w5/15

$$g(t) = f(\underset{h_1(t)}{\underset{\shortparallel}{e^t}}, \underset{h_2(t)}{\underset{\shortparallel}{e^{-t}}})$$

$$f(x,y) = \frac{x-y}{x+2y}$$

$g'(t)$ ... hledáme

$f: \mathbb{R}^2 \to \mathbb{R}$

$h: \mathbb{R} \to \mathbb{R}^2$

$$g'(t) = \frac{\partial f}{\partial x}\left(h_1(t), h_2(t)\right) \cdot h_1'(t) + \frac{\partial f}{\partial y}\left(h_1(t), h_2(t)\right) \cdot h_2'(t)$$

$$\frac{\partial f}{\partial x} = \frac{(x+2y)-(x-y)}{(x+2y)^2} = \frac{3y}{(x+2y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{-(x+2y)-(x-y)\cdot 2}{(x+2y)^2} = \frac{-3x}{(x+2y)^2}$$

$$h_1'(t) = e^t$$

$$h_2'(t) = -e^{-t}$$

$$= \frac{3e^{-t}}{(e^t+2e^{-t})^2} \cdot e^t + \frac{-3e^t}{(e^t+2e^{-t})^2} \cdot (-e^{-t}) = \frac{3e^{-t+t} + 3e^{t-t}}{(e^t+2e^{-t})^2} = \frac{6}{(e^t+2e^{-t})^2}$$

Taylorův polynom

$$T_k(x) = \sum_{i=0}^{k} \frac{1}{i!} \nabla^i_{x-a} f(a)$$

Hessova matice

$$H_f(a) = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2}(a) & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \vdots & & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{pmatrix}$$

$$\nabla^2_h f(a) = h \cdot \left(H_f(a) \cdot h \right)$$

$$\underbrace{\underbrace{\underbrace{f(a)}_{T_0(x)} + (x-a)\cdot \nabla f(a)}_{T_1(x)} + \frac{1}{2}(x-a)\left(H_f(a)(x-a)\right)}_{T_2(x)}$$

1a)

$f(x,y) = x^2 + xy - y$, $a = (1,2)$

$$T_2(x,y) = f(a) + \left((x,y) - (1,2)\right) \cdot \nabla f(a) + \frac{1}{2}\left((x,y) - (1,2)\right) \cdot \left(H_f(a) \cdot \left((x,y) - (1,2)\right)\right)$$

$$\nabla f(x,y) = (2x + y,\ x - 1)$$

$\nabla f(1,2) = (4,0)$

$$H_f(x,y) = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} \quad \vdots \quad H_f(a) = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$$

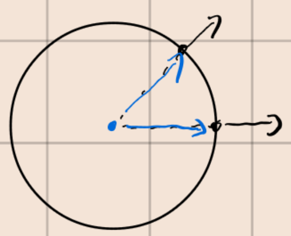
$$T_2(x,y) = 1 + (x-1, y-2)\cdot(4,0) + \frac{1}{2}(x-1, y-2)\cdot\left(\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}\cdot\begin{pmatrix} x-1 \\ y-2 \end{pmatrix}\right) =$$

$$= 4x - 4 + \frac{1}{2}(x-1, y-2)\cdot\begin{pmatrix} 2x-2+y-2 \\ x-1 \end{pmatrix} = -3 + 4y + \frac{1}{2}(2x^2 - 4x + yx - 2x + 4 - y + xy$$

$$- y - 2x + 2) = -3 + 4x + x^2 - 2x + \frac{xy}{2} - x + 2 - \frac{y}{2} + \frac{xy}{2} - \frac{y}{2} - x + 1 =$$

$$= x^2 + xy - y$$

2) $x^2 + y^2 + z^2 = 1$



3) $\frac{x^2}{16} + \frac{y^2}{9} - \frac{z^2}{8} = 0$, $a = (4, 3, 4)$

$f(x,y,z) = \frac{x^2}{16} + \frac{y^2}{9} - \frac{z^2}{8}$

$\operatorname{Lev}(f, 0)$ ... je zadaná plocha

$\nabla f(a)\cdot(x-a) = 0$ ... je tečná rovina

$$\nabla f(x,y,z) = \left(\frac{2x}{16}, \frac{2y}{9}, \frac{-2z}{8}\right) = \left(\frac{x}{8}, \frac{2y}{9}, \frac{-z}{4}\right)$$

$$\nabla f(a) = \left(\frac{1}{2}, \frac{2}{3}, -1\right)$$

$$\left(\frac{1}{2}, \frac{2}{3}, -1\right) \cdot (x-4, y-3, z-4) = 0$$

$$\frac{x}{2} - 2 + \frac{2}{3}y - 2 - z + 4 = 0$$

$$\frac{x}{2} + \frac{2}{3}y - z = 0$$

4 a) $2x^2 + y^2 + z^2 = 7$

$x^2 + y^2 + z^2 - 2x + 4y + 3 = 0$, $a = (0, -1, 0)$

$f(x,y,z) = 2x^2 + y^2 + z^2$, $\mathrm{lev}(f, 1)$

$g(x,y,z) = x^2 + y^2 + z^2 - 2x + 4y + 3$, $\mathrm{lev}(g, 0)$

$$\nabla f(x,y,z) = (4x, 2y, 2z)$$

$$\nabla g\,(2x-2, 2y+4, 2z)$$

$$\nabla f(0,-1,0) = (0,-2,0)$$

$$\nabla g(0,-1,0) = (-2, 2, 0)$$

$\nabla g(\ldots, 1, 0) = (\ldots, \ldots)$

$$\cos\alpha = \frac{|(0,-2,0)\cdot(-2,2,0)|}{\|(0,-2,0)\|\cdot\|(-2,2,0)\|} = \frac{4}{2\cdot 2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\alpha = \frac{\pi}{4}$$

5) $3x^2 + y^2 + 3z^2 = 1$ ... elipsoid

$-12x + 2y + 6z = 0$ ... rovn. rovina

$n = (-12, 2, 6)$

$\Rightarrow$ všechny rovn. roviny mají $n = c\cdot(-12,2,6)$, $c \in \mathbb{R}\setminus\{0\}$

$f(x,y,z) = 3x^2 + y^2 + 3z^2$, $\mathrm{lev}(f,1)$ ... zadaný elipsoid

$\nabla f(x,y,z)$ je n tečné roviny

$\nabla f(x,y,z) = (6x, 2y, 6z)$

$(6x, 2y, 6z) = (-12, 2, 6)\cdot c$

$$\left.\begin{aligned} x &= -2c \\ y &= c \\ z &= c \end{aligned}\right\}$$ body na elipsoidu $(-2c, c, c)$ jsou řešením

dosazení:

$3\cdot 4c^2 + c^2 + 3c^2 = 1$

$16c^2 = 1$

$c = \pm\frac{1}{4}$

tedy odpovídají body

$$\left(-\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right), \left(\frac{1}{2}, -\frac{1}{4}, -\frac{1}{4}\right)$$