NTNU
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EXERCISE 5

GRADED

Ch. 4.1, 5.3

TMR4182 MARINE DYNAMICS

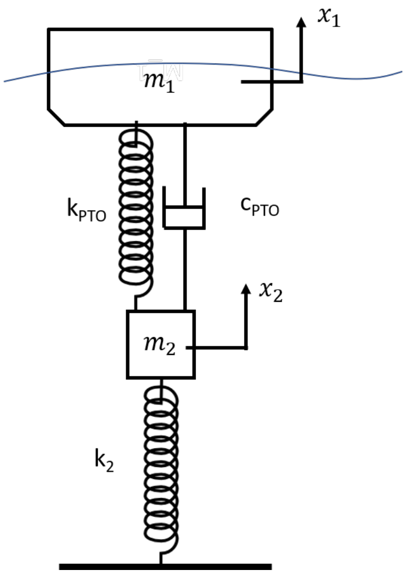
MDOF SYSTEMS

The purpose of this exercise is to set up and solve the forced equations of motion for systems with multiple degrees of freedom. This exercise shall be carried out in **groups** of (maximum) 3 students. One set of codes and one report should be submitted by each group.

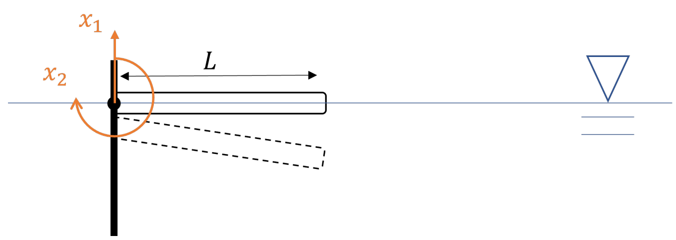
In this exercise, Parts 1 and 2a should be solved by hand, while the remaining parts of the exercise should be solved by via computer code. The code for Part 2b should be submitted (in .m or .py format) so that it can be tested automatically.

1 Equation of motion (20%)

For each marine system in Fig. 1, you should formulate the equations of motion in matrix form. You will need to introduce any relevant variables that you need and define these mathematically. Each system is described in greater detail below.



(a) Heaving wave energy converter



(b) floating pier

Figure 1: MDOF marine systems

System a) represents a wave energy converter. There is a buoy with mass m_1 at the sea surface. It is connected to a second body (with mass m_2) through a power takeoff system which consists of a spring (stiffness k_{PTO}) and dashpot (damping coefficient c_{PTO}). The second body is well submerged and connected to the seabed through a spring (stiffness k_2). You can assume that the second body is so far below the surface that no significant wave loads act on it, and that there is negligible hydrodynamic interaction between the two bodies.

System b) represents a floating pier which can rotate and translate. You may assign the pier mass (m), rotational inertia about its center ($I = mL^2/12$), draft (T) and breadth (B), and introduce any other variables you might need. You can disregard damping and excitation for this example. Note that the illustrated rotations are greatly exaggerated - we don't expect the float to fully submerge. Write the equations of motion using the indicated degrees of freedom. Is there a better choice of the degrees of freedom?

Hint: first write the hydrostatic stiffness as though it were a mechanical stiffness. There is a translational stiffness in heave $C_33 = \rho g L b$, and a rotational stiffness in pitch (about the center) $C_{55} = \rho g I_{wp} - mgz_g + \rho \nabla g z_B$, where I_{wp} is the waterplane moment of inertia, z_g is the vertical location of the center of gravity, and z_B is the vertical location of the center of buoyancy. You can assume that the float is symmetric about the middle.

2 Solving MDOF equations of motion (80%)

Consider the mechanical system shown in Fig. 2.

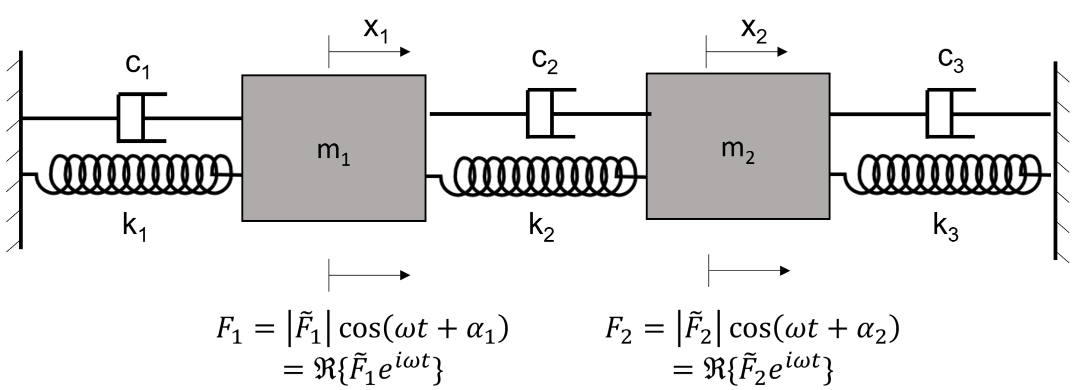


Figure 2: Two carts connected by springs and dashpots.

(a) (5%) Write the equation of motion of the system in matrix form.

(b) (25%) Write a python or matlab code to solve the equation of motion for given numerical inputs to find the *amplitude* of motion of each of the masses for given system parameters and forcing at a single frequency ω. Your code should be a standalone function which can be called in the format:

Matlab:

```
function [x1,x2] = twocarts(m1,m2,k1,k2,k3,c1,c2,c3,F1_abs,F1_phase,
    ↪ F2_abs,F2_phase,omega)
```

Python:

```
def twocarts(m1,m2,k1,k2,k3,c1,c2,c3,F1_abs,F1_phase,F2_abs,F2_phase,
    ↪ omega):
```

```
    # your code to find x1, x2 here
    return x1,x2
```

The inputs and outputs are defined in Table 1, including some unique default numerical values for each group.

Variable definition	Variable in code	Value	Unit
Cart 1 mass m_1	m1	10	kg
Cart 2 mass m_1	m2	Group number	kg
Spring 1 stiffness k_1	k1	Day of the month of the youngest group member's birthday	N/m
Spring 2 stiffness k_2	k2	3	N/m
Spring 3 stiffness k_3	k3	Group number/2	N/m
Damper 1 coefficient c_1	c1	0.2	Ns/m
Damper 2 coefficient c_2	c2	0.2	Ns/m
Damper 3 coefficient c_3	c3	0.2	Ns/m
Force 1 amplitude $\lvert\tilde{F}_1\rvert$	F1_abs	Day of the month of the oldest group member's birthday	N
Force 2 amplitude $\lvert\tilde{F}_2\rvert$	F2_abs	10	N
Force 1 phase α_1	F1_phase	0	rad
Force 2 phase α_2	F2_phase	0	rad
Force frequency ω	omega	-	rad
Cart 1 motion amplitude x_1	x1	(output)	m
Cart 2 motion amplitude x_2	x2	(output)	m

Table 1: System parameters.

Make sure to include the values for your group in the report!

(c) (10%) Write a python or matlab code to find the natural frequencies and mode shapes of the system, using the default numerical inputs for your group from Table 1. Report the results and describe the physical interpretation of the mode shapes.

Hint 1

In python, the scipy linear algebra library could be useful for finding eigenvalues and eigenvectors. For example, the script:

```
import numpy as np
import scipy.linalg as la

M = np.array([[1,0],[0,1]])
K = np.array([[2,-2],[-2,4]])
```

```
D,V = la.eig(K,M)
```

will return a list called `D` containing the eigenvalues (ω^2) and a matrix called `V` containing the eigenvectors.
In matlab, you might use `eigs`.

(d) (10%) Using the default numerical values for your group, plot the amplitude of motion of each of the masses for a range of frequencies from 0.01 rad/s to 5 rad/s. Briefly describe why the pattern looks the way it does.

(e) (15%) Now, your task is to maximize the amplitude of motion of mass m_2 (x2 in your code) by modifying the phases of the force inputs (F1_phase and F2_phase in your code) for a selected forcing frequency which is in between the two natural frequencies.

A brute force approach, where you try all possible combinations, may be the most convenient. If you have a special interest in learning about optimization methods, you can alternatively consider applying `fmincon` in matlab or `minimize` in python.

Explain how you solved the problem and discuss your results. Include a figure showing the time series of the 1) forces applied and 2) resulting motions of both carts as a part of your discussion.

(f) (15%) Finally, we will mimic some hydrodynamic effects in our system by introducing variations in the forcing phase and amplitude as a function of frequency. (In reality, of course, we would also have frequency dependence in the added mass and damping terms, but we will focus only on the forcing variation here for simplicity).

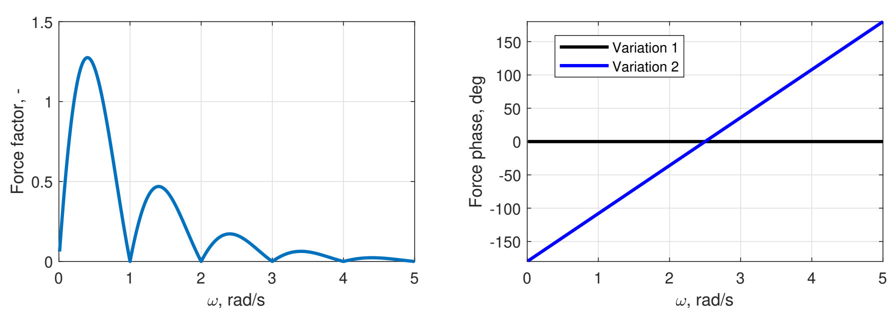


Figure 3: Frequency-dependent factor for $|\tilde{F}_1|$ and force 1 phase variations

Report the motion amplitudes of the two carts considering the provided frequency dependence in Fig. 3 applied as a factor on force $|\tilde{F}_1|$ for two cases: one with constant zero phase, and one with phase linearly varying from -180 degrees to 180 degrees (for $0 < \omega < 5$ rad/s). The force amplitude and phase on mass 2 can be kept constant as in Table 1.

Mathematically, this frequency-dependent forcing factor is is given as:

$$2e^{-\omega}|\sin(\pi\omega)|. \tag{1}$$

Report on the motion responses for the two masses.