$$x+2y+2z=1$$
$$x+y+z=1$$
$$x-y+z=5$$
$$(1,-2,2)$$

$$x=1-2y-2z$$
$$y=1-x-z$$
$$z=5-x+y$$

Jacobiho metoda

$$(x_0,y_0,z_0)=(2,0,0)$$
$$x_k=1-2y_{k-1}-2\cdot z_{k-1}$$
$$y_k=1-x_{k-1}-z_{k-1}$$
$$z_k=5-x_{k-1}+y_{k-1}$$

	x_k	y_k	z_k
0	2	0	0
1	1	-1	3
2	-3	-3	3

matice metody:

$$B_j=\begin{pmatrix}0&-2&-2\\-1&0&-1\\-1&1&0\end{pmatrix}$$

Gauss-seidl

$$(x_0,y_0,z_0)=(2,0,0)$$
$$x_k=1-2y_{k-1}-2z_{k-1}$$
$$y_k=1-x_k-z_{k-1}$$
$$z_k=5-x_k+y_k$$

	x_k	y_k	z_k
0	2	0	0
1	1	0	4
2	-7	4	16
3	-39	24	68

$$B_{GS}=\begin{pmatrix}0&-2&-2\\0&2&2\\0&4&3\end{pmatrix}$$

$$|B_{GS}|=-\lambda(2-\lambda)(3-\lambda)+4\lambda=$$
$$-\lambda^3+5\lambda^2-2\lambda=0$$
$$\lambda_1=0$$
$$\lambda_2=0{,}438$$
$$\lambda_3=4{,}562$$

Samostatně

$$x+2y+z=-1$$
$$x+y+4z=2$$
$$2x+y+z=2$$
$$x_0=(2,0,0)$$
$$x=-1-2y-z$$

Jacobiho metoda

$$x_k=-1-2y_{k-1}-z_{k-1}$$
$$y_k=2-x_{k-1}-4z_{k-1}$$
$$z_k=2-2x_{k-1}-y_{k-1}$$

$y = 2 - x - 4z$

$z = 2 - 2x - y$

k	x_k	y_k	z_k
0	2	0	0
1	-1	0	-2
2	1	11	4

$$B_j = \begin{pmatrix} 0 & -2 & -1 \\ -1 & 0 & -4 \\ -2 & -1 & 0 \end{pmatrix}$$

Gauss-seidl metoda

$$x_k = -1 - 2y_{k-1} - z_{k-1}$$

$$y_k = 2 - x_k - 4z_{k-1}$$

$$z_k = 2 - 2x_k - y_k$$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 4 \\ 2 & 1 & 1 \end{pmatrix}$$ $\to$ převod na silnou diagonálu

$$A \cdot \vec{x} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$$

$$A' = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 4 \end{pmatrix}$$

$$x = (2 - y - z) \cdot \frac{1}{2}$$

$$y = (-1 - x - z) \cdot \frac{1}{2}$$

$$z = (7 - x - y) \cdot \frac{1}{4}$$

konverguje $\to$ asi $(1, -2, 2)$

Gauss-seidel s relaxací (nefunguje)

$$x_k = (-1 - 2y_{k-1} - z_{k-1}) \cdot w + (1-w) \cdot x_{k-1}$$

$$y_k = (7 - x_k - 4 z_{k-1}) \cdot w + (1-w) \cdot y_{k-1}$$
$$z_k = (2 - 2x_k - y_k) \cdot w + (1-w) \cdot z_{k-1}$$

$$// \; x_k = \varphi(x_{k-1}) \cdot \lambda + (1-\lambda) \cdot x_{k-1} \; //$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & -1 & 1 & 5 \end{array}\right)$$

$(1, -2, 2)$

$$x_k = 1 - y_{k-1} - z_{k-1}$$
$$y_k = 1 - x_k - 2 z_{k-1}$$
$$z_k = 5 - x_{k-1} + y_{k-1}$$

	x_k	y_k	z_k
1	1	-2	2
2	1		
3			

s relaxací:

$$x_k = (1 - y_{k-1} - z_{k-1}) \cdot w + (1-w) \cdot x_{k-1}$$
$$y_k = (1 - x_k - 2 z_{k-1}) \cdot \frac{1}{2} \cdot w + (1-w) \cdot y_{k-1}$$
$$z_k = (5 - x_{k-1} + y_{k-1}) \cdot w + (1-w) \cdot z_{k-1}$$

$\longrightarrow$ tady to konverguje

$$A = \begin{pmatrix} 1 & 3 & -2 & -3 \\ -1 & 0 & 1 & 0 \\ -1 & -2 & 1 & 2 \end{pmatrix}$$

$\lambda_1 = 2$

$\lambda_2 = 1$

$\lambda_3 = 0$

$\lambda_4 = -1$

$$A \cdot \vec{x} = \lambda \cdot \vec{x} \Rightarrow \vec{x} = \frac{1}{\lambda} \cdot A \cdot \vec{x}$$

$\vec{x}_0 = (0, 1, 0, 0)$

$\lambda = 2$

$$\vec{x}_k = \frac{1}{2} \cdot A \cdot \vec{x}_{k-1}$$

$$\Downarrow$$

$$\vec{x} = (1, 0, 1, -1)$$