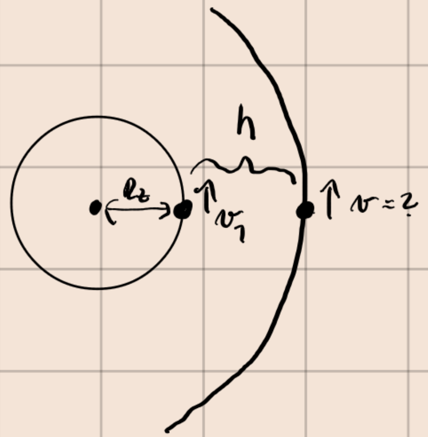
Gravitační pole

9.1 Geostacionární družice



$\omega = \omega_{země}$

$h = ?$

$v = ?$

$F_d = F_G$

$$\cancel{m} \cdot \omega^2 r = \varkappa \cdot \frac{\cancel{m} \cdot M_z}{r^2}$$

gravitační konstanta

$$r^3 \frac{4\pi^2}{T_z^2} = \varkappa \cdot M_z \quad \Rightarrow \quad r = \sqrt[3]{\frac{\varkappa \cdot M_z \cdot T_z^2}{4\pi^2}}$$

$T_z = 1 \text{ den}$

$r = R_z + h$

$$h = \sqrt[3]{\frac{\varkappa M_z T_z^2}{4\pi^2}} - R_z$$

$h = 35\,833 \text{ km}$

$v = \omega \cdot r$

$$v = \frac{2\pi}{T_z} \cdot \left(\varkappa \cdot M_z\right)^{1/3} \cdot \frac{T_z^{2/3}}{(2\pi)^{2/3}}$$

$$v = \frac{2^{1/3} \pi^{1/3}}{T_z^{1/3}} \left(\varkappa M_z\right)^{1/3}$$

$$v = \sqrt[3]{\frac{2\pi \varkappa M_z}{T_z}} = 3 \text{ km/s}$$

Keplerovy zákony

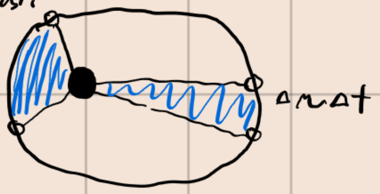
1. trajektorie planet jsou elipsy, v ohnisku je slunce

$L = \vec{r} \times \vec{p}$

2. $$w = \frac{dS}{dt} = \frac{d\left(\frac{1}{2}|\vec{r} \times d\vec{r}|\right)}{dt} = \frac{1}{2}\left|\vec{r} \times \frac{d\vec{r}}{dt}\right| = \frac{\frac{1}{2}|\vec{r} \times \vec{v}\, m|}{m} = \frac{|L|}{2m} = \text{konst.}$$

zákon zachování

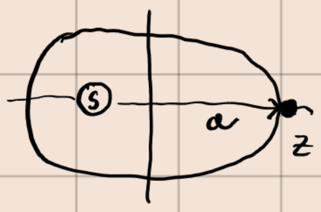
momentu hybnosti



$$r \cdot v \cdot m = L$$

$$\frac{dL}{dt} = \frac{d(\vec{r} \times m\vec{v})}{dt} = \frac{d\vec{r}}{dt} \times m\vec{v}$$

3.



$$\frac{T^2}{a^3} = \text{konst.}$$ (pro stejný orbitální systém)

upgrade 3.kz pro kružnicovou trajektorii

$$m\omega^2 r = \varkappa \frac{m \cdot M}{r^2}$$

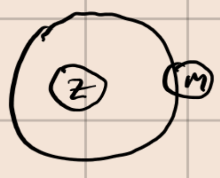
$$\frac{4\pi^2 r^3}{T^2} = \varkappa \cdot M$$

$$\text{konst.} = \frac{r^3}{T^2 M_\odot} = \frac{\varkappa}{4\pi^3}$$

9.9

$a_G = 1070\,000\,\text{km}$

$T_G = 7{,}15$ dne $= 846\,100$ s



$a_M = 384400\,\text{km}$

$T_M = 27{,}32$ dne

$$\frac{M_j}{M_z} = ? \qquad \frac{T_G^2 \cdot M_j}{a_G^3} = \frac{T_M^2}{a_M^3} \cdot M_z$$

$$\frac{M_j}{M_z} = \frac{T_M^2}{T_G^2} = \frac{a_G^3}{a_M^3} = 314$$

$\boxed{9.4 - \text{úkol}}$

9.6

1) $v_0 = ?$ → traj. kružnice

2) $v_0 = ?$ → těsně minu zemi

$$F_d = F_G$$

$$m \cdot \frac{v^2}{R_0} = \varkappa \frac{m \cdot M_Z}{R_0^2}$$

$$v = \sqrt{\frac{\varkappa M_Z}{R_0}}$$

$R_0 = R_Z$

1. kosmická rychlost

$$v_I = \sqrt{\frac{\varkappa M_Z}{R_Z}}$$

1) $L = \text{konst.}$

$r \cdot m \cdot v = \text{konst.}$

$$R_0 \cdot \cancel{m} \cdot v_0 = R_Z \cancel{m} \cdot v_p$$

$$\boxed{R_0 \cdot v_0 = R_Z \cdot v_p}$$

1. rce

$$E_{pot} = -\int \vec{F_G}\, d\vec{r} = -\int F_G\, dr$$

$$= + \int \varkappa \frac{m M_Z}{r^2}\, dr = - \varkappa \frac{m \cdot M_Z}{r}$$

$$E = E_{kin} + E_{pot}$$

$$= \frac{1}{2} m v^2 - \varkappa \cdot \frac{m \cdot M_Z}{r}$$

$$v_{II} = \sqrt{\frac{2 \varkappa M_Z}{R_Z}} =$$

2. kosmická (úniková) rychlost

zákon zachování mech. energie

$$\frac{1}{2} m \cdot v_0^2 - \frac{\varkappa\, m\, M_Z}{R_0} = \frac{1}{2} m \cdot v_p - \frac{\varkappa \cdot m \cdot M_Z}{R_Z}$$

$$R_0 \qquad\qquad R_z$$

2. věc

$$v_p^2 - v_0^2 = \frac{2 \cdot \varkappa \cdot m_z}{R_z} - \frac{2 \cdot \varkappa \cdot m_z}{R_0} = 2 \varkappa \cdot m_z \cdot \left(\frac{1}{R_z} - \frac{1}{R_0}\right) = 2 \varkappa m_z \left(\frac{R_0 - R_z}{R_z R_0}\right)$$

$$\left(\frac{R_0^2}{R_z^2} - 1\right) v_0^2 = \left(\frac{R_0^2 - R_z^2}{R_z^2}\right) \cdot v_0^2 = 2 \varkappa m_z \left(\frac{R_0 - R_z}{R_z \cdot R_0}\right)$$

$$v_0^2 = 2 \varkappa m_z \left(\frac{\cancel{R_0 - R_z}}{\cancel{R_z} R_0}\right)\left(\frac{R_z^{\cancel{2}}}{\underbrace{\cancel{R_0^2 - R_z^2}}_{(R_0 + R_z)}}\right)$$

$$v_0 = \sqrt{2 \varkappa m_z \frac{R_z}{R_0 (R_0 + R_z)}}$$

soustava hmotných bodů:

IMPULZOVÉ VĚTY

1\. IV: $\vec{I} = \Delta \vec{p} = \int \vec{F} \, dt$

$$\frac{d \sum \vec{p}}{dt} = \sum \vec{F} = \underbrace{\sum \vec{F}^{(i)}}_{= 0 \text{ díky 3.N.Z.}} + \sum \vec{F}^{(e)}$$

$$\frac{d \sum \vec{p}}{dt} = \sum \vec{F}^{(e)} = 0$$

Izolovaná soustava

$$\frac{d \sum \vec{p}}{dt} = 0$$ zákon zachování hybnosti

$\sum \vec{p} = \text{konst.}$

2\. I.V.

$\vec{M} = \vec{r} \times \vec{F}$

moment síly

$$\frac{d \sum \vec{L}}{dt} = \sum \vec{M}^{(e)}$$

$\vec{L} = \vec{r} \times \vec{p}$

moment hybnosti

2>MH

$$\frac{d\sum\vec{L}}{dt} = 0 \rightarrow \sum\vec{L} = \text{konst.}$$