1. $y'' - 5y' + 6y = e^{3x} + 12x - 4$

$y(0) = 2$

$y'(0) = 5$

$y'' - 5y' + 6y = 0$

$\lambda^2 - 5\lambda + 6 = 0$

$3 \quad 2$

$\lambda_1 = 3$

$\lambda_2 = 2$

$y_h = C_1 \cdot e^{2x} + C_2 \cdot e^{3x}$

$y_p = A \cdot x \cdot e^{3x} + Bx + C$

$y_p' = A \cdot (e^{3x} + 3xe^{3x}) + B$

$= Ae^{3x} + 3Axe^{3x} + B$

$y_p'' = 6Ae^{3x} + 9Axe^{3x}$

$6Ae^{3x} + 9Axe^{3x} - 5Ae^{3x} - 15Axe^{3x} + 5B + 6Axe^{3x}$

$+ 6Bx + 6C = e^{3x} + 12x - 4$

$Ae^{3x} + 6Bx + 6C - 5B = e^{3x} + 12x - 4$

$A = 1$

$B = 2$

$6C - 5B = -4 \rightarrow C = 1$

obec. řešení

$y = y_p + y_h$

$y = C_1 e^{2x} + C_2 e^{3x} + x e^{3x} + 2x + 1$

$y' = 2C_1 e^{2x} + 3C_2 e^{3x} + (e^{3x} + 3x e^{3x}) + 2$

$C_1 + C_2 + 1 = 2$ $\qquad$ $C_1 + C_2 = 1$

$2C_1 + 3C_2 + 1 + 2 = 5$ $\qquad$ $2C_1 + 3C_2 = 2$

$C_1 = 1$

$C_2 = 0$

$$\boxed{y = e^{2x} + x \cdot e^{3x} + 2x + 1}$$

2. $y''' + 9y' = 8\cos(x) - 9$

$y(0) = 13$

$y'(0) = 0$

$y''(0) = 0$

$y''' + 9y' = 0$

$\lambda^3 + 9\lambda = 0$

$\lambda(\lambda^2 + 9) = 0$

$\downarrow \quad \lambda_{2,3} = \pm\sqrt{-9}$

$\lambda_1 = 0$

$\lambda_1 = 0 \quad \lambda_2 = -3i \quad \lambda_2 = 3i$

$y_h = C_1 + C_2 \cdot \cos(3x) + C_3 \cdot \sin(3x)$

$y_p = A \cdot \cos(x) + B \cdot \sin(x) + Cx$

$y_p' = -A \sin(x) + B \cdot \cos(x) + C$

$y_p'' = -A \cdot \cos(x) + B \cdot \sin(x)$

$y_p''' = A \cdot \sin(x) - B \cdot \cos(x)$

$A \sin(x) - B\cos(x) - 9A\sin(x) + 9B\cos(x) + 9C = \ldots$

$(-8A)\sin(x) + (8B)\cos(x) + 9C = 8\cos(x) - 9$

$A = 0$

$8B = 8 \rightarrow B = 1$

$9C = -9 \rightarrow C = -1$

$y_p = \cos(x) - x$

$y = y_p + y_h$

$y = \cos(x) - x + C_1 + C_2 \cos(3x) + C_3 \sin(3x)$

$y(0) = 13$

$y'(0) = 0$

$$y''(0)=0$$

$$y' = -3C_2\sin(3x) + 3C_3\cos(3x) - \sin(x) - 1$$

$$y'' = -9C_2\cos(3x) - 9C_3\sin(3x) - \cos(x)$$

$$\Downarrow$$

$$\cos(x) - x + C_1 + C_2\cos(3x) + C_3\sin(3x) = 13$$

$$C_1 + C_2 + 1 = 13 \rightarrow C_1 + C_2 = 12$$

$$-3C_2\sin(3x) + 3C_3\cos(3x) - \sin(x) - 1 = 0$$

$$3C_3 - 1 = 0 \rightarrow C_3 = \frac{1}{3}$$

$$-9C_2\cos(3x) - 9C_3\sin(3x) - \cos(x) = 0$$

$$-9C_2 - 1 = 0 \rightarrow C_2 = -\frac{1}{9}$$

$$\rightarrow C_1 = 12 + \frac{1}{9} = \frac{109}{9}$$

$$\boxed{y = \frac{109}{9} - \frac{1}{9}\cos(3x) + \frac{1}{3}\sin(3x) + \cos(x) - x}$$