$$y'' + 2y' = 6e^{x} - 2e^{-2x} + 8\cos(2x) + 4x$$

$y(0) = 2 \qquad y'(0) = 2$

char. funkce:

$\lambda^2 + 2\lambda = 0$

$\lambda(\lambda + 2) = 0$

$\lambda_1 = 0$

$\lambda_2 = -2$

homogenní řešení:

$y_h = C_1 + C_2 \cdot e^{-2x}$

$y(0) = 2$

$y'(0) = 2$

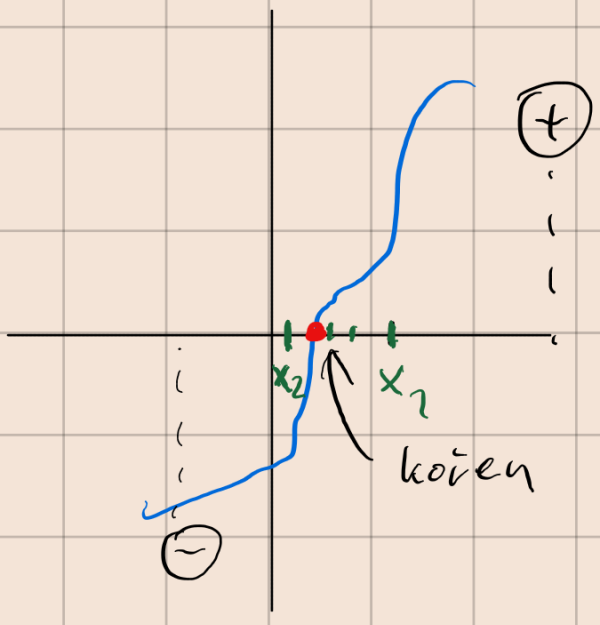
$y_0 = y_p + y_h$

Numerické řešení:

$f(x) = 0 \qquad x = ?$

$f(x) = \frac{x}{x^2 + 7}$

Bisekce (půlení intervalu)



$\langle -5 ; 3 \rangle \rightarrow f(-5) = \frac{-5}{25+1} = \ominus$

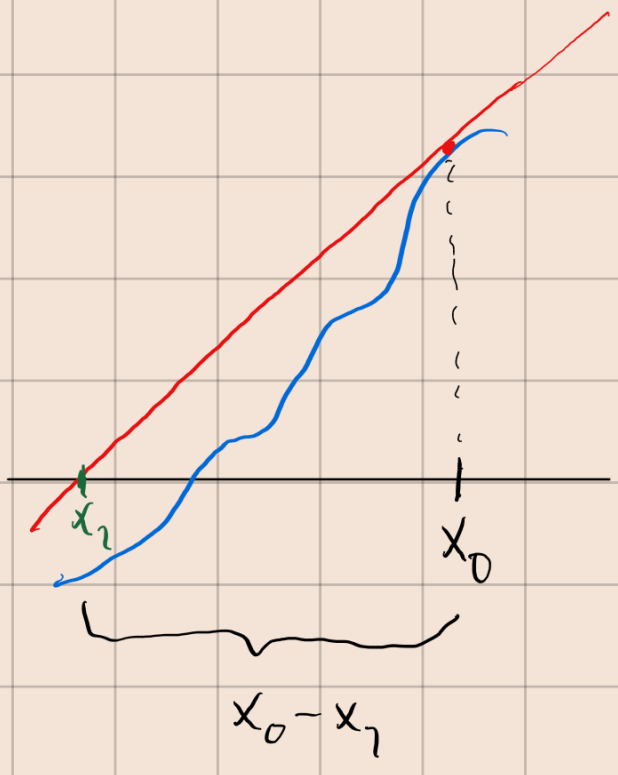
$f(3) = \frac{3}{9+1} = \oplus$

$x_1 = -1 \rightarrow f(-1)$ ⊖

$x_2 = 1 \rightarrow f(1)$ ⊕

$x_3 = 0 \rightarrow f(0) = 0$

Newtonova metoda



$$f'(x_0) = \frac{f(x_0)}{x_0 - x_1}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_n = x_{n-1} - \frac{f'(x_{n-1})}{f'(x_{n-1})}$$

$x_0 = \frac{1}{2}$

$$f'(x) = \frac{x^2 + 1 - 2x^2}{(x^2+1)^2}$$

$$x_1 = x_0 - \frac{x_{0-1}}{\left(x_{0-1}^2 + 1\right)^2}$$

$$x_{0-1}^5 + x_{0-1}$$

$$k \quad x_{k-1} \qquad \frac{x_{k-1}^2}{x_{k-1}^2+1} \qquad \frac{\quad}{1-x_{k-1}^2} = x_{k-1} - \frac{\quad}{1-x_{k-1}^2}$$

$$x_1 = \frac{1}{2} - \frac{\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)}{1-\frac{1}{4}} = \frac{1}{2} - \frac{5}{8}\cdot\frac{4}{3} = \frac{1}{2} - \frac{5}{6} = -\frac{1}{3}$$

$$x_2 = -\frac{1}{3} - \frac{\left(-\frac{1}{3}\right)^3 + \left(-\frac{1}{3}\right)}{1-\left(-\frac{1}{3}\right)^2} = \frac{1}{12}$$

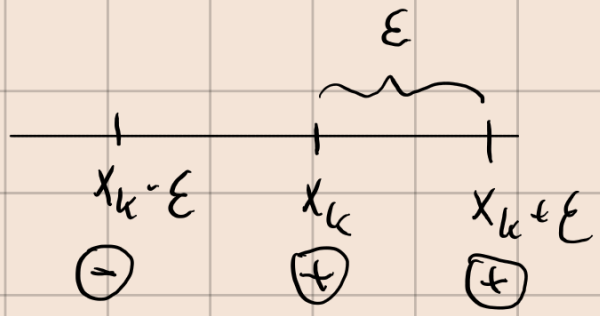
Je počet kroků dostatečný?

$\varepsilon < \frac{1}{4}$?

$f(x_k - \varepsilon) = f\left(\frac{1}{12} - \frac{1}{4}\right) = f\left(-\frac{1}{6}\right) = \ominus$

$f(x_k + \varepsilon) = f\left(\frac{1}{12} + \frac{1}{4}\right) = \oplus$

$f(x_k) = f\left(\frac{1}{12}\right) = \oplus$



Řád metody

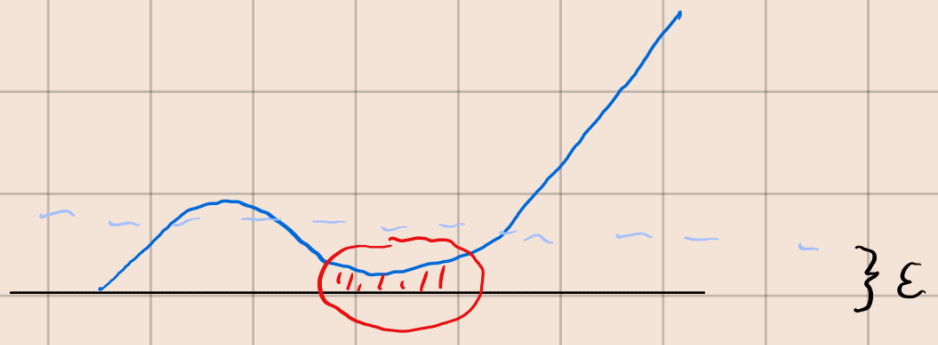
$E_k = r - x_k$

$|E_k| = C \cdot |E_{k-1}|$

I. $|x_k - x_{k-1}| < \varepsilon$ ... abs. podmínka

II. $\dfrac{|x_k - x_{k-1}|}{|x_n|} < \varepsilon$ ... rel. podmínka

III. $|f(x_k)| < \varepsilon$ ... hodnotová podmínka



$f(x) = 4 - \sqrt{x}$

$(1, 13)$ $\varepsilon = 0{,}0001$

$f(x) = \ln(x) + \sqrt{x} - 4 = 0$

$f'(x) = \frac{1}{x} + \frac{1}{2\sqrt{x}}$