LU4: Brute Force and KMP – String Searching

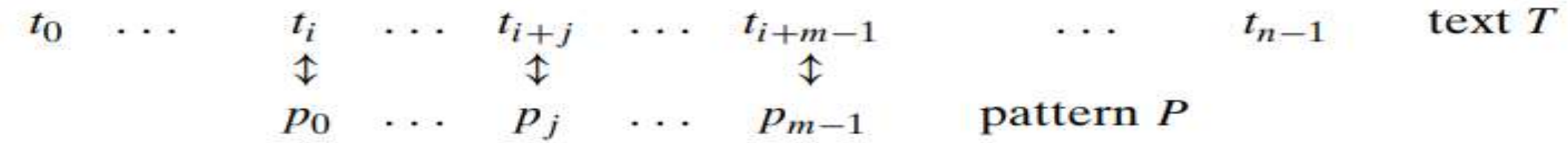
Brute Force– String Searching

Brute-Force : String Searching

- It is the process of **finding pattern in the larger text**. Also called as ***String Matching***.
    - *pattern*: a string of m characters to search for.
    - *text*: a (longer) string of n characters to search in.
- *problem*: given a string of n characters called the ***text*** and a string of m characters $(m \leq n)$ called the ***pattern,*** find a substring of the text that matches the pattern.
- Example:
    - Pattern: **Hi**
    - Text: Hello, Good Morning. **Hi**, Nice meeting you

Brute-Force : String Searching (Contd.,)

- Goal: To find i—the index of the leftmost character of the first matching substring in the text.



- such that

ti = p0, ti+1=p1, ...,ti+j = pj , ... ,ti+m−1 = pm−1

- Solution: If match occur, return index i. Otherwise return -1.

Brute-Force : String Searching (Contd.,)

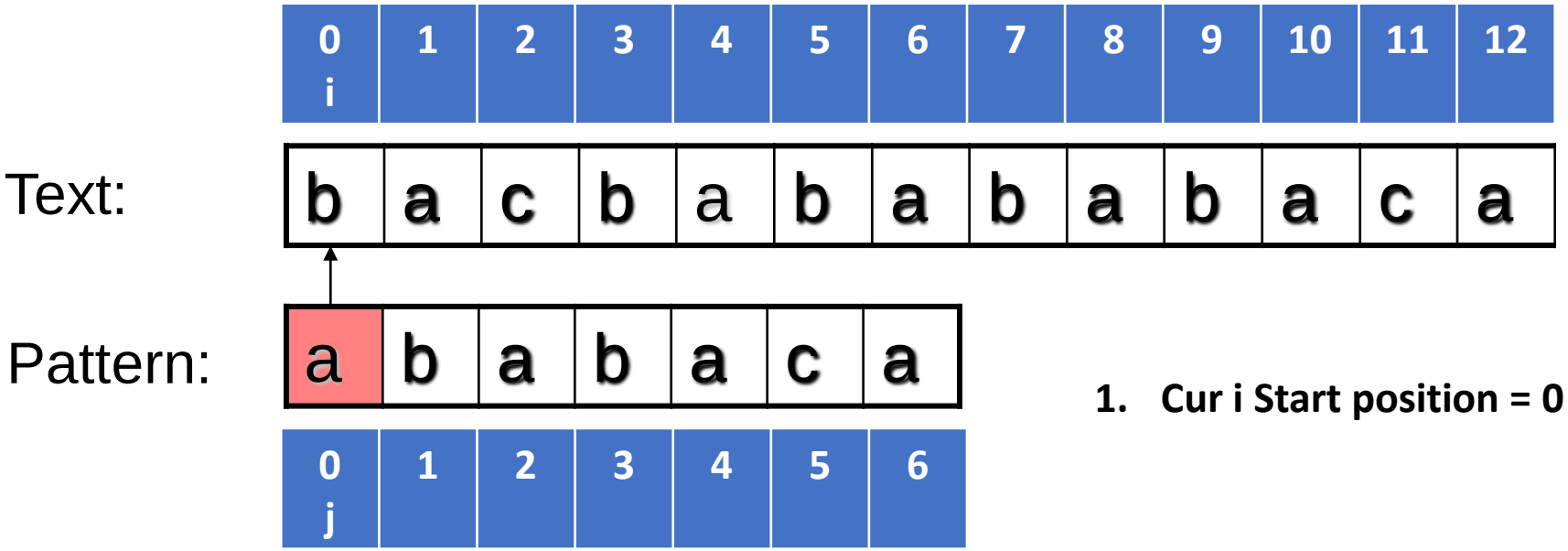
Step 1 Align pattern at beginning of text

Step 2 Moving from left to right, compare each character of pattern to the corresponding character in text until

- all characters are found to match (successful search); or
- a first mismatch is detected

Step 3 While pattern is not found and the text is not yet exhausted, **realign pattern one position to the right** and repeat Step 2

Brute-Force : String Searching-Example

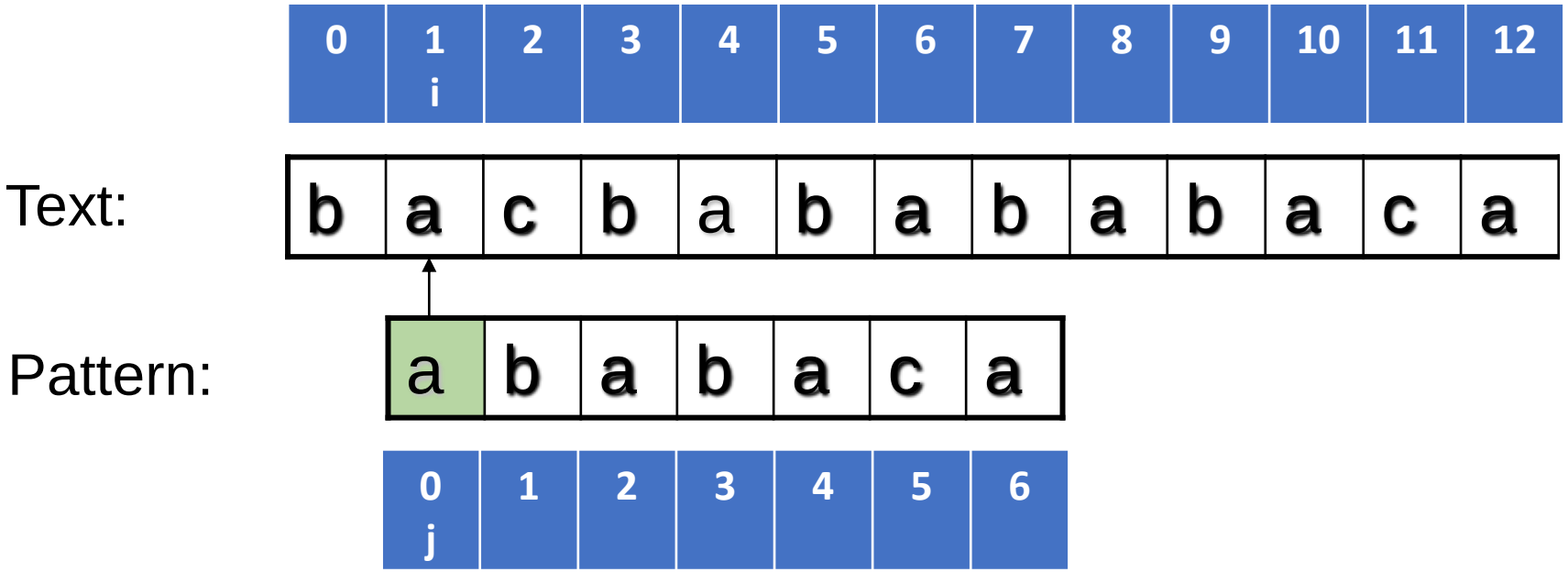


1. Cur i Start position = 0

Process:

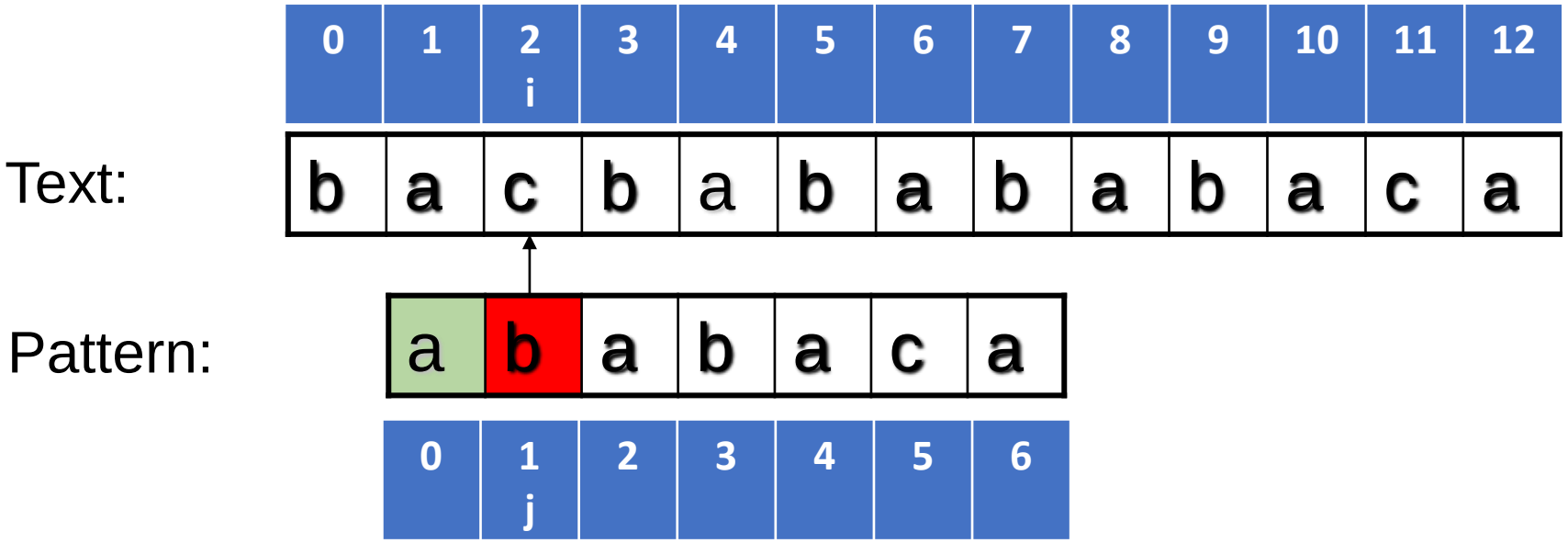
1. If there is a match, move i as well as j
2. Else
    1. If there is a mismatch, j resets to 0 in the next step
    2. Move i to the next position of previously started index

Brute-Force : String Searching-Example (Contd.,)



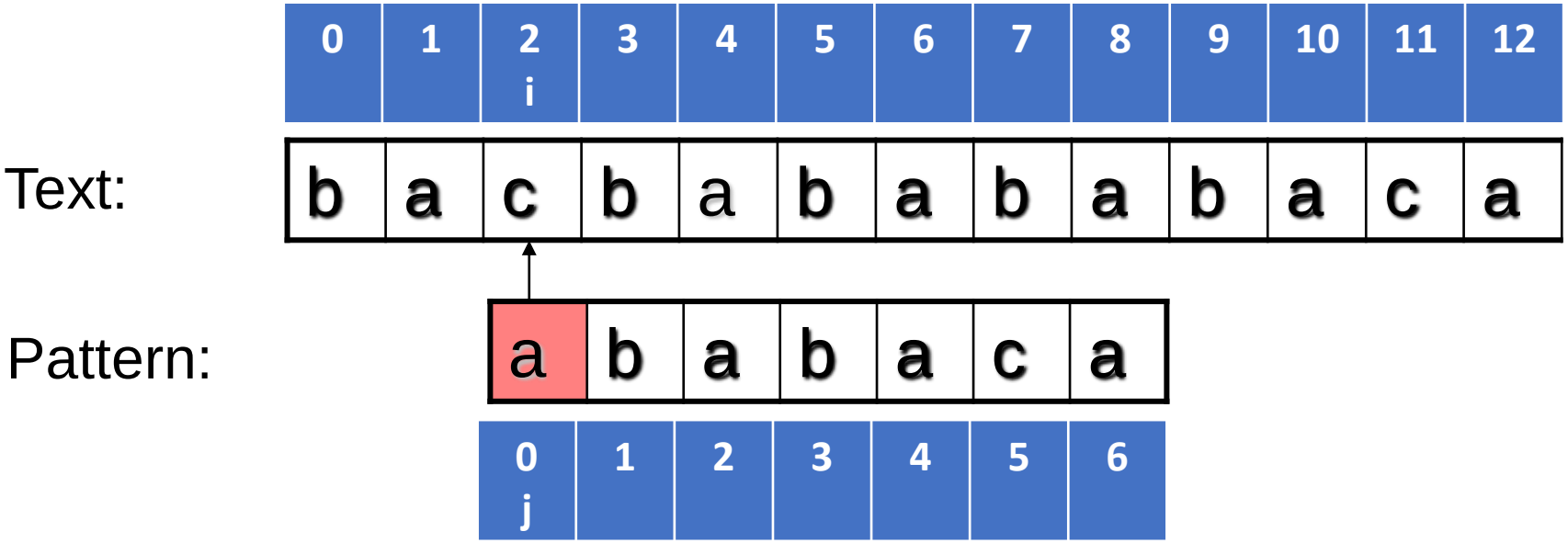
1. **Cur i Start position = 1**
2. **There is a match, move i as well as j**

Brute-Force : String Searching-Example (Contd.,)



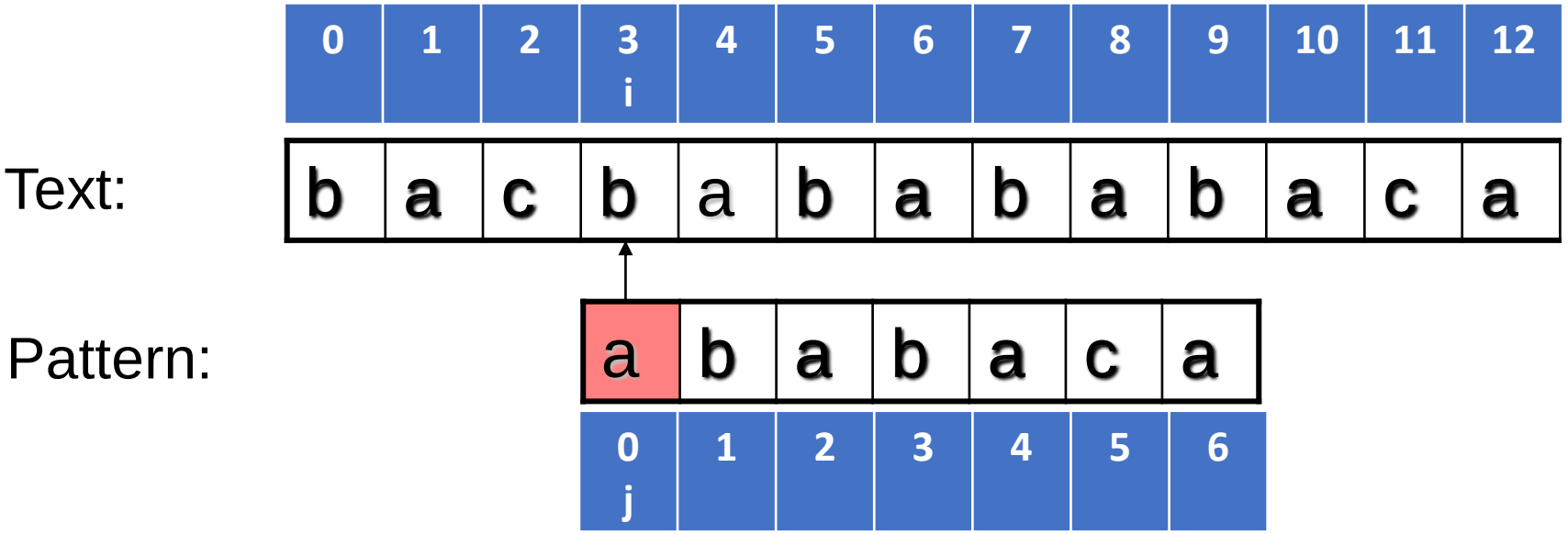
1. There is a mismatch, j resets to 0 in the next step
2. Move i to the next position = 2 (Cur. start position of i = 1, next position = Cur + 1 = 2)

Brute-Force : String Searching-Example (Contd.,)



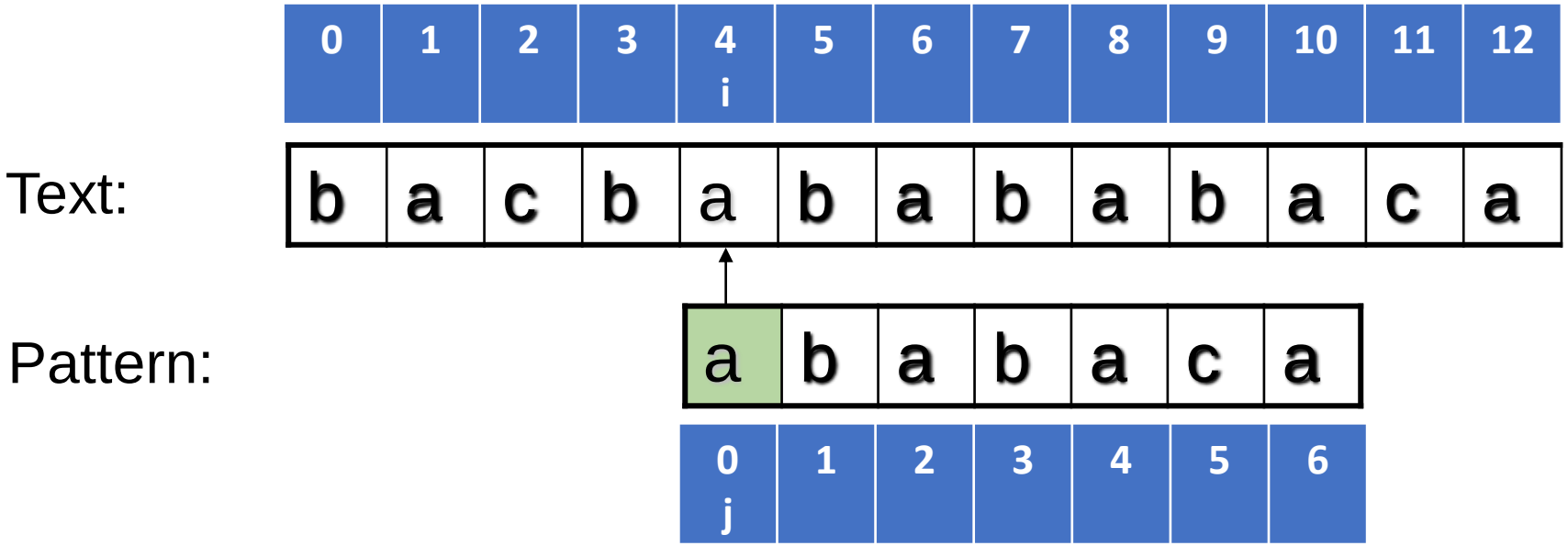
1. There is a mismatch, j resets to 0 in the next step
2. Move i to the next position = 3 (Cur. start position of i = 2, next position = cur + 1 = 3)

Brute-Force : String Searching-Example (Contd.,)



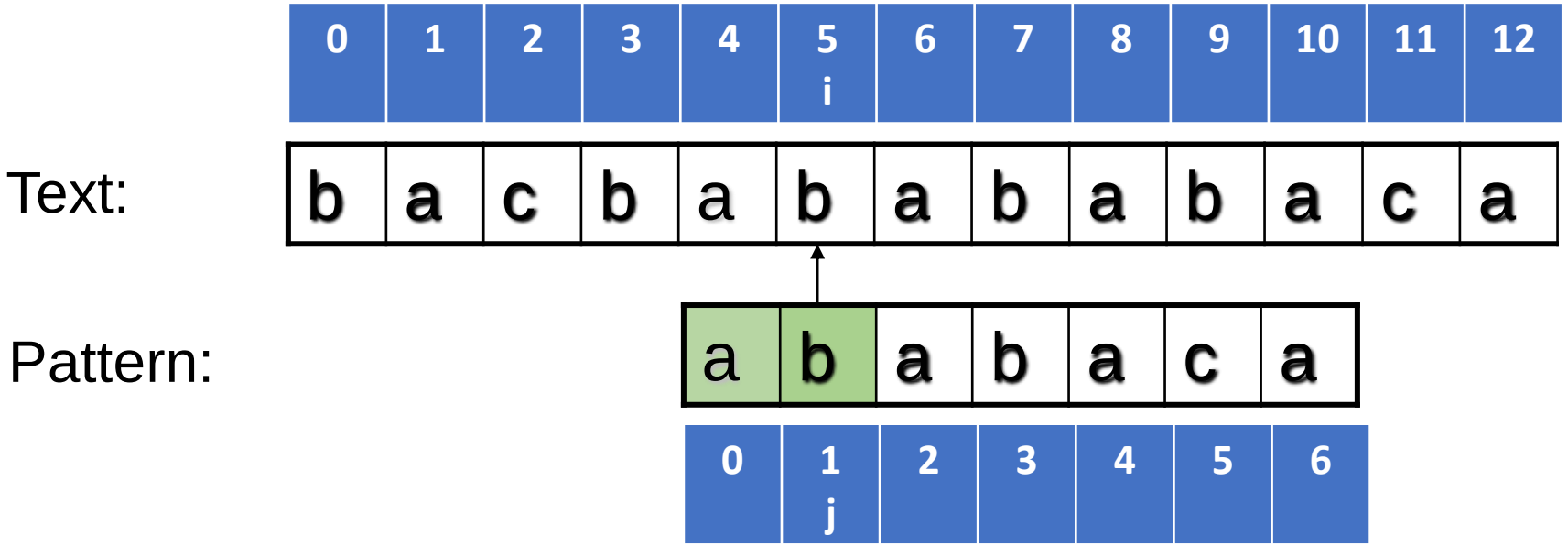
1. There is a mismatch, j resets to 0 in the next step
2. Move i to the next position = 4 (Prev. start position of i = 3, next position = prev + 1 = 4)

Brute-Force : String Searching-Example (Contd.,)



1. **Cur i Start position = 4**
2. **There is a match, move i as well as j**

Brute-Force : String Searching-Example (Contd.,)



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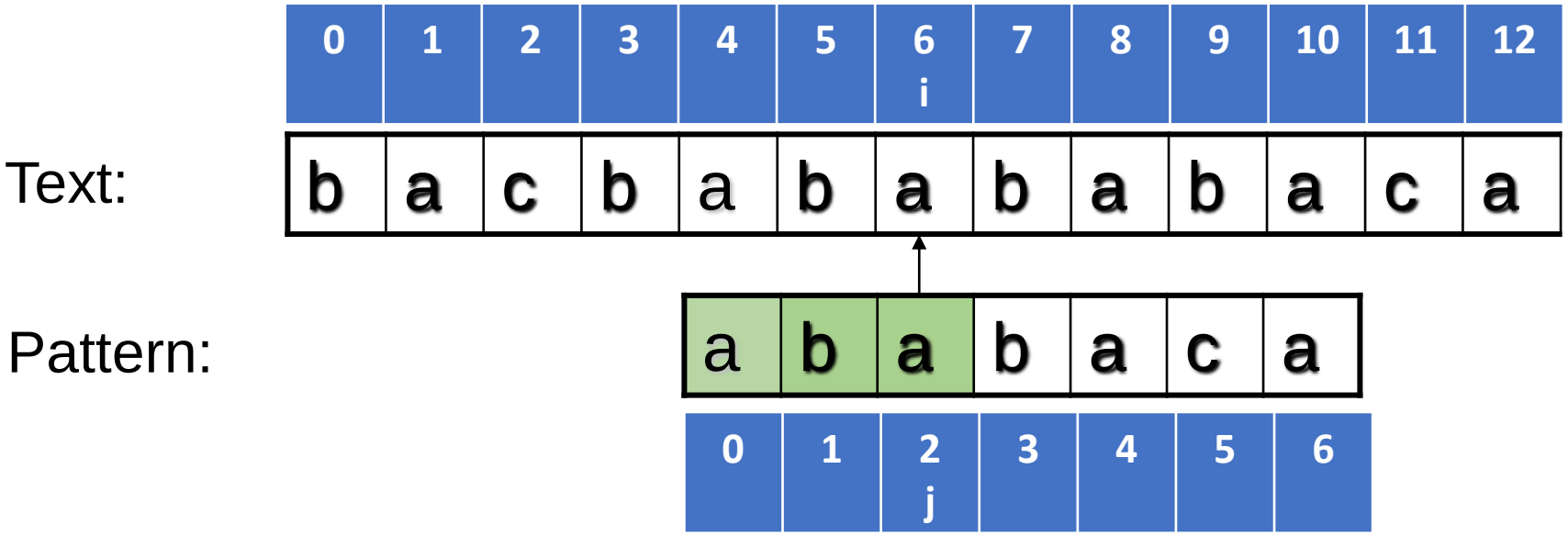
Brute-Force : String Searching-Example (Contd.,)



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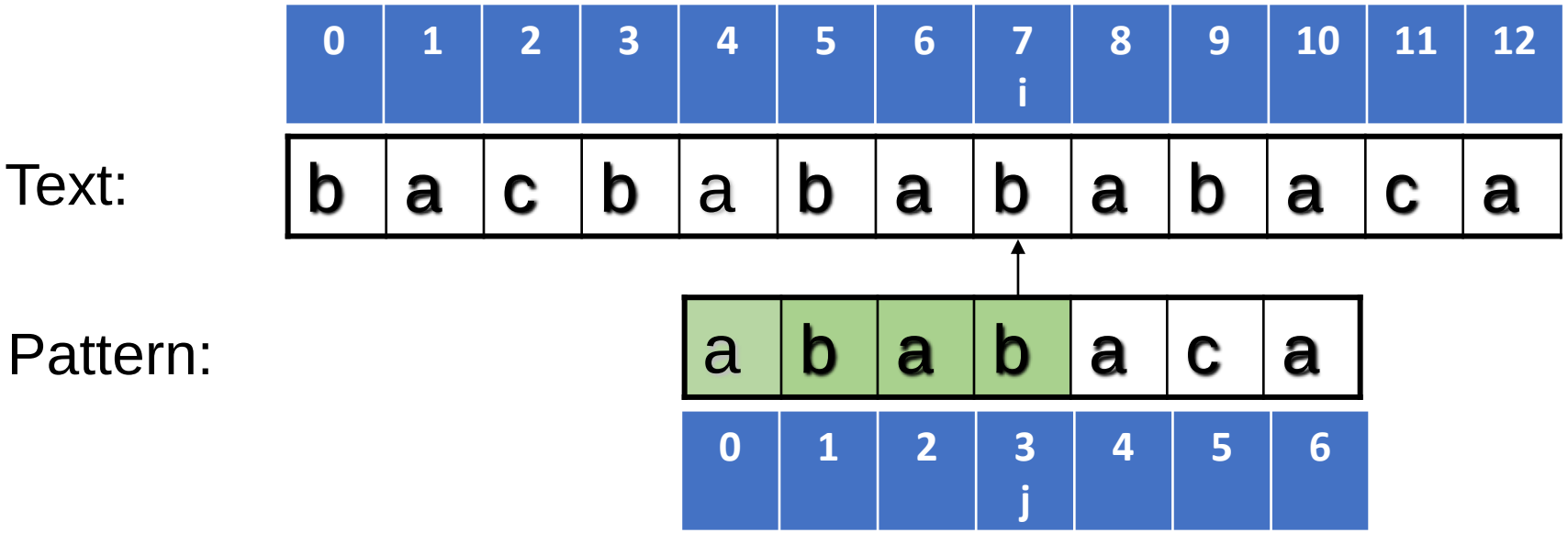
Brute-Force : String Searching-Example (Contd.,)



1. **Cur i Start position = 4**
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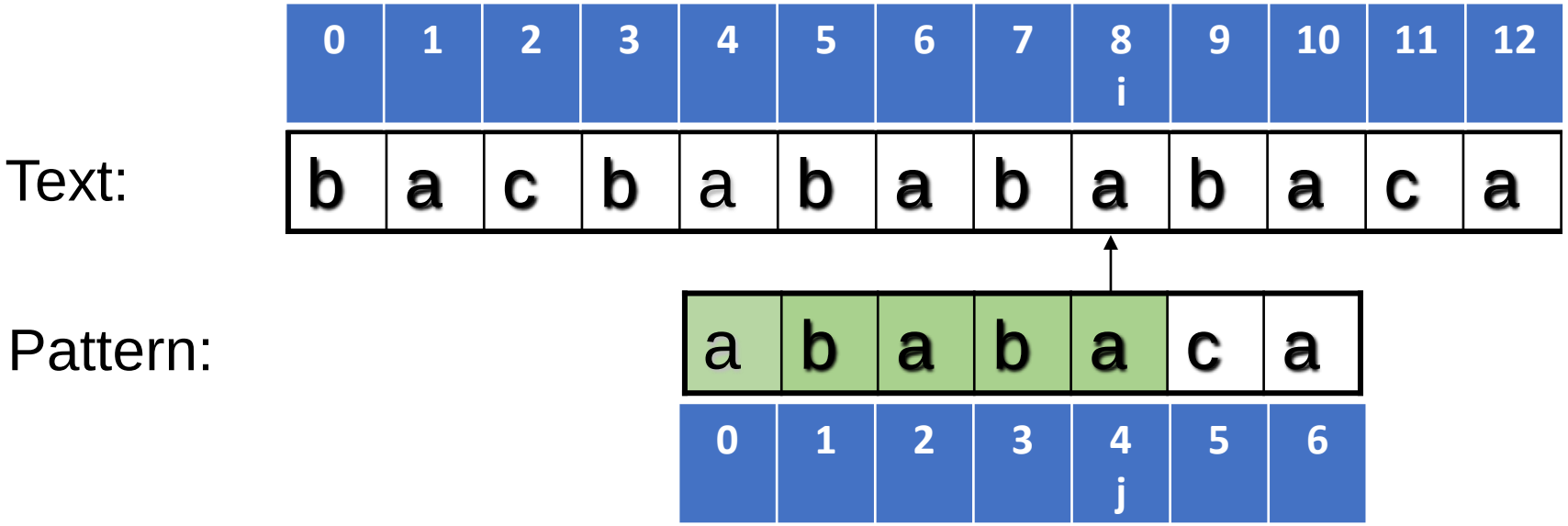
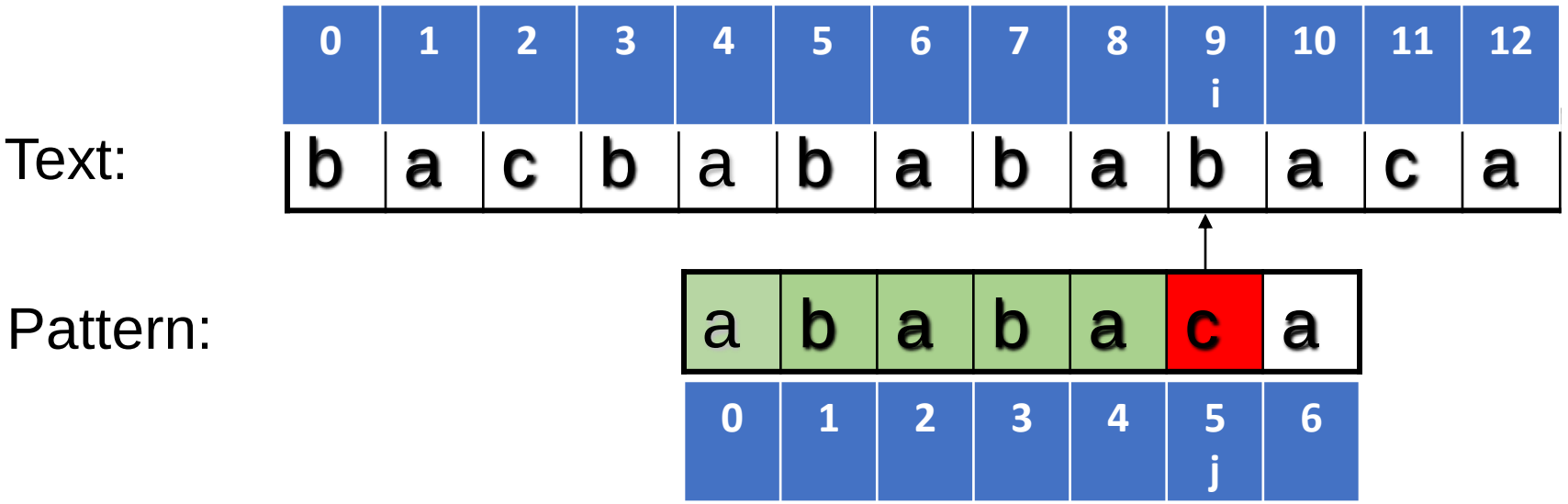
Brute-Force : String Searching-Example (Contd.,)



1. **Cur i Start position = 4**
2. **There is a match, move i as well as j**

Brute-Force : String Searching-Example (Contd.,)

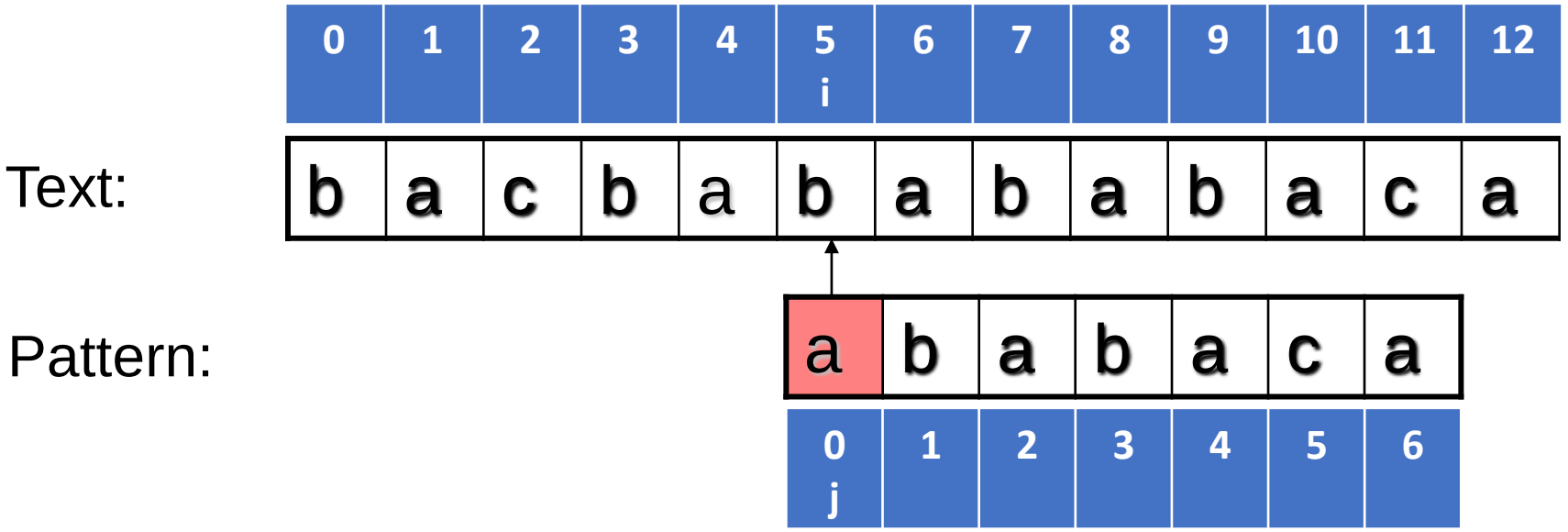


1. **There is a mismatch, j resets to 0 in the next step**
2. **Move i to the next position = 5 (Cur. start position of i = 4, next position = cur + 1 = 5)**

Drawback of **Bruteforce**:
Already i has reached **9**, when it finds mismatch, j sets to 0, but again i has to reset its index (**shifted back**) to earlier visited one . i.e. here the next shift of I will be **5**.

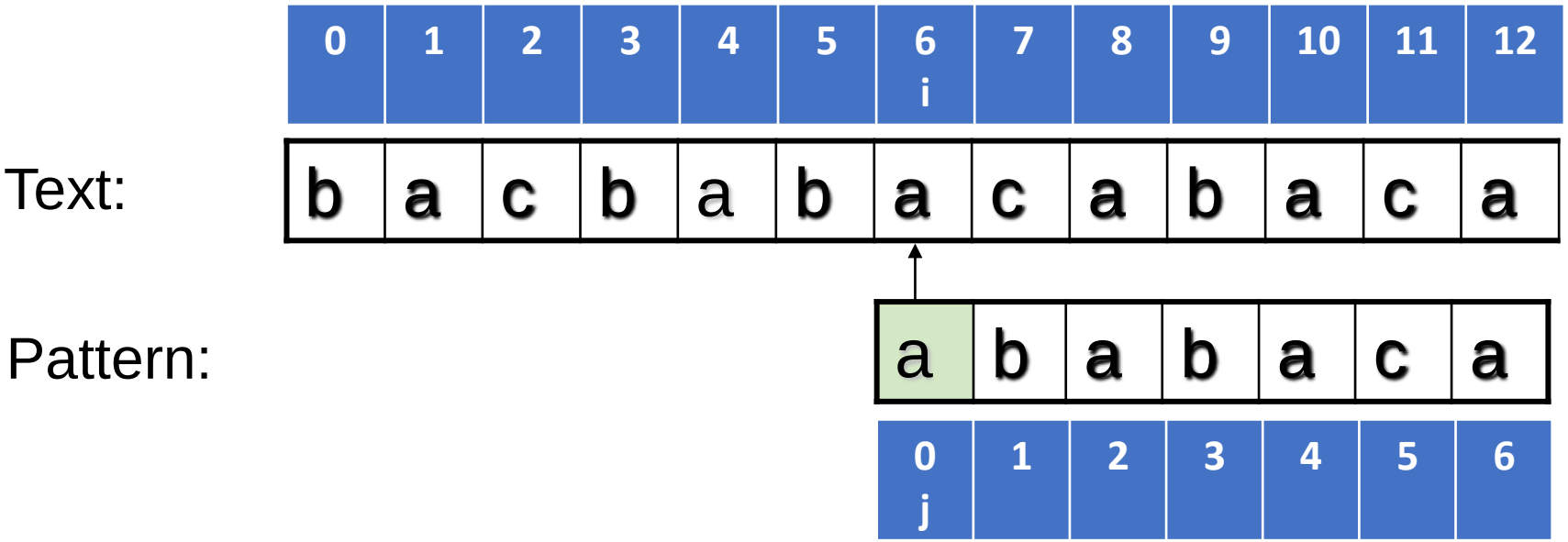
Here, I is shifting in the string. This is extra work in basic algorithm.

Brute-Force : String Searching-Example (Contd.,)



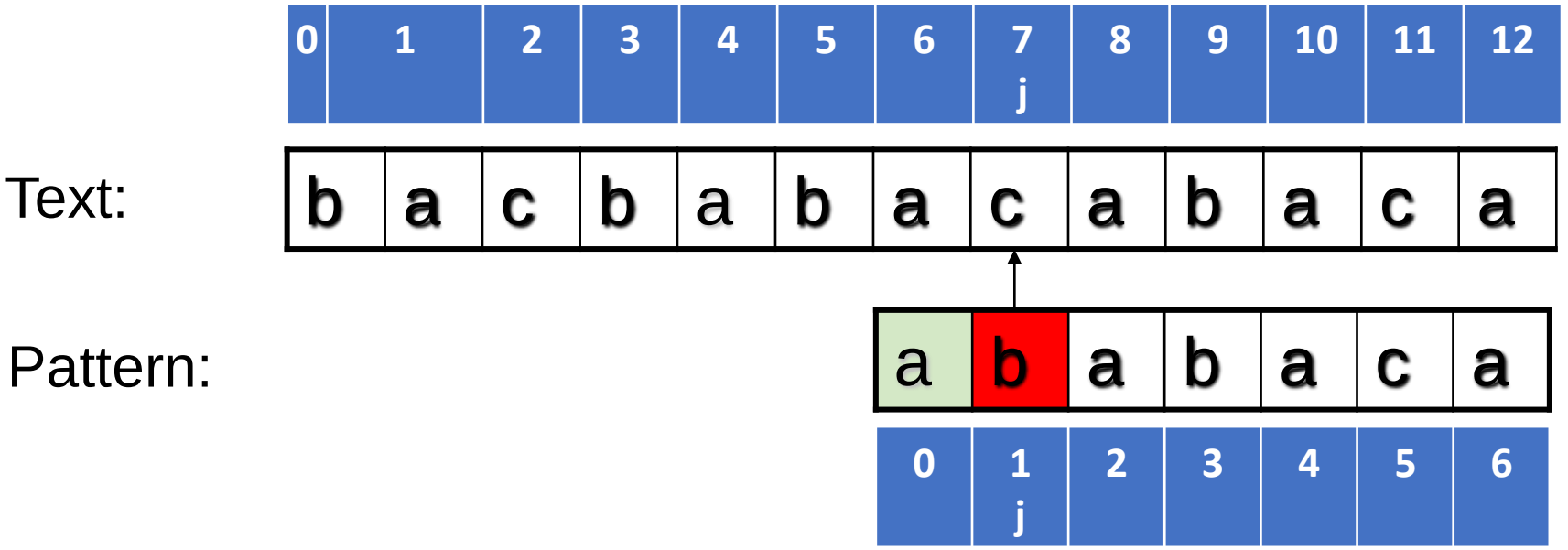
1. There is a mismatch, j resets to 0 in the next step
2. Move i to the next position = 6 (Cur. start position of i = 5, next position = cur + 1 = 6)

Brute-Force : String Searching-Example (Contd.,)



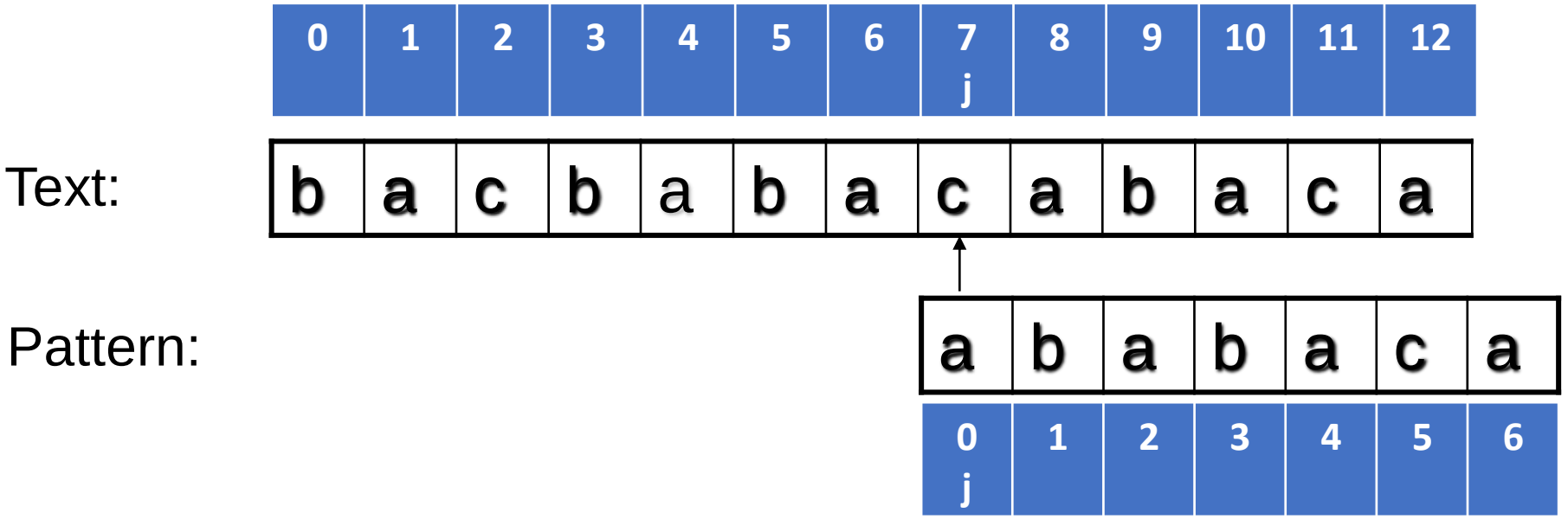
1. **Cur i Start position = 6**
2. **There is a match, move i as well as j**

Brute-Force : String Searching-Example (Contd.,)



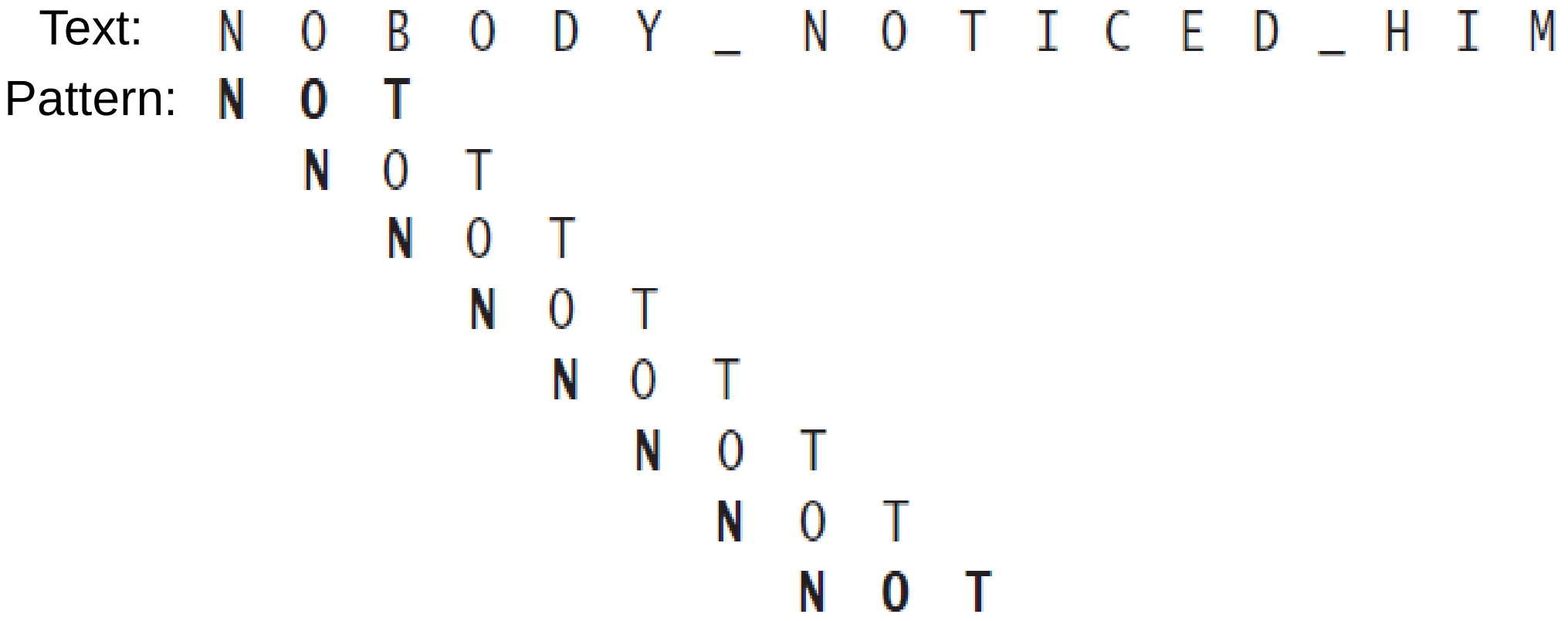
1. There is a mismatch, j resets to 0 in the next step
2. Move i to the next position = 7 (Cur. start position of i = 6, next position = cur + 1 = 7)

Brute-Force : String Searching-Example (Contd.,)



1. There is a mismatch, j resets to 0 in the next step
2. Move i to the next position = 7 (Cur. start position of i = 6, next position = cur + 1 = 7)
3. Since the length of the pattern exceeds the length of the remaining text to be compared, the process can be terminated.

Brute-Force : String Searching-Example (Contd.,)



Brute-Force : String Searching - Algorithm

ALGORITHM *BruteForceStringMatch*$(T[0..n-1],\ P[0..m-1])$

```
//Implements brute-force string matching
//Input: An array T[0..n − 1] of n characters representing a text and
//       an array P[0..m − 1] of m characters representing a pattern
//Output: The index of the first character in the text that starts a
//        matching substring or −1 if the search is unsuccessful
for i ← 0 to n − m do
    j ← 0
    while j < m and P[j] = T[i + j] do
        j ← j + 1
    if j = m return i
return −1
```

Brute-Force : String Searching – Algorithm (Contd.,)

Mathematical Analysis of algorithm:

- **Input Size:** n (as m<<n)
- **Basic Operation:** Comparison
- Does number of times basic operation execution depend only on input size?
  - No!!!!. It also depends on the specifics of the input.
  - Hence best, worst and average case efficiencies has to be examined.
- **Best Case: Ω(m)** as pattern of length m matches in the beginning of the text.
- **Average Case: Ω(n+m)** [with equal probability assumption, the average number of comparisons is proportional to the sum of the lengths of the text and the pattern]

  O(n) [for searching random text, efficiency is considerably better]

Brute-Force : String Searching – Algorithm (Contd.,)

Mathematical Analysis of algorithm: (Contd.,)

- **Worst Case**:

$$C(n) = \sum_{i=0}^{n-m} \sum_{j=0}^{m} 1$$

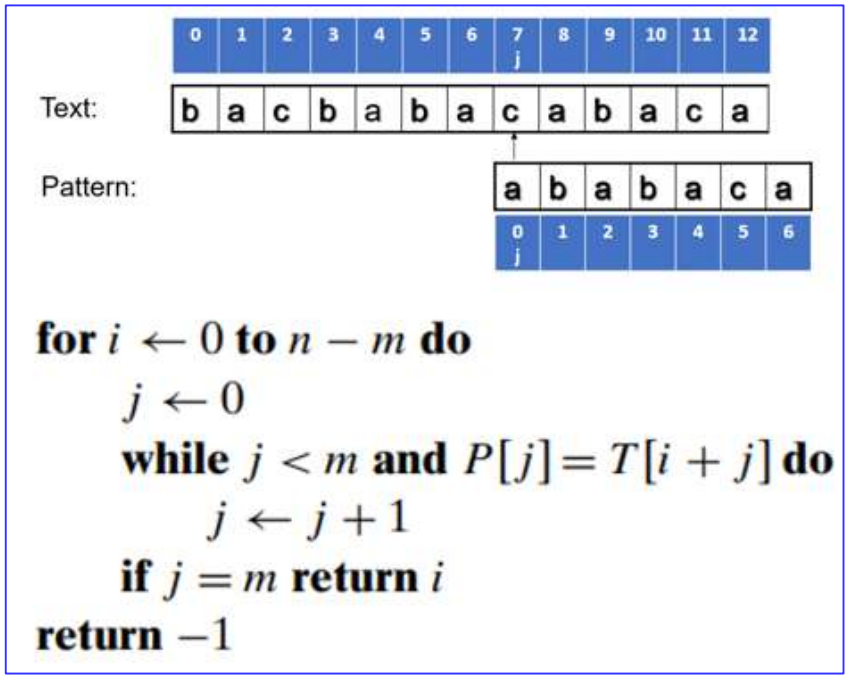
$$= \sum_{i=0}^{n-m} (m - 0 + 1)$$

$$= \text{m.} \sum_{i=0}^{n-m} 1$$

$$= \text{m(n-m-0+1)}$$

$$= \text{m.(n-m+1)}$$

Therefore, C(n) = O(nm)



It means that, it may have to make all *m* comparisons before shifting the pattern, and this can happen for (n–m+1) possible starting positions in the input text.

Exercises

- **Pattern:** 001011
  Text: 10010101101001100101111010

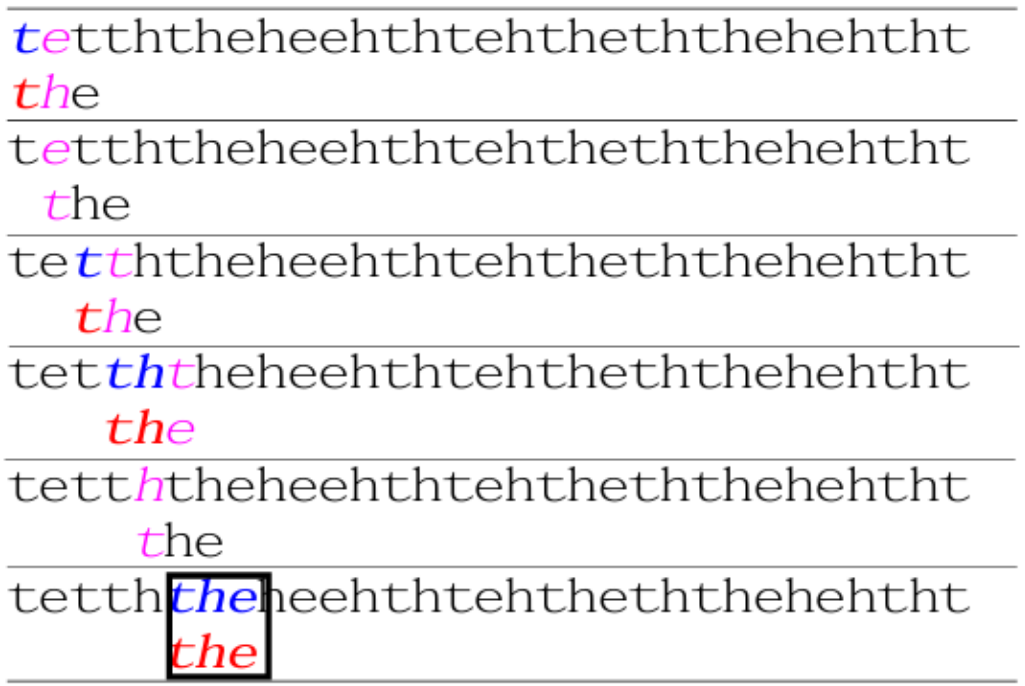
- **Pattern:** happy

  Text: It is never too late to have a happy childhood.

Brute Force Pseudo-Code

- Here's the pseudo-code

```
do
  if (text letter == pattern letter)
    compare next letter of pattern to next
    letter of text
  else
    move pattern down text by one letter
while (entire pattern found or end of text)
```



Brute Force-Complexity

- Given a pattern M characters in length, and a text N characters in length...
- **Worst case**: compares pattern to each substring of text of length M. For example, M=5.

1) *AAAAA*AAAAAAAAAAAAAAAAAAAAH
   AAAAH **5 comparisons made**
2) *AAAAA*AAAAAAAAAAAAAAAAAAAAH
    AAAAH **5 comparisons made**
3) *AAAAA*AAAAAAAAAAAAAAAAAAAAH
     AAAAH **5 comparisons made**
4) *AAAAA*AAAAAAAAAAAAAAAAAAAAH
      AAAAH **5 comparisons made**
5) *AAAAA*AAAAAAAAAAAAAAAAAAAAH
       AAAAH **5 comparisons made**

   ....

N) AAAAAAAAAAAAAAAAAAAAA*AAAAH*
   5 comparisons made *AAAAH*

- Total number of comparisons: M (N-M+1)
- Worst case time complexity: O(MN)

Brute Force-Complexity(cont.)

- Given a pattern M characters in length, and a text N characters in length...
- **Best case if pattern found**: Finds pattern in first M positions of text. For example, M=5.

1) *AAAAA*AAAAAAAAAAAAAAAAAAAAAH
   AAAAA **5 comparisons made**

- Total number of comparisons: M
- Best case time complexity: O(M)

Brute Force-Complexity(cont.)

- Given a pattern M characters in length, and a text N characters in length...
- **Best case if pattern not found**: Always mismatch on first character. For example, M=5.

1) *A*AAAAAAAAAAAAAAAAAAAAAAAAH
   *O*OOOH **1 comparison made**
2) A*A*AAAAAAAAAAAAAAAAAAAAAAAH
   *O*OOOH **1 comparison made**
3) AA*A*AAAAAAAAAAAAAAAAAAAAAAH
   *O*OOOH **1 comparison made**
4) AAA*A*AAAAAAAAAAAAAAAAAAAAAH
   *O*OOOH **1 comparison made**
5) AAAA*A*AAAAAAAAAAAAAAAAAAAAH
   *O*OOOH **1 comparison made**

   ...

N) AAAAAAAAAAAAAAAAAAAAAAA*A*AAAH
   1 comparison made *O*OOOH

- Total number of comparisons: N
- Best case time complexity: O(N)

Needle in a Haystack

- Search for the 10-symbol "needle" "helen hunt" in the Internet "haystack" with many TBs of data
- The brute force algorithm amounts to: "Look in this corner, now in this other corner, then over there, and so on."
- The Internet holds some 1^+ trillion pages, growing by billions a day; each page on average contains in excess of 10 KB, say

$m \cong 10$

$n \cong 10^{12} \times 10^4 = 10^{16}$ B = 10 PB (Petabyte) = 0.01 EB (Exabyte)

$O(mn) \cong 10^{17}$ comparisons $\Rightarrow 10^8$ s (> 3 years), with 10^9 comparisons / s

Needle in a Haystack

- The brute force matcher is inefficient because it totally ignores information gained about the text T for one value of the shift s when it considers other values of s.
- Such information can be quite valuable, however.
- For example:
  - say pattern P = "aaab"
  - and we find that s = 0 is valid
  - then can the shift s = 1 be valid?
  - what about shifts s = 2 or s = 3?
  - none are valid, since T[4] = b.

Needle in a Haystack

- For a particular pattern and unpredictable data strings, preprocess the pattern so that searching for it in various data strings becomes faster

Analogy: Magnetize the needle

To improve Brute force method on String matching

KMP method - Knuth Morris-Pratt Algorithm

Why is KMP a Dynamic Programming Algorithm?

No, **KMP (Knuth-Morris-Pratt) algorithm is not a Dynamic Programming (DP) algorithm**. Instead, it is a **greedy** string matching algorithm that uses preprocessing to achieve efficient pattern searching.

However, **the LPS (Longest Prefix Suffix) table construction in KMP has similarities to DP**, because:

- It builds the solution **incrementally** for each prefix of the pattern.
- It stores previously computed values (prefix lengths) and reuses them, avoiding redundant computations.

Why is KMP a Dynamic Programming Algorithm?

But **KMP does not follow the classical DP structure** because:

1. **No explicit recurrence relation**: DP problems typically have a recurrence relation (e.g., `dp[i]` depends on previous states in a well-defined way). KMP's LPS table updates based on conditions rather than a recurrence formula.
2. **No optimal substructure property**: In DP, solutions to subproblems contribute optimally to the final solution. In KMP, we only use the LPS table for efficient skipping, not for building an optimal solution.
3. **No overlapping subproblems**: DP typically solves subproblems repeatedly and stores results to avoid recomputation. KMP's LPS table does not recompute values—it builds the prefix function iteratively in a single pass.

Why is KMP a Dynamic Programming Algorithm?

KMP is a **preprocessing-based greedy algorithm**, not a dynamic programming algorithm. However, the way the **LPS table** is built has a **DP-like feel** because it stores and reuses information.

KMP – Knuth Morris-Pratt Algorithm

KMP – Knuth Morris-Pratt Algorithm

- For a given pattern, we can find prefix or suffix of the pattern
- What is the **prefix**?
- Need to take subset of the alphabets given in pattern. But we must **start from the beginning** of the pattern only
- How many characters that we can take for a subset?
  - **as many as u want.**

Pattern:

a	b	c	d	a	b	c

Prefix: a, ab, abc, abcd, and more

KMP – Knuth Morris-Pratt Algorithm

- For a given pattern, we can find prefix or suffix of the pattern
- What is the **Suffix**?
- Need to take subset of the alphabets given in pattern. But we must start from the **right hand side** of the pattern only
- How many characters that we can take for a subset?
  - **as many as u want.**

Pattern:

a	b	c	d	a	b	c

Suffix: c, bc, abc, dabc and more

KMP – Knuth Morris-Pratt Algorithm

- Idea of KMP: inside the pattern,
- Is there any prefix same as suffix?

Pattern:

a	b	c	d	a	b	c

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KMP – Knuth Morris-Pratt Algorithm

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Pattern:

a	b	c	d	a	b	c

Prefix: a, ab, abc, abcd, and more

Suffix: c, bc, abc, dabc and more

This is the prefix (abc) that also appears in suffix.

Idea of KMP: This is the observation what KMP algorithm will do on a pattern to avoid the no. of comparisons
i.e. Need to observe:- Is the beginning part of the pattern appearing again anywhere else in the pattern or not?

KMP – Knuth Morris-Pratt Algorithm

- In KMP algor, For a pattern, we generate a π table or LPS(Longest Prefix/Suffix) table

i Index always starts from 1:

	0	1	2	3	4	5	6	7	8	9
	j	i								
P1:	a	b	c	d	a	b	e	a	b	f
LPS/failure():	0									

```
if P[i] = P[j] then:
    j ← j + 1
    f[i] ← j
    i ← i + 1
else if j > 0 then:
    j ← f[j - 1]  // Backtrack using failure function
else:
    f[i] ← 0
    i ← i + 1
```

a is appearing anywhere ?
Together with a, b is also appearing.
ab is appearing 3 times.

What do we do now?
Prepare a LPS table.

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	1	2								
P1:	a	b	c	d	a	b	e	a	b	f
LPS:	0	0	0	0	1	2	0	1	2	0

Short way to prepare LPS table

a is appearing anywhere ?
Together with a, b is also appearing.
ab is appearing 3 times.

What do we do now?
Prepare a LPS table.
0 – first time appears, since no match
The 2nd "ab" matches with 1 and 2 indices.

KMP – Knuth Morris-Pratt Algorithm

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Index always **starts** from **1**:

1	2								

P1:

a	b	c	d	a	b	e	a	b	f

LPS/failure():

0	0	0	0	1	2	0	1	2	0

Short way to prepare LPS table

The failure function $f(j)$ is computed as follows:

1. $f(0) = 0$ → Always 0
2. $f(1) = 0$ → No proper prefix which is also a suffix
3. $f(2) = 0$ → No proper prefix which is also a suffix
4. $f(3) = 0$ → No proper prefix which is also a suffix
5. $f(4) = 1$ → Prefix "**a**" is also a suffix
6. $f(5) = 2$ → Prefix "**ab**" is also a suffix
7. $f(6) = 0$ → No proper prefix which is also a suffix
8. $f(7) = 1$ → Prefix "**a**" is also a suffix
9. $f(8) = 2$ → Prefix "**ab**" is also a suffix
10. $f(9) = 0$ → No proper prefix which is also a suffix

KMP Approach For Pattern Search

```
Algorithm KMPFailureFunction(P):
    Input: String P (pattern) with m characters
    Output: Failure function f for P

    f[0] ← 0   // First character has failure value 0
    j ← 0      // Tracks length of the longest prefix suffix

    i ← 1
    while i < m do:
        if P[i] = P[j] then:
            j ← j + 1
            f[i] ← j
            i ← i + 1
        else if j > 0 then:
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KMP – Knuth Morris-Pratt Algorithm

- In KMP algor, For a pattern, we generate a π table or LPS(Longest Prefix/Suffix) table

Index always starts from 1:	1	2	3							
P2:	a	b	c	d	e	a	b	f	a	b	c
LPS/failure():	0	0	0	0	0	1	2	0	1	2	3

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What do we do now?
Prepare a LPS table.
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The 2nd "ab" matches with 1 and 2 indices.

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Index always starts from 1:

1	2	3								

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a	b	c	d	e	a	b	f	a	b	c

LPS/failure():

0	0	0	0	0	1	2	0	1	2	3

1. $f(0) = 0 \rightarrow$ Always 0
2. $f(1) = 0 \rightarrow$ No proper prefix which is also a suffix
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9. $f(8) = 1 \rightarrow$ Prefix "**a**" is also a suffix
10. $f(9) = 2 \rightarrow$ Prefix "**ab**" is also a suffix
11. $f(10) = 3 \rightarrow$ Prefix "**abc**" is also a suffix

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Index always starts from 1:	1	2	3						
P3:	a	a	b	c	a	d	a	a	b	e
LPS/failure():	0	1	0	0	1	0	1	2	3	0

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P3:	a	a	b	c	a	d	a	a	b	e
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Index always starts from 1:	1	2	3						
P4:	a	a	a	a	b	a	a	c	d
LPS/failure():	0	1	2	3	0	1	2	0	0

3rd index a => matches with first “aa”
4th index a => matches with first “aaa”

a is appearing anywhere ?
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3rd index a => matches with first “aa”
4th index a => matches with first “aaa”

1. $f(0) = 0 \rightarrow$ Always 0
2. $f(1) = 1 \rightarrow$ Prefix "**a**" is also a suffix
3. $f(2) = 2 \rightarrow$ Prefix "**aa**" is also a suffix
4. $f(3) = 3 \rightarrow$ Prefix "**aaa**" is also a suffix
5. $f(4) = 0 \rightarrow$ No proper prefix which is also a suffix
6. $f(5) = 1 \rightarrow$ Prefix "**a**" is also a suffix
7. $f(6) = 2 \rightarrow$ Prefix "**aa**" is also a suffix
8. $f(7) = 0 \rightarrow$ No proper prefix which is also a suffix
9. $f(8) = 0 \rightarrow$ No proper prefix which is also a suffix

KMP – Knuth Morris-Pratt Algorithm

- In KMP algor, For a pattern, we generate a π table or LPS(Longest Prefix/Suffix) table

- Example:
  - value of the KMP failure function:

j	0	1	2	3	4	5
$P[j]$	a	b	a	b	a	c
$f(j)$						

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f(*j*)	0	0	1	2	3	0

The failure function $f(j)$ (or LPS array) is computed as follows:

1. $f(0) = 0$ (Always 0)
2. $f(1) = 0$ (No proper prefix which is also a suffix)
3. $f(2) = 1$ (Prefix "**a**" is also a suffix)
4. $f(3) = 2$ (Prefix "**ab**" is also a suffix)
5. $f(4) = 3$ (Prefix "**aba**" is also a suffix)
6. $f(5) = 0$ (No proper prefix which is also a suffix for "**ababac**")

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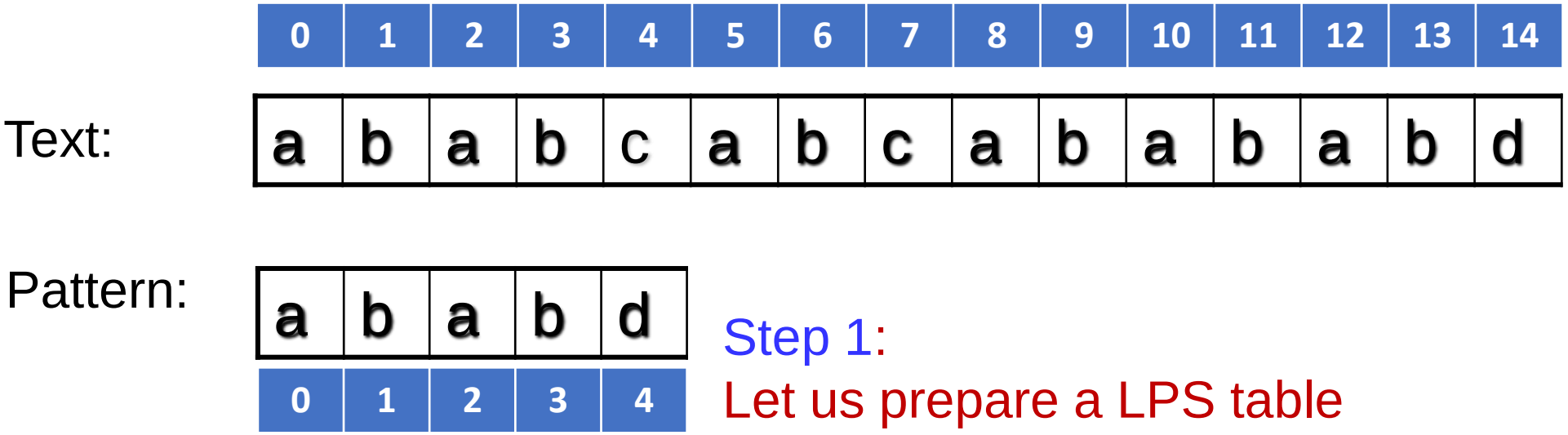
Working of KMP – Knuth Morris-Pratt Algorithm

- Explanation is based on tracing of algorithm.



Step 1:

Let us prepare a LPS table

Working of KMP – Knuth Morris-Pratt Algorithm

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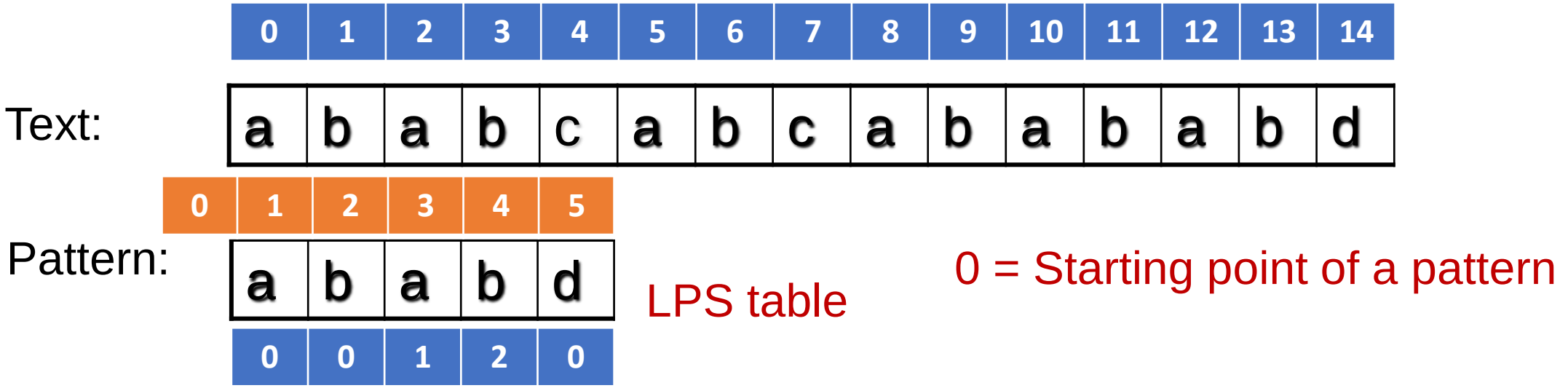

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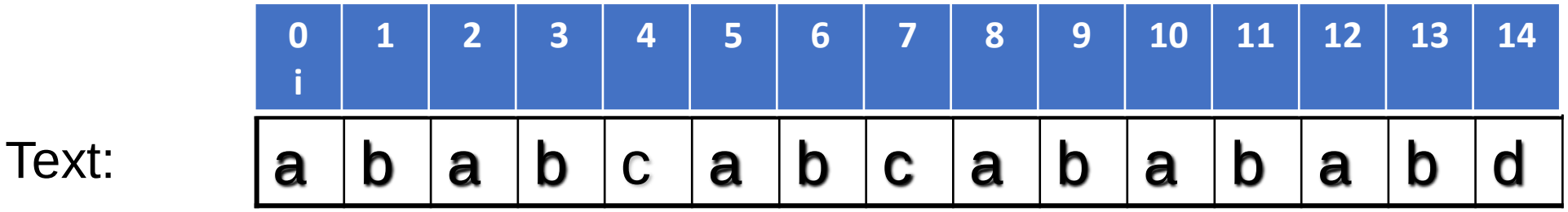

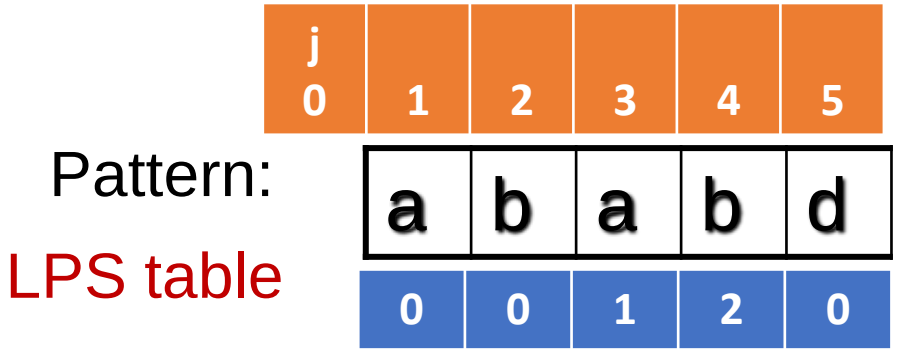

Compare char @(i) and char@(j+1)
If match, inc i and j

Working of KMP – Knuth Morris-Pratt Algorithm

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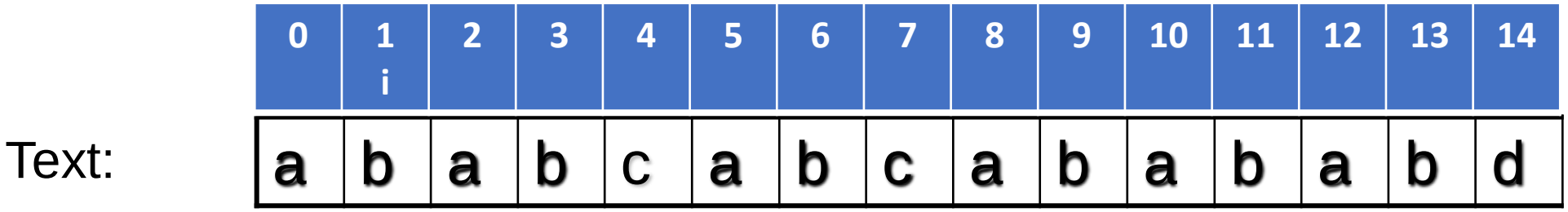

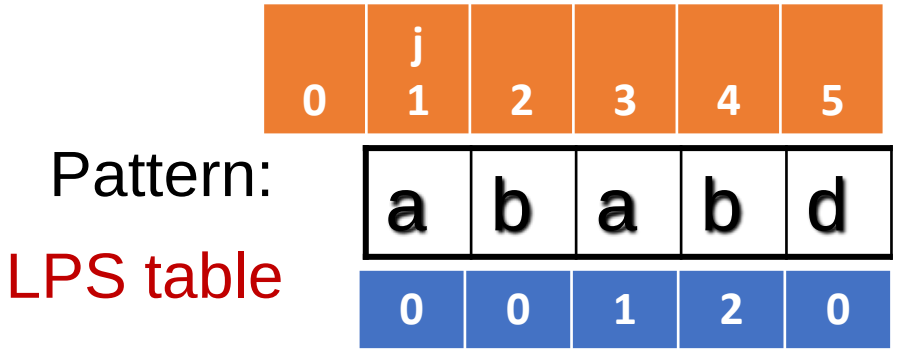

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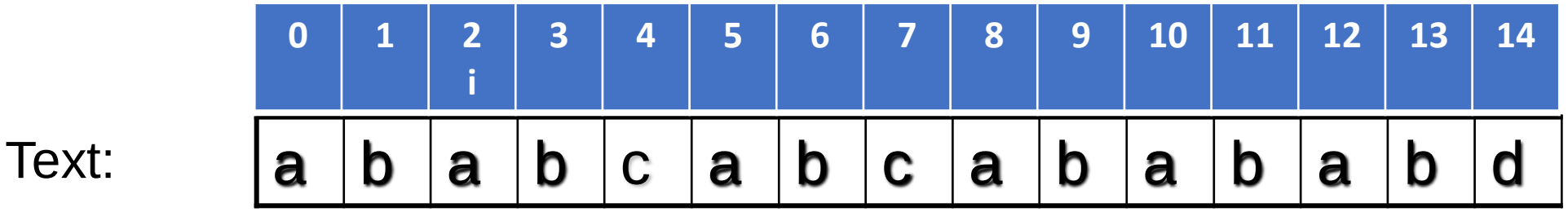

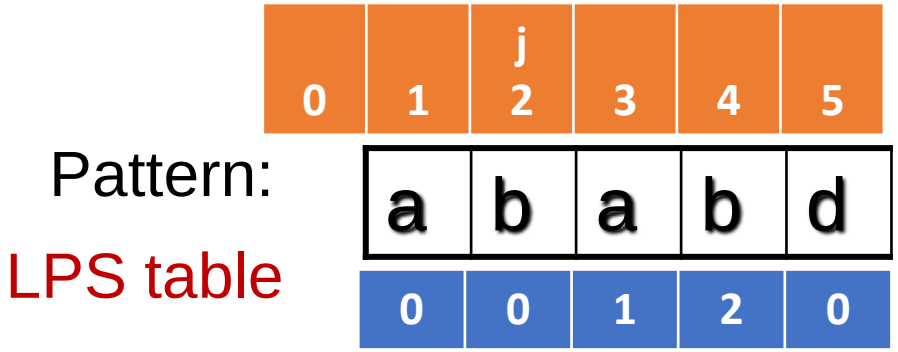

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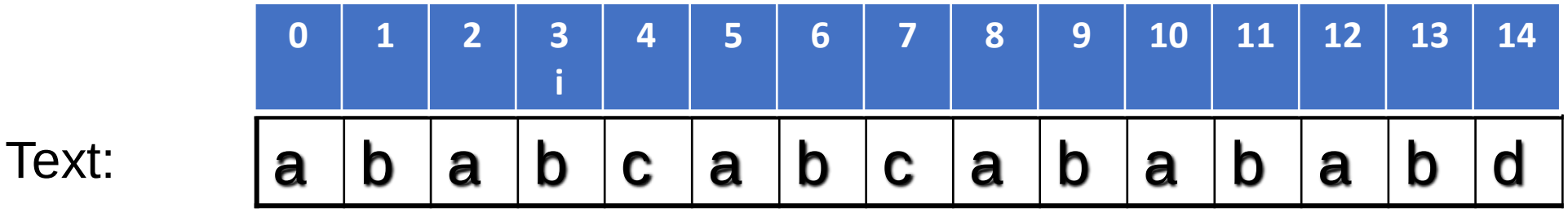

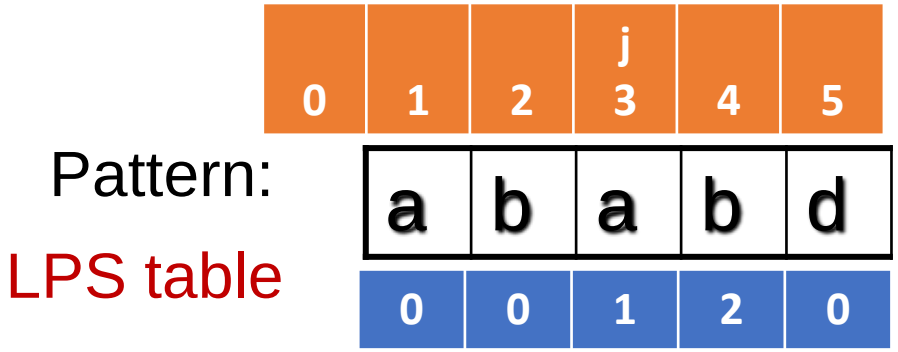

Compare char @(i) and char@(j+1)
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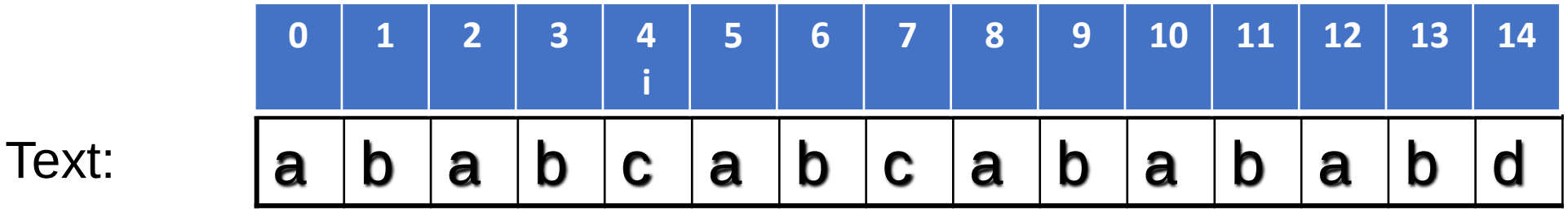
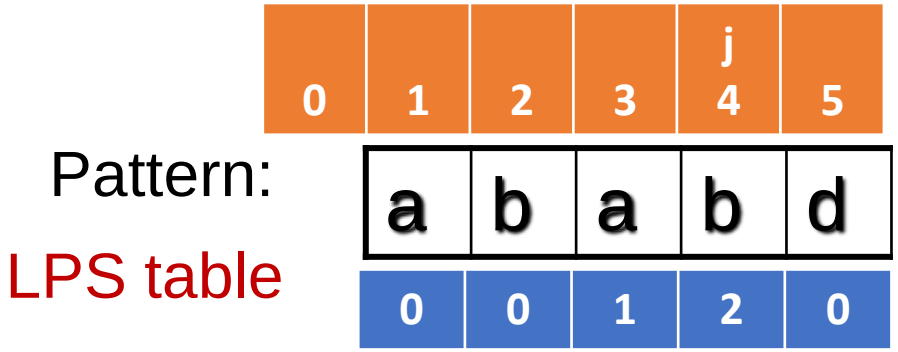




Compare char @(i) and char@(j+1)
If there is mismatch, then
Move j to table index (here = 2)

Working of KMP – Knuth Morris-Pratt Algorithm

- Explanation is based on tracing of algorithm.

Text:

0	1	2	3	4 i	5	6	7	8	9	10	11	12	13	14
a	b	a	b	c	a	b	c	a	b	a	b	a	b	d

Pattern:

0	1	j 2	3	4	5
a	b	a	b	d	

LPS table

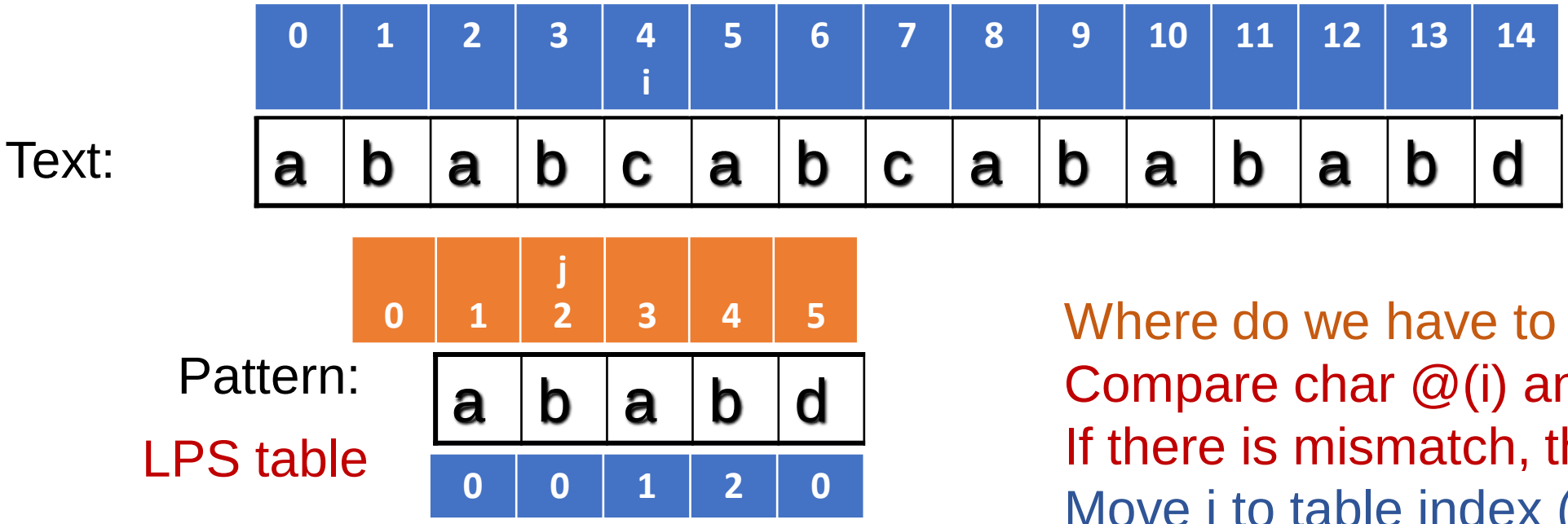
0	0	1	2	0

Compare chat @(i) and char@(j+1)
If there is mismatch, then
Move j to table index (here = 2).

Means, “abab” is repeating. “ab” came 2 times. Already we checked 1 time.
Where do we have to start?

Working of KMP – Knuth Morris-Pratt Algorithm

- Explanation is based on tracing of algorithm.



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Working of KMP – Knuth Morris-Pratt Algorithm

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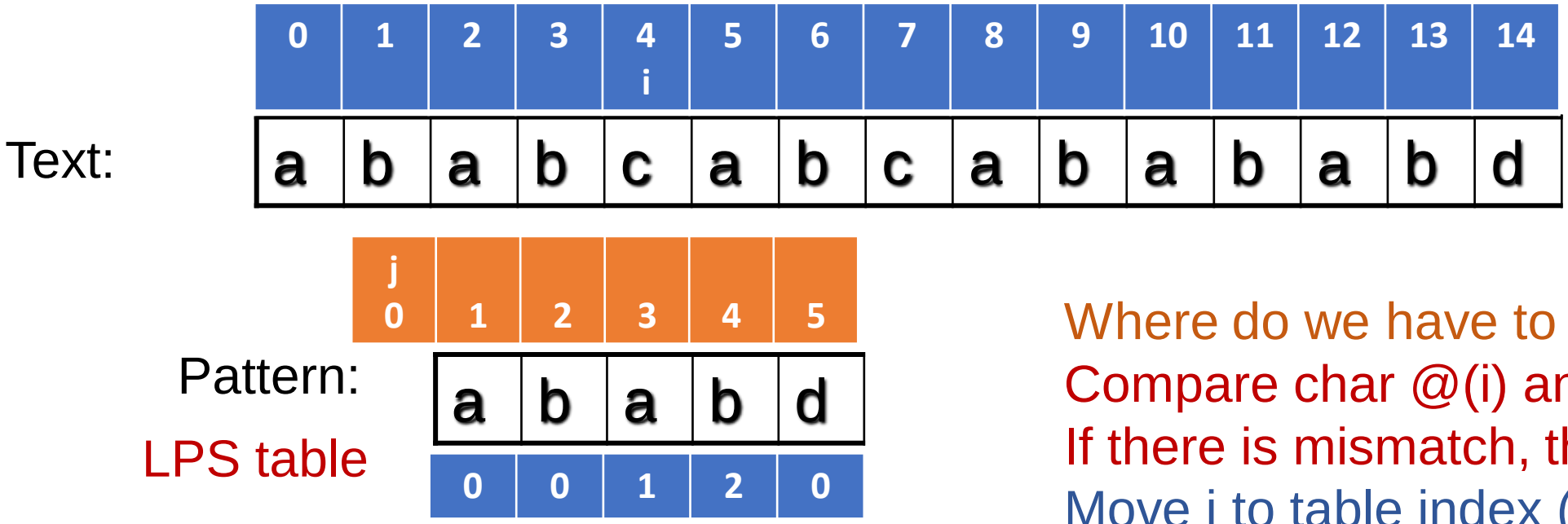

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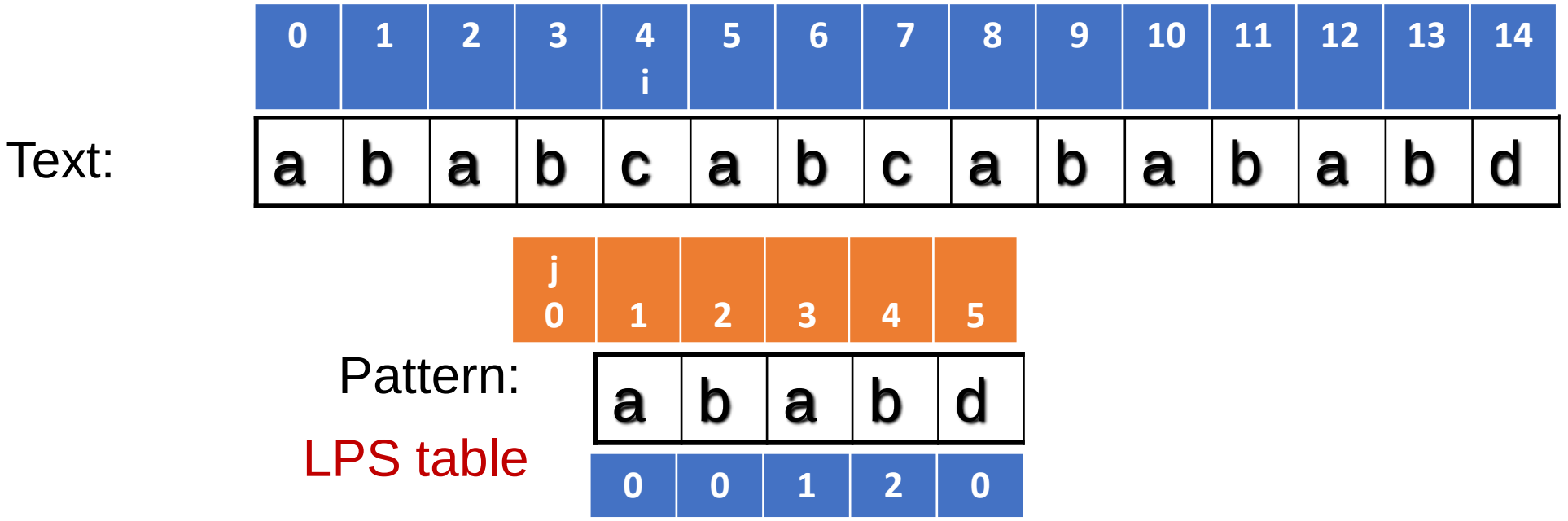


Where do we have to start?

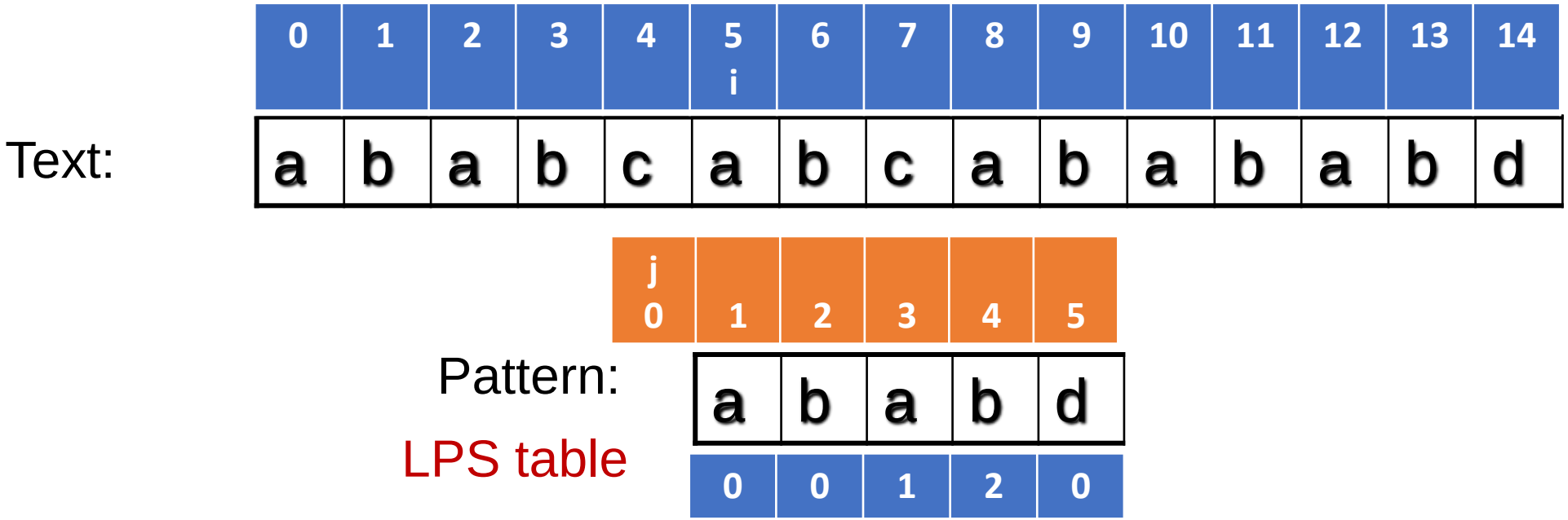
Compare char @(i) and char@(j+1)

Still there is a Mismatch, But **j** is on 0. So we can not move j to its left anymore.

So move **i** to its right.

Working of KMP – Knuth Morris-Pratt Algorithm

- Explanation is based on tracing of algorithm.



In case of mismatch, Only j is moving and i is not backtracking. i.e. i has moved to the next alphabet.

Compare char @(i) and char@(j+1)

Match found. So move i and j

Working of KMP – Knuth Morris-Pratt Algorithm

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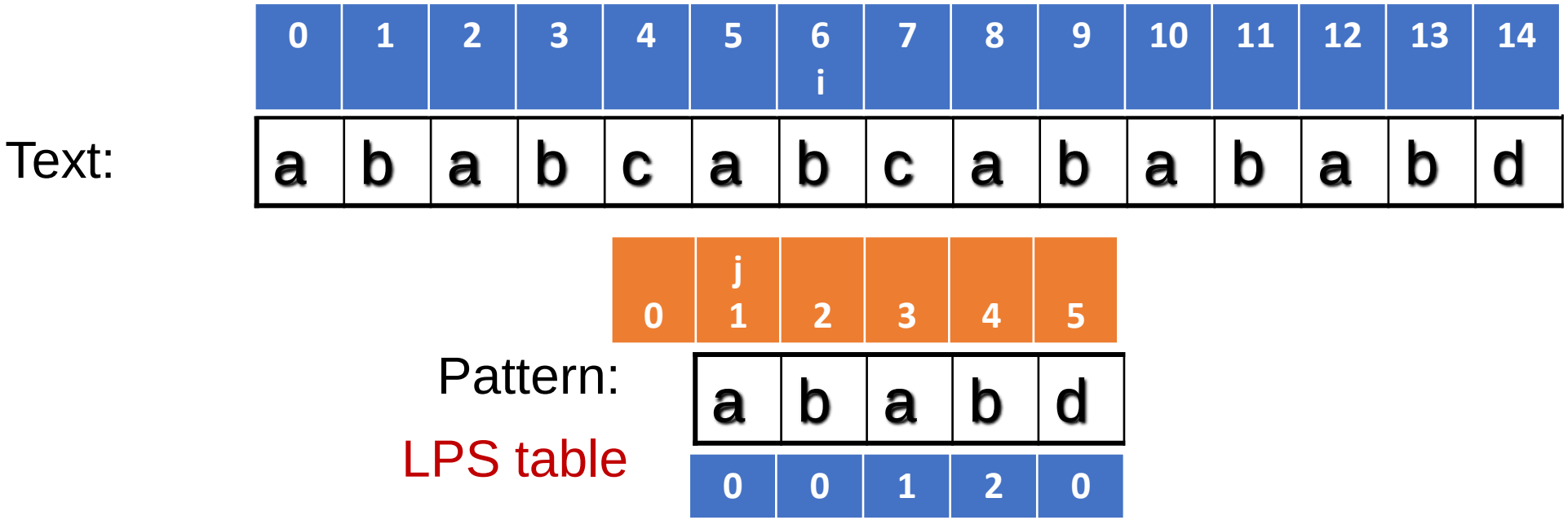

In case of mismatch, Only j is moving and i is not backtracking. i.e. i has moved to the next alphabet.

Compare char @(i) and char@(j+1)

Again, Match found. So move i and j

Working of KMP – Knuth Morris-Pratt Algorithm

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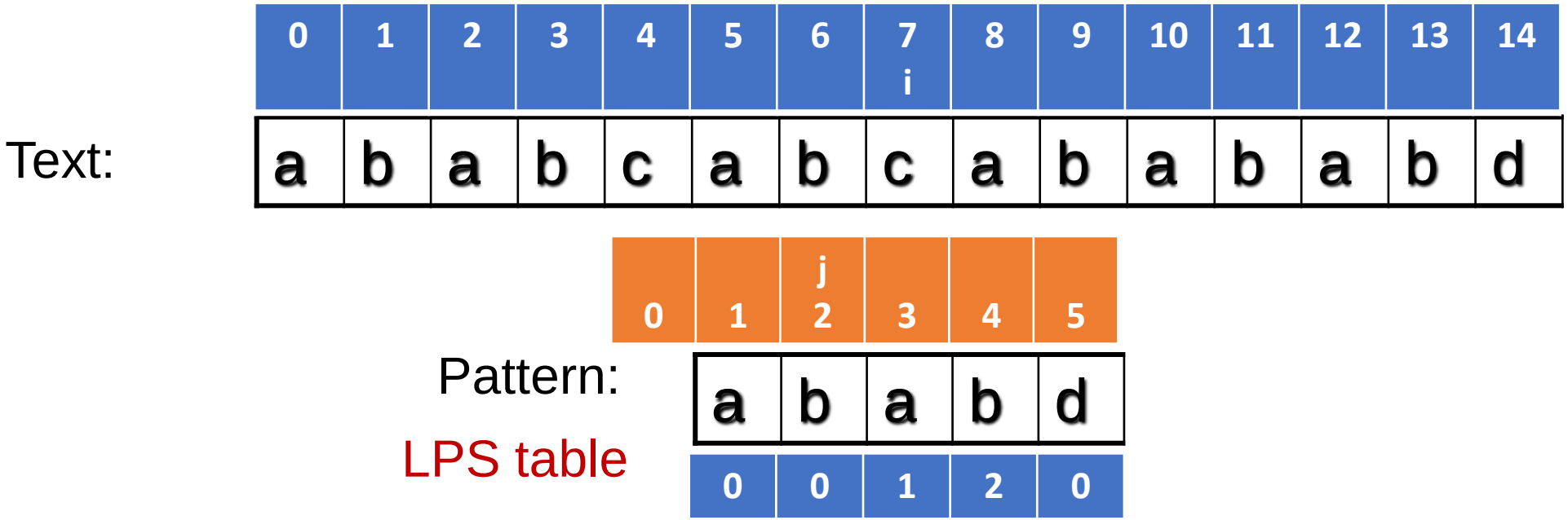

In case of mismatch, Only j is moving and i is not backtracking. i.e. i has moved to the next alphabet.

Compare char @(i) and char@(j+1)

MisMatch found. So move j to table index (here = 0)

Working of KMP – Knuth Morris-Pratt Algorithm

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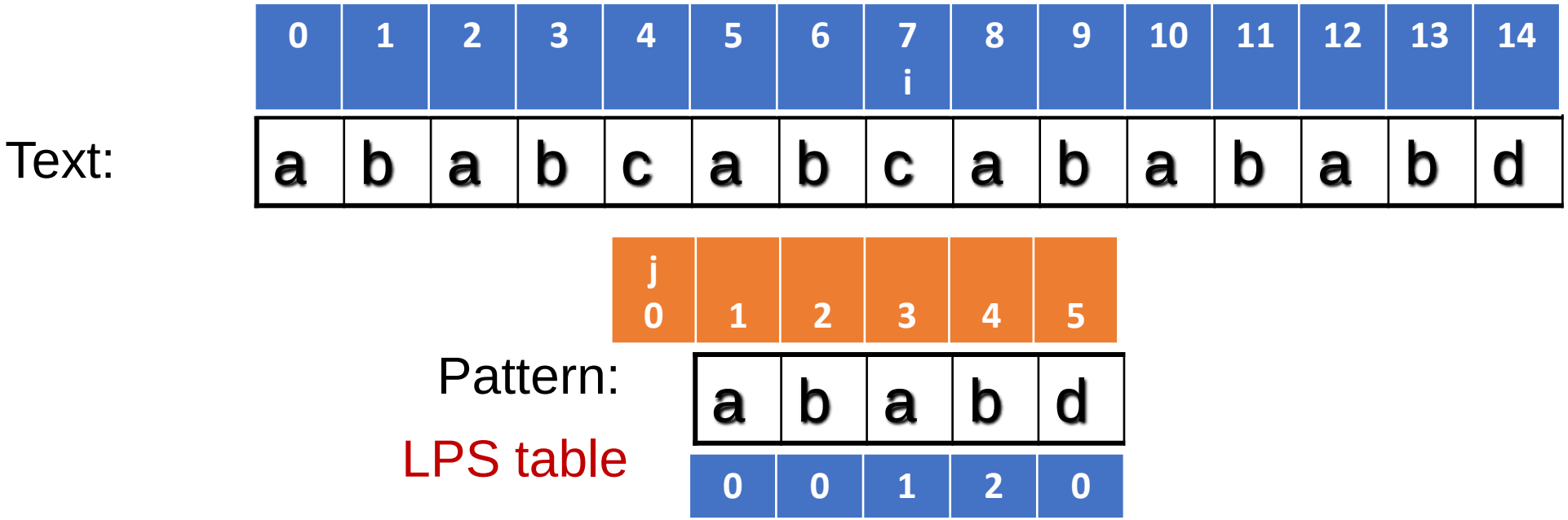

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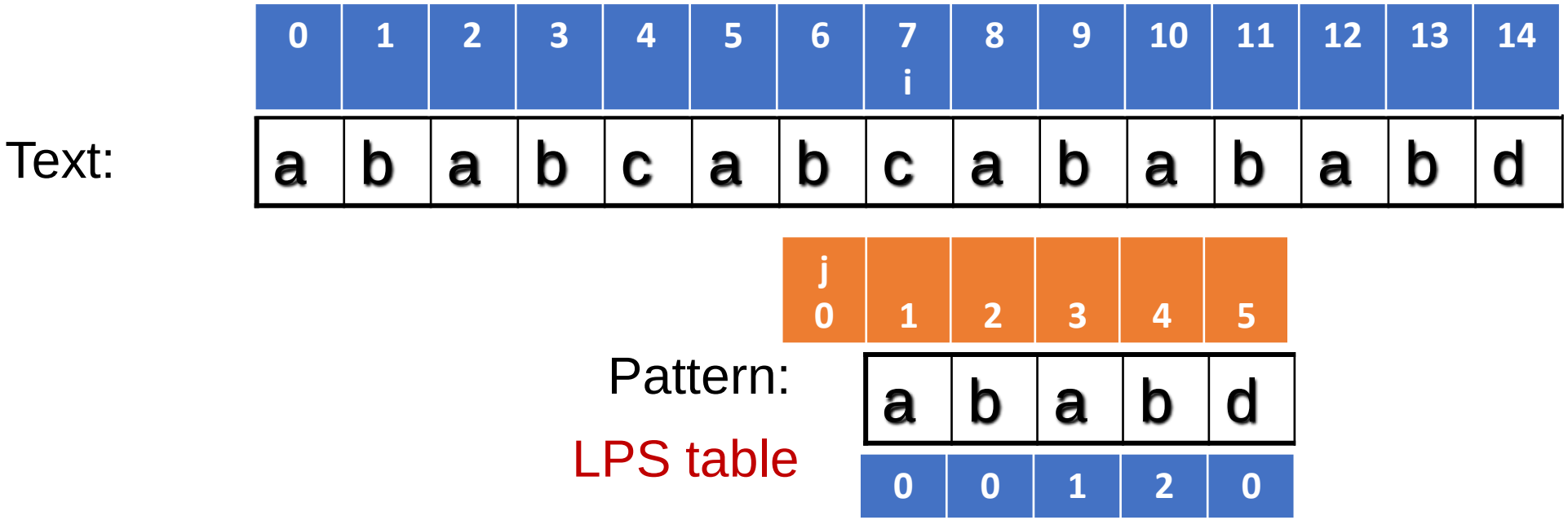

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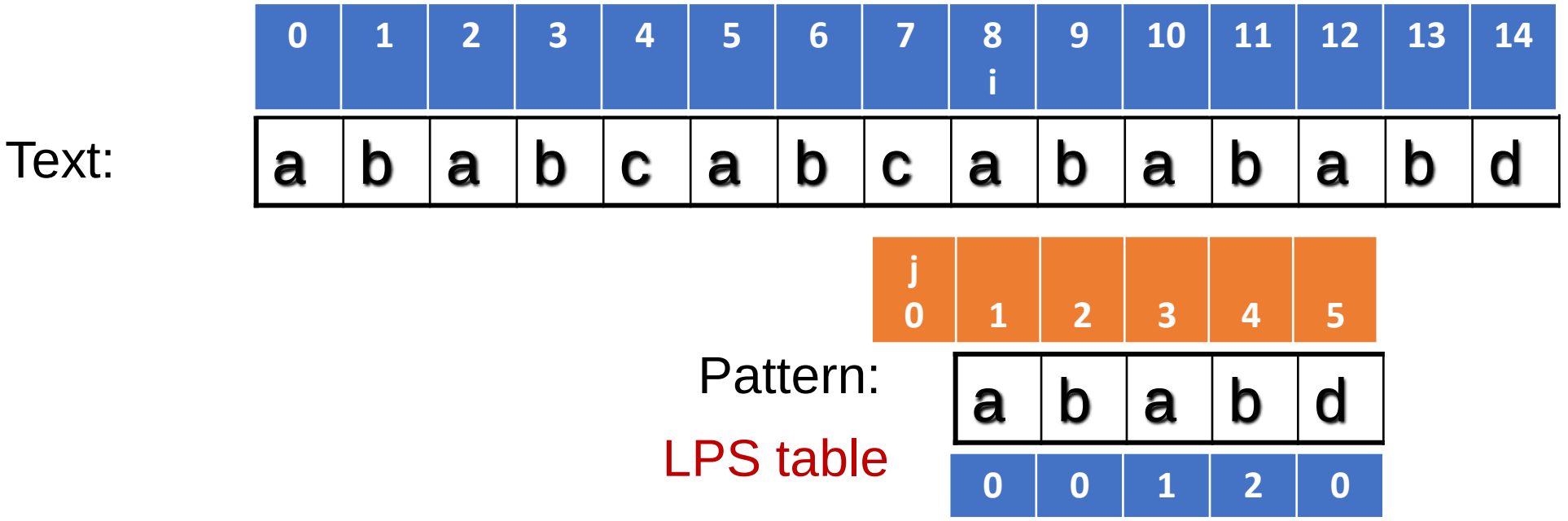

Compare char @(i) and char@(j+1)
Match found. Move i and j

Working of KMP – Knuth Morris-Pratt Algorithm

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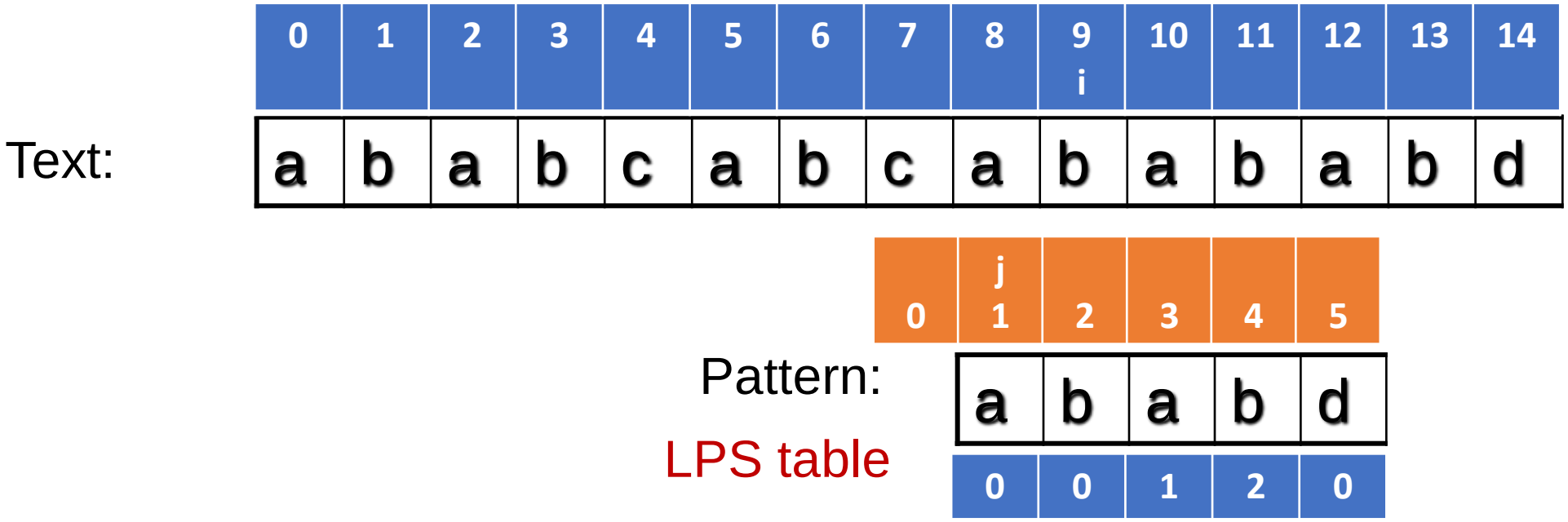

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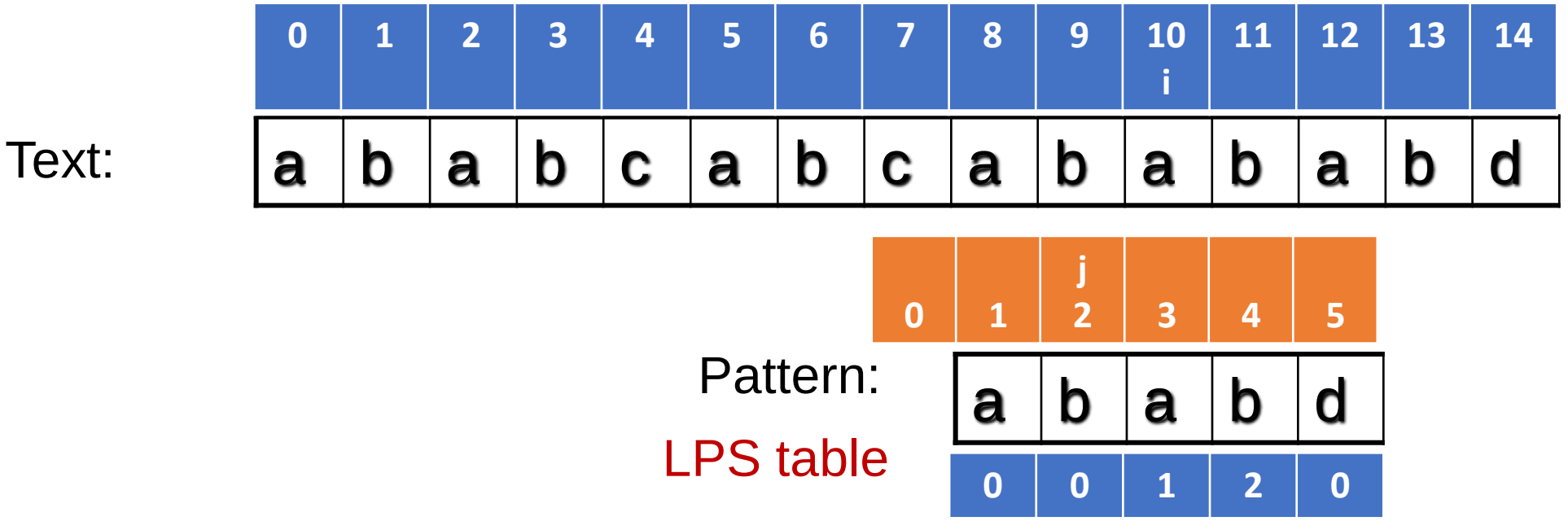

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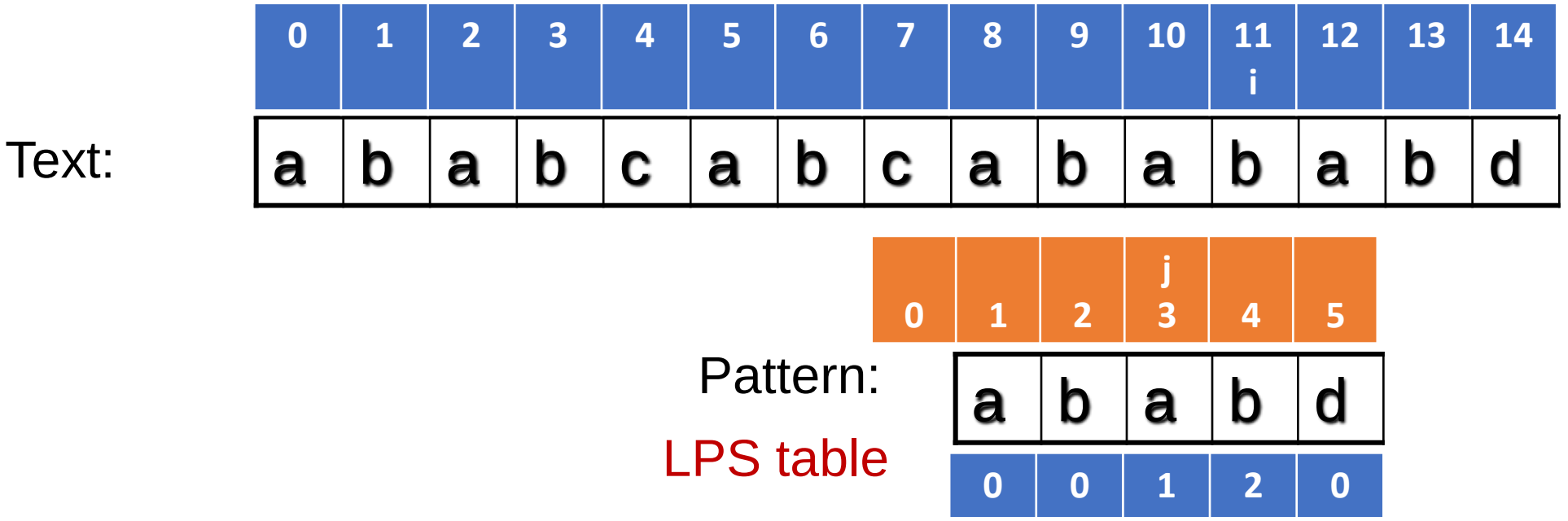

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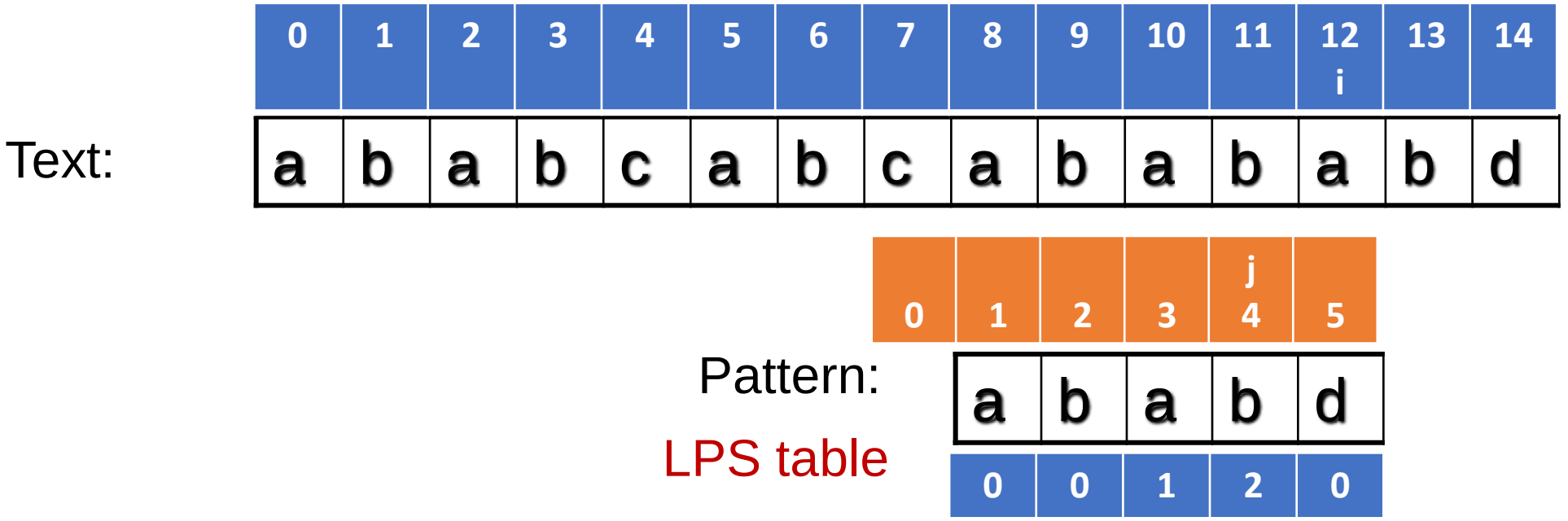

Compare char @(i) and char@(j+1)
MisMatch found. So move j to table index (here = 2)

Working of KMP – Knuth Morris-Pratt Algorithm

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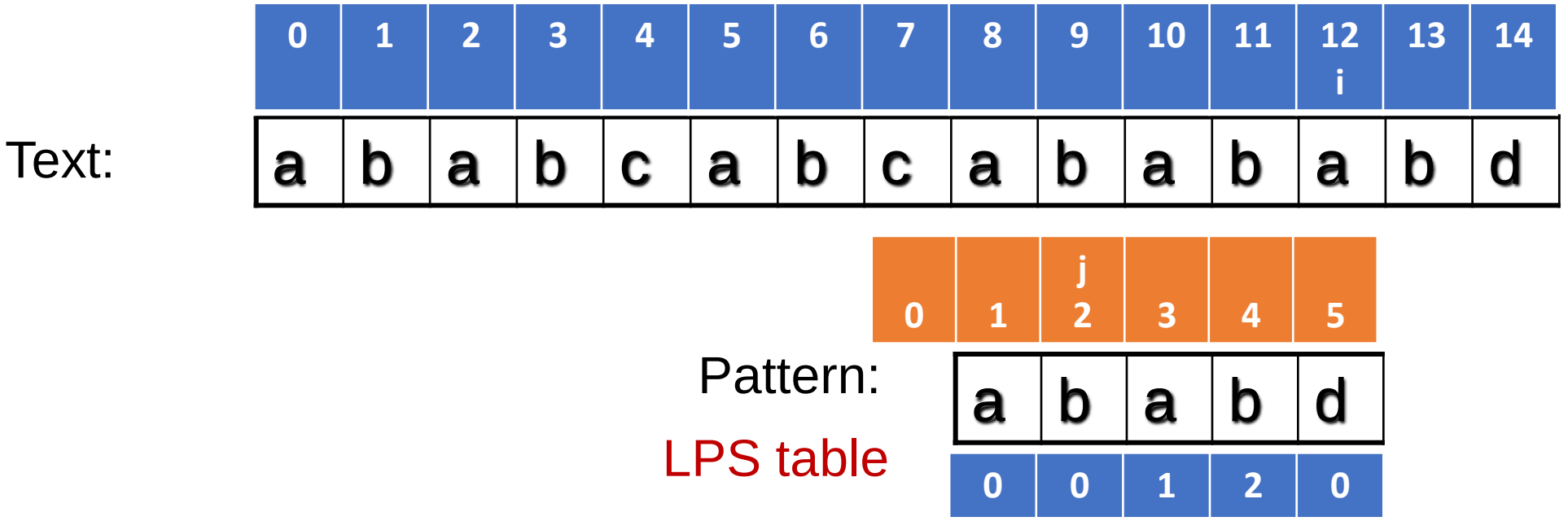

Compare char @(i) and char@(j+1)

Working of KMP – Knuth Morris-Pratt Algorithm

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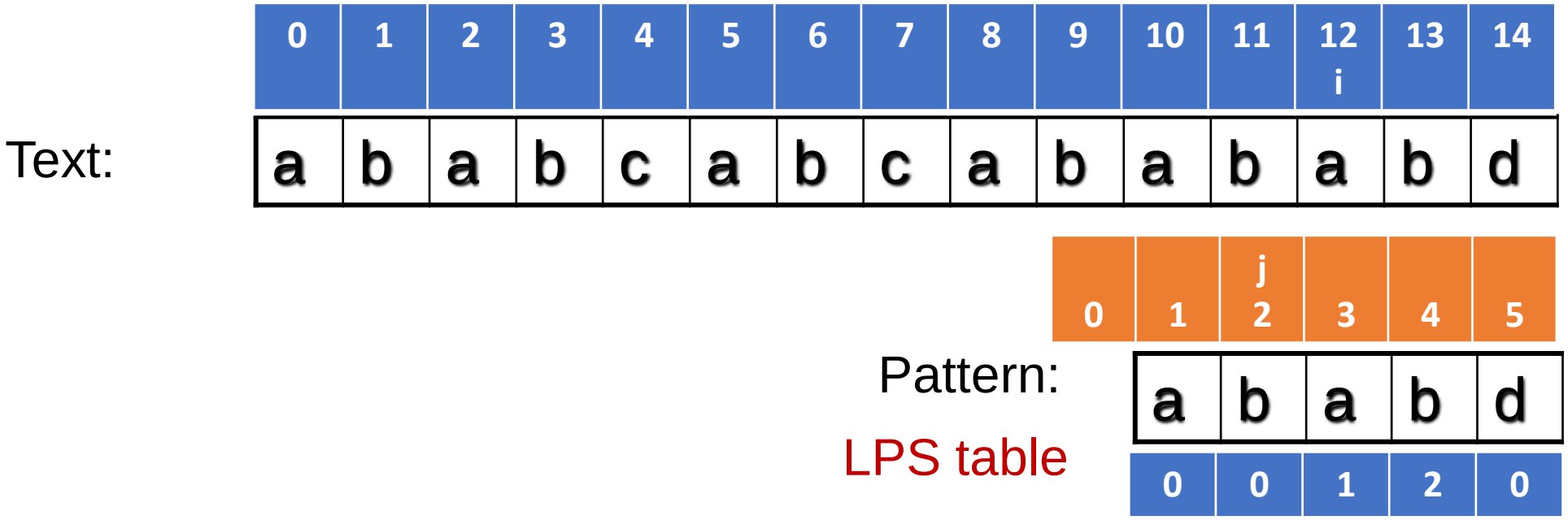

Since “ab” have come 2 times, we can skip the first time, we can start searching from the 2nd time onwards

Compare char @(i) and char@(j+1)
Match found. Move i and j

Working of KMP – Knuth Morris-Pratt Algorithm

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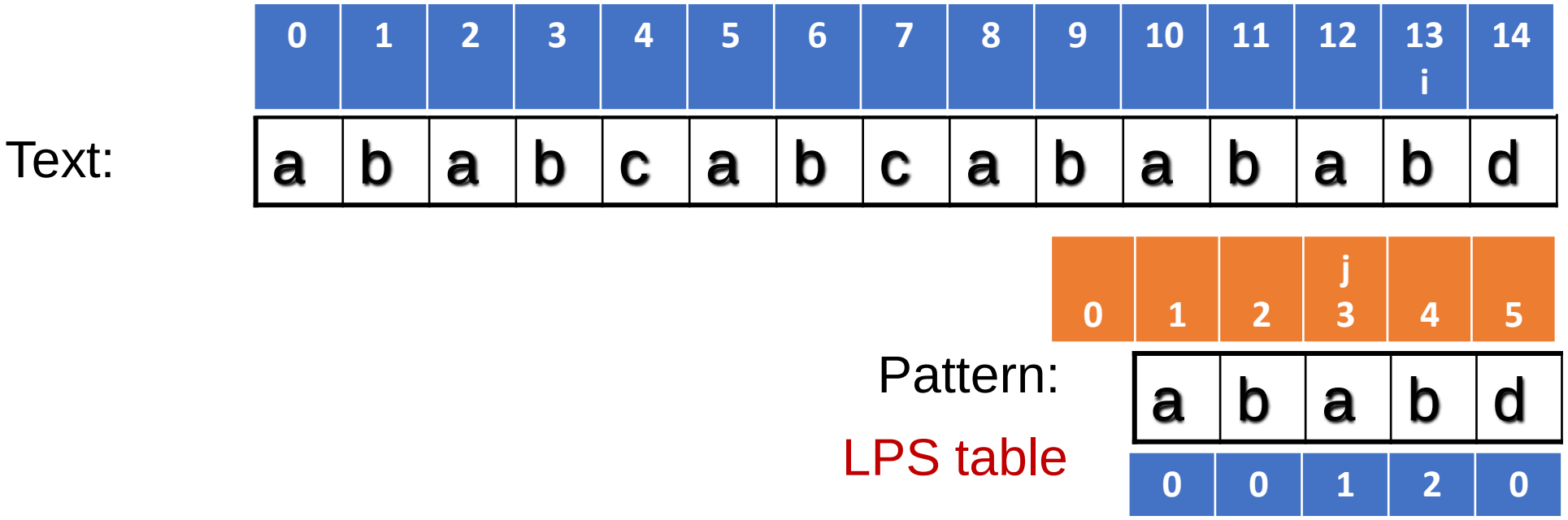

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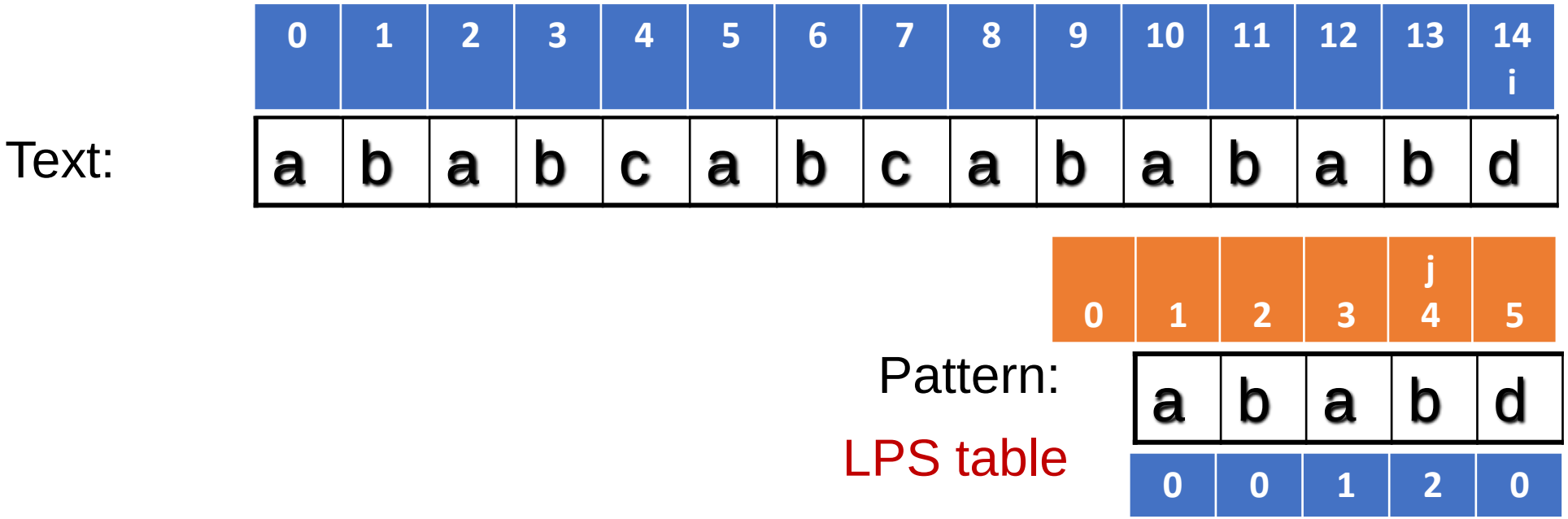

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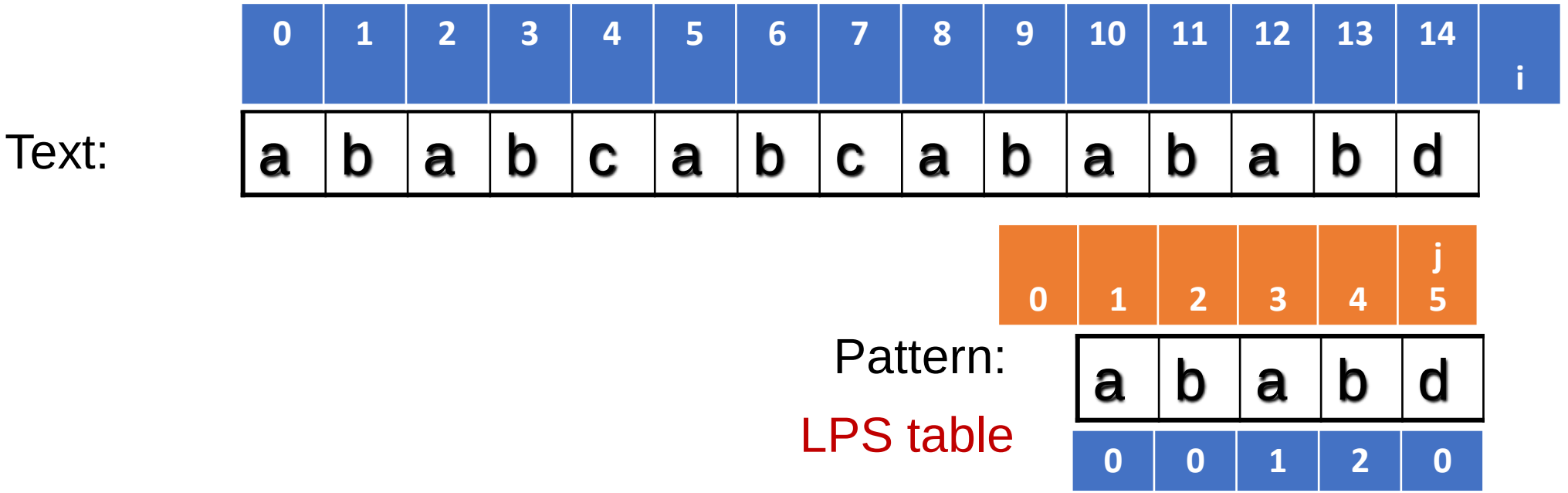


Since ‘i' has reached the end of the string,
Also, we reached the end of the pattern, So now Pattern available inside the string.

Idea behind KMP

- If the “beginning part” of the string is appearing again somewhere else in the string, then **comparison of characters** can be avoided (don’t compare the characters again)
- ‘i’ need not to be moved back again (Don’t move ‘i’ back again). [‘i’ is moved only in the basic naïve brute force method]

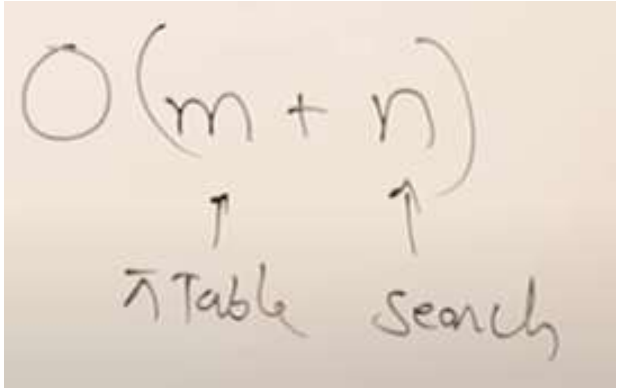
- Moving i only in the forward direction , never backtrack i

KMP - TimeComplexity

	0	1	2	3	4	5	6	7	8	9	10	11	12	13 i	14
Text:	a	b	a	b	c	a	b	c	a	b	a	b	a	b	d

Pattern:	a	b	a	b	d
	0	0	1	2	0

1. How many times that you are passing Text? 1 time
2. How many alphabets ? **n**
3. For preparing a table, parse through the pattern.
   How many alphabets in pattern? **m**



Naive Approach(Brute Force) For Pattern Search

```
void Patternsearch(char* pat, char* str) {
    int M = strlen(pat);
    int N = strlen(str);
    for (int i = 0; i <= N - M; i++) {
        int j;

        for (j = 0; j < M; j++)
            if (str[i + j] != pat[j])
                break;

        if (j == M)
            printf("Pattern found at index %d \n", i);
    }
}
```

KMP Approach For Pattern Search

```
Algorithm KMPFailureFunction(P):
    Input: String P (pattern) with m characters
    Output: Failure function f for P

    f[0] ← 0   // First character has failure value 0
    j ← 0      // Tracks length of the longest prefix suffix

    i ← 1
    while i < m do:
        if P[i] = P[j] then:
            j ← j + 1
            f[i] ← j
            i ← i + 1
        else if j > 0 then:
            j ← f[j - 1]  // Backtrack using failure function
        else:
            f[i] ← 0
            i ← i + 1
```

KMP Approach For Pattern Search (cont…)

```
Algorithm KMPMatch(T, P)
Input:
    - String T (text) of length n
    - String P (pattern) of length m
Output:
    - Index of the first occurrence of P in T, or -1 if P is not a substring of T.

1. Compute failure function f ← KMPFailureFunction(P)
2. i ← 0  (index for T)
3. j ← 0  (index for P)
4. while i < n do:
      a. if P[j] = T[i] then:
            - if j = m - 1 then:
                  return i - m + 1   {Pattern found}
            - i ← i + 1
            - j ← j + 1
      b. else if j > 0 then:
            - j ← f[j - 1]  {Use failure function to skip comparisons}
      c. else:
            - i ← i + 1
5. return -1  {Pattern not found}
```

KMP Algorithm : C++ Implementation

```
// Compute the LPS array (Unchanged)
void computeLPSArray(string pattern, vector<int>& lps) {
    int m = pattern.length();
    int length = 0;  // Length of the previous longest prefix
suffix
    lps[0] = 0;      // First element is always 0
    int i = 1;
```

```
while (i < m) {
        if (pattern[i] == pattern[length]) {
            length++;
            lps[i] = length;
            i++;
        } else {
            if (length != 0) {
                length = lps[length - 1];
            } else {
                lps[i] = 0;
                i++;
            }
        }
    }
}
```

KMP Algorithm : C++ Implementation (cont…)

```
// Modified KMP Search function with j+1 indexing
void KMPSearch(string text, string pattern) {
    int n = text.length();
    int m = pattern.length();
    vector<int> lps(m, 0);

    // Compute LPS array
    computeLPSArray(pattern, lps);

    int i = 1, j = 0;  // Start i at 1, j at 0

while (i < n) {
        while (j > 0 && pattern[j] != text[i]) {
            j = lps[j - 1];  // Backtrack using LPS
        }

        if (pattern[j] == text[i]) {
            j++;  // Move j forward
        }

        if (j == m) {  // Full match found
            cout << "Pattern found at index " << (i - j
+ 1) << endl;
            j = lps[j - 1];  // Correctly update j
        }
        i++; // Move to next character in text
    }
}
```

Drawbacks of Brute force

1. Redundant Comparisons

- **Brute Force:**
  - After a mismatch, it **resets the search** and compares characters again from the beginning of the pattern.
- **KMP:**
  - Uses the **LPS table** to skip unnecessary comparisons, avoiding redundant checks.

📌 **Example:**

Searching `"abab"` in `"abababab"`

- Brute force checks every position.
- KMP skips over repeated calculations using LPS.

Drawbacks of Brute force (contn...)

2. High Time Complexity in Worst Case

- **Brute Force:**
  - Worst-case time complexity: **O(n × m)** (where `n` is text length, `m` is pattern length).
  - In the worst case (e.g., searching `"aaaa"` in `"aaaaaaaaaaaa"`), the algorithm **restarts the comparison** from the next position every time a mismatch occurs.
- **KMP:**
  - Worst-case time complexity: **O(n + m)**.
  - Uses the **LPS (Longest Prefix Suffix) table** to avoid redundant comparisons, making it much faster.

📌 **Example:**

Searching `"abc"` in `"aaaaabc"`

- **Brute Force:** Will retry all comparisons when a mismatch happens.
- **KMP:** Uses previous comparisons to intelligently shift the pattern.

Drawbacks of Brute force (contn…)

3. Inefficiency with Large Text and Pattern

- **Brute Force:**
  - Performs **unoptimized shifting**, leading to **slow performance on large inputs**.
- **KMP:**
  - Uses **preprocessing (LPS table)**, making it **faster on large texts**.

📌 **Example:**

Searching `"ABABAC"` in `"ABABABABABABABAC"`

- **Brute Force:** May perform nearly `O(n × m)` comparisons.
- **KMP:** Will intelligently shift the pattern based on LPS values.

Drawbacks of Brute force (contn…)

4. Not Suitable for Real-World Large Text Searches

- **Brute Force:**
  - **Inefficient** for searching in **large texts** (e.g., DNA sequences, logs, books).
- **KMP:**
  - **Efficient** for searching in large datasets (e.g., substring search in documents, bioinformatics, and search engines).

5. Brute Force Cannot Utilize Previous Knowledge

- **Brute Force:**
  - Does **not** use any preprocessing. Every mismatch forces the algorithm to start fresh.
- **KMP:**
  - Precomputes **LPS table** once, then reuses it to **optimize pattern shifts**.

Final Comparison Table

Feature	Brute Force	KMP Algorithm
Worst-case Time Complexity	O(n × m)	O(n + m)
Redundant Comparisons	Yes (Resets search on mismatch)	No (Uses LPS table)
Efficiency for Large Text	Slow	Fast
Preprocessing Required	No	Yes (LPS table)
Best Use Case	Small inputs	Large texts & repeated searches

Example 2: O(n) Case (KMP Algorithm - Skipping Comparisons)

This occurs when the algorithm **uses a prefix table** to avoid unnecessary rechecking of characters.

Example:

- **Text (n = 10):** `"aaaaaaaaaa"`
- **Pattern (m = 3):** `"aaa"`
- **Algorithm:** Knuth-Morris-Pratt (KMP)

Modified KMP to get O(n) Time Complexity

Example 2: O(n) Case (KMP Algorithm - Skipping Comparisons)

This occurs when the algorithm **uses a prefix table** to avoid unnecessary rechecking of characters.

Example:

- **Text (n = 10):** `"aaaaaaaaaa"`
- **Pattern (m = 3):** `"aaa"`
- **Algorithm:** Knuth-Morris-Pratt (KMP)

Step	Text Window Being Checked	Pattern Matched?	Comparisons Made	Pattern Shift
1	"aaa"aaaaaaa	✅ Match	3 comparisons	Shift by 2 (from prefix table)
2	a"aaa"aaaaaa	✅ Match	1 comparison (skipped 2)	Shift by 2
3	aa"aaa"aaaaa	✅ Match	1 comparison (skipped 2)	Shift by 2
4	aaa"aaa"aaaa	✅ Match	1 comparison (skipped 2)	Shift by 2
5	aaaa"aaa"aaa	✅ Match	1 comparison (skipped 2)	Shift by 2
6	aaaaa"aaa"aa	✅ Match	1 comparison (skipped 2)	Shift by 2
7	aaaaaa"aaa"a	✅ Match	1 comparison (skipped 2)	Shift by 2
8	aaaaaaa"aaa"	✅ Match	1 comparison (skipped 2)	-
Total	-	Multiple matches	O(n) = 10 comparisons	-

Example 1: O(n + m) Case (Naïve Algorithm)

This occurs when the algorithm checks **every position** in the text, comparing all characters of the pattern at each step.

Example:

- **Text (n = 10):** `"abcdefghij"`
- **Pattern (m = 3):** `"abc"`
- **Algorithm:** Naïve String Matching

Step	Text Window Being Checked	Pattern Match?	Comparisons Made
1	`"abc" defghij`	✅ **Match found**	3 comparisons (all matched)
2	`a "bcd" efghij`	❌ No match	3 comparisons
3	`ab "cde" fghij`	❌ No match	3 comparisons
4	`abc "def" ghij`	❌ No match	3 comparisons
5	`abcd "efg" hij`	❌ No match	3 comparisons
6	`abcde "fgh" ij`	❌ No match	3 comparisons
7	`abcdef "ghi" j`	❌ No match	3 comparisons
8	`abcdefg "hij"`	❌ No match	3 comparisons
Total	-	**1 match found**	**O(n + m) = 10 + 3 = 13 comparisons**

Example 2: O(n) Case (KMP Algorithm - Skipping Comparisons)

✅ **Why O(n)?**

- Instead of rechecking already matched characters, **KMP skips comparisons** using a prefix table, ensuring the time complexity stays **O(n)**.

KMP algorithm

- The **Knuth-Morris-Pratt (KMP) Algorithm** optimizes pattern matching by using a **prefix table** (also called the **lps table** – longest proper prefix which is also a suffix). This helps avoid unnecessary rechecking of characters, allowing the algorithm to **shift the pattern efficiently**.
- O(n) Case (KMP Algorithm - Skipping Comparisons)
- This occurs when the algorithm **uses a prefix table** to avoid unnecessary rechecking of characters.

How Does the Prefix Table Work?

- The **prefix table (lps array)** stores information about the longest prefix of the pattern that is also a suffix.
- When a mismatch occurs, instead of shifting by 1 (like in the naïve approach), **KMP uses the lps table to determine how far it can shift**.
- This reduces redundant comparisons, making the algorithm run in **O(n) time** instead of **O(n + m).**

Example 3: O(n) Case (Boyer-Moore Algorithm - Large Jumps)

This occurs when the algorithm makes **large shifts** based on mismatches.

Example:

- **Text (n = 10):** "abcdefghij"
- **Pattern (m = 3):** "xyz"
- **Algorithm:** Boyer-Moore

Step	Text Window Being Checked	Last Character in Pattern	Mismatch Found?	Pattern Shift
1	`"abc"defghij`	`'c'` (not in pattern)	❌ Yes	Shift by **3**
2	`abc"def"ghij`	`'f'` (not in pattern)	❌ Yes	Shift by **3**
3	`abcdef"ghi"j`	`'i'` (not in pattern)	❌ Yes	Shift by **3**
4	`abcdefgh"ij"`	`'j'` (not in pattern)	❌ Yes	Shift by **3**
Total	-	-	**No matches found**	**O(n) = 4 comparisons**

Example 3: O(n) Case (Boyer-Moore Algorithm - Large Jumps)

This occurs when the algorithm makes **large shifts** based on mismatches.

Example:

- **Text (n = 10):** "abcdefghij"
- **Pattern (m = 3):** "xyz"
- **Algorithm:** Boyer-Moore

✅ **Why O(n)?**

- Boyer-Moore makes **large jumps** on mismatches, reducing the number of comparisons **to a fraction of n.**

Step	Text Window Being Checked	Last Character in Pattern	Mismatch Found?	Pattern Shift
1	`"abc"defghij`	`'c'` (not in pattern)	❌ Yes	Shift by **3**
2	`abc"def"ghij`	`'f'` (not in pattern)	❌ Yes	Shift by **3**
3	`abcdef"ghi"j`	`'i'` (not in pattern)	❌ Yes	Shift by **3**
4	`abcdefgh"ij"`	`'j'` (not in pattern)	❌ Yes	Shift by **3**
Total	-	-	**No matches found**	**O(n) = 4 comparisons**

Final Summary of Complexity Differences

Case	Algorithm	Pattern Matching Behavior	Time Complexity
O(n + m) (Slowest Case)	Naïve Algorithm	Checks every character in the text sequentially	**O(n + m)**
O(n) (Skipping Comparisons)	KMP Algorithm	Uses prefix table to skip unnecessary comparisons	**O(n)**
O(n) (Large Jumps on Mismatch)	Boyer-Moore Algorithm	Uses bad-character rule to jump ahead	**O(n)**

Summary

- **O(n + m) happens when the pattern is rechecked at every position.**
- **O(n) is possible when the algorithm efficiently skips unnecessary comparisons** (KMP) or **makes large jumps** on mismatches (Boyer-Moore).
- **Optimized algorithms reduce time complexity by avoiding redundant operations, leading to O(n) in practice.**