1) a)

$$\begin{pmatrix} 3 & -2 & 1 & | & 7 \\ 1 & -1 & 0 & | & 2 \\ -4 & 2 & 0 & | & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 0 & | & 2 \\ 3 & -2 & 1 & | & 7 \\ -4 & 2 & 0 & | & 6 \end{pmatrix} \begin{matrix} \\ R_2 - 3R_1 \\ R_3 + 4R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & -1 & 0 & | & 2 \\ 0 & 1 & 1 & | & 7 \\ 0 & -2 & 0 & | & -6 \end{pmatrix}_{R_3 + 2R_2} \sim \begin{pmatrix} 1 & -1 & 0 & | & 2 \\ 0 & 1 & 1 & | & 1 \\ 0 & 0 & 2 & | & 4 \end{pmatrix} \Rightarrow$$

$2z = 4$

$\underline{z = 2}$

$y + z = 1$

$\underline{y = -1}$

$x - y = 2$

$\underline{x = 1}$

b) $\det\begin{pmatrix} 1 & -1 & 0 & | & 2 \\ 0 & 1 & 1 & | & 1 \\ 0 & 0 & 2 & | & 4 \end{pmatrix} = 1 \cdot 1 \cdot 2 = \cancel{2} \quad -2$ (prohozený řádky)

c) $\begin{pmatrix} 4 \\ 1 \\ -4 \end{pmatrix} \overset{R_1 \Leftrightarrow R_2}{\sim} \begin{pmatrix} 1 \\ 4 \\ -4 \end{pmatrix} \begin{matrix} \\ R_2 - 3R_1 \\ R_3 - 4R_1 \end{matrix} \sim \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}_{R_3 + 2R_2} \sim \underline{\underline{\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}}}$

$$\rightarrow \begin{pmatrix} 1 & -1 & 0 & | & 1 \\ 0 & 1 & 1 & | & 1 \\ 0 & 0 & 2 & | & 2 \end{pmatrix}$$

$2z = 2 \rightarrow \underline{\underline{z = 1}}$

$y + z = 1$

$\underline{\underline{y = 0}}$

$x - y = 1 \rightarrow \underline{\underline{x = 1}}$

$3 \cdot 1 - 2 \cdot 0 + 1 = 4$

$3 + 1 = 4$ ✓

$1 - 0 = 1$ ✓

$-4 + 2 \cdot 0 = -4$ ✓

<u>sedí</u>

2) $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$

$\det(A - \lambda E) = 0$

$$\begin{vmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda) - 6 = \lambda^2 - 3\lambda - 4 = 0$$

$\Downarrow$

$\lambda_1 = 4$

$\underline{\underline{\lambda_2 = -1}}$

spekt. poloměr:

$\rho(A) = \max|\lambda| = 4$

řádková norma:

$= \max\left((|2|+|2|), (|3|+|2|) \right) = \max(3, 5) = 5$

sloupcová norma:

$= \max(4, 4) = 4$