Huffman Coding Procedure

INTRODUCTION

- Developed by D.A.Huffman in 1952
- Optimal prefix code
- Lossless Data Compression
- Variable Length Coding
- Widely used in mainstream compression formats such as JPEG,PNG,MP3,ZIP

Data Compression

Given a collection of letters, numbers, or other symbols, find the most efficient method to represent them using a binary code.

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Text!

T e x t !

R

G

B

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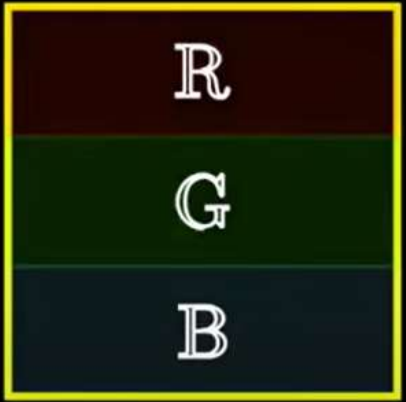
Text!

T e x t !



R

G

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Data Compression

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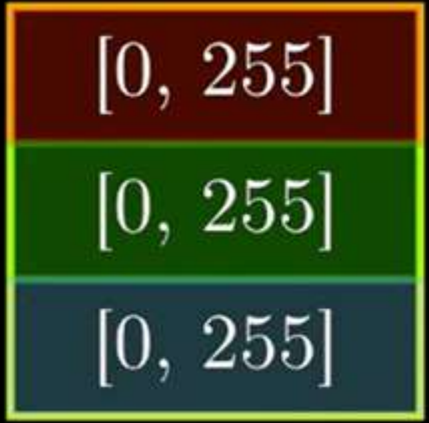


[0, 255]

[0, 255]

[0, 255]

Data Compression

Given a collection of letters, numbers, or other symbols, find the most efficient method to represent them using a binary code.

Text!

T e x t !

0 1 0 1 1 1 1 1

0 1 0 1 1 1 1 0

1 0 0 1 1 0 1 0

Huffman Code

variable length coding

fixed length coding

- Consider the following short text:

Eerie eyes seen near lake.

- Count up the occurrences of **all characters** in the text

a	10
b	101
c	110
c	111

1 0 1 0 1 1 0 1 1 1

Ahmed Fawzy

1 0 1 0 1 1 0 1 1 1

Huffman Code

Eerie eyes seen near lake.

- What **characters** are present?

E e r i space y s n a l k .

Huffman Code

Eerie eyes seen near lake.

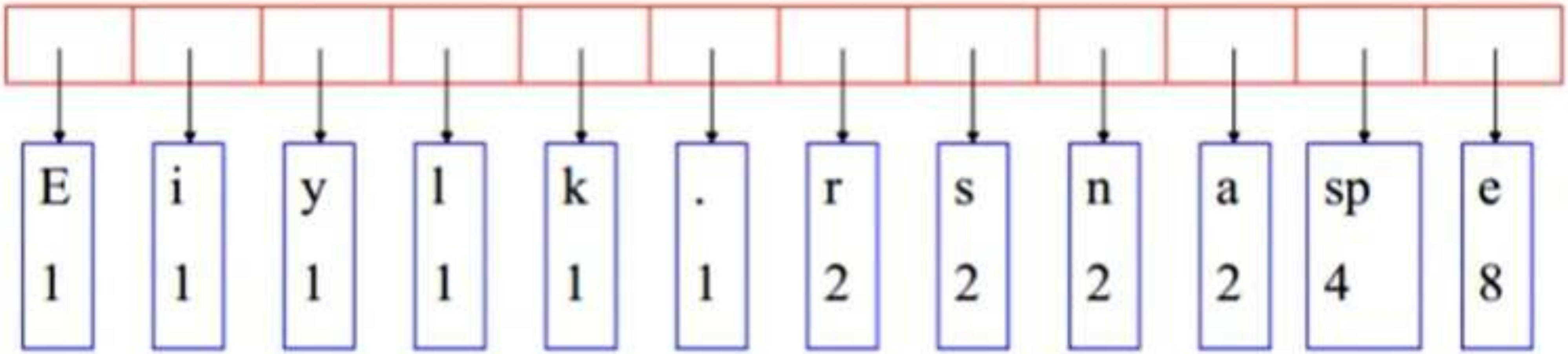
- What is the frequency of each character in the text?

Char	Freq.	Char	Freq.
E	1	*s*	2
e	8	*n*	2
r	2	*a*	2
i	1	*l*	1
space	4	*k*	1
y	1	.	1

Building Tree

- The queue after inserting all nodes

Eerie eyes seen near lake.

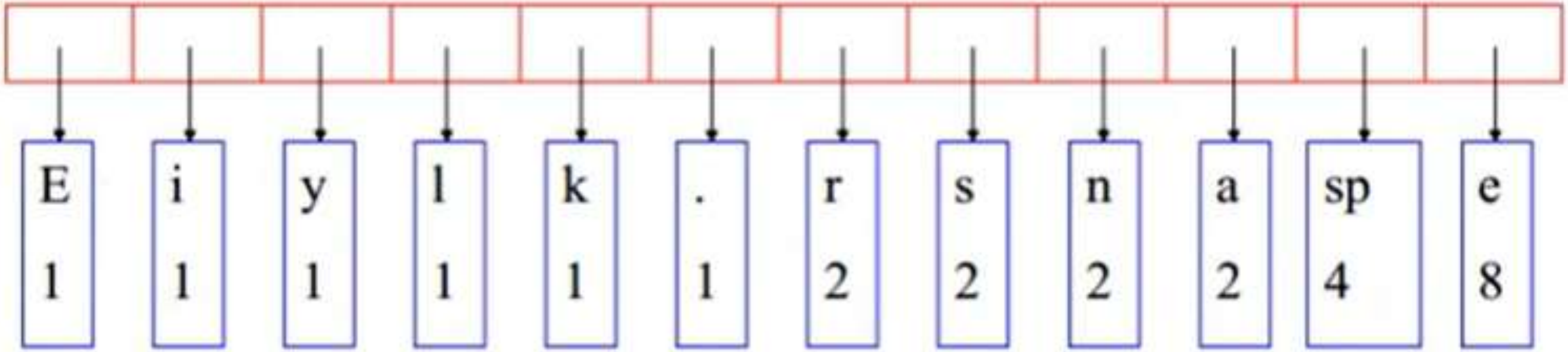


E	i	y	l	k	.	r	s	n	a	sp	e
1	1	1	1	1	1	2	2	2	2	4	8

Building Tree

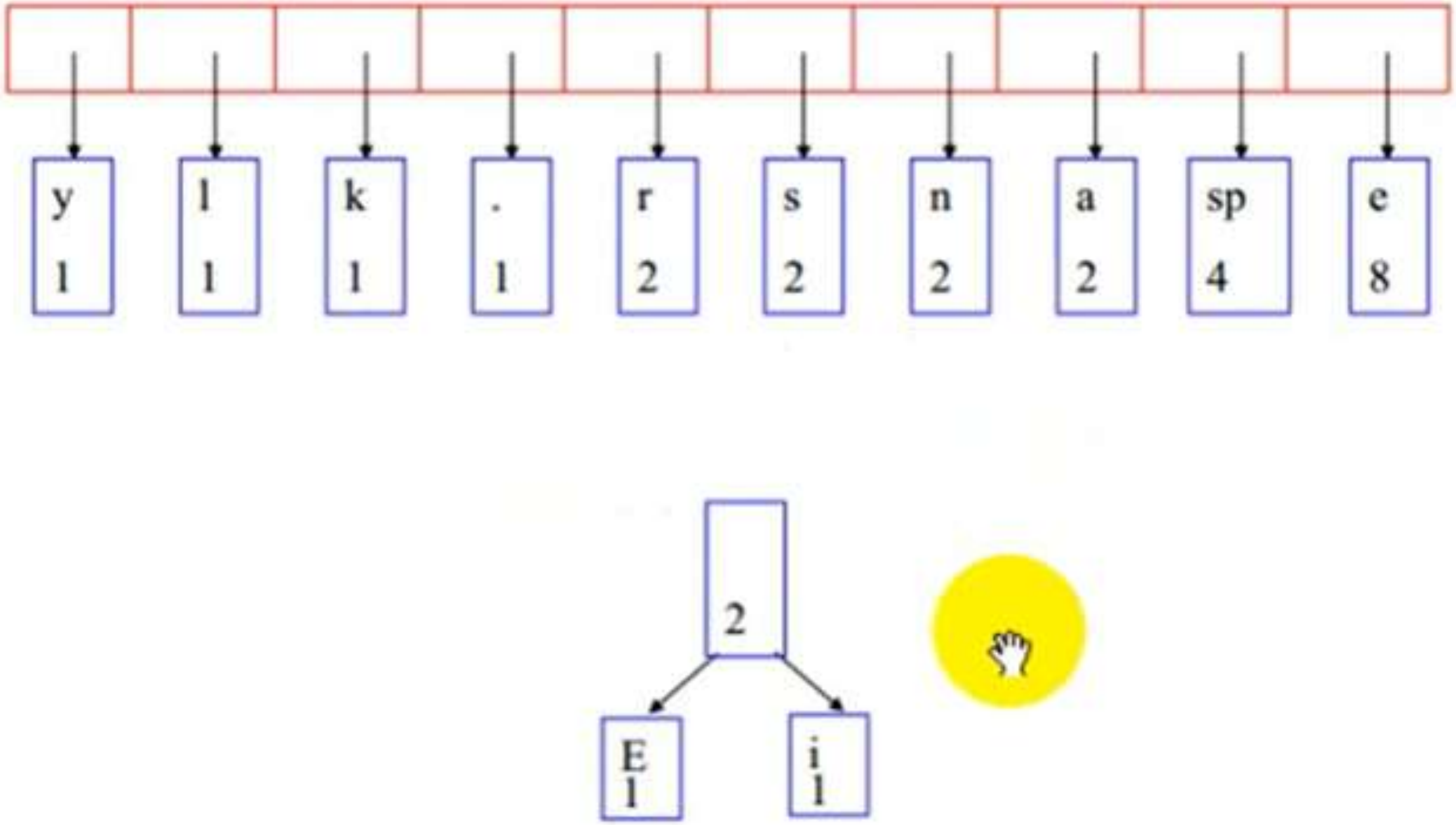
- The queue after inserting all nodes **Eerie eyes seen near lake.**

w:2

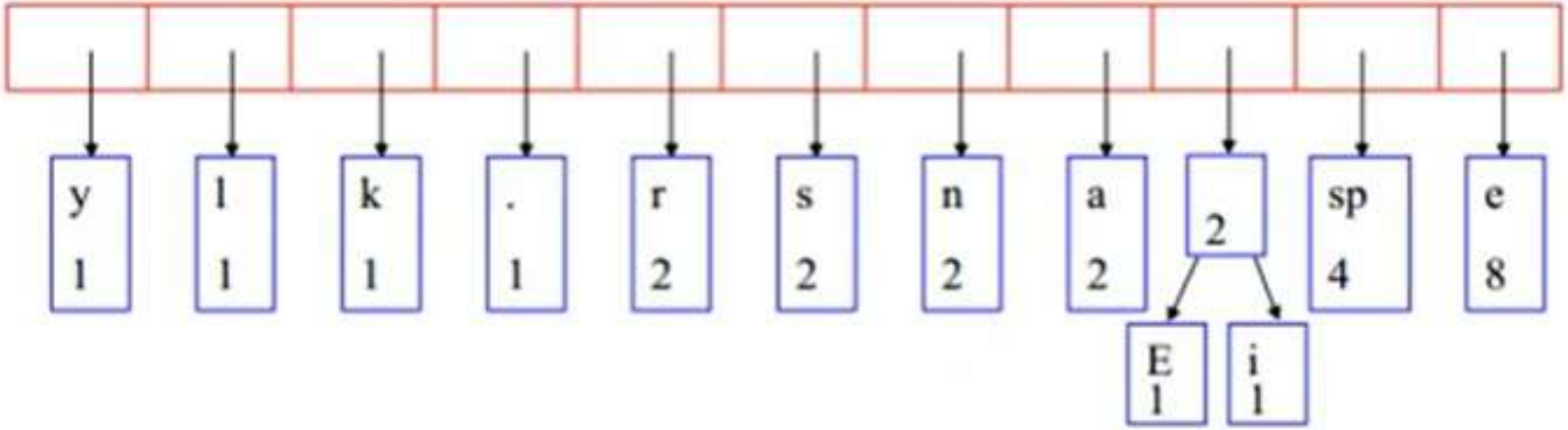


E	i	y	l	k	.	r	s	n	a	sp	e
1	1	1	1	1	1	2	2	2	2	4	8

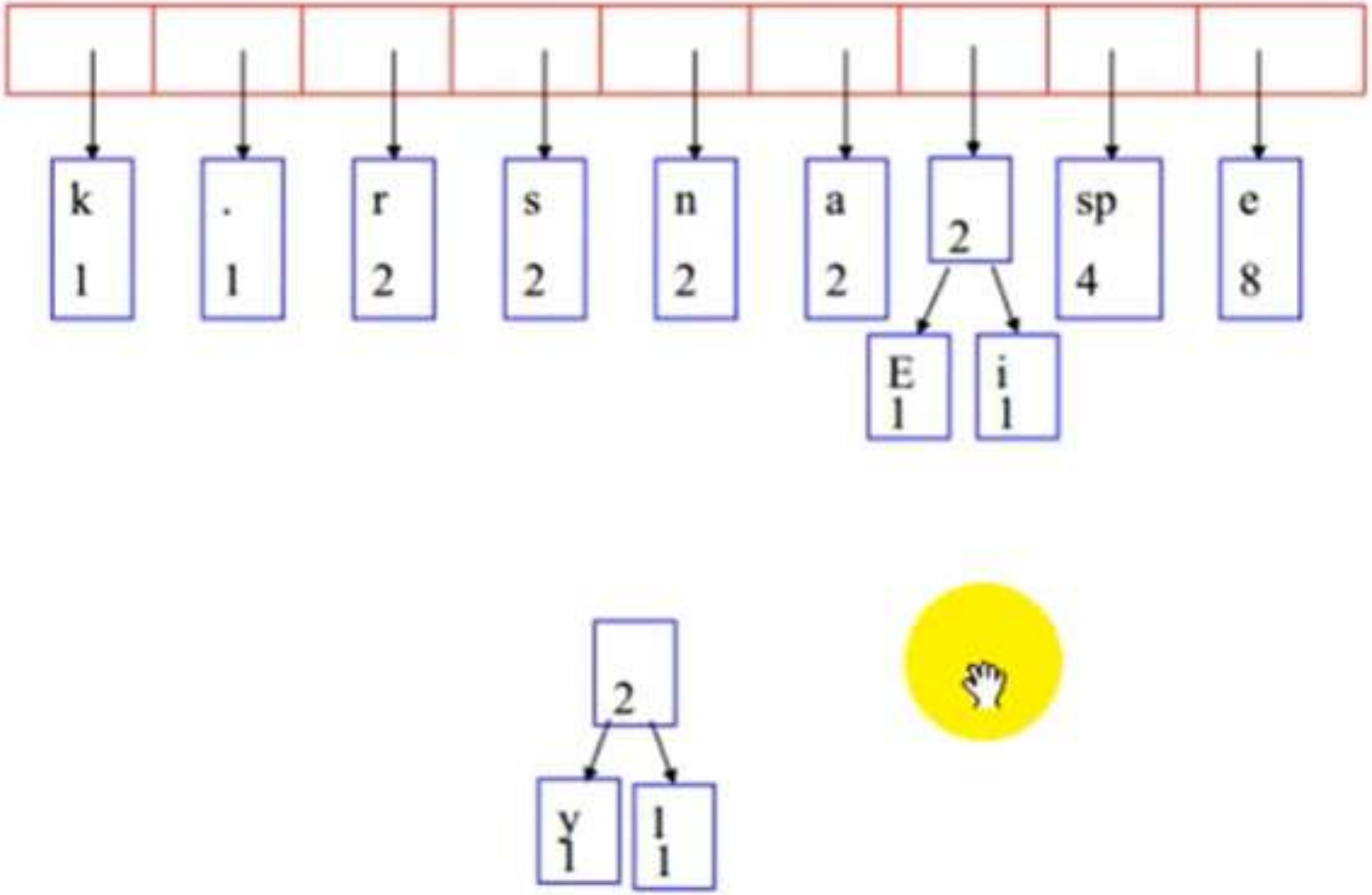
Building Tree



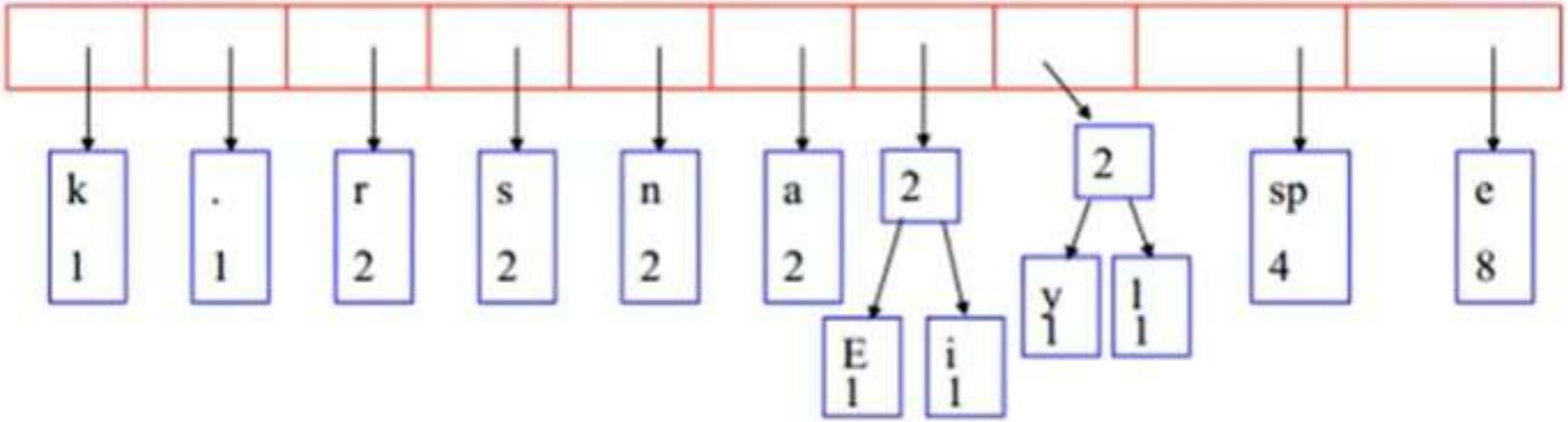
Building Tree



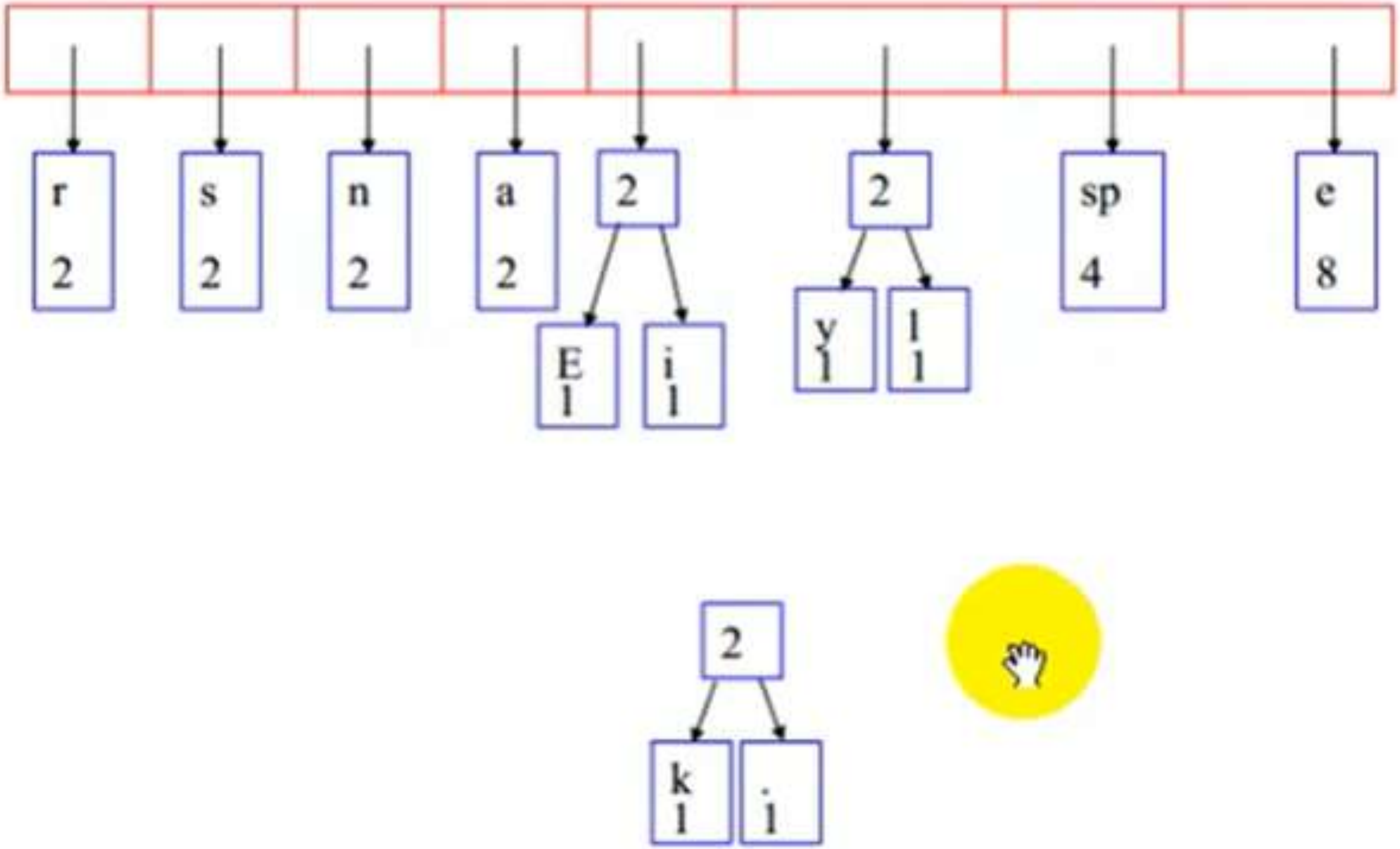
Building Tree



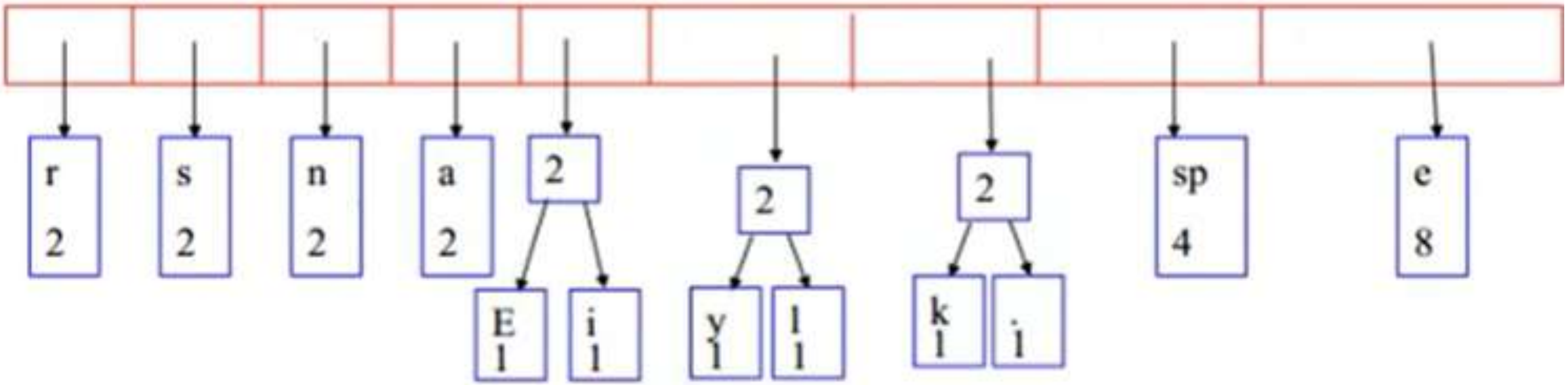
Building Tree



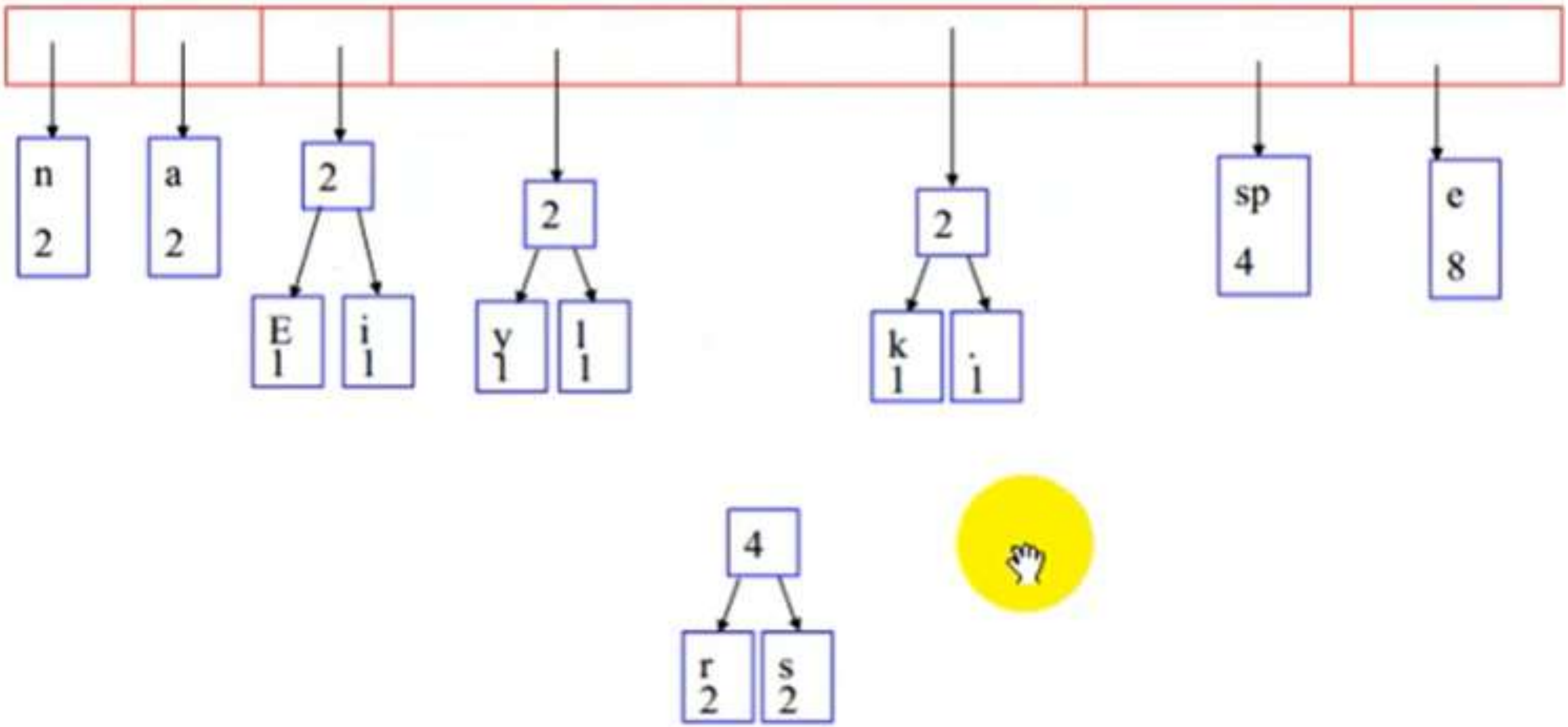
Building Tree



Building Tree



Building Tree



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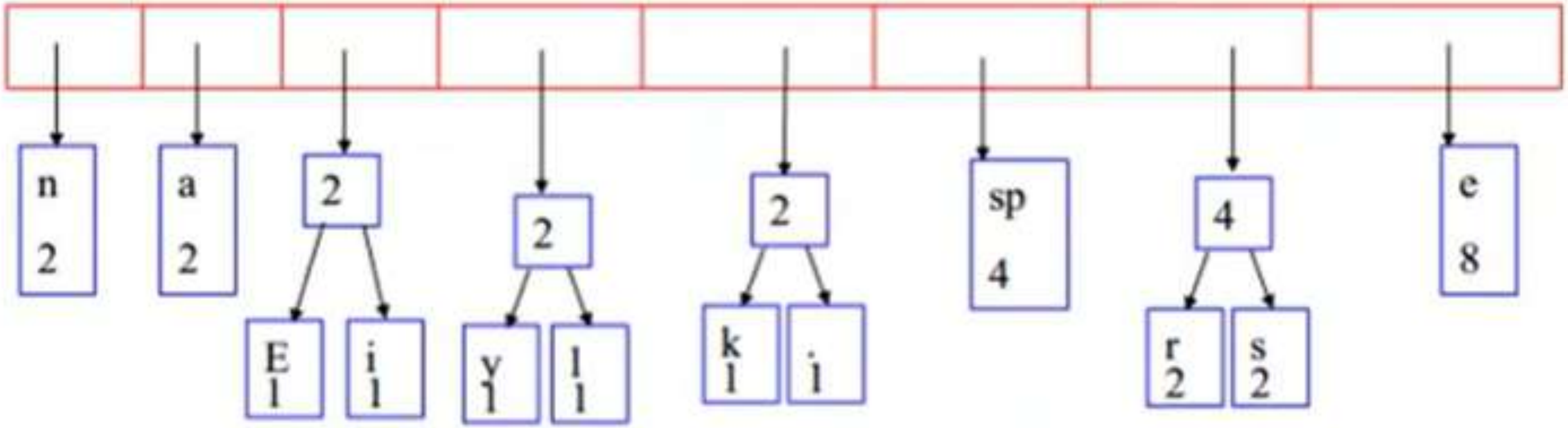

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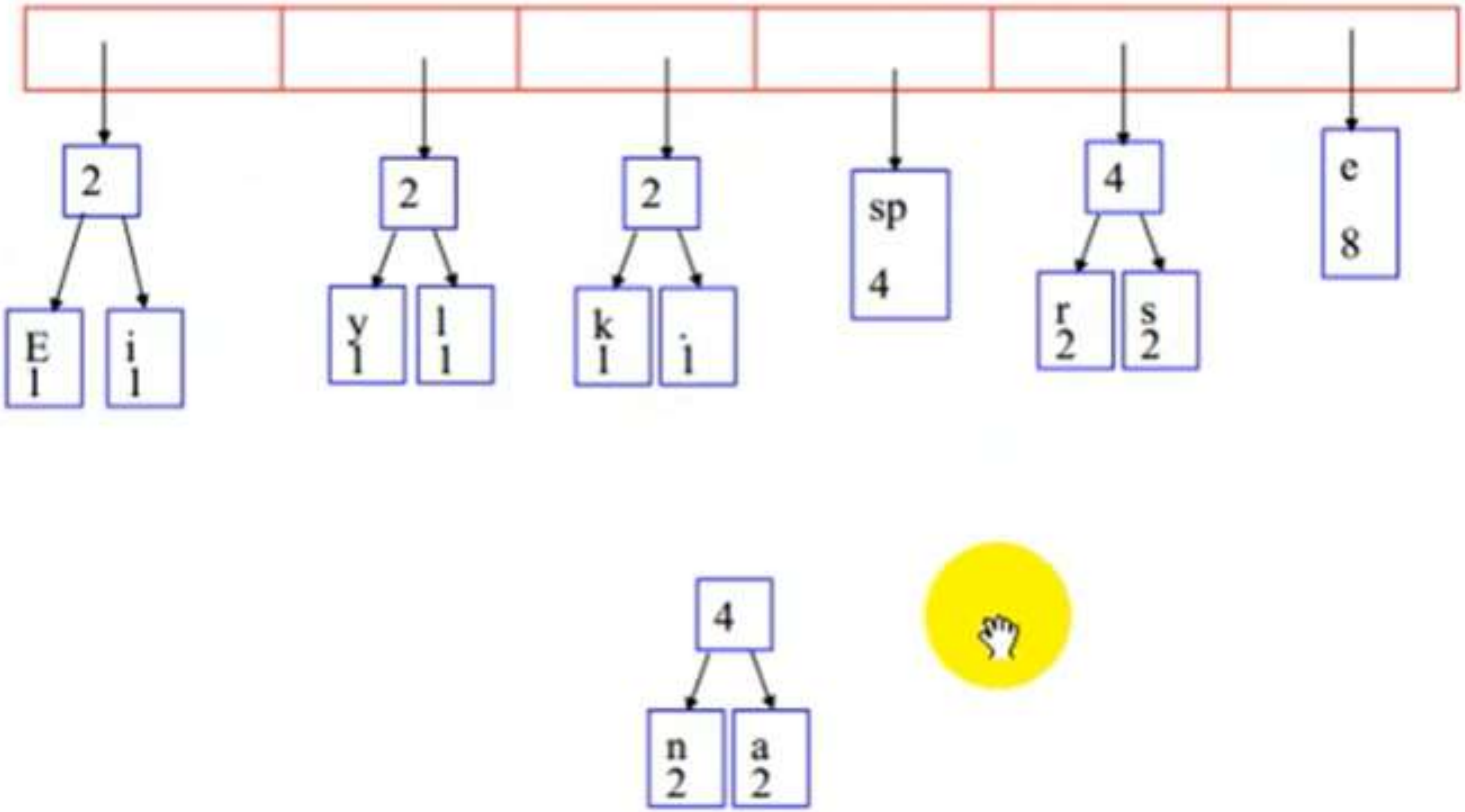
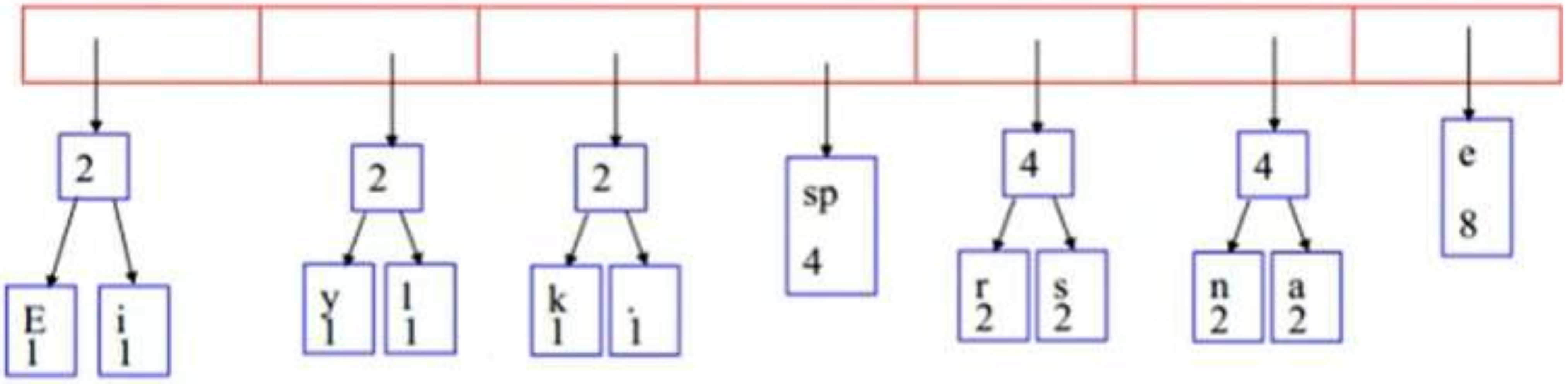
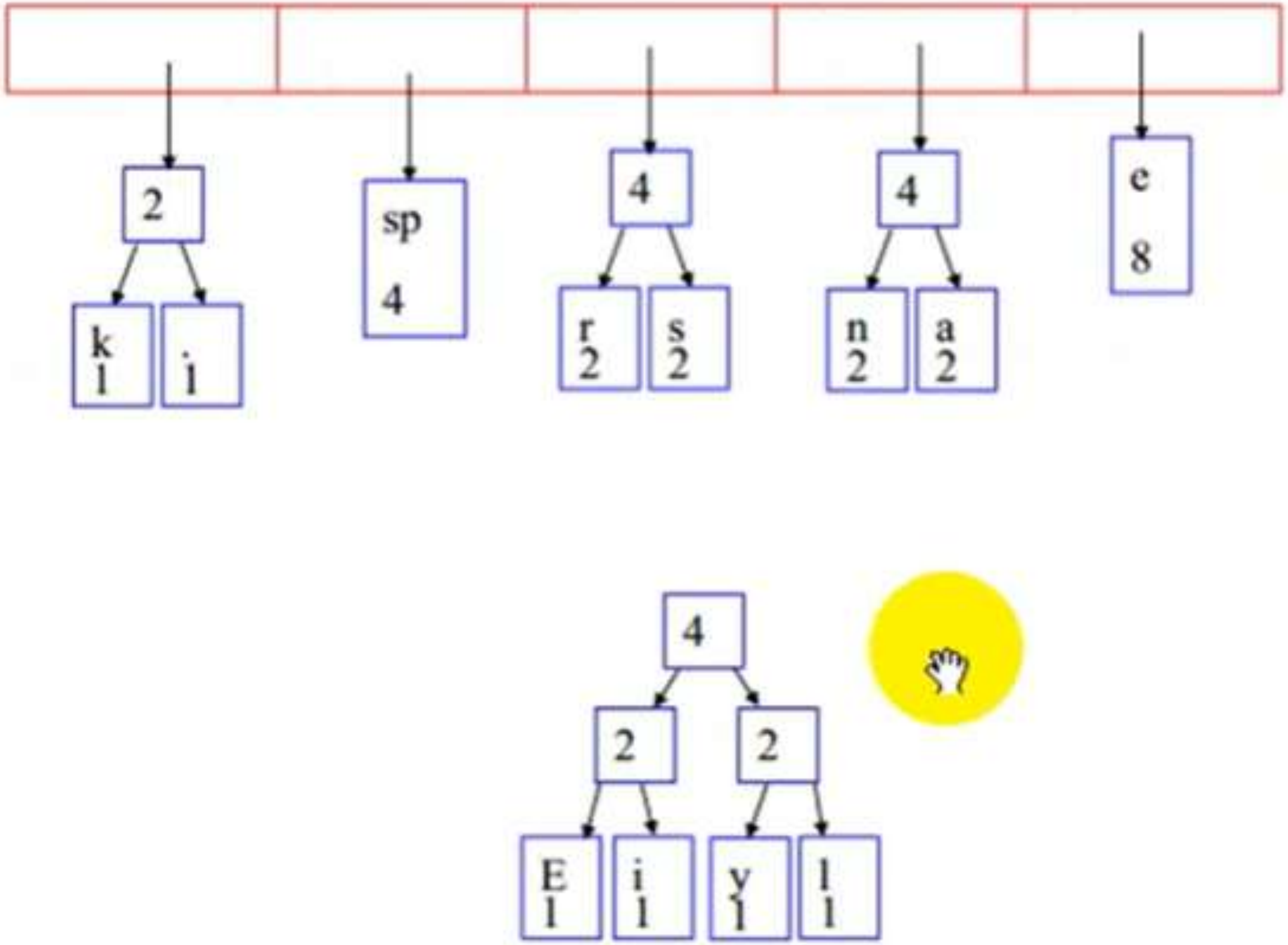


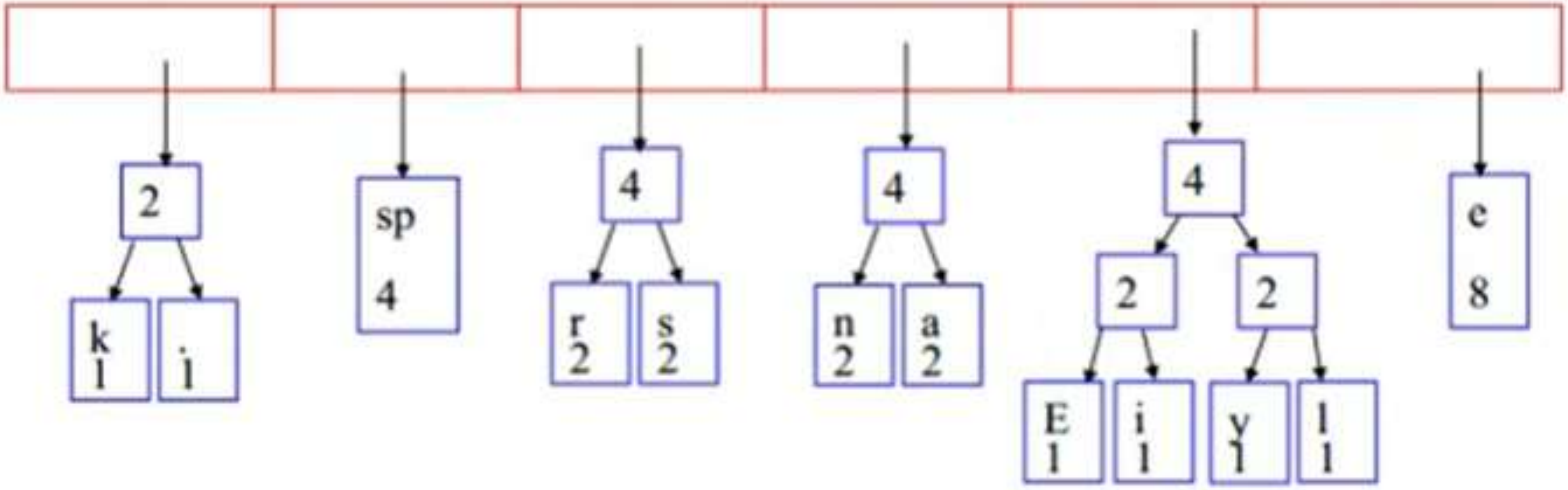
Building Tree



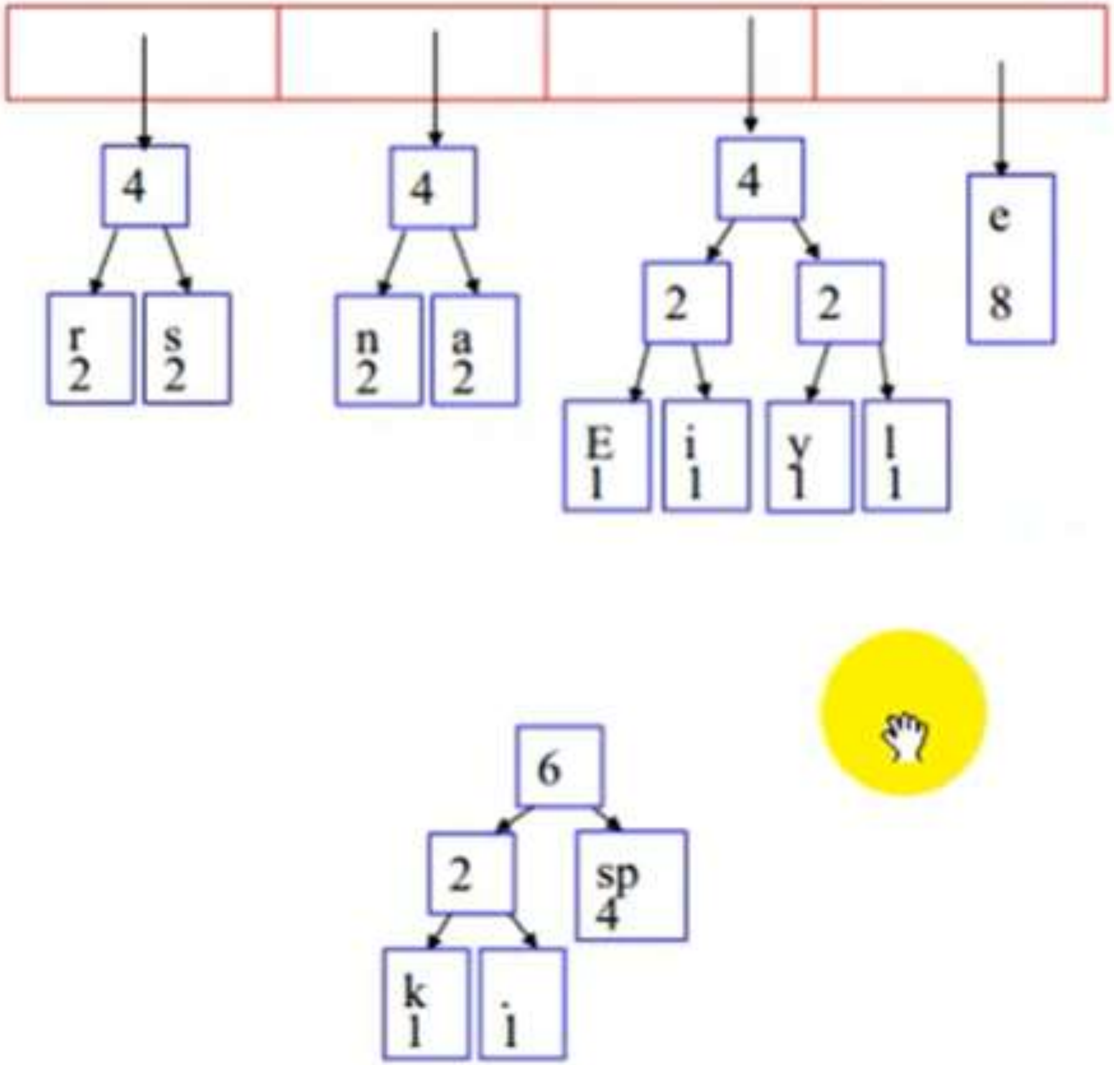
Building Tree



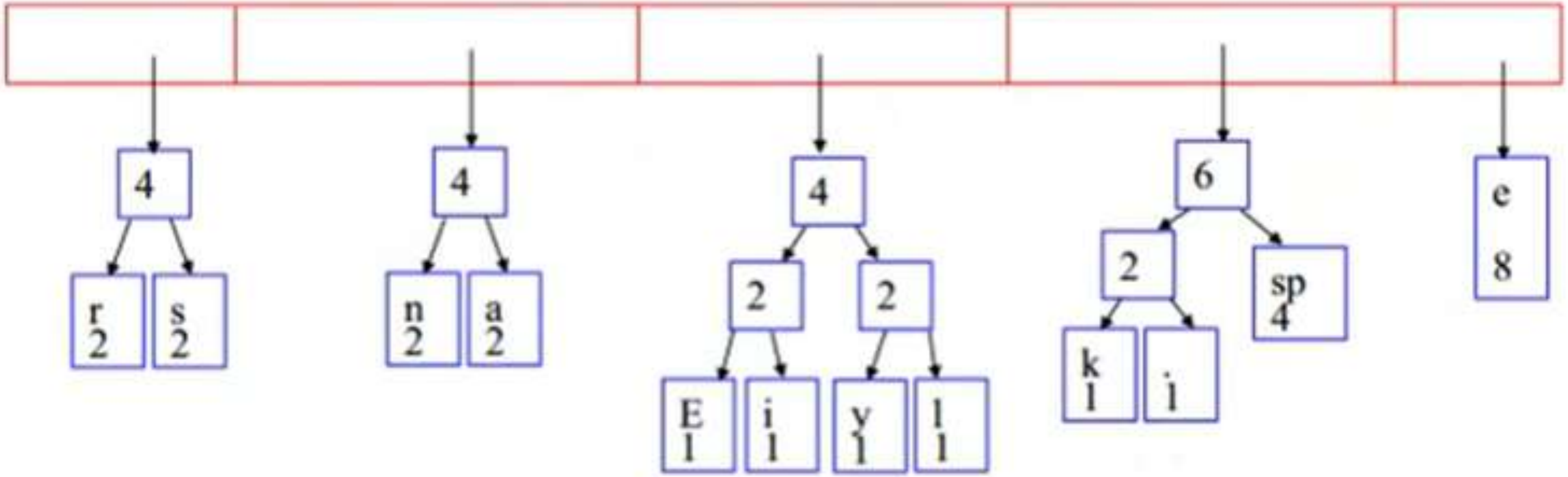
Building Tree



Building Tree



Building Tree



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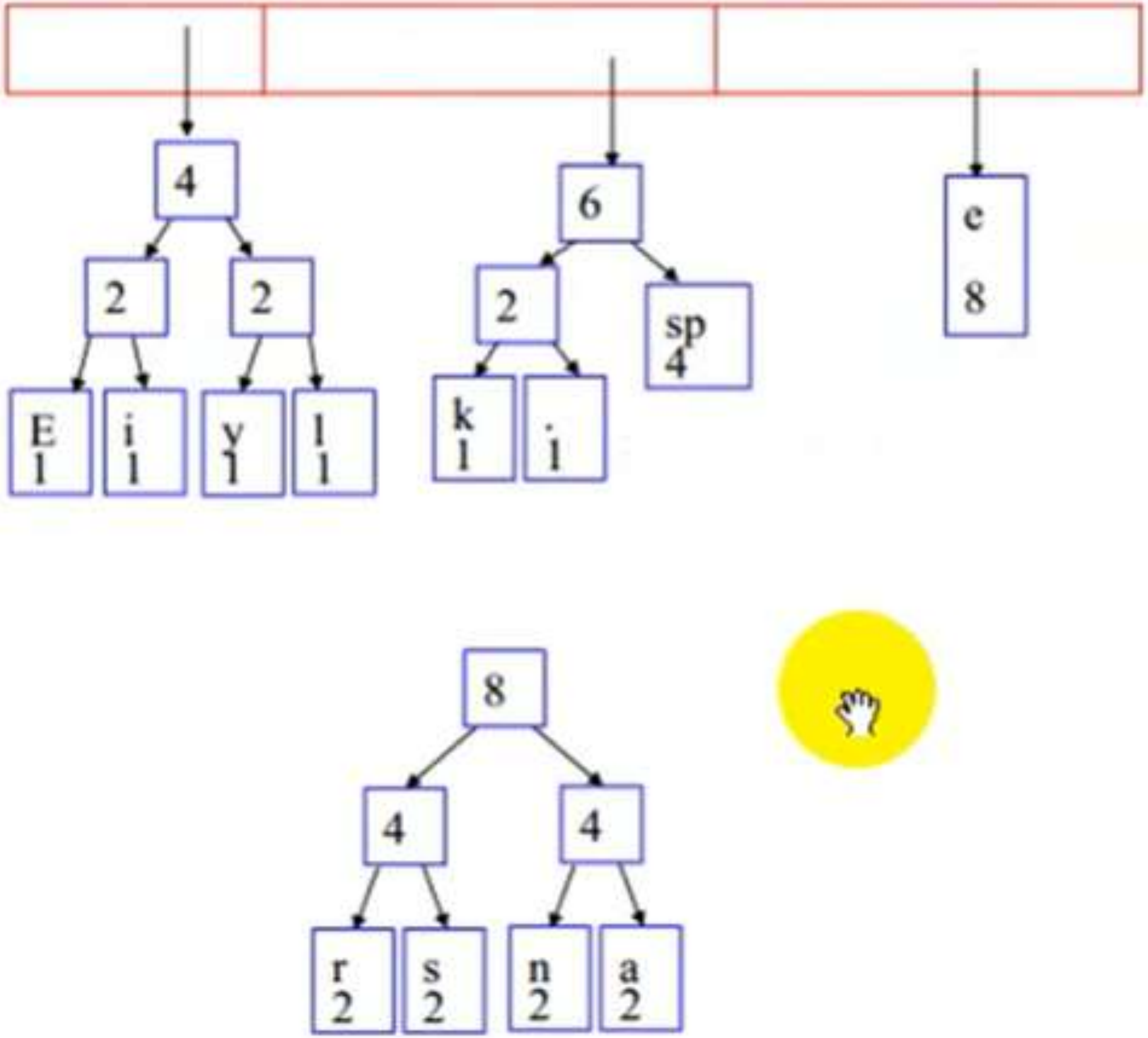
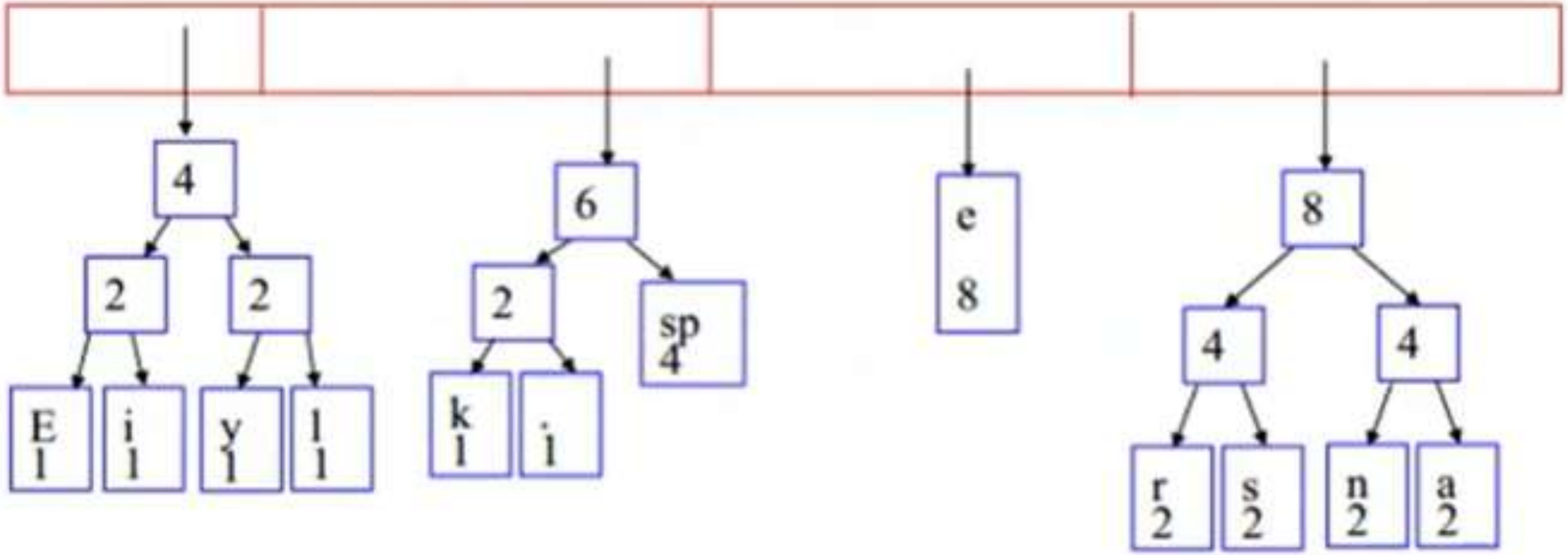


Building Tree



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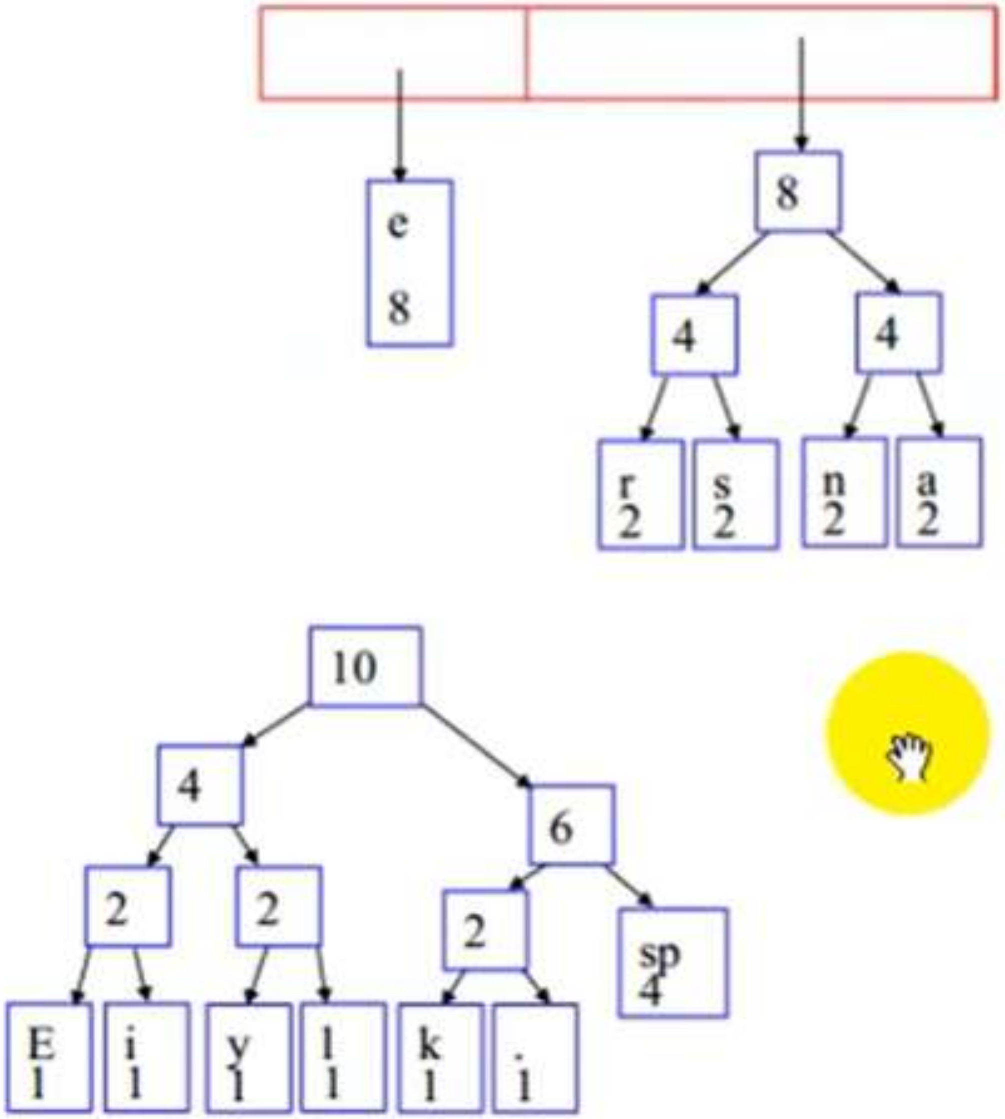

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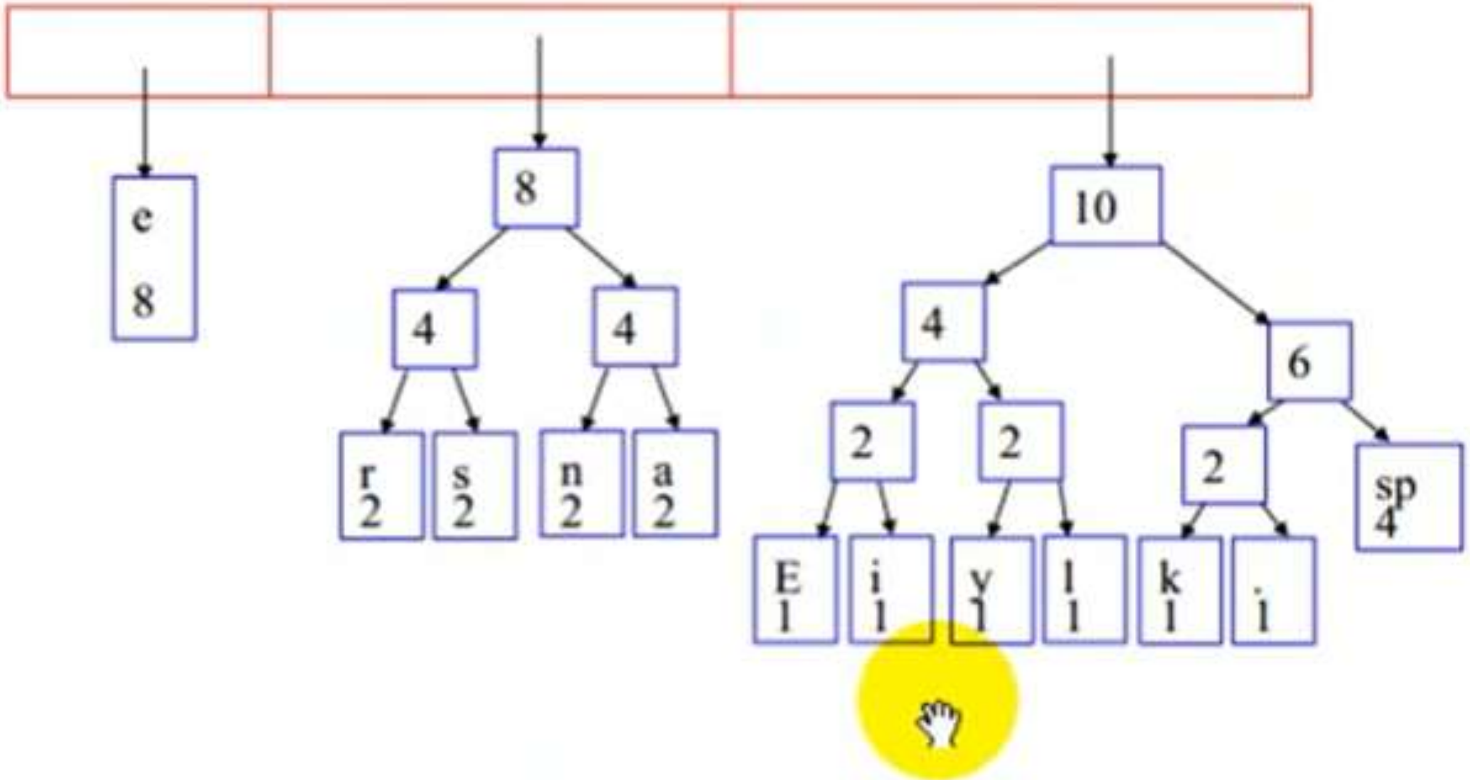

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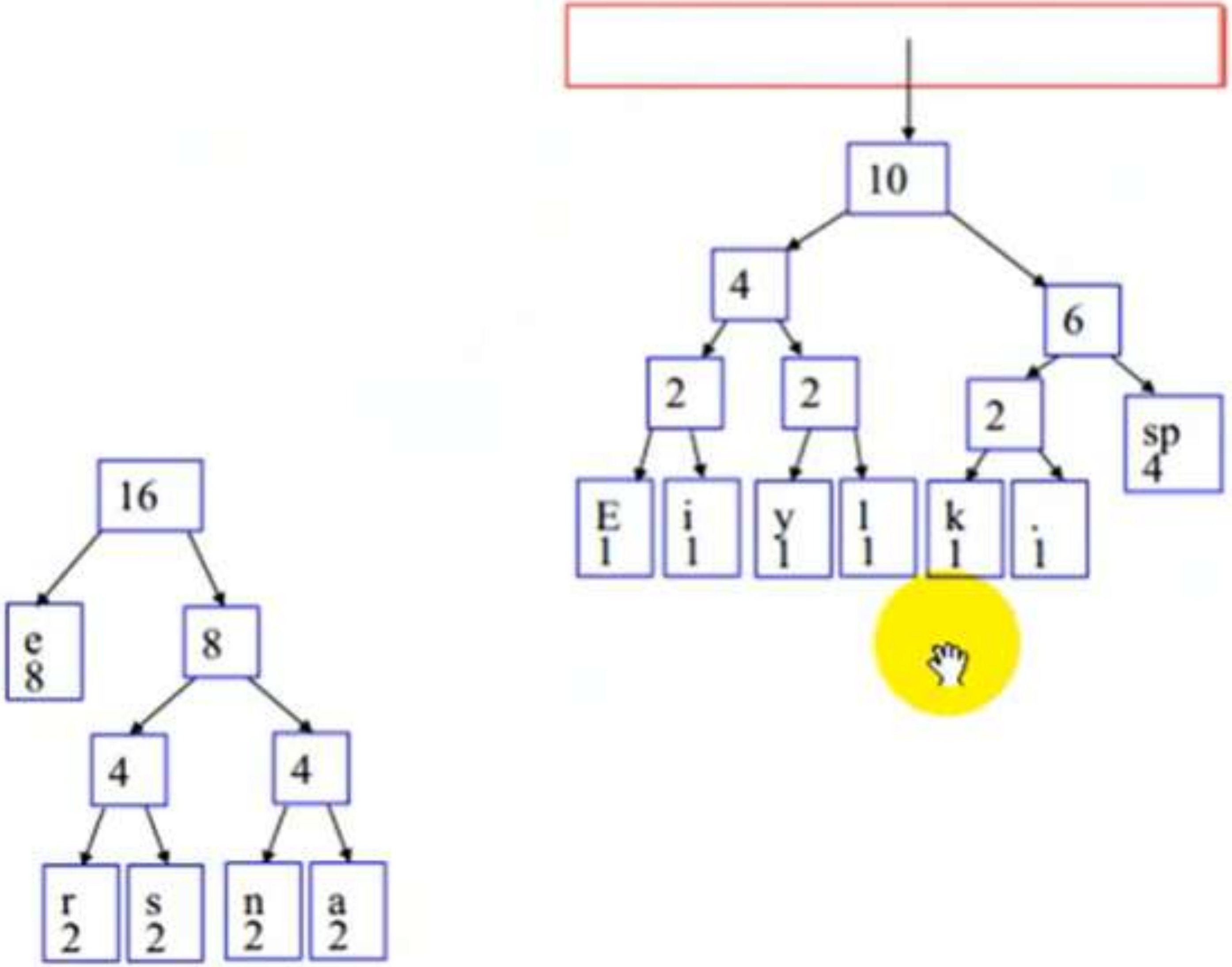

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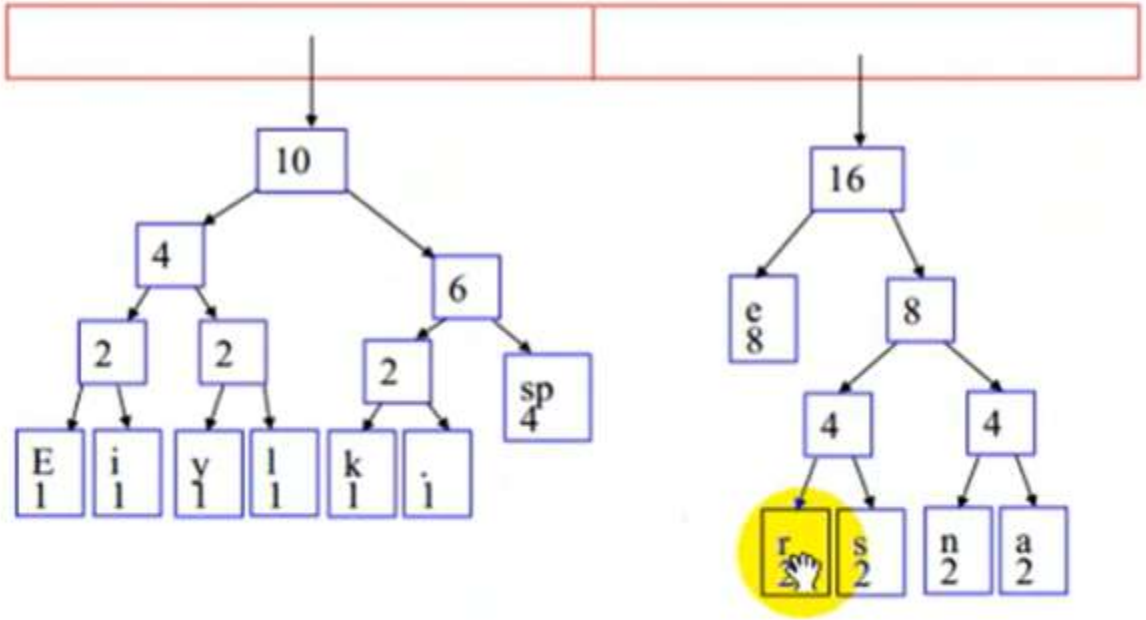

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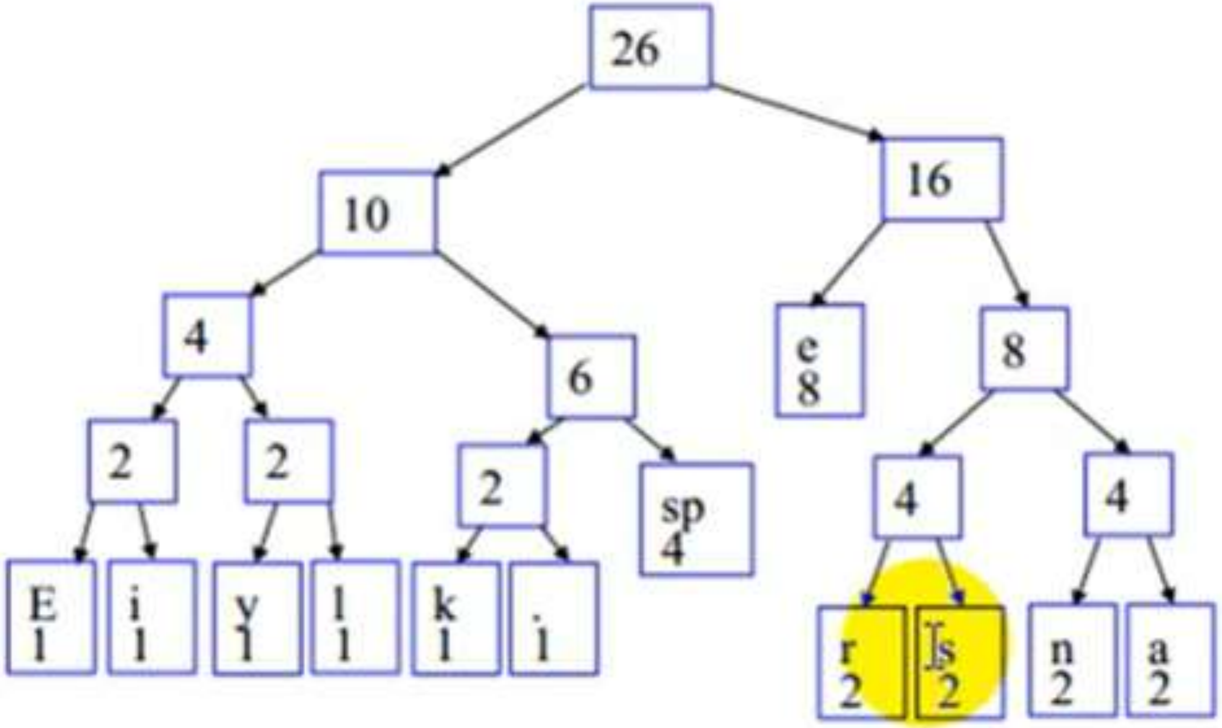
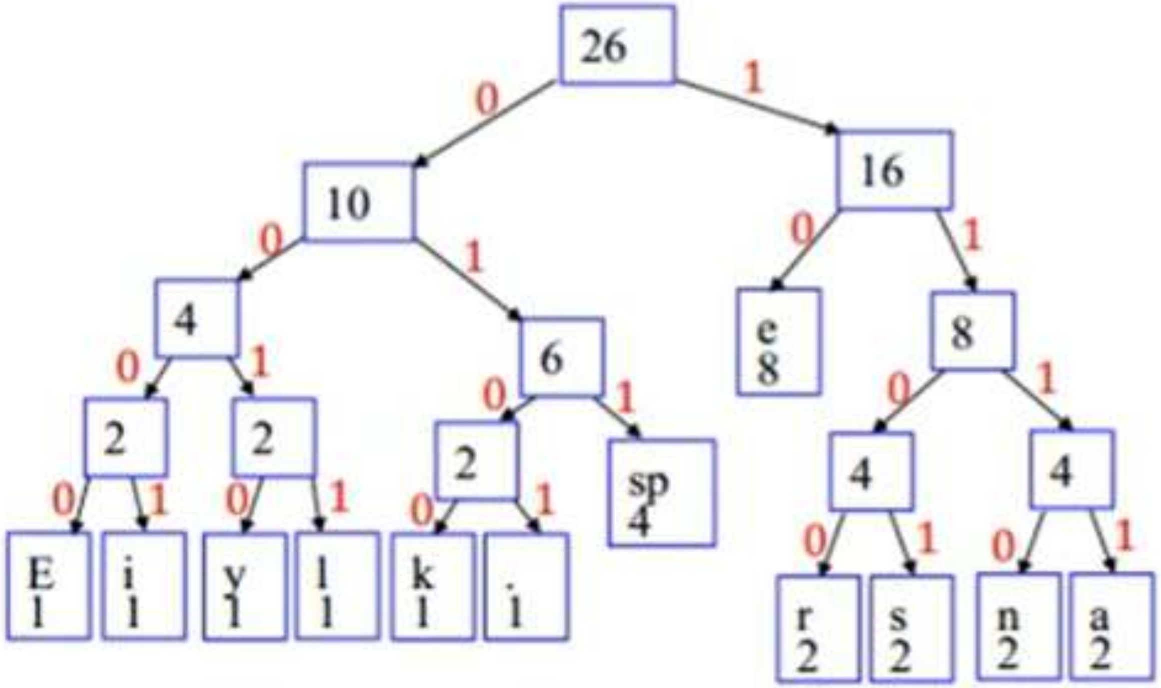


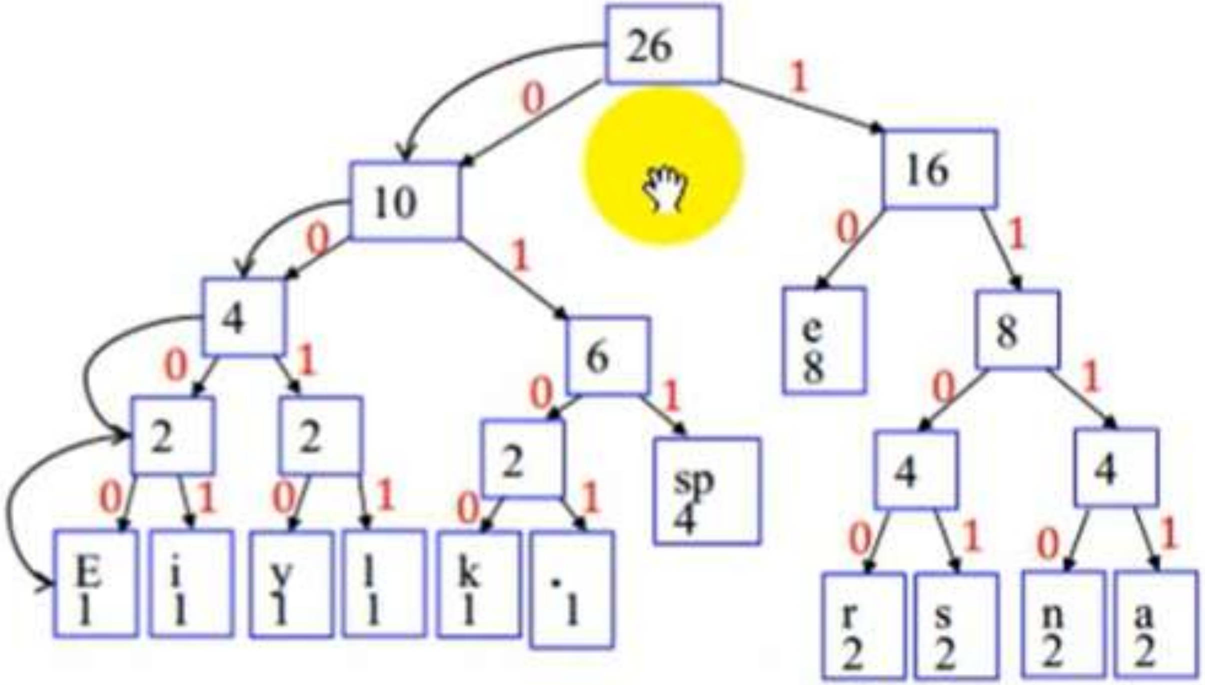
Building Tree

- $Left\ branch\ = 0$
- $Right\ branch\ = 1$



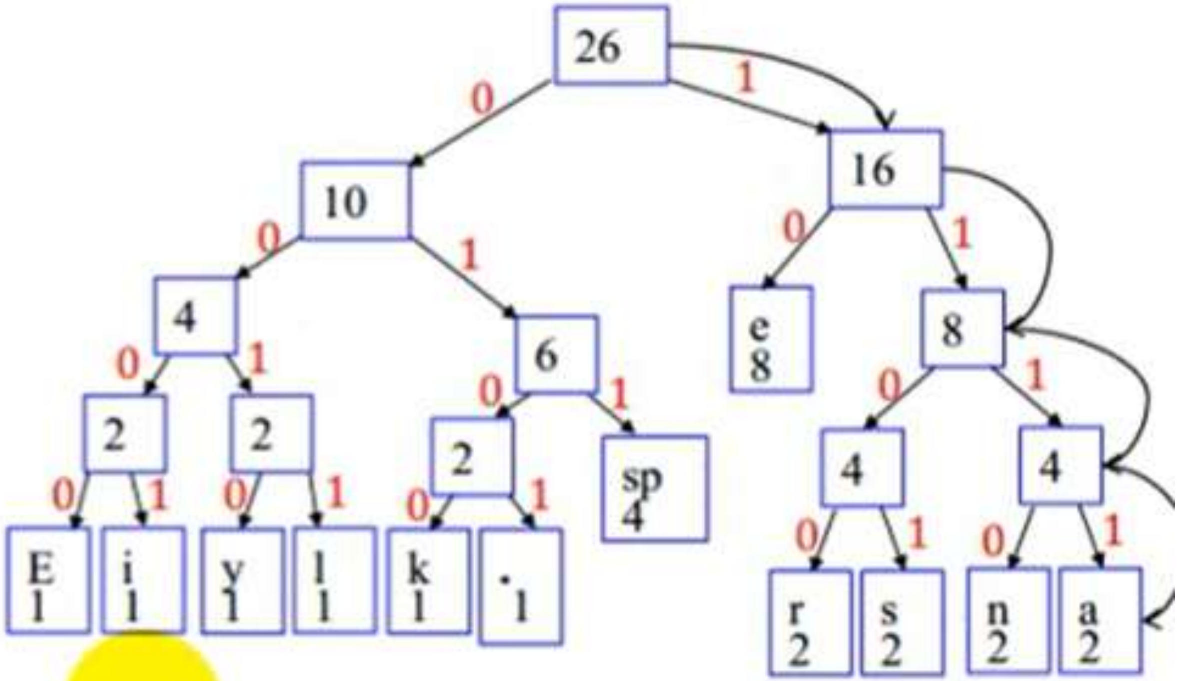
Traverse Tree for Codes

Char	Code
E	
i	
y	
l	
k	
.	
space	
e	
r	
s	
n	
a	



Traverse Tree for Codes

Char	Code
E	0000
i	0001
y	0010
l	0011
k	0100
.	0101
space	011
e	10
r	1100
s	1101
n	1110
a	1111



Encoding Text

Char	Code
E	0000
i	0001
y	0010
l	0011
k	0100
.	0101
space	011
e	10
r	1100
s	1101
n	1110
a	1111

Eerie eyes seen near lake.

Code:

0000

Encoding Text

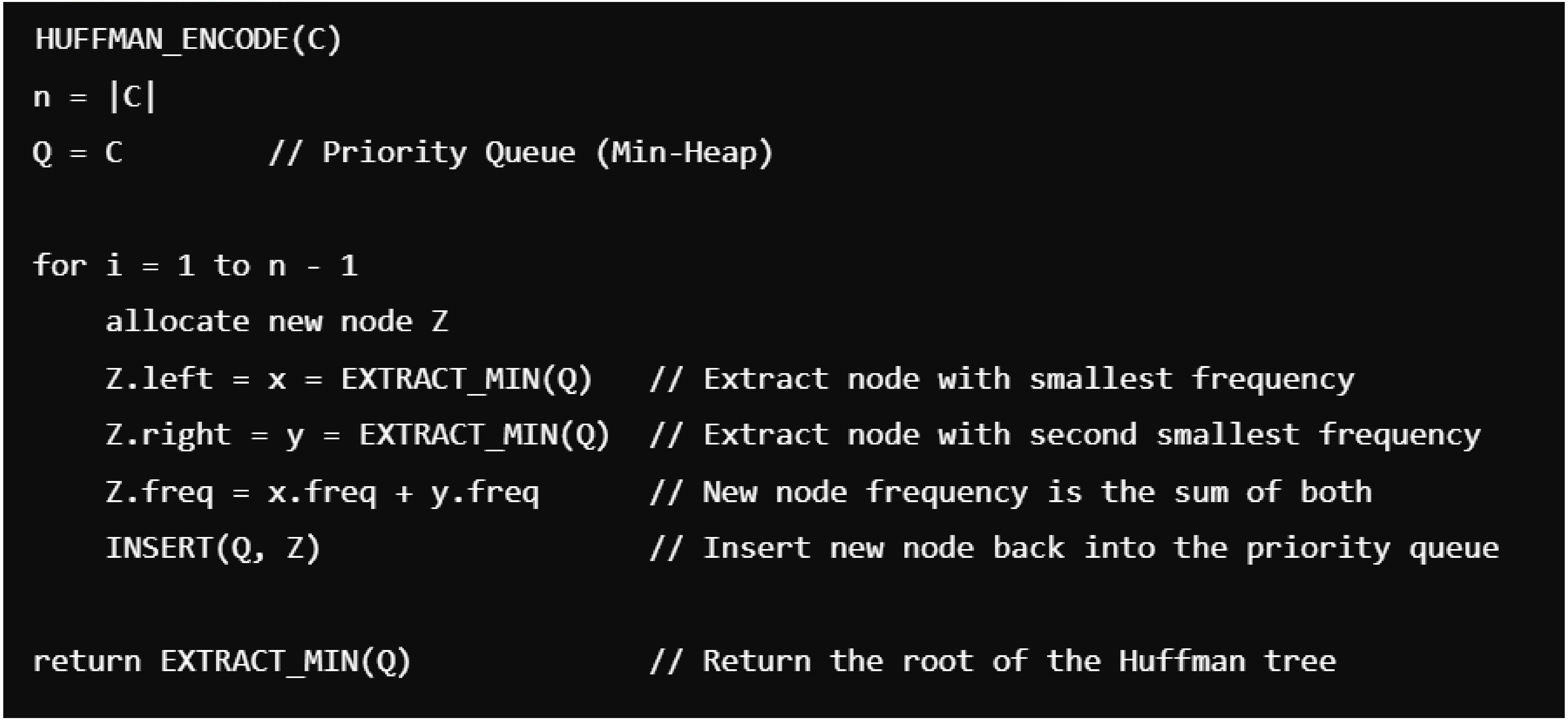
Char	Code
E	0000
i	0001
y	0010
l	0011
k	0100
.	0101
space	011
e	10
r	1100
s	1101
n	1110
a	1111

Eerie eyes seen near lake.

Code:

000010110000011001110001010
11010111101101011100111110
101111110001100111111010010
0101

Huffman Code



```
HUFFMAN_ENCODE(C)
n = |C|
Q = C        // Priority Queue (Min-Heap)

for i = 1 to n - 1
    allocate new node Z
    Z.left = x = EXTRACT_MIN(Q)   // Extract node with smallest frequency
    Z.right = y = EXTRACT_MIN(Q)  // Extract node with second smallest frequency
    Z.freq = x.freq + y.freq      // New node frequency is the sum of both
    INSERT(Q, Z)                  // Insert new node back into the priority queue

return EXTRACT_MIN(Q)             // Return the root of the Huffman tree
```

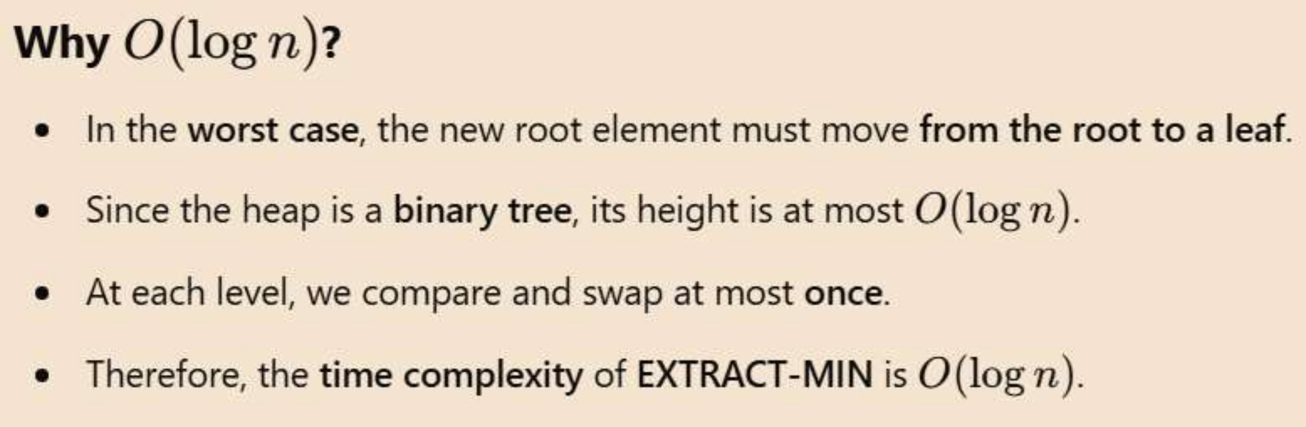
Huffman Code – Analysis

```
$HUFFAMN(C)$
    $n = |C| \longleftarrow 1$
    $Q = C \longleftarrow O(n) \quad BUILD-MIN-HEAP$
    $for \;\; i = 1 \;\; to \;\; n-1 \longleftarrow n-1$
        $allocate\ new\ node \;\; Z \longleftarrow n-1$
        $Z.left = x = EXTRACT-MIN(Q) \leftarrow (n-1)\lg n$
        $Z.right = y = EXTRACT-MIN(Q) \leftarrow (n-1)\lg n$
        $Z.freq = x.freq + y.freq \longleftarrow n-1$
        $INSERT(Q, Z) \longleftarrow (n-1)\lg n$
    $return \;\; EXTRACT-MIN(Q) \longleftarrow \lg n$
```

$\therefore T(n) = O(n \lg n)$

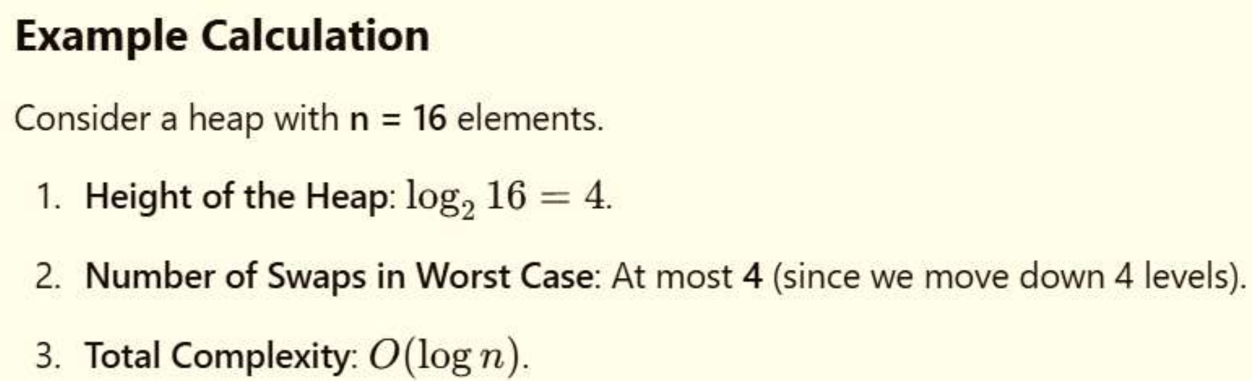
Steps in EXTRACT-MIN(Q)

1. **Removing the Root Element (Minimum)**
   - The minimum element is always at the root.
   - We remove it, which leaves a hole at the root.
2. **Replacing with the Last Element**
   - The last element in the heap is moved to the root position.
3. **Heapify (Heap Reordering Downward)**
   - Since the new root element might not maintain the **min-heap property**, we perform a **heapify-down** (or **percolate-down**) operation.
   - We swap the new root with the smaller of its two children, **recursively**, until the heap property is restored.



Why $O(\log n)$?

- In the **worst case**, the new root element must move **from the root to a leaf**.
- Since the heap is a **binary tree**, its height is at most $O(\log n)$.
- At each level, we compare and swap at most **once**.
- Therefore, the **time complexity** of EXTRACT-MIN is $O(\log n)$.

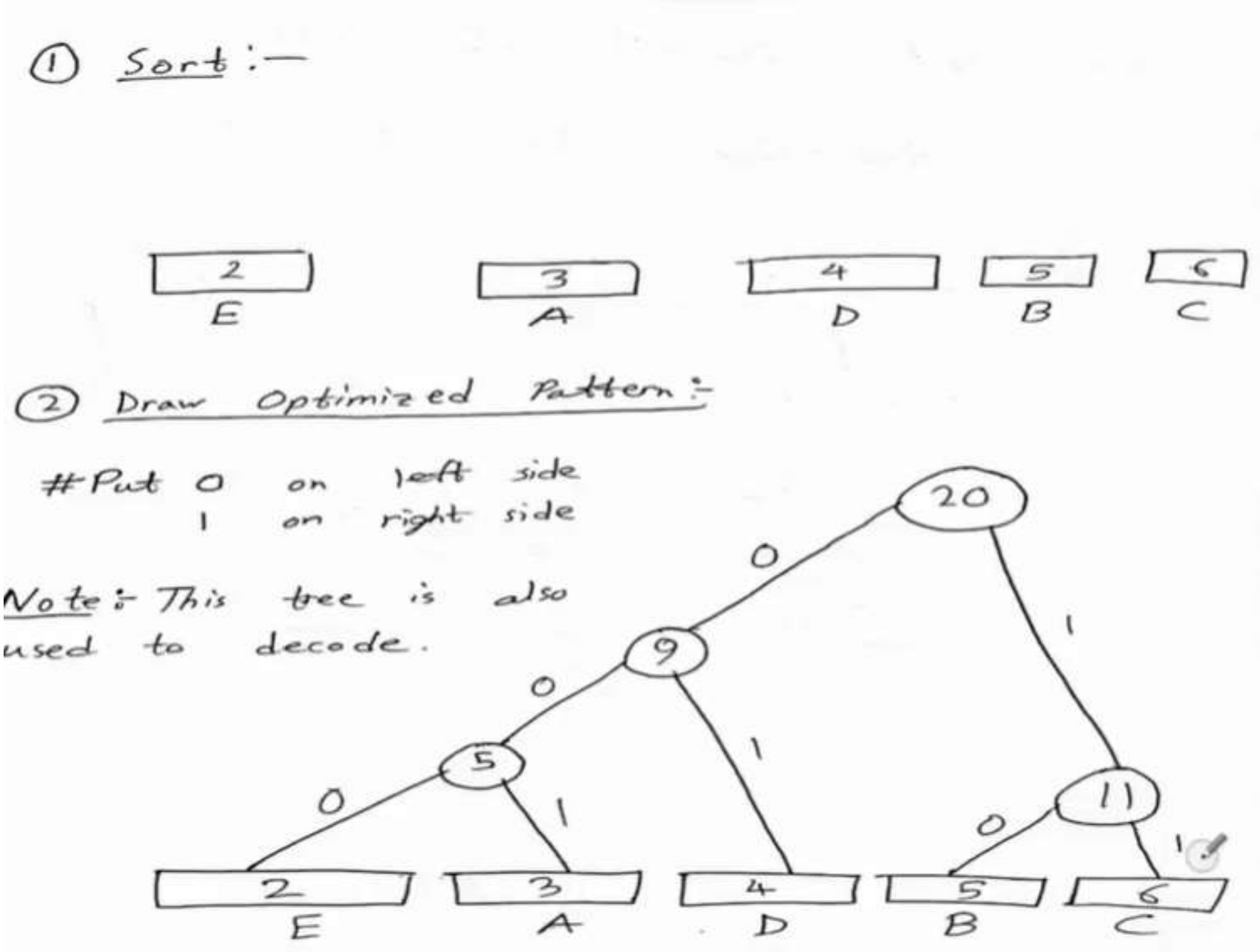


Example Calculation

Consider a heap with **n = 16** elements.

1. **Height of the Heap**: $\log_2 16 = 4$.
2. **Number of Swaps in Worst Case**: At most **4** (since we move down 4 levels).
3. **Total Complexity**: $O(\log n)$.

Hufman Decoding Procedure



① Sort:—

2	3	4	5	6
E	A	D	B	C

② Draw Optimized Pattern:-

#Put 0 on left side
1 on right side

Note:- This tree is also used to decode.

20
0
1
9
0
1
5
0
1
11
0
1
2
E
3
A
4
D
5
B
6
C

Hufman Decoding Procedure

① Sort :—

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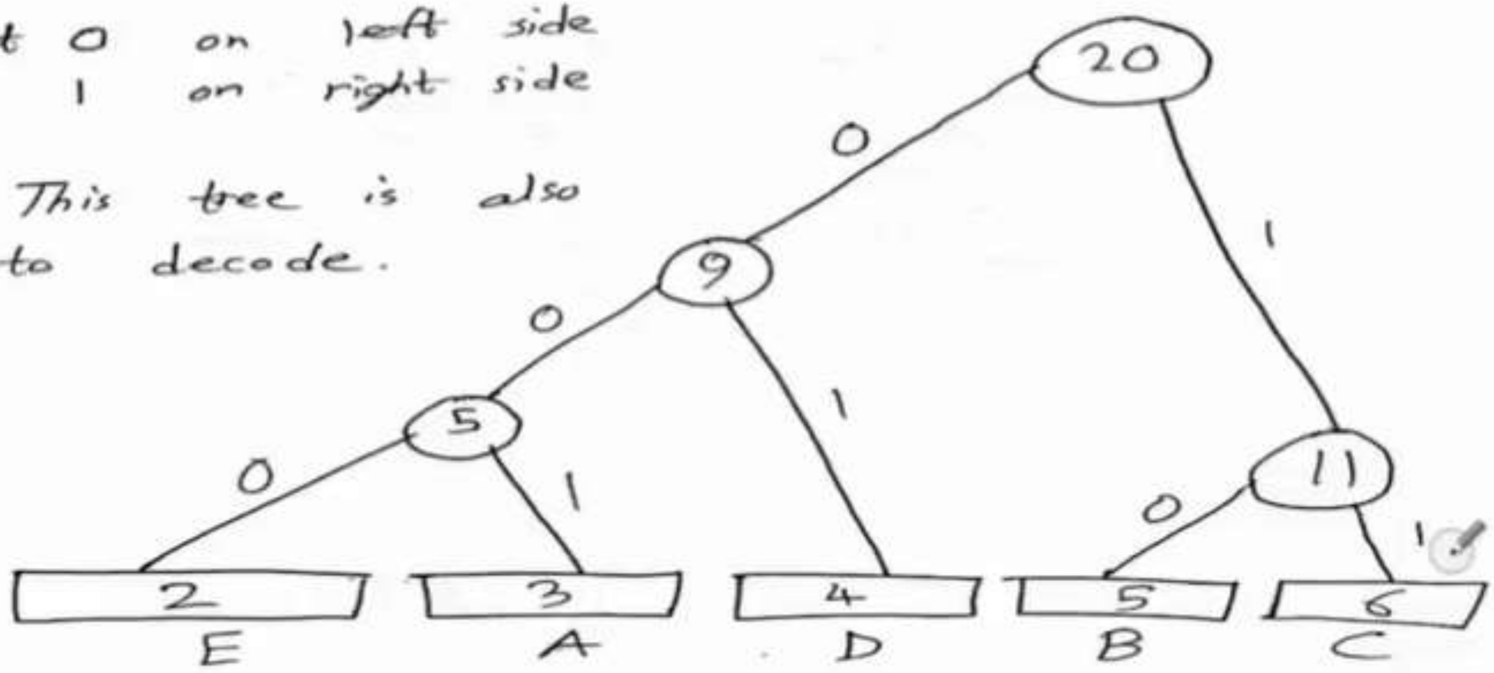
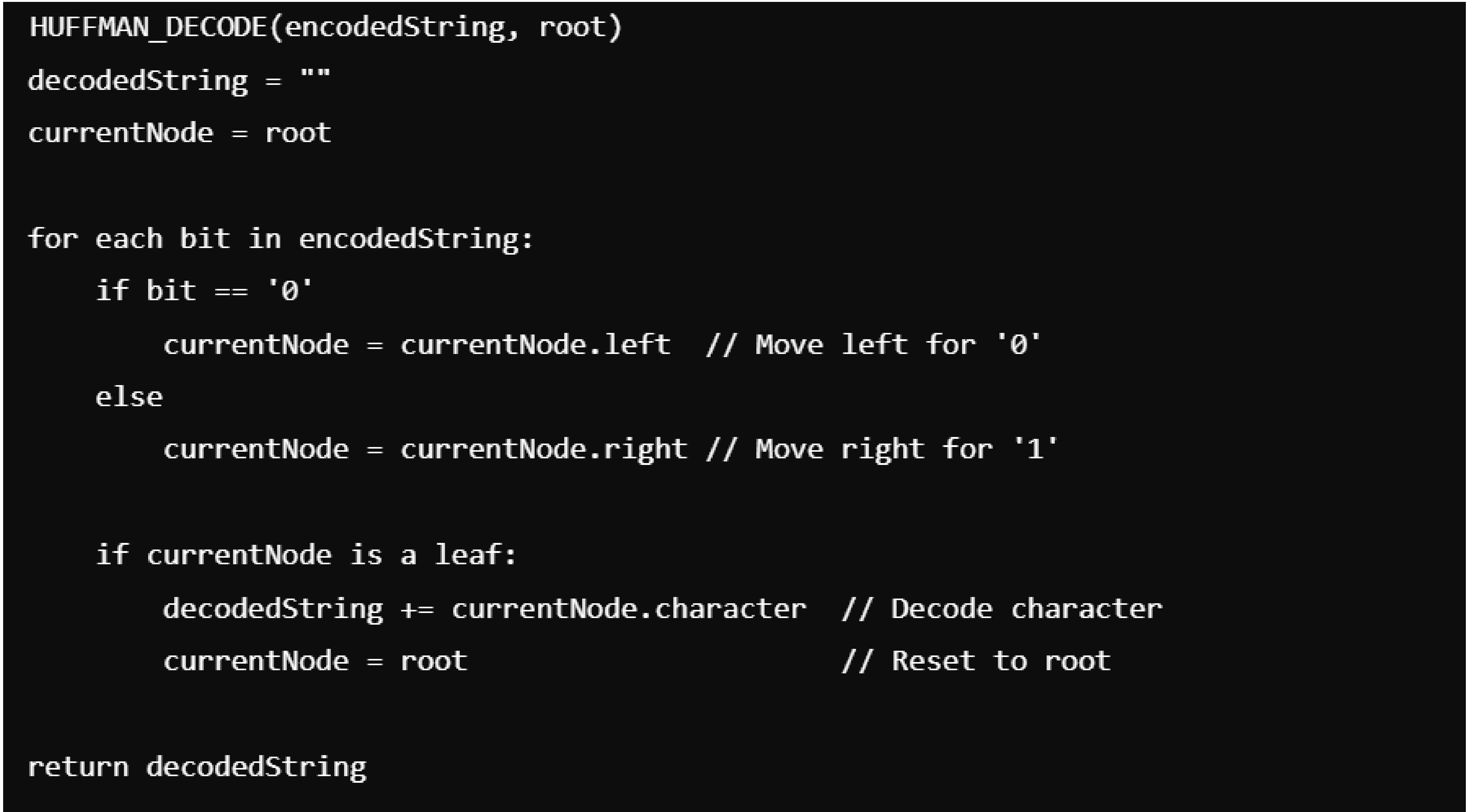


Decoding Algorithm



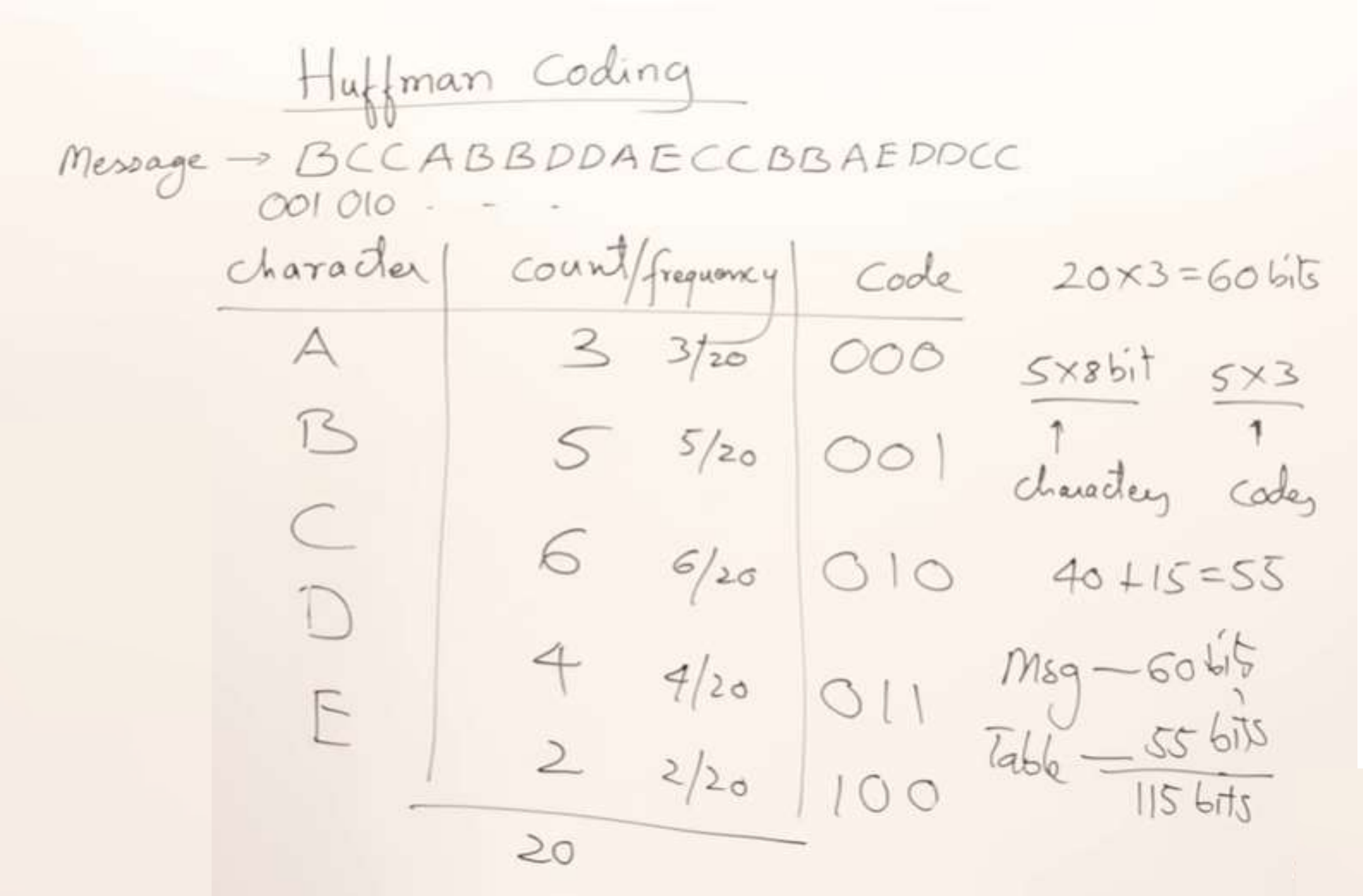
```
HUFFMAN_DECODE(encodedString, root)
decodedString = ""
currentNode = root

for each bit in encodedString:
    if bit == '0'
        currentNode = currentNode.left  // Move left for '0'
    else
        currentNode = currentNode.right // Move right for '1'

    if currentNode is a leaf:
        decodedString += currentNode.character  // Decode character
        currentNode = root                      // Reset to root

return decodedString
```

Overall Time Complexity:

- For each of the `m` bits in the encoded string, we may have to traverse up to the height of the tree `h`, which is **O(log n)**.
- Therefore, the total time complexity for decoding is:

$$\text{Time Complexity} = O(m \cdot h) = O(m \cdot \log n)$$

FIXED LENGTH CODE



Huffman Coding

Message → BCCABBDDAECCBBAEDDCC

001 010 . . .

character	count/frequency		Code
A	3	3/20	000
B	5	5/20	001
C	6	6/20	010
D	4	4/20	011
E	2	2/20	100
	20		

20×3=60 bits

5×8bit ↑ characters

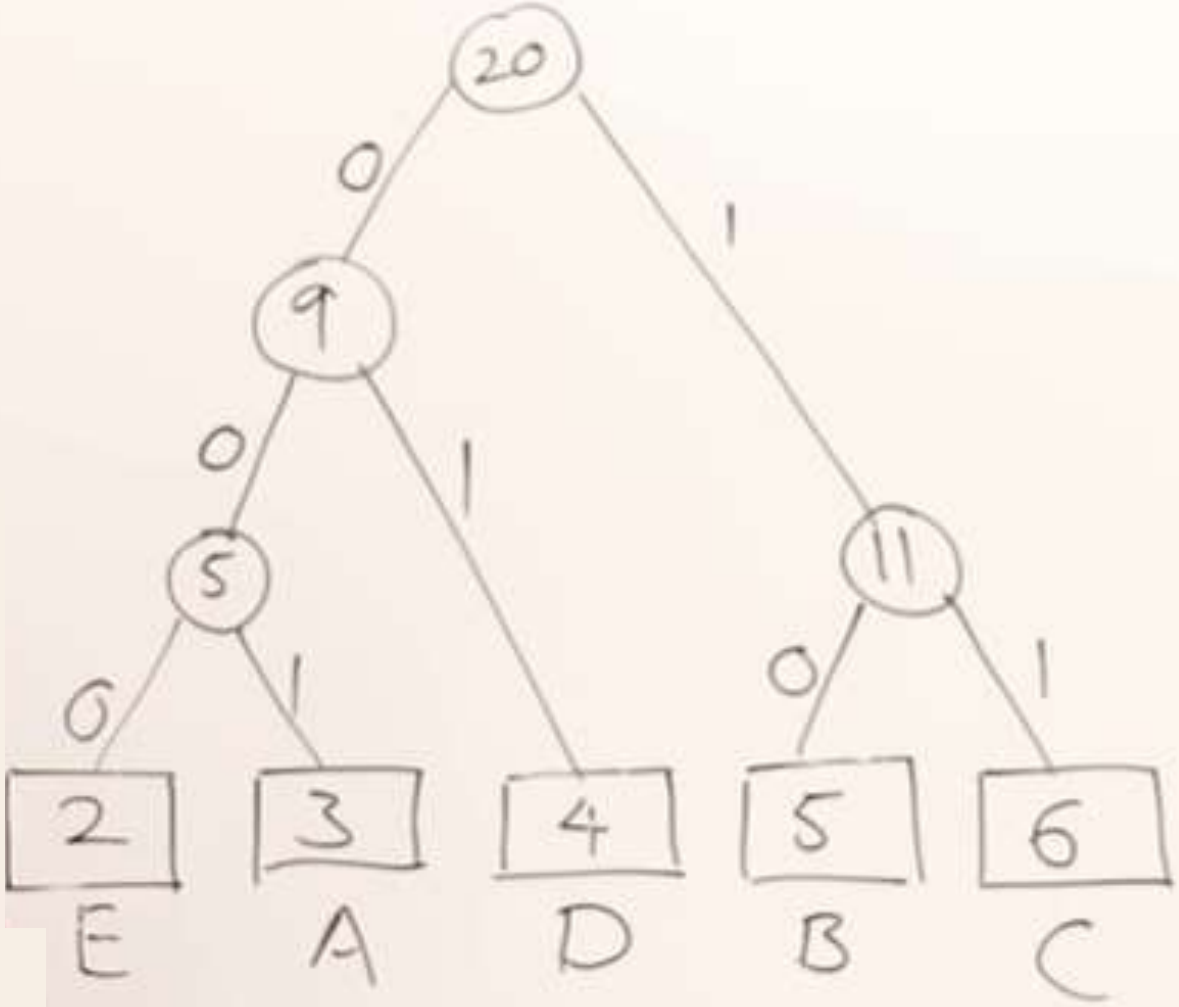
5×3 ↑ codes

40+15=55

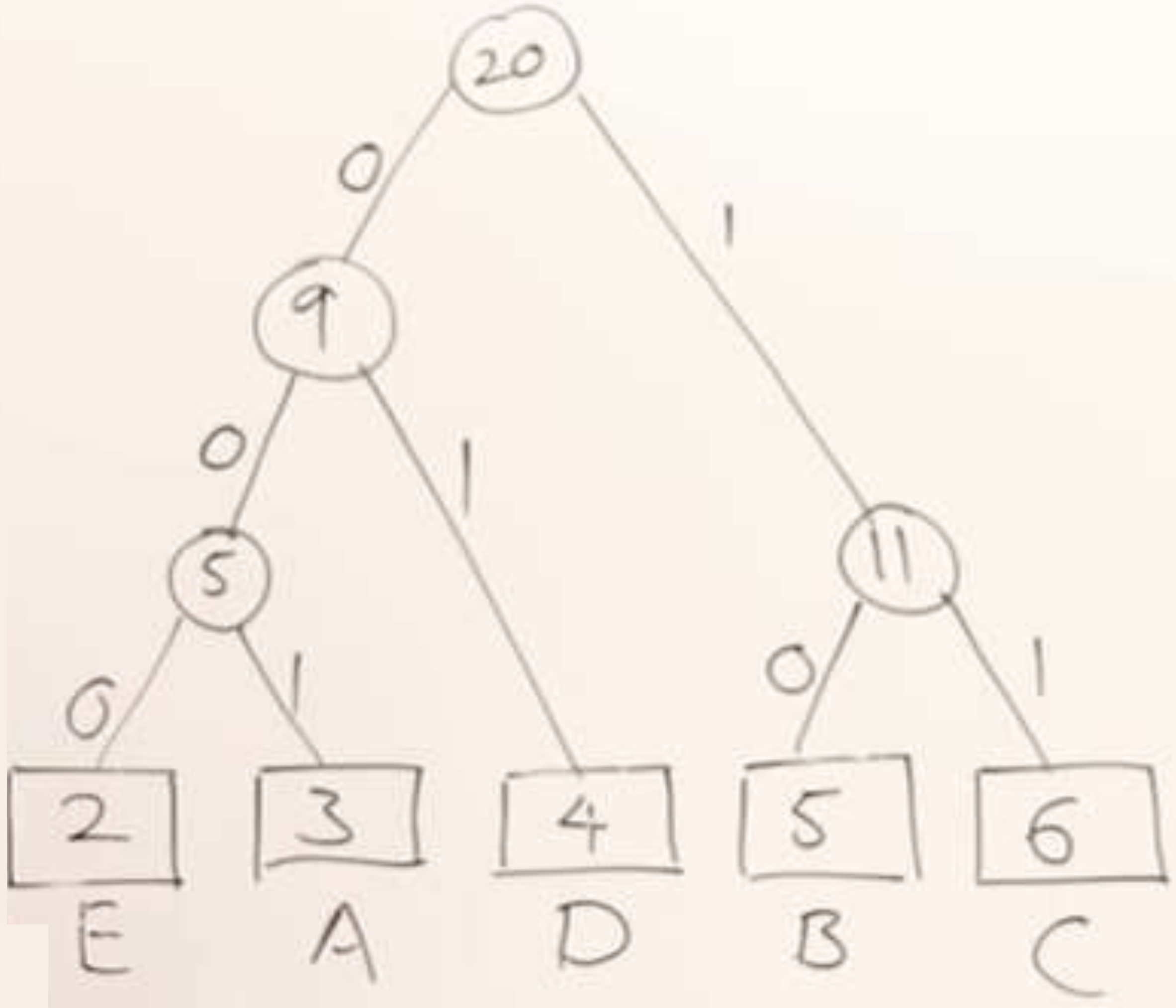
Msg — 60 bits

Table — 55 bits

115 bits



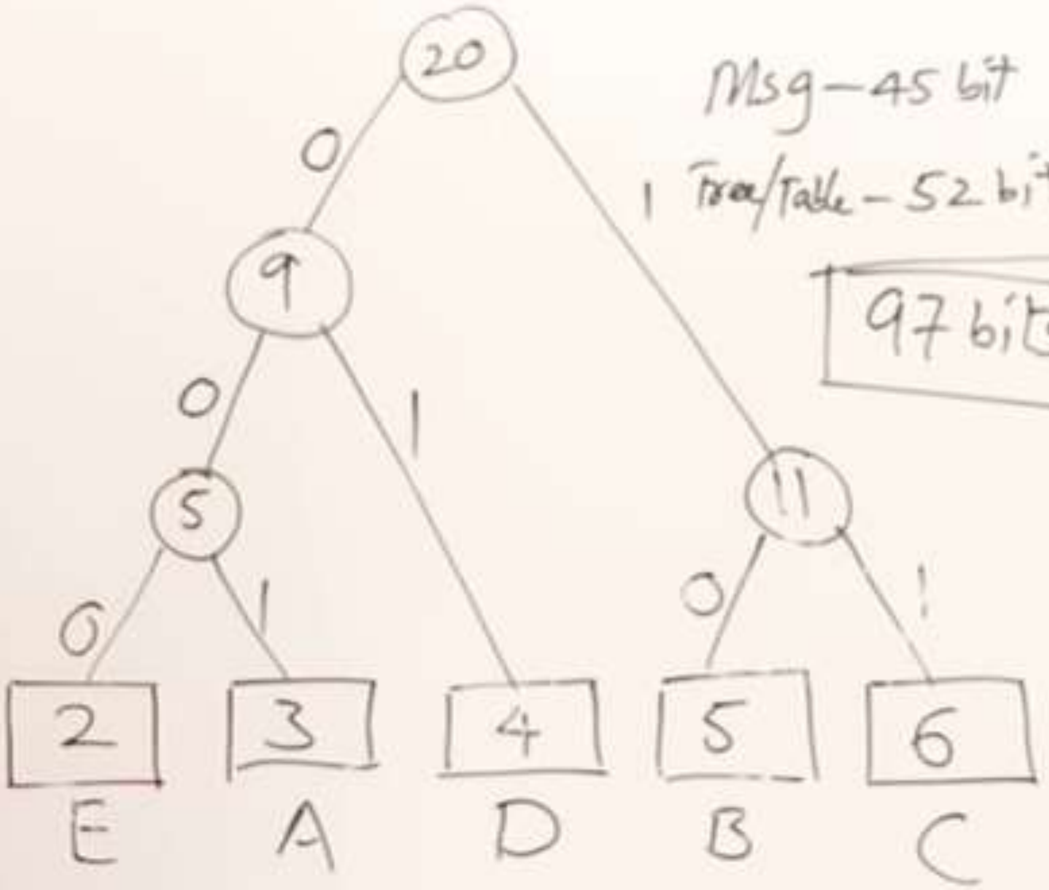
Char	count	Code
A	3	001
B	5	10
C	6	11
D	4	01
E	2	000
	20	



char	count	code	
A	3	001	3x3=9
B	5	10	5x2=10
C	6	11	6x2=12
D	4	01	4x2=8
E	2	000	2x3=6
	20		45 bits

Huffman Coding

Message → BCCABBDDAECCBBAEDDCC
10 11 11 001 10 10 01 01 - - -



Msg – 45 bit

Tree/Table – 52 bits

$\boxed{\text{97 bits}}$

Char	count	Code	
A	3	001	3×3=9
B	5	10	5×2=10
C	6	11	6×2=12
D	4	01	4×2=8
E	2	000	2×3=6
5x 8bit 40bits	20 +	12 bit	45 bits