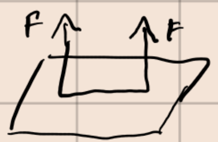
7.5 | žebřík

1. imp. věta - izol. s.

$$\frac{d\vec{p}}{dt} = 0 \rightarrow \sum \vec{p} = konst.$$
$$\searrow \sum \vec{F} = 0$$

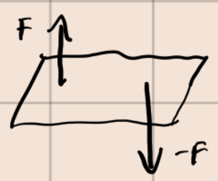
2. imp. věta - izol. s.

$$\frac{d\sum\vec{L}}{dt} = 0 \rightarrow \sum \vec{L} = konst.$$
$$\sum \vec{M} = 0$$



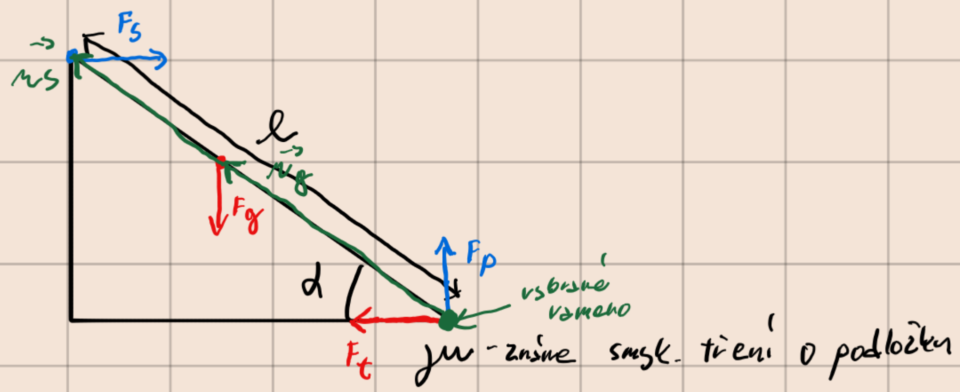
$\sum \vec{M} = 0$

$\sum \vec{F} \neq 0$



$\sum \vec{M} = 0$

$\sum \vec{F} = 0$



μ - značí smyk. tření o podložku

$\alpha(\mu) = ?$ ... aby se žebřík nehnul

1) $\sum \vec{F} = 0$

$$\vec{F}_s + \vec{F}_g + \vec{F}_p + \vec{F}_t = 0$$

x-složka: $+F_s - F_t = 0$ | y-složka: $-F_g + F_p = 0$

$\boxed{2)\ F_s - F_t = F}$ $\longrightarrow$ $F_s = \mu \cancel{F_g} = \cancel{F_g}$ $\longleftarrow$

$F_t \geq F_s$ $\to$ F_s ... $\frac{}{2tg\alpha}$ ①

2) $\sum \vec{M} = \vec{0}$ vůči lib. bodu otáčení

$\vec{M}_{F_s} + \vec{M}_{F_g} + \cancel{\vec{M}_{F_p}} + \cancel{\vec{M}_{F_t}} = \vec{0} \to \vec{r}_s \times \vec{F}_s + \vec{r}_g \times \vec{F}_g = \vec{0}$

$\to -l F_s \cdot \sin\alpha + \frac{l}{2} F_g \cos\alpha = 0$

$\to F_s \sin\alpha = \frac{F_g}{2}\cos\alpha$

$F_s \cdot tg\,\alpha = \frac{F_g}{2}$

$$\boxed{F_s = \frac{F_g}{2tg\alpha}}$$

$\vec{F}_s = (F_s, 0, 0)$

$\vec{r}_s = (-l\cdot\cos\alpha, l\cdot\sin\alpha, 0)$

$\vec{F}_g = (0, -F_g, 0)$

$\vec{r}_g = \left(-\frac{l}{2}\cos\alpha, \frac{l}{2}\sin\alpha, 0\right)$

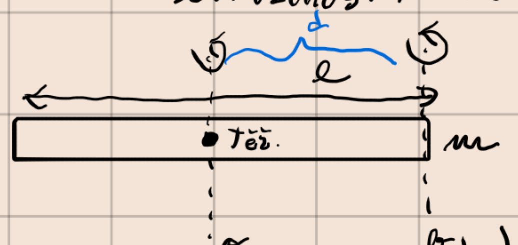
$\vec{M} = \vec{M}_z$

① $\mu \geq \frac{1}{2tg\alpha}$

$tg\,\alpha \geq \frac{1}{2\mu}$

$\alpha \geq \operatorname{arctg}\left(\frac{1}{2\mu}\right)$

Moment setrvačnosti homog. tenké tyče



a) $J_0 = ?$ kolem těžiště

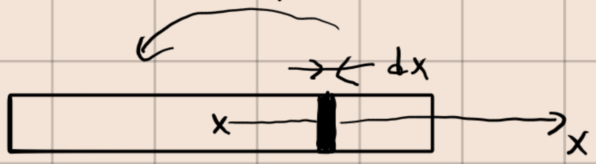
b) $J_1 = ?$ kolem okraje

a)

$J = \int r_\perp^2\, dm = \int^{l/2} r_\perp^2 \rho \cdot d \cdot V \overset{r_\perp = x}{=} \int_{-l/2}^{l/2} x^2 \rho \cdot S \cdot dx =$ ⊛

$\rho \cdot d \cdot V \quad -l/2$

$\rho \cdot S = \frac{m}{V} \cdot S = \frac{m}{l} = \lambda$ lin. hustota



$dV = S \cdot dx$

$$(*) = \frac{m}{l}\left[\frac{x^3}{3}\right]_{-l/2}^{l/2} = \frac{m}{l}\cdot\frac{l^3}{3\cdot 8} - \frac{m}{3l}\left(\frac{-l^3}{8}\right) = \frac{m}{l}\cdot\frac{l^3}{3\cdot 8}\cdot 2 = \frac{ml^3}{12}$$

$$J = \frac{1}{12} m l^2$$

b) Steinerova věta

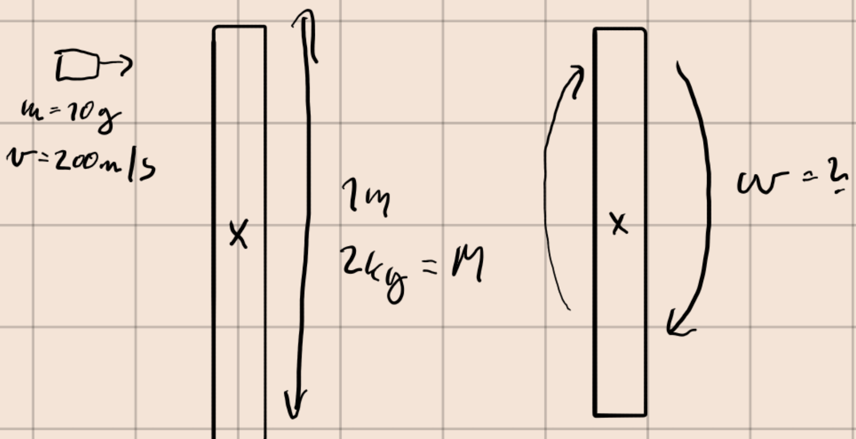
$$J = J_0 + m d^2$$

osa proch. jiným bodem $\parallel$ s J_0

osa proch. těžištěm

$$J_1 = \frac{1}{12} m l^2 + m\left(\frac{l}{2}\right)^2 = \frac{1}{3} m l^2$$

7.27)



ZZMH:

$$L_{střely} = \frac{l}{2} m v \longrightarrow \underset{v\ tyči}{L_{střely}} = \underset{(střela + tyč)}{L_{s+t}} = J_{s+T} \cdot \omega = (J_s + J_t)\omega$$

$L = J\omega$ ≐ jako $p = m.v$

Königova věta:

$$E_{kin} = \underbrace{\frac{1}{2} m v^2}_{translace} + \underbrace{\frac{1}{2} J \omega^2}_{rotace}$$

$$J_t = \frac{1}{12} m l^2$$

$J = \int r^2 dm = r^2 m \leftarrow$ moment setrvačnosti hmot. body

$$J = m R^2$$

$$\frac{l}{2} m v = \left(\frac{1}{12} M l^2 + \frac{1}{4} m l^2\right) \omega$$

$$\omega = \frac{6 m v}{(M + 3m) l} \doteq 6 \frac{1}{s}$$

Speciální teorie relativity

galileovské transformace

$$x' = x - vt \qquad v = v_x$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

axiomy STR:

1) rychlost světla ve všech inerc. soust. je stejná

2) všechny fyz. zákony mají ve všech inerc. soustavách stejný tvar

Lorenzovy transformace:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\gamma = \frac{1}{1 - \frac{v^2}{c^2}} \geq 1$$

Lorenz. faktor!

$$y' = y \,,\; z' = z$$

$$t' = \frac{\left(t - \frac{xv}{c^2}\right)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

dilatace času:

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{\ldots}}}$$

c^2

Kontrakce délek:

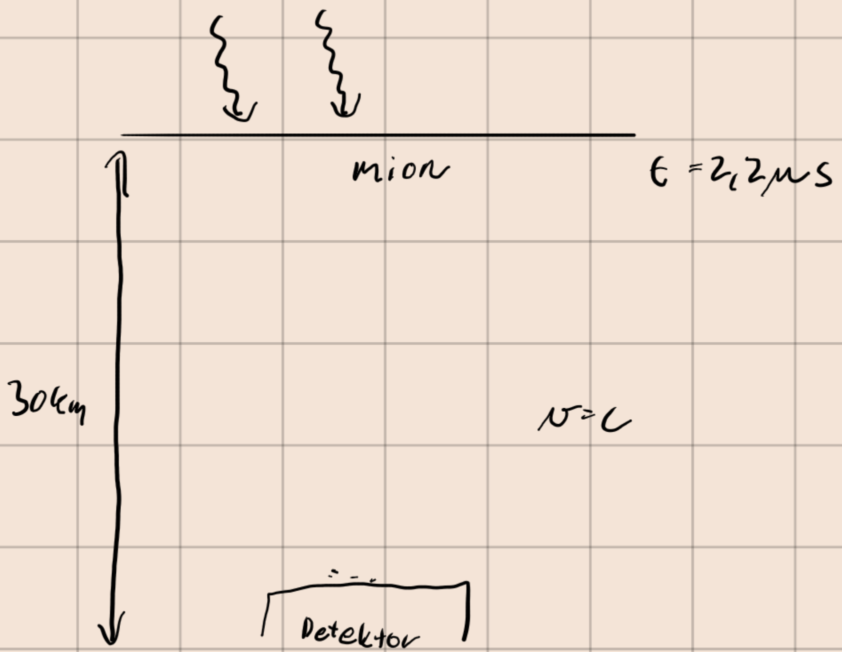
$$l' = l\sqrt{1-\frac{v^2}{c^2}}$$

sčítání rychlostí:

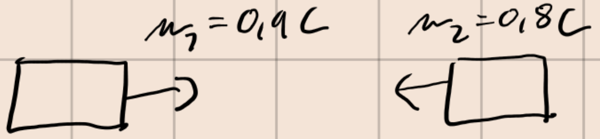
$$u = \frac{u'+v}{1+\frac{u'v}{c^2}}$$

$$u = \frac{(c+v)\cancel{c}}{\cancel{c}+\frac{\cancel{c}v}{\cancel{c}^2}} = c$$

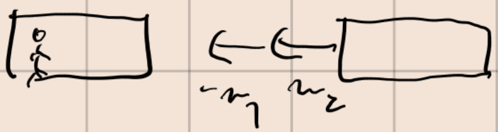
10.2



10.4)



relativní rychlost



$$\Delta u = \frac{u_2 - u_1}{1 + \frac{u_1 u_2}{c^2}} = \frac{|u_2| + |u_1|}{1 + \frac{|u_1||u_2|}{c^2}} = 0{,}988\,c$$

$$E_{kin} \neq \frac{1}{2} m(v)\, v^2$$

relativistická hmotnost:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Celk. energie při pohybu:

$$E = m \cdot c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

klidová energie:

$E_0 = m_0 c^2$

kin. energie:

$$E_{kin} = E - E_0 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0 c^2$$

$\boxed{\text{úkol 10.10}}$

Průlet galaxií

$$\longleftrightarrow$$

$l = 100\,000$ LY

$\rightsquigarrow$ $3 \cdot 10^8$ let

proton: $E = 10^{10}$ GeV

$E_0 = 938$ MeV

doba průletu

a) soustava spojená s galaxií

b) soustava spojená s protonem

$$E = \frac{E_0}{\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow v = c\sqrt{1-\frac{E_0^2}{E^2}}$$

$v \doteq c$

E_0

$$\frac{E_0}{E} = \sqrt{1-\frac{v^2}{c^2}}$$

proton: $\Delta t_p = \frac{d}{c} = 100\,000$ let

$$\Delta t_p = \frac{d'}{c} = \frac{d}{c}\sqrt{1-\frac{v^2}{c^2}} = \frac{d}{c}\cdot\frac{E_0}{E} = 5\,\text{min}$$