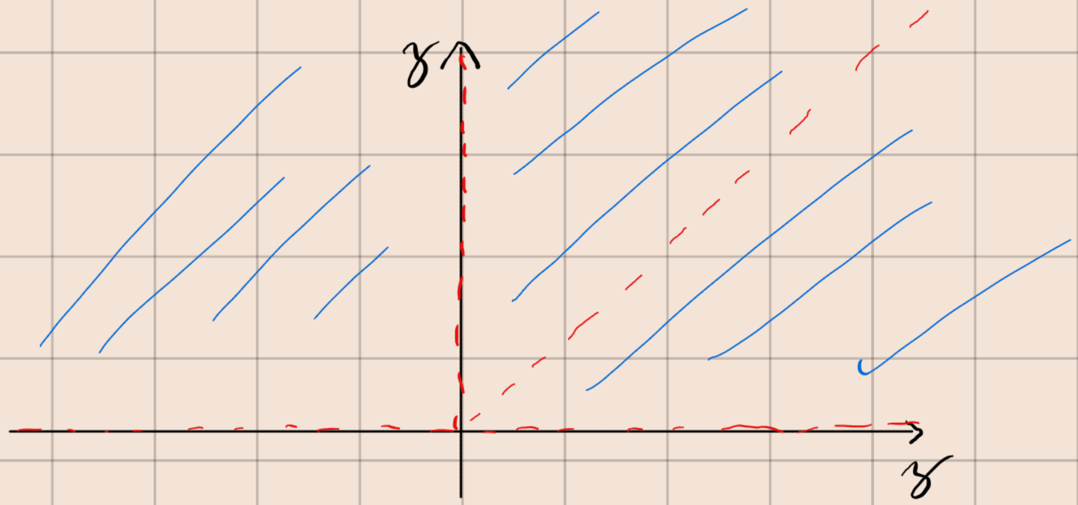
1b) $\dfrac{\ln(x^2y)}{\sqrt{|y-x|}}$ $\rightarrow x^2y>0 \quad x\neq 0, y>0$

$\rightarrow y-x\neq 0$

$x\neq y$

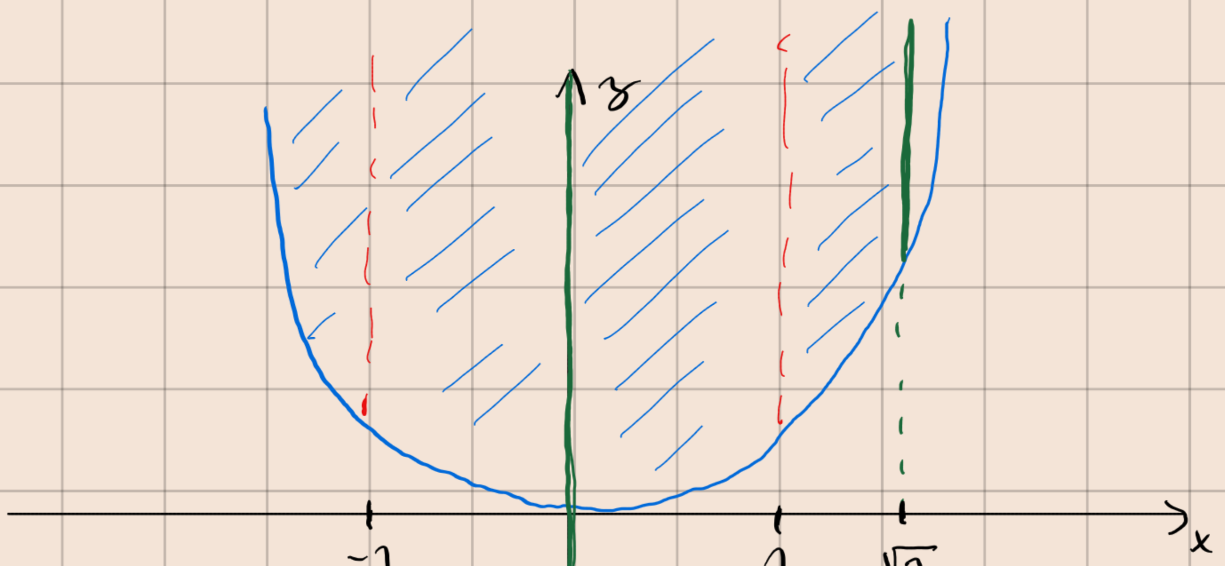
$D=\{(x,y)\in\mathbb{R}^2, \; x\neq 0, y>0, x\neq y\}$



2) $f(x,y)=\dfrac{\sqrt{y-x^2}}{1-x^2}$ $\rightarrow x\neq\pm 1$

$\sqrt{y-x^2}\geq 0 \qquad y-x^2\geq 0$

$y\geq x^2$



3) $f(x,y) = \dfrac{1}{\sin(\pi(x+y))}$

D a hladina výšky 1

$\sin(\pi(x+y)) \neq 0$

$\pi(x+y) \neq k\pi$

$x+y \neq k \in \mathbb{Z}$

$D = \{(x,y) \in \mathbb{R}^2,\ x+y \neq k,\ k \in \mathbb{Z}\}$

hladina:

$\dfrac{1}{\sin(\pi(x+y))} = 1 \iff \sin(\pi(x+y)) = 1$

$\pi(x+y) = \frac{\pi}{2} + 2k\pi$

$x+y = \frac{1}{2} + 2k$

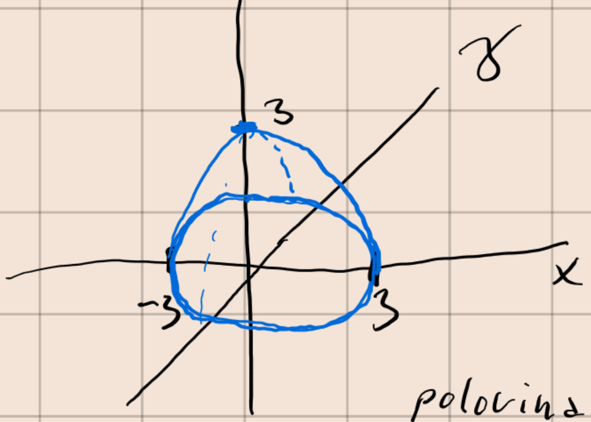
$\mathrm{Lev}(f(1)) = \{(x, \frac{1}{2}+2k-x),\ x \in \mathbb{R},\ k \in \mathbb{Z}\}$

4) $f(x,y) = \sqrt{9-x^2-y^2}$

$9-x^2-y^2 \geq 0$

$9 \geq x^2+y^2$

$D = B(0,3)$



polovina sféry

$$\sqrt{9-x^2-y^2}=z$$
$$9-x^2-y^2=z^2$$
$$9=x^2+y^2+z^2$$

• hladina $\mathrm{lev}(f,c),\ c\geq 0$

$$\left.\begin{aligned}\sqrt{9-x^2-y^2}&=c\\ 9-x^2-y^2&=c^2\\ 9-c^2&=x^2+y^2\end{aligned}\right\}\ \mathrm{lev}(f,c)=\begin{cases} k\left(0,\sqrt{9-c^2}\right), & c\in[0,3]\\ \emptyset, & c>3\end{cases}$$

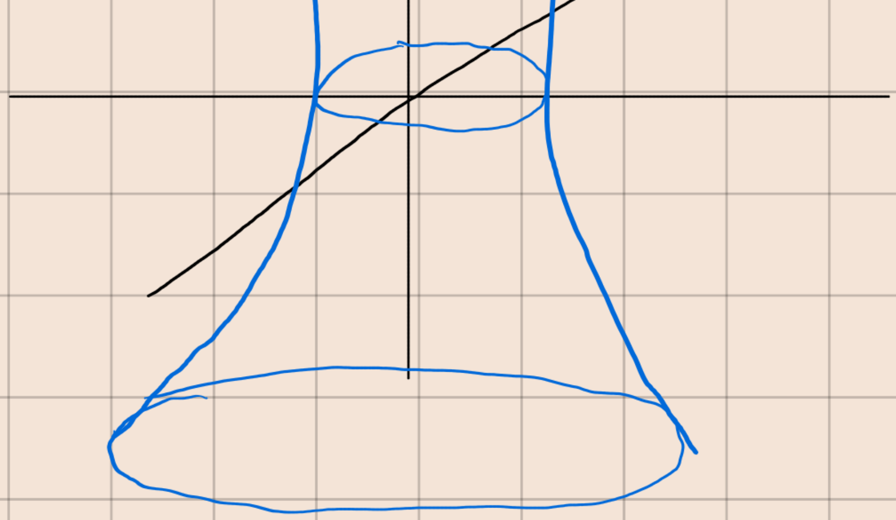
5) $f(x,y,z)=x^2+y^2-z^2$

$$x^2+y^2-z^2=1$$
$$x^2+y^2=1+z^2$$

poloměry kružnic: $\sqrt{1+z^2}$

v 0: 1

v ∞: $|z|$



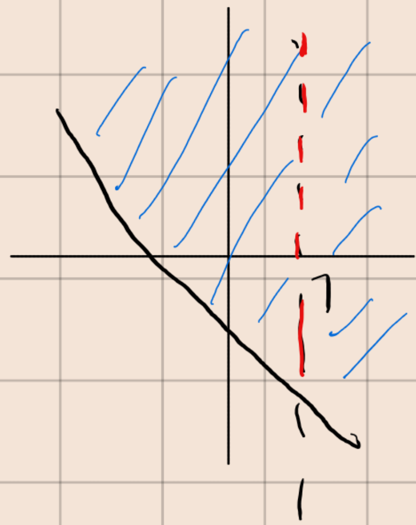
D, H, hladiny c

6) a) $f(x,y) = \dfrac{\sqrt{x+y+1}}{x-1}$

$x \neq 1$

$x+y+1 \geq 0$

$x+y \geq -1$



$D = \{(x,y) \in \mathbb{R}^2,\ x \neq 1,\ y \geq -x-1\}$

$f(D)$:

$x=0 \quad f(0,y) = -\sqrt{y+1},\ -\sqrt{y+1}\,([-1,\infty)) = (-\infty, 0]$

$x=2 \quad f(2,y) = \sqrt{y+3},\ \sqrt{y+3}\,([-3,\infty)) = [0,\infty)$

$f(D) = \mathbb{R}$

hladiny:

$\dfrac{\sqrt{x+y+1}}{x-1} = c$

$x \neq 1$

$\sqrt{x+y+1} = c(x-1)$

$$x+y+1 = c^2(x-1)^2$$

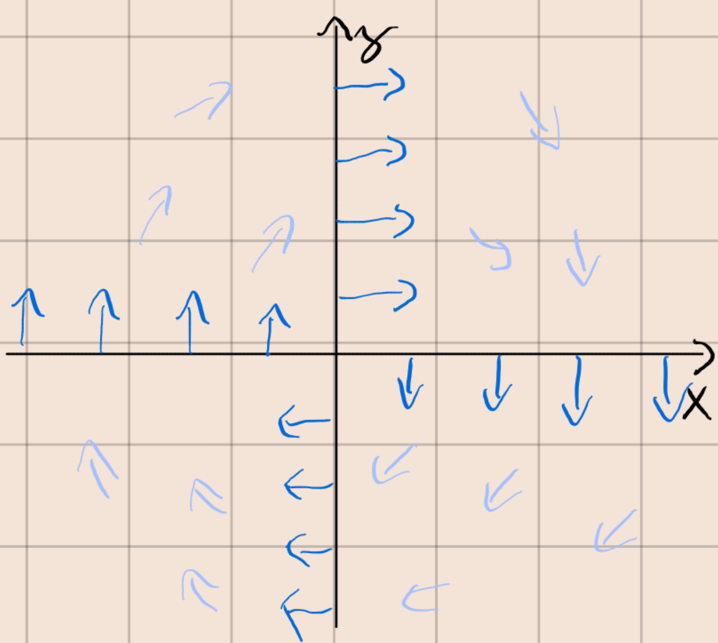
$$y = c^2(x-1)^2 - x - 1$$

$$\operatorname{lev}(f,c) \begin{cases} \{(x, c^2(x-1)^2 - x - 1),\ x > 1\}, & c > 0 \\ \{(x, -x-1),\ x \neq 1\} & c = 0 \\ \{(x, c^2(x-1)^2 - x - 1),\ x < 1\}, & c < 0 \end{cases}$$

1) určete def. obor vektorového pole

$$\vec{F}(x,y) = \left(\frac{y}{\sqrt{x^2+y^2}} , \frac{-x}{\sqrt{x^2+y^2}} \right)$$

$$D = \mathbb{R}^2 \setminus \{(0,0)\}$$



$$x = 0 : F(0,y) = \left(\frac{y}{\sqrt{y^2}}, 0\right) = \left(\frac{y}{|y|}, 0\right) = (\operatorname{sgn}(y), 0)$$

$$y = 0 : F(x,0) = (0, -\operatorname{sgn}(x))$$

8) $f(x,y,z) = \left(\operatorname{arctg} y,\ \dfrac{\ln|z|}{x}\right)$

$D = \{(x,y,z) \in \mathbb{R}^3,\ x \neq 0,\ z > 0\}$

$f(D) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times \mathbb{R}$

$f^{-1}((0,0)) = \{(x,0,1),\ x \in \mathbb{R} \setminus \{0\}\}$