$\vec{r}$ ... polohový vektor

$\vec{p} = m \cdot \vec{v}$ ... hybnost

$\vec{v} = \frac{dr}{dt}$ ... rychlost

$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2 r}{dt}$ ... zrychlení

$\vec{e_v}$ ... jednotkový vektor ve směru rychlosti

$\frac{d\vec{e_v}}{dt} = 0$ ... přímočarý pohyb

$\frac{d\vec{v}}{dt} = 0$ ... rovnoměrný pohyb

$\frac{d\vec{v}}{dt} > 0$ ... rovnoměrně zrychlený pohyb

rovnoměrný přímočarý pohyb:

$v = konst$

$v = \frac{ds}{dt}$

$$ds = v \cdot dt \quad \Big| \int_{s_0}^{s} \int_{t_0}^{t}$$

$$s - s_0 = v \cdot (t - t_0)$$

$$s(t) = s_0 + v(t - t_0)$$

$a =$ konst.

$$a = \frac{dv}{dt}$$

$$a \cdot dt = dv \quad \int$$

$$\int a \cdot dt = v$$

$$at + C_1 = v$$

$$v_0 = v(t_0) = at_0 + C_1$$

$$C_1 = v_0 - at_0$$

$$v(t) = at + v_0 - at_0 = a(t - t_0) + v_0$$

$$v = \frac{ds}{dt}$$

$$v \cdot dt = ds \quad \Big| \int$$

$$\int v(t) \cdot dt = s(t)$$

$$s(t) = \int a\cdot(t-t_0) + v_0 \; dt$$

$$s(t) = a\cdot\int(t-t_0)\,dt + \int v_0\,dt$$

$$s(t) = a\cdot\frac{(t-t_0)^2}{2} + v_0(t-t_0) + C_2$$

$$C_2 = s_0$$

$$s(t) = s_0 + v_0(t-t_0) + \frac{a}{2}(t-t_0)^2$$

(2.1) autobusy:

$\Delta l = (c-v)\,T_v$ ... ve směru chůze

$\Delta l = (c+v)\,T_p$ ... proti směru

$\Delta l = c\cdot T_{dr}$

$$T_{dr} = \frac{\Delta l}{c} = \frac{(c-v)T_v}{c} = \frac{c-v}{c}T_v =$$

$$\left(1-\frac{v}{c}\right)T_v = T_{dr}$$

... porovnat & dosadit poměr

1.4 rozměrová analýza

$$l_p \sim C \cdot c^{\alpha} \cdot \hbar^{\beta} \cdot G^{\gamma}$$

m^1 $\quad m \cdot s^{-1}$ $\quad kg \cdot m^2 \cdot s^{-1}$ $\quad kg^{-1} \cdot m^3 \cdot s^{-2}$

$m: 1 = \alpha + 2\beta + 3\gamma$

$kg: 0 = \beta - \gamma$

$s: 0 = -\alpha - \beta - 2\gamma$

$\alpha = -\frac{3}{2}$

$\beta = \frac{1}{2}$

$\gamma = \frac{1}{2}$

2.12 Pohyb s proměnným zrychlením

$a_0 = 10\, m \cdot s^{-2}$ ... $t = 0$, $\vec{v} = 0$ $\vec{r} = 0$

$a = 0$ ...... $t = T = 20 s$

$\vec{r}$ a $\vec{v}$ v čase T

$a(t) = a_0 - kt$ ... rovn. klesá

$a(T) = 0$

$\Rightarrow a_0 - kT = 0$ ... koncový stav

$$k = \frac{a_0}{t}$$

$$a(t) = a_0 - \frac{a_0}{t} t = a_0 \left(1 - \frac{t}{t} \right)$$

$$a(t) = \frac{d v(t)}{dt}$$

$$d v(t) = a(t)\, dt$$

$$v(t) = \int a(t)\, dt$$

$$v(t) = \int a_0 \left(1 - \frac{t}{t} \right) dt$$

$$v(t) = a_0 \int 1 - \frac{t}{T}\, dt + C$$

$$= a_0 \left(t - \frac{1}{T} \int t \right) + C$$

$$= a_0 \left(t - \frac{1}{T} \frac{t^2}{2} \right) + C$$

$$v(t) - v_0 = a_0 t - a_0 \frac{t^2}{2T}$$

$$v(t) = \frac{ds}{dt}$$

$$ds = v(t)\, dt$$

$$s(t) = \int v(t)\, dt$$

$s(t) = \int$

$$s(t) = \int a_0 t - a_0 \frac{t^2}{2T} \, dt$$

$$s(t) = a_0 \frac{t^2}{2} - \frac{a_0}{2T} \int t^2 \, dt$$

$$s(t) = a_0 \frac{t^2}{2} - \frac{a_0}{2T} \frac{t^3}{3} + C$$

$$s(t) - s_0 = \frac{a_0 t^2}{2} - a_0 \frac{t^3}{6T}$$

2.15 - kamínek

$$\vec{u} = (u, 0, 0)$$

$$x(t) = R \cdot \omega t - R \cdot \sin(\omega t)$$

poloměr kola

úhlová rychlost

$$\omega = \frac{u}{R}$$

$$x(t) = R \frac{\vec{u}}{R} \cdot t - R \cdot \sin(\omega t)$$

$$x(t) = \vec{u} \cdot t - R \cdot \sin(\omega t)$$

$$z(t) = R - R \cdot \cos(\omega t)$$

$$\dot{x} = v_x = (\vec{u} \cdot t - R \cdot \sin(\omega t))'$$

$$v_x(t) = u - R\omega \cdot \cos(\omega t)$$

$$a_x(t) = \ddot{x} = 0 + R \cdot \omega^2 \cdot \sin(\omega t)$$

$$\vec{a} = \sqrt{(R\omega^2 \cdot \cos(\omega t))^2 + (u - R\omega \cos(\omega t))^2}$$

volný pád:

$$v(t) = \vec{a} \cdot t =$$

$$v(t) = \int a(t)\,dt$$

$$\vec{v}(t) = a \cdot (t - t_0) + v_0$$

$$\vec{v}(t) = v_0 + at$$

$$v = \frac{d\vec{r}}{dt}$$

$$r = \int v \, dt$$

$$r = \int v_0 + at \, dt$$

$$r = v_0 t + r_1 + a \cdot \frac{t^2}{2}$$

$$r = r_1 + v_0 t + \frac{1}{2} a t^2$$

b) svislý vrh

to samé, ale s poč. rychlostí
$(0, 0, v_z)$

c) vodorovný vrh

to samé, ale $\vec{v_0} = (v_x, 0, 0)$

d) šikmý vrh

$$v_0 = (\overset{x}{v_0 \cdot \cos\alpha} ; \overset{z}{v_0 \cdot \sin\alpha})$$

$$z(t) = h + \underbrace{v_0}_{} t \cdot \underbrace{\sin\alpha}_{} - \frac{1}{2} g t^2$$

$$x(t) = v_0 t \cdot \cos\alpha$$

2.22 - opice

$$\vec{v}_{střela} = (v_0 \cos\alpha, 0, v_0 \sin\alpha)$$

souřadnice vrhu = souřadnice při pádu opice

Tečné a normálové zrychlení

vod. vrh

$$\vec{v}_0 = (v_0, 0, 0)$$

$$\vec{a} = (0, 0, -g)$$

$$\vec{r}_0 = (0, 0, h)$$

$$\vec{v}(t) = (v_0, 0, -gt)$$

$$\|\vec{v}\| = \sqrt{v_0^2 + (gt)^2}$$

$$a_t(t) = \frac{d\|\vec{v}\|}{dt}$$

$$a_n(t) = \sqrt{a^2 - a_t^2}$$

$$\frac{1}{\cancel{2}} \cdot \frac{1}{\sqrt{v_0^2 + (gt)^2}} \cdot g^2 \cancel{2} t$$

$$= \frac{g^2 t}{\sqrt{v_0^2 + (gt)^2}}$$

kruhový pohyb

$r^2 = r \cdot r =$ konst.

$$\frac{dr}{dt} \cdot r = \dot{r} \cdot r + r \cdot \dot{r} = 0$$

$v \cdot r = 0 \rightarrow v$ je kolmý

na poloměr

$$\Delta\varphi = \frac{\Delta s}{r} = \frac{v \cdot \Delta t}{r}$$

středový úhel

$$\boxed{\omega = \frac{\Delta\varphi}{\Delta t}}$$

úhlová rychlost

$$\boxed{v = \omega r}$$

okamžitá obvodová rychlost

$$\boxed{T = \frac{2\pi r}{v} = \frac{2\pi r}{\omega r} = \frac{2\pi}{\omega}}$$

perioda

$$f = \frac{1}{T}$$

2.28 Rumpál

$$\left.\begin{aligned} s &= \frac{1}{2}kt^2 \\ v &= \dot{s} = kt \\ a_{posunu} &= \ddot{s} = k \end{aligned}\right\} \text{kbelík}$$

$$\left.\begin{aligned} a &= R \cdot \alpha \\ \alpha &= \frac{a}{R} = \frac{k}{R} \end{aligned}\right\} \text{rumpál}$$

úhlové zrychlení

$$a_t = R\alpha = R \cdot \frac{k}{R} = k$$

$$a_n = \frac{v^2}{R}$$

3.6 - nakloněná rovina

$$F_g \cdot \sin\alpha = \mu \cdot F_g \cdot \cos\alpha$$

3.9 - Loď

$$\vec{F} = m \cdot \vec{a}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 2c^2(t - t_2)$$

$$v = c^2(t - t_2)^2$$

$$(t - t_2)^2 = \frac{v}{c^2}$$

$$(t - t_2) = -\sqrt{\frac{v}{c}}$$

$$F = m \cdot 2c^2\left(-\frac{\sqrt{v}}{c}\right) = -2mc\sqrt{v}$$

- newtonovy zákony

3.37 Řeka

vychlost řeky : $v = 5 km/h$
šířka řeky $d = 1 km$

$\varphi = 45^0$ n. š. 45. rovnoběžka

$$F_{cor} = -2m \vec{\omega} \times v$$

vychlost řeky

úhl. vychlost
rot. země:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{86400} \doteq 7{,}292 \cdot 10^{-5} \, rad \cdot s^{-1}$$

$$|F_{cor}| = 2 \cdot m \cdot v \cdot \omega \sin(\pi - \varphi)$$

$$= 2 m v \omega \sin \varphi$$

$$F_p = g \cdot \frac{\Delta h}{d}$$

$$F_c = F_p \; \dots$$

$$\frac{2 d v \omega \sin \varphi}{g} = \Delta h$$

g

$= 75 \, mm$

$$P_0 = \vec{F} \cdot \vec{v} = m \cdot \frac{d\vec{v}}{dt} \cdot v =$$

$$m \cdot \frac{d}{dt}(v \cdot v) = m \cdot 2v \cdot \frac{dv}{dt}$$

TODO

$$A = \int_0^s F \cdot ds$$

$$E_{kin} = \frac{1}{2} m \cdot g^2$$

$$E_{pot} = m \cdot g \cdot h$$

3.17 - Artista

Ab$_s$ nospod (:

$F_0 = F_g$

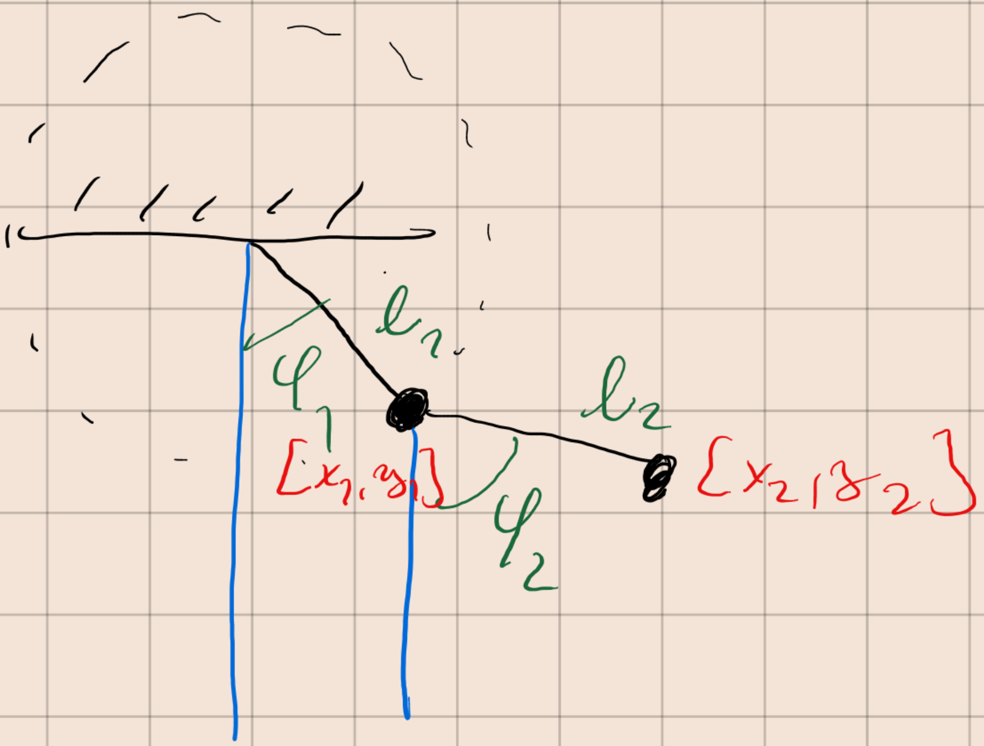
$$F_0 = \boxed{m \cdot \frac{v^2}{R}} = m \cdot g \rightarrow v^2 \geq Rg$$

$$\frac{1}{2}\cancel{m}v^2 + \cancel{m}\cdot g\cdot(2R) = \cancel{m}\cdot g\cdot h$$

$$\frac{1}{2}Rg + g2R = gh$$

$$h = 2R + \frac{R}{2} = \frac{5}{2}R$$

$$\frac{1}{2}mv^2 = A_t = \mu\cdot m\cdot g$$



$$x_1 = l_1\cdot\sin\varphi_1$$

$$y_1 = -l_1\cdot\cos\varphi_1$$

$$x_2 = l_1 \cdot \sin \varphi_1 + l_2 \cdot \sin \varphi_2$$

$$y_2 = -l_1 \cdot \cos \varphi_1 - l_2 \cdot \cos \varphi_2$$

$$\dot{x}_1 = l_1 \cdot \cos \varphi_1 \cdot \dot{\varphi}_1$$

$$\dot{y}_1 = l_1 \cdot \sin \varphi_1 \cdot \dot{\varphi}_1$$

$$\dot{x}_2 = l_1 \cdot \cos \varphi_1 \cdot \dot{\varphi}_1 + l_2 \cdot \cos \varphi_2 \cdot \dot{\varphi}_2$$

$$\dot{y}_2 = l_1 \cdot \sin \varphi_1 \cdot \dot{\varphi}_1 + l_2 \cdot \sin \varphi_2 \cdot \dot{\varphi}_2$$

$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} m_1 \cdot \left(x_1^2 + y_1^2\right) + \frac{1}{2} m_2 \left(x_2^2 + y_2^2\right)$$

....

$$E_k = \frac{1}{2}(m_1 + m_2) l_1^2 \dot{\varphi}_1^2 + \frac{1}{2} m_2 \Big(l_2^2 \varphi_2^2 + 2 l_1 \cdot l_2 \dot{\varphi}_1 \dot{\varphi}_2 \cos(\varphi_1 - \varphi_2)\Big)$$

$$E_p = m_1 \cdot g \cdot y_1 + m_2 \cdot g \cdot y_2$$

$$= -(m_1 + m_2) g\, l_1 \cdot \cos\varphi_1 - m_2 g l_2 \cos\varphi_2$$

$$\boxed{L = E_k - E_p}$$

$$L = \frac{1}{2}(m_1 + m_2) l_1^2 \dot{\varphi}_1^2 + \frac{1}{2} m_2 [l_2^2 \dot{\varphi}_2^2 + 2 l_1 \cdot$$

$$\cdot l_2 \dot{\varphi}_1 \dot{\varphi}_2 \cos(\varphi_1 - \varphi_2) + (m_1 + m_2) g\, l_1 \cdot$$

$$\cdot \cos\varphi_1 + m_2 \cdot g \cdot l_2 \cdot \cos\varphi_2$$

Lagr. rovnice 2. druhu

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_{1,2}}\right) - \frac{\partial L}{\partial q_{1,2}} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}_2}\right) - \frac{\partial L}{\partial \varphi_2} = 0$$

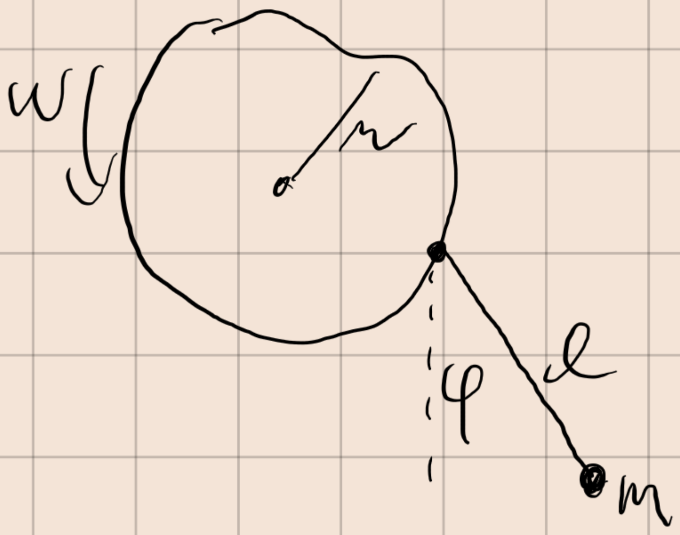
$$\frac{\partial L}{\partial \varphi_1} = -m_2 l_1 l_2 \dot{\varphi}_1 \dot{\varphi}_2 \sin(\varphi_1 - \varphi_2) - (m_1 + m_2) g l_1 \sin \varphi_1$$

$$\frac{\partial L}{\partial \varphi_2} = m_2 l_1 l_2 \dot{\varphi}_1 \dot{\varphi}_2 \cdot \sin(\varphi_1 - \varphi_2) - m_2 g l_2 \sin \varphi_2$$

Hamiltonian

$$H = \sum_i p_i q_i - L$$

$$p_i = \frac{\partial L}{\partial \dot{q}_i} \qquad q_i = \text{velicina}$$



$$x = r \cdot \cos(\omega t) + l \cdot \sin(\varphi)$$

$$y = r \cdot \sin(\omega t) + l \cdot \sin(\varphi)$$

$$\dot{x} = -r \cdot \sin(\omega t) \cdot \omega + l \cdot \cos(\varphi) \cdot \dot{\varphi}$$

$$\dot{y} = r \cdot \cos(\omega t) \cdot \omega - l \cos(\varphi) \dot{\varphi}$$

$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} m \cdot (\dot{x}^2 + \dot{y}^2) =$$

$$= \frac{1}{2} m \left[(r\omega)^2 + l^2 \dot{\varphi}^2 + 2 r \omega l \dot{\varphi} \sin(\omega t + \varphi) \right]$$

$$E_p = -m \cdot g \cdot h = -m \cdot g \cdot y =$$

$$-m \cdot g\left(r \cdot \sin(\omega t) - l \cdot \cos(\varphi)\right)\right) =$$

$$= -m \cdot g \cdot r \cdot \sin(\omega t) + m \cdot g \cdot l \cdot \cos(\varphi)$$

$$L = \frac{1}{2} m \left(r^2 \omega^2 + l^2 \dot{\varphi}^2 + 2 \ldots \right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = 0$$

$$\frac{\partial L}{\partial \dot{\varphi}} = m l^2 \dot{\varphi} + m r \omega l \cdot \sin(\omega t + \varphi)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = m l^2 \ddot{\varphi} + m r \omega l \left(\omega + \dot{\varphi}\right) \cos(\omega t + \varphi))$$

$$\frac{\partial L}{\partial \varphi} = m r \omega l \dot{\varphi} \cdot \cos(\omega t + \varphi) - g \cdot \sin\varphi$$