenost, když si jsou nejblíže

Ferda v čase t

$$x_{F0} + v_F \cdot t$$

počáteční poloha Ferdy

Beruška v čase t:

$$v_B \cdot t \quad (\text{protože z } (0,0))$$

vzdálenost mezi B a F v čase t:

$$L(t) = \sqrt{(x_{F0} - v_F \cdot t)^2 + v_B^2 \cdot t^2}$$

minimální vzdálenost:

minimální vzdálenost

hledám stacionární bod, kde se mi bude měnit klesání vzájemné vzdálenosti na růst:

$$\frac{dl(t)}{dt} = 0$$

$$\frac{dl(t)}{dt} = \frac{1}{2}\cdot\frac{2(w_{F0} - v_F\cdot t)(-v_F) + 2(v_B t)\, v_B}{\sqrt{(w_{F0} - v_F\cdot t)^2 + v_B^2 t^2}}$$

$$= \frac{\cancel{2}(v_F^2 t + v_B^2 t - v_F w_{F0})}{\cancel{2}\sqrt{(w_{F0} - v_F\cdot t)^2 + v_B^2 t^2}}$$

$$\frac{v_F^2 t + v_B^2 t - v_F w_{F0}}{\sqrt{(w_{F0} - v_F\cdot t)^2 + v_B^2 t^2}} = 0$$

$$v_F^2 t + v_B^2 t - v_F w_{F0} = 0$$

$$t(v_F^2 + v_B^2) = v_F w_{F0}$$

$$t_n = \frac{v_F w_{F0}}{v_F^2 + v_B^2}$$

čas minimální vzdálenosti

$$l_n = \sqrt{\left(w_{F0} - v_F\cdot\frac{v_F\cdot w_{F0}}{\dots}\right)^2 + \left(v_B\,\frac{v_F\cdot w_{F0}}{\dots}\right)^2}$$

$$\sqrt{\left(v_{F0} \cdots v_F \frac{\cdots}{v_F^2+v_B^2}\right)^2 + \left(v_B \frac{\cdots}{v_F^2+v_B^2}\right)^2} =$$

$$= \sqrt{\left(v_{F0}\left(1-\frac{v_F^2}{v_B^2+v_F^2}\right)\right)^2 + \left(v_B \frac{v_F - v_{F0}}{v_F^2+v_B^2}\right)^2} =$$

$$= \sqrt{v_{F0}^2 \frac{v_B^4}{(v_B^2+v_F^2)^2} + v_{F0}^2 \frac{v_B^2 v_F^2}{(v_B^2+v_F^2)^2}}$$

$$= \sqrt{v_{F0}^2 \frac{v_B^4+v_B^2 v_F^2}{(v_B^2+v_F^2)^2}} = v_{F0} \frac{\sqrt{v_B^4+v_B^2 v_F^2}}{v_B^2+v_F^2} =$$

$$= v_{F0} \frac{v_B \sqrt{v_B^2+v_F^2}}{v_B^2+v_F^2} = \frac{v_{F0} \cdot v_B}{\sqrt{v_B^2+v_F^2}} \cdot \frac{\sqrt{v_B^2+v_F^2}}{v_B^2+v_F^2}$$

$$= \frac{v_{F0} \cdot v_B}{\sqrt{v_B^2+v_F^2}}$$