Určete jádro lin. zobrazení:

$$f(ax^2+bx+c) = (a-c)x^2+b$$

hledám $\forall a,b,c,d \in \mathbb{R}$, aby $f(a,b,c,d) = 0$

$$(a-c)x^2+b=0$$

když $(a-c)=0$

a $b=0$:

tedy: $\ker(f) = \left\{ \begin{pmatrix} a \\ 0 \\ a \end{pmatrix} \middle| \, a \in \mathbb{R} \right\} = \operatorname{span}\left(\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right)$

Dokažte linearitu:

Lineární:
1) uzavřené pro sčítání

2) uzavřené pro násobení skalárem

1) a) $f\big((a_1x^2+b_1x+c_1)+(a_2x^2+b_2x+c_2)\big) = f\big((a_1+a_2)x^2+(b_1+b_2)x+(c_1+c_2)\big)$

$= \big((a_1+a_2)-(c_1+c_2)\big)x^2 + (b_1+b_2) = (a_1+a_2-c_1-c_2)x^2+b_1+b_2$

$= a_1x^2+a_2x^2-c_1x^2-c_2x^2+b_1+b_2$

$=$

b) $f(a_1x^2+b_1x+c_1)+f(a_2x^2+b_2x+c_2) =$

$(a_1-c_1)x^2+b_1+(a_2-c_2)x^2+b_2 =$

$= a_1x^2-c_1x^2+b_1+a_2x^2-c_2x^2+b_2$

$= a_1x^2+a_2x^2-c_1x^2-c_2x^2+b_1+b_2$

2) a) $f\big(\alpha(ax^2+bx+c)\big) = f(\alpha ax^2+\alpha bx+\alpha c)$

$$= (\alpha a - \alpha c)x^2 + \alpha b = \alpha a x^2 - \alpha c x^2 + \alpha b$$

$$\alpha \cdot f(ax^2+bx+c) = \alpha \cdot \big((a-c)x^2 + b\big) =$$

$$= \alpha(ax^2 - cx^2 + b) = \alpha a x^2 - \alpha c x^2 + \alpha b$$

$$=$$

Random úkol

$$X = \begin{pmatrix} 0 & -2 & 1 \\ 0 & 1 & 1 \\ 5 & -2 & 9 \end{pmatrix}$$

$$Y = \begin{pmatrix} 1 & 0 & -1 \\ -2 & 3 & 2 \\ 3 & 1 & 2 \end{pmatrix}$$

$$\text{coord}_Y\left(\begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}\right) = a\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + b\begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + c\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & -2 & 1 \\ -2 & 3 & 2 & 0 & 1 & 1 \\ 3 & 1 & 2 & 5 & -2 & 9 \end{array}\right) \Rightarrow$$

$$T_{X \mapsto Y} = \cancel{\begin{pmatrix} 0 & -2 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}} = \begin{pmatrix} 1 & -1 & 2 \\ 0 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$B = x_2 + x_3 = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 9 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 7 \end{pmatrix} \quad \text{vůči } X$$

$$A = T_{X \mapsto Y} \cdot B = \begin{pmatrix} 1 & -1 & 2 \\ 0 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} -1-2+14 \\ 0-2+7 \\ \dots \end{pmatrix} = \begin{pmatrix} 11 \\ 5 \\ \dots \end{pmatrix}$$

$$\ldots + 2 + 5 \qquad \begin{pmatrix} 8 \end{pmatrix}$$

$$\mathrm{coord}_\gamma(\vec{v}) =$$