c) $x(n) = x(n/2) + n$ for $n > 1$, $x(1) = 1$ solve for $n = 2k$

$x(1) = 1$

$\underline{n=2}$:-

$n = 2^1 = 2$

$$x(2) = x(2/2) + 2$$
$$= x(1) + 2$$
$$\boxed{x(2) = 3}$$

$\underline{n=4}$:-

$n = 2^2 = 4$

$$x(4) = x(4/2) + 4$$
$$= x(2) + 4$$
$$= 3 + 4$$
$$\boxed{x(4) = 7}$$

$\underline{n=8}$:-

$n = 2^3 = 8$

$$x(8) = x(8/2) + 8$$
$$= x(4) + 8$$
$$= 7 + 8$$
$$\boxed{x(8) = 15}$$

For this recursion relation, $2^n - 1 = 2^{2k} - 1$

for $\boxed{n = 2^k}$, $\boxed{x(2k) = 2^{2k} - 1}$

d) $x(n) = x(n/3) + 1$ for $n > 1$, $x(1) = 1$, $n = 3^k$

$x(1) = 1$

$\underline{n=3}$:-

$n = 3^1 = 3$

$$x(3) = x(3/3) + 1$$
$$= x(1) + 1$$
$$= 1 + 2$$
$$\boxed{x(3) = 2}$$

$\underline{n=9}$:-

$n = 3^2 = 9$

$$x(9) = x(9/3) + 1$$
$$= x(3) + 1$$
$$= 2 + 1$$
$$\boxed{x(9) = 3}$$

$\underline{n=27}$:-

$n = 3^3 = 27$

$$x(27) = x(27/3) + 1$$
$$= x(9) + 1$$
$$= 3 + 1$$
$$\boxed{x(27) = 4}$$

for this recurrance relation,

$$\boxed{x(n) = \cancel{\log}^{k+1}_{\cancel{3}} \cancel{n}}$$

$$\boxed{x(3^k) = \cancel{\log}^{k+1}_{\cancel{3}} \cancel{3^k}}$$

2. Evaluate the following recurrences completely

i) $T(n) = T(n/2) + 1$, where $n = 2k$ for all $k \geq 0$.

$T(n) = T(n/2) + 1$

Since $n = 2^k$, we can rewrite $n/2$ as 2^{k-1}.

$T(2^k) = T(2^{k-1}) + 1$

$T(2^k) = T(2^{k-2}) + 1 + 1$

$T(2^k) = T(2^{k-3}) + 1 + 1 + 1 \ldots$

$T(2^k) = T(1) + k$

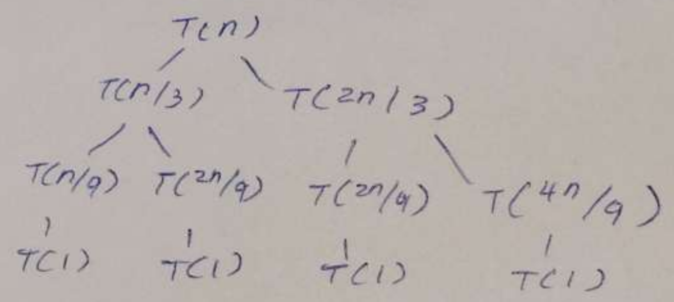
Since $T(1)$ is constant, lets call it 'a'

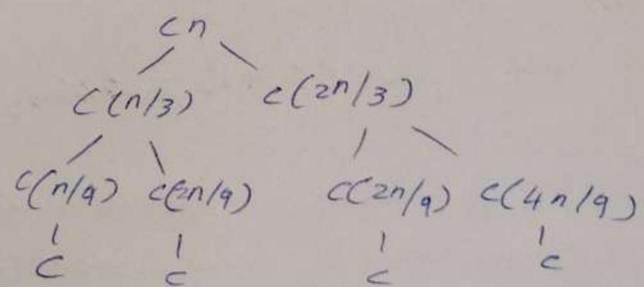
$T(2^k) = a + k$

$\therefore T(n) = a + \log_2(n)$

ii) $T(n) = T(n/3) + T(2n/3) + cn$

$T(n) = T(n/3) + T(2n/3) + cn$





length $= \log_3 n$ (divided by 3)

$T(n) = cn \log_3 n$

$\Rightarrow \omega(n \log n)$

3. consider following algorithm

min1[A[0...n-1])
if n=1 return A[0] ⇒1
Else temp = min1[A[0...n-2]]
if temp ≤ A[n-1] return temp
else
return A[n-1] - n - 1

a) what does this algorithm compute?

b) set up a recurrence relation for algorithm basic operations count and solve it.

a) This algorithm computes minimum element in an array A of size n. If $i<n$, $A[i]$ is smaller than all elements, then, $A[j]$, $j=i+1$ to $n-1$, then it returns $A[i]$. It also returns the left most minimal element.

b) main by comparison occurs during recursion.

So, $T(n) = T(n-1)+1$, when $n>1$ (one comparison at every step except, $n=1$)

$T(1) = 0$ (no compare when $n=1$)

$T(n) = T(1)+(n-1)*1$

$= 0+(n-1)$

$= n-1$

∴ Time complexity $= O(n)$.

4. Analyze Order of growth

i) $F(n)=2n^2+5$ and $g(n)=7n$ use $\Omega(g(n))$ notation.

$n=1$

$F(1)=2(1)^2+5=7$

$g(1)=7$

$n=2$

$F(2)=2(2)^2+5=13$

$g(2)=7\times2=14$

$n=3$

$F(3)=2(3)^2+5=23$

$g(3)=21$

$n=1$, $7=7$

$n=2$, $13=14$

$n=3$ $23=21$

$n\geq3$

$F(n)\geq g(n).C$

F(n) is always greater than or equal to $c.g(n)$ when, n value is greater or equal to 3

∴ $F(n)=\Omega(g(n))$

F(n) grows more than g(n) from below asymptotically.

[redacted]

Design and Analysis of Algorithm for polynomial problems

1. Solve the following recurrance relations :-

a) $x(n) = x(n-1)+5$ for $n>1$ $x(1)=0$

$x(1)=0$

Substitute

$n=2$:- $x(2) = x(2-1)+5$
$= x(1)+5$
$= 0+5$
$\boxed{x(2) = 5}$

$n=3$:- $x(3) = x(3-1)+5$
$= x(2)+5$
$= 5+5$
$\boxed{x(3) = 10}$

$n=4$:- $x(4) = x(4-1)+5$
$= x(3)+5$
$= 10+5$
$\boxed{x(4) = 15}$

For this recurrance relation, each term is 5 more than prev

$\boxed{\text{So, } x(n) = 5n-5 \text{ for } n>1}$

b) $x(n) = 3x(n-1)$ for $n>1$ $x(1)=4$

$x(1)=4$

Substitute:

$n=2$:- $x(2) = 3x(2-1)$
$= 3x(1)$
$= 3(4)$
$\boxed{x(2) = 12}$

$n=3$:- $x(3) = 3x(3-1)$
$= 3x(2)$
$= 3(12)$
$\boxed{x(3) = 36}$

$n=4$:- $x(4) = 3x(4-1)$
$= 3x(3)$
$= 3(36)$
$\boxed{x(4) = 108}$

For this recurrance relation, each term is 3 times t

$\boxed{\text{So, } x(n) = 4 * 3^{n-1} \text{ for } n>1}$