Journal of
Microscopy

Journal of Microscopy, Vol. 243, Pt 2 2011, pp. 141–153
Received 6 September 2010; accepted 3 January 2011
doi: 10.1111/j.1365-2818.2010.03487.x

Multiscale toggle contrast operator-based mineral image enhancement

X. BAI & F. ZHOU
Image Processing Center, Beijing University of Aeronautics and Astronautics, Beijing, China

Key words. Mathematical morphology, mineral image enhancement, multiscale, toggle contrast operator.

Summary

Mineral image is one type of crucial data for mineral research and analysis. However, some mineral images are not clear in some cases. To efficiently enhance mineral images and therefore benefit the applications of these images, a multiscale toggle contrast operator based algorithm is proposed in this paper. First, the toggle contrast operator is discussed. Secondly, the multiscale toggle contrast operator using structuring elements with different sizes is given. Thirdly, the multiscale toggle contrast operator is used to enhance the original image at different scales. Finally, the final enhanced image is constructed from the multiscale enhanced results. Because multiscale structuring elements are used, the algorithm performs efficiently and produces few noises. Experimental results show that the proposed algorithm is efficient for mineral image enhancement. More importantly, the proposed algorithm could be also used in other types of images, such as visual image, medical image and so on, for image enhancement.

Introduction

Mineral images obtained by scanning electron microscopy, transmission electron microscopy, scanning tunnelling microscopy and some other microscopy equipments are important data for mineral analysis. However, many obtained mineral images are blurred by imaging equipment or operators, which may affect the mineral applications, such as mineral property analysis, mineral identification, and so on (Schultze-Lam *et al.*, 1992; Carter *et al.*, 2000; Cheng *et al.*, 2006). If the mineral images could be enhanced, the enhanced images would be more efficient for mineral applications. So, it would be useful if these blurred images could be enhanced and become clear. And, the image enhancement technique could be used for mineral image enhancement.

Image enhancement has been an important image processing technique. Many algorithms have been proposed for image enhancement, such as histogram-based algorithms (Wan & Shi, 2007), fuzzy logic-based algorithms (Farbiz *et al.*, 2000), diffusion-based algorithms (Gilboa *et al.*, 2004), frequency-based algorithms (Agaian *et al.*, 2001), wavelet-based algorithms (Mencattini *et al.*, 2008), mathematical morphology-based algorithms (Soille, 2003), and so on. Among them, mathematical morphology-based algorithms are one class of widely used algorithms, and top-hat transform (De *et al.*, 2006; Bai *et al.*, 2010) and toggle contrast operator (Kramer *et al.*, 1975; Lester *et al.*, 1980; Meyer & Serra, 1989; Schavemaker *et al.*, 2000; Dorini *et al.*, 2007a,b; Maragos, 2005) are two mainly used operators in mathematical morphology-based algorithms. The main operations in top-hat transform are opening and closing which could smooth image details, then the performance of top-hat transform may be affected and may be not good for mineral image enhancement. Toggle contrast operator could sharpen image and make the original blurred image clear. So, toggle contrast operator may be an efficient tool for mineral image enhancement. However, toggle contrast operator using structuring element with small size could not perform very efficiently and toggle contrast operator using structuring element with large size may produce many noises. To make the toggle contrast operator efficient and suppress the possible produced noises, the multiscale technique (Matheron, 1975; Boomgaard, 1992; Jackway, 1992; Jackway *et al.*, 1994; Jackway, 1995, 1998; Jackway *et al.*, 1996; Jalba *et al.*, 2004; De *et al.*, 2006; Oliveira *et al.*, 2008) by using multiscale structuring elements with the same shape and increasing sizes should be used.

Based on this idea, the multiscale toggle contrast operator is proposed and applied for mineral image enhancement in this paper. First, the toggle contrast operator based on dilation and erosion is introduced and discussed. Secondly, the multiscale toggle contrast operator using structuring elements with the

Correspondence to: X. Bai, Image Processing Center, Beijing University of Aeronautics and Astronautics, 100191 Beijing, China. Tel: +86-10-82338048; fax: +86-10-82316502; e-mail: jackybxz163@163.com

same shape and increasing sizes is given. Thirdly, after the original image is enhanced by using multiscale toggle contrast operator, the enhanced results of each scale are combined together to construct the final enhanced result. Because the final result is constructed from the multiscale toggle contrast operator using structuring elements with different sizes, the algorithm could perform efficiently and produce very few noises. Experimental results verified that the proposed algorithm was efficient for mineral image enhancement. Moreover, the proposed algorithm could be also used in other types of images for image enhancement.

Toggle contrast operator

Mathematical morphology is based on geometry and set theory. Most of morphological operations work with two sets: the original image to be processed and the structuring element. Let f and B represent a grey scale image and structuring element, respectively. The two basic operations which are dilation ($\oplus$) and erosion (Θ) of $f(x, y)$ using $B(i, j)$ are defined as follows:

$$f \oplus B(x, y) = \max_{i,j}(f(x-i, y-j) + B(i, j)),$$

$$f \Theta B(x, y) = \min_{i,j}(f(x+i, y+j) - B(i, j)).$$

Toggle mappings are one type of important morphological operators. The result of toggle mapping is a selection output. Two important factors in toggle mappings are primitives and decision rules (Meyer & Serra, 1989; Serra, 1988). The result of toggle mapping for each pixel is one of the values of the same pixels in the primitives following the decision rules. By using morphological operations as primitives and through finding efficient decision rules, different toggle mappings could be constructed and efficiently used for image processing (Serra *et al.*, 1992; Terol-Villalobos, 1996; Terol-Villalobos *et al.*, 1998). One type of toggle mapping, which is also called toggle contrast operator, is defined as follows by using morphological dilation and erosion as the primitives.

$$TCO(x, y) = \begin{cases} f \oplus B(x, y), & \text{if } f \oplus B(x, y) - f(x, y) < f(x, y) - f \Theta B(x, y) \\ f \Theta B(x, y), & \text{if } f \oplus B(x, y) - f(x, y) > f(x, y) - f \Theta B(x, y) \\ f(x, y), & else. \end{cases}$$

This definition of toggle contrast operator indicates that each pixel of the enhanced image using toggle contrast operator is selectively replaced by the same pixel in the result of dilation or erosion with grey value close to the grey value of the same pixel in the original image.

Mineral image enhancement

Multiscale toggle contrast operator

Mineral image usually has many blurred image details. TCO could sharpen image and make blurred image details clear. So, TCO is used for mineral image enhancement in this paper. Dilation or erosion enlarges or shrinks image regions corresponding to the size of the used structuring element. Then, comparing with the original image, the result of dilation or erosion mainly changes the marginal region of image regions. Therefore, TCO will not only smooth image regions, but also replace the grey values of the marginal region with the grey values of the result of dilation or erosion. This will sharpen image and achieve the purpose of image enhancement, which makes the original blurred mineral image clear.

However, TCO using structuring element with small size could not perform very efficiently whereas TCO using structuring element with large size will produce many noises. Fig. 1(a) is an example of original mineral image. Fig. 1(b) is the enhanced result using B_1 with radius size 1, which is very similar as the original image and not efficiently enhanced. Fig. 1(c) is the enhanced result using B_{11} with radius size 11. Although the original image is efficiently enhanced, some noises are also produced, especially at the edge region of image regions, which will destroy image details. Moreover, if the size of structuring element is very large, the produced noises will be more.

To be efficient and suppress noises, multiscale structuring elements should be used. Then, the multiscale enhanced results using multiscale structuring elements could be used to construct the final enhanced result, which is efficient and has few noises.

Multiscale operators were first proposed by Witkin (1983) for image processing, which use Gaussian convolution to generate scale space. After that, the discrete equivalent of the Gaussian linear scale space gives one way of using scale space theory in digital space (Lindeberg, 1994). The first possible size (scale)-dependent operation is used by Matheron in the study of granulometries (Matheron, 1975). Then, size-dependent morphological operations have been extended and used in different applications (Jackway, 1998; De *et al.*, 2006). Types of scale spaces using morphological operations, such as multiscale dilation and erosion (Jackway *et al.*, 1996), multiscale opening and closing (Jackway, 1992) and some extension theories (Boomgaard, 1992; Jackway *et al.*, 1994; Jackway, 1995), are proposed and used in image processing. Among these morphological scale spaces, the morphological scale space based on dilation and erosion has been extended to be well used in image processing, such as cell segmentation (Dorini *et al.*, 2007b). However, because dilation and erosion are sensitive to noisy contours, the multiscale property may be affected in real images with noises (Rivas-Araiza *et al.*, 2008). Fortunately, in our application, mineral images are mainly

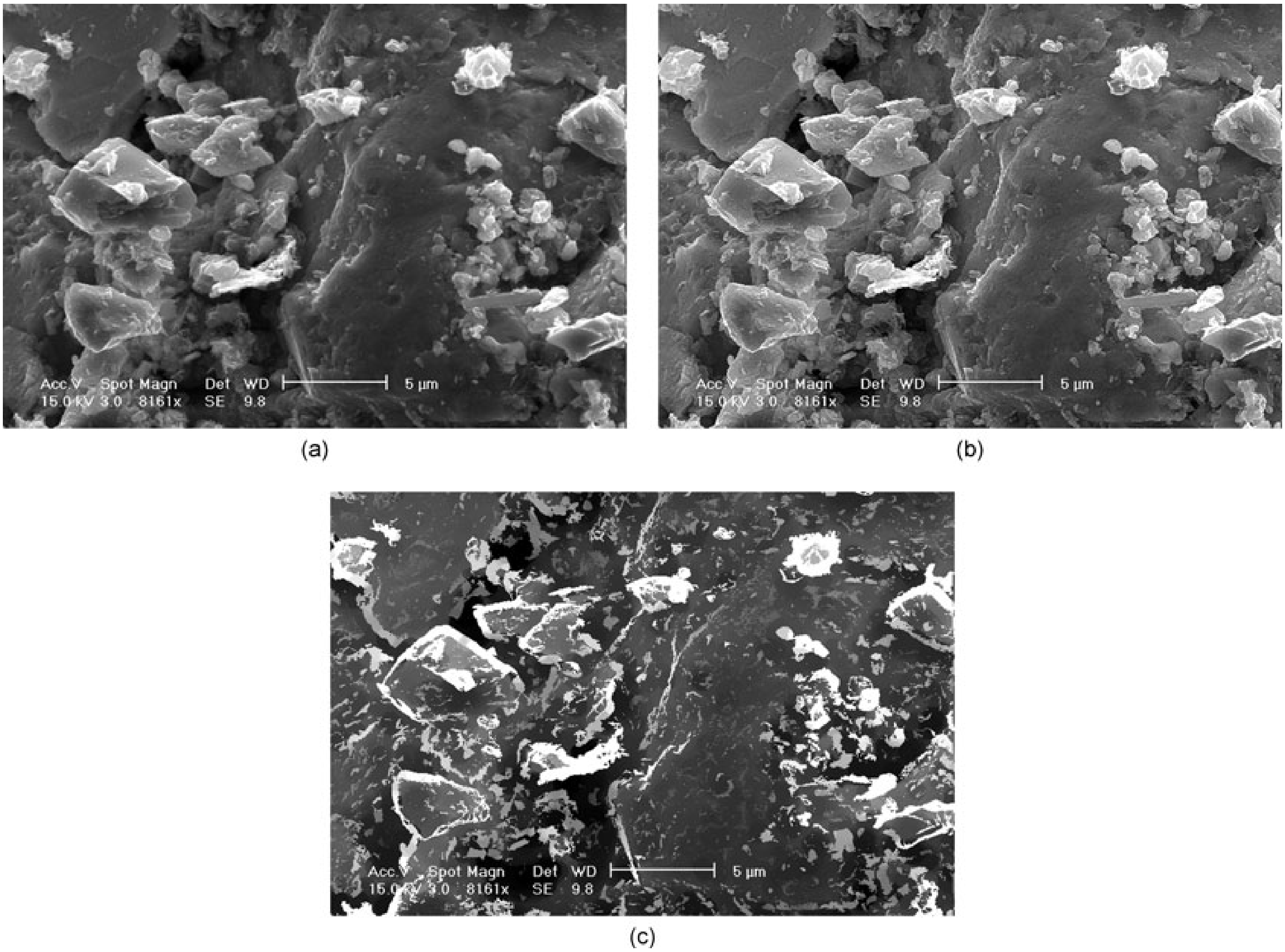


Fig. 1. Enhanced result of toggle contrast operator at different scales. (a) Original image, (b) enhanced result using B_1 and (c) enhanced result using B_{11}.

blurred and do not contain very heavy noises. So, although the multiscale property may be affected, the extracted useful information at each scale will be not significantly different from the ideal situation. Moreover, multiscale dilation and erosion have been efficiently used in image processing, such as cell segmentation (Dorini *et al.*, 2007b). Therefore, in our paper, multiscale dilation and erosion are also used to form multiscale toggle contrast operator for mineral image enhancement.

Suppose there are n scales of structuring elements with the same shape and increasing sizes, denoted by $B_0, B_1, \ldots, B_{n-1}$, should be used. $B_s = \underbrace{B_0 \oplus B_0 \ldots \oplus B_0}_{\text{dilation} \quad s \quad \text{times}}$, $0 \le s \le n-1$. The size of B_s is s.

Using multiscale structuring elements, multiscale dilation and erosion of $f(x, y)$ using $B_s(i, j)$ could be defined as follows:

$$f \oplus B_s(x, y) = \max_{i,j}(f(x - i, y - j) + B_s(i, j)),$$

$$f \ \Theta \ B_s(x, y) = \max_{i,j}(f(x + i, y + j) - B_s(i, j)).$$

Based on the dilation and erosion results at scale s, the toggle contrast operator at scale s could be calculated as follows:

$$\mathrm{TCO}_s(x, y) = \begin{cases} f \oplus B_s(x, y), \text{ if } & f \oplus B_s(x, y) - f(x, y) \\ & < f(x, y) - f \Theta B_s(x, y) \\ f \Theta B_s(x, y), \text{ if } & f \oplus B_s(x, y) - f(x, y) \\ & > f(x, y) - f \Theta B_s(x, y) \\ f(x, y), else. & \end{cases}$$

Then, through varying s, the multiscale enhanced results by using TCO_s could be obtained.

Image enhancement

Different enhanced results using toggle contrast operator at multiscales have different properties. Enhanced result at low scale is not efficient but has very few noises because a structuring element with small size is used. Enhanced result at high scale is efficient but may produce many noises because a structuring element with large size is used. One basic idea of constructing the final result is pixel-wise averaging of the multiscale enhanced results as follows:

$$R = \frac{\sum_{s=0}^{n-1} \mathrm{TCO}_s}{n}.$$

R is the final enhanced result of mineral image. Because of the averaging, the efficiently enhanced result at high scale will be combined into the enhanced result at low scale, which will make the final result efficiency. Moreover, because of the averaging, the produced noises only existed at high scale will be suppressed. All of these means, R is an operator to combine the priorities of multiscale results of toggle contrast operator. Therefore, the final result will be efficient and has very few noises. This would be very useful for the mineral applications, such as mineral property analysis, mineral identification, and so on.

Also, because the values of the same pixel in the results of dilation and erosion may be very different, toggle contrast operator may produce jumps and oscillations when they are iterated. However, in our algorithm, the sizes of the used scales gradually increase and the size difference of the neighbouring scale is only 1. So, the effect of jumps and oscillations is small. Moreover, using pixel-wise averaging on all the extracted multiscale features to calculate the final result will suppress the effect of jumps and oscillations. Therefore, the proposed algorithm is efficient for mineral image enhancement.

Parameter specification

Shape of structuring element. Dilation and erosion enlarge or shrink image regions corresponding to the shape and size of the structuring element. The effect of the size of the structuring element is suppressed by the multiscale structuring elements with different sizes. Some common used shapes in structuring element are square, rectangle, hexogen and circle. Square, rectangle and hexogen do not have smoothing edges and may produce block effect, which destroys the shapes of image regions for minerals. Especially, when the size of structuring element is large, the block effect will be heavy. The circle shape could suppress the block effect because of the smooth edge. Therefore, we choose the circle as the shape of the used structuring element in this paper.

Number of scales. The number of scales n is an important parameter in this algorithm. A very large n may generate enhanced results having heavy noises at many high scales, which will result in more noises in the final image. A very small n may generate inefficient enhanced result. Different numbers of scales have been used in our experiments and we have found that using 5, 6 or 7 as the number of scales may give satisfying results on all the images. Therefore, we choose 6 as the number of scales in this paper. That is $n = 6$ in this paper.

Implementation

Based on the analysis earlier, the whole procedure of the multiscale toggle contrast operator for mineral image enhancement is demonstrated as follows. First, for each scale s $(0 \le s \le n-1)$, the dilation and erosion results using structuring element B_s with size s are calculated. Then, for each scale s $(0 \le s \le n-1)$, the result of toggle contrast operator is calculated following the definition of TCO_s. Finally, after all the results of toggle contrast operator are calculated, the final enhanced result is constructed through the pixel-wise averaging on all the calculated multiscale results produced by toggle contrast operator. The implementation of the algorithm is demonstrated in Fig. 2.

Algorithm

For s from 0 to $n-1$;

Calculate the dilation and erosion results of f using structuring element B_s: $f \oplus B_i$ and $f \Theta B_s$;

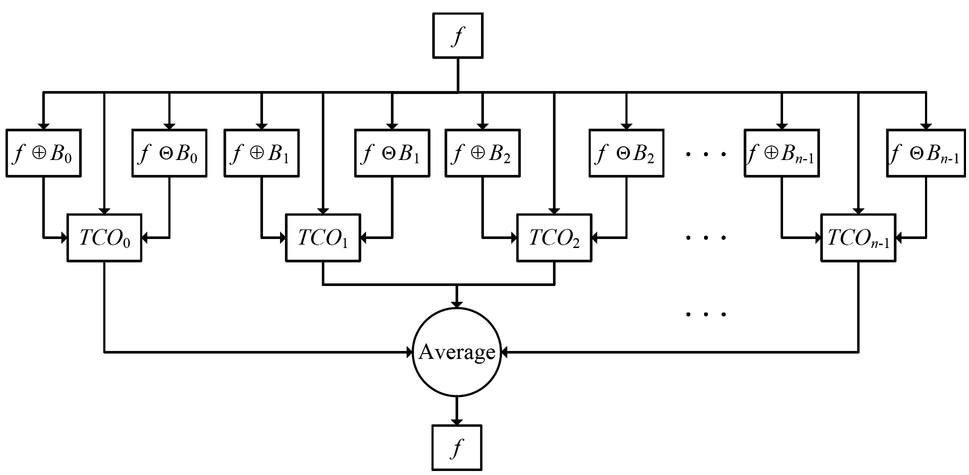


Fig. 2. Implementation of the proposed algorithm.

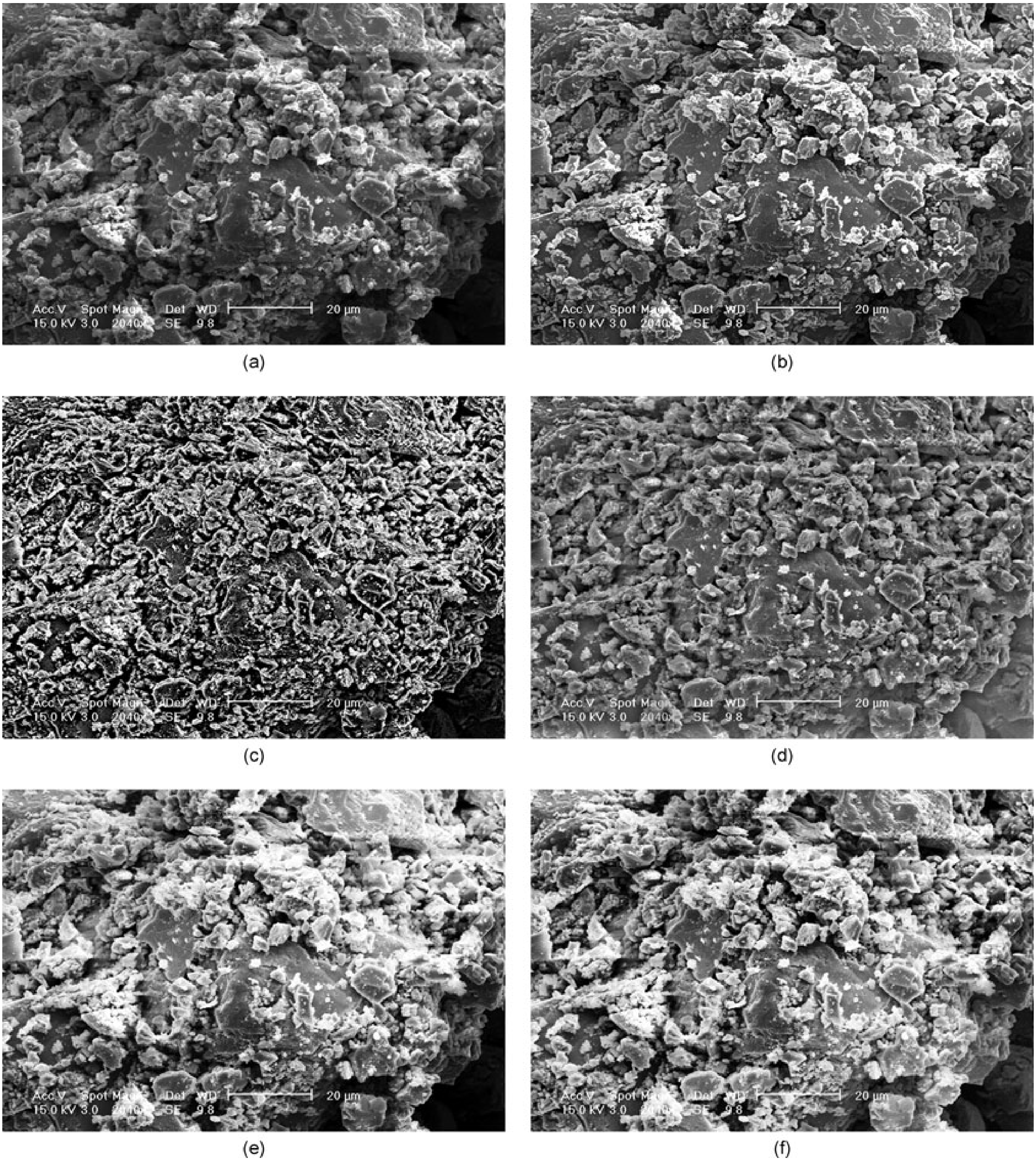


Fig. 3. Comparison result 1. (a) Original image, (b) enhanced result by the proposed algorithm, (c) enhanced result by MSM, (d) enhanced result by WF, (e) enhanced result by HE and (f) enhanced result by CLAHE.

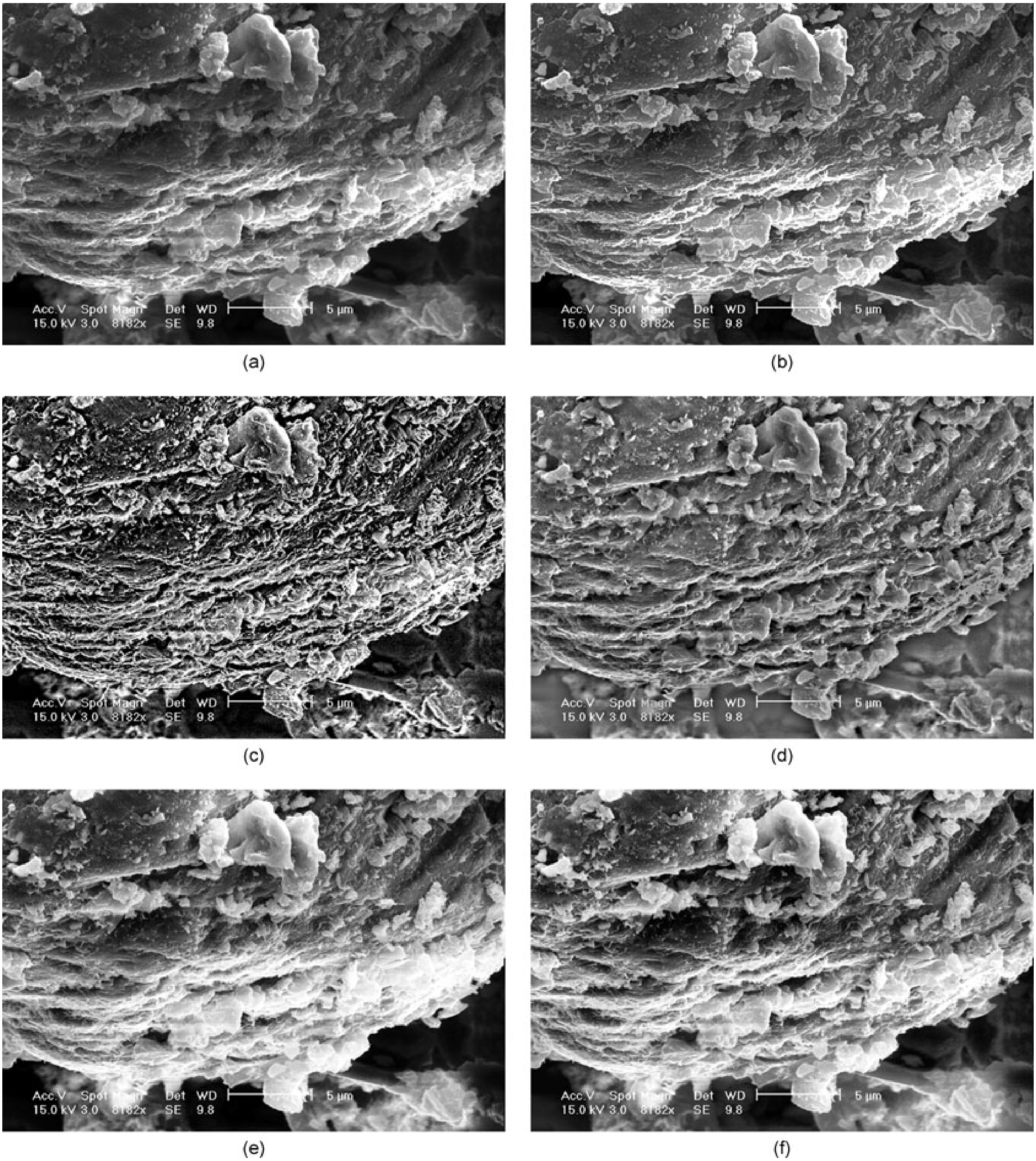


Fig. 4. Comparison result 2. (a) Original image, (b) enhanced result by the proposed algorithm, (c) enhanced result by MSM, (d) enhanced result by WF, (e) enhanced result by HE and (f) enhanced result by CLAHE.

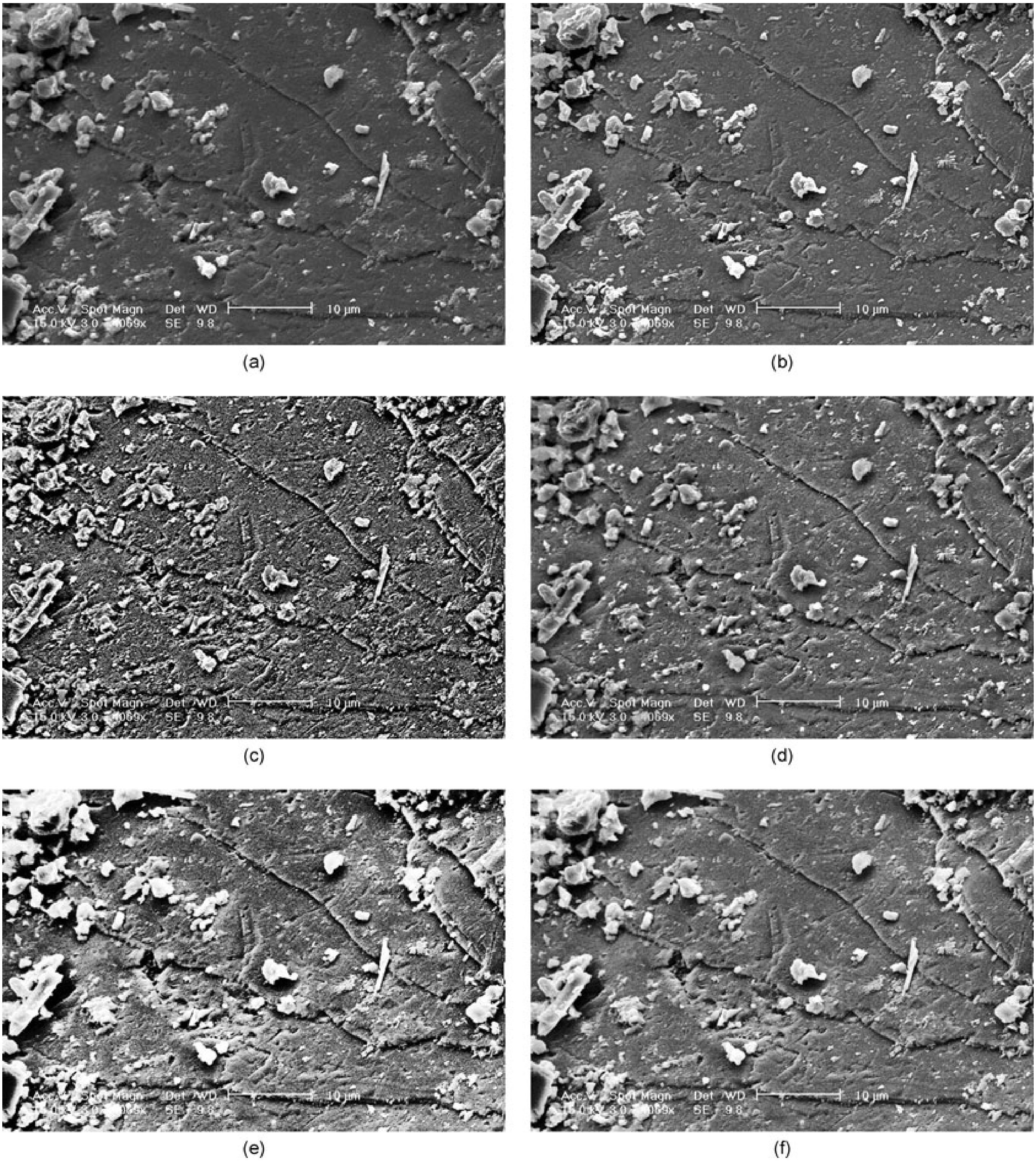


Fig. 5. Comparison result 3. (a) Original image, (b) enhanced result by the proposed algorithm, (c) enhanced result by MSM, (d) enhanced result by WF, (e) enhanced result by HE and (f) enhanced result by CLAHE.

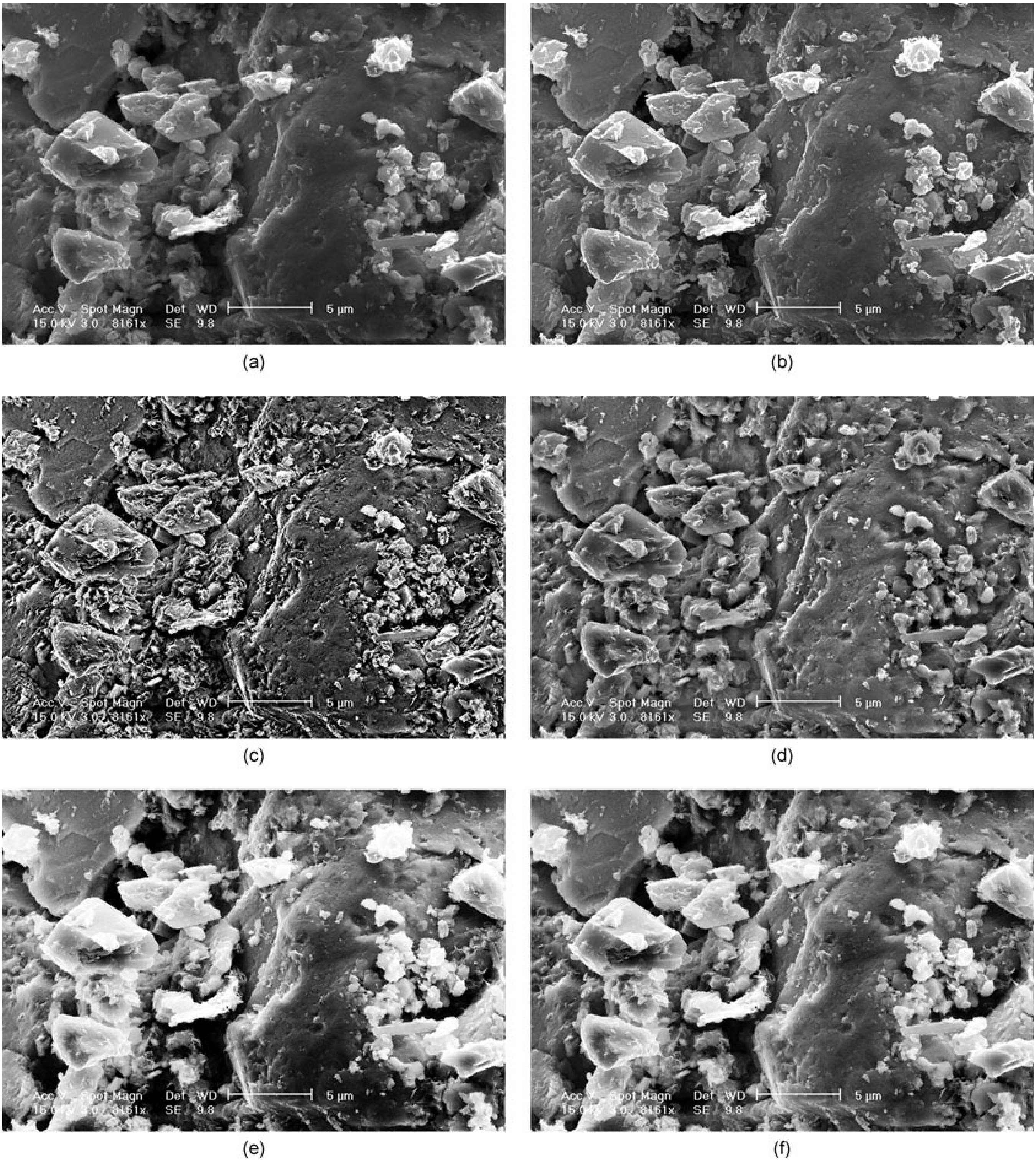
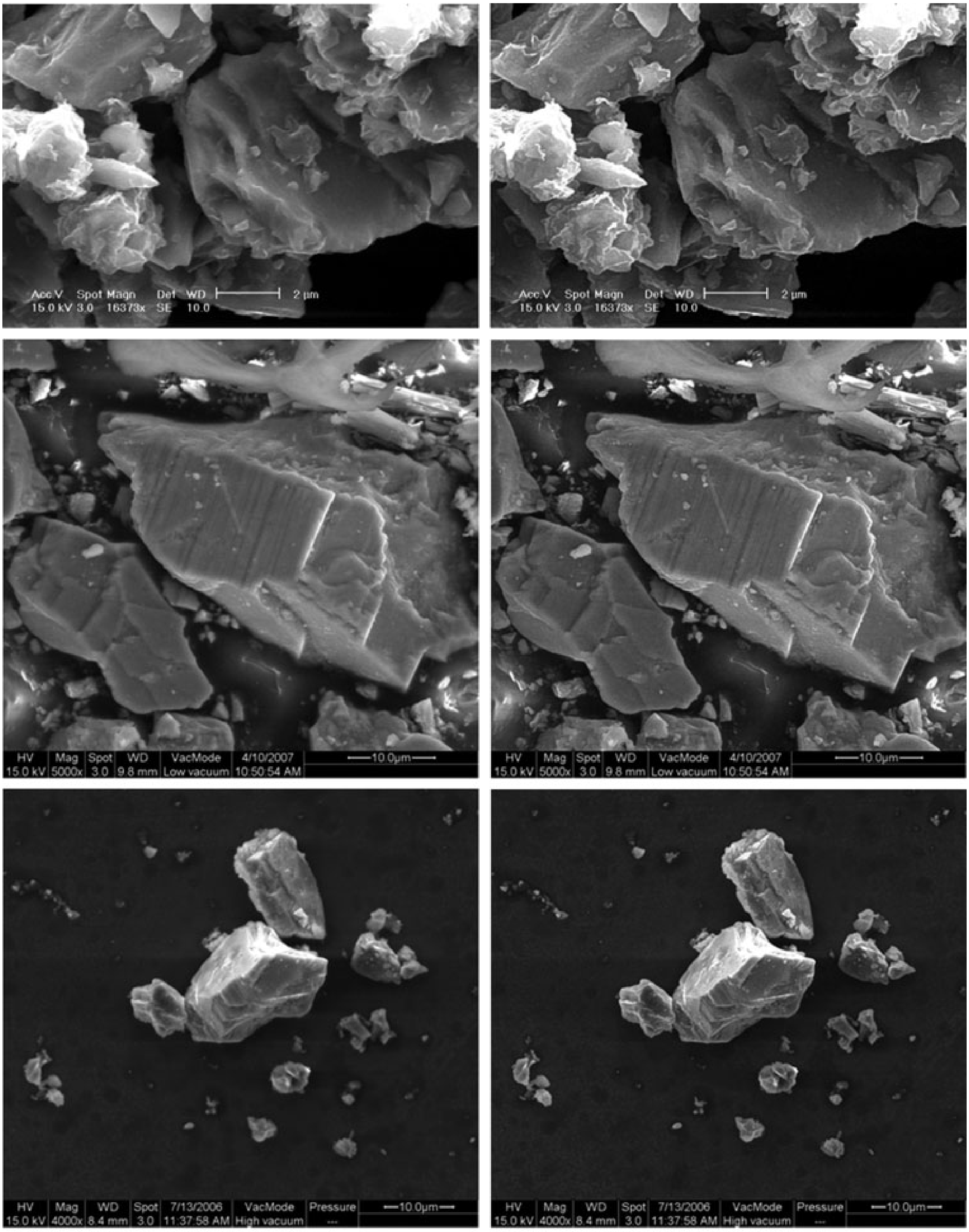
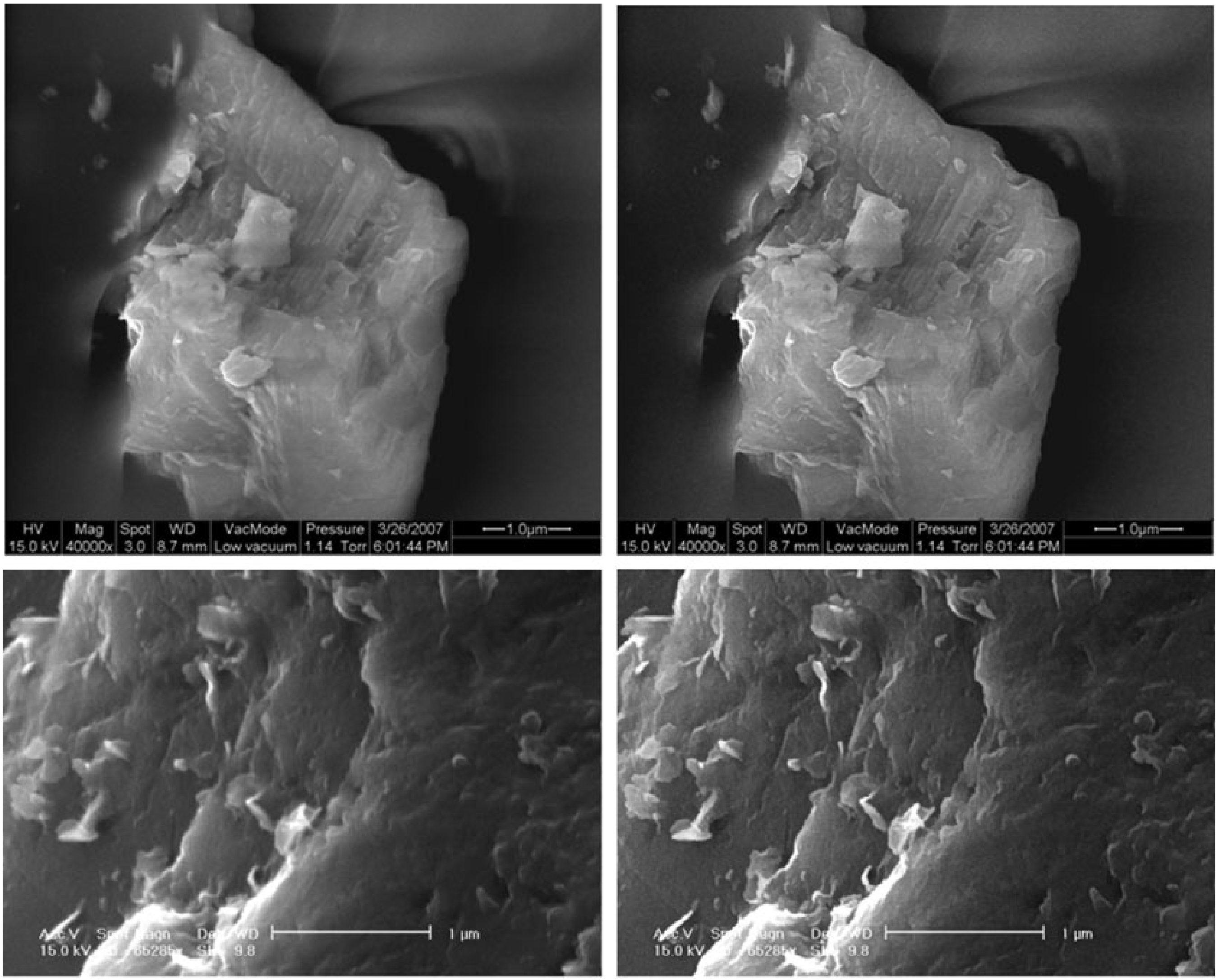


Fig. 6. Comparison result 4. (a) Original image, (b) enhanced result by the proposed algorithm, (c) enhanced result by MSM, (d) enhanced result by WF, (e) enhanced result by HE and (f) enhanced result by CLAHE.



(a) Original images (b) Enhanced results

Fig. 7. Enhanced examples of some mineral images. (a) Original images and (b) enhanced results.



(a) Original images (b) Enhanced results

Fig. 8. Enhanced examples of some more mineral images. (a) Original images and (b) enhanced results.

Calculate the toggle contrast operator result of this scale TCO_s;

End;

Calculate the final enhanced result of f using all scales of toggle contrast operator: $R = \frac{\sum_{s=0}^{n-1} TCO_s}{n}$.

Experimental results

To verify the efficiency of the proposed algorithm for mineral image enhancement, different images from four mineral image sets including more than 90 mineral images are used in this paper. And, some results are shown later. Moreover, to show the efficiency of the proposed algorithm, some other widely used algorithms including multiscale morphological approach (MSM), wallis filter algorithm (WF), histogram equalization algorithm (HE) and contrast limited adaptive histogram equalization algorithm (CLAHE) are also applied on the original images, and the results are used to do the comparison. In mineral image, the grey distribution of the image is very important information for mineral analysis. So, it would be good to both enhance the image and well keep the grey distribution of the enhanced image.

Fig. 3 is an example of mineral image enhancement. MSM produces many noises and many regions are over enhanced. HE and CLAHE over enhance many bright regions of the original image. Then, the over enhanced regions will affect the mineral analysis. WF makes the grey values of different regions very similar, which may change the mineral property demonstrated by the original image and affect the application of the enhanced result. The proposed algorithm keeps the grey distribution of the original image and the enhanced image is

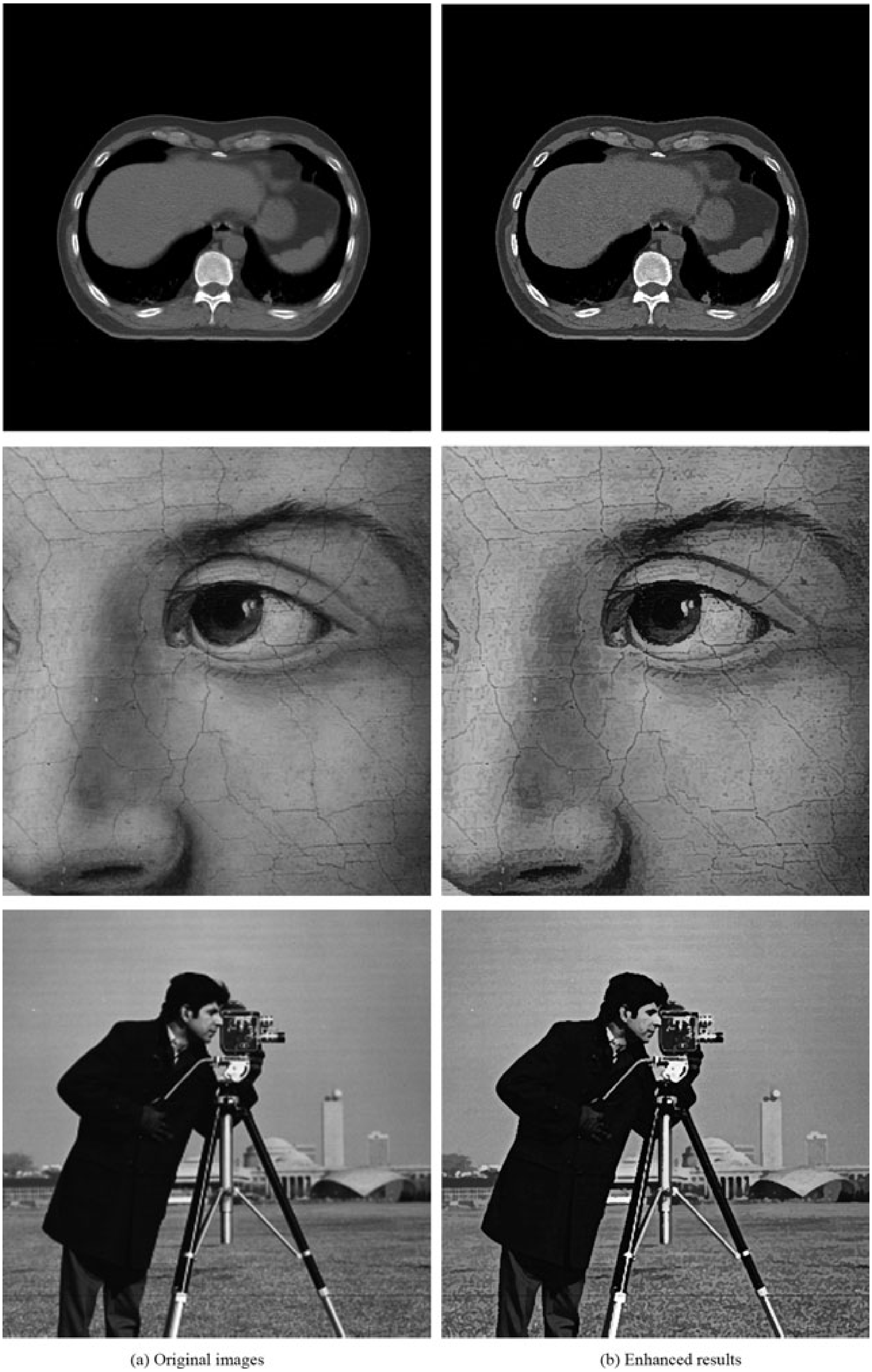


Fig. 9. Enhanced examples of other types of images. (a) Original images and (b) enhanced results.

very clearer than the original image, which not only maintains the mineral property demonstrated by the original image, but also benefits the mineral analysis. This would be very useful for the applications of the enhanced image.

Fig. 4 is another example of mineral images. There are many bright image regions in the original image. And, there are many unclear edge detail regions. HE and CLAHE over enhance the bright regions and MSM over enhances the edge regions. These enhanced results are even worse than the original image. WF changes the grey distributions of the original image. The proposed algorithm keeps the information of the original image very well and makes the enhanced image clearer than the original image, which performs better than other algorithms.

Figs 5 and 6 are another two comparison results. HE and CLAHE over enhance many regions, which makes the over enhanced images even worse than the original images. MSM and WF could enhance the original images, but MSM produces many noises and WF changes the grey distribution of the original images, which may affect the applications of the enhanced images for mineral analysis. The enhanced results of the proposed algorithm are clearer than the original images and the properties of image regions of the original images are also well preserved. Therefore, the proposed algorithm achieves the best performance.

Some other experimental results by using our algorithm are demonstrated in Figs 7 and 8. The original images are not very clear. Especially, the edge regions of image regions are not very clear. However, after our algorithm, the enhanced results become brighter and the edge regions are very clearer than the original images, which would be more benefit for mineral analysis.

Moreover, the proposed algorithm could be also used to enhance other types of images, such as medical images, visual images, and so on. And, some examples are demonstrated in Fig. 9. These results show that the blurred image details in the original images could be enhanced and these regions are also clearer than the original images. Therefore, the enhanced images are efficient.

All of these experimental results show that, the image details and edge regions of mineral images become clearer than the original images after the proposed algorithm, which makes the enhanced images more useful for mineral applications. Moreover, the proposed algorithm could be also used in other types of images. Therefore, the proposed algorithm could be an efficient algorithm for image enhancement.

Conclusions

Blurred mineral image may be not very efficient for mineral applications. Morphological toggle contrast operator is one important operator for image enhancement. But, the size of the used structuring element in toggle contrast operation may affect the performance of toggle contrast operator. To be efficient for mineral image enhancement, a multiscale toggle contrast operator based algorithm is proposed in this paper for mineral image enhancement. Multiscale toggle contrast operator using structuring elements with the same shape and increasing sizes enhances the original image at different scales. Then, the enhanced results at different scales are combined to form the final enhancement result. Because multiscale structuring elements are used, the effect of structuring element is suppressed. Experimental results on different mineral images show that the proposed algorithm is very efficient for mineral image enhancement. More importantly, the algorithm could be also used to enhance other types of images, such as visual image, medical image, and so on.

Acknowledgements

The authors thank the anonymous reviewers for their very constructive comments. The authors are grateful to Dr. Yan Li at Peking University, Beijing, China, for providing the mineral image sets. This work is partly supported by the National Natural Science Foundation of China (60902056) and Natural Science Foundation of Beijing.

References

Agaian, S., Panetta, K. & Grigoryan, A. (2001) Transform-based image enhancement algorithms with performance measure. *IEEE Trans. Image Process.* **10**, 267–382.

Bai, X. & Zhou, F. (2010) Analysis of new top-hat transformation and the application for infrared dim small target detection. *Pattern Recogn.* **43**, 2145–2156.

Boomgaard, R. (1992) *Mathematical morphology: extensions towards computer vision.* PhD Thesis, University of Amsterdam, The Netherlands.

Carter, D., Scully A., Hatton, P., Davies R. & Aaron J. (2000) Cryopreservation and image enhancement of juvenile and adult dentine mineral. *Histochem. J.* **32**, 253–261.

Cheng, Q., Jing, L. & Panahi A. (2006) Principal component analysis with optimum order sample correlation coefficient for image enhancement. *Int. J. Remote Sens.* **27**, 3387–3401.

De, I., Chanda, B. & Chattopadhyay, B. (2006) Enhancing effective depth-of-field by image fusion using mathematical morphology. *Image Vision Comput.* **24**, 1278–1287.

Dorini, L. & Leite, N. (2007a) A scale-space toggle operator for morphological segmentation. *Proceedings of the 8th International Symposium on Mathematical Morphology*, Rio de Janeiro, Brazil, October 10–13, pp. 101–112.

Dorini, L., Minetto, R. & Leite, N. (2007b) White blood cell segmentation using morphological operators and scale-space analysis. *Proceedings of the XX Brazilian Symposium on Computer Graphics and Image Processing*, Belo Horizonte, Minas Gerais, Brazil, October 7–10, pp. 294–301.

Farbiz, F., Bager, M., Motamedi, S. & Hagan, M. (2000) A new fuzzy logic filter for image enhancement. *IEEE Trans. Syst. Man. Cy. B* **30**, 110–119.

Gilboa, G., Sochen, N. & Zeevi, Y. (2004) Image enhancement and denoising by complex diffusion processes. *IEEE Trans. Pattern Anal.* **26**, 1020–1036.

Jackway, P. (1992) Scale-space properties of the multiscale morphological closing-opening filter. *Proceedings of International Conference on Image Processing*, Singapore, September, pp. 278–281.

Jackway, P. (1995) On dimensionality in multiscale morphological scale-space with elliptical poweroid structuring functions. *J. Vis. Commun. Image R.* **6**, 189–195.

Jackway, P. (1998) On the scale-space theorem of Chen and Yan. *IEEE Trans. Pattern Anal.* **20**, 351–352.

Jackway, P. & Deriche, M. (1996) Scale-space properties of the multiscale morphological dilation-erosion. *IEEE Trans. Pattern Anal.* **18**, 38–51.

Jackway, P., Boles, W. & Deriche, M. (1994) Morphological scale space fingerprints and their use in object recognition in range image, *Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing*, Adelaide, April 19–22, pp. V5–V8.

Jalba, A., Wilkinson, M. & Roerdink, J. (2004) Morphological hat-transform scale spaces and their use in pattern classification. *Pattern Recogn.* **37**, 901–915.

Kramer, H. & Bruckner, J. (1975) Iterations of non-linear transformations for enhancement on digital images. *Pattern Recogn.* **7**, 53–58.

Lester, J., Brenner, J. & Selles, W. (1980) Local transforms for biomedical image analysis. *Comput. Graph. Image Process.* **13**, 17–30.

Lindeberg, T. (1994) *Scale-Space Theory in Computer Vision*. Kluwer Academic, Dordrecht, The Netherlands.

Maragos, P. (2005) Morphological filtering for image enhancement and feature detection. *The Image and Video Processing Handbook* (ed. by A. C. Bovik), 2nd edn, pp. 135–156. Elsevier Academic Press, Amsterdam, The Netherlands.

Matheron, G. (1975) *Random Sets and Integral Geometry*. Wiley, New York, USA.

Mencattini, A., Salmeri, M., Lojacono, R., Frigerio, M. & Caselli, F. (2008) Mammographic images enhancement and denoising for breast cancer detection using dyadic wavelet processing. *IEEE Trans. Instrum. Meas.* **57**, 1422–1430.

Meyer, F. & Serra, J. (1989) Contrasts and activity lattice. *Signal Process.* **16**, 303–317.

Oliveira, M. & Leite, N. (2008) A multiscale directional operator and morphological tools for reconnecting broken ridges in fingerprint images. *Pattern Recogn.* **41**, 367–377.

Rivas-Araiza, E., Mendiola-Santibanez, J. & Herrera-Ruiz, G. (2008) Morphological multiscale fingerprints from connected transformations. *Signal Process.* **88**, 1125–1133.

Serra, J. (1988) Toggle mappings. *Technical Report N-18/88/MM*, Centre de Morphologie Mathematique, ENSMP, Fontainebleau, France.

Serra, J. & Vincent, L. (1992) An overview of morphological filtering. *Circ. Syst. Signal. Process.* **11**, 47–108.

Schavemaker, J., Reinders, M., Gerbrands, J. & Backer, E. (2000) Image sharpening by morphological filtering. *Pattern Recogn.* **33**, 997–1012.

Schultze-Lam, S., Harauz, G. & Beveridge, T. (1992) Participation of a cyanobacterial S layer in fine-grain mineral formation. *J. Bacteriol.* **174**, 7971–7981.

Soille, P. (2003) *Morphological Image Analysis-Principle and Applications*. Springer, Germany.

Terol-Villalobos, I. (1996) Nonincreasing filters using morphological gradient criteria. *Opt. Eng.* **35**, 3172–3182.

Terol-Villalobos, I. & Cruz-Mandujano, J. (1998) Contrast enhancement and image segmentation using a class of morphological nonincreasing filters. *J. Electron. Imaging* **7**, 641–654.

Wan, Y. & Shi, D. (2007) Joint exact histogram specification and image enhancement through the wavelet transform. *IEEE Trans. Image Process.* **16**, 2245–2250.

Witkin, A. (1983) Scale space filtering, *Proceedings of the 8th International Joint Conference on Artificial Intelligence*, Morgan-Kaufmann, Palo Alto, CA, USA, pp. 1019–1022.