

$$\begin{array}{ccccc}
 \tilde{\mathbf{U}} & \xrightarrow[\textstyle / \bar{\mathbf{T}}_{\mathbf{m},\mathbf{n}}]{\mathbf{f}} & \mathbf{M} & \xleftarrow{\mathbf{G}} & \\
 \downarrow & & \downarrow & & \\
 \pi_1(\tilde{\mathbf{U}}, \tilde{\mathbf{x}}_0) & \xrightarrow{\mathbf{f}_*} & \mathrm{H}_1(\mathbf{M}, \mathbb{Q}) & \xleftarrow{\mathbf{G}_*} & \\
 & \searrow \Omega_{\mathbf{u},\mathbf{v}} & \downarrow \tilde{dir} & & \\
 \bar{\mathbf{T}}_{\mathbf{m},\mathbf{n}} \xleftarrow{\sim} \mathbb{Z}^2 & & \mathbb{Z}^2 & \xleftarrow{\mathbf{G}'} & 
 \end{array}$$