

Computational Modeling of Social Systems

Modeling Segregation Week 2

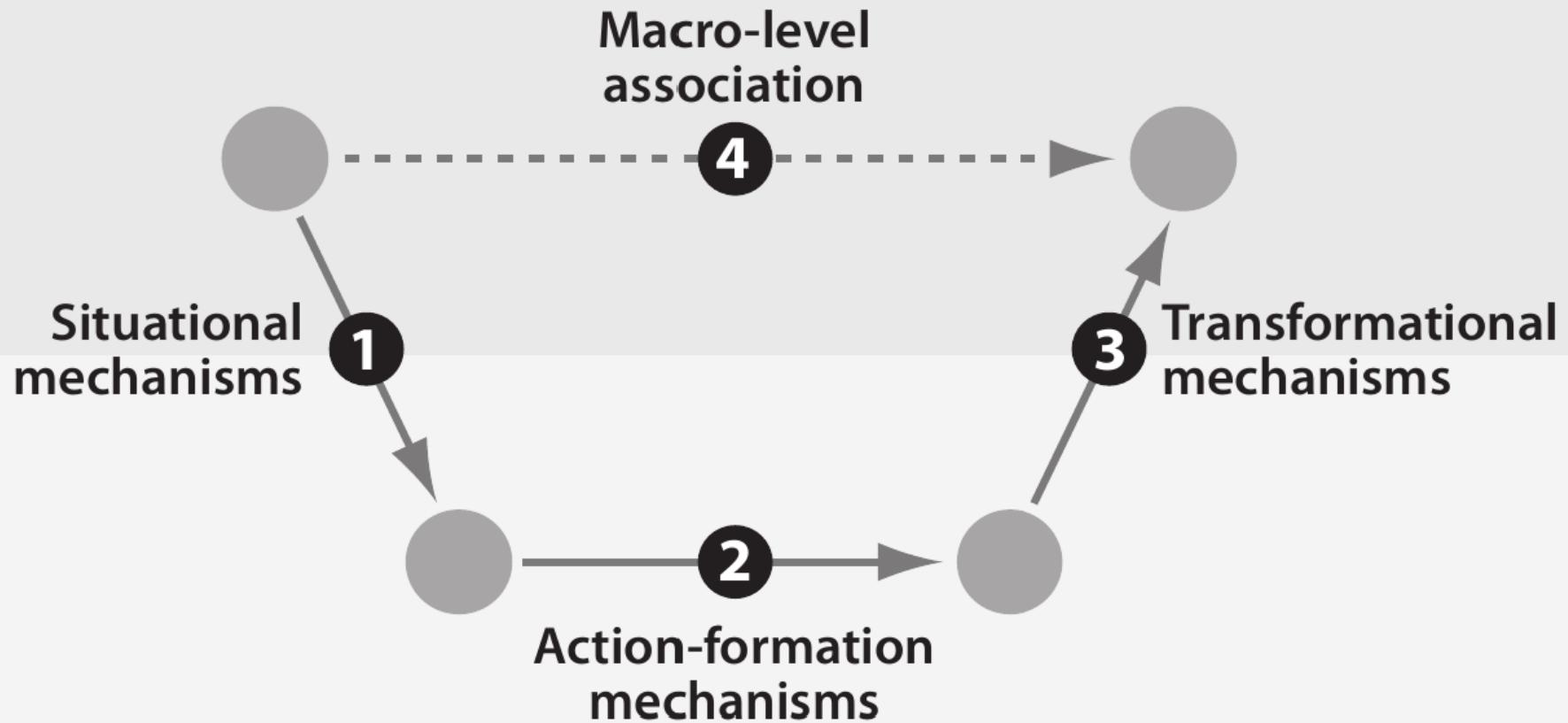
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Graz
13.03.2024

Recap

ABM is a computational tool that allows us to simulate the **micro-level** behavior of agents in simple societies and observe the **macro-level** outcome. This allows us to draw a **causal link** between micro to macro.

The micro-macro gap



*Causal Mechanisms in the Social Sciences. Peter Hedström and Petri Ylikoski.
Annual Review of Sociology, 2010.*

Beyond exploration: computational theory and framing

ABM are for analysis, not just exploration

- Exploring what happens in a simulation is fine, but ABM can do much more!
- **Behavior calibration** of individual agents with experiments or surveys: integrating social and behavioral findings in an ABM
- **Analysis of observable outcomes** versus parameters of behavior or alternative mechanisms/policies
- **Testing outcomes** with large-scale data (e.g. digital traces from computational social systems), across conditions and over time

From factors to actors: Computational Sociology and Agent-Based Modeling.
Michael Macy and Robert Willer. Annual Review of Sociology, 2002.

Properties of good ABM

- They **model causation**: agent actions and conditions have counterfactuals, and dynamics are not ad hoc to get an outcome.
- They have **measurable outcomes**: collective behavior can be aggregated into one or more quantities that can be measured in many simulations and across conditions.
- They have **quantifiable designs**: individual dynamics are based on metrics that can be tested with empirical methods (e.g. experiments, surveys)
- Are **minimal and modular**: you can test the sensitivity of outcomes with different blocks of dynamics and include only what is necessary.
Ockham's razor principle: if you have two competing ideas to explain the same phenomenon, you should prefer the simpler one.
- **Stochasticity**, randomness, and path-dependencies are considered.

Overview of today's lecture

1. Segregation
2. Schelling's segregation model
3. Analyzing Schelling's model
4. Cellular Automata: The game of life

Urban segregation: definition

Urban segregation is the **unequal** distribution of different social groups in the urban space, based mainly on occupation, income and education, as well as on gender and ethnicity. -- The Future of Cities. European Research Commission

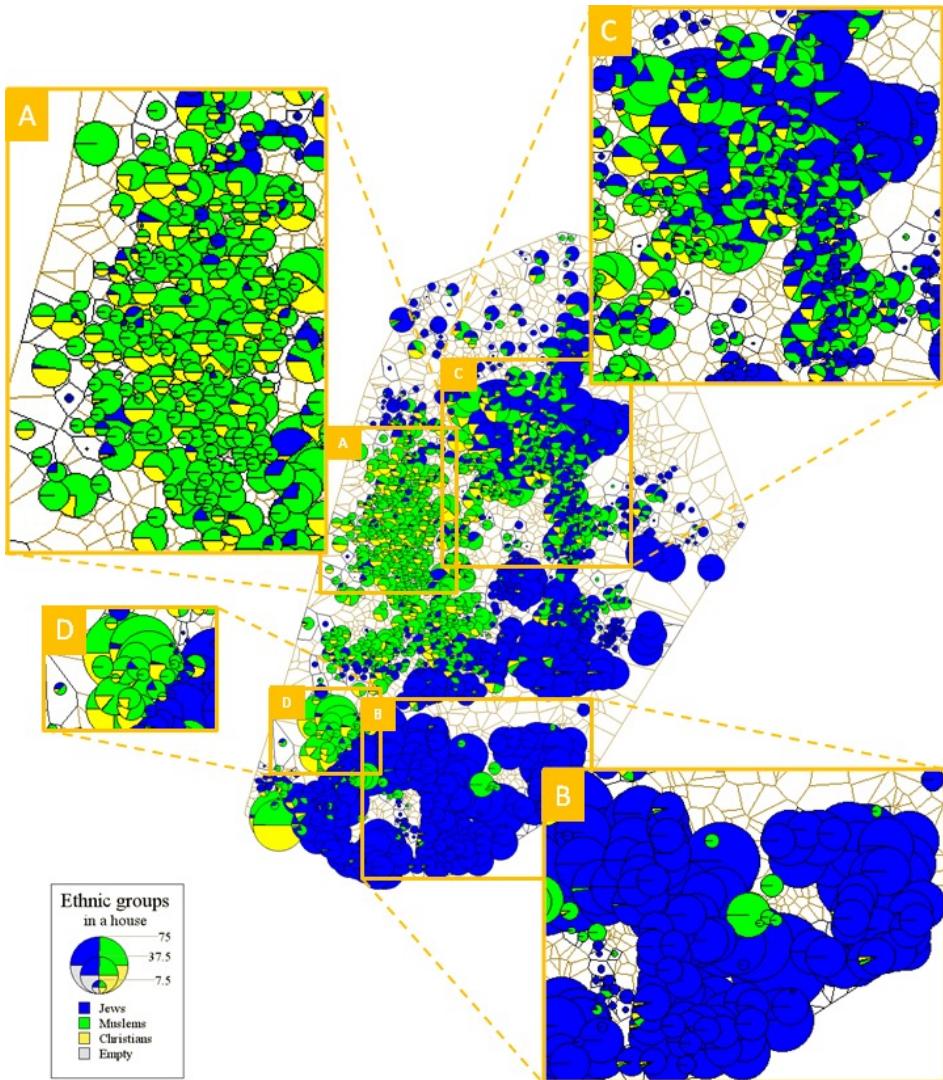


Segregation example: District of Columbia, US



America is more diverse than ever — but still segregated. The Washington Post

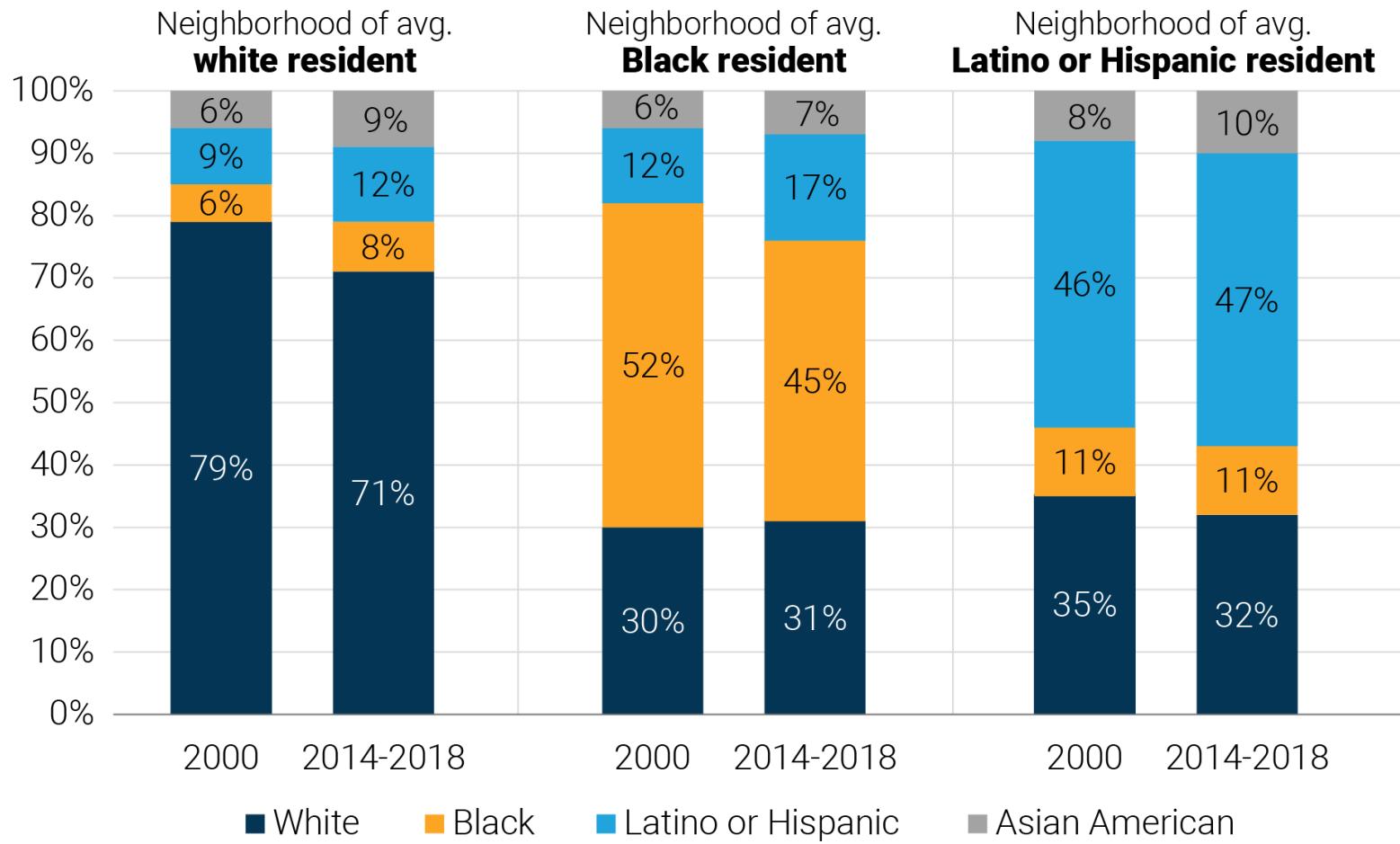
Segregation example: Yaffo, Israel



- Neighborhood units with shares of ethnic groups as pie charts
- Share of Jewish, Muslim, and Christian inhabitants
- Segregation: Jewish vs non-Jewish areas
- Observation: some areas are less segregated (area C)

Hatna & Benenson. JASSS, 2012

Residential segregation persistence



The Great Real Estate Reset. Loh, Coes, & Buthe, 2020

Increasing income segregation

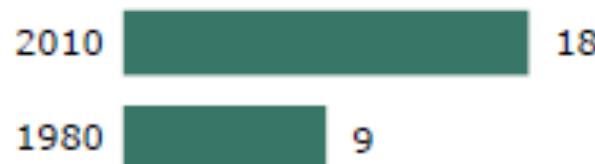
Share of Lower-Income and Upper-Income Households Who Live Mainly Among Themselves, 1980 and 2010

%

More lower-income households live in majority low-income tracts ...

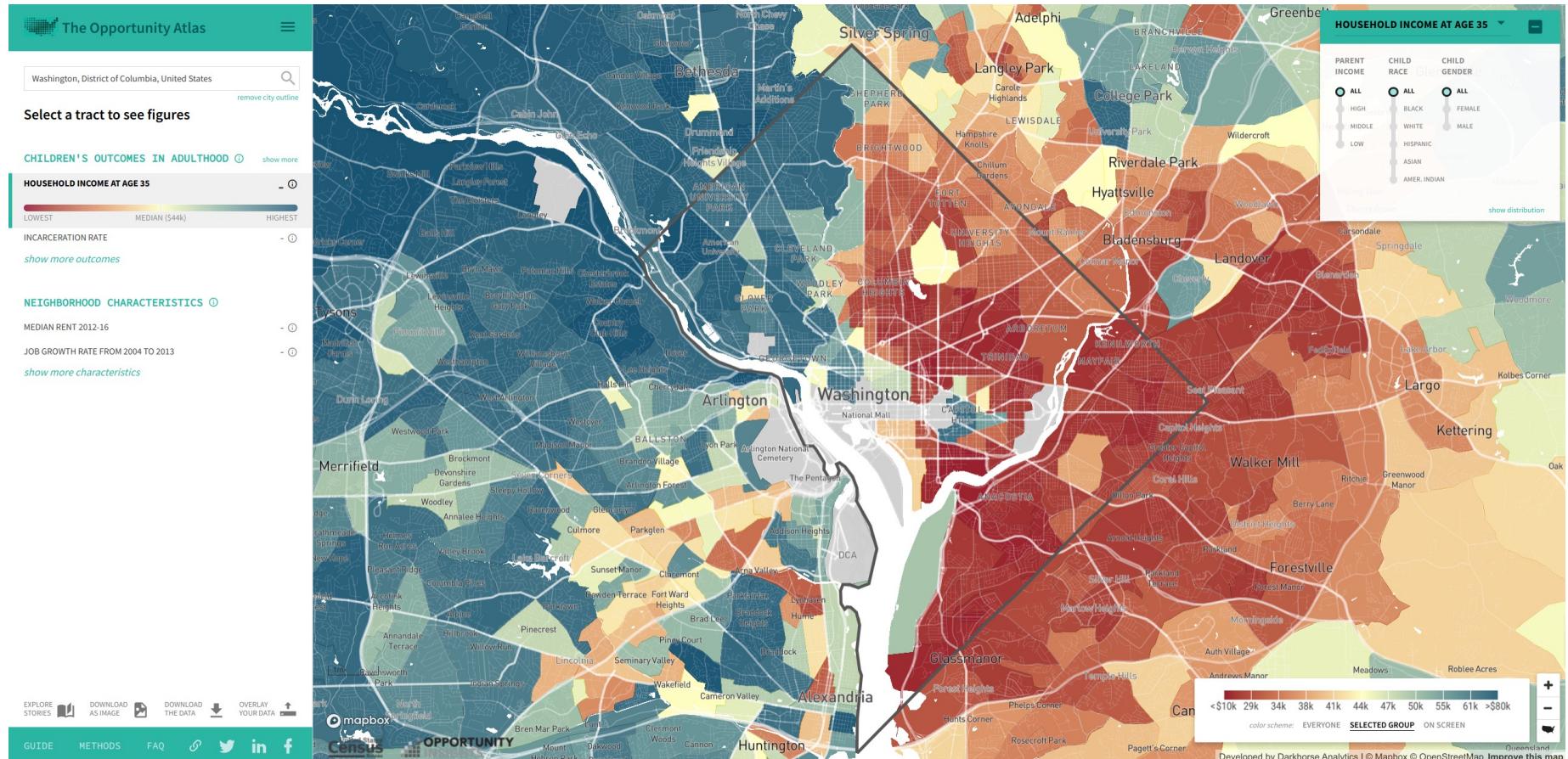


... and more upper-income households live in majority upper-income census tracts



The Rise of Residential Segregation by Income. Pew Research Center.

ZIP code at birth predicts life outcomes



<https://www.opportunityatlas.org/>

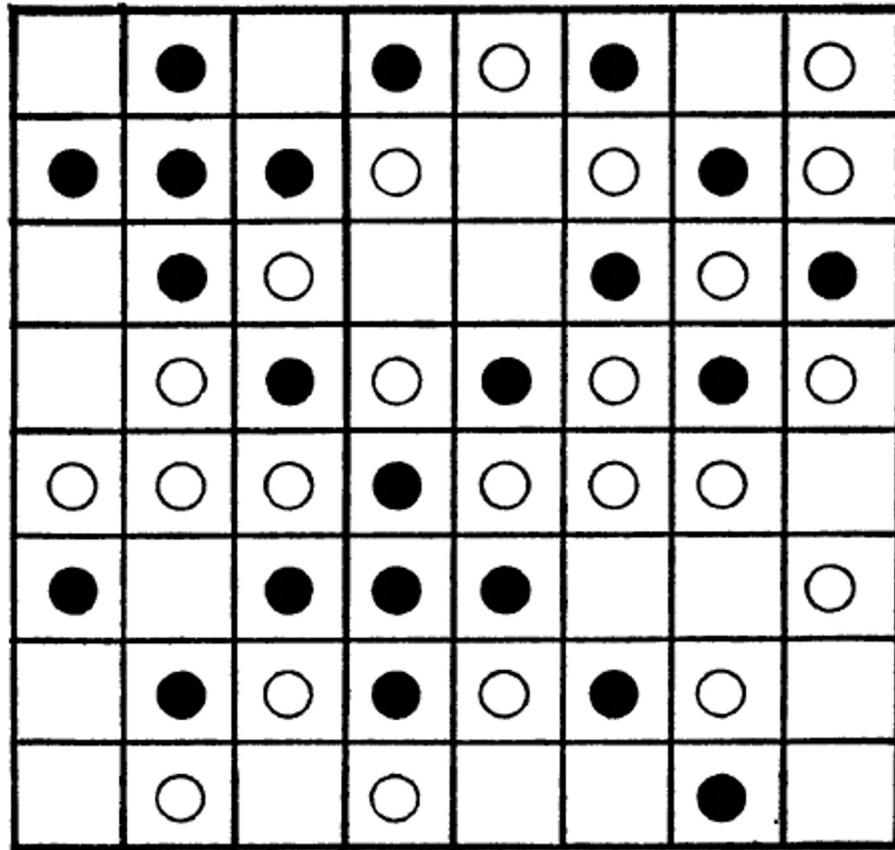
Questions about the origin of segregation

- Top-down segregation: Discriminatory policies
- Bottom-up segregation: Individual location choices influenced by:
 - Price of housing and services
 - Access to religious centers and education
 - Tenant exclusion and bias
- What is the role of inhabitant intolerance to different neighbors?
- **Without central discrimination, can segregation emerge even when individuals tolerate living in a neighborhood in which they are in the minority?**

Overview

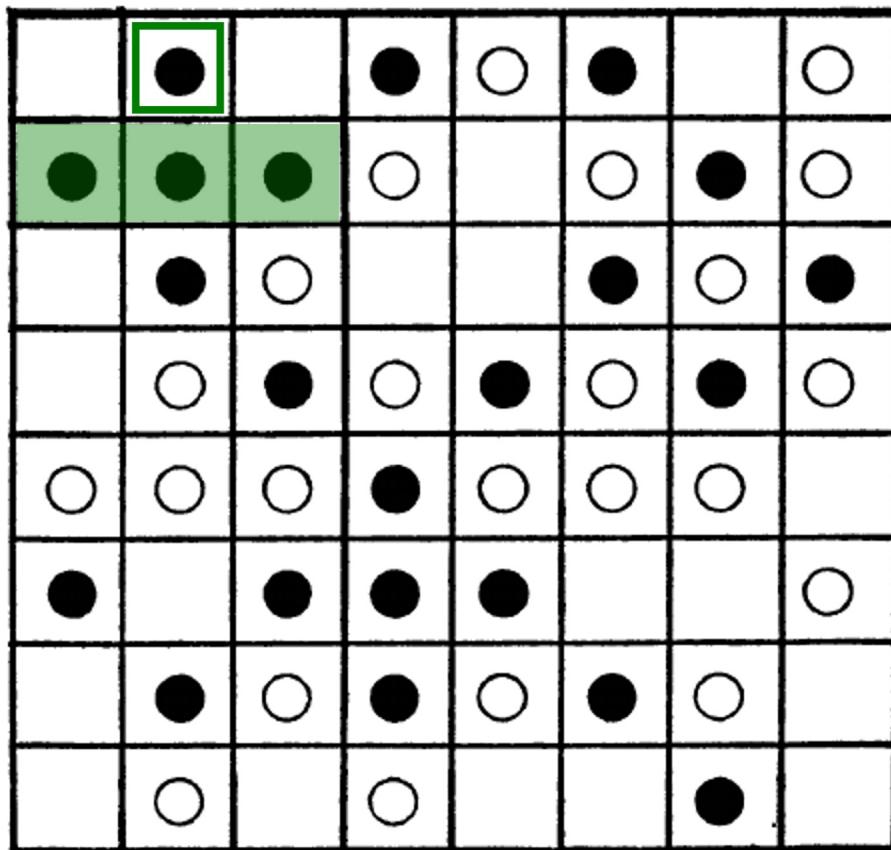
1. Segregation
- 2. Schelling's segregation model**
3. Analyzing Schelling's model
4. Cellular Automata: The game of life

Schelling's model setup



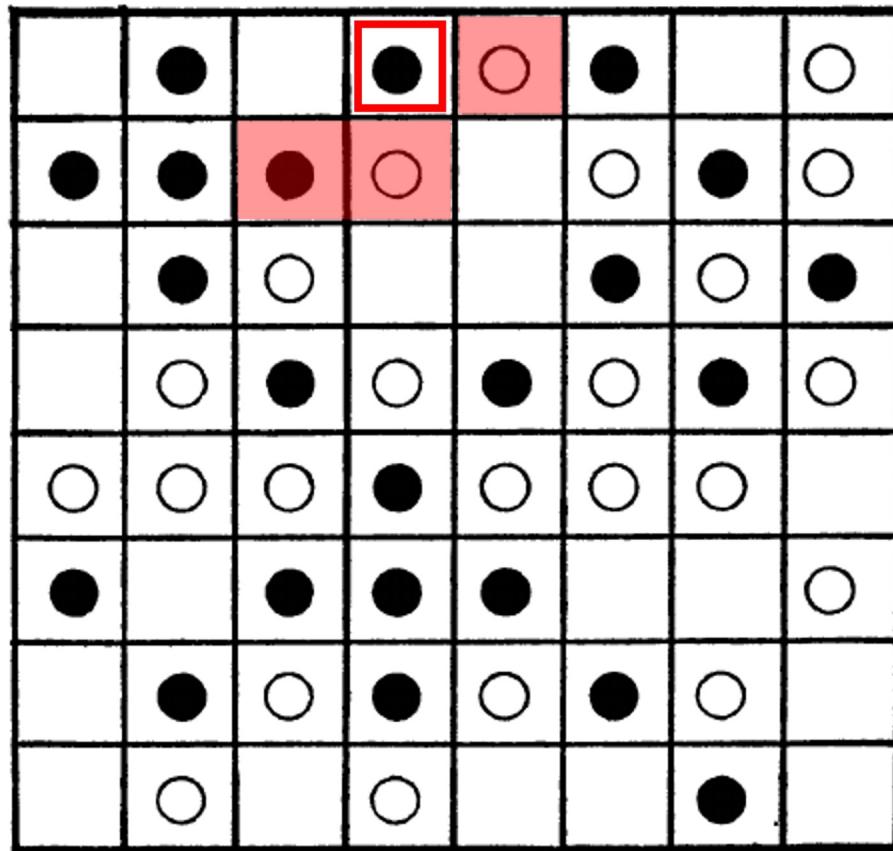
- Agents of two kinds, similar number (here 22 and 23)
- Low but nonzero fraction of empty cells (here 19)
- Cells have a neighborhood of 8 cells ($3 \times 3 - 1$)
- Border and corner cells have smaller neighborhood
- Agents are aware of the fraction of similar agents in their neighborhood: f
- F measures intolerance
- Agents are satisfied with $f \geq F$

Relocation example



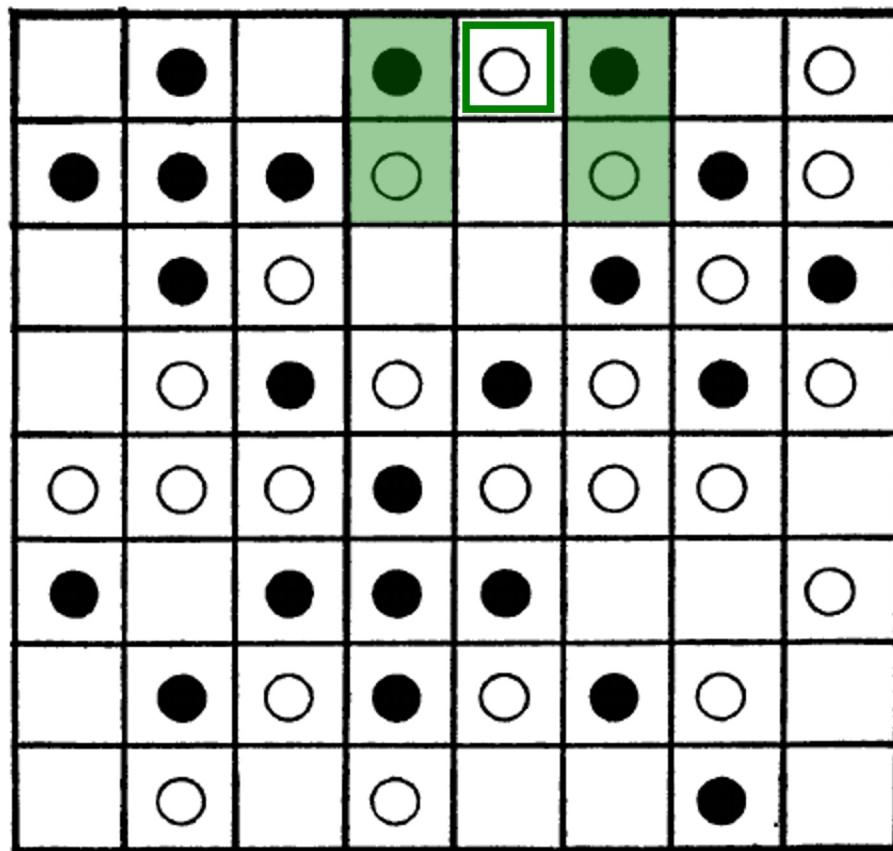
- $F=0.35$
- 3/3 neighbors are equal:
 - agent stays
- Remember:
 - Only occupied neighbor cells count
 - Cells close to the border have smaller neighborhoods

Relocation example



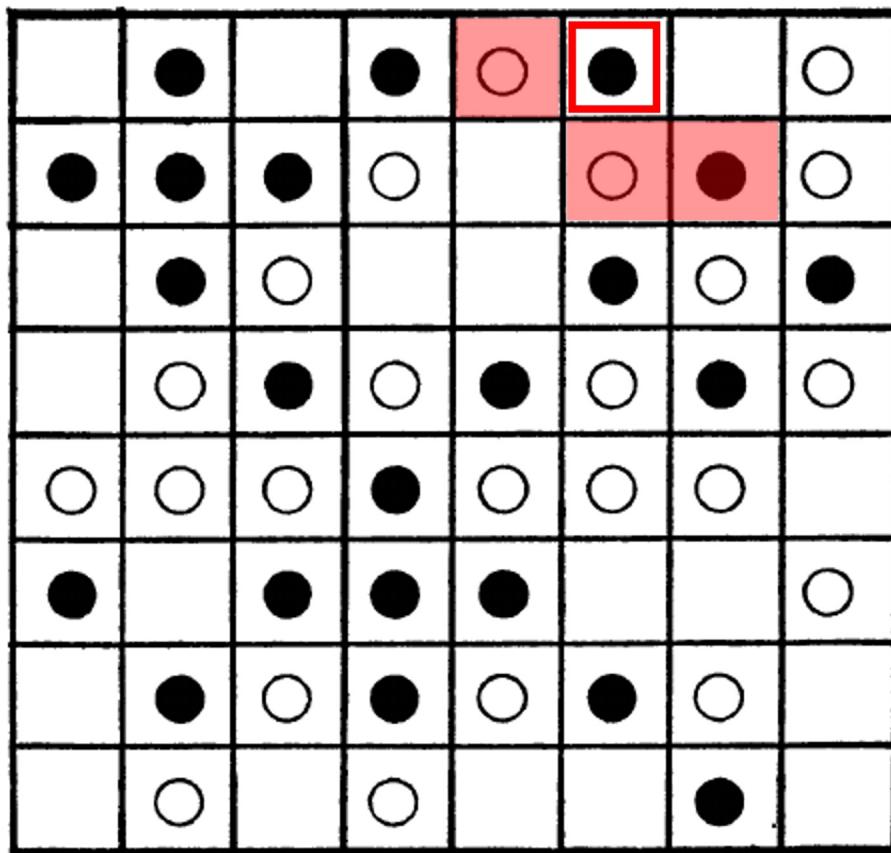
- $F=0.35$
- 1/3 neighbors are equal:
 - $1/3 < 0.35$
 - agent marked for relocation

Relocation example



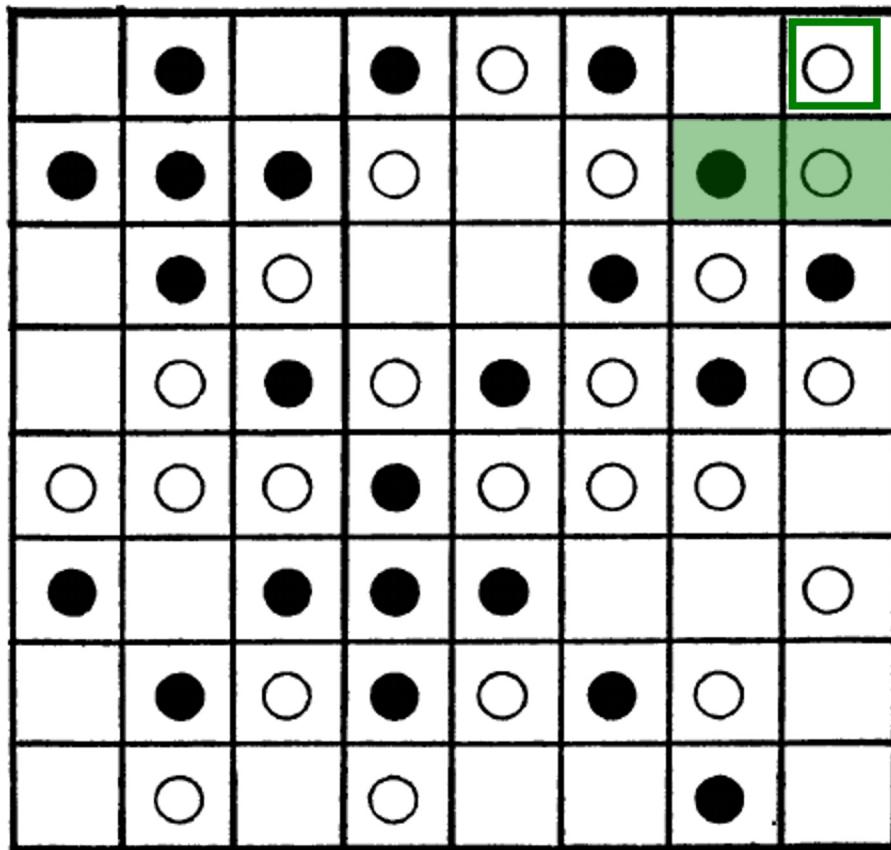
- $F=0.35$
- 2/4 neighbors are equal:
 - $0.5 > 0.35$
 - Agent stays

Relocation example



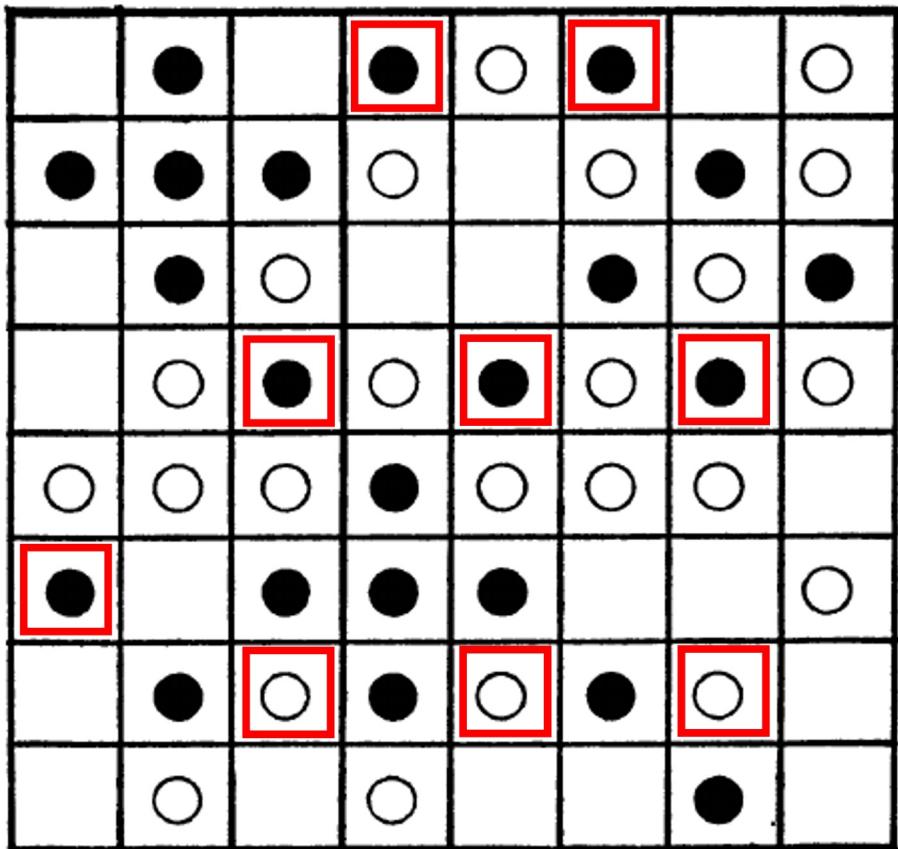
- $F=0.35$
- 1/3 neighbors are equal:
 - $1/3 < 0.35$
 - agent marked for relocation

Relocation example



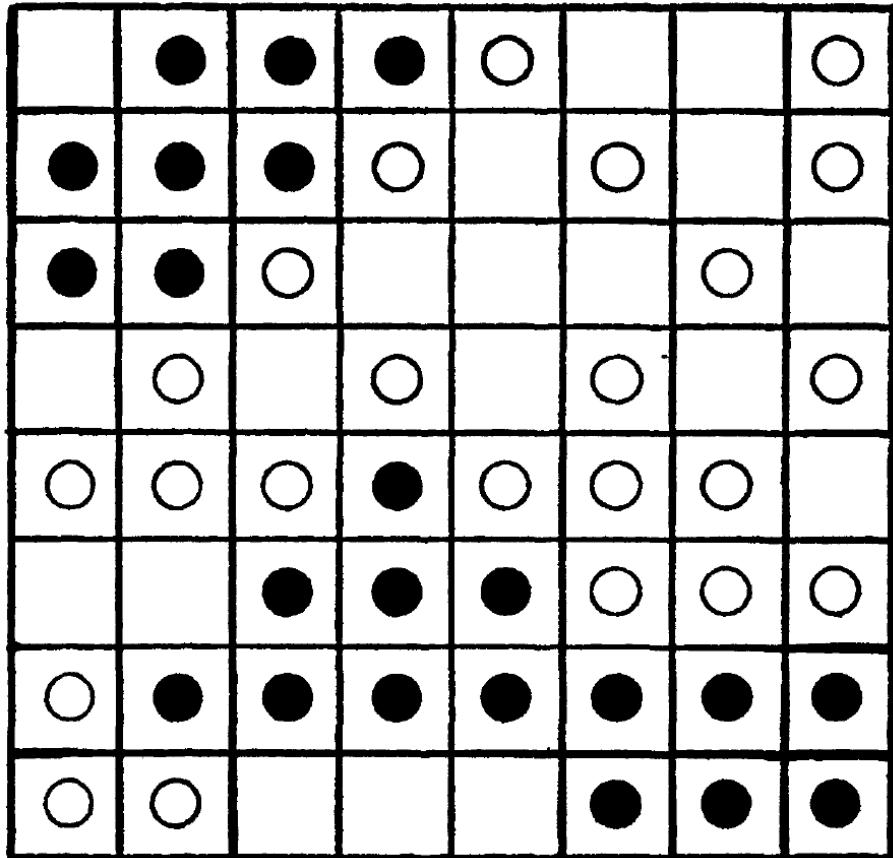
- $F=0.35$
- 1/2 neighbors are equal:
 - $0.5 > 0.35$
 - agent stays
- Note:
 - Corner cells have at most three neighbors

Relocation example



- "Unhappy" agents with $f < F$
- Relocation is based on a sequence and a rule where to jump
- Some variability in this step:
 - Random sequence or ordered
- Options about jumps:
 - a) closest happy free spot
 - b) happiest free spot: $\max(f)$, even if new f still below F
 - c) random free spot

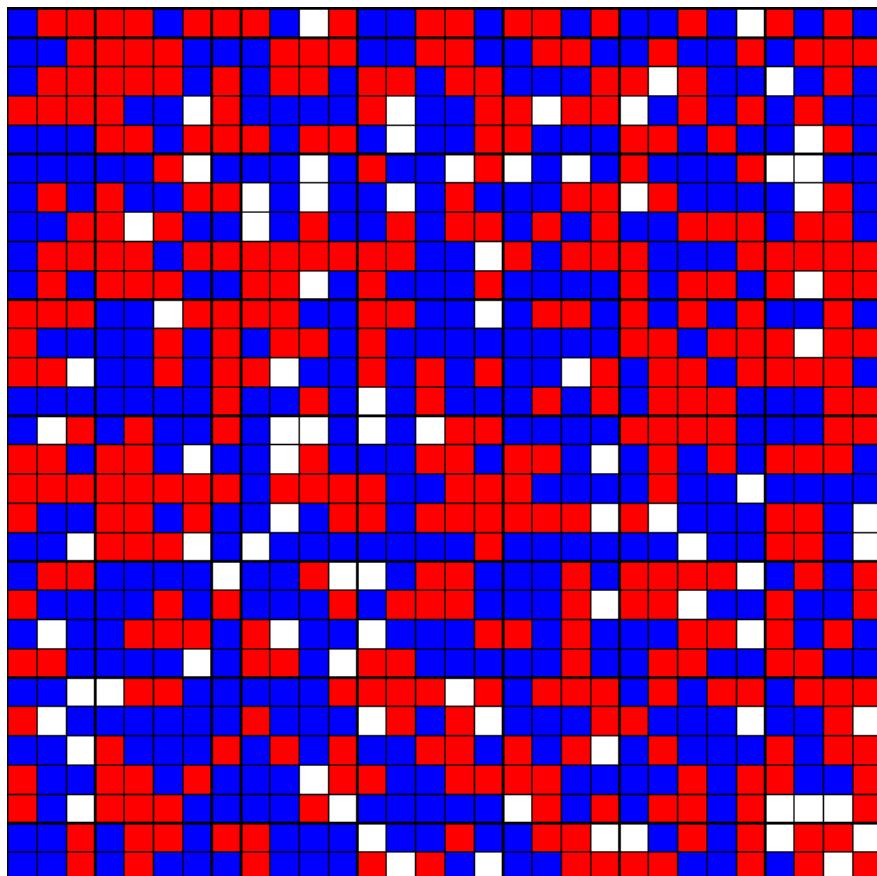
Simulation results



Even when $F=0.35$ is not that high!

- State after running a few iterations
- All agents are satisfied ($f \geq F$)
- No more relocations take place (simulation ends)
- Result looks segregated
 - Two large black regions
 - Rest formed by scattered white agents

Simulation configuration



Round 0

Satisfied 0 %

Reset

Similar: 30%

Start

Red/Blue: 50/50%

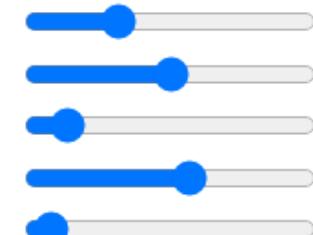
Stop

Empty: 10%

Step

Size: 50x50

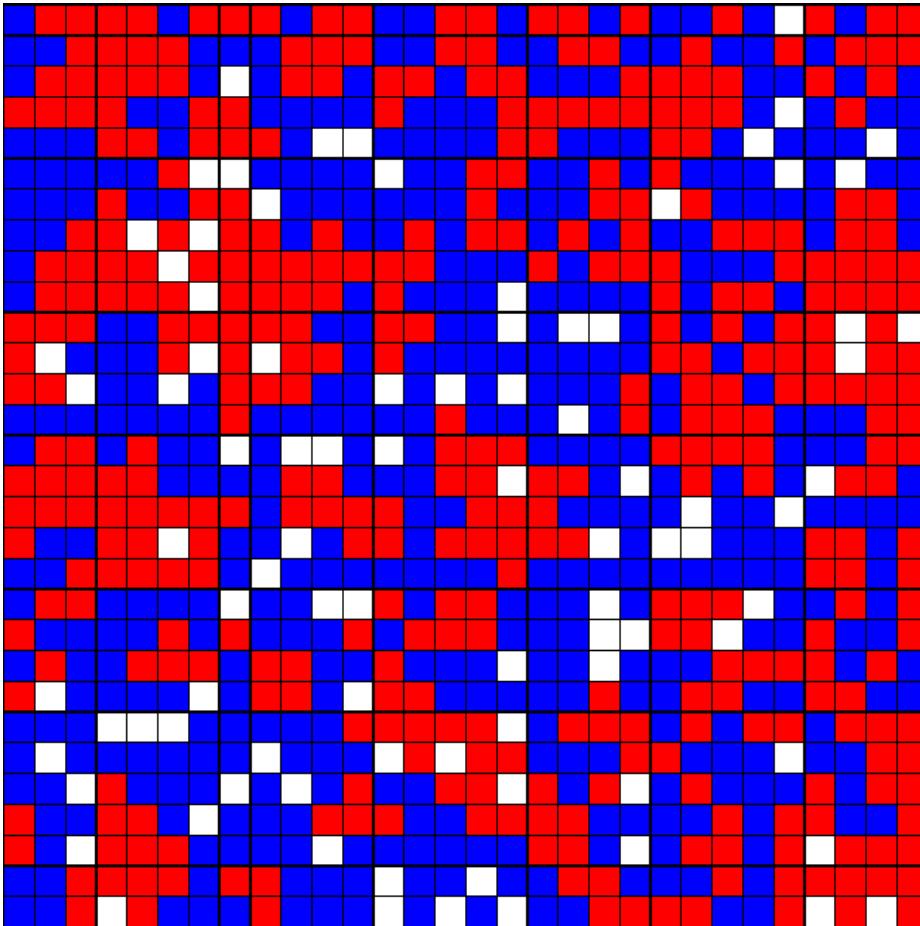
Delay: 100 ms



<http://nifty.stanford.edu/2014/mccown-schelling-model-segregation/>

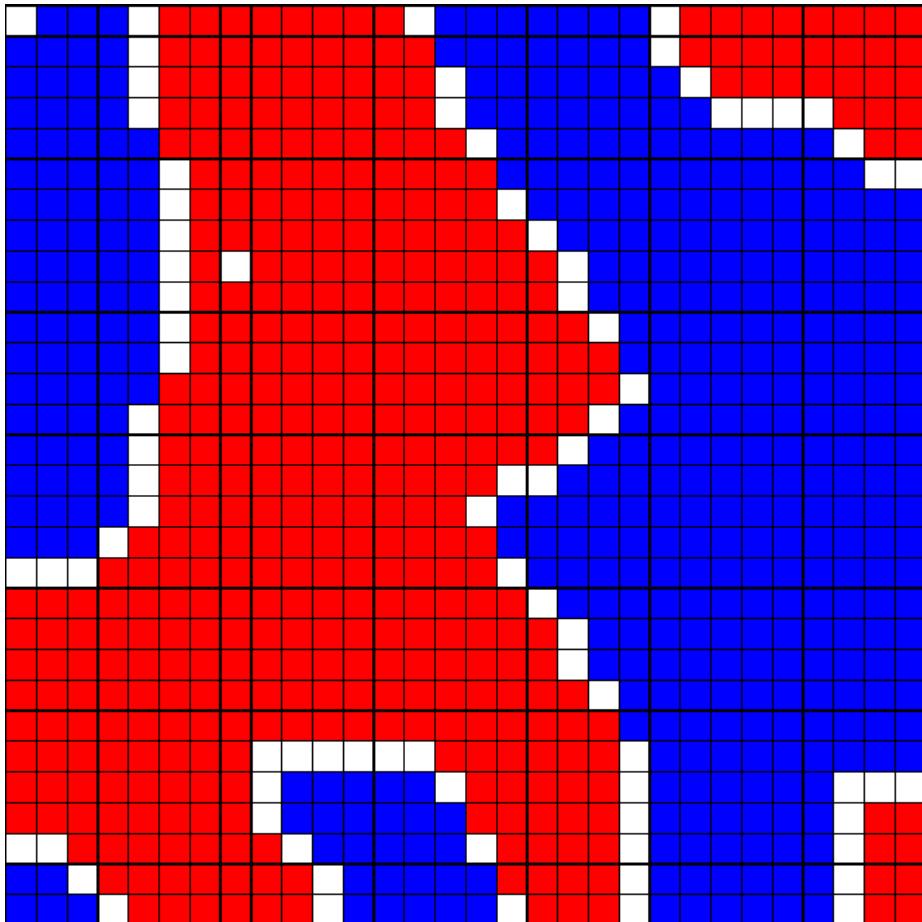
- Random initial configuration
- Parameters can be changed
- Reporting % satisfied agents and simulation round

High tolerance case (low F)



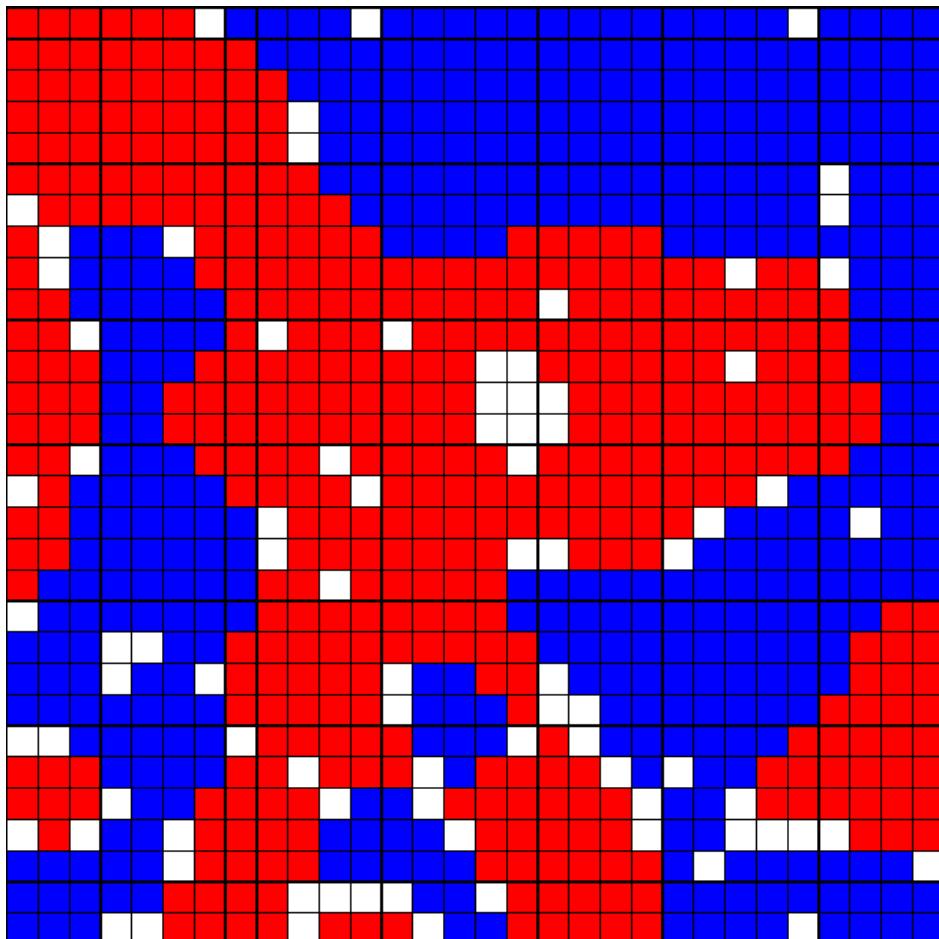
- $F=0.2$
- Result very similar to initial configuration
- No apparent segregation
- All agents satisfied soon

High intolerance case



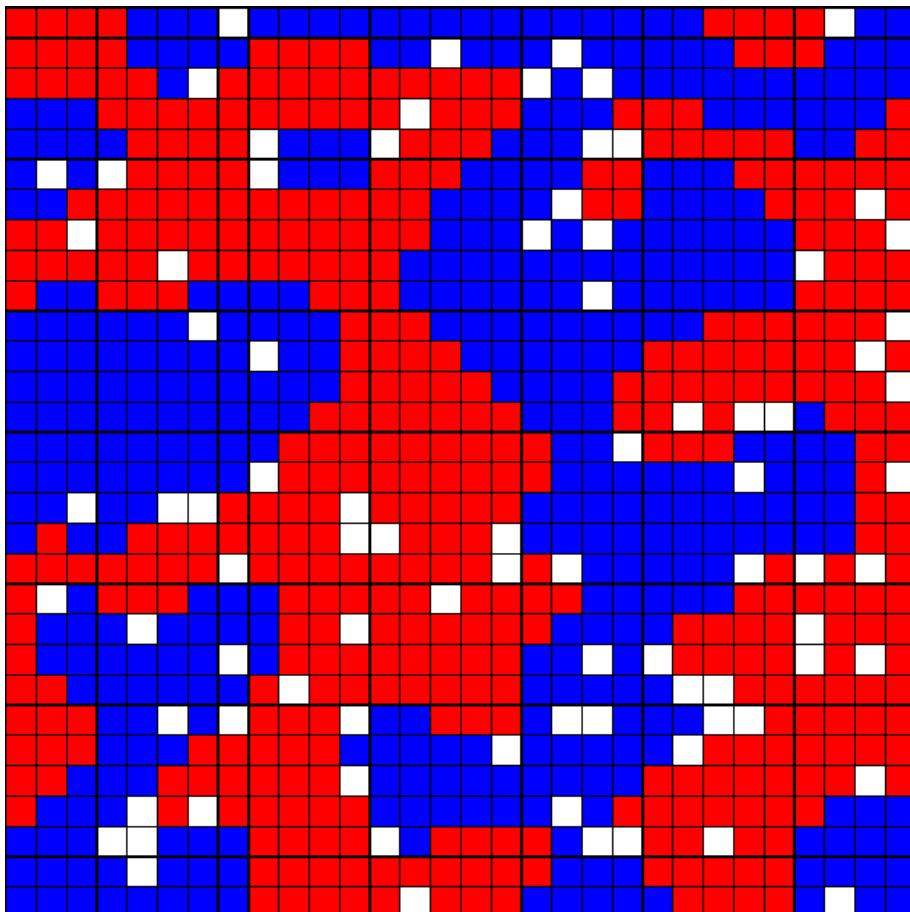
- $F=0.6$
- Result is very segregated
- Empty cells form borders between clusters
- Several iterations are necessary for all agents to be satisfied

0.5 intolerance case



- $F=0.5$
- Result is substantially segregated
- Empty cells form borders between clusters
- Several iterations are necessary for all agents to be satisfied

0.4 intolerance case



- $F=0.4$
- Result is also segregated
- **Agents are OK living in minority!**
- All agents can be satisfied

Overview

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- 3. Analyzing Schelling's model**
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Measuring segregation: Moran's I

$$I = \frac{M \sum_i \sum_j w_{i,j} (x_i - \bar{x})(x_j - \bar{x})}{(\sum_i \sum_j w_{i,j}) \sum_i (x_i - \bar{x})^2}$$

- M : Number of occupied cells
- $w_{i,j}$: adjacency matrix of cells
 - $w_{i,j} = 1$ if i is a neighbor of j , otherwise $w_{i,j} = 0$
- x_i : color value of occupied cell i
 - $x_i = 1$ if blue agent in it, $x_i = 0$ if red agent in it
 - We only count over occupied cells, ignore empty ones
- \bar{x} : mean value of x , i.e. fraction of blue agents
- Also known as spatial autocorrelation

How to calculate Moran's I

We rewrite I as $I = \frac{MC}{W\sigma}$, where:

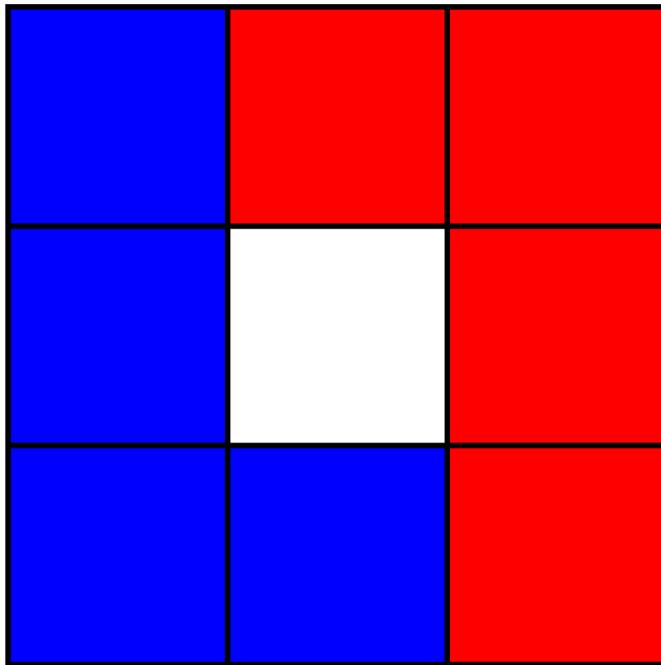
$$C = \sum_i \sum_j w_{i,j} (x_i - \bar{x})(x_j - \bar{x})$$

$$W = \sum_i \sum_j w_{i,j}$$

$$\sigma = \sum_i (x_i - \bar{x})^2$$

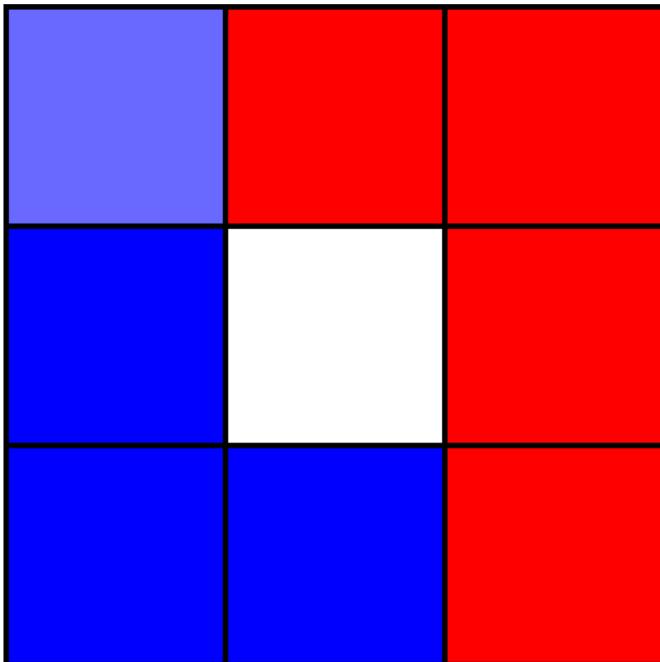
With a loop over the cells, we can iteratively calculate C , W , and σ

Moran's I example



- Example: 9 cells
- 3x3 neighborhood
- 4 red and 4 blue agents
 - $x_i=0$ if i occupied by blue
 - $x_i=1$ if i occupied by red
 - $\text{Avg}(x)=0.5$
 - $M=8$

Moran's I example



- First cell: blue ($x_1 = 0$)

- Contribution to σ :

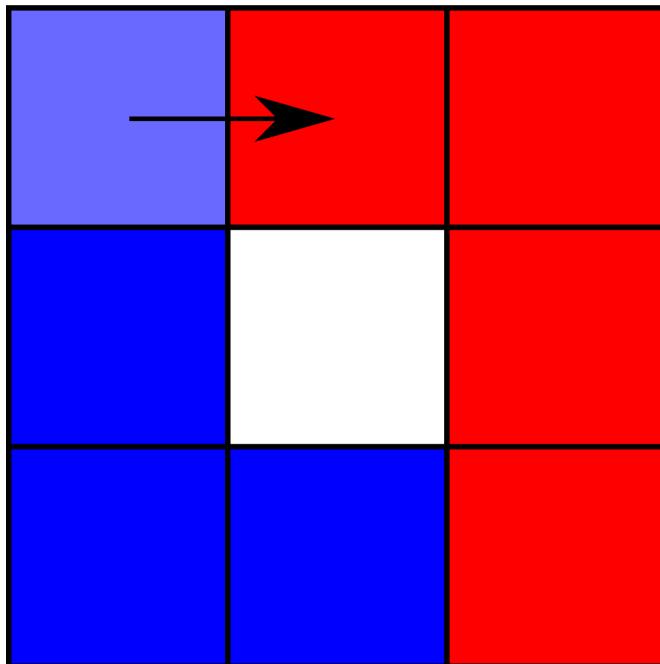
$$\sigma \leftarrow \sigma + (x_i - \bar{x})^2 \leftarrow \sigma + (0 - 0.5)^2$$

- Contribution to W : 2 neighbors

$$W \leftarrow W + 2$$

- Empty neighborhood cell doesn't count!

Moran's I example



- First neighbor (cell 2): contribution to C

$$C \leftarrow C + w_{1,2}(x_1 - \bar{x})(x_2 - \bar{x})$$

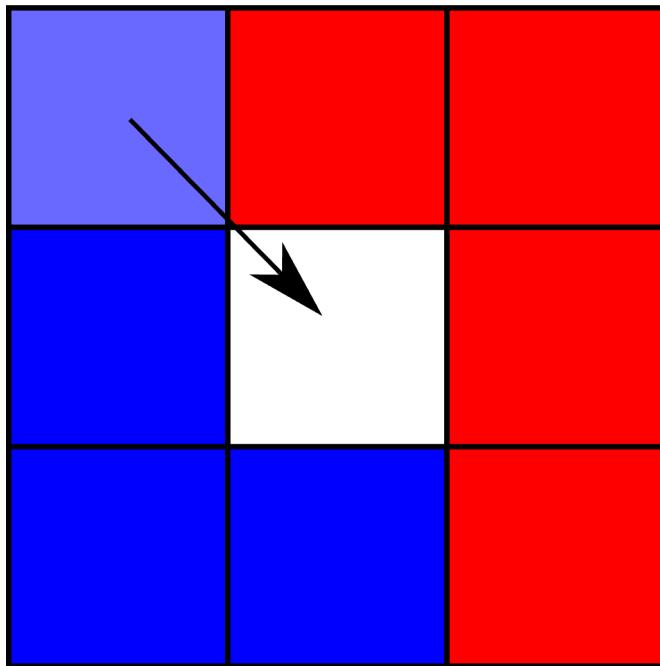
- Because cells 1 and 2 are neighbors, $w_{1,2} = 1$

$$C \leftarrow C + 1(0 - 0.5)(1 - 0.5)$$

$$C \leftarrow C - 0.25$$

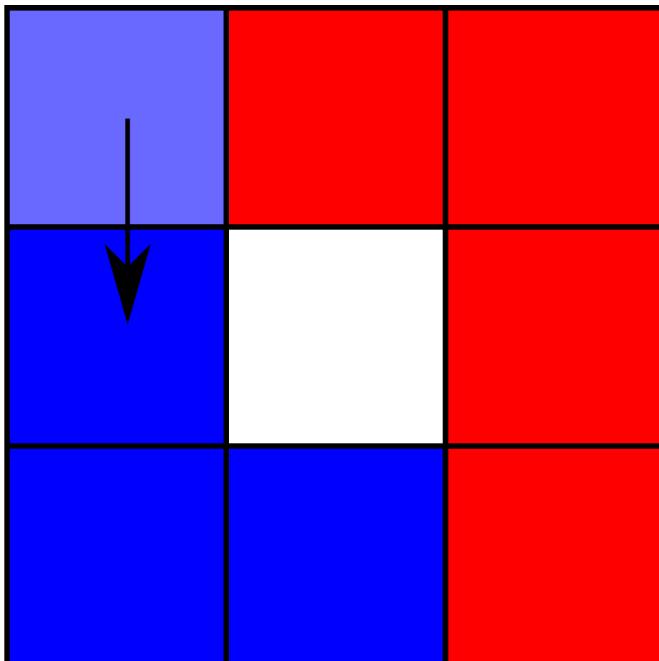
- Different neighbors reduce I

Moran's I example



- Second neighbor (cell 5)
- Does not contribute to C because it is empty

Moran's I example



- First neighbor (cell 4): contribution to C

$$C \leftarrow C + w_{1,4}(x_1 - \bar{x})(x_4 - \bar{x})$$

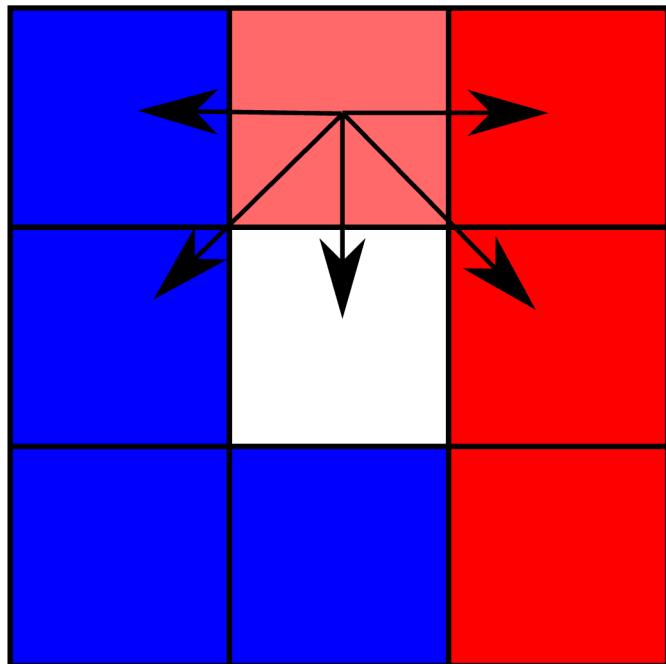
- Because cells 1 and 4 are neighbors, $w_{1,4} = 1$

$$C \leftarrow C + 1(0 - 0.5)(0 - 0.5)$$

$$C \leftarrow C + 0.25$$

- Equal neighbors increase I

Moran's I example



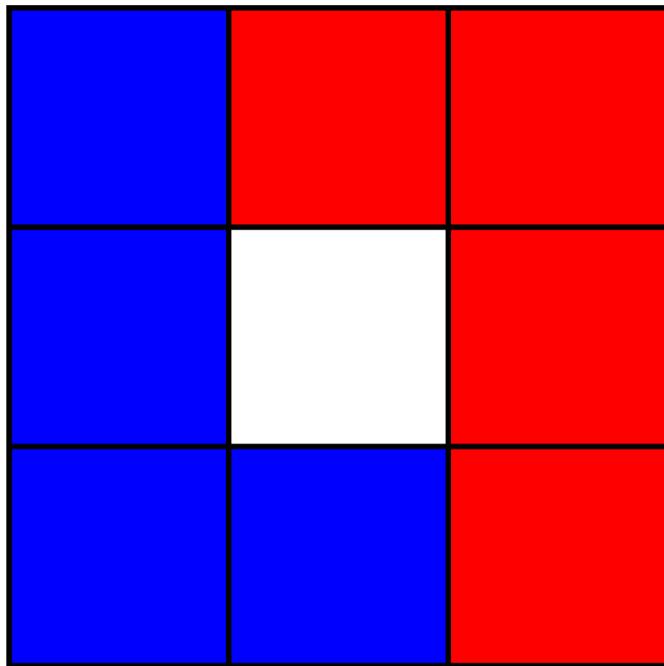
- Second cell: red ($x_2 = 1$)
- Contribution to σ and W

$$\sigma \leftarrow \sigma + (x_i - \bar{x})^2 \leftarrow \sigma + (1 - 0.5)^2$$

$$W \leftarrow W + 4$$

- Four contributions to C :
- $$C \leftarrow C + 0.25 + 0.25 - 0.25 - 0.25$$

Moran's I example



$$M = 8$$

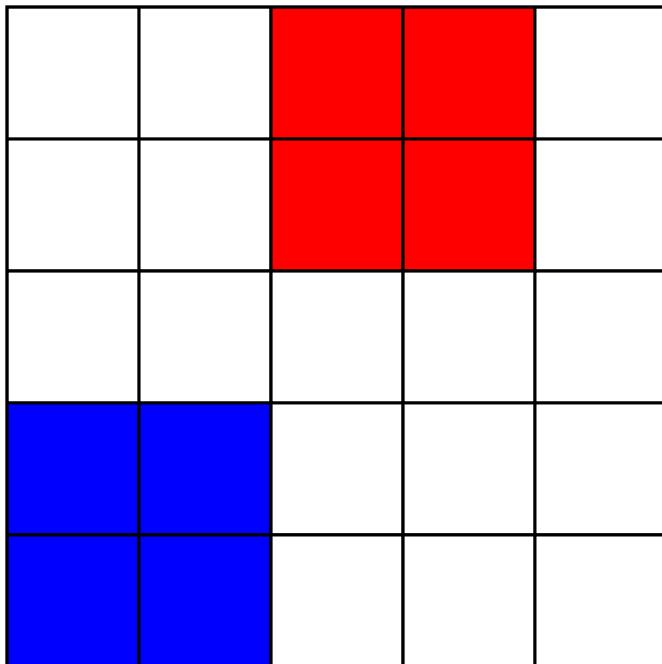
$$C = 16 * 0.25 - 8 * 0.25 = 2$$

$$W = 24$$

$$\sigma = 0.25 * 8 = 2$$

$$I = \frac{MC}{W\sigma} = \frac{8 * 2}{24 * 2} = 0.3$$

Maximum Moran's I example

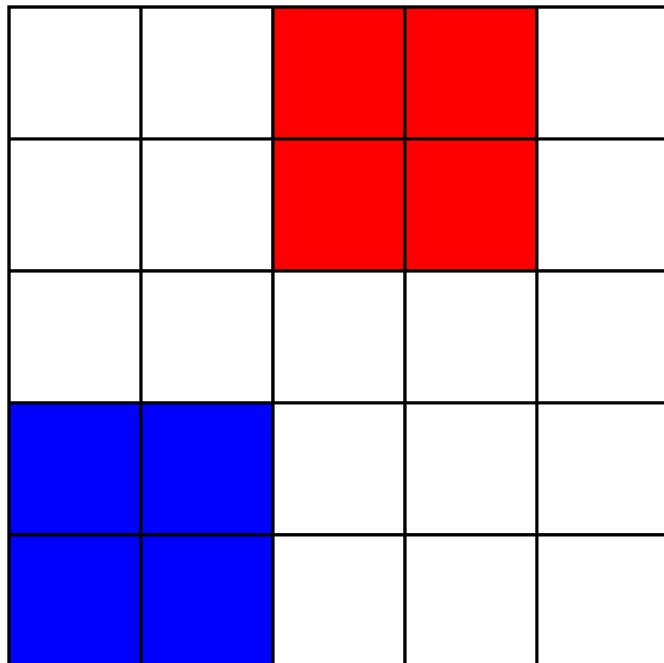


Calculate the Moran Index for this

Hint: team up in a group of 2 and split the task:

C , W , sigma

Maximum Moran's I example



$$M = 8$$

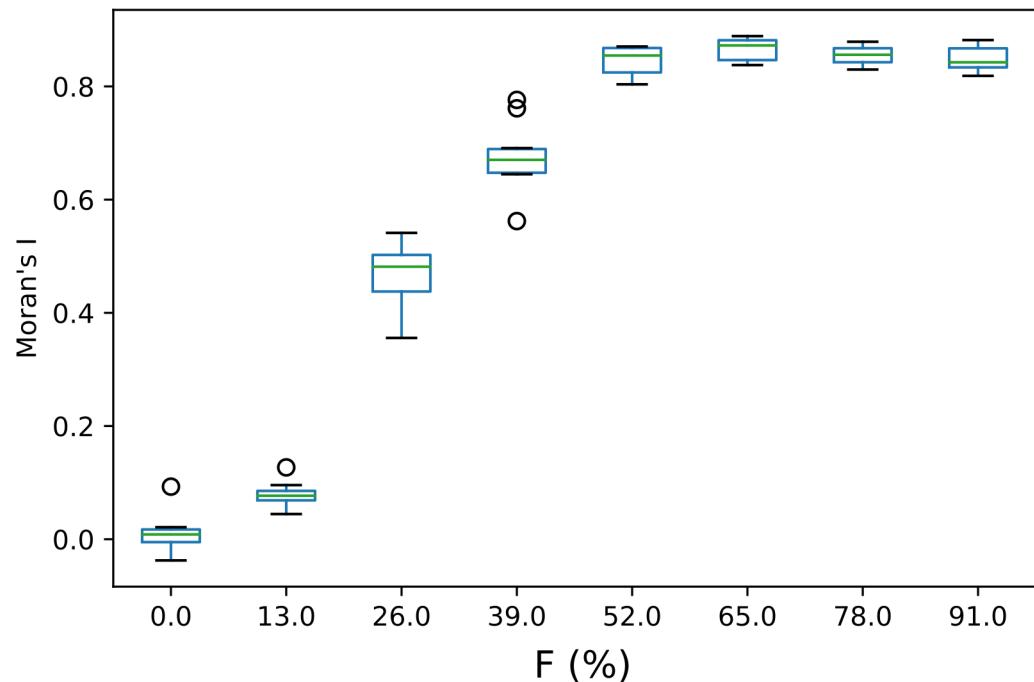
$$C = 24 * 0.25 = 6$$

$$W = 24$$

$$\sigma = 0.25 * 8 = 2$$

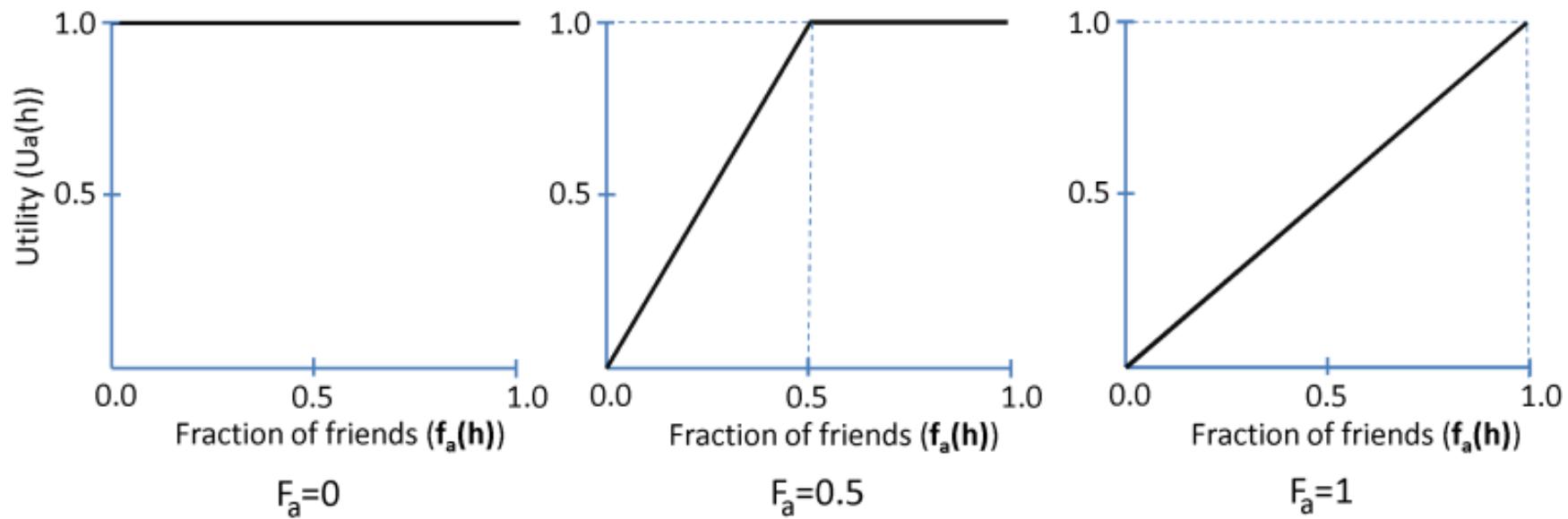
$$I = \frac{MC}{W\sigma} = \frac{8 * 6}{24 * 2} = 1$$

Segregation versus tolerance



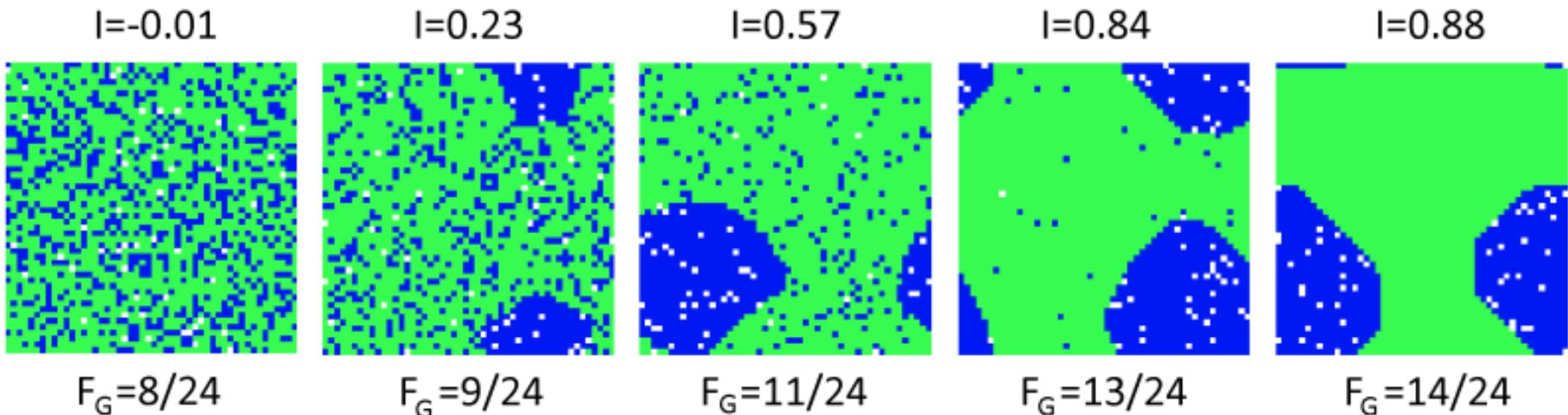
- 3x3 neighborhood (up to 8 neighbors), torus edges
- Boxplots of Moran I after convergence in several simulations
- Moran's I stays low for low F values
- Sharp increase above two neighbors for F
- Substantial segregation for $F > 0.33$

Generalizing relocation rules



- Modeling the utility of an agent under with certain fraction of similar agents
- Probability to relocate every iteration: $1 - \text{utility}$
- Happiness is measured as total utility

Coexistence between segregation and mixtures



- Generalization of model: different number of agents per color and different thresholds for each color
 - Model reproduces observed clusters of one group with mixed regions
- The Schelling Model of Ethnic Residential Dynamics: Beyond the Integrated - Segregated Dichotomy of Patterns. Erez Hatna and Itzhak Benenson. Journal of Artificial Societies and Social Simulation, 2012

Thomas C. Schelling Facts

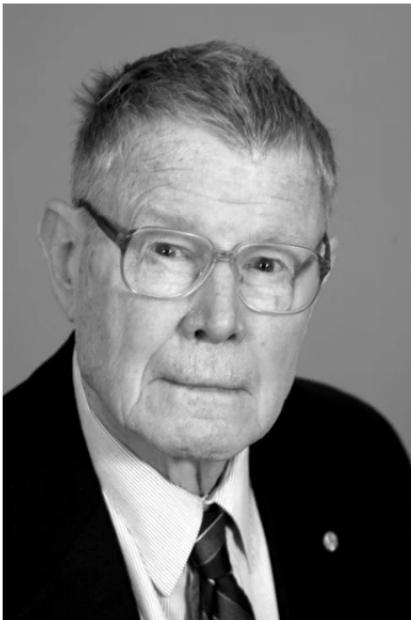


Photo: T. Zadig

Thomas C. Schelling
The Sveriges Riksbank Prize in Economic Sciences in
Memory of Alfred Nobel 2005

Born: 14 April 1921, Oakland, CA, USA

Died: 13 December 2016, Bethesda, MD, USA

Affiliation at the time of the award: University of Maryland,
Department of Economics and School of Public Policy,
College Park, MD, USA

Prize motivation: “for having enhanced our understanding
of conflict and cooperation through game-theory analysis”

Prize share: 1/2

Overview

1. Segregation
2. Schelling's segregation model
3. Analyzing Schelling's model
- 4. Cellular Automata: The game of life**

Cellular Automata

- agents \equiv cells, arranged on a predefined lattice
- formal structure of Cellular Automata
 - finite or infinite grid of identical cells $i=1, \dots, L$
 - cell with internal state $\theta_i(t) \in \{\theta_1, \dots, \theta_n\}$
 - neighborhood configuration: $\beta_i(t) \in \{\beta_1, \dots, \beta_m\}$
 - discrete dynamics for the state variable at $t+1$:

$$\theta_i(t+1) = F[\theta_i(t), \beta_i(t)]$$

- characteristics:
 - discrete space, time, and states
 - same transition rule applies to all cells
 - *each cell is an agent (!)*

Conway's Game of Life

- History

- **John Conway** (1970): design a self-replicating system
- 2010: first self-replicating structure in *Life*, called *Gemini*
- 2013: first replicator that creates a bigger copy of itself (self-similarity)

- Rules of the Game

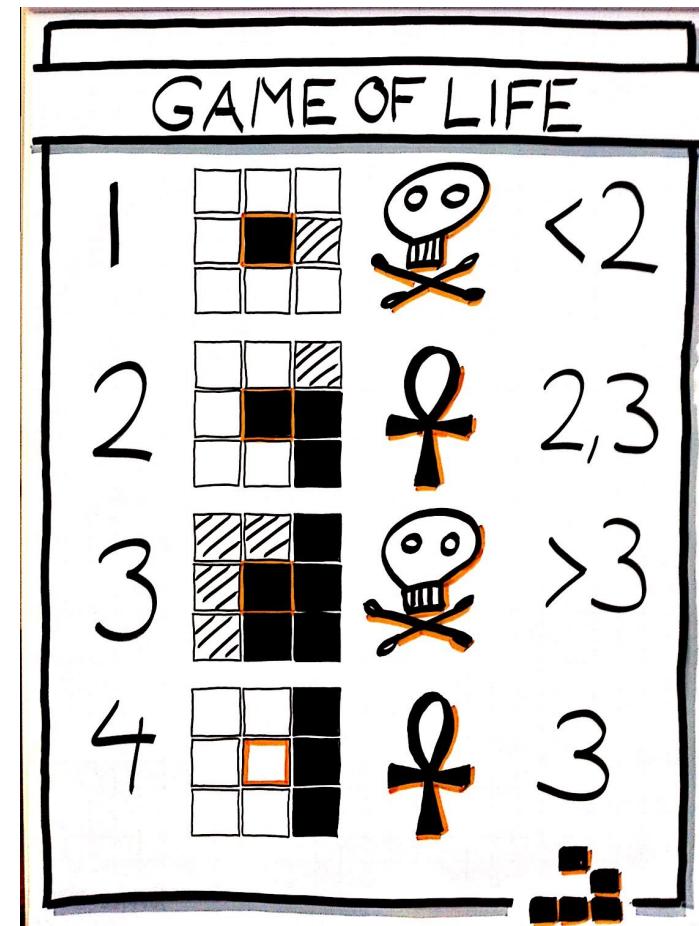
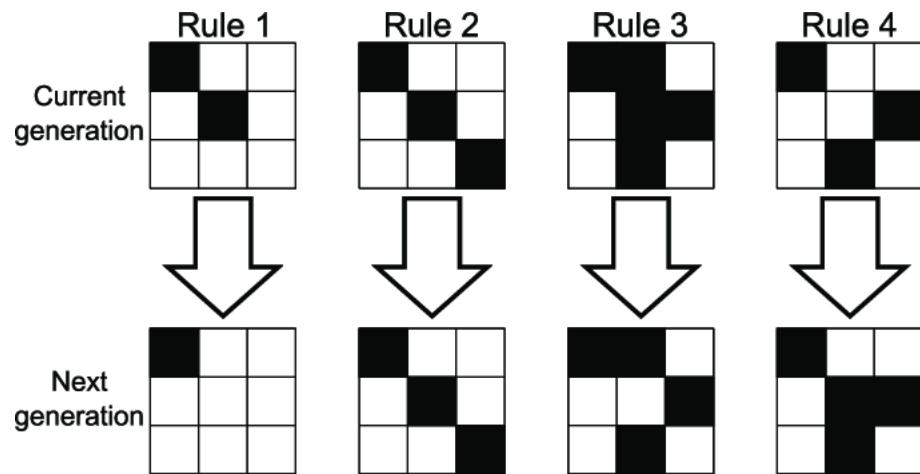
- 2D rectangular infinite grid with "dead" and "alive" cells
- a dead cell becomes alive if 3 neighbours are alive - **reproduction**
- an alive cell dies if less than 2 neighbours are alive - **underpopulation**
- an alive cell dies if more than 3 neighbours are alive - **overcrowding**

- Importance of Conway's Game of Life

- complex behaviour emerges from simple rules for agents
- early modeling of self-organization

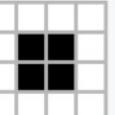
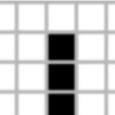
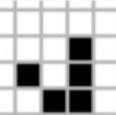
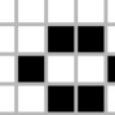
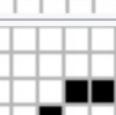
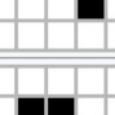
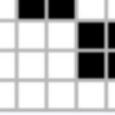
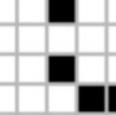
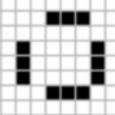
To play the game: <https://playgameoflife.com>

Rules of the Game of Life



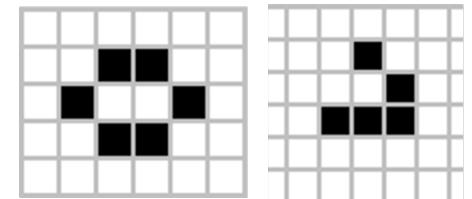
Evolution of structures

Same rules, different outcomes

Still lifes		Oscillators		Spaceships	
Block		Blinker (period 2)		Glider	
Bee-hive		Toad (period 2)		Light-weight spaceship (LWSS)	
Loaf		Beacon (period 2)		Middle-weight spaceship (MWSS)	
Boat		Pulsar (period 3)		Heavy-weight spaceship (HWSS)	
Tub		Penta-decathlon (period 15)			

- **Still lifes:** static patterns (e.g. 'beehive')
- oscillators: repeating patterns
- **spaceships:** translating patterns (e.g. 'glider')
- **guns:** stationary patterns that emit spaceships
- **puffers:** translating patterns that leave a trail of debris behind

Evolution of structures



the Gemini replicator: <http://youtu.be/A8B5MbHPIH0>

A whole library of structures is available
at <https://playgameoflife.com/lexicon>

You can even setup Game of Life in the Game of
life: <http://youtu.be/xP5-ileKXE8>

Quiz

- Is there residential segregation in Graz? If so, on what attributes?
- What other contexts can have segregation besides the residential case?
- What are the simplifications of Schelling's model?
- How is Schelling's model different to the formulation of cellular automata?
- Do you know any other cellular automata besides the game of life?