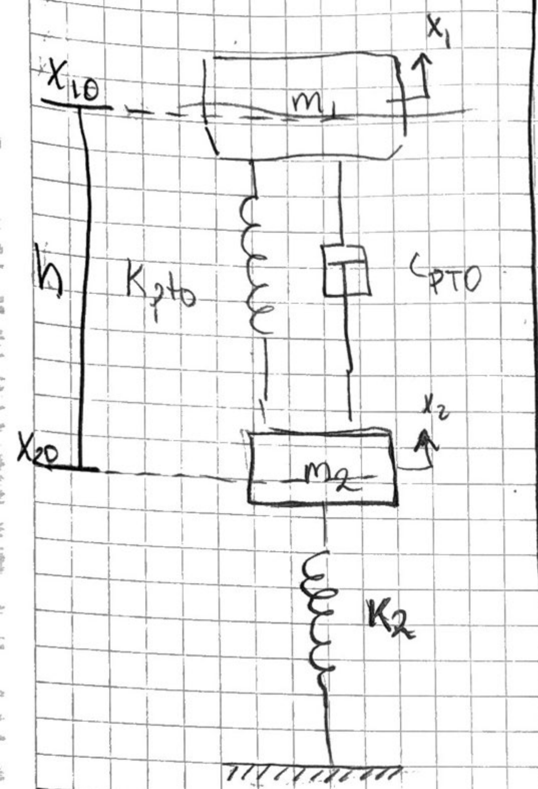

Equation of motion body 1

$\Sigma F = m\ddot{x}_1 \rightarrow F_{waves} + F_{KPTO} + F_{CPTO} = m\ddot{x}_1$

$m\ddot{x}_1 - F_{waves} - F_{KPTO} - F_{CPTO} = 0$

Current, wind and tidal forces are neglected.

$F_{KPTO} = K_{PTO}(X_2 - X_1)$

$F_{CPTO} = C_{PTO}(\dot{X}_2 - \dot{X}_1)$

Why $(X_2 - X_1)$ and not $(X_1 - X_2)$?
because if we imagine x_2 being constant, positive increase in x_1, would cause the forces F_{KPTO} and F_{CPTO} to decrease in value. That is because the factors C_{PTO} and $K_{pto} > 0$.
From newtons 2nd law $ma - F = 0$ whenever force is increasing the acceleration must also increase to maintain the equilibrium. Or in other words acceleration is proportional with force.

Equation of motion body 2

$\Sigma F = m\ddot{x}_2 \rightarrow F_{K2} + F_{KPTO} + F_{CPTO} = m\ddot{x}_2$

$F_{K2} = K_2(-x_2)$

F_{KPTO} and F_{CPTO} depend on the distance between body 1 and body 2.

$\Rightarrow (X_1 - X_2)$ or $\cancel{(X_2 - X_1)}$?

Imagine holding body 1 fixed and increasing x_2
$\ddot{X}_2$ is positive, and the spring is getting compressed

resisting further changes. same with the damper. when increasing velocity then the damper will oppose the change.

$$\Sigma F = m\ddot{x}_2 \rightarrow F_{KPTO} + F_{CPTO} + F_{K2} = m\ddot{x}_2$$

$$K_2(-x_2) + C_{PTO}(\dot{x}_1 - \dot{x}_2) + K_{PTO}(x_1 - x_2) = m\ddot{x}_2$$

$$\begin{cases} m_1\ddot{x}_1 - F_{KPTO} - F_{CPTO} = 0 \\ m_2\ddot{x}_2 - F_{KPTO} - F_{CPTO} - F_{K2} = 0 \end{cases}$$

$$\begin{cases} m_1\ddot{x}_1 - K_{PTO}(x_2 - x_1) - C_{PTO}(\dot{x}_2 - \dot{x}_1) = 0 \\ m_2\ddot{x}_2 - K_{PTO}(x_1 - x_2) - C_{PTO}(\dot{x}_1 - \dot{x}_2) - K_2(-x_2) = 0 \end{cases}$$

$$\begin{cases} m_1\ddot{x}_1 + C_{PTO}\dot{x}_1 - C_{PTO}\dot{x}_2 + K_{PTO}x_1 - K_{PTO}x_2 = 0 \\ m_2\ddot{x}_2 - C_{PTO}\dot{x}_1 + C_{PTO}\dot{x}_2 - K_{PTO}x_1 + (K_{PTO} + K_2)x_2 = 0 \end{cases}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\ddot{x} + C_{PTO}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\dot{x} + \begin{bmatrix} K_{PTO} & -K_{PTO} \\ -K_{PTO} & K_{PTO} + K_2 \end{bmatrix}x = \vec{0}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\ddot{x} + C_{PTO}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\dot{x} + K_{PTO}\begin{bmatrix} 1 & -1 \\ -1 & 1 + \frac{K_2}{K_{PTO}} \end{bmatrix}x = \vec{0}$$

This system is missing now the exitation force coming from waves. since we're in water, and we have hydrodynamical forces we need to add their coefficients

$$\begin{bmatrix} m_1 + A & 0 \\ 0 & m_2 \end{bmatrix}\ddot{x} + C_{PTO}\begin{bmatrix} 1+B & -1 \\ -1 & 1 \end{bmatrix}\dot{x} + K_{PTO}\begin{bmatrix} 1+C & -1 \\ -1 & 1 + \frac{K_2}{K_{PTO}} \end{bmatrix}x = \begin{bmatrix} F_{ex} \\ 0 \end{bmatrix}$$