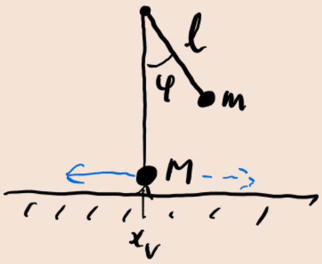
4.5 – kyvadlo na vozíku



kyvadlo:

$$x = x_v + l.\sin(\varphi)$$

$$y = -\cos(\varphi)$$

vozík:

$$x = x_v$$

$$y = 0$$

$$\dot{x} = \dot{x}_v + l.\cos(\varphi).\dot{\varphi}$$

$$\dot{y} = l.\sin(\varphi).\dot{\varphi}$$

$$E_{k_v} = \frac{1}{2}M.x_v^2$$

$$E_{k_k} = \frac{1}{2}m.(\dot{x}^2 + \dot{y}^2)$$

$$E_{k_k} = \frac{1}{2}m\left(\dot{x}_v^2 + 2l\dot{x}.\cos\varphi.\dot{\varphi} + l^2\dot{\varphi}^2\right)$$

$$E_k = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}\left(\dot{x}_v^2 + 2l\dot{x}.\cos(\varphi).\dot{\varphi} + l^2\dot{\varphi}^2\right)$$

$$= \frac{1}{2}(M+m)\dot{x}_v^2 + m(\dot{x}.\cos(\varphi).\dot{\varphi} + \frac{1}{2}m.l^2.\dot{\varphi}^2$$

$$E_p = -m.g.y = m.g.\cos(\varphi)$$

$$L = E_k - E_p = \frac{1}{2}(M+m)\dot{x}_v^2 + m(\dot{x}.\cos(\varphi).\dot{\varphi} + \frac{1}{2}m.l^2.\dot{\varphi}^2 - m.g.l.\cos(\varphi)$$

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$$x: \frac{d}{dx}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$\frac{\partial L}{\partial \dot{x}} = (M+m)\dot{x}_v + m.l.\cos(\varphi).\dot{\varphi}$$

$$\frac{d}{dx}\left(\frac{\partial L}{\partial \dot{x}}\right) = (M+m)\ddot{x}_v + m.l\left(-\sin(\varphi).\dot{\varphi}^2 + \cos(\varphi).\ddot{\varphi}\right)$$

$$\frac{\partial L}{\partial x} = 0$$

$$\boxed{(M+m)\ddot{x}_v + m.l\left(-\dot{\varphi}^2.\sin(\varphi) + \ddot{\varphi}.\cos(\varphi)\right) = 0}$$

$$\varphi: \quad \frac{d}{dx}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = 0$$

$$\frac{\partial L}{\partial \dot{\varphi}} = m.\ell.\dot{x}_v.\cos(\varphi) + m.\ell^2\dot{\varphi}$$

$$\frac{d}{dx}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = m.\ell.\ddot{x}.\cos(\varphi) - m.\ell.\dot{x}.\dot{\varphi}.\sin\varphi + m.\ell^2.\ddot{\varphi}$$

$$\frac{\partial L}{\partial \varphi} = -m.\ell.\dot{x}.\dot{\varphi}.\sin(\varphi) + m.g.\ell.\sin(\varphi)$$

$$m.\ell.\ddot{x}.\cos(\varphi) - m.\ell.\dot{x}.\dot{\varphi}.\sin(\varphi) + m.\ell^2.\ddot{\varphi} + m.\ell.\dot{x}.\dot{\varphi}.\sin(\varphi)$$
$$- m.g.\ell.\sin(\varphi) = 0$$

$$m.\ell.\ddot{x}.\cos(\varphi) + m.\ell^2.\ddot{\varphi} - m.g.\ell.\sin(\varphi) = 0$$

$$\boxed{\ddot{x}.\cos(\varphi) + \ell.\ddot{\varphi} - \ell.\sin(\varphi) = 0}$$