Existence, Definedness

and the semantics of possibility and necessity

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… is to discuss Professor Williamson's treatment of *necessitism* and *contingentism*, in the light of a hypersequent semantics for quantified modal logic.

The Plan

Modal Model Theory and the Barcan Formula

Sequents & Defining Rules

Hypersequents, Possibility and Necessiy

Quantifiers, Definedness & the Barcan Formula

The Status of Contingentist Models

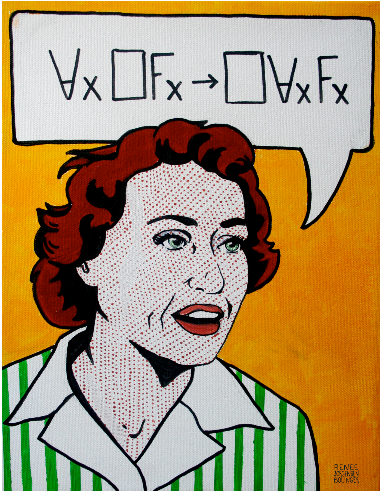
Quantifiers—wider and still wider

Epistemic Modals

Concluding Thoughts

MODAL MODEL THEORY AND THE BARCAN FORMULA

Ruth Barcan and the Barcan Formula



Portrait by *Renee Jorgensen Bolinger*

Just add $(\forall L)$, $(\forall R)$, $(\Box L)$ & $(\Box R)$ and stir.

Barcan Derivations

$$\dfrac{\dfrac{\dfrac{\dfrac{Fa \succ Fa}{\Box Fa \succ \quad | \quad \succ Fa}\,[\Box L]}{\forall x \Box Fx \succ \quad | \quad \succ Fa}\,[\forall L]}{\forall x \Box Fx \succ \quad | \quad \succ \forall x Fx}\,[\forall R]}{\forall x \Box Fx \succ \Box \forall x Fx}\,[\Box R]$$

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$$

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$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{Fa \succ Fa}{\Box Fa \succ \quad \mid \quad \succ Fa}\,[\Box L]}{\forall x \Box Fx \succ \quad \mid \quad \succ Fa}\,[\forall L]}{\forall x \Box Fx \succ \quad \mid \quad \succ \forall x Fx}\,[\forall R]}{\forall x \Box Fx \succ \Box \forall x Fx}\,[\Box R]
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{Fa \succ Fa}{Fa \succ \quad \mid \quad \succ \Diamond Fa}\,[\Diamond R]}{Fa \succ \quad \mid \quad \succ \exists x \Diamond Fx}\,[\exists R]}{\exists x Fx \succ \quad \mid \quad \succ \exists x \Diamond Fx}\,[\exists L]}{\Diamond \exists x Fx \succ \exists x \Diamond Fx}\,[\Diamond L]
$$

Falsifying the Barcan Formula

$$\begin{array}{l} w_1 \\ \hline \Diamond(\exists x)Fx \end{array}$$

Falsifying the Barcan Formula

w_1	
$\Diamond(\exists x)Fx$	$(\exists x)Fx$

Falsifying the Barcan Formula

w_1	w_2
$\Diamond(\exists x)Fx$	$(\exists x)Fx$
	Fa

Falsifying the Barcan Formula

w_1	w_2
$\Diamond(\exists x)Fx$	$(\exists x)Fx$
	Fa
$\neg(\exists x)\Diamond Fx$	

w_1	w_2
$\Diamond(\exists x)Fx$	$(\exists x)Fx$
	Fa
$\neg(\exists x)\Diamond Fx$	
$\neg\Diamond Fb$	

Falsifying the Barcan Formula

w_1	w_2
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	Fa
$\neg(\exists x)\Diamond Fx$	
$\neg\Diamond Fb$	$\neg Fb$

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$\neg(\exists x)\Diamond Fx$	
$\neg\Diamond Fb$	$\neg Fb$
$\neg\Diamond Fc$	

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$\vdots$	$\vdots$

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The *domain of quantification* for $\exists$ must vary from world to world.

Questions about such models

- If w_1 is the actual world, what *are* those objects in the domains of other worlds, like w_2?
- Is there meant to be a single *intended model*? If so, what is it?
- If we collect together all the domains, we can *define* a necessitist quantifier.
- *Metaphysical Universality*

SEQUENTS & DEFINING RULES

$$\Gamma \succ \Delta$$

Don't assert each element of Γ
and deny each element of Δ.

Identity: $A \succ A$

Structural Rules

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Weakening: $$\frac{\Gamma \succ \Delta}{\Gamma, A \succ \Delta} \qquad \frac{\Gamma \succ \Delta}{\Gamma \succ A, \Delta}$$

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Contraction: $$\frac{\Gamma, A, A \succ \Delta}{\Gamma, A \succ \Delta} \qquad \frac{\Gamma \succ A, A, \Delta}{\Gamma \succ A, \Delta}$$

Cut: $$\frac{\Gamma \succ A, \Delta \quad \Gamma, A \succ \Delta}{\Gamma \succ \Delta}$$

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Structural rules govern declarative sentences *as such*.

With Left/Right rules?

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \land B \succ \Delta}\ [\land L] \qquad \frac{\Gamma \succ A, \Delta \quad \Gamma \succ B, \Delta}{\Gamma \succ A \land B, \Delta}\ [\land R]$$

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$$\frac{\Gamma, B \succ \Delta}{\Gamma, A \mathbin{tonk} B \succ \Delta}\,[tonkL] \qquad \frac{\Gamma \succ A, \Delta}{\Gamma \succ A \mathbin{tonk} B, \Delta}\,[tonkR]$$

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Desideratum #1: $\succ_{\mathcal{L}'}$ is *conservative:* $(\succ_{\mathcal{L}'})|_{\mathcal{L}}$ is $\succ_{\mathcal{L}}$.

Desideratum #2: Concepts are defined *uniquely*.

A Defining Rule

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \land B \succ \Delta}\ [\land Df]$$

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$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \wedge B \succ \Delta}\ [\wedge Df]$$

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Identity and *Cut* determine the behaviour of conjunctions on the *right*.

$$\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\dfrac{}{A \wedge B \succ A \wedge B}[Id]}{A, B \succ A \wedge B}[\wedge Df]}{\Gamma, A \succ A \wedge B, \Delta}[Cut]}{\Gamma \succ A \wedge B, \Delta}[Cut]$$

From [$\wedge Df$] to [$\wedge L/R$]

$$
\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\dfrac{}{A \wedge B \succ A \wedge B}\,[Id]}{\color{red}{A, B \succ A \wedge B}}\,[\wedge Df]}{\Gamma, A \succ A \wedge B, \Delta}\,[Cut]}{\Gamma \succ A \wedge B, \Delta}\,[Cut]
$$

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$$\dfrac{\Gamma \succ A, \Delta \qquad \Gamma \succ B, \Delta}{\Gamma \succ A \wedge B, \Delta}\,[\wedge R]$$

And Back

$$\dfrac{\dfrac{A \succ A \quad B \succ B}{A, B \succ A \wedge B}\,[\wedge R] \qquad \Gamma, A \wedge B \succ \Delta}{\Gamma, A, B \succ \Delta}\,[Cut]$$

I want to see how this works for modal operators, and examine their interaction with the quantifiers.

Why this is important

Explaining *why* the modal operators have the logical properties they exhibit is an open question.

… possible worlds, in the sense of possible states of affairs are not *really* individuals (just as numbers are not *really* individuals).

To say that a state of affairs obtains is just to say that something is the case; to say that something is a possible state of affairs is just to say that something could be the case; and to say that something is the case 'in' a possible state of affairs is just to say that the thing in question would necessarily be the case if that state of affairs obtained, i.e. if something else were the case …

We understand 'truth in states of affairs' because we understand 'necessarily'; not *vice versa*.
— "Worlds, Times and Selves" (1969)

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- (Why does possibility distribute of disjunction, necessity over disjunction? Why do the modalities *work* like normal modal logics?)
- If modality is *primitive* we have no explanation.
- If modality is governed by the rules introduced here, then we can see *why* possible worlds are useful, and model the behaviour of modal concepts.

HYPERSEQUENTS, POSSIBILITY AND NECESSITY

Modal Reasoning involves *Shifts*

Suppose it's necessary that p and necessary that q.

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Suppose it's necessary that p and necessary that q.
Is it necessary that both p and q?

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So, no matter how things go, we have p and q.

Modal Reasoning involves *Shifts*

Suppose it's necessary that p and necessary that q.
Is it necessary that both p and q?

Could we *avoid* p and q?

Consider any way it could go:

Since it's necessary that p, here we have p.

Since it's necessary that q, here we have q.

So, we have both p and q.

So, no matter how things go, we have p and q.

So the conjunction p and q is necessary.

Exposing the Structure of that Deduction

$$
\cfrac{\cfrac{\cfrac{\cfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \;\mid\; \succ p}\,[\Box Df] \qquad \cfrac{\Box p, \Box q \succ \Box q}{\Box p, \Box q \succ \;\mid\; \succ q}\,[\Box Df]}{\Box p, \Box q \succ \;\mid\; \succ p \wedge q}\,[\wedge R]}{\Box p, \Box q \succ \Box(p \wedge q)}\,[\Box Df]}{\Box p \wedge \Box q \succ \Box(p \wedge q)}\,[\wedge Df]
$$

Exposing the Structure of that Deduction

$$\dfrac{\dfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \ \mid\ \succ p}[\Box Df] \qquad \dfrac{\Box p, \Box q \succ \Box q}{\Box p, \Box q \succ \ \mid\ \succ q}[\Box Df]}{\dfrac{\dfrac{\Box p, \Box q \succ \ \mid\ \succ p \land q}{\Box p, \Box q \succ \Box(p \land q)}[\Box Df]}{\color{red}{\Box p \land \Box q} \succ \Box(p \land q)}[\land Df]}[\land R]$$

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$$

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\dfrac{
  \dfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \quad \mid \quad \succ p}[\Box Df]
  \qquad
  \dfrac{\Box p, \Box q \succ \Box q}{\color{red}{\Box p, \Box q \succ \quad \mid \quad \succ q}}[\Box Df]
}{
  \dfrac{\dfrac{\Box p, \Box q \succ \quad \mid \quad \succ p \land q}{\Box p, \Box q \succ \Box(p \land q)}[\Box Df]}{\Box p \land \Box q \succ \Box(p \land q)}[\land Df]
}[\land R]
$$

Exposing the Structure of that Deduction

$$
\dfrac{
\dfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \quad | \quad \succ p}[\Box Df]
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}{
\dfrac{
\dfrac{\Box p, \Box q \succ \quad | \quad \succ \color{red}{p \land q}}{\Box p, \Box q \succ \Box(p \land q)}[\Box Df]
}{\Box p \land \Box q \succ \Box(p \land q)}[\land Df]
}[\land R]
$$

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$$

$$\Box p, \Box q \succ \quad \mid \quad \succ p \land q$$

Don't assert $\Box p$ and $\Box q$ in one 'zone'
and deny $p \land q$ in another.

$$\Gamma \succ \Delta \mid \Gamma' \succ \Delta'$$

Don't assert each member of Γ
and deny each member of Δ in one 'zone'
and assert each member of Γ'
and deny each member of Δ' in another.

Defining Rules for $\Box$ and $\Diamond$

$$\frac{\Gamma \succ \Delta \mid\ \succ A \mid \mathcal{H}}{\Gamma \succ \Box A, \Delta \mid \mathcal{H}}\,[\Box Df] \qquad \frac{\Gamma \succ \Delta \mid A \succ\ \mid \mathcal{H}}{\Gamma, \Diamond A \succ \Delta \mid \mathcal{H}}\,[\Diamond Df]$$

QUANTIFICATION, DEFINEDNESS & THE BARCAN FORMULA

The Standard Quantifier Rules

$$\frac{\Gamma \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta}\,[\forall Df] \qquad \frac{\Gamma, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta}\,[\exists Df]$$

Deriving the Barcan Formula

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(\forall x)\Box Fx \succ (\forall x)\Box Fx}{(\forall x)\Box Fx \succ \Box Fn}\,[\forall Df]}{(\forall x)\Box Fx \succ \quad | \quad \succ Fn}\,[\Box Df]}{(\forall x)\Box Fx \succ \quad | \quad \succ (\forall x)Fx}\,[\forall Df]}{(\forall x)\Box Fx \succ \Box(\forall x)Fx}\,[\Box Df]}{\succ (\forall x)\Box Fx \supset \Box(\forall x)Fx}\,[\supset Df]
$$

Where the derivation breaks down

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(\forall x)\Box Fx \succ (\forall x)\Box Fx}{(\forall x)\Box Fx \succ \Box Fn}\,[\forall Df]}{(\forall x)\Box Fx \succ \quad\mid\quad \succ Fn}\,[\Box Df]}{(\forall x)\Box Fx \succ \quad\mid\quad \succ (\forall x)Fx}\,[\forall Df]}{(\forall x)\Box Fx \succ \Box(\forall x)Fx}\,[\Box Df]}{\succ (\forall x)\Box Fx \supset \Box(\forall x)Fx}\,[\supset Df]$$

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$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(\forall x)\Box Fx \succ (\forall x)\Box Fx}{(\forall x)\Box Fx \succ \Box Fn}\,[\forall Df]}{\color{red}{(\forall x)\Box Fx \succ \quad \mid \quad \succ Fn}}\,[\Box Df]}{(\forall x)\Box Fx \succ \quad \mid \quad \succ (\forall x)Fx}\,[\forall Df]}{(\forall x)\Box Fx \succ \Box(\forall x)Fx}\,[\Box Df]}{\succ (\forall x)\Box Fx \supset \Box(\forall x)Fx}\,[\supset Df]$$

Pro and *Con* attitudes to Terms

To rule a term *in* is to take it as suitable
to substitute into a quantifier,
i.e., to take the term to *denote*.

To rule a term *out* is to take it as unsuitable
to substitute into a quantifier,
i.e., to take the term to *not denote*.

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to substitute into a quantifier,
i.e., to take the term to *not denote*.

We add terms to the LHS and RHS of sequents $\Gamma \succ \Delta$.

Structural Rules remain as before

Identity: $\mathsf{X} \succ \mathsf{X}$

Weakening: $\dfrac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma, \mathsf{X} \succ \Delta]} \quad \dfrac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \mathsf{X}, \Delta]}$

Contraction: $\dfrac{\mathcal{H}[\Gamma, \mathsf{X}, \mathsf{X} \succ \Delta]}{\mathcal{H}[\Gamma, \mathsf{X} \succ \Delta]} \quad \dfrac{\mathcal{H}[\Gamma \succ \mathsf{X}, \mathsf{X}, \Delta]}{\mathcal{H}[\Gamma \succ \mathsf{X}, \Delta]}$

Cut: $\dfrac{\mathcal{H}[\Gamma \succ \mathsf{X}, \Delta] \quad \mathcal{H}[\Gamma, \mathsf{X} \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta]}$

Here X is either a sentence or a term.

...and there are some more

Ext. Weak.: $$\frac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta \mid \Gamma' \succ \Delta']} \quad \frac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta \parallel \Gamma' \succ \Delta']}$$

Ext. Contr.: $$\frac{\mathcal{H}[\Gamma \succ \Delta \mid \Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta]} \quad \frac{\mathcal{H}[\mathcal{S} \parallel \mathcal{S}]}{\mathcal{H}[\mathcal{S}]}$$

Quantifier Rules, allowing for non-denoting terms

$$\frac{\mathcal{H}[\Gamma, n \succ A(n), \Delta]}{\mathcal{H}[\Gamma \succ (\forall x)A(x), \Delta]} [\forall Df] \qquad \frac{\mathcal{H}[\Gamma, n, A(n) \succ \Delta]}{\mathcal{H}[\Gamma, (\exists x)A(x) \succ \Delta]} [\exists Df]$$

Now you *can't* derive the Barcan Formula

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$$(\forall x)\Box Fx \succ \Box(\forall x)Fx \quad | \quad \succ (\forall x)Fx$$

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$$(\forall x)\Box Fx \succ \Box(\forall x)Fx \quad | \quad b \succ Fb, (\forall x)Fx$$

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$$(\forall x)\Box Fx \succ b, Fb, \Box(\forall x)Fx \;\mid\; b \succ Fb, (\forall x)Fx$$

Now you *can't* derive the Barcan Formula

$$a, (\forall x)\Box Fx \succ b, Fb, \Box(\forall x)Fx \mid a, b \succ Fb, (\forall x)Fx$$

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$$a, \Box Fa, (\forall x)\Box Fx \succ b, Fb, \Box(\forall x)Fx \;\mid\; a, b \succ Fb, (\forall x)Fx$$

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This hypersequent is underivable…

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$$a, Fa, \Box Fa, (\forall x)\Box Fx \succ b, Fb, \Box(\forall x)Fx \mid a, b, Fa \succ Fb, (\forall x)Fx$$

This hypersequent is underivable…

…and it's *fully refined*.

Positions

- A *finite* position is an underivable hypersequent.

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- An arbitrary position is a set (*indicative* alternatives) of sets (*subjunctive* alternatives) of pairs of sets of formulas or terms (*components*).

Fully Refined Positions

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  - If $\Box A$ is in the LHS of a component, A is in the LHS of every subjunctive alternative of that component.
  - If $\Box A$ is in the RHS of a component, A is in the RHS of some subjunctive alternative of that component.

Models

Fully refinied positions are examples of *models*, with variable domains and the expected truth conditions for the connectives, quantifiers and modal operators.

Soundness and Completeness

- Any derivable hypersequent (using *Cut*) holds in all models.

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- Any hypersequent that cannot be derived (without *Cut*) can be extended into a fully refined position.
- That fully refined position determines a model in which the hypersequent does not hold.
- So the models are adequate for the logic.
- And in the logic, the *Cut* rule is admissible in the *Cut*-free system.

THE STATUS OF CONTINGENTIST MODELS

The structure of modal concepts is explained
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Worlds (and their domains) are idealised positions.

There is no single intended model.

Given a language, any coherent position $\Gamma \succ \Delta$ can be canonically extended into a model.

QUANTIFIERS: WIDER AND STILL WIDER

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On most approaches to contingentism, it can be *defined*.

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On most approaches to contingentism, it can be *defined*.

This proof theoretical semantics is no different in that regard....

We have *Outer* Quantification

$$\frac{\mathcal{H}(n \succ \ \mid\ \Gamma \succ A(n), \Delta)}{\mathcal{H}(\Gamma \succ (\forall^{\Diamond} x)A(x), \Delta)} \ [\forall^{\Diamond} Df] \qquad \frac{\mathcal{H}(n \succ \ \mid\ \Gamma, A(n) \succ \Delta)}{\mathcal{H}(\Gamma, (\exists^{\Diamond} x)A(x) \succ \Delta)} \ [\exists^{\Diamond} Df]$$

for which the substituted term need be defined in *some* zone.

The Barcan Formula is Derivable

$$\cfrac{\cfrac{\cfrac{\cfrac{(\forall^{\Diamond}x)\Box A(x) \succ (\forall^{\Diamond}x)\Box A(x)}{n \succ \mid (\forall^{\Diamond}x)\Box A(x) \succ \Box A(n)}\,[\forall^{\Diamond}Df]}{n \succ \mid (\forall^{\Diamond}x)\Box A(x) \succ \mid \succ A(n)}\,[\Box Df]}{(\forall^{\Diamond}x)\Box A(x) \succ \mid \succ (\forall^{\Diamond}x)A(x)}\,[\forall^{\Diamond}Df]}{(\forall^{\Diamond}x)\Box A(x) \succ \Box(\forall^{\Diamond}x)A(x)}\,[\Box Df]$$

But we also have *Way Out* Quantification

$$\frac{\mathcal{H}[\Gamma \succ A(n), \Delta]}{\mathcal{H}[\Gamma \succ (\Pi x)A(x), \Delta]} \; [\Pi Df] \qquad \frac{\mathcal{H}[\Gamma, A(n) \succ \Delta]}{\mathcal{H}[\Gamma, (\Sigma x)A(x) \succ \Delta]} \; [\Sigma Df]$$

for which the term need not be defined *anywhere*.

What's so bad about undefined terms?

For a restriction to completely free logic undermines the application of scientific method by permitting one to hold on to a universal generalization after one of its instances has been refuted: one denies Ga but still asserts $\forall x Gx$ by also denying $\exists y\ a = y$, still retaining the constant a in the language. We assume that the formal languages under consideration in this chapter are well designed in the relevant sense, so that metaphysical universality implies truth. For our present aim is neither to model natural languages, for example in their use of fictional and mythological names...nor to stick to what is knowable a priori in some sense, which might exclude whether some names refer. Rather, our business is to clarify the structure of metaphysical universality in a broadly scientific spirit. Non-referring uses of 'Pegasus' have no more place in such an enquiry than they have in physics or zoology. Of course, the term 'phlogiston' did occur in scientific language, but if it failed to refer (rather than referring to an empty kind) then its presence in any scientific theory was a defect in that theory. Consequently, we should not distort our formal language by allowing for such a term. *MLM, pages 131–132.*

Are *these* scientific terms?

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We should not distort our formal language by *banning* these terms.

SOLOMON FEFERMAN*

DEFINEDNESS

ABSTRACT. Questions of definedness are ubiquitous in mathematics. Informally, these involve reasoning about expressions which may or may not have a value. This paper surveys work on logics in which such reasoning can be carried out directly, especially in computational contexts. It begins with a general logic of "partial terms", continues with partial combinatory and lambda calculi, and concludes with an expressively rich theory of partial functions and polymorphic types, where termination of functional programs can be established in a natural way.

Erkenntnis 43: 295–320, 1995.

t is defined

$$t{\downarrow}$$

t is defined

$$\mathrm{t}\!\downarrow$$

$$\frac{\Gamma, \mathrm{t} \succ \Delta \mid \mathcal{H}}{\Gamma, \mathrm{t}\!\downarrow \succ \Delta \mid \mathcal{H}}\ [{\downarrow}Df]$$

$(\forall x)$ is a restricted $(\forall^{\Diamond} x)$

$$
\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\Gamma \succ (\forall x)A(x), \Delta \mid \mathcal{H}}{\Gamma, n \succ A(n), \Delta \mid \mathcal{H}}\ [\forall Df]}{n \succ \;\mid\; \Gamma, n \succ A(n), \Delta \mid \mathcal{H}}\ [K,W]}{n \succ \;\mid\; \Gamma, n{\downarrow} \succ A(n), \Delta \mid \mathcal{H}}\ [{\downarrow}Df]}{n \succ \;\mid\; \Gamma \succ n{\downarrow} \supset A(n), \Delta \mid \mathcal{H}}\ [\supset Df]}{\Gamma \succ (\forall^{\Diamond} x)(x{\downarrow} \supset A(x)), \Delta \mid \mathcal{H}}\ [\forall^{\Diamond} Df]
$$

$(\forall^{\Diamond} x)$ is a restricted (Πx)

$$
\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\Gamma \succ (\forall^{\Diamond} x)A(x), \Delta \mid \mathcal{H}}{n \succ \mid \Gamma \succ A(n), \Delta \mid \mathcal{H}}[\forall^{\Diamond} Df]}{n{\downarrow} \succ \mid \Gamma \succ A(n), \Delta \mid \mathcal{H}}[{\downarrow}Df]}{\Gamma, \Diamond n{\downarrow} \succ A(n), \Delta \mid \mathcal{H}}[\Diamond Df]}{\Gamma \succ \Diamond n{\downarrow} \supset A(n), \Delta \mid \mathcal{H}}[\supset Df]}{\Gamma \succ (\Pi x)(\Diamond x{\downarrow} \supset A(x)), \Delta \mid \mathcal{H}}[\Pi Df]
$$

EPISTEMIC MODALS

SUBJUNCTIVE: suppose things go differently.
or *had gone* differently.

INDICATIVE: suppose I'm *wrong* and that actually . . .

SPECIAL COMMEMORATIVE SECTION

The New York Times.

LATE CITY EDITION

KENNEDY IS KILLED BY SNIPER AS HE RIDES IN CAR IN DALLAS; JOHNSON SWORN IN ON PLANE

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- Suppose Oswald *didn't* shoot JFK.

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STEREOSCOPIC VISION:

Persons, Freedom, and Two Spaces of Material Inference

Mark Lance
Georgetown University

W. Heath White
University of North Carolina at Wilmington

Freedom, oh freedom, well that's just some people talkin'.
— The Eagles

What is a person, as opposed to a non-person? One might begin to address the question by appealing to a second distinction: between *agents*, characterized by the ability to act freely and intentionally, and mere patients, caught up in events but in no sense authors of the happenings involving them. An alternative way to address the question appeals to a third distinction: between *subjects* — bearers of rights and responsibilities, commitments and entitlements, makers of claims, thinkers of thoughts, issuers of orders, and posers of questions — and mere objects, graspable or evaluable by subjects but not themselves graspers or evaluators.

We take it as a methodological point of departure that these three distinctions are largely coextensive, indeed coextensive in conceptually central cases. Granted, these distinctions can come apart. One might think that 'person' applies to anything that is worthy of a distinctive sort of moral respect and think this applicable to some fetuses or the deeply infirm elderly. Even if the particular respect due such beings is importantly different from "what we owe each other", such respect could still be thought to be of the kind distinctively due people, and think this even while holding that such people lack agentive or subjective capacity. Similarly, one might think dogs or various severely impaired humans to be attenuated subjects but not agents.

Without taking any particular stand on such examples, our methodological hypothesis is that such cases, if they exist, are understood as persons (agents, subjects) essentially by reference to paradigm cases and, indeed, to a single paradigm within which person/non-person, subject/object, and agent/patient are conceptually connected.[1] Stated

1. For one detailed development of this sort of paradigm-riff structure, and a defense of the possibility of concepts essentially governed by such a structure, see Lance and Little (2004). Discussions with Hilda Lindeman have helped

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- DISAGREEMENT: We *disagree*. We have reason to come to shared positions.

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- We do *many different and strange things* in our messy zone-shifting practices, but we can isolate a particular convention or practice, idealise it, to see what we could do following *those* rules.
  – (*Analogies*: $\forall x$ from first order logic and natural language's 'all.' Frictionless planes. etc.)

Example Subjunctive Shifts

Oswald *did* shoot JFK, but suppose he *hadn't*? How would history have gone differently then?

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$$\mathrm{oSk} \succ_{@} \quad \mid \; @\mathrm{oSk} \succ \mathrm{oSk}$$

We open up a zone for consideration, in which we deny oSk, while keeping track of the initial zone where we assert it.

(And if we like, we can assert @oSk in the zone under the counterfactual supposition.)

Disagreement and Indicative Shifting

I think that Oswald shot JFK, but you don't.

I consider what it would mean for you to be right.

If you're right, Oswald *actually didn't* shoot JFK.

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Epistemic alternatives interact differently with actuality.

$$\mathrm{oSk} \succ_{@} \quad \Longrightarrow \quad \succ_{@} \mathrm{oSk}$$

I think that Hesperus is Phosphorous, but I recognise that *you don't*. I don't take *you* to be *inconsistent* or misusing names.

Indicative Shifting

I think that Hesperus is Phosphorous, but I recognise that *you don't*. I don't take *you* to be *inconsistent* or misusing names.

We *can't* derive this:

$$a = b \succ \quad \Longrightarrow \quad Fa \succ Fb$$

It's coherent for you to assert Fa and deny Fb even if I take it that $a = b$, and it's coherent for me to consider an alternative in which $a \neq b$ even if I take it that $a = b$.

- Let's take any zone found by indicatively shifting from here (or from anywhere from here, etc.) to be an alternative indicative context from any *other* context indicatively shifted from here.

Idealised Indicative Shifts

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- This is as *liberal* as possible about what counts as an *alternative* from any alternative zone.
- This gives us a motivation for a richer family of hypersequents.
- … and the scope to define a rigorous formal kind of "indicative necessity" (or idealised *a priori knowability*).

Two Dimensional Hypersequents

$$
\begin{array}{ccccccccc}
X_1^1 \succ_@ Y_1^1 & | & X_2^1 \succ Y_2^1 & | & \cdots & | & X_{m_1}^1 \succ Y_{m_1}^1 & \| \\
X_1^2 \succ_@ Y_1^2 & | & X_2^2 \succ Y_2^2 & | & \cdots & | & X_{m_2}^2 \succ Y_{m_2}^2 & \| \\
\vdots & & \vdots & & & & \vdots & \\
X_1^n \succ_@ Y_1^n & | & X_2^n \succ Y_2^n & | & \cdots & | & X_{m_n}^n \succ Y_{m_n}^n &
\end{array}
$$

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$$
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X_1^2 \succ_{@} Y_1^2 & \mid & X_2^2 \succ Y_2^2 & \mid & \cdots & \mid & X_{m_2}^2 \succ Y_{m_2}^2 & \| \\
\vdots & & \vdots & & & & \vdots & \\
X_1^n \succ_{@} Y_1^n & \mid & X_2^n \succ Y_2^n & \mid & \cdots & \mid & X_{m_n}^n \succ Y_{m_n}^n &
\end{array}
$$

Think of these as *scorecards*, keeping track of assertions and denials.

Notation

$$\mathcal{H}[\Gamma \succ \Delta]$$

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$$\mathcal{H}[\Gamma \succ \Delta \ \| \ \Gamma' \succ \Delta']$$

$$\frac{\mathcal{H}[\Gamma \succ \Delta \mid \ \succ A]}{\mathcal{H}[\Gamma \succ \Box A, \Delta]}\,[\Box Df]$$

Defining Rule for @

$$\frac{\mathcal{H}[\Gamma, A \succ_{@} \Delta \mid \Gamma' \succ \Delta']}{\mathcal{H}[\Gamma \succ_{@} \Delta \mid \Gamma', @A \succ \Delta']} \; [@Df]$$

Defining Rule for [e]

$$\dfrac{\mathcal{H}[\Gamma \succ \Delta \;\;\|\;\; \succ_{@} A]}{\mathcal{H}[\Gamma \succ [\mathsf{e}]A, \Delta]}\;[[\mathsf{e}]Df]$$

Example Derivation

$$
\begin{array}{cl}
\succ \quad \mid \ [\mathrm{e}]A \succ [\mathrm{e}]A & \\ \hline
\succ \quad \mid \ [\mathrm{e}]A \succ \quad \| \quad \succ_{@} A & [[\mathrm{e}]Df] \\ \hline
\succ \quad \mid \ [\mathrm{e}]A \succ \quad \| \quad \succ_{@} @A & [@Df] \\ \hline
\succ \quad \mid \ [\mathrm{e}]A \succ [\mathrm{e}]@A & [[\mathrm{e}]Df] \\ \hline
\succ \quad \mid \quad \succ [\mathrm{e}]A \supset [\mathrm{e}]@A & [\supset Df] \\ \hline
\succ \Box([\mathrm{e}]A \supset [\mathrm{e}]@A) & [\Box Df]
\end{array}
$$

Epistemic Barcan Formula

$$(\forall x)[e]Fx \succ [e](\forall x)Fx$$

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$$(\forall x)[e]Fx \succ [e](\forall x)Fx$$

$$\langle e\rangle(\exists x)Fx \succ (\exists x)\langle e\rangle Fx$$

Suppose $\langle e \rangle (\exists x)(\exists y)(Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$.

Suppose $\langle e\rangle(\exists x)(\exists y)(Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$.

Do we have $(\exists x)\langle e\rangle((\exists y)(Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$?

Morning Star and Evening Star

Suppose $\langle e\rangle(\exists x)(\exists y)(Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$.

Do we have $(\exists x)\langle e\rangle((\exists y)(Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$?

And $(\exists x)(\exists y)\langle e\rangle(Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$?

Suppose $\langle e \rangle (\exists x)(\exists y)(Mx \;\&\; Ey \;\&\; x \neq y)$.

Do we have $(\exists x)\langle e \rangle ((\exists y)(Mx \;\&\; Ey \;\&\; x \neq y)$?

And $(\exists x)(\exists y)\langle e \rangle (Mx \;\&\; Ey \;\&\; x \neq y)$?

What could such x and y *be*?

CONCLUDING THOUGHTS

I offer *compliments* and *complements*

- COMPLIMENTS: Williamson has shown us that the tools of modal logic are a fruitful setting for understanding modal metaphysics. We *need* the discipline of a formal theory to
  - Keep ourselves honest.
  - Help us understand exactly what theory it is we're adopting.
  - Articulate the consequences of our theories and the possibilities for interpreting them.

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  Modal Logic as Metaphysics is very fruitful example of the *best* work in this tradition.

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  Modal Logic as Metaphysics is very fruitful example of the *best* work in this tradition.

- COMPLEMENTS: I've tried bring to bear the neglected half of logic—*proof theory*—to the task of modal semantics.

Thank you, Tim!

THANK YOU!

http://consequently.org/presentation/2016/existence-definedness-awpl-tplc

@consequently on Twitter