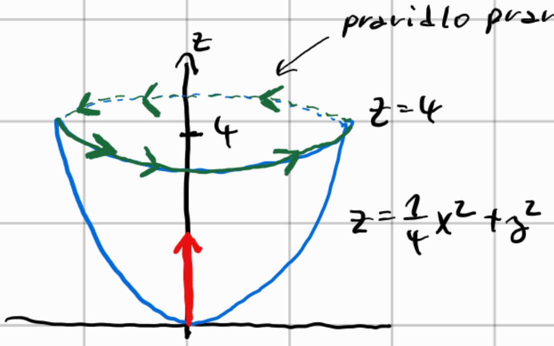
7c)

$$\int_{(S,N)} \nabla \times \left(6yz, 5x, yze^{x^2}\right) d\sigma \overset{\text{stokes}}{=\!=} \int_{(C,\tau)} \left(6yz, 5x, yze^{x^2}\right) ds = \circledast$$



$$x = 4\cos t$$
$$y = 2\sin t$$
$$z = 4$$
$$4 = \frac{1}{4}x^2 + y^2$$

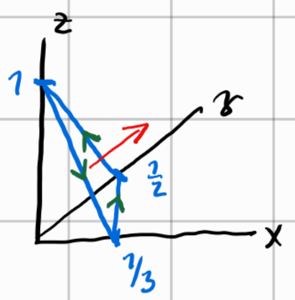
$$\varphi(t) = \left(4\cos t, 2\sin t, 4\right), \ t \in [0, 2\pi]$$
$$\varphi'(t) = \left(-4\sin t, 2\cos t, 0\right)$$

$$\circledast = \int_0^{2\pi} \left(6 \cdot 2\sin t \cdot 4, 20\cos t, \dots\right) \cdot \left(-4\sin t, 2\cos t, 0\right) dt =$$

$$= \int_0^{2\pi} \left(-12 \cdot 16 \cdot \sin^2 t + 40\cos^2 t\right) dt = -192\pi + 40\pi = -152\pi$$

8a) $$\int_{(C,J)} \left(1, x+y+z, xy-\sqrt{z}\right) \cdot ds \overset{\text{stokes}}{=\!=} \int_{(S,N)} \nabla \times \left(1, x+yz, xy-\sqrt{z}\right) \cdot d\sigma =$$

$$= \int_{(S,N)} \left(x-y, 0-y, 1-0\right) d\sigma = \circledast$$



$$\phi(u,v) = \left(\tfrac{1}{3},0,0\right) + u\left(\left(0,\tfrac{1}{2},0\right) - \left(\tfrac{1}{3},0,0\right)\right) + v\left(\left(0,0,1\right) - \left(\tfrac{1}{3},0,0\right)\right) =$$

$$= \left(\frac{1}{3} - \frac{1}{3}u - \frac{1}{3}v, \frac{u}{2}, v\right), \ u \in [0,1], \ v \in [0, 1-u]$$

$$\frac{\partial \phi}{\partial u} \times \frac{\partial \phi}{\partial v} = \left(-\frac{1}{3}, \frac{1}{2}, 0\right) \times \left(-\frac{1}{3}, 0, 1\right) = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right)$$

$$\circledast = \int_0^1\int_0^{1-u}\left(\frac{1}{3}-\frac{u}{3}-\frac{v}{3}-\frac{u}{2}, 1-\frac{u}{2}, 1\right)\cdot\left(\frac{1}{2},\frac{1}{3},\frac{1}{6}\right)\,dv\,du =$$

$$=\int_0^1\int_0^{1-u}\left(\frac{1}{6}-\frac{u}{6}-\frac{v}{6}-\frac{u}{4}-\frac{u}{6}+\frac{1}{6}\right)dv\,du=\int_0^1\int_0^{1-u}\left(\frac{1}{3}-\frac{7u}{12}-\frac{v}{6}\right)dv\,du$$

$$=\frac{1}{24}$$

Řady

1d) $\displaystyle\sum_{n=0}^{\infty}\frac{(-4)^n n}{n+1}x^{2n}$

poloměr konvergence: zkoumáme $\displaystyle\sum_{n=0}^{\infty}\left|\frac{(-4)^n n}{n+1}\right| r^{2n}$

$$\lim_{n\to\infty}\sqrt[n]{\left|\frac{(-4)^n\cdot n}{n+1}\right| r^{2n}}=\lim_{n\to\infty}\frac{4\cdot\sqrt[n]{n}\cdot r^2}{\sqrt[n]{n+1}}=\lim_{n\to\infty}4r^2\,\frac{\overbrace{\sqrt[n]{n}}^{\to 1}}{\underbrace{\sqrt[n]{n+1}}_{\to 1}}=4r^2$$

$$=4r^2\begin{cases}<1 & \text{pro } r<\frac{1}{2} \quad \sum_{n=1}|a_n| r^{2n} \text{ konverguje}\\ =1 & \text{pro } r=\frac{1}{2}\\ >1 & \text{pro } r>\frac{1}{2} \quad \sum_{n=1}|a_n| r^{2n} \text{ diverguje}\end{cases}\Bigg\}\ R=\frac{1}{2}$$

$\Rightarrow \sum a_n x^{2n}$ konv. ABS na $\left[-\frac{1}{2},\frac{1}{2}\right]$

2b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n(1+n)}{3^n\,n!}(x-2)^n$

poloměr konvergence: pomocí podíl. kritéria:

$$\lim_{n\to\infty}\frac{\left|\frac{(-1)^{n+1}(n+2)}{3^n\cdot 3\cdot(n+1)n!}\right| r^{n+1}}{\left|\frac{(-1)^n(n+1)}{3^n\cdot n!}\right| r^n}=\lim_{n\to\infty}\frac{(n+2)}{3(n+1)^2}\cdot r=\lim_{n\to\infty}\frac{r}{3}\,\frac{n+2}{n^2+2n+1}=0$$

$0<1 \ \forall r \in [0, \infty) \Rightarrow R = \infty$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n (1+n)}{3^n \, n!} (x-2)^n \quad \text{konv. ABS. } \forall x \in \mathbb{R}$$

$$\int \left(\sum_{n=0}^{\infty} \frac{(-1)^n (1+n)}{3^n . n!} (x-2)^n \right) = \sum_{n=0}^{\infty} \frac{(-1)^n (1+n)}{3^n . n!} \int (x-2)^n \, dx =$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \cancel{(1+n)}}{3^n . n!} \cdot \frac{(x-2)^{n+1}}{\cancel{n+1}} = (x-2) \sum_{n=0}^{\infty} \frac{\left(\frac{(-1)^n (x-2)}{3} \right)^n}{n!} =$$

$$= (x-2) . e^{-\frac{x-2}{3}}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n (1+n)}{3^n \, n!} (x-2)^n = \left(\int \Sigma \right)' = \left((x-2) . e^{-\frac{x-2}{3}} \right)' = e^{-\frac{x-2}{3}} + (x-2) e^{-\frac{x-2}{3}} \left(-\frac{1}{3} \right) =$$

$$= e^{-\frac{x-2}{3}} \left(1 - \frac{x-2}{3} \right) = e^{-\frac{x-2}{3}} = e^{-\frac{x-2}{3}} \left(\frac{5}{3} - \frac{x}{3} \right), x \in \mathbb{R}$$

2c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n (n+1)} x^{n+1}$

poloměr konvergence: podílové kritérium:

$$\lim_{n \to \infty} \frac{\left| \frac{1}{2^{n+1} (n+2)} \right| r^{n+2}}{\left| \frac{1}{2^n (n+1)} \right| r^{n+1}} = \lim_{n \to \infty} \frac{r (n+1)}{2 (n+2)} = \frac{r}{2} \lim_{n \to \infty} \frac{n+1}{n+2} = \frac{r}{2}$$

$$\frac{n}{2}\begin{cases} <1 \text{ pro } n<2 & \text{konverguje} \\ =1 \text{ pro } n=2 \\ >1 \text{ pro } n>2 \end{cases} \Bigg\} R=2$$

Řada ABS. konverguje pro $(-2,2)$

součet:

$$\left(\sum_{n=1}^{\infty}\frac{1}{2^n(n+1)}x^{n+1}\right)' = \sum_{n=1}^{\infty}\frac{1}{2^n\cancel{(n+1)}}\cdot\cancel{(n+1)}x^n = \sum_{n=1}^{\infty}\left(\frac{x}{2}\right)^n =$$

$$\underset{\text{geom. řada}}{=} \underbrace{\frac{1}{1-\frac{x}{2}}}_{=\sum_0^{\infty}} \underbrace{-1}_{n=0} = \frac{1}{\frac{2-x}{2}} - 1 = \frac{2-2+x}{2-x} = \frac{x}{2-x}, \; x\in(-2,2)$$

$$\sum_{n=1}^{\infty} a_n x^{n+1} = \int\left(\Sigma\right)' = \int\frac{x}{2-x}\,dx = \int\frac{2(-2+x)}{2-x}\,dx = \int\left(\frac{2}{2-x}-1\right)dx =$$

$$= -2\ln|2-x| - x + C$$

dosadíme $x = x_0\ (=0)$

$$\left.\begin{aligned} &\sum \frac{1}{2^n(n+1)}0^{n+1} = 0 \\ &-2\ln 2 + C \end{aligned}\right\} \begin{aligned} 0 &= -2\ln 2 + C \\ 2\ln 2 &= C \end{aligned}$$

$$\Rightarrow \sum_{n=1}^{\infty} = -x - 2\ln|2-x| + \ln 4, \; x\in(-2,2)$$

3a) $f(x) = 3^x = e^{x \cdot \ln 3}\ e^{(x+3)\ln 3 - 3\ln 3}$

$x_0 = -3 \ldots$ chceme $(x+3)^n$

$$= e^{-3\ln 3} \cdot e^{(x+3)\ln 3} = 3^{-3} \sum_{n=0}^{3} \frac{(x+3)^n (\ln 3)^n}{n!}, \quad x \in \mathbb{R}$$

3d) $f(x) = \dfrac{1}{2x-5} = \dfrac{1}{2(x-2)-5+2} = \dfrac{1}{-3+2(x-1)} = -\dfrac{1}{3} \cdot \dfrac{1}{1 - \underbrace{\frac{2(x-1)}{3}}_{q}}$

$$= -\frac{1}{3} \sum_{n=0}^{\infty} \left(\frac{2(x-1)}{3} \right)^n = - \sum_{n=1}^{\infty} \frac{2^n}{3^{n+1}} (x-1)^n, \quad \left| \frac{2(x-1)}{3} \right| < 1$$

$$|x-1| < \frac{3}{2}$$

$$x \in \left(-\frac{1}{2}, \frac{5}{2} \right)$$