$$\int \frac{\sin^2 x+2}{\cos(x)+\cos x.\sin(x)}\,dx = \left| \begin{array}{l} \text{Sub.} \\ \sin x = y \\ \cos x\,dx = dy \end{array} \right| = \int \frac{(\sin^2 x+2).\cos x}{\underbrace{\cos^2 x}_{1-\sin^2 x}+\cos^2 x\,\sin(x)}\,dx =$$

$$\int \frac{y^2+2}{(1-y^2)(1+y)}\,dy = \int \frac{y^2+2}{(1-y)(1+y)^2}\,dy = -\int \frac{y^2+2}{(y-1)(y+1)^2}\,dy =$$

$$= -\frac{3}{2(\sin(x)+1)} - \frac{1}{4}\ln(1-\sin(x)) + C, \quad x \in \left(-\frac{\pi}{2}+k\pi, \frac{\pi}{2}+k\pi\right), k \in \mathbb{Z}$$

$$\int \frac{\sin^6 x+\cos^5 x}{\sin x.\cos^3 x+\sin^3 x}\,dx = \left| \begin{array}{l} \text{Sub.} \\ \cos x = t \\ \sin x\,dx = dt \end{array} \right.$$

$$\int \sin^5 x.\cos^3 x\,dx = \left| \begin{array}{l} \text{Sub.} \\ \sin x = t \\ \cos x\,dx = dt \end{array} \right| = \frac{1}{6}\sin^6 x - \frac{1}{8}\sin^8 x + C, \; x \in \mathbb{R}$$

$$\int_0^\pi x.\cos(x)\,dx = \left| \begin{array}{ll} \text{P.P.} & \\ u = x & v' = \cos x \\ u' = 1 & v = \sin x \end{array} \right| = \Big[x.\sin\Big]_0^\pi - \int_0^\pi 1 - \sin x\,dx = (0-0) - \Big[-\cos x\Big]_0^\pi = -1-1 = -2$$

Doma

$$\int_0^\pi x.\sin(2x)\,dx = \dots = -\frac{\pi}{2}$$

$$\int_0^{\frac{1}{2}\ln 3} \frac{e^x}{e^{2x}+1}\,dx = \left| \begin{array}{l} \text{Sub.} \\ e^x = t \\ e^x dx = dt \end{array} \right| = \int_1^{\sqrt{3}} \frac{1}{t^2+1} = \Big[\operatorname{arctg} t\Big]_1^{\sqrt{3}} = \operatorname{arctg}(\sqrt{3}) - \operatorname{arctg}(1)$$

$$e^{\frac{1}{2}\ln 3} = \sqrt{3}$$

$$e^0 = 1$$

$$= \frac{\pi}{3} - \frac{\pi}{4} =$$

$$= \frac{\pi}{12}$$

Nevlastní integrál

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b_-) - F(a^+)$$, pokud rozdíl upravo existuje

$-\infty \leq a < b \leq \infty$

Doma:

$$\int_3^{\infty} \frac{2x-1}{x^2-x}\,dx = \dots = [+\infty]$$

$$\int_0^{\infty} \frac{e^{2x}}{e^{4x}+1}\,dx = \left|\begin{array}{l} \text{Sub.} \\ e^{2x} = t \\ 2e^{2x}dx = dt \end{array}\right| = \frac{1}{2}\int_1^{\infty} \frac{1}{t^2+1}\,dt = \frac{1}{2}\Big[\operatorname{arctg} t\Big]_1^{\infty} = \frac{1}{2}\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \frac{\pi}{8}$$

$dx = \frac{dt}{2e^{2x}}$

$D(f) = \mathbb{R}$

spojitá

$(0,\infty \in \mathbb{R})$

$$\int_0^{\frac{1}{2}} \frac{1}{x(\ln^2 2x + 1)} = \left|\begin{array}{l} \text{Sub.} \\ \ln 2x = u \\ \frac{1}{x}\,dx = du \end{array}\right| = \int_{-\infty}^{0} \frac{1}{u^2+1}\,du = \Big[\operatorname{arctg}\Big]_{-\infty}^{0} = 0 - \left(-\frac{\pi}{2}\right) = \frac{\pi}{2}$$

$D(f) = (0,\infty)$

$(0,\frac{1}{2}) \in D(f)$