$$y' = f(y)\, g(x)$$

$$y'(x) = f(y(x))\, g(x)$$

$f(y) \neq 0$

$$\frac{y'}{f(y)} = g(x)$$

matematicky

$$\frac{y'(x)}{f(y(x))} = g(x)$$

$$\int \frac{y'(x)}{f(y(x))}\, dx = \int g(x)\, dx$$

$$\int \frac{y'(x)}{f(y(x))}\, dx = \left| \begin{array}{l} y = y(x) \\ dy = y'(x)\, dx \end{array} \right| = \int \frac{1}{f(y)}\, dy$$

nematematicky

$$\frac{\frac{dy}{dx}}{f(y)} = g(x)$$

$$\int \frac{dy}{f(y)} = \int g(x)\, dx$$

1

$$y' = \frac{3y}{x-1} \qquad x \neq 1$$

$y \neq 0$ $\qquad$ $y(x) = 0$ ... stac. řešení na $(-\infty, 1)$ nebo $(1, \infty)$

$$\frac{y'}{3y} = \frac{1}{x-1}$$

$$\int \frac{1}{3y}\, dy = \int \frac{1}{x-1}\, dx$$

$$\frac{1}{3} \ln|y| = \ln|x-1| + C, \quad C \in \mathbb{R}$$

$\ln|y| = 3\ln|x-1| + C$

$|y| = e^{3\cdot\ln|x-1| + c} = e^{c}\cdot e^{\ln|x-1|^3} = e^{c}\cdot|x-1|^3$

$y = \pm e^{c}\cdot|x-1|^3 = (\pm e^{c})\cdot(x-1)^3, \; x \neq 1$

$= D(x-1)^3, \; D = \mathbb{R}\setminus\{0\}$

Řešení: $\underbrace{y(x) = 0 \text{ nebo } y(x) = \pm e^{c}(x-1)^3, x\neq 1}_{= D\cdot(x-1)^3,\; D\neq 0,\; x\neq 1}$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad}$$

$$y(x) = \tilde{D}(x-1)^3, \; \tilde{D}\in\mathbb{R}, \; x\neq 1$$

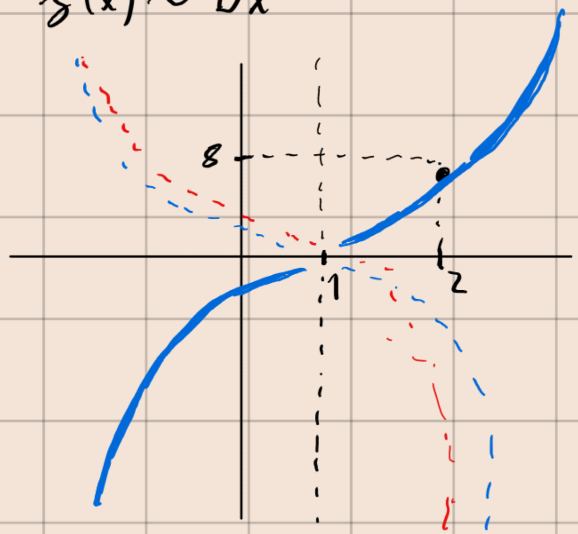
Asymptotický růst v $+\infty$ $\quad y(x) \sim \tilde{D}x^3$

Počáteční podmínky:

a) $y(2) = 8$

$8 = \tilde{D}(2-1)^3 = \tilde{D}$

$\Rightarrow y(x) = 8(x-1)^3, \; x\in(1,\infty)$



b) $y(-1) = 8$

$8 = \tilde{D}(-1-1)^3 = \tilde{D}(-2)^3 = \tilde{D}(-8)$

$\Rightarrow y(x) = -(x-1)^3, \; x\in(-\infty,1)$

c) $y(1) = 8$

$\Rightarrow$ řešení neexistuje

(2)

$$2y' = \frac{e^x}{y} \qquad \boxed{y \neq 0}$$

$$2yy' = e^x$$

$$\int 2y\,dy = \int e^x\,dx$$

$$y^2 = e^x + C$$

$$y = \pm\sqrt{e^x + C}\ , \ C \in \mathbb{R}$$

def. obor

$$e^x > -C$$

- $C \geq 0,\ x \in \mathbb{R}$
- $C < 0,\ x > \ln(-C)$

Asymptotický růst v $+\infty$:

$$y(x) \sim \pm e^{\frac{x}{2}}$$

Počáteční podmínky:

a) $y(0) = 2$

$$2 = \sqrt{1 + C}$$

$$4 = 1 + C$$

$$\boxed{C = 3}$$

$$y = \sqrt{e^x + 3}\ , \ x \in \mathbb{R}$$

b) $y(0) = -1$

$$1 = -\sqrt{e^0 + C}$$

$$C = 0$$

$$y = -\sqrt{e^x} = -e^{\frac{x}{2}}\ , \ x \in \mathbb{R}$$

c) $y(1) = 0$

$y \neq 0 \Rightarrow$ nelze

d) $y(1) = 1$

$$1 = \sqrt{e + C}$$

$$1 = e + C$$

$$C = 1 - e$$

$$y = \sqrt{e^x + 1 - e}\ , \ x > \ln(e - 1)$$

(3)

$$y' = 1 - y$$

$$\frac{y'}{1-y} = 1$$

$$\int \frac{1}{1-y}\, dy = \int 1\, dx \qquad 1 - y \neq 0$$

$$\underline{y \neq 1} \quad \to y(x) = 1,\ x \in \mathbb{R}$$

$$-\ln(1-y) = x + c$$

$$\ln|1-y| = -x + c$$

$$|1-y| = e^{-x+c}$$

$$|1-y| = e^{-x} + e^{c}$$

$$1 - y = \pm e^{-x} \cdot e^{-c}$$

$$y = 1 \pm e^{-c} \cdot e^{-x}$$

$$y = 1 \pm D \cdot e^{-x}, \quad D \in \mathbb{R}$$
$$x \in \mathbb{R}$$

<u>asympt. rieš. v +∞</u>

$1 \pm D \cdot e^{-x} \to \infty$:

$1 \pm D \cdot \frac{e}{x} \to \infty$:

$\to \underline{y(x) \sim 1}$

<u>Part. riešeni:</u>

$$y(0) = 0$$

$$0 = 1 + D e^{0}$$

$$0 = 1 + D \quad \to \quad y = 1 - e^{-x},\ x \in \mathbb{R}$$

$$D = -1$$

$x' = 2t \cdot x^2$

$$\frac{x'}{x^2} = 2t \qquad \text{(bez podm.)}$$

$$\int \frac{1}{x^2}\, dx = 2\int t\, dt$$

$$\ln|x^2| = 2 \cdot t^2$$

$$|x^2| = e^{2t^2}$$

$$x^2 = e^{2t^2}$$

$$x = \pm\sqrt{e^{2t^2}}$$

a) $x(1) = 0$

b) $x(-2) = 1$

c) $x(-1) = -1$