1) $y' = \dfrac{2xy}{x^2-1}$ $\quad \Big| \; x^2-1 \neq 0$

$\qquad\qquad\qquad\qquad x^2 \neq 1$

$\qquad\qquad\qquad\qquad x \neq \pm 1$

$$\frac{y'}{2y} = \frac{x}{x^2-1}$$

$$\int \frac{1}{2y}\,dy = \int \frac{x}{x^2-1}\,dx \leftarrow \text{subst.}$$

$$\frac{1}{2}\ln|y| = \frac{1}{2}\ln|x^2-1| + C \quad , C \in \mathbb{R}$$

$$\ln|y| = \ln|x^2-1| + C$$

$$y = \pm e^{\ln|x^2-1| + C}$$

$$|y| = e^C \cdot e^{\ln|x^2-1|}$$

$$y = \pm e^C \cdot (x^2-1) \,, x \neq 1 , x \neq -1$$

$$y = D(x^2-1)$$

$$\rightarrow \; y(x) = D(x^2-1) \,, D \in \mathbb{R} \,, x \neq \pm 1$$

a) $y(0) = 1$

$$1 = D(0-1) = -D \quad \rightarrow D = -1$$

$$y(x) = -(x^2-1) \,, \quad x \in (-1, 1)$$

b) $y(2) = -3$

$-3 = D(4-1) = 3D$

$D = -1$

$y(x) = -(x^2-1)\,, x \in (1, \infty)$

c) $y(-3) = 16$

$16 = D(9-1) = 8D$

$D = 2$

$y(x) = 2(x^2-1)\,, \; x \in (-\infty, -1)$

d) $y(2) = 0$

$0 = D(4-1) = 3D$

$D = 0$

$y(x) = 0 \;, x \in (1, \infty)$

2) $y' = \dfrac{e^x}{e^y}$

$y' \cdot e^y = e^x$

$$\int e^{y}\,dy = \int e^{x}\,dx$$

$$e^{y} = e^{x} + C$$

$$\rightarrow y(x) = \ln\left(e^{x} + C\right), \quad C \in \mathbb{R}$$

Asymptotický růst v $+\infty$:

$$\ln\left(e^{x} + \cancel{C}\right) \xrightarrow[x \to \infty]{} \infty$$

toto je dominantní

tedy: y se chová jako x