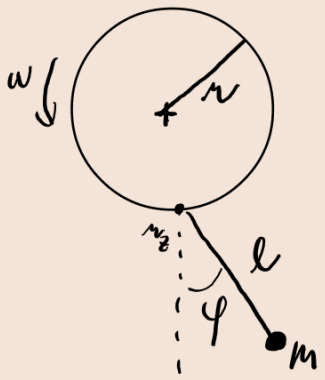
4.2 – kyvadlo na rotujícím kotouči



bod na kružnici:

$x_z = r.\cos(wt)$

$y_z = r.\sin(wt)$

konec kyv. rel. k bodu na kružnici:

$x_k = l.\sin(\varphi)$

$y_k = -l.\cos(\varphi)$

$x = r.\cos(wt) + l.\sin(\varphi)$ $\qquad \dot{x} = -r.\sin(wt).w + l.\cos(\varphi).\dot{\varphi}$

$y = r.\sin(wt) - l.\cos(\varphi)$ $\qquad \dot{y} = r.\cos(wt).w + l.\sin(\varphi).\dot{\varphi}$

$L = E_k - E_p$

$E_k = \frac{1}{2}mv^2$ $\qquad E_p = -m.g.y = -m.g(r.\sin(wt) - l.\cos(\varphi))$

$\vec{v}^2 = \dot{x}^2 + \dot{y}^2$

$\dot{x}^2 = r^2w^2\sin^2(wt) - 2.r.w.\sin(wt).l.\cos(\varphi).\dot{\varphi} + l^2.\cos^2(\varphi).\dot{\varphi}^2$

$\dot{y}^2 = r^2.w^2.\cos^2(wt) + 2.rw.\cos(wt).l.\sin(\varphi).\dot{\varphi} + l^2.\sin^2(\varphi).\dot{\varphi}^2$

$\vec{v}^2 = r^2.w^2.(\sin^2(wt) + \cos^2(wt)) + l^2.\dot{\varphi}^2(\cos^2\varphi + \sin^2\varphi) - 2rwl.\sin(wt).$

$.\cos\varphi.\dot{\varphi} + 2rwl.\cos(wt).\sin\varphi.\dot{\varphi}$

$\vec{v}^2 = r^2.w^2 + l^2.\dot{\varphi}^2 - 2rwl.\sin(wt).\cos(\varphi).\dot{\varphi} + 2rwl.\cos(wt).$

$.\sin(\varphi).\dot{\varphi}$

$\vec{v}^2 = r^2.w^2 + l^2\dot{\varphi}^2 + 2rwl\,\dot{\varphi}\sin(\varphi + wt)$

$E_k = \frac{1}{2}m\left(r^2w^2 + l^2.\dot{\varphi}^2 + 2rwl.\dot{\varphi}.\sin(\varphi + wt)\right)$

$$L = \frac{1}{2}m\left(r^2w^2 + l^2.\dot{\varphi}^2 + 2rwl.\dot{\varphi}.\sin(\varphi + wt)\right) + m.g\left(r.\sin(wt) - l.\cos(\varphi)\right)$$

$L = \frac{1}{2}m\left(r^2w^2 + l^2\dot{\varphi}^2 + 2rwl\,\dot{\varphi}\sin(\varphi + wt)\right) + m.g.r.\sin(wt) - m.g.l.\cos(\varphi)$

$L = \frac{1}{2}m(\dots)$

Lagr. rovnice II. druhu:

$$\varphi: \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = 0$$

$$\frac{\partial L}{\partial \dot{\varphi}} = m l^2 \dot{\varphi} + m \cdot r \cdot w \cdot l \cdot \sin(\varphi + wt)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = m l^2 \ddot{\varphi} + m \cdot r \cdot w \cdot l \cdot \left(\cos(\varphi + wt) \cdot w + \cos(\varphi + wt) \cdot \dot{\varphi}\right)$$

$$\frac{\partial L}{\partial \varphi} = m \cdot r \cdot w \cdot l \cdot \varphi \cdot \cos(\varphi + wt) + m \cdot g \cdot l \cdot \sin\varphi$$

$$m l^2 \ddot{\varphi} + m \cdot r \cdot w \cdot l \cdot \left(\cos(\varphi + wt) \cdot w + \cos(\varphi + wt) \cdot \dot{\varphi}\right) - m r w l \cdot \cos(\varphi + wt) - m \cdot g \cdot l \cdot \sin\varphi = 0$$

$$m l^2 \ddot{\varphi} + m r w^2 \cdot l \cdot \cos(\varphi + wt) - m \cdot g \cdot l \cdot \sin(\varphi) = 0$$

$$\boxed{l \cdot \ddot{\varphi} + r \cdot w^2 \cdot \cos(\varphi + wt) - g \cdot \sin(\varphi) = 0}$$