Lagrangián

$$L(\vec{q}, \dot{\vec{q}}, t) = T - U$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

↑ zobec. hybnost

konzervativní síla

$$\vec{F} = -\nabla U$$

$$\vec{\nabla} \times F = 0$$

$$-\mathrm{rot}(\mathrm{grad}(U)) = 0$$

existují i síly:

- gyroskopická
- disipativní: $F_{dis} = -k \cdot \vec{v}$

RAYLEIGHOVA FCE

$$R = \frac{1}{2}k\left(v_x^2 + v_y^2 + v_z^2\right)$$

$$F_{dis_x} = -\frac{\partial R}{\partial v_x} = -k v_x$$

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

zobec. energie:

$$\mathcal{E} = \sum_i \frac{\partial L}{\partial \dot{q}_i} \cdot \dot{q}_i - L$$

$$\frac{d\mathcal{E}}{dt} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\dot{q} + \frac{\partial L}{\partial \dot{q}}\ddot{q} - \frac{\partial L}{\partial q}\cdot\dot{q} - \frac{\partial L}{\partial \dot{q}}\ddot{q} - \frac{\partial L}{dt}$$

$$= \frac{d\mathcal{E}}{dt} = \frac{-\partial L}{\partial t}$$

$$dL = \frac{\partial L}{\partial q} \cdot \frac{dq}{dt} + \frac{dL}{d\dot{q}} \cdot \ldots$$

q_i na kterém nezávisí ... cyklické souřadnice

$$p_q = \frac{\partial L}{\partial \dot{q}} = \text{konst}$$ ... hybnost se zachovává → integrál pohybu

Hamiltonián

$$H(\vec{q}, \vec{p}, t) = \sum_i p_i \dot{q}_i - L$$

kanonické hamiltonovy rovnice

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \cdot \frac{dp_i}{dt} = \dot{p}_i = \frac{-\partial H}{\partial q_i}$$

[illegible]

(ř.:) Volný hmotný bod

$\bullet m$ $\quad q_1 = x$

$q_2 = y$

$q_3 = z$

$L = T - U$

$T = \frac{1}{2} m \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2 \right)$ $\dashleftarrow$

odvození

$\vec{F} \cdot \vec{v} = m \cdot \frac{d\vec{v}}{dt} \cdot \vec{v} = \frac{1}{2} \frac{d(\vec{v} \cdot \vec{v})}{dt} = \frac{d}{dt}\left(\frac{1}{2} m \cdot v^2 \right)$

$U = 0$ (nebo konst.)

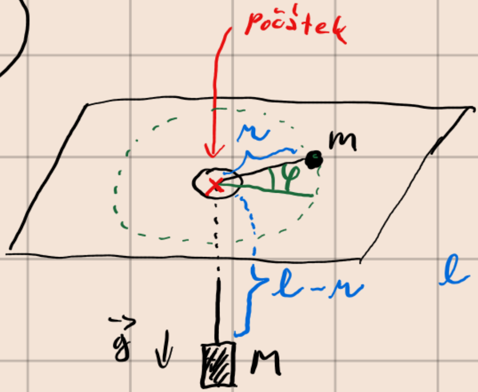
$T = L$

$x: \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = 0$

$\frac{d}{dt}\left(m \cdot \dot{x} \right) = 0 \longrightarrow \left. \begin{matrix} q = x \\ q = y \\ q = z \end{matrix} \right\}$ ... cyklické souřadnice

$\left. \begin{matrix} p_x \\ p_y \\ p_z \end{matrix} \right\} = \text{konst.}$

4.8



l ... celková délka provázku

stupně volnosti

x, y

r, φ

$|r_k| + |z_z| = l$

$S = 3N - R = 6$

máme 4 vazby

$\Rightarrow$ 2 stupně volnosti

$q_1 = \varphi$

$q_2 = r$

kulička   závaží

- $z_k = 0$
- $x_k = r \cdot \cos\varphi$
- $y_k = r \cdot \sin\varphi$

- $x_z = 0$
- $y_z = 0$
- $z_z = -(l - r)$

rychlosti:

kulička

$$\dot{x}_k = \dot{r} \cdot \cos\varphi - r \cdot \dot{\varphi} \cdot \sin\varphi$$

$$\dot{y}_k = \dot{r} \cdot \sin\varphi + r\,\dot{\varphi} \cdot \cos\varphi$$

$$\dot{z}_k = 0$$

$$\dot{x}_z = 0$$

$$\dot{y}_z = 0$$

$$\dot{z}_z = -(0 - \dot{r}) = \dot{r}$$

$$L = \sum_i T_i - \sum_i U_i = T_k + T_z - U_k - U_z$$

$$L = \frac{1}{2} m \left(\dot{x}_k^2 + \dot{y}_k^2 + 0\right) + \frac{1}{2} M \dot{z}_z^2 - M g z_z$$

$$\dot{y}_k^2 + \dot{x}_k^2 = \dot{r}^2 \cdot \cos^2\varphi + r^2 \dot{\varphi}^2 \sin\varphi$$
$$\cancel{- 2 r \dot{r} \dot{\varphi} \sin\varphi \cos\varphi}$$
$$+ \dot{r}^2 \sin^2\varphi + r^2 \dot{\varphi}^2 \cos\varphi$$
$$\cancel{+ 2 r \dot{r} \dot{\varphi} \sin\varphi \cos\varphi}$$

$$L = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m \cdot r^2 \dot{\varphi}^2 + \frac{1}{2} M \dot{r}^2 + M g (l - r)$$

$$r: \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0$$

$+Mg$

$$\frac{d}{dt}\left((m + M)\dot{r}\right) - m r \dot{\varphi}^2 - M g (0 - 1) = 0$$

$$(m + M)\ddot{r} = m \cdot r \cdot \dot{\varphi}^2 - M g$$

$$\varphi: \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = 0$$

$$\frac{d}{dt}\left(m.r^2\dot{\varphi}\right)=0$$

φ je cyklická souřadnice

$p_\varphi = m r^2 \dot{\varphi}$ = konst.... integrál pohybu

$$\rightarrow m 2 r.\dot{r}\dot{\varphi} + m r^2 \ddot{\varphi} = 0$$

úkol: 4.5

Gaussův zákon:

$$\vec{F}_g = -G.\frac{m.M}{r^2}\cdot\frac{\vec{r}}{r} \rightarrow U = -\int \vec{F}.d\vec{r} = -G\frac{m.M}{r}$$

$$\rightarrow \vec{k} = \frac{\vec{F}}{m} = -G\frac{M}{r^2}\frac{\vec{r}}{r} \xrightarrow{k=\nabla\phi} \phi = \frac{U}{m} = -\frac{GM}{r}$$

$$\vec{k} = \frac{\vec{F}}{m}$$

$$\oiint \vec{k}\, d\vec{S} = -4\pi G \iiint \rho\, dV$$

$$\iint \sigma\, dS$$

$$\int \lambda\, dl$$

$$k.4\pi r^2 = -4\pi G M(r)$$

$$k(r) = -G\frac{M(r)}{r^2}$$

$$F(r) = -G\frac{m\cdot M(r)}{r^2}$$