① $\vec{n} = (-1, 2, 1)$ ... norm. vektor roviny

$\vec{b} = (1, 2, 1)$ ... bod v rovině

$$-1x + 2y + 1z + d = 0$$

$$-1 + 4 + 1 = -d$$

$$-4 = d$$

$\to \quad -x + 2y + z - 4 = 0$

②

$a = (3, 1, -2)$
$b = (2, 3, 1)$ } který je blíže rovině:

rovina: $x - y = 0$

$\vec{n} = (1, -1, 0)$

p_A: $x = 3 + t$
$y = 1 - t$
$z = -2$

v rovině

$$3 + t - 1 + t = 0$$

$$2 + 2t = 0$$

$$2(t + 1) = 0$$

$t = -1$ ... průsečík

$$\|A - p_A(-1)\| = \sqrt{(3-2)^2 + (1-2)^2 + (-2+2)^2} = \sqrt{2}$$

b) vzorcem

$$\|B - R\| = \frac{|a.p_1 + b.p_2 + c.p_3|}{\sqrt{a^2 + b^2 + c^2}} \quad \dots = \frac{\sqrt{2}}{2}$$

tedy B je blíže než A.

5) $\vec{a} = (2,3,0)$ 2 3

$\vec{b} = (1,0,3)$ 1 0

chci
$$\begin{cases} \vec{v} = a \times b = (9-0,\, 0-6,\, 0-3) = (9,-6,-3) \\ \|v\| = \sqrt{9^2+(-6)^2+(-3)^2} = \sqrt{81+36+9} = \sqrt{126} \\ \vec{v} \perp a, b \end{cases}$$

$\vec{v} \cdot \vec{a} = 2.9 - 3.6 = 0 \Rightarrow$ je kolmý!

...

7) $M = \{(x,y) \in \mathbb{R}^2 \mid |y| \leq 1 - x^2\}$

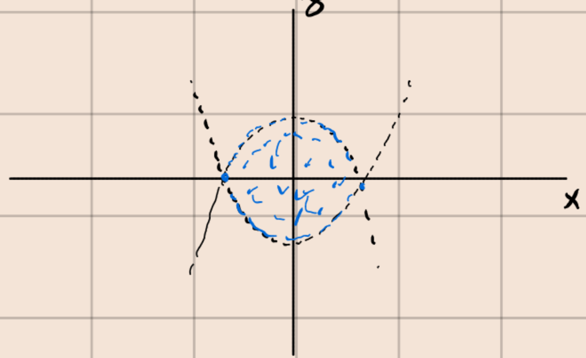
$y \geq 0$

$y \leq 1 - x^2$

$y \leq 0$

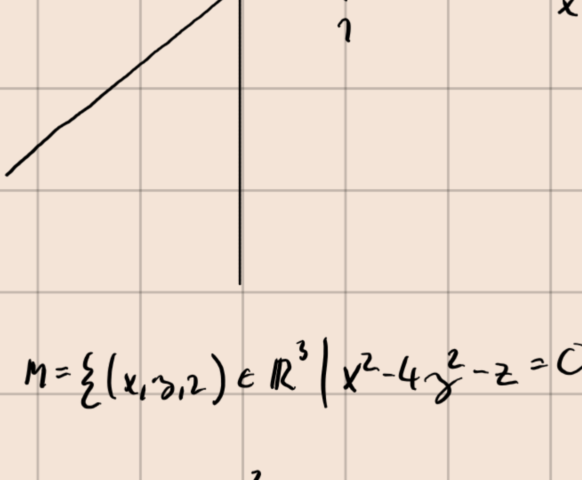
$-y < 1 - x^2$

$y > x^2 + 1$



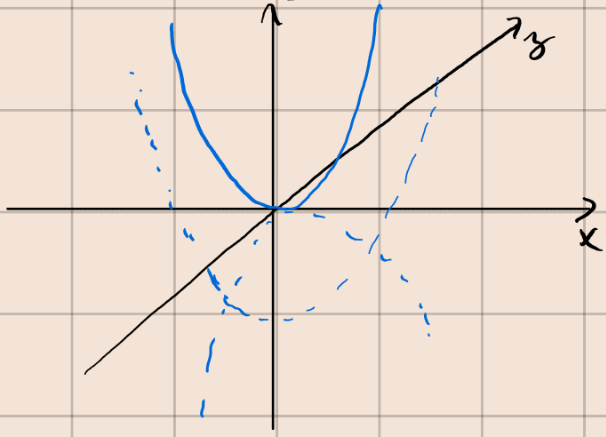
8)

a) $M = \{(x,y,z) \in \mathbb{R}^3 \mid x \geq 0, y \geq 0, z \geq 0, x+y+z \leq 1\}$



b) $M = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 - 4y^2 - z = 0\}$

z $z = x^2 - 4y^2$



pro $|y|>0$ se posouvá vrchol paraboly dolů.

→ hyperbolický paraboloid

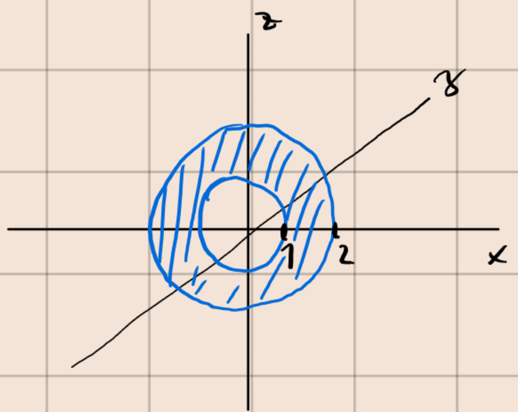
c) $M = \{(x,y,z) \in \mathbb{R}^3 \mid 4x^2 + y^2 + z^2 - 24x - 8y + 4z + 55 = 0\}$

$$(2x)^2 - 24x + 36 + y^2 - 8y + 16 + z^2 + 4z + 4 = 1$$

$$(2x-6)^2 + (y-4)^2 + (z+2)^2 = 1$$

→ elipsoid

d) $M = \{(x,y,z) \in \mathbb{R}^3 \mid 1 \le x^2 + y^2 + z^2 \le 4$



→ mezisféří

10)

a) $\vec{x}_k = \left(\frac{1}{k}, \frac{k-3k^2}{k+k^2}\right)$

$$\left.\begin{aligned} \lim_{k\to\infty} \frac{1}{k} &= 0 \\ \lim_{k\to\infty} \frac{k-3k^2}{k+k^2} &= -3 \end{aligned}\right\} \text{konverguje} \quad \lim \vec{x}_k = (0,-3)$$

b) $\vec{x}_k = (1, \sin(\pi k), k)$

$\lim 1 = 1$

$x \to \infty$

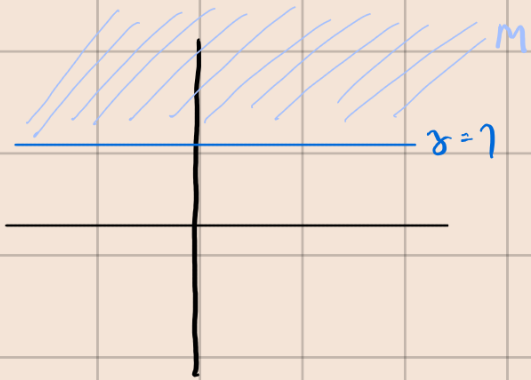
$\lim\limits_{x \to \infty} \sin(\pi k) = 0$

$\lim\limits_{x \to \infty} k = \infty$

} diverguje

11)

a) $M = \{(x,y) \in \mathbb{R}^2 \mid y > 1\}$



$\operatorname{Int} M = M$

↳ je otevřená

$\partial M = \{(x,y) \in \mathbb{R}^2 \mid y = 1\}$

$\bar{M} = M \cup \partial M = \{(x,y) \in \mathbb{R}^2 \mid y \geq 1\}$

hromadné body $= \bar{M}$

izolované body $= \{\}$

b) M je přímka v rovině

$\operatorname{Int} M = \{\}$

$\partial M = M$

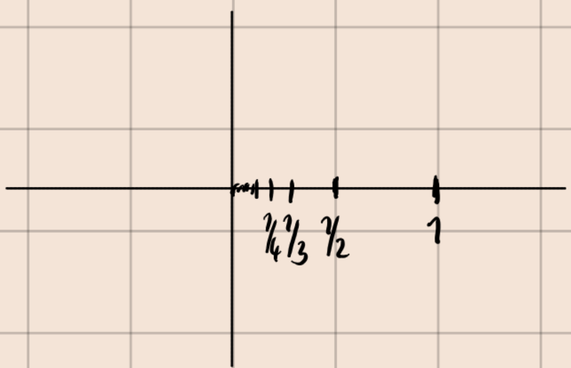
$\bar{M} = M \cup \partial M = M$

↳ M je uzavřená

hromadné body $= M$

izolované body $= \{\}$

c) $M = \{(\frac{1}{n}, 0), n \in \mathbb{N}\}$



$\operatorname{Int} M = \{\}$

$\partial M = M \cup \{(0,0)\}$

$\bar{M} = M \cup \partial M = M \cup \{(0,0)\}$

→ není ani otevřená ani uzavřená

hromadné body $= \{(0,0)\}$

izolované body $= M$