1) $$\int_{-\frac{\pi}{3}}^{0} \frac{3\cos x \cdot \sin x - 2\sin x}{\cos^3 x - 2\sin^2 x + 2}\,dx = \int_{-\frac{\pi}{3}}^{0} \frac{\sin x\left(3\cos x - 2\right)}{\cos^3 x - 2\sin^2 x + 2}\,dx =$$

$$\left|\begin{matrix}\text{sub:}\\ u = \cos x\\ du = -\sin x\,dx\end{matrix}\right| = \int_{\frac{1}{2}}^{1} \frac{-(3u-2)}{u^3 - 2(1-u^2) + 2}\,du = \int_{\frac{1}{2}}^{1} \frac{-(3u-2)}{u^3 - 2 + 2u^2 + 2}\,du =$$

$$= -\int_{\frac{1}{2}}^{1} \frac{3u-2}{u^2(u+2)}\,du = (*)$$

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$$\frac{A}{u} + \frac{B}{u^2} + \frac{C}{u+2} = \frac{3u-2}{u^2(u+2)}$$

$$Au(u+2) = Au^2 + 2Au$$

$$B(u+2) = Bu + 2B$$

$$Cu^2 = Cu^2$$

$$(A+C)u^2 + (2A+B)u + 2B = 3u - 2$$

u^2: $A + C = 0$

u: $2A + B = 3$

c: $2B = -2 \rightarrow \underline{B = -1}$

$2A - 1 = 3 \rightarrow 2A = 4 \rightarrow \underline{A = 2}$

$A + C = 0 \rightarrow 2 + C = 0 \rightarrow \underline{C = -2}$

$$(*) = -\int_{\frac{1}{2}}^{1}\left(\frac{2}{u} - \frac{1}{u^2} - \frac{2}{u+2}\right)du = -\Big[2\ln|u| + \frac{1}{u} - 2\ln|u+2|\Big]_{\frac{1}{2}}^{1} =$$

$$= 2\ln 1 + 1 - 2\ln 3 - 2\ln\frac{1}{2} + 2 - 2\ln\frac{5}{2} = 1 - 2\ln 3 - (-2\ln 2 + 2 - 2\ln 5 + 2\ln 2) =$$

$$= 1 - 2\ln 3 - 2 + 2\ln 5 = -1 - 2\ln 3 + 2\ln 5 = \boxed{-1 + 2\ln\frac{5}{3}}$$

2) $$\int_{0}^{\frac{\pi}{2}} \frac{2\cos x - 6\cos x\cdot\sin x - 2\cos^3 x}{6 - \cos^2 x + 2\sin x}\,dx = \int_{0}^{\frac{\pi}{2}} \frac{\cos x\,\overbrace{\left(2 - 6\sin x - 2\cos^2 x\right)}^{2-6\sin x - 2(1-\sin^2 x)}}{6 - \cos^2 x + 2\sin x}\,dx =$$

$$= \int_{0}^{\frac{\pi}{2}} \frac{-2\cos x\left(3\sin x - \sin^2 x\right)}{6 - \cos^2 x + 2\sin x}\,dx = \left|\begin{matrix}\text{sub:}\\ u = \sin x\\ du = \cos x\,dx\end{matrix}\right| =$$

$$= \int_{0}^{1} \frac{2u^2 - 6u}{u^2 + 2u + 5}\,du \overset{\text{dělení}}{=} \int_{0}^{1} 2 - \frac{10u + 10}{u^2 + 2u + 5}\,du =$$

$$= 2 - \int_{0}^{1} \frac{10u + 10}{u^2 + 2u + 5}\,du = 2 - 5\int_{0}^{1} \frac{2u + 2}{u^2 + 2u + 5}\,du =$$

$$= 2 - 5\Big[\ln\left(u^2 + 2u + 5\right)\Big]_{0}^{1} = 2 - 5(\ln 8 - \ln 5) = \boxed{2 - 5\ln\frac{8}{5}}$$

③

$$\int_2^{2e^2} \frac{6\ln\frac{x}{2}-4}{x\left(\ln^2\frac{x}{2}-4\ln\frac{x}{2}+8\right)}\,dx = \left|\begin{array}{l} \text{Sub.} \\ u=\ln\frac{x}{2} \\ x=2e^u \\ dx=2e^u\,du = x\,du \end{array}\right| =$$

$$= \int_0^2 \frac{6u-4}{u^2-4u+8}\,du = \left|\begin{array}{l} \text{jmen.} \\ (u^2-4u+8)' = 2u-4 \\ \text{čit.} \\ 6u-4 = 3(2u-4)+8 \end{array}\right| =$$

$$= \int_0^2 \frac{3(2u-4)}{u^2-4u+8}\,du + \int_0^2 \frac{8}{u^2-4u+8}\,du =$$

$$= 3\cdot\left[\ln(u^2-4u+8)\right]_0^2 + \int_0^2 \frac{8}{(u-2)^2+4}\,du = \left|\begin{array}{l} \text{Sub.} \\ v=u-2 \\ dv=du \end{array}\right| =$$

$$= 3\left[\ln(u^2-4u+8)\right]_0^2 + \int_{-2}^0 \frac{8}{v^2+4}\,dv$$

$$= 3\left[\ln(u^2-4u+8)\right]_0^2 + 8\int_{-2}^0 \frac{1}{v^2+4}\,dv = 3\left[\ln(u^2+4u+8)\right]_0^2 + 8\cdot\left[\frac{1}{2}\operatorname{arctg}\frac{v}{2}\right]_{-2}^0$$

$$= \underbrace{3(\ln(4)-\ln(8))}_{3\ln\frac{4}{8} = 3\ln\frac{1}{2} = -3\ln 2} + 4(\operatorname{arctg}(0) - \operatorname{arctg}(-1)) = -3\ln 2 + 4\left(0+\frac{\pi}{4}\right)$$

$$= \boxed{-3\ln 2 + \pi}$$

4

$$\int_0^{\ln 2} \frac{e^{4x}+7e^{2x}}{e^{4x}+2e^{2x}-3}\,dx = \left| \begin{array}{l} \text{Sub:} \\ u=e^{2x} \\ du=2e^{2x}\,dx \\ \quad =2u\,dx \end{array} \right| = \int_1^4 \frac{u^2+7u}{u^2+2u+3}\cdot\frac{1}{2u}\,du =$$

$$= \frac{1}{2}\int_1^4 \frac{u^2+7u}{(u^2+2u-3)u}\,du = \frac{1}{2}\int_1^4 \frac{u^2+7u}{u(u+3)(u-1)}\,du = (*)$$

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$$\frac{u^2+7u}{u(u+3)(u-1)} = \frac{A}{u} + \frac{B}{u+3} + \frac{C}{u-1}$$

$$u^2+7u = A(u+3)(u-1) + Bu(u-1) + Cu(u+3)$$

$$0 = A(3)(-1) \rightarrow \underline{A=0}$$

$$1^2+7 = B\cdot 1\cdot 0 + C\cdot 1\cdot 4 \rightarrow 4C=8 \rightarrow \underline{C=2}$$

$$(-3)^2+7\cdot(-3) = B\cdot(-3)\cdot(-4) \rightarrow 9-21=12B \rightarrow \underline{B=-1}$$

$(*)$

$$= \frac{1}{2}\int_1^4 \frac{-1}{u+3} + \frac{2}{u-1}\,du = \frac{1}{2}\Big[-\ln|u+3| + 2\ln|u-1|\Big]_1^4$$

$$= -\ln 7 + 2\ln 3 + \ln 4 - \underline{2\ln 0} \text{ nelze}$$

$\Rightarrow$ Funkce není Riemannovsky integrovatelná