Na zápočet: 35 b., z toho 20/40 b. z písemek.

Rozměrová analýza

1.2 přesýpací hodiny

$$\frac{\Delta m}{\Delta t} = \frac{\Delta m}{\Delta t}\Big(\underset{\text{objem}}{S},\ g,\ \underset{\text{hustota}}{\rho}\Big)$$

$$\frac{\Delta m}{\Delta t} = \text{konst.}\ \rho^{\alpha} \cdot S^{\beta} \cdot g^{\gamma} \quad *$$

$$\left[\frac{\Delta m}{\Delta t}\right] = kg^{1} \cdot s^{-1} = kg^{\alpha} \cdot m^{-3\alpha} \cdot m^{2\beta} \cdot m^{\gamma} \cdot s^{-2\gamma} = \left[\rho^{\alpha} \cdot S^{\beta} \cdot g^{\gamma}\right]$$

kg: $\boxed{1 = \alpha}$

s: $-1 = -2\gamma$

m: $0 = -3\alpha + 2\beta + \gamma$

$-3 + 2\beta + \gamma = 0$

$-2\gamma = -1$

$\boxed{\gamma = 1/2}$

$-3 + 2\beta + 1/2 = 0$

$2\beta = \frac{5}{2}$

$\boxed{\beta = \frac{5}{4}}$

$$* = \text{konst.}\ \rho \cdot S^{5/4} \cdot \sqrt{g}$$

Podle vědeckého článku: průřez díry, průřez zrn

$$\frac{\Delta m}{\Delta t} = \text{konst.}\ \rho \cdot \sqrt{g}\left(\left(\underset{\text{průřez díry}}{D} - \text{konst}\ \underset{\text{průřez zrn}}{d}\right)^{2}\right)^{5/4}$$

1.3 gravitační tlak

$$P_g \sim \text{konst.}\ M^{\alpha} \cdot R^{\beta} \cdot G^{\gamma}$$

$$[P_g] = Pa = kg^{1}\ m^{-1}\ s^{-2} = kg^{\alpha}\ m^{\beta}\ kg^{-\gamma}\ m^{3\gamma}\ s^{-2\gamma}$$

$$[P_g] = Pa = kg \cdot m \cdot s = \underbrace{kg} \cdot \underbrace{m} \cdot \underbrace{kg \cdot m \cdot s}$$

$$[p] = \frac{F}{S} = N \cdot m^{-2}$$

$$[F] = [m \cdot a] = N = kg \cdot m \cdot s^{-2}$$

$$\left(F = G \cdot \frac{m \cdot M}{r^2}\right) \quad [G] = \left[\frac{F \cdot r^2}{m \cdot M}\right] = N \cdot m^2 \cdot kg^{-2} = kg \cdot m \cdot s^{-2} \cdot m^2 \cdot kg^{-2} = \underbrace{kg^{-1} \cdot m^3 \cdot s^{-2}}$$

$$kg: 1 = \alpha - \gamma \rightarrow \underbrace{\alpha = 2}$$

$$m: -1 = \beta + 3\gamma \rightarrow -1 = \beta + 3 \rightarrow \underbrace{\beta = -4}$$

$$s: -2 = -2\gamma \rightarrow \underbrace{\gamma = 1}$$

$$\longrightarrow P_g = \frac{m^2}{R^4} G$$

Úkol: 1.4

vyberte si 1 planckovu jednotku:

$x_p, E_p, (m_p)$

redukovaná planckova konstanta

$$x_p \sim konst \cdot c^\alpha \cdot \hbar^\beta \cdot G^\gamma$$

$c = 3 \cdot 10^8 \, m \cdot s^{-1}$

$\hbar = 1{,}05 \cdot 10^{-34} \, J \cdot s$

$G = 6{,}67 \cdot 10^{-11} \, N \cdot kg^{-2} \cdot m^2$

Planckova délka:

$$l_p \sim konst. \, c^\alpha \cdot \hbar^\beta \cdot G^\gamma$$

$[l_p] = m^1$

$[c] = m \cdot s^{-1}$

$[\hbar] = J \cdot s = N^1 \cdot m^1 \cdot s^1$

$[G] = N^1 \cdot kg^{-2} \cdot m^2$

$[F] = m \cdot a = kg^1 \cdot m^1 \cdot s^{-2}$

Kinematika hmotného bodu

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt}$$

velikost

jednotkový směrový vektor

$$\vec{v} = v \cdot \vec{e}_v$$

$$\frac{d\vec{v}}{dt} = \frac{dv}{dt} \cdot \vec{e}_v + v \frac{d\vec{e}_v}{dt}$$

$$\frac{d\vec{e}_v}{dt} = 0 \rightarrow \text{přímočarý}$$

$$\frac{dv}{dt} \neq 0 \rightarrow \text{konst.} \rightarrow a \neq 0 \text{ rovnoměrně zrychlený}$$

$$= 0 \rightarrow \text{rovnoměrný pohyb}$$

a) rovnoměrný pohyb přímočarý

$$v = \text{konst.}$$

$$v = \frac{ds}{dt} \rightarrow ds = v.dt \Big/ \int_{s_0}^{s} \int_{t_0}^{t}$$

$$s - s_0 = v(t - t_0)$$

$$s(t) = s_0 + v(t - t_0)$$

b) rovnoměrně zrychlený pohyb

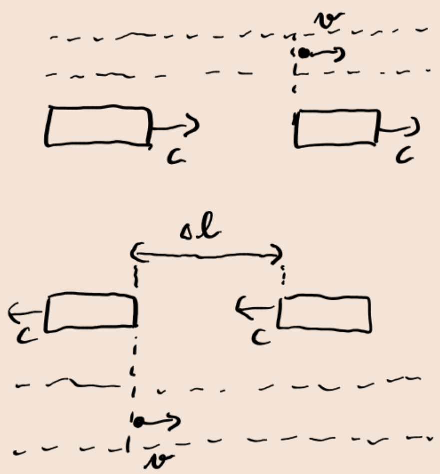
$$a = \text{konst} = \frac{dv}{dt}$$

$$dv = a.dt \;\Big|\; \int$$

$$v(t) - v_0 = \int_{t_0}^{t} a\,dt = a.(t - t_0)$$

$$v(t) = v_0 + a(t - t_0)$$

(2.1) Autobusy na Strahov



$T_v = 13\text{min } 30\text{sec}$

a) $\Delta l = (c - v) T_v$

$=$

b) $\Delta l = (c + v) T_p$

$T_p = 10\text{min } 48\text{s}$

$$\Delta l = c.T_{av}$$

$$T_{av} = \left(1 - \frac{v}{c}\right) T_v$$

$$c.T_v - v.T_v = c.T_p + v.T_p$$

$$cT_v - cT_p = v.T_p + vT_v$$

$$c(T_v - T_p) = v(T_p + T_v)$$

$$\frac{c}{v} = \frac{(T_p + T_v)}{(T_v - T_p)} \rightarrow \frac{v}{c} = \frac{(T_v - T_p)}{(T_p + T_v)}$$

$$T_{zy} = \left(1 - \frac{T_v - T_p}{T_p + T_v}\right) \cdot T_v = \frac{2 \cdot T_v \cdot T_p}{T_v + T_p}$$

$T_v = 780 + 30 = 810\ s$

$T_p = 648$

$$T_{zy} = \frac{2 \cdot 810 \cdot 648}{1458} = \frac{810 \cdot 648}{729} =$$

$$\begin{array}{r} 648 \\ 810 \\ \hline 1458 \end{array}$$

Planckova délka:

$l_p \sim konst. \; c^{\alpha} \cdot \hbar^{\beta} \cdot G^{\gamma}$

$$m^1 \cdot kg^0 \cdot s^0 = m^{\alpha} \cdot s^{-\alpha} \cdot kg^{\beta} \cdot m^{2\beta} \cdot s^{-\beta} \cdot kg^{-\gamma} \cdot m^{3\gamma} \cdot s^{-2\gamma}$$

$[l_p] = m^1$

- $[c] = m \cdot s^{-1}$
- $[\hbar] = J \cdot s = N^1 \cdot m^1 \cdot s^1 = kg^1 \cdot m^2 \cdot s^{-1}$
- $[G] = N^1 \cdot kg^{-2} \cdot m^2 = kg^{-1} \cdot m^3 \cdot s^{-2}$

$[F] = m \cdot a = kg^1 \cdot m^1 \cdot s^{-2}$

$m: \; 1 = \alpha + 2\beta + 3\gamma$

$kg: \; 0 = \beta - \gamma \rightarrow \gamma = \beta$

$s: \; 0 = -\alpha - \beta - 2\gamma$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ -1 & -1 & -2 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{array}\right)$$

$\boxed{\gamma = \frac{1}{2}}$

$\boxed{\beta = \frac{1}{2}}$

$\alpha + 3/2 = 0$

$\boxed{\alpha = -3/2}$

$$c^{-\frac{3}{2}} \cdot \hbar^{\frac{1}{2}} \cdot \gamma^{\frac{1}{2}} = \sqrt{\frac{\hbar \cdot G}{c^3}}$$

$$l_p = \frac{1{,}054571817 \cdot 10^{-34} \cdot 6{,}67430 \cdot 10^{-11}}{\quad}$$

$\sqrt{}$ 299792458

$l_p = 1{,}616255 \cdot 10^{-35}\,m$