MANIPAL ACADEMY of HIGHER EDUCATION
(Institution of Eminence Deemed to be University)

MDS5204/ Segment 1

CLASSIFICATION OF STOCHASTIC PROCESSES

Table of Contents

Introduction

The classification of Stochastic Process has been done based on Parameter space and State space. Parameter space and State space has been further classified based on the discrete and continuous property of time and state space.

Learning Objectives

At the end of this topic, you will be able to:

- Explain the discrete and continuous state space and parameter space
- Classify the types of stochastic processes

1. Stochastic Processes

A stochastic process is a family or set of ordered random variables. The outcome of a deterministic system is fixed, while the outcome of a stochastic process is uncertain. The dynamics of stochastic processes are described by random variables and probability distributions.

The order of random variables is indicated by indexing each random variable in the family by a subscript. The ordering is a result of the random variables being observed over time. Referring to the examples of the previous session, the maximum number of outcomes obtained while rolling a dice was the random variable where indexing was done over the number of throws of the dice. X_t is the notation for the random variable that models the value of the stochastic process at time *t*. The random variables in the set can be dependent or independent of each other, reflecting the nature of the process being modelled. In all stochastic modelling, a sample of data is collected from the process being modelled.

2. Classification of Stochastic Process

A stochastic process can be defined as a family of random variables, $\{X(t): t \in T\}$, where t denotes time, that is, at every time t in the set T, a random number $X(t)$ is observed. The stochastic process is classified based on parameter space and state space. Each of these is further classified based on their discreet and continuous nature.

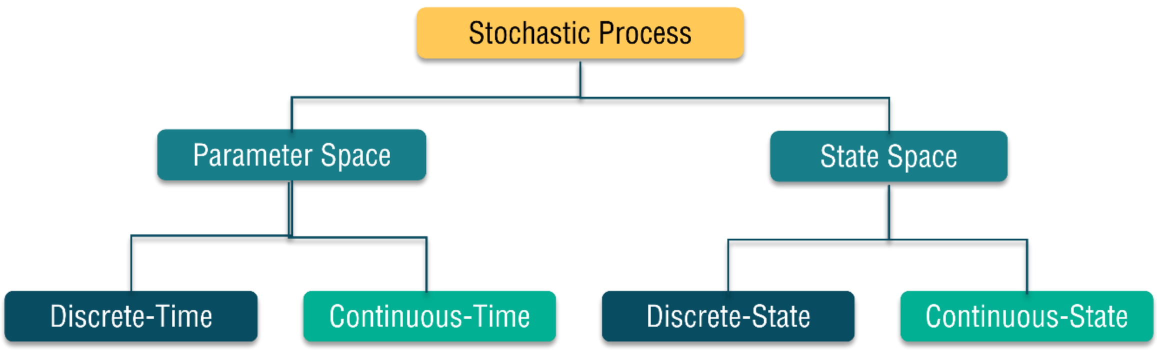

Figure 1: Classification of Stochastic Process

2.1 Parameter Space

- **Discrete-Time Process**
  $\{ X(t): t \in T\}$ is a discrete-time process if the set T is finite or countable. Here T is the parameter space and is discrete in nature. In practice, this means $T = \{0, 1, 2, 3, \dots\}$. In other words, in a discrete-time process $\{X(0), X(1), X(2), \dots\}$ a random variable is associated with every time $0,1,2,3, \dots$
- **Continuous-Time Process**
  $\{X(t): t \in T\}$ is a continuous-time process if the set T is not finite or countable. In practice, this means $T = [0, \infty), \, or \, T = [0, K)$ for some K. Thus, a continuous-time process is $\{X(t): t \in T\}$ has a random number $X(t)$ associated with every instant in time. Note that, $X(t)$ need not change at every instant in time, but it is allowed to change at any time. That is, $X(t)$ does not just change at $t = 0, 1, 2, 3, \dots$, as seen in discrete-time process.

2.2 State Space

The State Space (S) is the set of real values that $X(t)$ can take (a random variable can take). This collection of values is known as the state space. While tossing a coin, the state spaces are head and tail and while rolling a dice, state spaces are all the possible outcomes - 1, 2, 3, 4, 5 and 6. Every $X(t)$ takes a value in $\mathbb{R}$, but S will often be a smaller set $S \subseteq \mathbb{R}$. For example, if $X(t)$ is the outcome of a coin tossed at time t, then the state space is $S = \{0,1\}$. This $\{0,1\}$ represents the real number values for the sample space of random variable X. The stochastic process can be in the state space S (the set of states). The state space (S) is discrete if it is finite or countable. Otherwise, it is continuous.

2.3 Discrete and Continuous Property

The classification of a stochastic process depends on the two possible values of the state space and the parameter space which is discrete and continuous property.

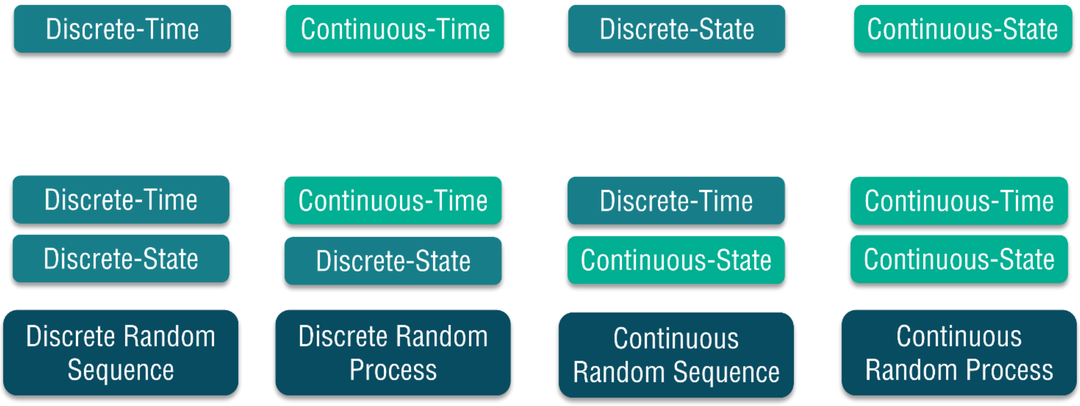

Figure 2: Classification of Stochastic Process

Following are the classification of the Stochastic process:

1. **Discrete random sequence:** It is the stochastic process in which both state and parameter space is discrete in nature, i.e., Discrete-Time and Discrete-State processes. Hence, another name for Discrete Random Sequence is Discrete-Time and Discrete-State process.

   Let us look at a few examples of discrete random sequences.

   Example 1: Let the number of individuals in a population at the end of year t be $X(t)$. Here, the random variable is number of individuals in a population at the end of year t, and t is the indexing parameter. Then the stochastic process $\{X(t): t \in T\}$ is a discreet random sequence where, time $T = \{0,1,2,3, \dots\}$ and the state space $S = \{0,1,2,3, \dots\}$. T is a set of values that are discrete in nature and the state space is also discrete in nature.

   Example 2: Suppose, a motor insurance company reviews the status of its customers yearly, then there are three levels of discounts given $(0\%,\ 10\%,\ 25\%)$ depending on the accident record of the driver. Let $X(t)$ be the percentage of discount at the end of year t. The stochastic process is given by $\{X(t): t \in T\}$, where $T = \{0,1,2,3, \dots\}$ and the state space $S = \{0,10,25\}$. Here, the parameter space that is T and the state space S as both are discrete in nature.

2. **Discrete random process:** When the parameter space is continuous in nature and the state space is discrete in nature, then such stochastic process is known as discrete random process.

   Let us look at a few examples of discrete random processes.

   Example 1: Let the number of incoming calls in an interval $[0, t]$ be $X(t)$, where $X(t)$ is the random variable and $[0, t]$ is the interval over which the number of incoming calls is recorded. Then, the stochastic process given by the collection of these random variables $\{ X(t): t \in T\}$, has $T = \{t: 0 \leq t < \infty\}$ and the state space $S = \{0,1, \dots\}$. Here, parameter space is continuous and state space is discrete in nature.

   Example 2: Let the number of cars parked at a commercial centre in the time interval $[0, t]$ be $X(t)$. Therefore, the stochastic process is given by $\{ X(t): t \in T\}$, where $T = \{t: 0 \leq t < \infty\}$ and the state space $S = \{0,1, \dots\}$. Here, the parameter space is continuous in nature and the state space is discrete in nature because it represents the number of cars.

3. **Continuous random sequence:** When the parameter space is discrete in nature and the state space is continuous in nature, then such stochastic process is known as continuous random sequence.

   Let us look at a few examples of continuous random sequences.

   Example 1: The share price for an asset at the close of trading on day 't' with $T = \{0,1,2, \dots\}$ and $S = \{x: 0 \leq x < \infty\}$. Here, share prices can take any value from the real number system which makes it continuous in nature.

   Example 2: Let $X(n)$ represent the maximum temperature in a certain place on the earth's surface on the nth day, where $T = \{1,2,3, \dots\}$ and say, $S = [-90,60]$ degree celsius. Here, the state space is continuous in nature, whereas the parameter space can only take the discrete values.

4. **Continuous random process:** When the state space and parameter space both are continuous in nature, then the stochastic process is known as continuous random process.

Let us look at a few examples of continuous random sequences.

Example 1: Let $X(t)$ be the intensity of sun in a certain location on the earth's surface at time t, where $T = [0, \infty)$, and say $S = [0{,}120\,000]$lux. Here, both parameter space and state space are continuous in nature. Therefore, this is a Continuous-Time, Continuous-State Stochastic Process or Continuous Random Process.

Example 2: The stochastic process of temperature in a particular city at time t *is* given as $\{X(t),\ t \geq 0\}$. Here, $T = [0, \infty)$, and $S = \{x|-50 < x < 60\}$, where both state space and parameter space are continuous in nature.

Example 3: The stochastic process of water level in a dam observed at time *t* is given as $\{X(t),\ t \geq 0\}$. Here, $T = [0, \infty)$, and $S = [0, \infty)$. In this, both state space and parameter space are continuous in nature because water level and time are both continuous in nature.

3. Summary

In this topic, we discussed:

- A Stochastic Process is defined based on the Parameter and State space.
- Parameter and State Space can be discrete or continuous in nature.
- There are four types of stochastic processes based on the discreet and continuous values of Parameter and the State space.

