$$x^2 = \frac{1}{x+1}$$

$$x_0 = 0, \quad \varepsilon = 0{,}0001$$

Newton: $n = 5$, $0{,}7548777$

$\varphi(x) = x$ ... pevný bod

$$x^2 - \frac{1}{x+1} = 0$$

$$x^2 - \frac{1}{x+1} + x = x$$

$$\varphi(x) = x^2 - \frac{1}{x+1} + x$$

$$x_{k+1} = \varphi(x_k)$$

1. $\varphi(x) = x^2 - \frac{1}{x+1} + x$

$$x_{k+1} = \varphi(x_k)$$

a) $x_0 = 0$

$x_1 = -1$

$x_2 = (-1)^2 - \frac{1}{-1+1} - 1 = ?$

b) $x_0 = 1$

$x_1 = \frac{3}{2}$

$x_2 = \frac{9}{4} - \frac{2}{5} + \frac{3}{2}$ = diverguje

$$\varphi'(x) = 2x + \frac{1}{(x+1)^2} + 1$$

$$|\varphi'(1)| = 2 \cdot 1 + \frac{1}{4} \quad \times$$

2.

$$x^2 = \frac{1}{x+1}$$

$$x = \frac{1}{\sqrt{x+1}}$$

$$\varphi(x) = \frac{1}{\sqrt{x+1}}$$

$$x_{k+1} = \frac{1}{\sqrt{x_k+1}}$$

$x_0 = 0$

$n = 8$

$x_8 = 0{,}7549$

3. $x^2 = \frac{1}{x+1} \Rightarrow x+1 = \frac{1}{x^2} \Rightarrow x = \frac{1}{x^2} - 1$

$$\varphi(x) = \frac{1}{x^2} - 1$$

$$\varphi'(x) = -\frac{2}{x^3}$$

$$|\varphi'(x)| = \frac{2}{|x^3|}$$

$x_0 = 0$ nelze

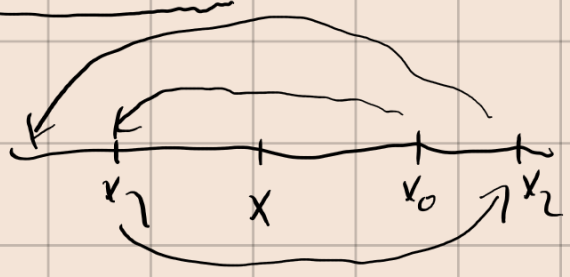
$x_0 = 1 \Rightarrow x_2 =$ nelze

$|\varphi(1)| = 2 > 1$ nelze

$x_0 = \frac{1}{2}$

$\rightsquigarrow 0{,}5 ; 3 ; -0{,}88 ; \dots 0{,}07 ; 739$ diverguje

metoda relaxace:



$$x_1 = \varphi(x_0) \cdot \lambda + (1-\lambda) x_0$$

$$x_0 = \frac{1}{2}$$

$$\lambda = \frac{1}{2}$$

$$\frac{1}{2};\ 1{,}75;\ 0{,}53;\ 1{,}49;\ 0{,}47;\ 1{,}99;\ 0{,}62;\ 1;\ 1;$$

$$\varphi_\lambda(x) = \lambda \cdot \varphi(x) + (1-\lambda)x$$

$$\varphi'_\lambda(x) = \lambda \cdot \frac{-2}{x^3} + 1 - \lambda = 0$$

$$\lambda = \frac{1}{\frac{2}{x^3} + 1} = \frac{x^3}{2 + x^3} \qquad x_0 = \frac{1}{2} \rightarrow \lambda = \frac{\left(\frac{1}{2}\right)^3}{2 + \left(\frac{1}{2}\right)^3} = \frac{1}{9}$$

$$\rightarrow 0{,}5;\ 0{,}777;\ 0{,}7639;\ 0{,}7583$$

$$x - 3 = \frac{1}{x^2}$$

$$x_0 = 1$$

1. metoda

$$\varphi(x) = \frac{1}{x^2} + 3$$

2. metoda

$$x^2 = \frac{1}{x-3}$$

$x_{k+1} = \varphi(x_k)$

$x_0 = 7$

$x_1 = \frac{1}{7} + 3 = 3$

$x_2 = \frac{1}{9} + 3 = \frac{28}{9}$

$x_3 = \frac{9^2}{28^2}$ ... moc

$\varphi(x) = x^{-2} + 3$

$\varphi'(x) = -2x^{-3}$

$= \frac{-2}{x^3}$

$\varphi'(7) = \frac{-2}{7} = -2 \to |-2| > 7$

... keine chance

$\varphi_\lambda(x) = \lambda \varphi(x) + (1-\lambda) \cdot x$

$\varphi_\lambda(x_0) = 0$

$\lambda\left(\frac{1}{7} + 3\right) + (1-\lambda) \cdot 7 = 0$

$9\lambda + 7 - \lambda = 0$

$3\lambda = -7$

$\lambda = -\frac{7}{3}$

$x_0 = \varphi_\lambda(x) = -\frac{7}{3} \cdot \left(\frac{1}{x^2} + 3\right) + \left(1 + \frac{7}{3}\right) \cdot x$

$x_1 = \varphi_\lambda(7) = -\frac{7}{3}\left(4\right) + \left(1 + \frac{7}{3}\right)$

$= -\frac{4}{3} + \frac{5}{3} = \frac{1}{7}$

$\varphi(x) = \frac{1}{\sqrt{x-3}}$

$x_0 = 7$

$x_1 = \frac{1}{\sqrt{7-3}} = \frac{1}{\sqrt{2}}$

$x_2 = \frac{1}{\sqrt{\frac{1}{\sqrt{2}} - 3}} = \frac{1}{\sqrt{\frac{1-3\sqrt{2}}{\sqrt{2}}}} = \frac{2}{\sqrt{1-3\sqrt{2}}}$

$x_3 \sim$

$\varphi(x) = (x-3)^{-\frac{1}{2}}$

$\varphi'(x) = x \cdot \left(-\frac{1}{2}\right)(x-3)^{-\frac{3}{2}}$

$$x_2 = \varphi_1\left(\frac{2}{3}\right) = -\frac{1}{3}\left(9+3\right) + \frac{4}{3}\cdot\frac{2}{3}$$

$$= -\frac{12}{3} + \frac{4}{3} = -\frac{8}{3}$$