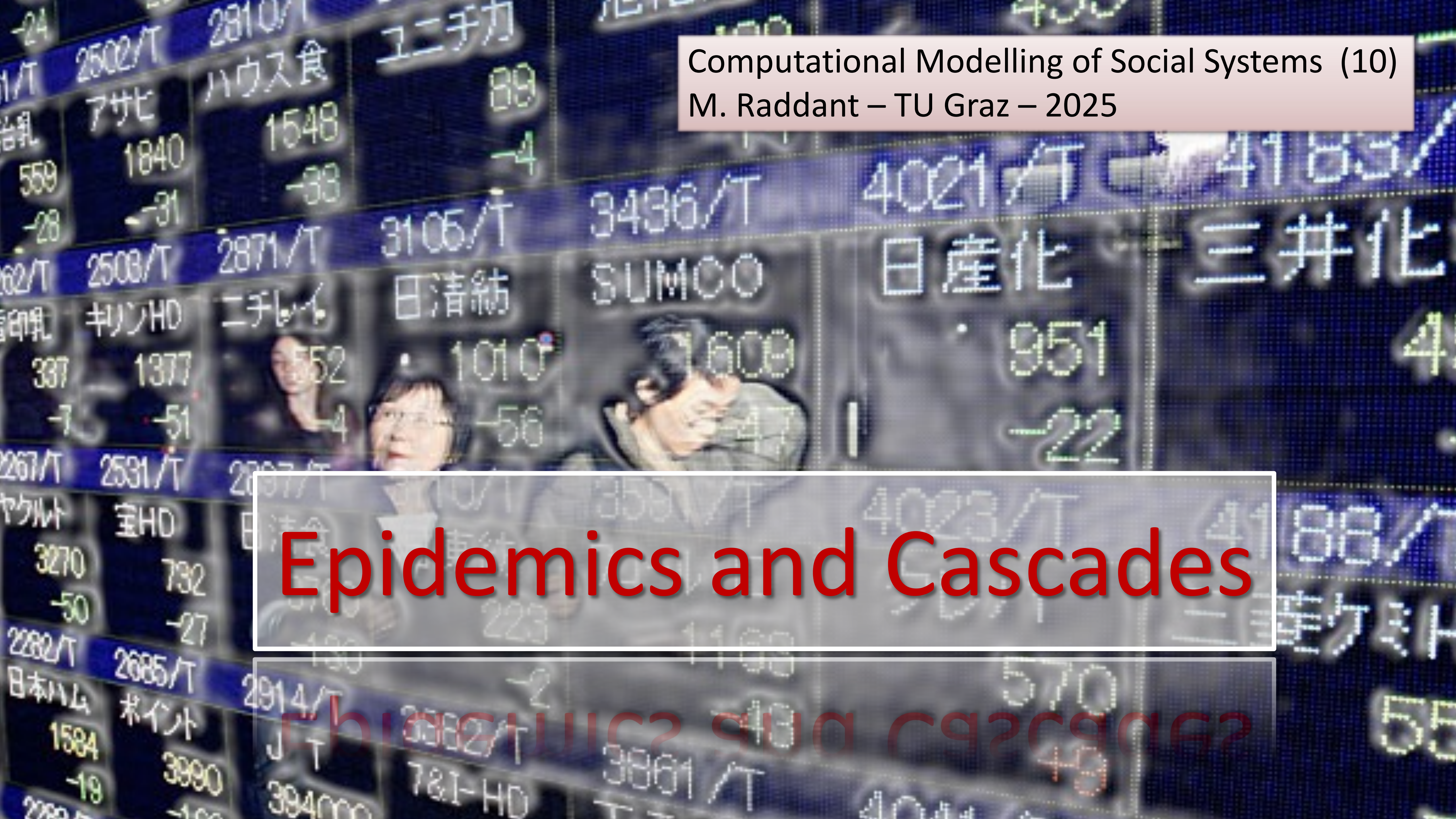


Computational Modelling of Social Systems (10)
M. Raddant – TU Graz – 2025

Epidemics and Cascades

Today

The connection of structure and function in networks

…and why networks can be really bad: about epidemics, cascades and risk.

- Percolation
- SI-R-S Model
- Epidemics
- Financial contagion, systemic risk

Background literature for this lecture: Chapter 17 from the book 'Networks, An Introduction' by M. Newman, Oxford 2010

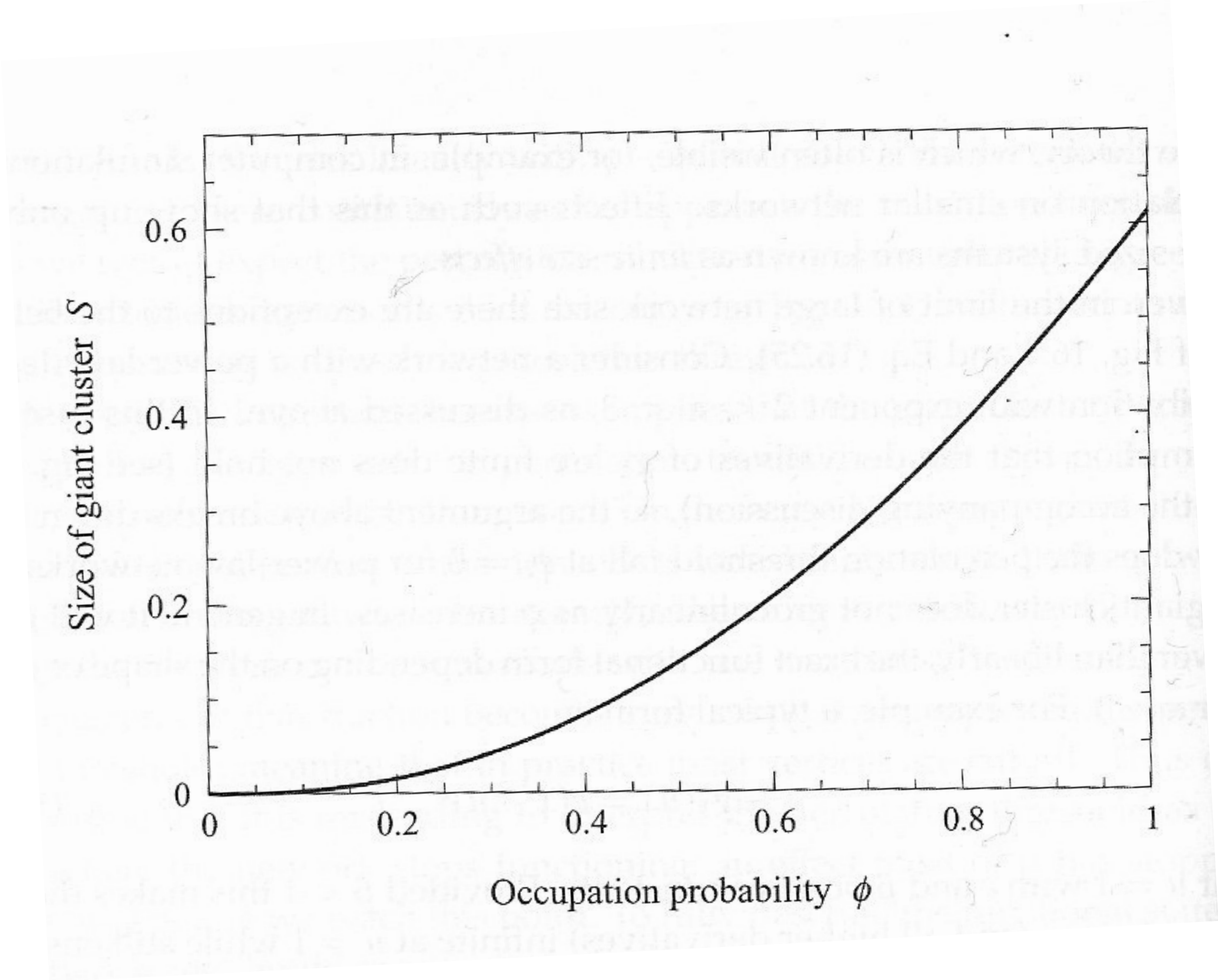
Percolation

- **Percolation** refers to a process where single parts of a network fail or are removed
- **Site percolation**: removal of vertices
- **Bond percolation**: removal of edges
- A lot of examples exist, where real world networks have an ongoing percolation process or where human action can be interpreted as a percolation effect

- **Technical infrastructure** and the failure of single components:
  - Nodes of the power grid
  - Routers in the internet
  - Junctions and crossings in transportation networks
- Animals and humans with **infections diseases**, where the **immunization** is an act of percolation

- Hence, in a percolation model vertices can be function or dysfunctional. The terminology for this is “**occupation**”
- The occupation probability $\boldsymbol{\varphi}$, can vary between ***0*** and ***1***
- For most networks one can observe a **percolation transition**, which means that at some critical level for $\boldsymbol{\varphi}$ a „disfunctional“ collection of unconnected clusters form a **giant cluster**

Figure from Newman (2010)



- We will skip the theoretical treatment of percolation models here and go straight to some application:
- Often it is possible to analyze networks by **simulating percolation**, and to get some insights into the stability of a network and critical parts.
- A straight forward approach is to generate variations of the original network with a random set of the **vertices removed** and to calculate the mean of the size of the largest component $\boldsymbol{S(\varphi)}$

- This can be achieved by repeatedly using a **breach-first based** algorithm to find the largest component and calculate its size
- The repeated calculation of the clusters and sizes however, is rather time consuming
- It **is more efficient** to work the other way around: start with an empty network and "switching on" vertices in a random order several times

- Use an algorithm like this:
- Start with an **empty network** and choose a **random order** to add vertices (with its edges), give each cluster a label
- After adding a vertex, check it the **labels** of now connected vertices are the same. If not, give them the same name
- **Update** the number of different labels (# clusters) and the maximum of items with the same label (giant component size)
- Repeat until all vertices are added
- Repeat for different random orders

Epidemics

- Models of epidemics on networks are widely used to analyze the spread of diseases like HIV and the flu.
- In the basic version however, there is no network, and we only look at the interaction of the healthy and infected part of the population.
- This is called the **SI** model, short for **susceptible** and **infected**

- The basic model assumes that the transmission of a disease takes place in a kind of mass interaction, so that everybody has the same chance to be infected
- Assume a **population** where ***S(t)*** describes the expected number of people who are susceptible and ***X(t)*** the expected number of infected at time ***t***
- Assume that people **meet at random** with a rate of **β** **contacts per time unit**

- A susceptible gets infected only when he meets an infected person
- The **population** size is given by $\boldsymbol{n}$ (a constant)
- Probability to **pick** a susceptible is $\boldsymbol{S/n}$
- An infected has contact with $\boldsymbol{\beta S/n}$ susceptible per time unit
- Since the average number of infected is defined as $\boldsymbol{X}$, the average of **new infections** is $\boldsymbol{\beta SX/n}$
- We can write this as a differential equation

- The rate of change of ***X***

$$\frac{dX}{dt} = \beta \frac{SX}{n}$$

- and in the same manner for the susceptible ***S***

$$\frac{dS}{dt} = -\beta \frac{SX}{n}$$

- We can define ***s*** and ***x*** as variables to represent the fractions, such that

  s=S/n and ***x=X/n***

- We can than rewrite the equations as

$$\frac{ds}{dt} = -\beta s x$$

$$\frac{dx}{dt} = \beta s x$$

- Since ***n=S+X*** or ***1=s+x*** , we can write ***s=1-x***

- This leaves us with the **logistic growth** equation

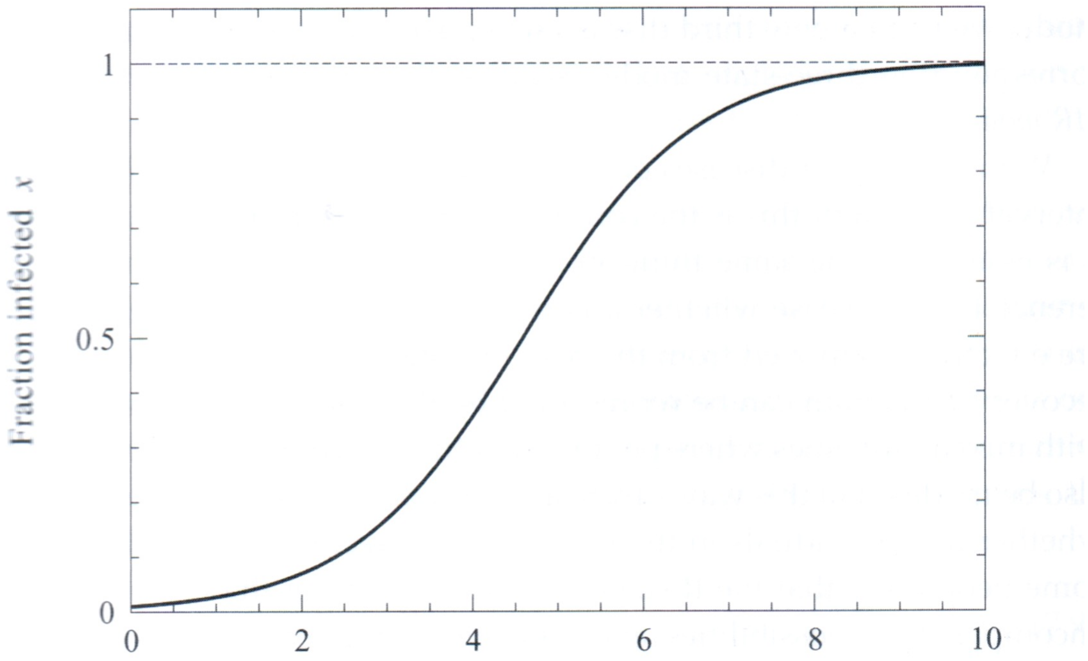
$$\frac{dx}{dt} = \beta(1 - x)x$$

- The solution to this is

$$x(t) = \frac{x_0 e^{\beta t}}{1 - x_0 + x_0 e^{\beta t}}$$

- Which will produce an S-shaped curve

Figure from Newman (2010)



- This is probably not the most realistic representation of a disease, since it mostly does not infect the entire population. Also part of the population is or may become immune. If the speed is slow also other effects will become important in these dynamics.

SIR

- One extension to the SI model is to add the feature of **recovery**. This means that some time after the infection people can recover from the disease.
- Mostly this also means that they cannot get infected again
- This leads us to the **Susceptible-Infected-Recovered model**

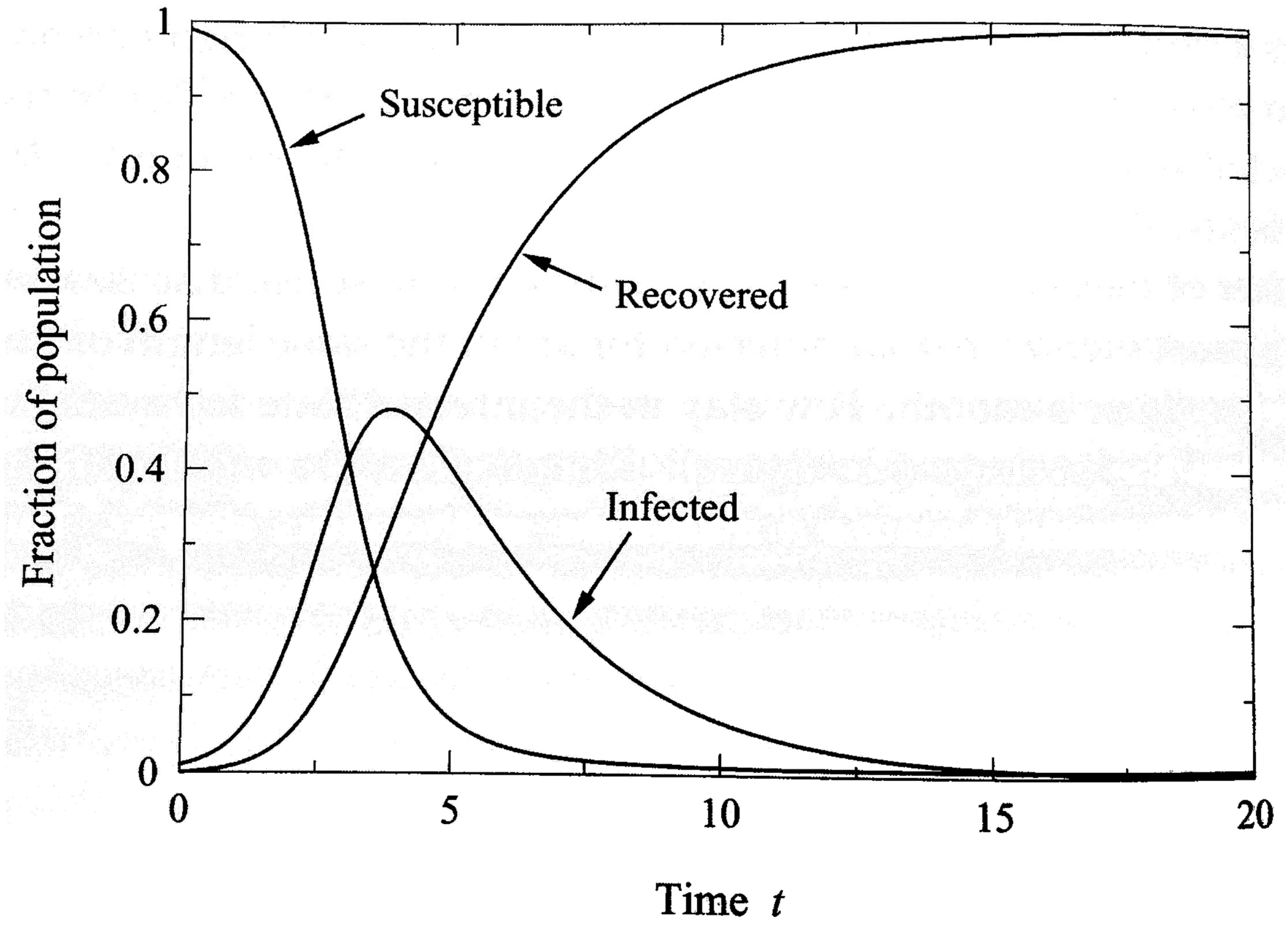
- We add a second stage to our model where infected people can **recover** (or die) at a rate **γ**
- The equations of the SIR model are

$$\frac{ds}{dt} = -\beta s x$$

$$\frac{dx}{dt} = \beta s x - \gamma x$$

$$\frac{dr}{dt} = \gamma x$$

Figure from Newman (2010)



- This modified model can be used to calculate the outbreak size of a disease (the number of people ever infected)

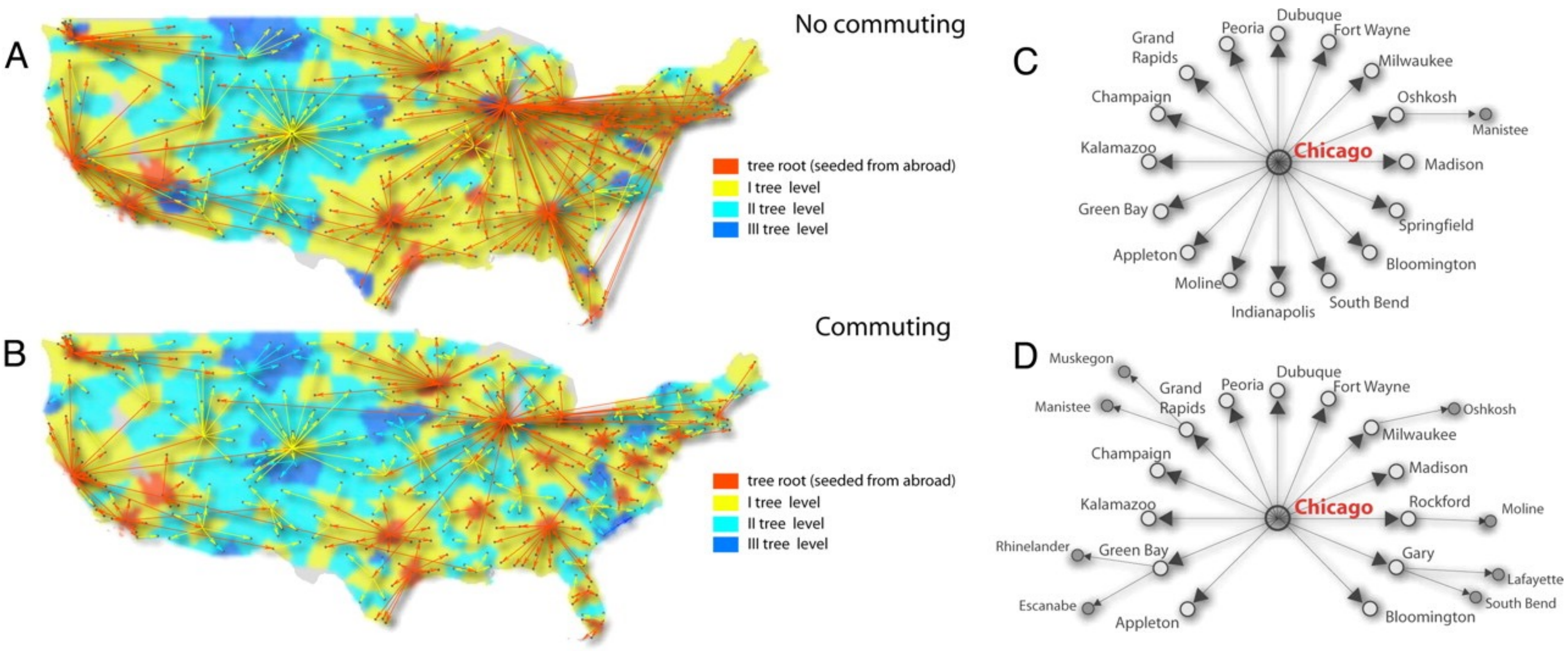
- Yet another variation is the **SIS model**, which allows for **re-infection**
- In the simplest version this means that people continuously switch between the two states

- In the **SIRS model** people can recover from a disease and develop **temporary immunity**
- After some time, when immunity is lost they become susceptible again

Epidemics on networks

- Taking these models to networks means to get rid of the assumption that all people can meet with the same probability, instead we can approximate the **real contacts** by information about the **location** of people, and their regular **commuting** patterns, like going to work, meeting neighbors, and neighbors.
- Where we do not have information about individuals we can **model groups**, e.g., the behavior of children, workforce, senior citizens, together with their group specific infection risk

Epidemic invasive tree.



Epidemic invasive tree. (A and B) Geographical representation of the continental U.S. epidemic invasion tree with only airline traffic (A) and when both airline traffic and commuting are considered (B). Red represents the roots (i.e., the first cities that were seeded from abroad), and, as we move down the tree, the colors change from yellow to dark blue. The arrows representing the edges of the tree are colored as the parent node. (C and D) We also provide a schematic representation of the invasion tree rooted at Chicago when only flights are considered (C) and with both air traffic and commuting (D). As demonstrated in both examples, the spreading pathway is completely dominated by the airline hubs as the only sources of imported seeds. However, the hierarchy is broken by the introduction of commuting flows as the number of shells around the airline hubs and the branches at the secondary nodes increase.

Balcan D et al. PNAS 2009;106:21484-21489

PNAS

Risks in Financial Markets

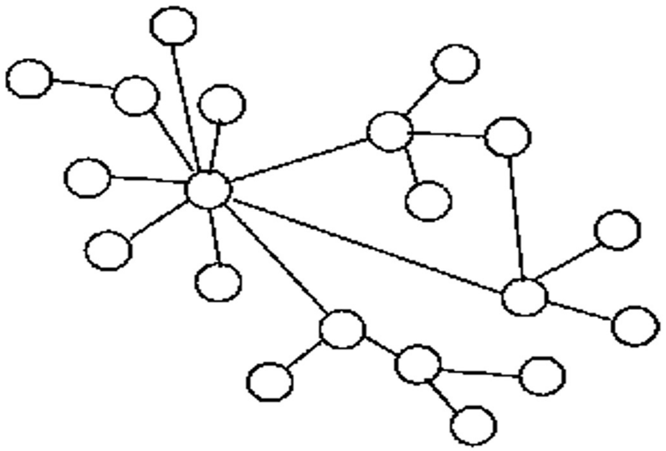
- Counterparty risk
  - default of a institution you have a deal with
- Settlement risk
  - failure to deliver a security or cash value
- Market risk
  - losses due to change of the price
- Liquidity risk
  - inability to convert assets in the short term
- Systemic risk
  - collapse of a financial system

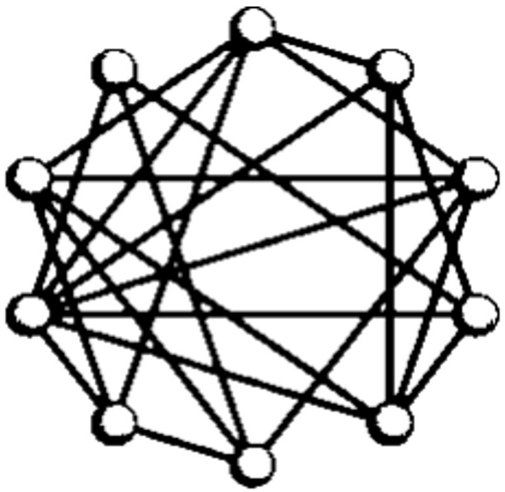
More on **Systemic Risk**

- The concept of ***systemic risk*** is rooted in the analysis of disease spreading, i.e., has is origins in Biology and Ecology
- In financial systems, a ***systemic event*** is defined to happen when information about an institution (including its default) leads to subsequent failure of one or several other institutions or markets
- Mostly, the case in which subsequent failures lead to a default are labeled ***strong systemic events***, or ***contagion***

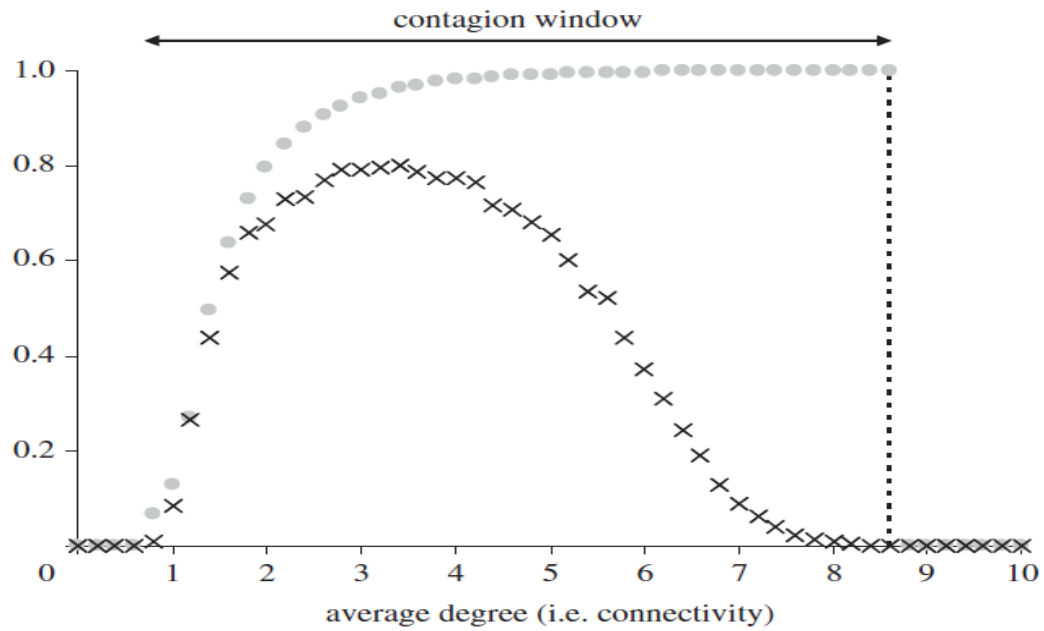
Stability of the interbank network

- An important part of interbank networks exists in the form of the interbank loan market, where banks provide liquidity to each other via overnight loans
- Interbank networks are (approximately) scale-free networks, which are less susceptible to most shocks than (Bernoulli) random graphs (where the connectivity is homogenous)
- This makes the outcome of shocks difficult to predict





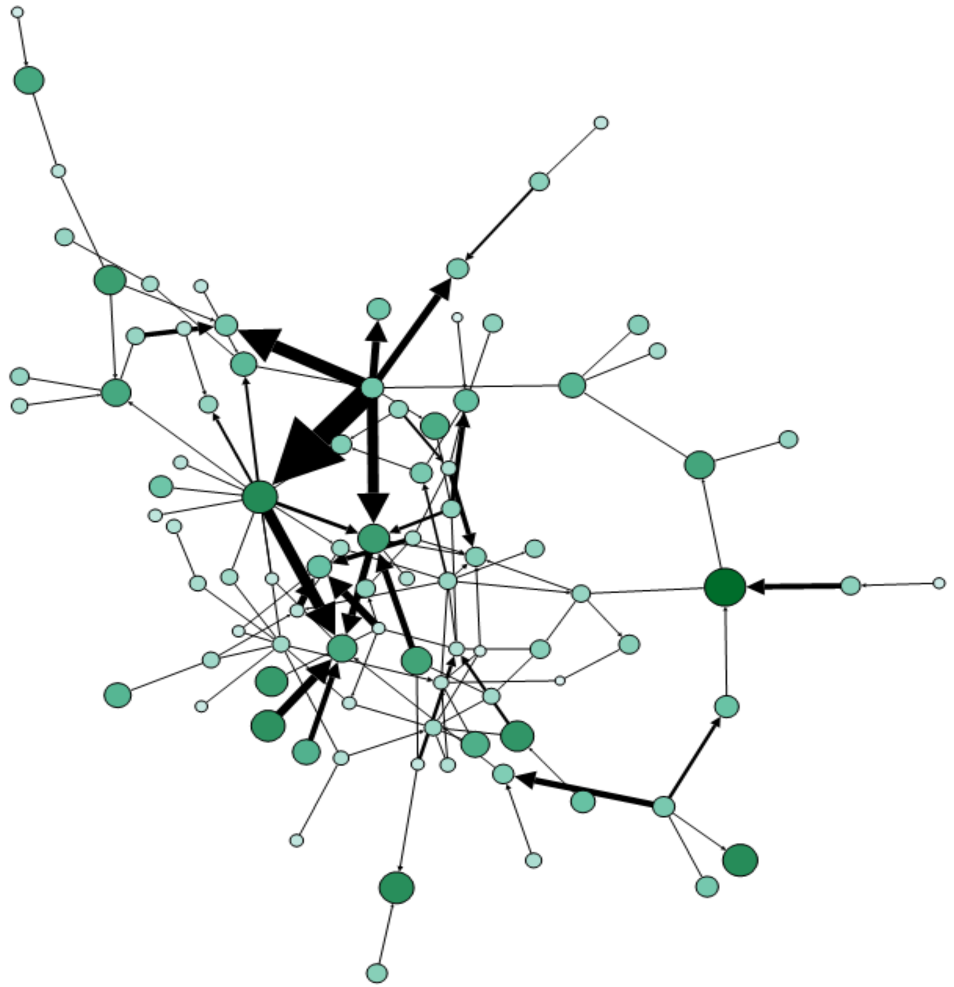
- Question: Is connectivity stabilizing?
- Answer: Mostly
  - once we reach a certain threshold
  - and if shocks are not excessive



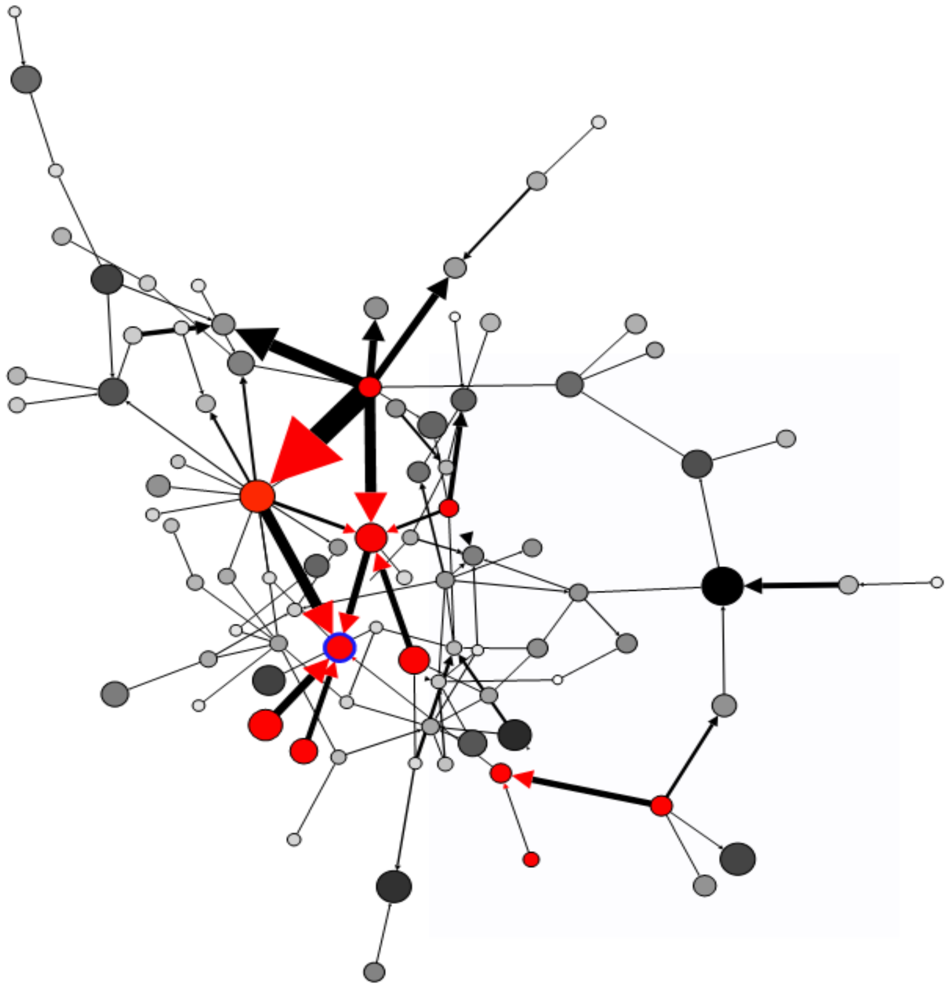
source: Gai and Kapadia, Proc R Soc A, 2010

Cascade simulations with empirical data

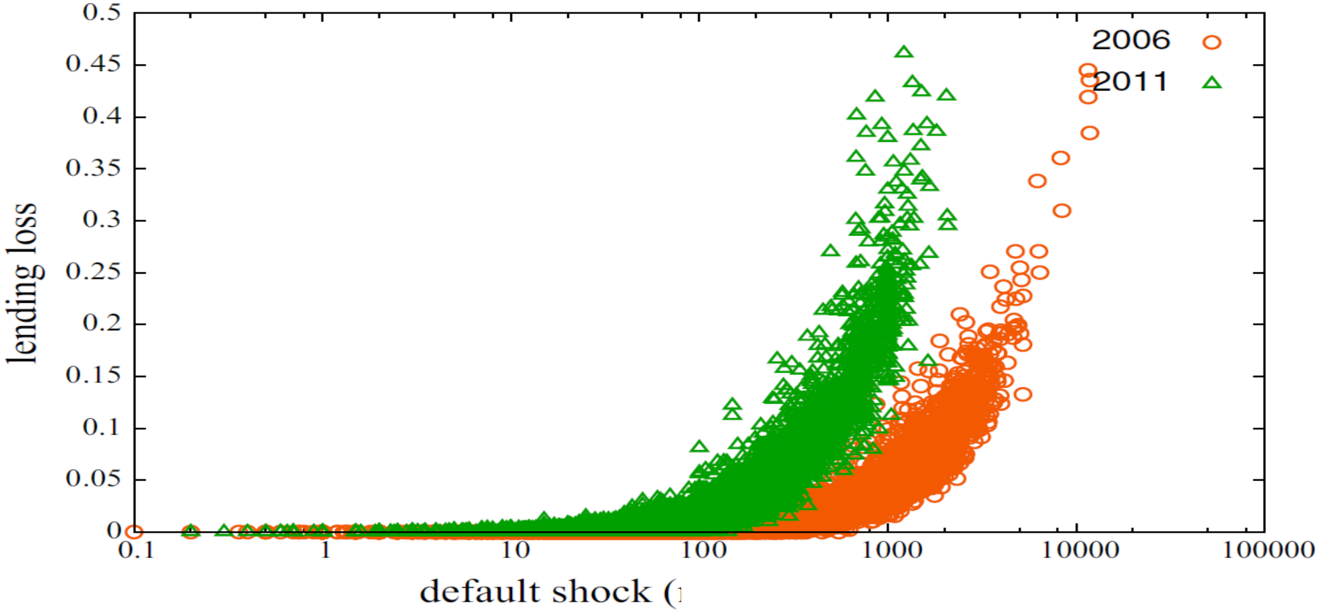
One week snapshot of the network of Italian overnight loan market E-mid. Nodes are banks and links are loans. Node size is according to their degree and link size is according to the loan.



Interbank network under default shock. The bank with the blue circle defaults and it gives shock to its lenders.

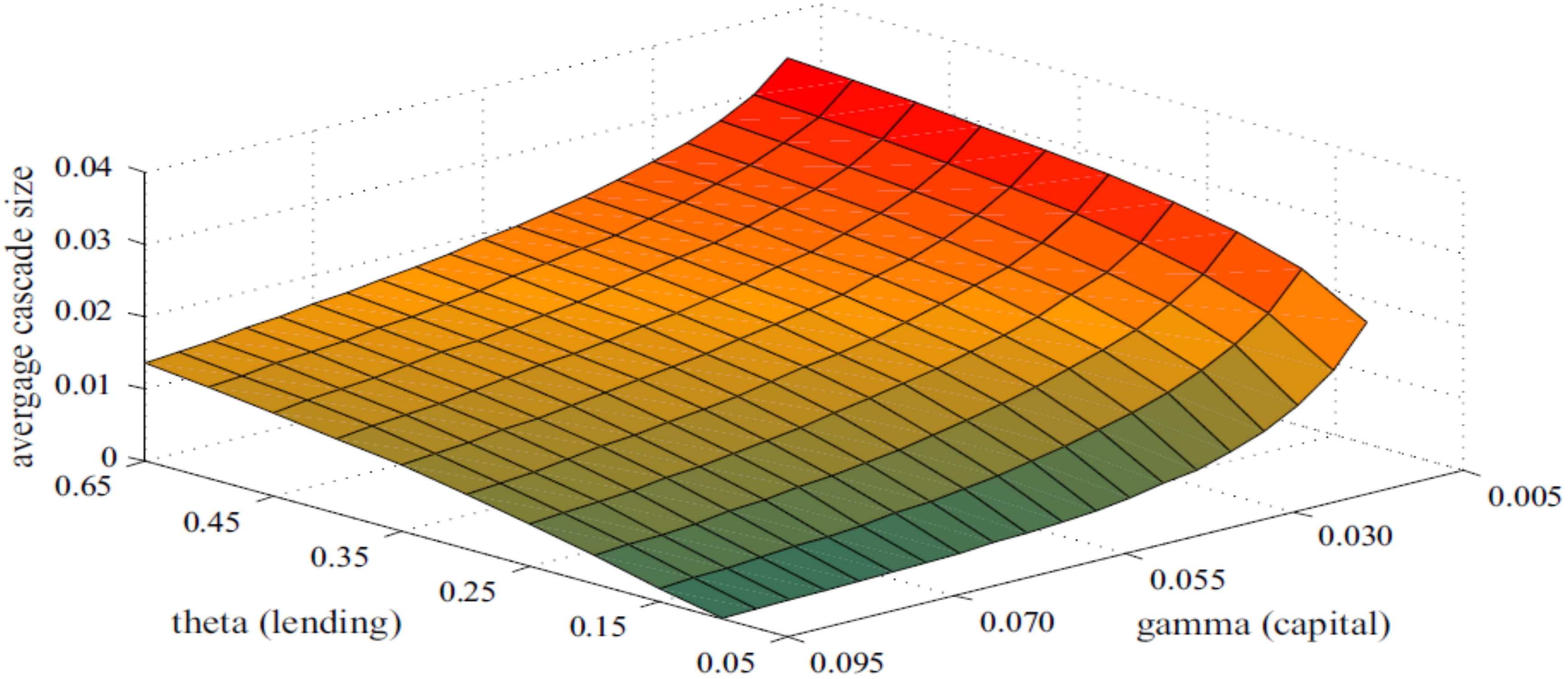


Karimi/Raddant (2016), Cascades in Real Financial Markets, CompEcon .



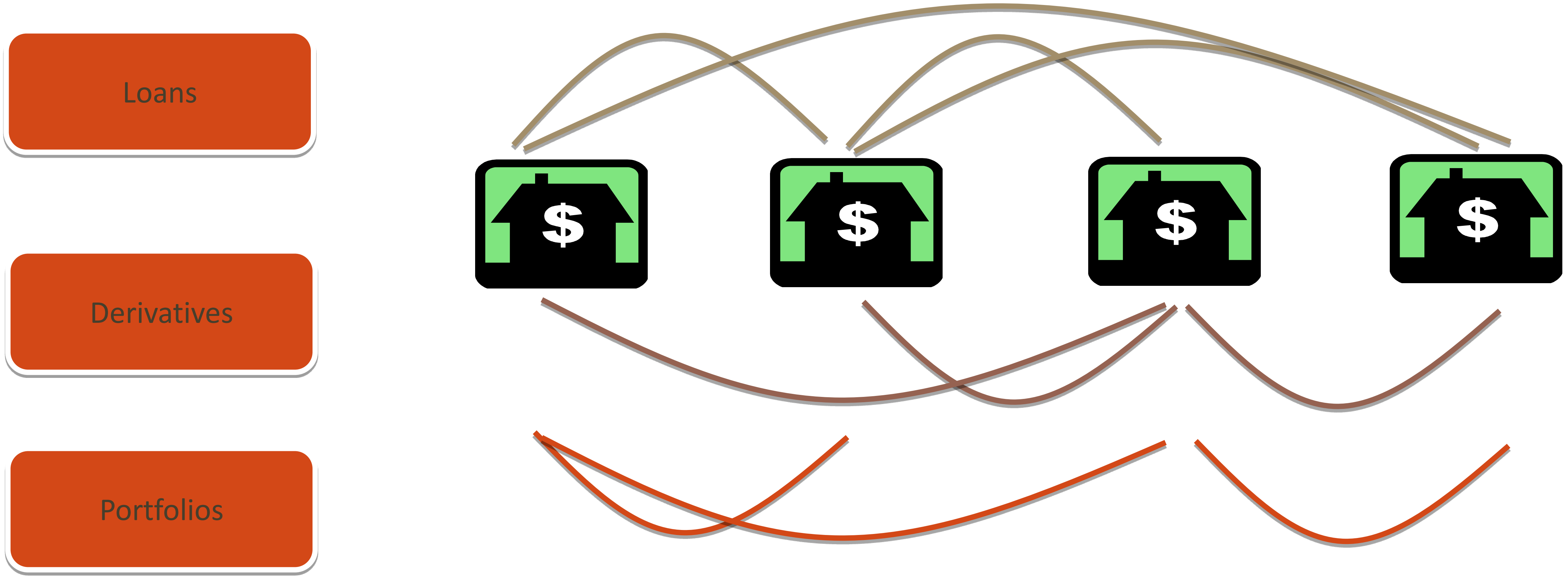
Only very few shocks lead to default cascades, smaller shocks are mostly negligible

The level of defaults depends heavily on the parameterization of the model



The Financial Multiplex

Long chains of defaults can mostly not be reproduced by cascade simulations in loan networks. Also the dispersion of shocks to different regions demands more complex structures. Including the holdings of securities and assuming some dynamics for their price seems like a useful way to reproduce the effect of macro shocks.



G-SIBs as of November 2023[10] allocated to buckets corresponding to required levels of additional capital buffers

Bucket[11]	G-SIBs in alphabetical order within each bucket
5 (3.5%)	(Empty)
4 (2.5%)	JP Morgan Chase
3 (2.0%)	Bank of America Citigroup HSBC
2 (1.5%)	Agricultural Bank of China Bank of China Barclays BNP Paribas China Construction Bank Deutsche Bank Goldman Sachs Industrial and Commercial Bank of China Mitsubishi UFJ FG UBS
1 (1.0%)	Bank of Communications (BoCom) Bank of New York Mellon Groupe BPCE Groupe Crédit Agricole ING Mizuho FG Morgan Stanley Royal Bank of Canada Santander Société Générale Standard Chartered State Street Sumitomo Mitsui FG Toronto Dominion Wells Fargo

Financial market regulation after 2008

Some regulation based on the centrality of banks: Additional capital requirements of "SIFIS"

G-SIBs as of November 2023, allocated to buckets corresponding to required level of additional loss absorbency

FSB, Nov 2023

Games on Networks

Today

Network Science and Game Theory

- Game Theory basics
- Games on fixed networks
- Game Theory and network structure

Background literature for this lecture: A. Wilhite, Economic Activity on fixed Networks, in: Handbook of Computational Economics, Vol. 2, L. Tesfatsion and K.L. Judd (ed.), Elsevier 2006.

Describing games

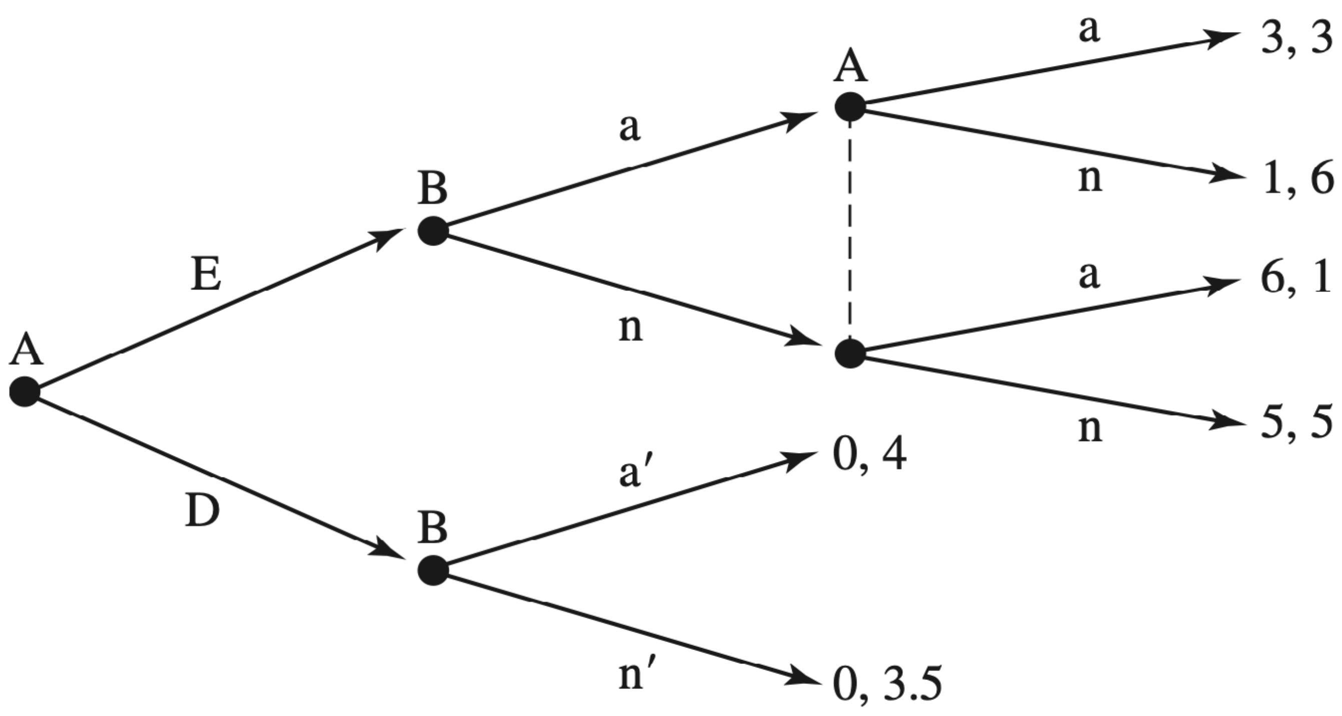
- list of players
- description of what players can do
- description of what they know when
- outcomes dependent on actions
- preferences over outcomes

Strategies

- A strategy is a complete contingent plan for a player in the game

The extensive form

Firm A decides whether to enter firm B's industry. Firm B observes this decision. If firm A enters, then the two firms simultaneously decide whether to advertise. Otherwise, firm B alone decides whether to advertise. With two firms in the market, the firms earn profits of $3 million each if they both advertise and $5 million if they both do not advertise. If only one firm advertises, then it earns $6 million and the other earns $1 million. When firm B is solely in the industry, it earns $4 million if it advertises and $3.5 million if it does not advertise. Firm A earns nothing if it does not enter.



Background literature Game Theory: J. Watson, Strategy (3rd edition), 2013, W.W. Norton

Normal-form games

Matrix representation: each row corresponds to a strategy of player 1, each column corresponds to a strategy of player 2.

Each cell of the matrix corresponds to a strategy profile and contains the payoffs for each player.

		2	
1		H	T
	H	1, -1	-1, 1
	T	-1, 1	1, -1

Matching Pennies

		2	
1		A	B
	A	1, 1	0, 0
	B	0, 0	1, 1

Coordination

		2	
1		C	D
	C	4, 4	0, 7
	D	7, 0	0.1, 0.1

Prisoners' Dilemma

Prisoners' Dilemma, best responses

		Player 2	
		Cooperate (C)	Defect (D)
Player 1	Cooperate (C)	4, 4	0, 7
	Defect (D)	7, 0	0.1, 0.1

- Assume this game to be played once
- Consider the best responses:
  - If 1 cooperates, 2 would defect
  - If 1 defects, 2 would defect
  - the same for 2 (symmetric game)
- Hence: Cooperation is not an equilibrium

- How to achieve cooperation?

- Not possible in a one shot game!
- Possible in repeated games
- Possible by including the group payoff into the individual's utility function (which means to change the payoffs)
- Possible if one assumes some kind of learning (or strategies like 'tit for tat')

Network Version

Game rules

7 different networks

Synchronous updating

Neighborhood of 5 where possible

50 runs per network

- Start: every agent is randomly assigned strategy **C** or **D**
- Each agent plays this strategy with his neighbors (and himself)
- Observe the total payoff of the neighbors and adopt the strategy that yielded the highest payoff (for the next round)

Results on the complete network

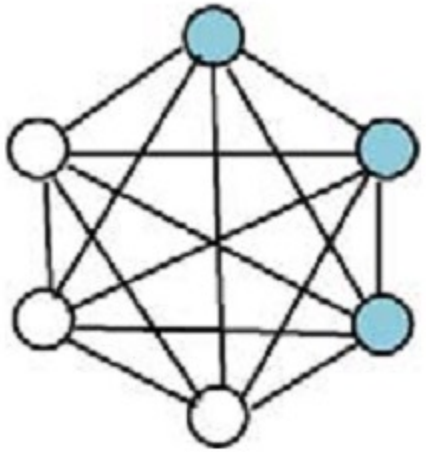


Figure from Wilhite (2006)

- Since the network is complete, just check the total payoff for **C** and **D**
- Assume ***n*** agents, among them ***m*** defectors
- Strategy **C** yields **4 (*n*-*m*)**
- Strategy **D** yields **7 (*n*-*m*) + 0.1 *m***
- As long as there is one defector, everybody will adopt **D** ☹

Results on the star network

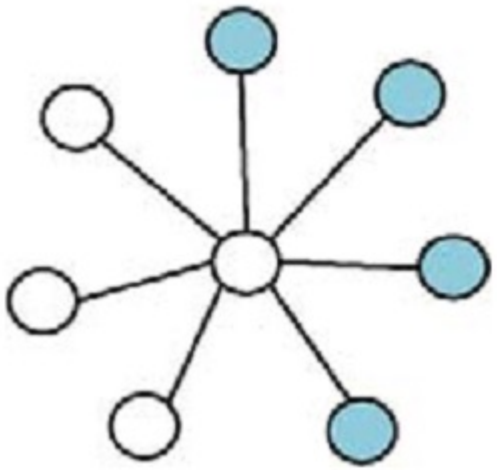


Figure from Wilhite (2006)

- Outcomes depend heavily on the center agent
- **Assume** the center agent starts with **D**
- He earns **7(*n*-*m*)+0.1 *m***
- Periphery earns **0** or **0.1**
- The periphery will adopt **D** in the next round and everybody will **defect**

In the paper the notation n/m changes!

Results on the star network (2)

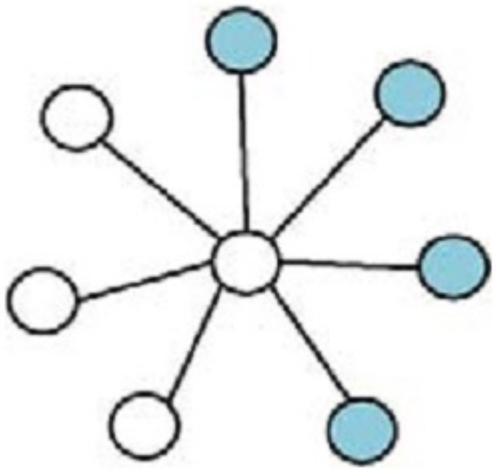


Figure from Wilhite (2006)

- **Assume** the center agent starts with **C**
- He earns **4 *(n-m)***
- Periphery earns **7** each when playing **D**
- The periphery will adopt **C** if $n\text{-}m \geq 2$

- Otherwise the center and the remaining periphery agent will switch to **D**

Results on the ring

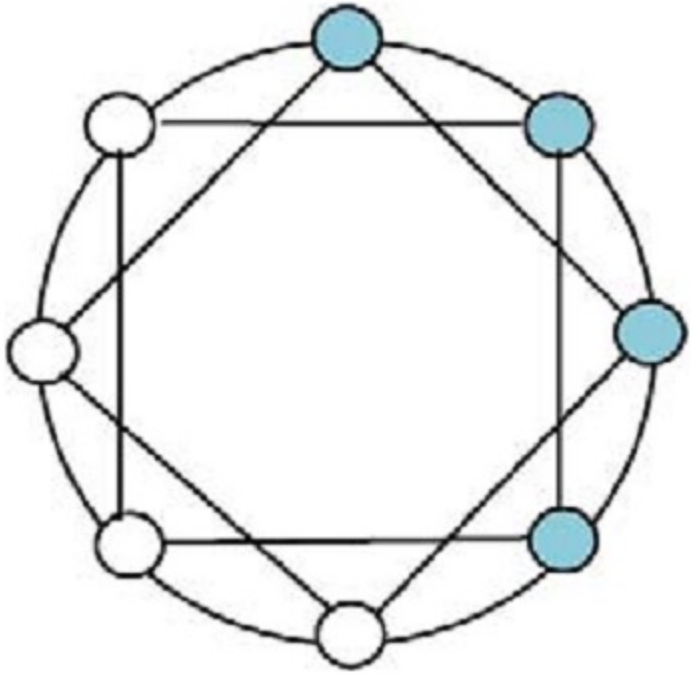


Figure from Wilhite (2006)

- Assume a neighborhood of 5
- If there is no string with more than 3 cooperators, **D** spreads
- Otherwise: **C** spreads, see the payoffs for the **C** agent at the border

… D	D	C	C	C	**C**	D	D	D	D …
7.4	14.3	12	16	16	12	14.3	7.4	0.5	0.5

- The adjacent **D** agent has a higher payoff, but the **C** agent to the left is even better
- The border **D** agent will also adopt **C**

Results on the ring (2)

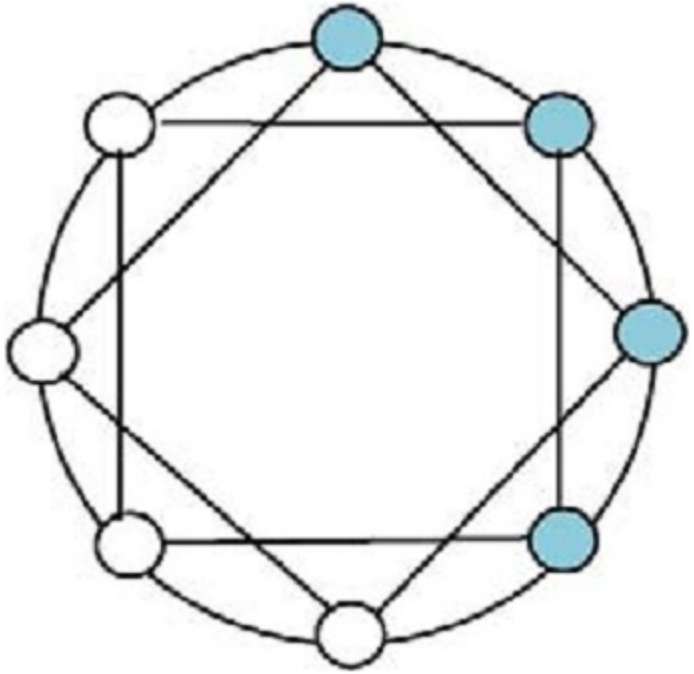


Figure from Wilhite (2006)

- Will defectors vanish?
- Assume only one is left:

… C	C	C	C	**D**	C	C	C	C	C …	
20	20	16	16	28.1	16	16	20	20	20	

- The 4 adjacent cooperators will switch to **D** in the next round

… C	C	D	D	D	D	**D**	C	C	C …
16	12	14.3	7.4	.5	7.4	14.3	12	16	20

- But now again the border **D** agent will switch → cycle

Scale-free

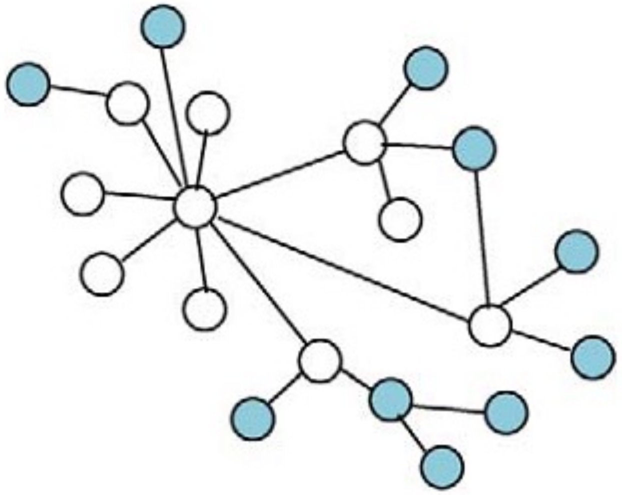


Figure from Wilhite (2006)

- Both strategies survive but cooperators are in the majority
- Hubs have a coordinating effect, since large clusters with defectors will not survive, they will mostly cooperate
- Defectors can survive in the periphery, where their cooperative neighbors will not switch to **D** themselves

Results and Limitations

- The results (convergence to only **C**/**D** or mixed equilibrium) in this game depend on the network structure
- Agents choose strategies from a local optimum
- But: agents are myopic and do not learn optimally even in a local context

Exchange in Networks

- Another interesting experiment is to study trade on these network structures (barter economy)
- Assume that two goods are distributed unevenly on the network (initial allocation of 150 / 1500 units)
- All agents ***i*** prefer to consume both goods, i.e.

$$U^i = g_1^i g_2^i$$

- Agents trade bilaterally if an exchange will increase utility for both
- We have reached an equilibrium when the marginal rates of substitution converge

$$mrs^i = \frac{U'\left(g_1^i\right)}{U'\left(g_2^i\right)} = \frac{g_2^i}{g_1^i}$$

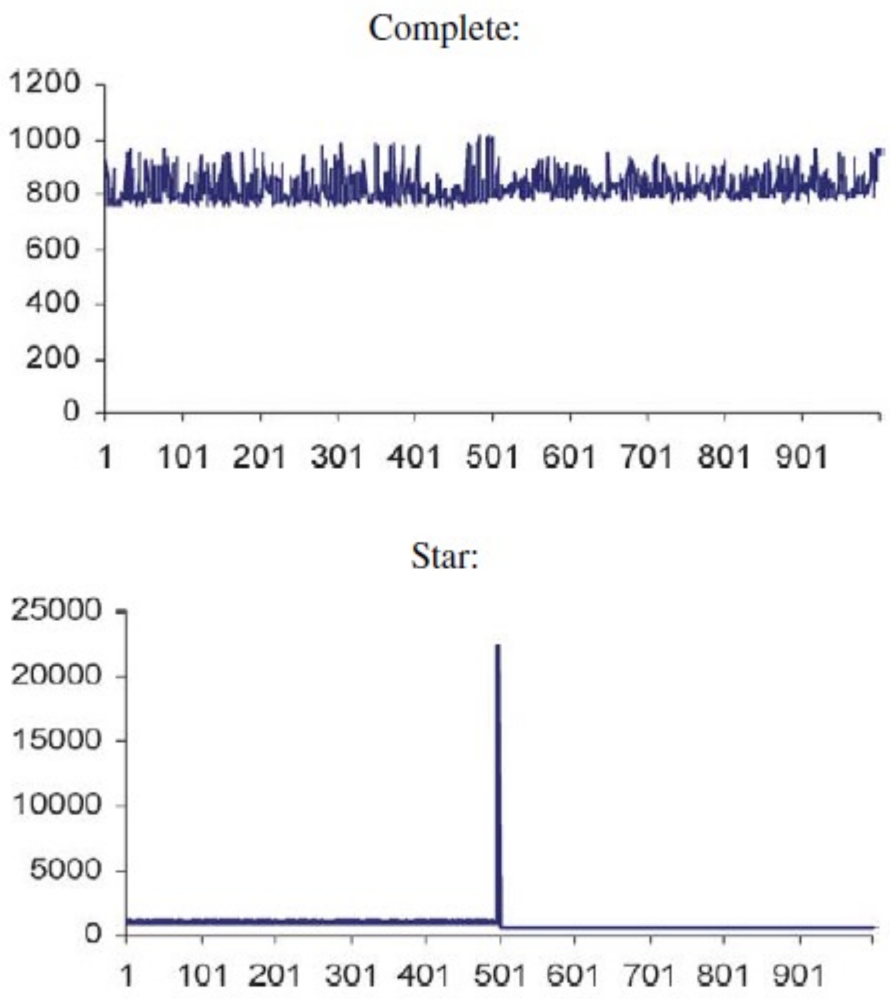
- The trading algorithm randomly picks one agent, who will then pick the neighbor with the best offer for trading
- The local price between agent ***i*** and ***j*** is given by

$$p_{i,j} = \frac{g_2^i + g_2^j}{g_1^i + g_1^j}$$

Income distribution in equilibrium

Complete network: fast convergence with high homogeneity

Star: efficient, but produces one peak

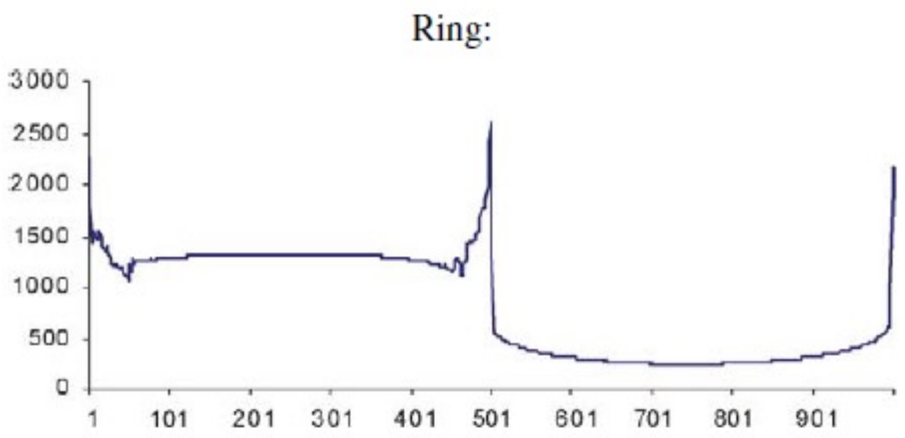


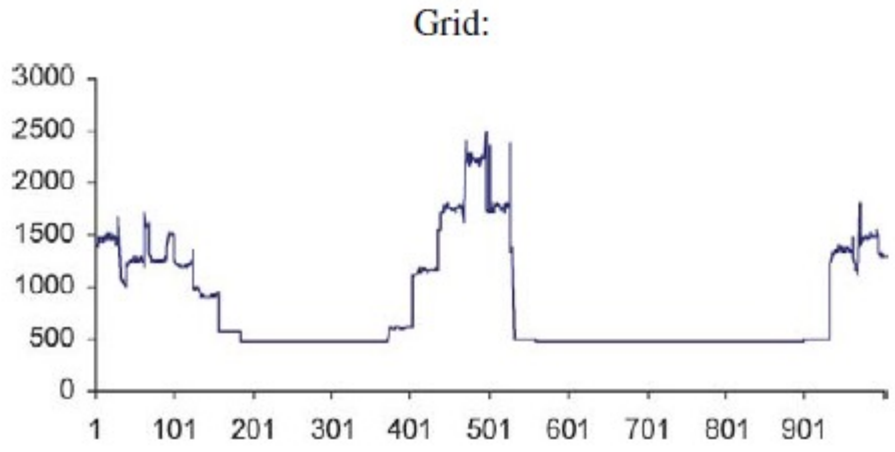
Figures from Wilhite (2006)

Income distribution in equilibrium (2)

Ring: endowments in *g1* and *g2* are on different sides of the ring. The border regions earns from trade flow.

Similar results hold for the grid network.





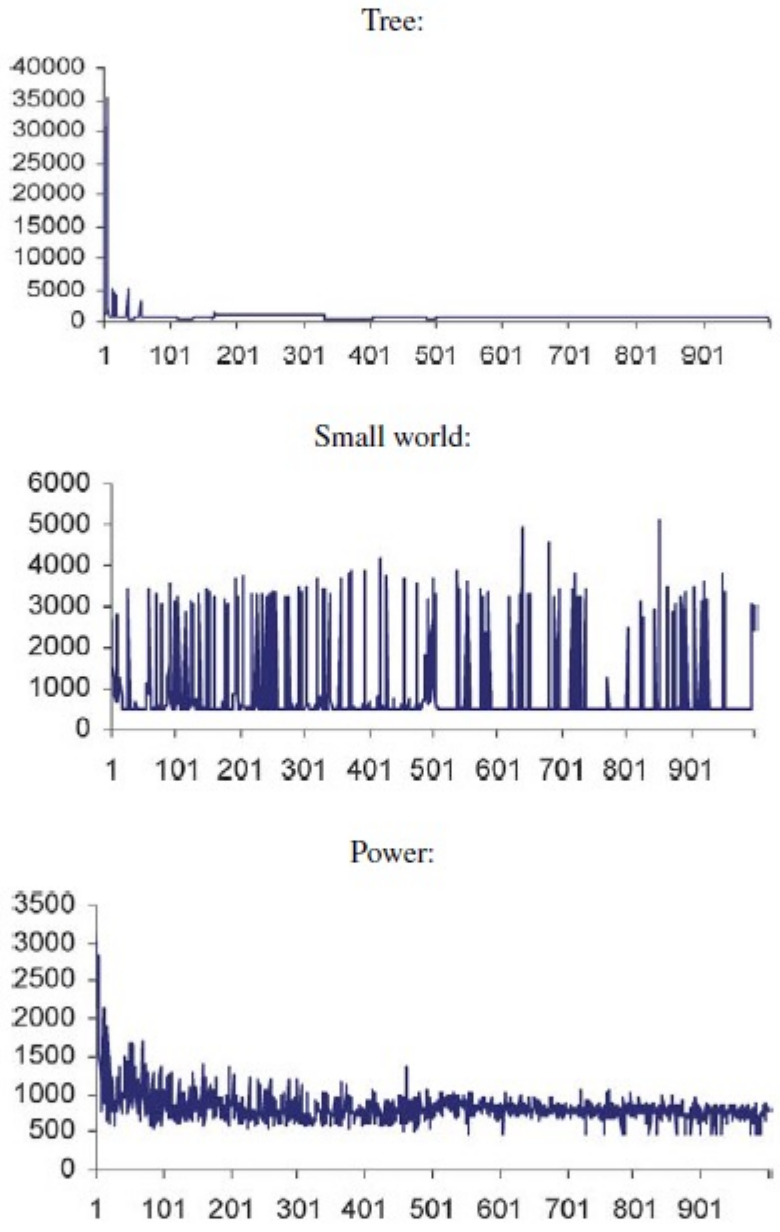
Figures from Wilhite (2006)

Income distribution in equilibrium (3)

Tree: Top level agents prosper if goods are abundant in different branches

Small world: many spikes, but lower dispersion

Scale-free: some hubs gain from their position, still rather even distribution



Figures from Wilhite (2006)

Network Formation and Games

- Instead of looking at fixed networks, a different approach is to analyze how networks emerge from games
- Assuming that a link resembles cooperation or some other game action, how does the network look like, when agents play a repeated game?

- These games can be cooperative or non-cooperative games
- Links might be costly or free
- Connections might be unidirectional or bidirectional
- Payoffs can be aggregated or counted as a one shot game
- Agents might discount future payoffs
- Agents might or might not learn from the past

More games on networks...

- In most papers the described games are based on individual incentives of the agents (opposed to centrally planned networks)
- Jackson and Wolinsky (1996) let agents play a game where each side has to agree to have a link. Such games normally do not result in stable networks (often even empty networks).
- Bala and Goyal (2000) assume a non-cooperative game, where a link is costly and there is some kind of learning

Jackson, M. and A. Wolinsky (1996), A Strategic Model of Economic and social Networks, Journal of Economic Theory 71, 44-74.

- Specifically, assume that all agents have pieces of information, which they can trade by forming a link ('communicating')
- Each agent chooses a strategy, describing to which of the other agents he links
- Each agents receives a payoff dependent on the information it accesses minus the costs of forming the links
- Information can also be accessed indirectly, through 'friends of friends'
- One-way and two-way flow is analyzed

- Results for the one-way flow version:
- The ring and the star are the only stable Nash networks

FIGURE 3A.—The star and the wheel (one-way model).

Bala, V. and S. Goyal (2000), A Noncooperative Model of Network Formation, Econometrica 68(5), 1181-1229

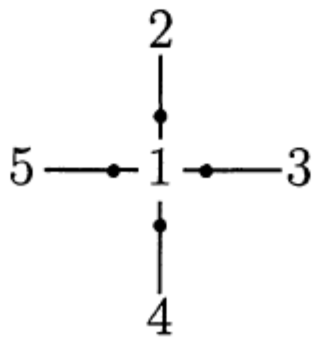


FIGURE 5A.—Center-sponsored.

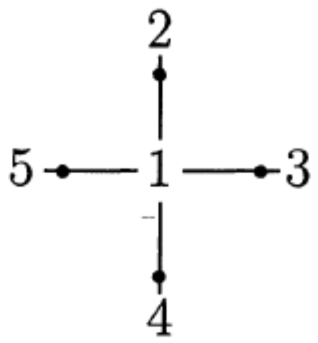


FIGURE 5B.—Periphery-sponsored.

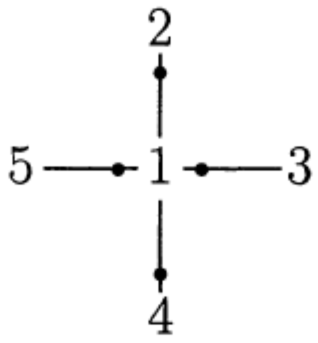
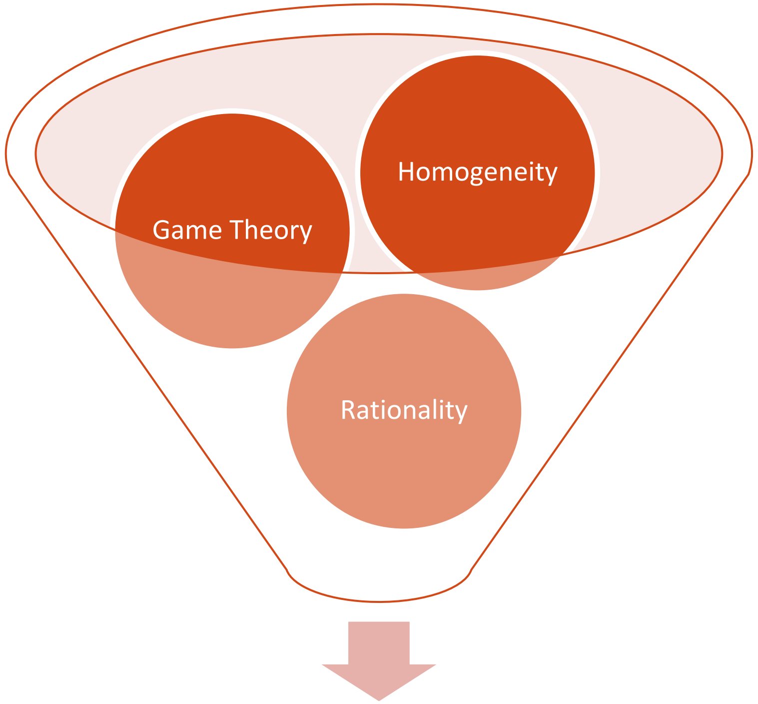


FIGURE 5C.—Mixed-type.

For the two-way flow model the outcome are star-like networks which differ in who is paying for the link

Figure from Bala and Goyal (2000)

Results and Problems



Realistic Structure?

- Economic and social activity can be modelled with networks
- Problem 1: Fixed networks are unrealistic
- Problem 2: Networks that emerge when applying traditional economic concepts or Game Theory are highly stylized
- Conjecture: Networks do not emerge out of a rational best response game or alike ☹