Problem Set 1

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Question 1: Plot the series

GDPC1 from FRED

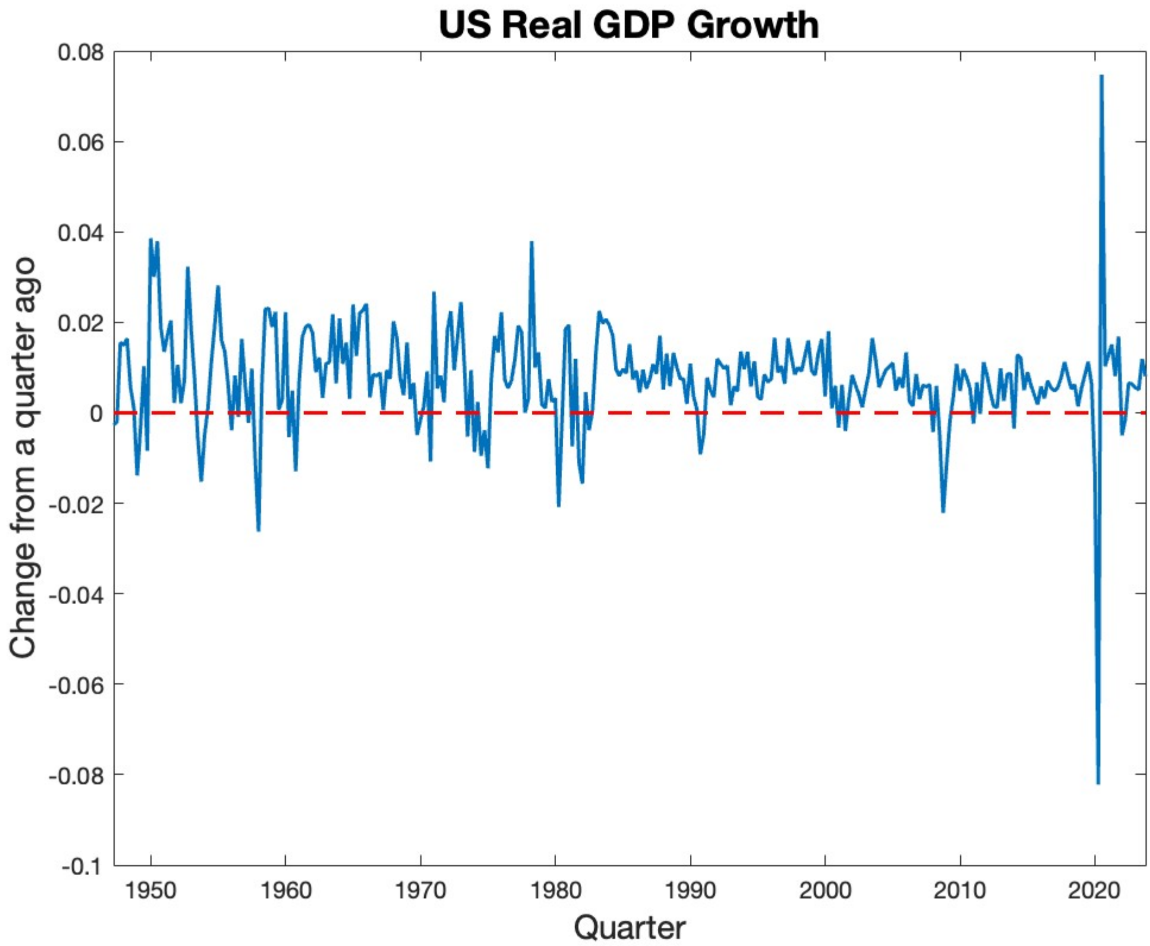

Figure 1: Quarterly growth in U.S real GDP

Question 2-4: AR(1)

Parameter estimation

$$y_t = c + \phi y_{t-1} + \epsilon_t$$

An OLS estimation using the GDP growth data for 307 quarters yields the following parameters:

$$\hat{c} = 0.0066; \hat{\phi} = 0.1333$$

$\hat{\sigma^2}$ is computed using the formula: Residual Sum of Squares/degrees of freedom and it is found to be $1.2393 * 10^{-4}$

Auto-correlation Function

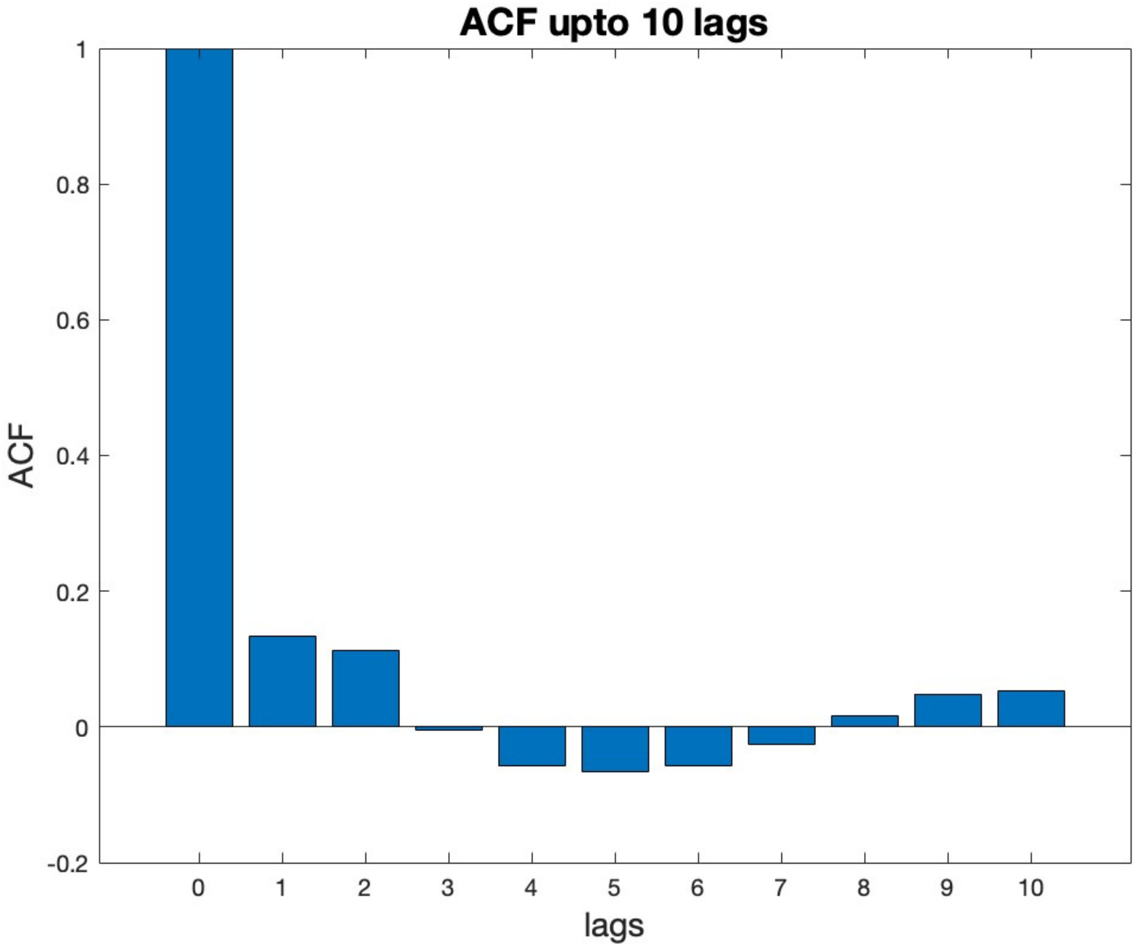

Figure 2: ACF

This is estimated using the autocorr function in MATLAB. Figure 2 shows that the ACF decays as the lag increases to 10.

Wold Representation - MA(1)
Figure 3 shows that the MA(1) coefficients decay as the lags rise to 100.

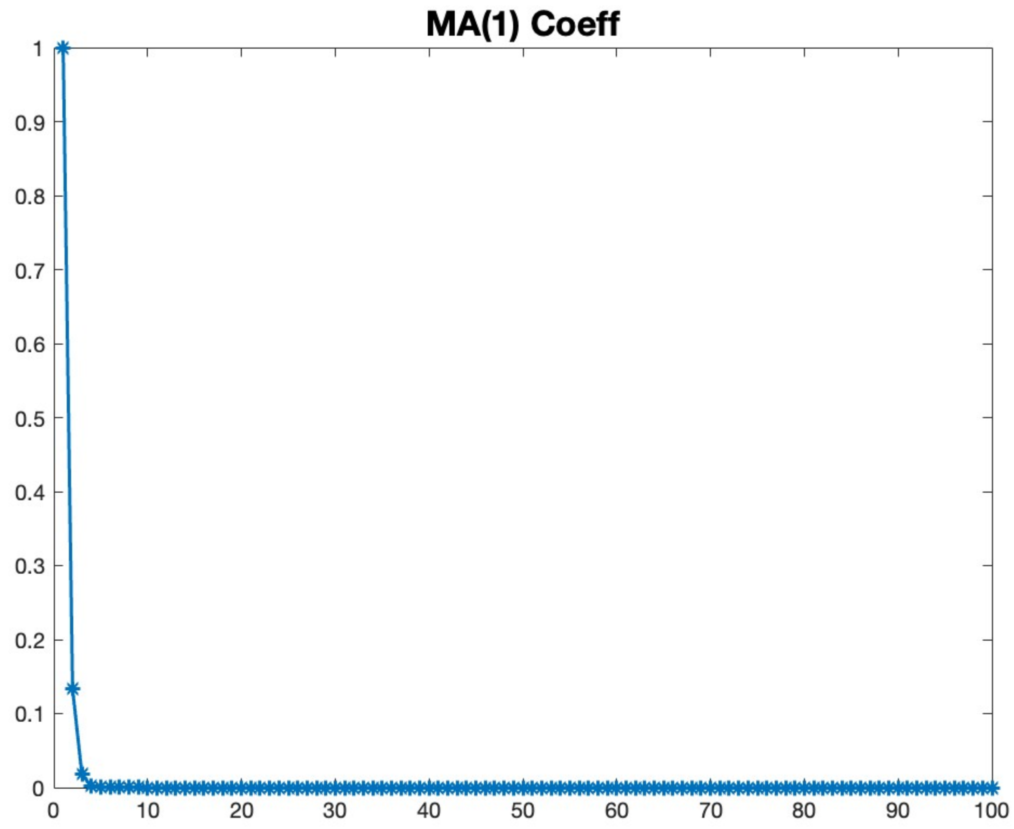

Figure 3: Plot of the MA(1) coefficients

Question 5-8: AR(2)

Parameter estimation

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \epsilon_t$$

An OLS estimation using the GDP growth data for 307 quarters yields the following parameters:

$$\hat{c} = 0.0060$$

$$\hat{\phi}_1 = 0.1181$$

$$\hat{\phi}_2 = 0.0968$$

$\hat{\sigma^2}$ is computed using the formula: Residual Sum of Squares/degrees of freedom

$$= 0.0377/206$$

$$= 1.2335 * 10^{-4}$$

Roots of the polynomial
The characteristic equation of the AR(2) process is as follows:

$$(1 - \phi_1 Z - \phi_2 Z^2)$$

The roots of the equation are found to be

$$Z_1 = -3.8812; Z_2 = 2.6611$$

Conditions for stationary and causal process
Causality: An AR(p) process given by $\theta(L) * Y_t = \epsilon_t$ is causal if and only if $\theta(L)$ not equal to 0 for all z belonging to C such that |z| < or = 1.
Stationarity: An AR(p) process given by $\theta(L) * Y_t = \epsilon_t$ has a stationary solution if and only if $\theta(L)$ not equal to 0 for all z belonging to C such that |z| = 1.
For an AR(2) process to be causal and stationary, the absolute value of the roots of the characteristic equation must lie outside the unit circle or |z| > 1. In our case, both $|Z_1|$ and $|Z_2|$ are greater than 1, therefore the given process is causal and stationary.

Wold Representation - MA(2)

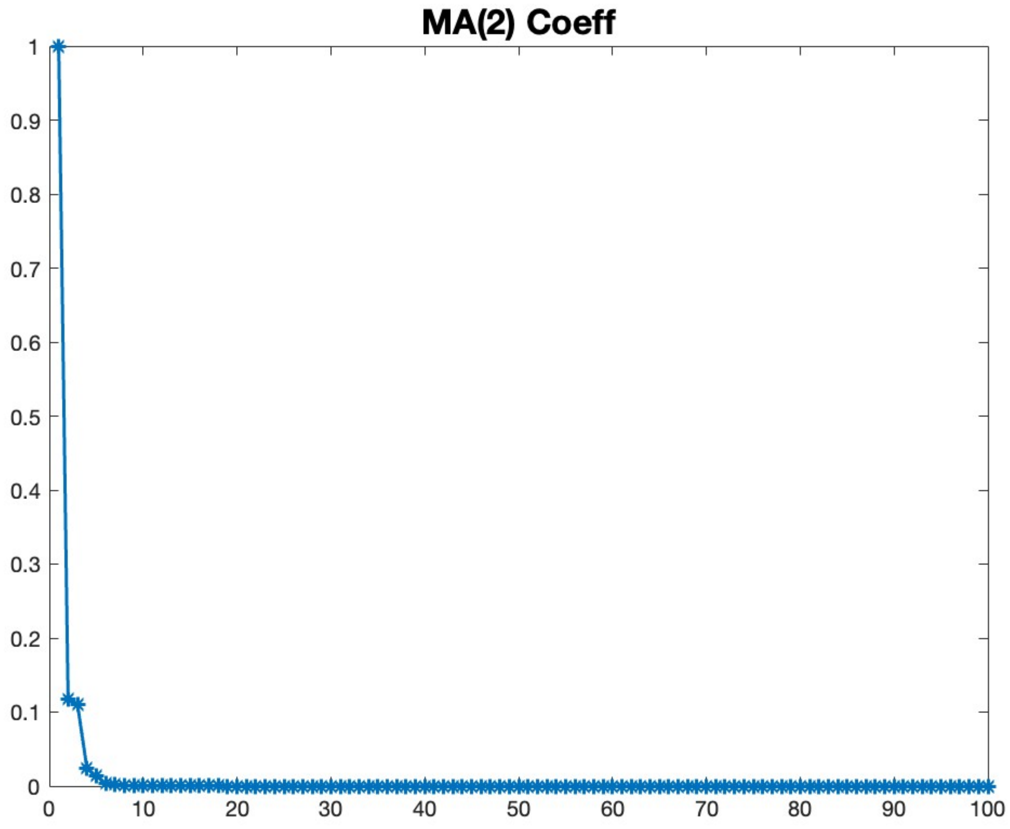

Figure 4: Plot of the MA(2) coefficients

Figure 4 shows that the MA(2) coefficients decay or tend to zero as the lags rise to 100.

Question 9-10: MA(2)

$$y_t = c + \epsilon_t + 1.2\epsilon_{t-1} + 2\epsilon_{t-2}$$

Why stationary?
MA processes are inherently stationary under the assumption of ϵ_t having white noise with zero mean and σ^2 variance. In other words, this process has a finite and constant mean and variance. The autocovariance function is independent of t. In the given process,

$$E(y_t) = c$$
$$Var(y_t) = \sigma^2 * (1 + 1.44 + 4) = 6.44\sigma^2$$
$$\gamma(1) = 3.66 * \sigma^2$$
$$\gamma(2) = 2 * \sigma^2$$
$$\gamma(2) = 0 for j > 2$$

Roots of the polynomial
The characteristic equation of the MA(2) process is as follows:

$$(1 + 1.2Z + 2Z^2)$$

We need to find L such that the equation is zero. This quadratic equation can simply be solved using the formula:

$$Z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The roots of the equation are found to be

$$Z_1 = -0.3000 + 0.6403i$$

$$Z_2 = -0.3000 - 0.6403i$$

Invertibility
A MA process defined by the equation $Y_t = \theta(L) * \epsilon_t$ is invertible if and only if $\theta(z)$ not equal to 0 for all z belongs to C such that |z| < or = 1.
The modulus value of the root of the characteristic equation is 0.7071 and it is less than 1. Therefore, the given MA(2) process is not invertible.