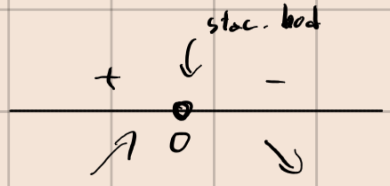
$f(x) = \frac{1}{x^2+1}$ | $D_f = \mathbb{R}$, spojitá

sudá

$f'(x) = \frac{0-2x}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2} = \left((x^2+1)^{-1}\right)' = \frac{-2x}{(x^2+1)^2}$ | $D_{f'} = \mathbb{R}$



$f''(x) = \left(\frac{-2x}{(x^2+1)^2}\right)' = \frac{-2(x^2+1)^2 - (-2x) \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4}$

$= \frac{2(3x^2+1)}{(x^2+1)^3}$

ze spojitosti:

f je rostoucí: $(-\infty; 0\rangle$

f je klesající: $\langle 0; \infty)$

má ostré lok. maximum $f(0) = 1$

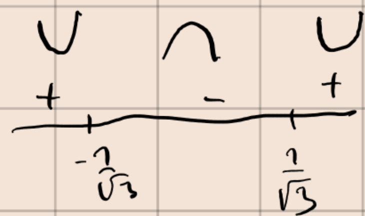
$f''(x) = 0$

$2(3x^2+1) = 0$

$3x^2 = -1$

$x = \pm\sqrt{\frac{1}{3}}$

$x = \pm\frac{1}{\sqrt{3}}$



minima a maxima funkce

$f(0) = 1$ $\max\limits_{\mathbb{R}} f$

$\left.\begin{array}{l} \lim\limits_{x \to -\infty} = \frac{1}{\infty} = 0 \\ \\ \lim\limits_{x \to \infty} = \frac{1}{\infty} = 0 \end{array}\right\}$ $\min\limits_{\mathbb{R}} f$ neexistuje

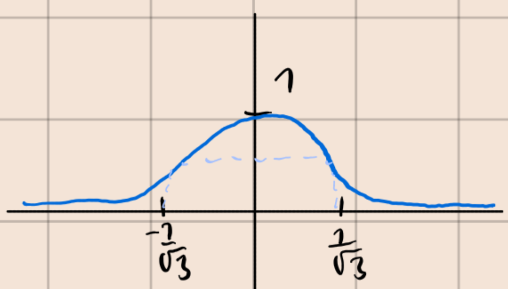
$f''(x) > 0 \Leftrightarrow 3x^2 > 1 \Leftrightarrow x^2 > \frac{1}{3}$

ze spojitosti:

f je ryze konvexní na $(-\infty, -\frac{1}{\sqrt{3}}\rangle$

a na $\langle \frac{1}{\sqrt{3}}; \infty)$

f je ryze konkávní na $\langle -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\rangle$



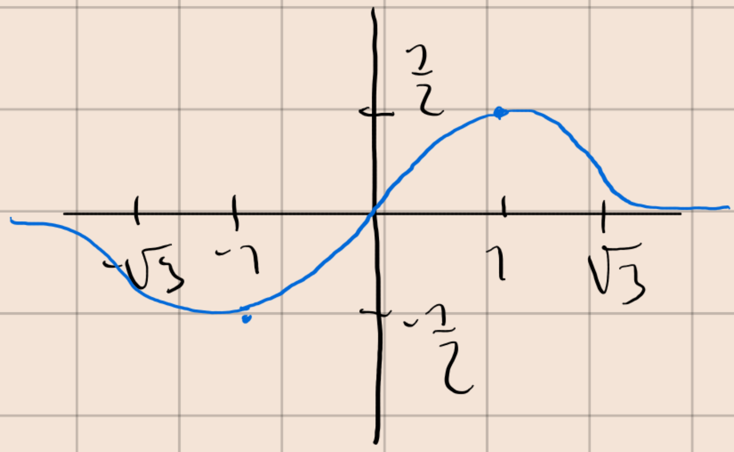
graf f má inflexní body:

$\left[\frac{1}{\sqrt{3}}, \frac{3}{4}\right]$ $\left[-\frac{1}{\sqrt{3}}, \frac{3}{4}\right]$

$\left[\sqrt{3}, \infty\right], \left[\sqrt{3}, \infty\right]$

Doma:

$$f(x) = \frac{x}{x^2+1}$$



$$f(x) = \frac{|x-1|}{x^2}, \quad D_f = \mathbb{R} \setminus \{0\}$$

spojitá na D_f

$$f(x) = \begin{cases} \frac{x-1}{x^2} & x \geq 1 \\ \frac{1-x}{x^2} & x \leq 1, x \neq 0 \end{cases}$$

$$f'(x) = \frac{1 \cdot x^2 - (x-1)2x}{x^4} = \frac{2-x}{x^3} \quad x > 1$$

$$\frac{x-2}{x^3} \quad x < 1 \;\; x \neq 0$$

$\underline{x > 1}$

$f'(x) = \frac{2-x}{x^3}$

$f'(x) = 0$ pro $x = 2$

$f'(x) > 0$ pro $x \in (1;2)$

$f'(x) < 0$ pro $x \in (2;\infty)$

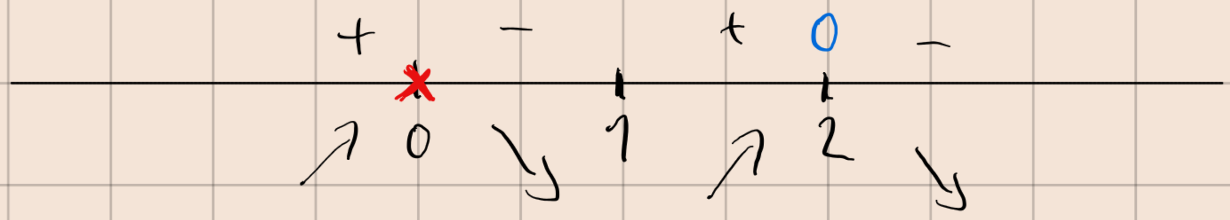
$\underline{x < 1}$

$f'(x) = \frac{x-2}{x^3}$

$f'(x) = 0$ neexistuje

$f'(x) > 0$

(



$f'(1)$: $\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^-} \frac{x-2}{x^3} = -1$

$\lim_{x \to 1^+} f'(x) = \lim_{x \to 1^-} \frac{x^2}{x^3} = 1$

$\Big\}$ $f'(1)$ neex.

ze spojitosti:

- f je rostoucí na $(-\infty;0)$ a na $\langle 1;2\rangle$

- f je klesající na $(0,1\rangle$ a na $\langle 2;\infty)$

- f má ostré lok. minimum $f(1) = 0$

- f má lok. maximum $f(2) = \frac{1}{4}$

$\underbrace{x>1}$ $\underbrace{x<1}$

$f''(x) = \left(\frac{2-x}{x^3}\right)' = \frac{2(x-3)}{x^4}$ $f''(x) = \frac{2(3-x)}{x^4}$

$f''(x) > 0 \quad x > 3$ $f''(x) = 0$ nenastává

$f''(x) < 0 \quad x \in (0,3)$ $f''(x) > 0$ na $(-\infty,0)$

$f''(x) = 0 \quad x = 3$ na $(0,1)$

f''	$+$		$+$		$-$		$+$
f	$\cup$	0	$\cup$	1	$\cap$	3	$\cup$

f je konvexní na $(-\infty;0)$, $(0,1\rangle$, $\langle 3;\infty)$

f je ryze konkávní na $\langle 1;3\rangle$

asymptoty:

vodorovné:

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x-1}{x^2} = \lim_{x\to\infty} \frac{1-\frac{1}{x}}{x} = \left|\frac{1}{\infty}\right| = 0$$

asymptota v $+\infty$ s rovnicí $y=0$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1-x}{x^2} = 0$$

asymptota v $-\infty$ s rovnicí $y=0$

svislé:

$$\lim_{x\to 0^+} f(x) = \left|\frac{1}{0^+}\right| = \infty$$

↳ f má svislou asymptotu $x=0$

šikmé

$$y = px + q \quad \begin{cases} p = \lim\limits_{x \to x_0} \dfrac{f(x)}{x} \\ q = \lim\limits_{x \to x_0} \left(f(x) - px\right) \end{cases}$$

$$\lim_{x \to x_0} \left(f(x) - (px + q) \right) = 0$$