$$\begin{cases} x^2+2y^2=1 \\ xy>1 \end{cases}$$

① body v rovině $x+y+z=1$ nejblíže k bodu $(2,0,-3)$

$$\tilde{f}(x,y,z)=\|(x,y,z)-(2,0,-3)\|=\sqrt{(x-2)^2+y^2+(z+3)^2}$$

$$f(x,y,z)=\|(x,y,z)-(2,0,-3)\|^2=(x-2)^2+y^2+(z+3)^2$$

$$M=\{(x,y,z)\in\mathbb{R}^3 \mid x+y+z=1\}$$

ať $A=(\alpha,\beta,\gamma)\in M$

A je bod minima $\tilde{f}$ na M, t.j. $\tilde{f}(A)\leq f(x,y,z)$ $\forall(x,y,z)\in M$

$\Leftrightarrow$ A je bod min f na M, t.j. $f(A)\leq f(x,y,z)$ $\forall(x,y,z)\in M$

$g(x,y,z)=x+y+z-1$ ... vazbová funkce

$$\nabla f+\lambda\nabla g=0$$
$$g=0$$

$$2(x-2)+\lambda\cdot 1=0$$
$$2y+\lambda\cdot 1=0 \quad \dots \lambda=-2y$$
$$2(z+3)+\lambda\cdot 1=0$$
$$x+y+z=1$$

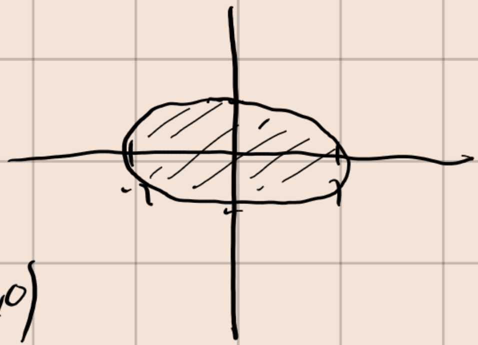
$$2x-4-2y=0 \qquad y=x-2$$
$$2z+6-2y=0$$
$$x+y+z=1$$

$$2z+10-2x=0 \quad \dots x=z+5$$
$$2x+z=3$$

$$3z+10=3$$
$$z=-\frac{7}{3}$$
$$x=\frac{8}{3}$$
$$y=\frac{2}{3}$$

$\left(\frac{8}{3},\frac{2}{3},-\frac{7}{3}\right)$ ... podezřelý bod z extrému je jen jeden minimum uvnitř existuje $\Rightarrow$ je to minimum

Př.: $f(x,y) = e^{-xy}$, $M = \{(x,y) \in \mathbb{R}^2 \mid x^2+4y^2 \leq 1\}$



- $(x,y) \in \operatorname{int} M$:

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= -y e^{-xy} = 0 \\ \frac{\partial f}{\partial y} &= -x e^{-xy} = 0 \end{aligned}\right\} (x,y) = (0,0)$$

- $(x,y) \in \partial M = \{(x,y) \in \mathbb{R}^2 \mid x^2+4y^2 = 1\}$

$$\left.\begin{aligned} -y e^{-xy} + \lambda \cdot 2x &= 0 \qquad / \cdot 4y \\ -x \cdot e^{-xy} + \lambda 8y &= 0 \qquad / \cdot (-x) \end{aligned}\right\} \oplus \Rightarrow (-4y^2 + x^2) e^{-xy} = 0$$

$$x^2 + 4y^2 = 1$$

$-4y^2 + x^2 = 0 \quad \dots \quad x^2 = 4y^2$

$4y^2 + x^2 = 1$

$4y^2 + 4y^2 = 1$

$8y^2 = 1$

$y = \pm \sqrt{\frac{1}{8}}$

$x = \pm \frac{1}{\sqrt{2}}$

podezřelé body:

$A = (0,0)$ $\qquad f(A) = 1 = e^0$

$B = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{8}}\right)$ $\qquad f(B) = e^{-\frac{1}{4}}$ ... minimum

$C = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{8}}\right)$ $\qquad f(C) = e^{\frac{1}{4}}$ ... maximum

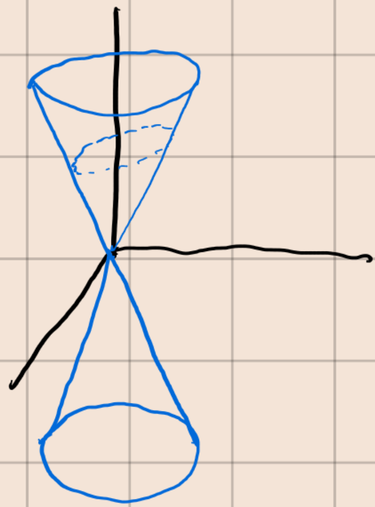
$D = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{8}}\right)$ $\qquad f(D) = e^{\frac{1}{4}}$ ... maximum

$E = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{8}}\right)$ $\qquad f(E) = e^{-\frac{1}{4}}$ ... minimum

Př.: Nalezněte nejvyšší bod na křivce, která je průnikem

ploch: $x^2+y^2-z^2=0$

$x+2z=4$



$f(x,y,z)=z$

$M=\{(x,y,z)\in\mathbb{R}^3 \mid x^2+y^2-z^2=0,\ x+2z=4\}$

$0+\lambda\cdot 2x+\mu\cdot 1=0$ (1)

$0+\lambda\cdot 2y+\mu\cdot 0=0 \ \dots\ 2\lambda y=0$ (2)

$1+\lambda(-2z)+\mu\cdot 2=0$ (3)

$x^2+y^2-z^2=0$ (4)

$x+2z=4$ (5)

- $\lambda=0 \Rightarrow \mu=0$: spor s (3)
- $\lambda\neq 0,\ y=0$: $\left.\begin{array}{l}(4)\Rightarrow x^2=z^2\\ (5)\Rightarrow x=4-2z\end{array}\right\} \Rightarrow 16-16z+3z^2=0$

$D=16^2-4\cdot 16\cdot 3=16(16-12)=16\cdot 4$

$z=\frac{16\pm 8}{6}=\begin{cases}4 & ✓\\ \frac{4}{3} & ✗\end{cases}$

Hledaný bod je:

$(-4,0,4)$