

①

$$\int (6x^3 - 5\sqrt[3]{x^2}) \ln 2x \, dx =$$

$$\int (6x^3 - 5x^{2/3}) \ln 2x = \int (6x^3 - 5x^{2/3}) (\ln x + \ln 2) \, dx$$

$$= \int (6x^3 \ln 2 + 6x^3 \ln x - 5x^{2/3} \ln 2 - 5x^{2/3} \ln x) \, dx$$

$$= \underbrace{6 \ln 2 \int x^3 \, dx}_{= 6 \ln 2 \cdot \frac{x^4}{4}} + \underbrace{6 \int x^3 \ln x \, dx}_{\textcircled{1}} - \underbrace{5 \ln 2 \int x^{2/3} \, dx}_{= 5 \ln 2 \cdot \frac{3}{5} x^{5/3}} - \underbrace{5 \int x^{2/3} \ln x \, dx}_{\textcircled{2}}$$

① $\int x^3 \ln x \, dx = \left| \begin{array}{l} u = \ln x \quad dv = x^3 \, dx \\ du = \frac{1}{x} \, dx \quad v = \frac{x^4}{4} \end{array} \right| = \frac{x^4}{4} \ln x -$

$$\int \frac{x^4}{4} \cdot \frac{1}{x} \, dx = \frac{x^4}{4} \ln x - \frac{1}{4} \int x^3 \, dx = \frac{x^4}{4} \ln x - \frac{1}{16} x^4$$

② $\int x^{2/3} \ln x = \left| \begin{array}{l} u = \ln x \quad dv = x^{2/3} \\ du = \frac{1}{x} \quad v = \frac{3}{5} x^{5/3} \end{array} \right| = \frac{3}{5} x^{5/3} \ln x -$

$$\int \frac{3}{5} x^{5/3} \cdot \frac{1}{x} \, dx = \frac{3}{5} x^{5/3} \ln x - \frac{9}{25} x^{5/3}$$

$$= 6 \ln 2 \cdot \frac{x^4}{4} + 6 \left(\frac{x^4}{4} \ln x - \frac{1}{16} x^4 \right) - 5 \ln 2 \cdot \frac{3}{5} x^{5/3} - 5 \left(\frac{3}{5} x^{5/3} \ln x - \frac{9}{25} x^{5/3} \right)$$

$$= \frac{3}{2} \ln 2 \cdot x^4 + \frac{3}{2} x^4 \ln x - \frac{3}{8} x^4 - 3 \ln 2 \cdot x^{5/3} - 3 x^{5/3} \ln x + \frac{9}{5} x^{5/3} + C$$

PP

② $\int (4x-3) \sin(5x) \, dx = \left| \begin{array}{l} u = 4x-3 \quad dv = \sin(5x) \\ du = 4 \quad v = -\frac{1}{5} \cos(5x) \end{array} \right|$

$$\begin{aligned}
 & \int (4x-3) \sin(5x) dx \quad \left| \begin{array}{l} du = 4 dx \\ v = -\frac{1}{5} \cos(5x) \end{array} \right| \\
 &= (4x-3) \left(-\frac{1}{5} \cos(5x) \right) - \int \left(-\frac{1}{5} \cos(5x) \right) \cdot 4 dx \\
 &= -\frac{1}{5} (4x-3) \cos(5x) + \frac{4}{5} \int \cos(5x) dx \\
 &= -\frac{1}{5} (4x-3) \cos(5x) + \frac{4}{5} \cdot \frac{1}{5} \sin(5x) + C \\
 &= -\frac{1}{5} (4x-3) \cos(5x) + \frac{4}{25} \sin(5x) + C
 \end{aligned}$$

$$(3) \quad \int (2-7x) \cos \frac{x}{4} dx = \left| \begin{array}{ll} u = 2-7x & dv = \cos \frac{x}{4} \\ du = -7 & v = 4 \sin \frac{x}{4} \end{array} \right|$$

$$\begin{aligned}
 &= (2-7x) \cdot 4 \sin \left(\frac{x}{4} \right) - \int 4 \sin \left(\frac{x}{4} \right) \cdot (-7) dx \\
 &= 4(2-7x) \cdot \sin \left(\frac{x}{4} \right) + 28 \int \sin \frac{x}{4} dx \\
 &= 4(2-7x) \cdot \sin \left(\frac{x}{4} \right) - 112 \cos \left(\frac{x}{4} \right) + C
 \end{aligned}$$

$$(4) \quad \int (2x+5) e^{-4x} dx = \left| \begin{array}{ll} \text{pp} \\ u = 2x+5 & dv = e^{-4x} \\ du = 2 & v = -\frac{1}{4} e^{-4x} \end{array} \right| =$$

$$\begin{aligned}
 &= (2x+5) \left(-\frac{1}{4} e^{-4x} \right) - \int \left(-\frac{1}{4} e^{-4x} \right) \cdot 2 dx = \\
 &= -\frac{1}{4} (2x+5) e^{-4x} + \frac{1}{2} \int e^{-4x} dx =
 \end{aligned}$$

$$= -\frac{1}{4} (2x+5) e^{-4x} + \frac{1}{2} \left(-\frac{1}{4} e^{-4x} \right) = -\frac{1}{4} (2x+5) e^{-4x} - \frac{1}{8} e^{-4x}$$

$$4(x+3)e$$

$$\cdot \frac{1}{2} (4e)$$

$$4(x+3)e$$

$$\cdot \frac{1}{8} e$$

