

Def. obor, Limits v hraničních bodech def. oboru

$$1. f(x) = \frac{\ln(2-x)}{x^2-2x-3}$$

def. obor:

$$2-x > 0 \Rightarrow x < 2$$

$$x^2-2x-3 = (x-3)(x+1) \neq 0$$

$$\begin{aligned} &\rightarrow x \neq 3 \\ &\rightarrow x \neq -1 \end{aligned}$$

$$D_f = (-\infty; -1) \cup (-1; 2)$$

f

$$a) \lim_{x \rightarrow (-1)^-} \frac{\ln(2-x)}{(x+1)(x-3)} = \frac{\ln(2-x)}{0^+} = \infty$$

Handwritten notes for (a):
 - $x \rightarrow -1^-$ leads to $\ln(3) > 0$
 - $x+1 \rightarrow 0^-$
 - $x-3 \rightarrow -4$

$$b) \lim_{x \rightarrow (-1)^+} \frac{\ln(2-x)}{(x+1)(x-3)} = \frac{\ln(2-x)}{0^-} = -\infty$$

Handwritten notes for (b):
 - $x \rightarrow -1^+$ leads to $\ln(3) > 0$
 - $x+1 \rightarrow 0^+$
 - $x-3 \rightarrow -4$

$$c) \lim_{x \rightarrow 2^-} \frac{\ln(2-x)}{(x+1)(x-3)} = \frac{-\infty}{-3} = \infty$$

Handwritten notes for (c):
 - $x \rightarrow 2^-$ leads to $\ln(0^+) \rightarrow -\infty$
 - $x+1 \rightarrow 3$
 - $x-3 \rightarrow -1$

$$d) \lim_{x \rightarrow -\infty} \frac{\ln(2-x)}{(x+1)(x-3)} = \left[\text{nepr. tvar } \frac{\infty}{\infty} \right] \stackrel{\text{L'H}}{=} \frac{\frac{-1}{2-x}}{2x-2}$$

Handwritten notes for (d):
 - $x \rightarrow -\infty$ leads to $\ln(2-x) \rightarrow \infty$
 - $x+1 \rightarrow \infty$
 - $x-3 \rightarrow \infty$

$$\lim_{z \rightarrow 2} \frac{-1}{(2-x)(2x-2)} = \frac{-1}{-\infty} \stackrel{\text{over. miz.}}{=} 0$$

$$2. f(x) = \frac{\cos 2x - 1}{2^x - 1}$$

$$2^x - 1 = 0$$

$$2^x = 1$$

$$2^x = 2^0$$

$$\rightarrow x = 0$$

$$D_f = (-\infty; 0) \cup (0; \infty)$$

$$a) \lim_{x \rightarrow -\infty} \frac{\cos(2x) - 1}{2^x - 1} \stackrel{\text{osciluje}}{=} \text{lim. neexistuje}$$

$$b) \lim_{x \rightarrow \infty} \frac{\cos(2x) - 1}{2^x - 1} \stackrel{\text{over. miz.}}{=} 0$$

$$c) \lim_{x \rightarrow 0+} \frac{\cos(2x) - 1}{2^x - 1} = \left[\text{nevec. l'ozsaz } \frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0+} \frac{-2 \sin(2x)}{2^x \ln(2)}$$

$$d) \lim_{x \rightarrow 0^+} \frac{\cos(2x) - 1}{2^x - 1} = \left[\text{applic. di L'Hôpital} \right] \lim_{x \rightarrow 0^+} \frac{-2\sin(2x)}{2^x \cdot \ln(2)} = 0$$