# Lagrangeův formalismus


Princip minima akce $S$

$\delta S = 0 \;\to\; \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \qquad i \in \{1, 2, \dots s\}$

pro každý stupeň volnosti

Akce

$S = \int L(\vec{q}, \dot{\vec{q}}, t)\, dt$

pohyb. rovnice, rovnice sil

Lagrangeova funkce

zobecněné souřadnice

zobec. rychlost

počet stupňů volnosti:

$= 3N - V$ počet holonomních vazeb

$h(x, y, z, t)$

- $z = 0$
- $x^2 + y^2 = \ell^2$

$3 - 2 = 1$ stupeň volnosti

diference

$dz = 0$

$2x\,dx + 2y\,dy = 0$

$L = T - U = E_{kin} - E_{pot}.$

$L = L(\varphi, \dot{\varphi}, \;)$

Příklad:

$L = T - U = T(\dot{x}) - U(x)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$

$\frac{\partial \frac{1}{2} m (v_x^2 + v_y^2)}{\;} = \partial\left(\frac{1}{2} m (\dot{x}^2 + \dot{y}^2)\right) - U \quad = m\dot{x} = \frac{\partial L}{\partial \dot{x}} = p_i$

$\partial v_x \qquad \partial \dot{x} \qquad \partial x$

zobecněná hybnost,

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

$$-\frac{\partial U}{\partial x} = \frac{\partial \left( T(\dot{x}) - U(x) \right)}{\partial x} = \frac{\partial L}{\partial x}$$

## 4.3 | Pružina + kyvadlo


- $z = 0 \leftarrow$ jediná holonomní vazba
- $x^2 + y^2 = l^2(t)$

$3 - 1 = 2$ stupně volnosti

$q_1 = l$

$q_2 = \varphi$

$x = l \cdot \sin\varphi$

$y = -l \cdot \cos\varphi$

$z = 0$

$\dot{x} = \dot{l} \cdot \sin\varphi + l \cdot \dot{\varphi} \cos\varphi$

$\dot{y} = -\dot{l} \cdot \cos\varphi - l \cdot \dot{\varphi} \cdot \sin\varphi$

$$\dot{x}^2 + \dot{y}^2 = \dot{l}^2 \sin^2\varphi + l^2 \cdot \dot{\varphi}^2 \cdot \cos^2\varphi \cancel{+ 2 l \dot{l} \dot{\varphi} \sin\varphi \cdot \cos\varphi}$$
$$+ \dot{l}^2 \cos^2\varphi + l^2 \dot{\varphi}^2 \sin\varphi \cancel{- 2 l \dot{l} \dot{\varphi} \sin\varphi \cdot \cos\varphi}$$

$L = T - U = \frac{1}{2} m \left( \dot{x}^2 + \dot{y}^2 + \dot{z}^2 \right) - U$

$E_{kin} = \frac{1}{2} m \cdot \dot{l}^2 + \frac{1}{2} m l^2 \dot{\varphi}^2$

$E_{pot} = m \cdot g \cdot h + \frac{1}{2} k \cdot (l - l_0)^2 = m \cdot g \cdot l \cdot \cos\varphi + \frac{1}{2} k (l - l_0)^2$

$L = \frac{1}{2} m \cdot \dot{l}^2 + \frac{1}{2} m \cdot l^2 \dot{\varphi}^2 + m \cdot g \cdot l \cdot \cos\varphi - \frac{1}{2} k (l - l_0)^2$

$q_2 = \varphi \rightarrow \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\varphi}} \right) - \frac{\partial L}{\partial \varphi} = 0$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\varphi}\right) - \frac{\partial L}{\partial \varphi} = 0$$

$$\frac{d}{dt}\left(m.l^2.\dot\varphi\right) - m.g.l(-\sin\varphi) = 0$$

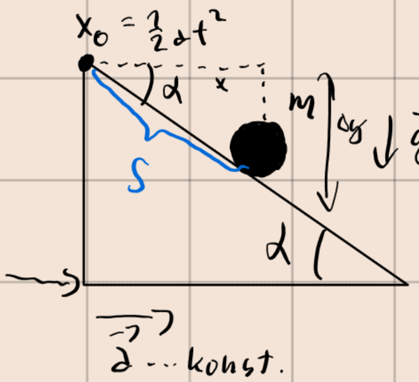
$$m.2.l.\dot l.\dot\varphi + m\,l^2\ddot\varphi + m.g.l.\sin\varphi = 0$$

$$q_2 = l \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot l}\right) - \frac{\partial L}{\partial l} = 0$$

$$\frac{d}{dt}\left(m.\dot l\right) - m\,l.\dot\varphi^2 - m.g\cos\varphi + k(l - l_0) = 0$$

$$m\ddot l = m\,l\,\dot\varphi^2 + m.g.\cos\varphi + k.(l - l_0)$$

## 4.61




pro jaký úhel $\alpha$ se kulička nepohybuje

- $z = 0$
- $\mathrm{tg}\,\alpha = \frac{y}{x}$

$$x = x_0(t) + s.\cos(\alpha)$$

$$x = \frac{1}{2}at^2 + s.\cos(\alpha)$$

$$y = h - s.\sin(\alpha)$$

$$\dot z = 0$$

$$\dot x = at + \dot s.\cos\alpha$$

$$\dot y = -\dot s.\sin\alpha$$

$$\dot x^2 + \dot y^2 = a^2t^2 + \dot s^2.\cos^2\alpha + 2at.\dot s.\cos\alpha + \dot s^2\sin^2\alpha$$

$$E_{kin} = \frac{1}{2}m\left(\dot x^2 + \dot y^2 + \dot z^2\right) = \frac{1}{2}m.a^2.t^2 + \frac{1}{2}m.\dot s^2 + \frac{1}{2}m2at\dot s.\cos\alpha$$

$$E_{pot} = m.g.y = m.g.h - m.g.s.\sin\alpha$$

$$L = \frac{1}{2}m.a^2t^2 + \frac{1}{2}m.\dot s^2 + m.a.t.\dot s.\cos\alpha - m.g.h - m.g.s.\sin\alpha$$

$$q = s \to \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{s}}\right) - \frac{\partial L}{\partial s} = 0$$

$$\frac{d}{dt}\left(m \cdot \dot{s} + m \cdot g \cdot t \cdot \cos\alpha\right) - m \cdot g \cdot \sin\alpha = 0$$

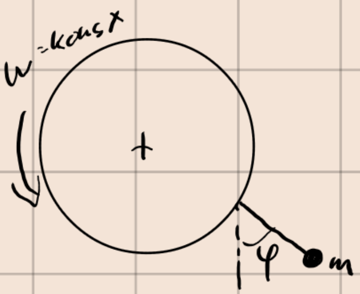
$$m \cdot \ddot{s} + m \cdot a \cdot \cos\alpha - m \cdot g \cdot \sin\alpha = 0$$

$$\hookrightarrow \ddot{s} = 0$$

$$m \cdot a \cdot \cos\alpha = m \cdot g \cdot \sin\alpha$$

$$\frac{a}{g} = \operatorname{tg}\alpha$$

$\boxed{\text{úkol: 4.2}}$




1 stupeň volnosti

$q = \varphi$

$$x = x_0 + l \cdot \sin\varphi$$
$$y = y_0 - l \cdot \cos\varphi$$
$$x_0 = R \cdot \cos \omega t$$
$$y_0 = R \cdot \sin \omega t$$

## Hamiltonův formalismus

$$H(\vec{q}, \vec{p}, t) = \sum_i p_i \dot{q}_i - L$$

HAMILTONOVSKÁ FCE.

$$\vec{p} = m(v_x, v_y, v_z)$$

$$m \cdot v_x \cdot v_x + m \cdot v_y \cdot v_y + m \cdot v_z \cdot v_z$$

$$H = \underbrace{mv^2}_{2T} - \underbrace{L}_{T-U} = 2T - T + U = T + U$$

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

Hamilt. kanonické rovnice:

$\partial H$

$$p_i = -\frac{\partial H}{\partial q_i}$$

$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$

5.2)


$q_1 = \ell$

$q_2 = \varphi$

$$L = \frac{1}{2} m \dot{\ell}^2 + \frac{1}{2} m . \ell^2 . \dot{\varphi}^2 + m.g.\ell \cos\varphi - \frac{1}{2} k (\ell - \ell_0)^2$$

$$p_\ell = \frac{\partial L}{\partial \dot{\ell}} = m . \dot{\ell}$$

$$p_\varphi = \frac{\partial L}{\partial \dot{\varphi}} = m . \ell^2 . \dot{\varphi}$$

$$H = m . \dot{\ell}^2 + m . \ell^2 \dot{\varphi}^2 - \frac{1}{2} m . \dot{\ell}^2 - \frac{1}{2} m \ell^2 \dot{\varphi}^2 - m . g\, \ell . \cos\varphi + \frac{1}{2} k (\ell - \ell_0)^2$$

$$H = \frac{1}{2} m . \dot{\ell}^2 + \frac{1}{2} m \ell^2 \dot{\varphi}^2 - m.g.\ell.\cos\varphi + \frac{1}{2} k (\ell - \ell_0)^2$$

$$\dot{\ell}^2 = \frac{p.\ell^2}{m^2}$$

$$\dot{\varphi}^2 = \frac{p\varphi^2}{m^2 \ell^4}$$

$$H = \frac{p.\ell^2}{2m} + \frac{p\varphi^2}{2m\ell^2} - m.g.\ell.\cos\varphi + \frac{1}{2} k (\ell - \ell_0)^2$$

$$\dot{p}_\ell = \frac{-\partial H}{\partial \ell} = \frac{p\varphi^2}{m\ell^3} + mg.\cos\varphi - k.(\ell - \ell_0)$$

$$\dot{p}\varphi = \frac{-\partial H}{\partial \varphi} = -m.g.\ell.\sin\varphi$$

$$\dot{\ell} = \frac{\partial H}{\partial p.\ell} = \frac{p\ell}{m}$$

$$\dot{\varphi} = \frac{\partial H}{\partial p\varphi} = \frac{p\varphi}{m.\ell^2}$$