Computational Modeling of Social Systems

Basics of spreading: Opinion Dynamics
Week 5

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Graz

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Recap so far …

Block 1: Fundamentals of agent-based modelling

- Basics of agent-based modelling: the micro-macro gap
    - Tutorial: ABM basics in Python with Mesa (session 1)

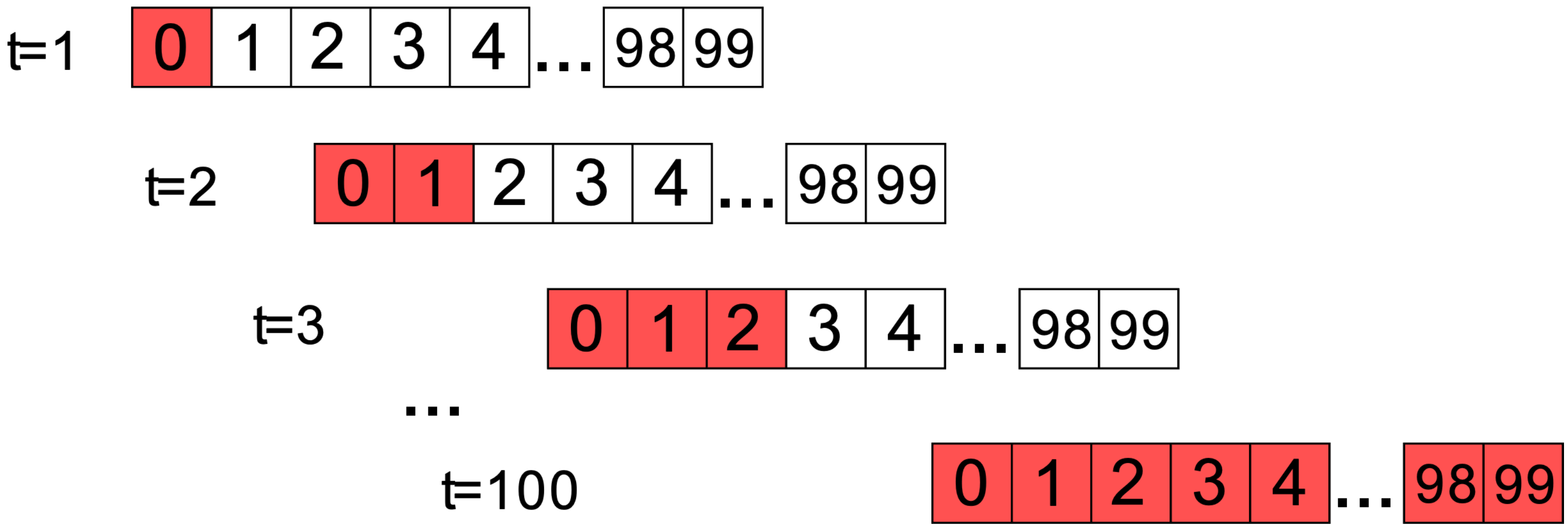
- Modelling segregation: Schelling's model
    - Tutorial: ABM basics in Python with Mesa (session 2)
- Modelling cultures
    - Exercise 1: Schelling's model and Pandas (session 1)

Block 2: Opinion dynamics

- Basics of spreading: Granovetter's threshold model
    - Exercise 1: Schelling's model and Pandas (session 2)
- **Today:** Opinion dynamics: more complex scenarios

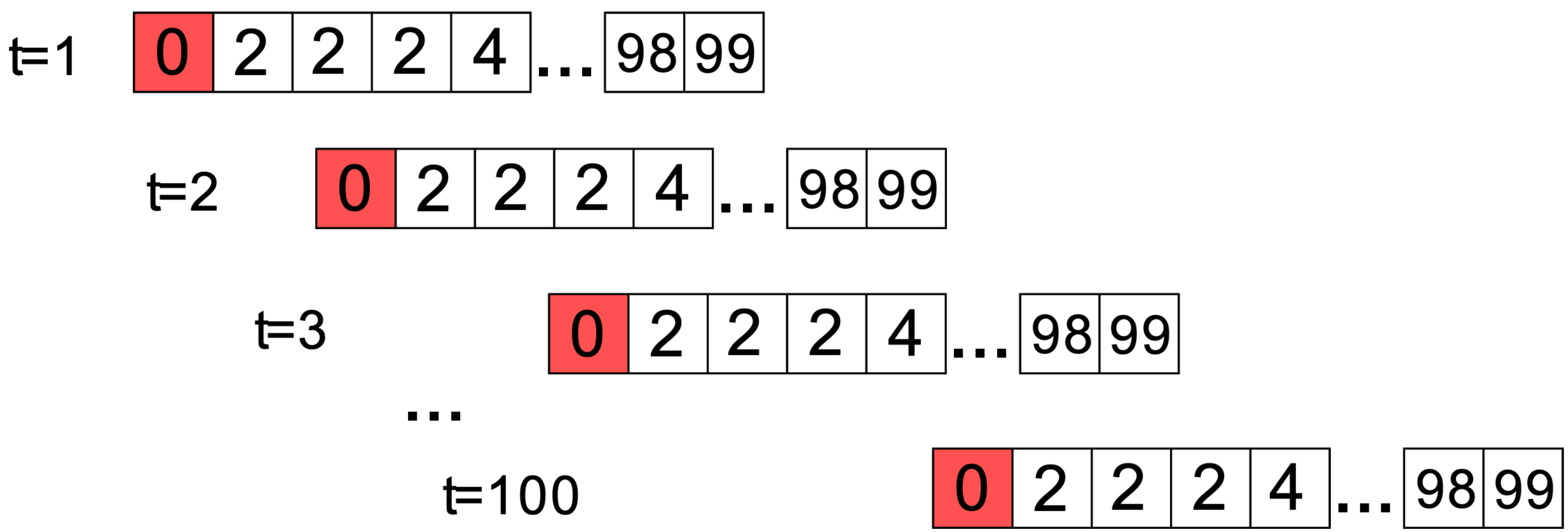
- Next week: Disease spreading: Guest lecture by Prof. Jana Lasser

Toy example of Granovetter's model



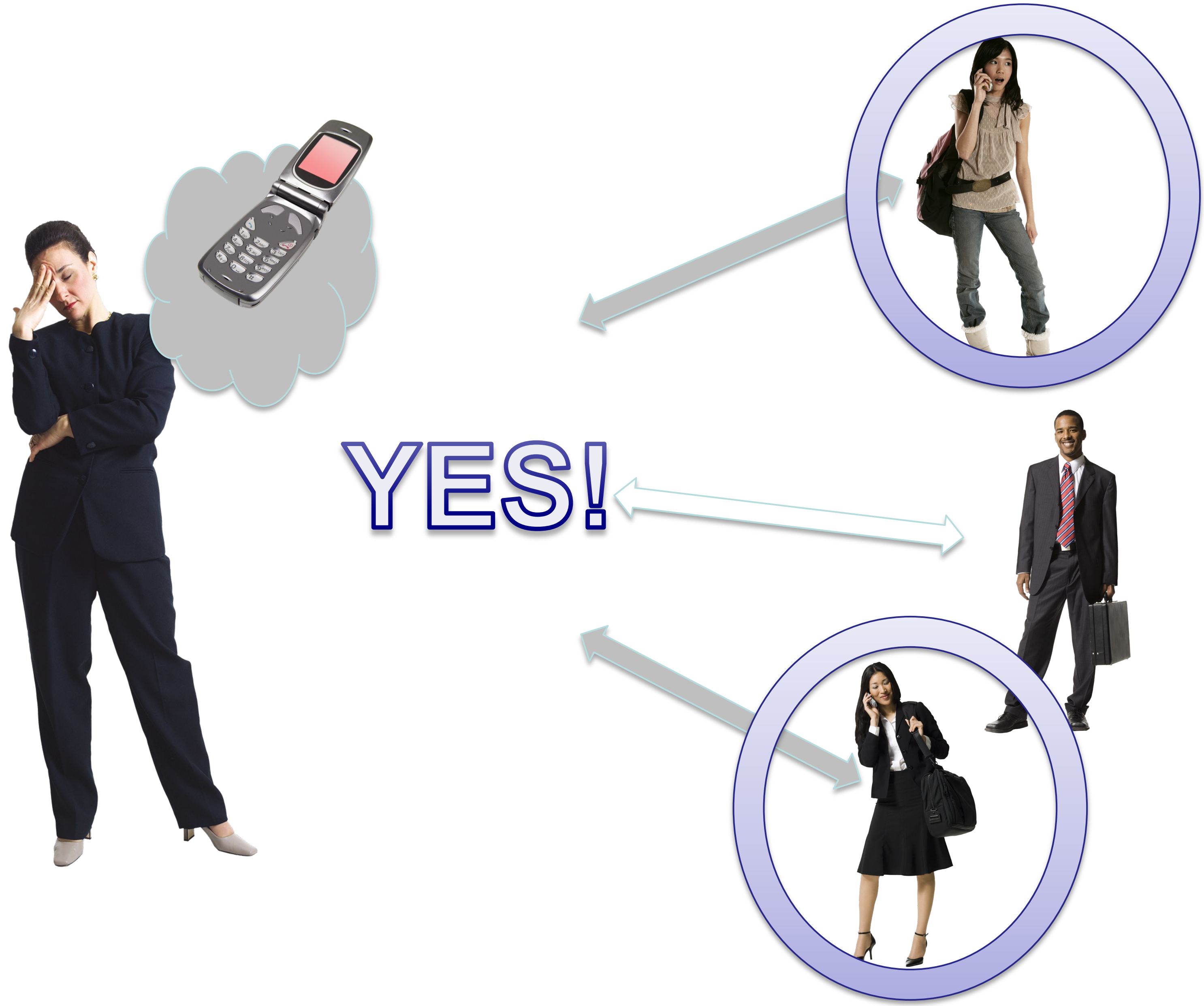
- 100 Agents
- Uniform sequence of thresholds with integer values [0,99]
- The first agent activates, then the second, and so on
- One agent joins per iteration and all agents are active at the end

Toy example version 2



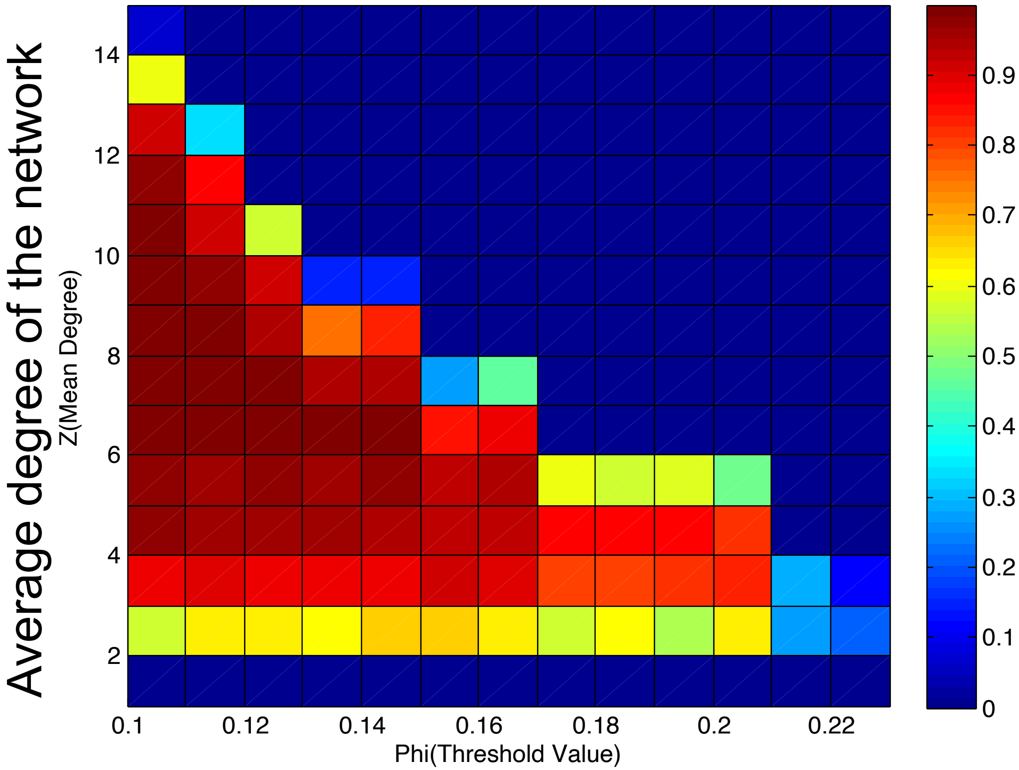
- Same example as before but agents with thresholds 1 and 3 now have threshold 2
- First agent activates and the simulation ends
- Radically different outcome for minimal change in thresholds!
- **Deducing preference distributions from collective outcomes is risky**

Granovetter Threshold model in networks



$$\Phi = 0.4$$

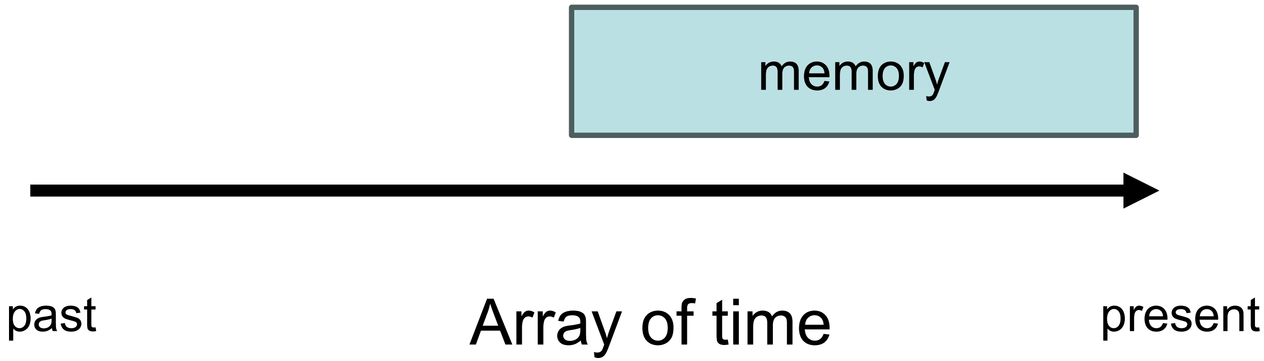
Granovetter Threshold model in networks



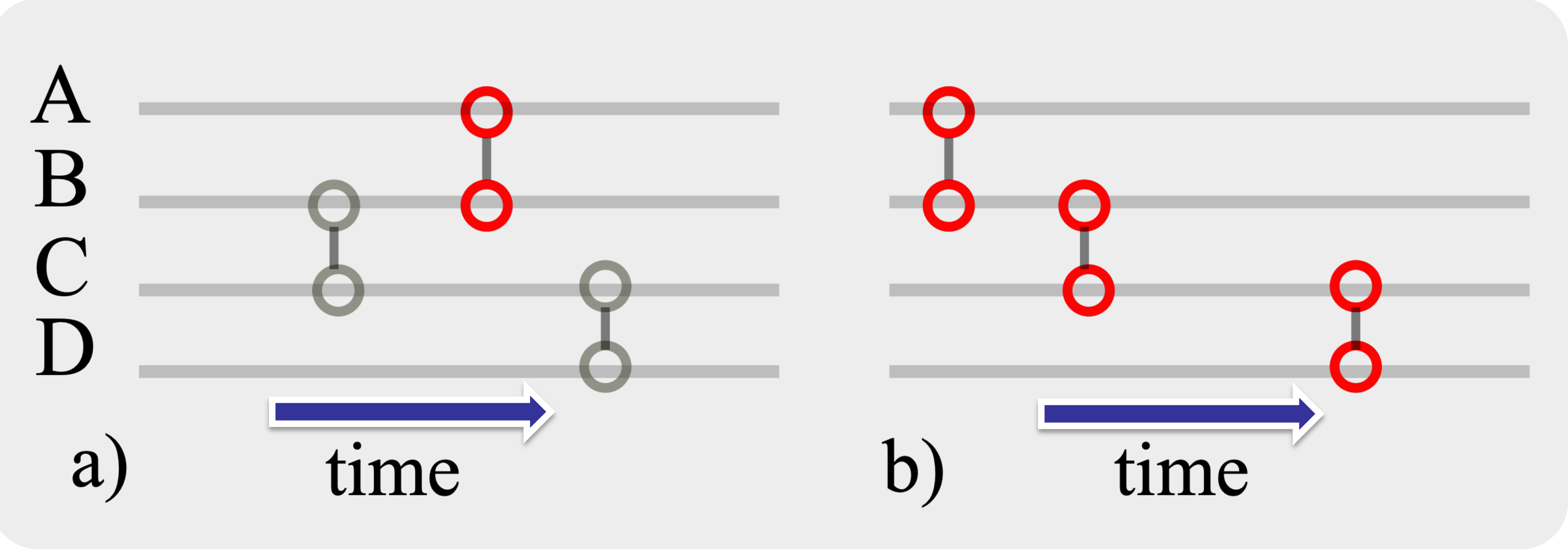
$$\Phi \le f_i = \frac{a_i}{n_i}$$

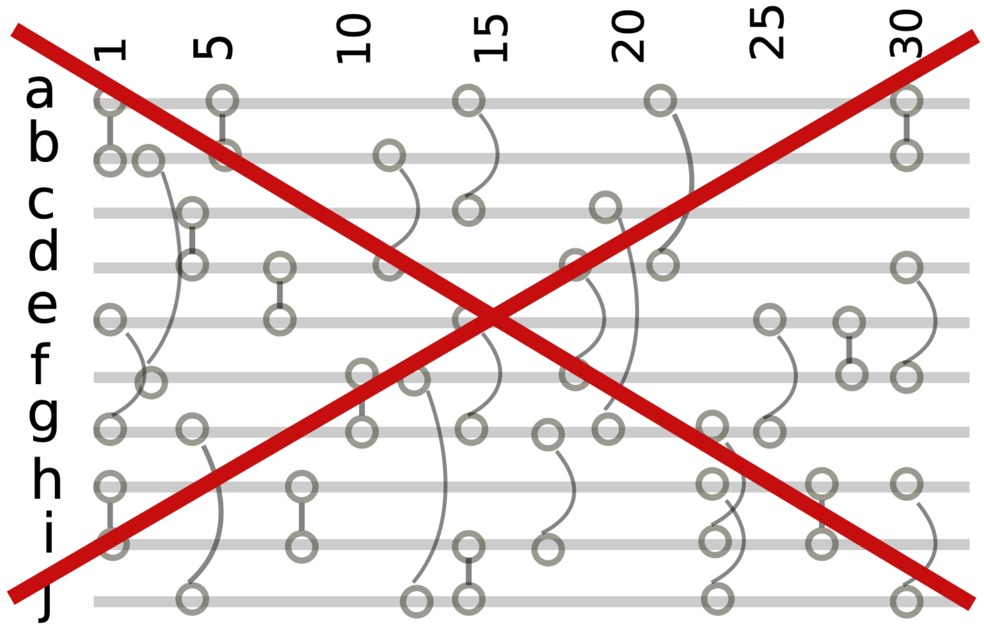
D. Watts, *PNAS (2002)*
Karimi and Holme, *Physica A (2013)*

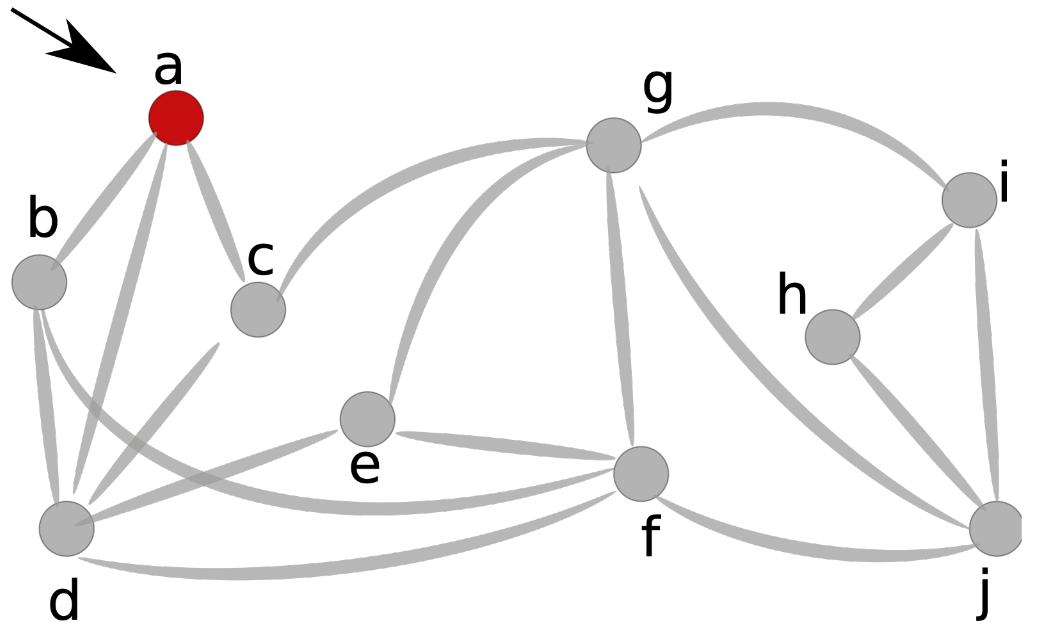
Effect of time and memory



Effect of time in networks







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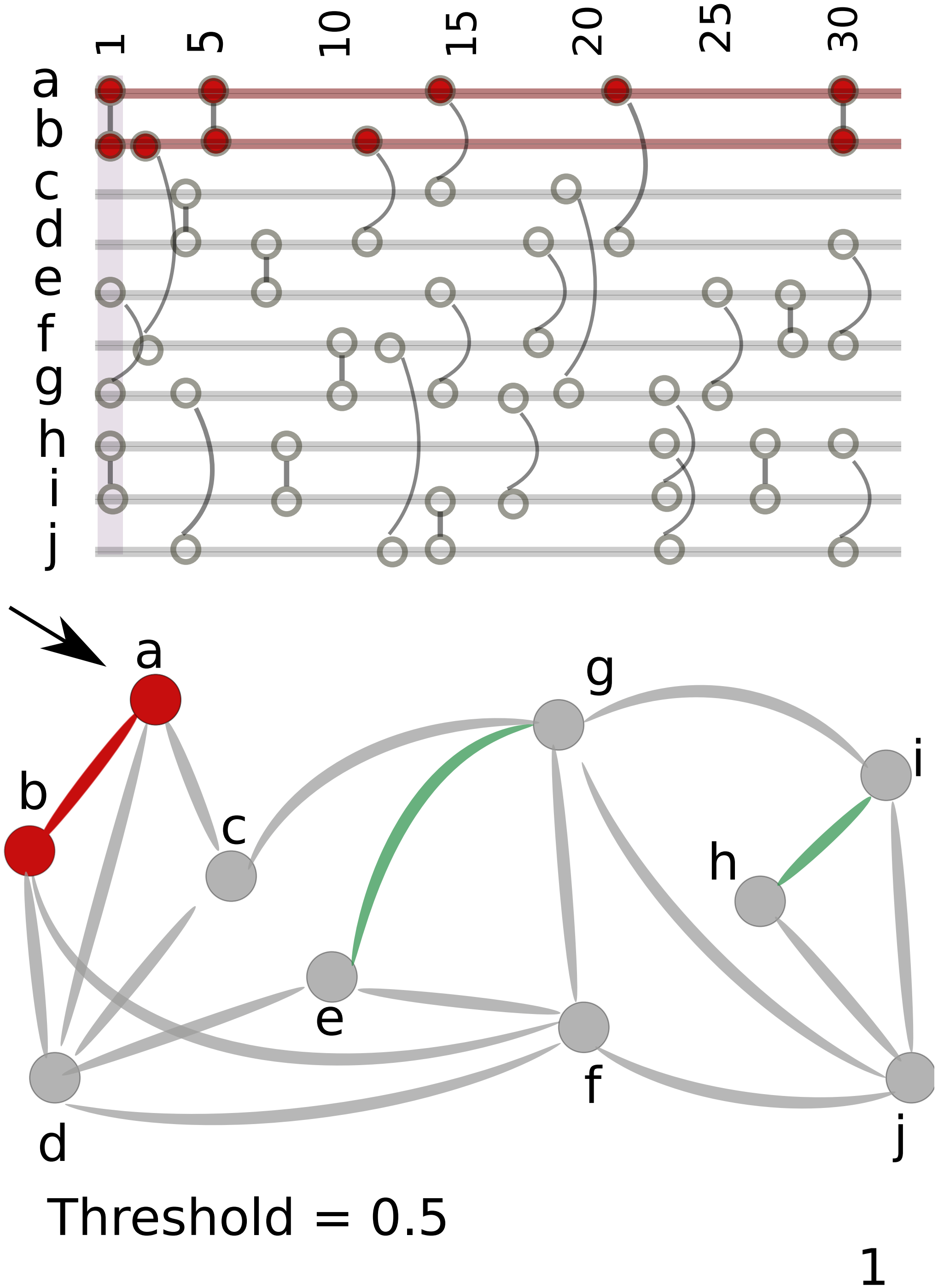
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Threshold = 0.5
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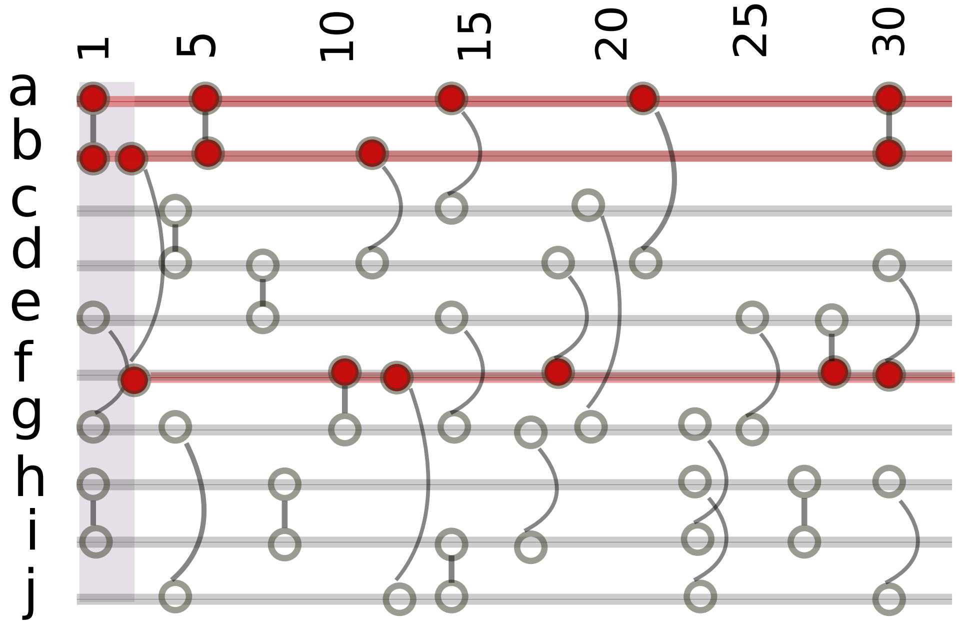

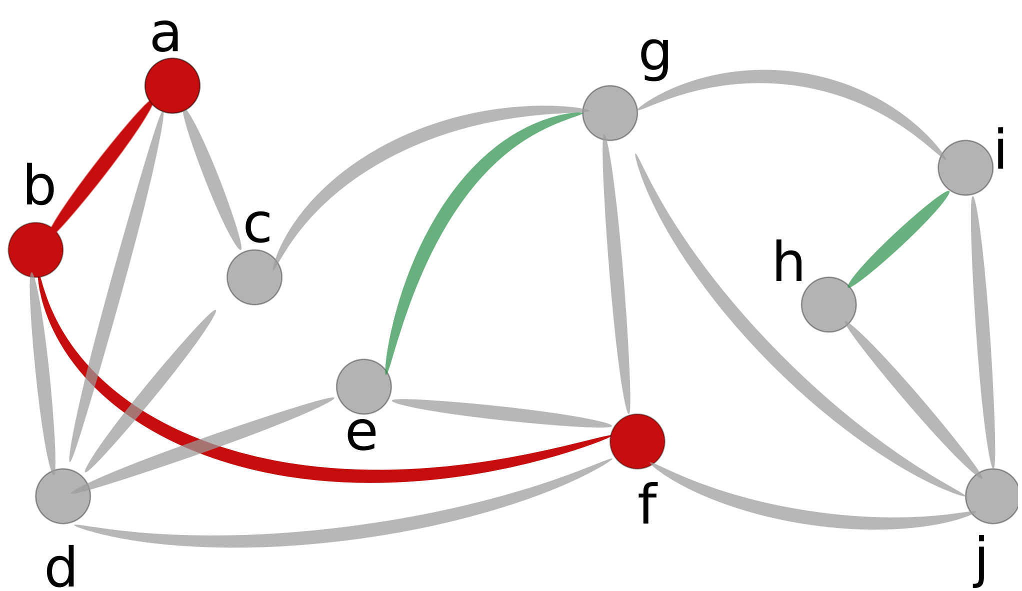

Threshold = 0.5

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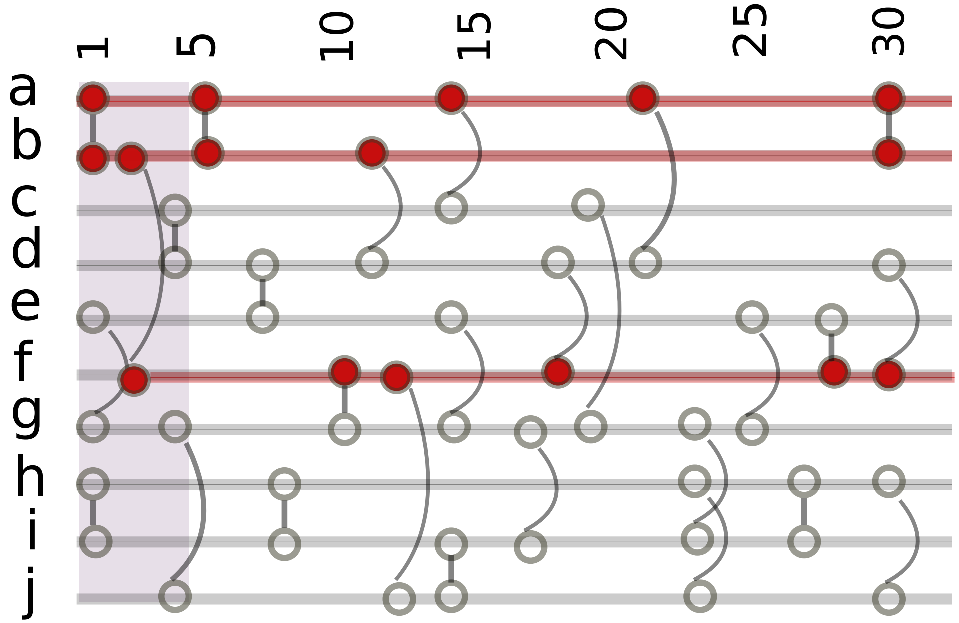

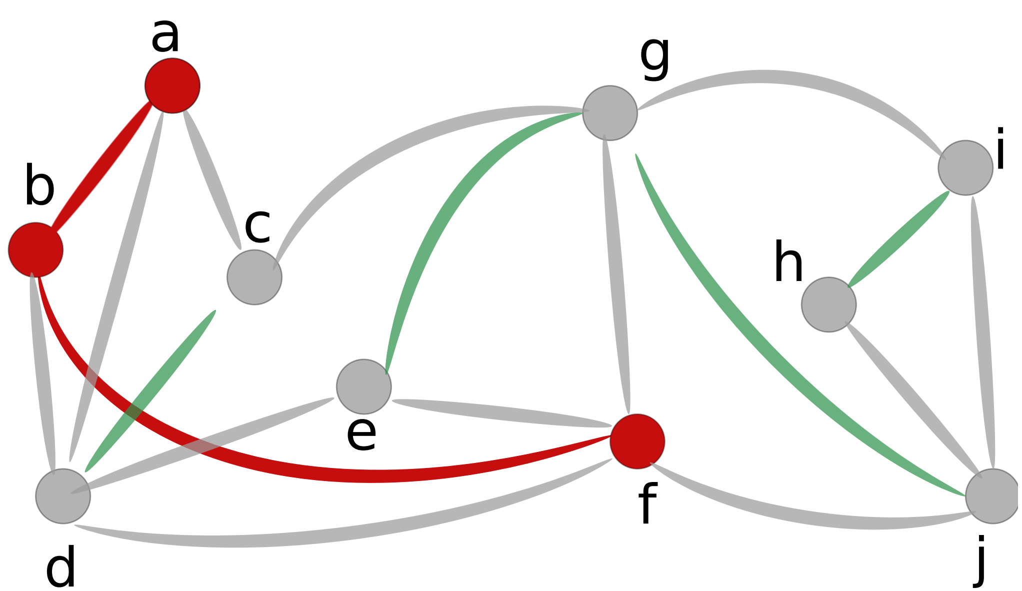

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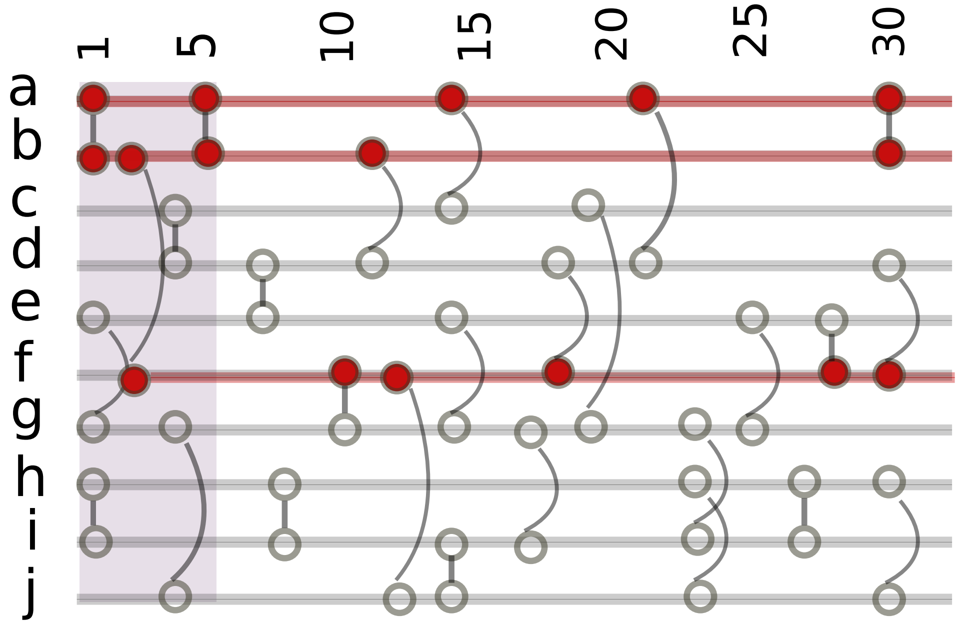

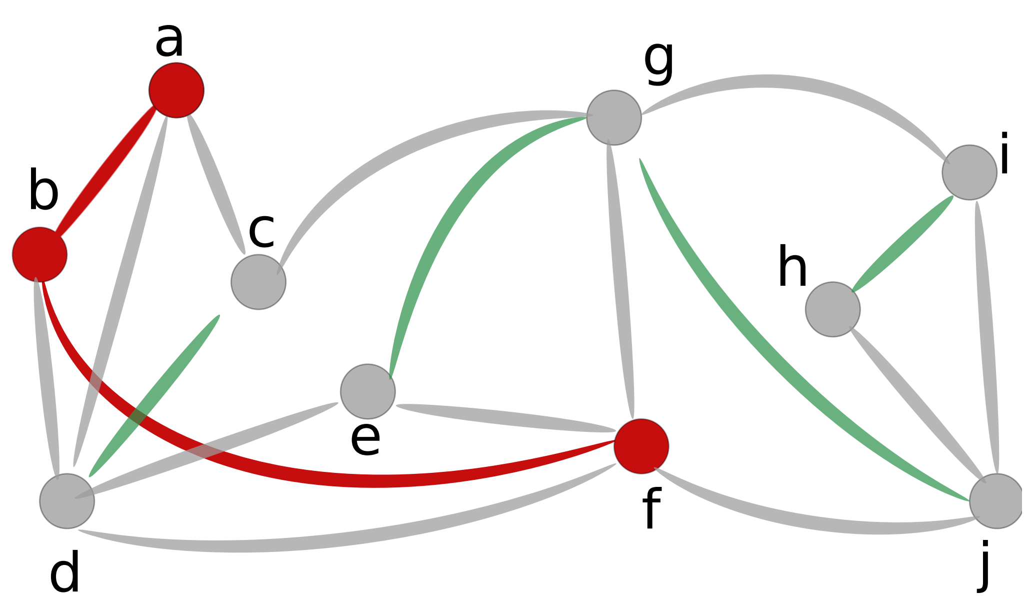

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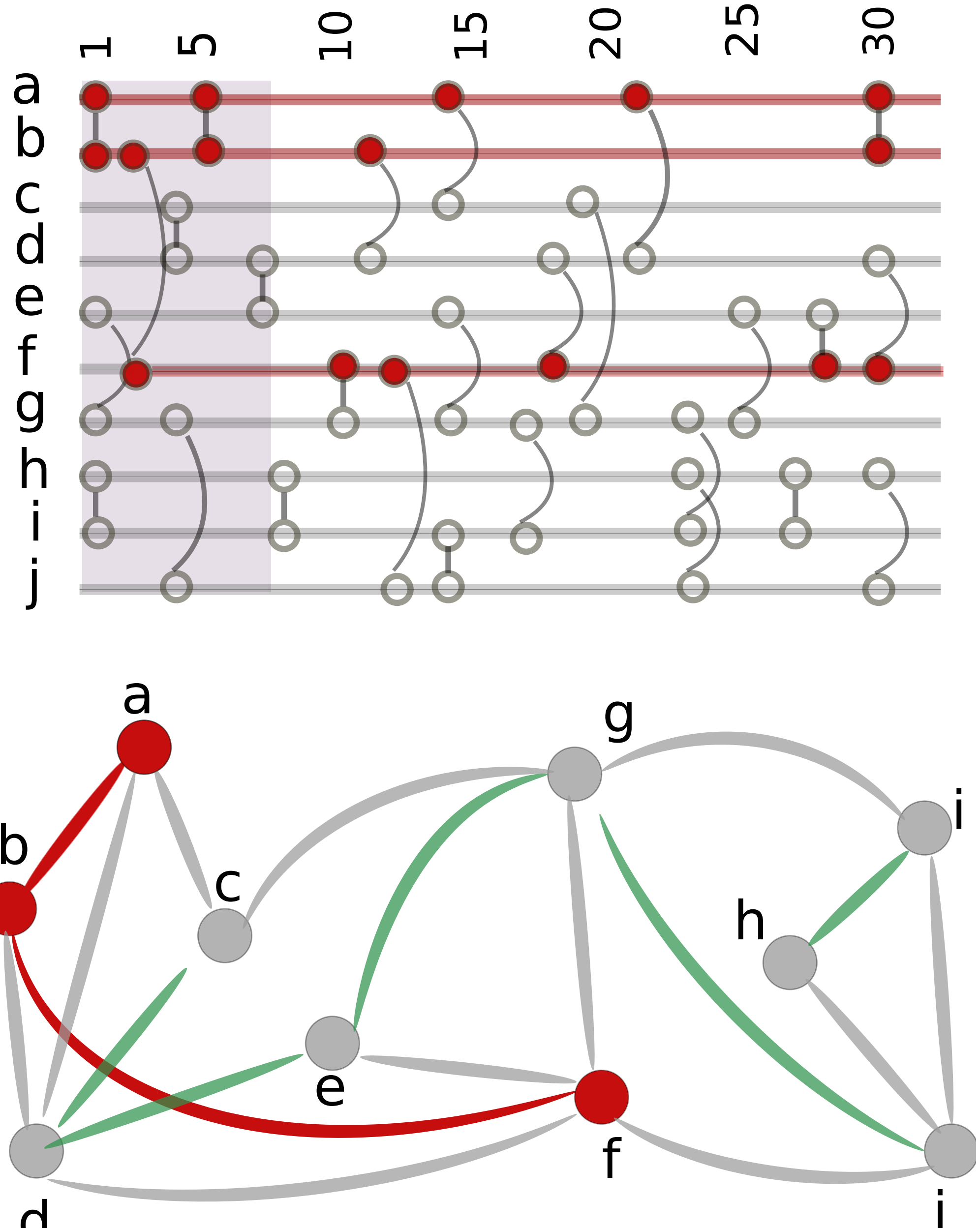

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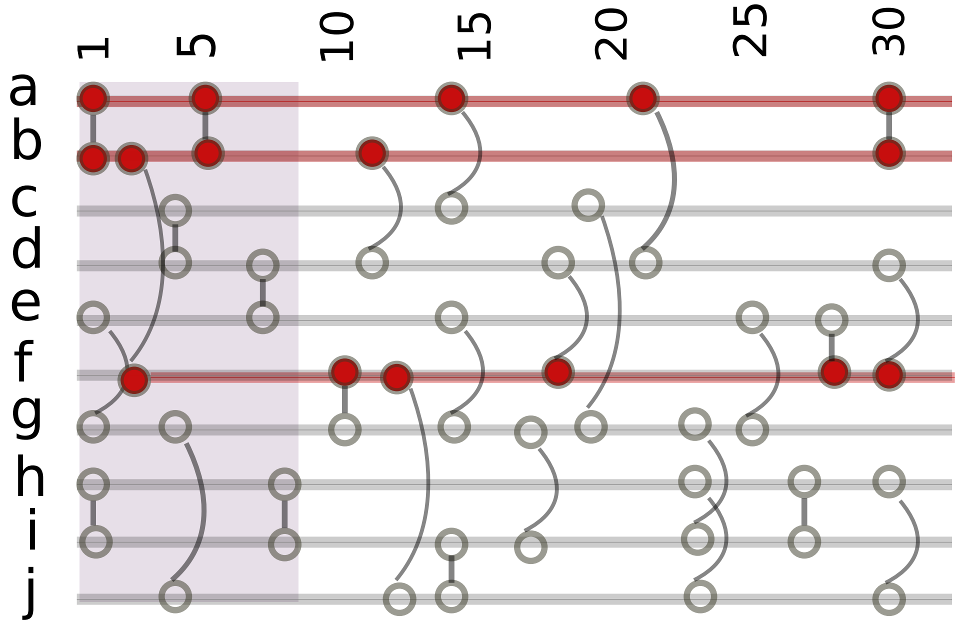

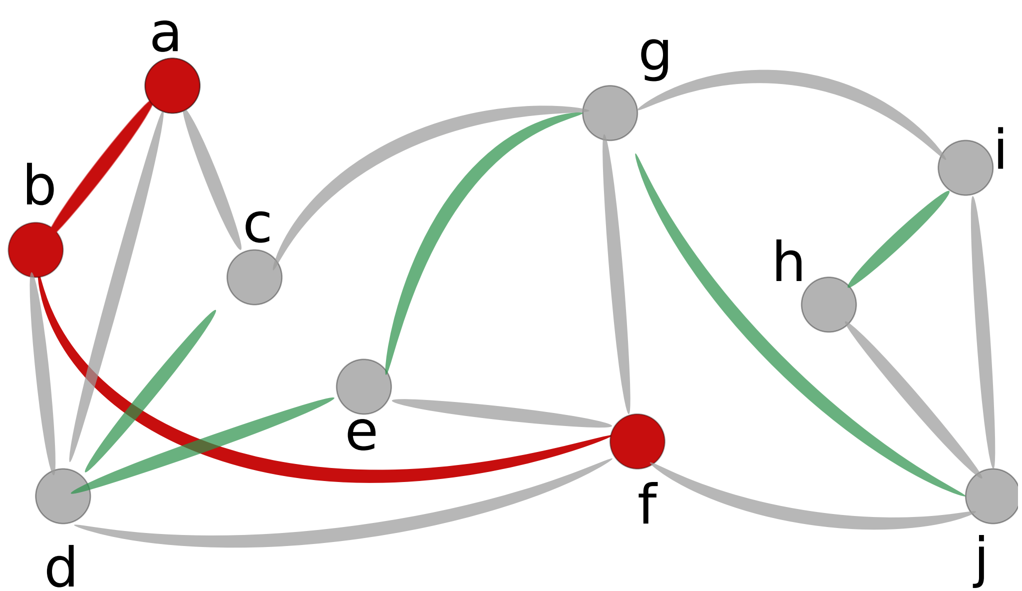

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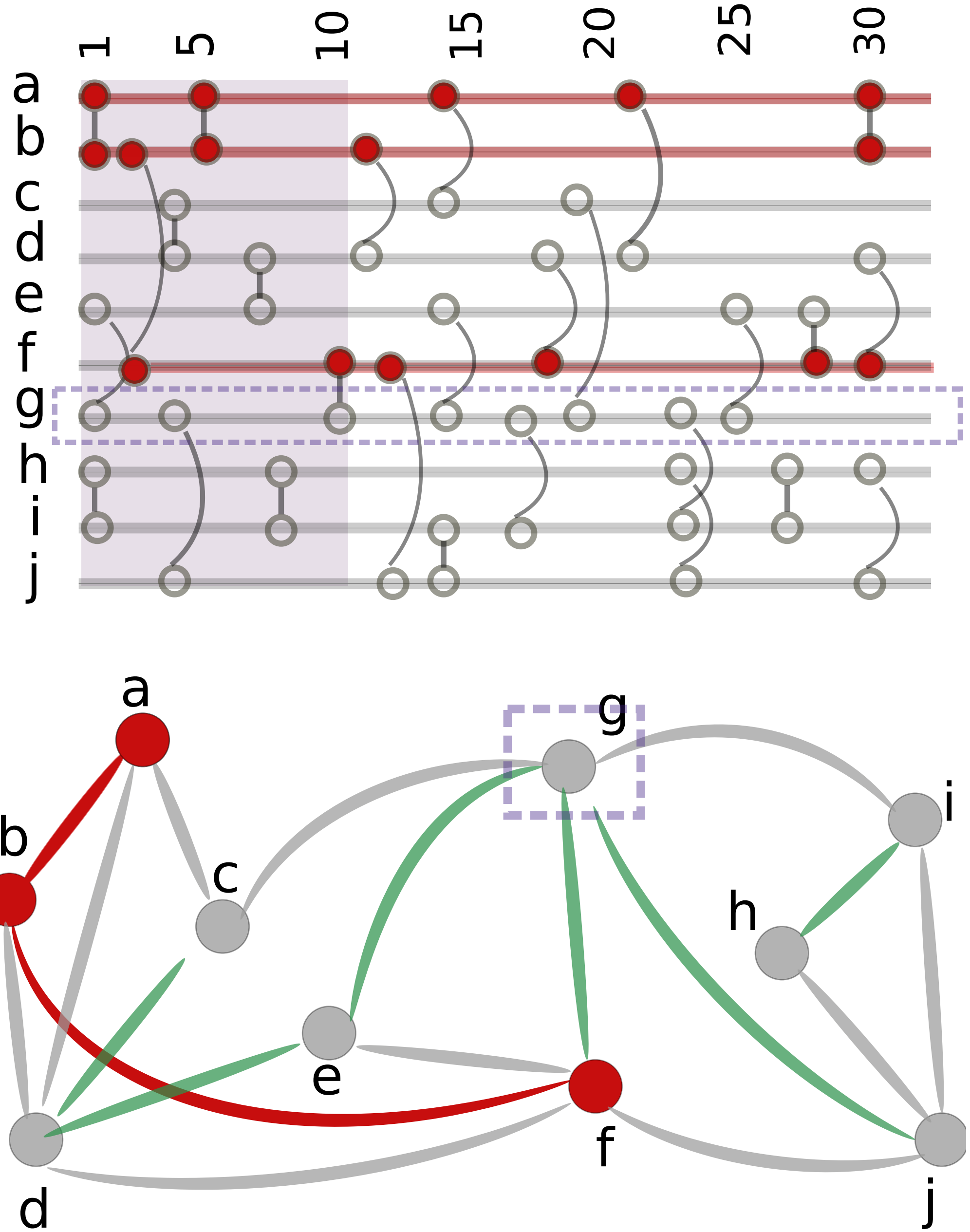

Threshold = 0.5

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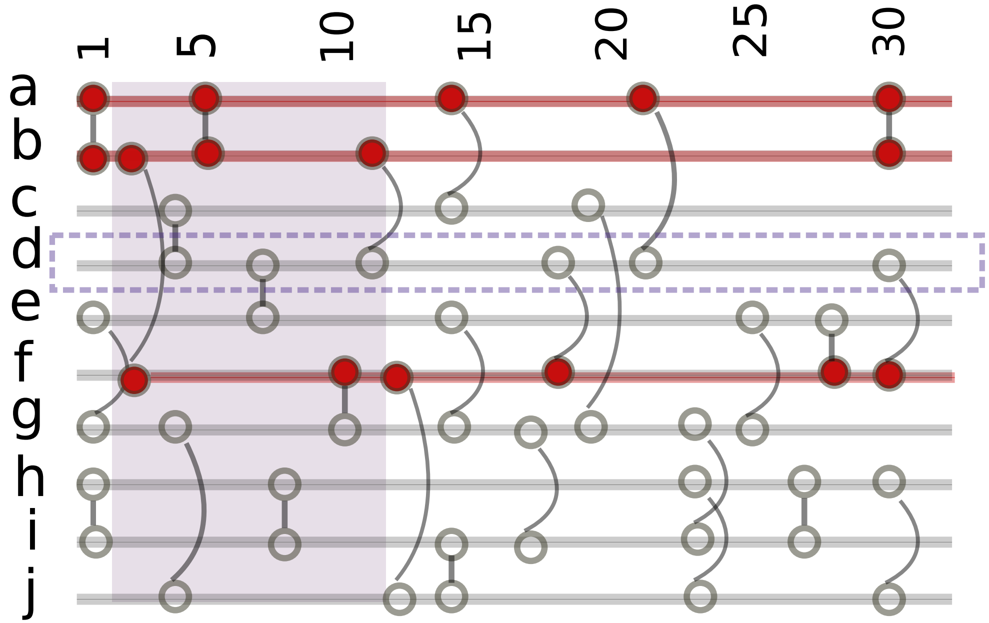

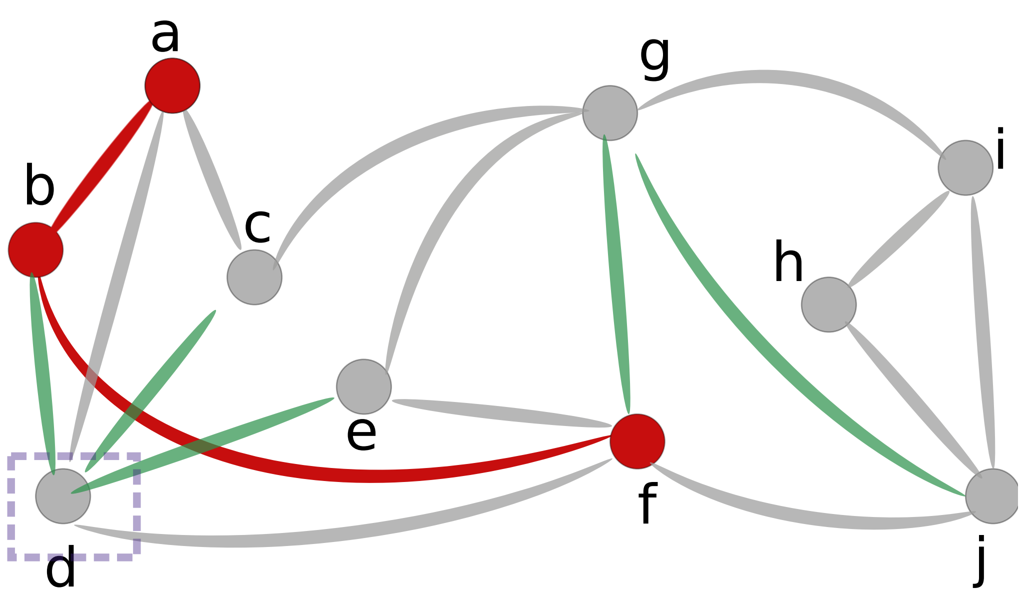

Threshold = 0.5

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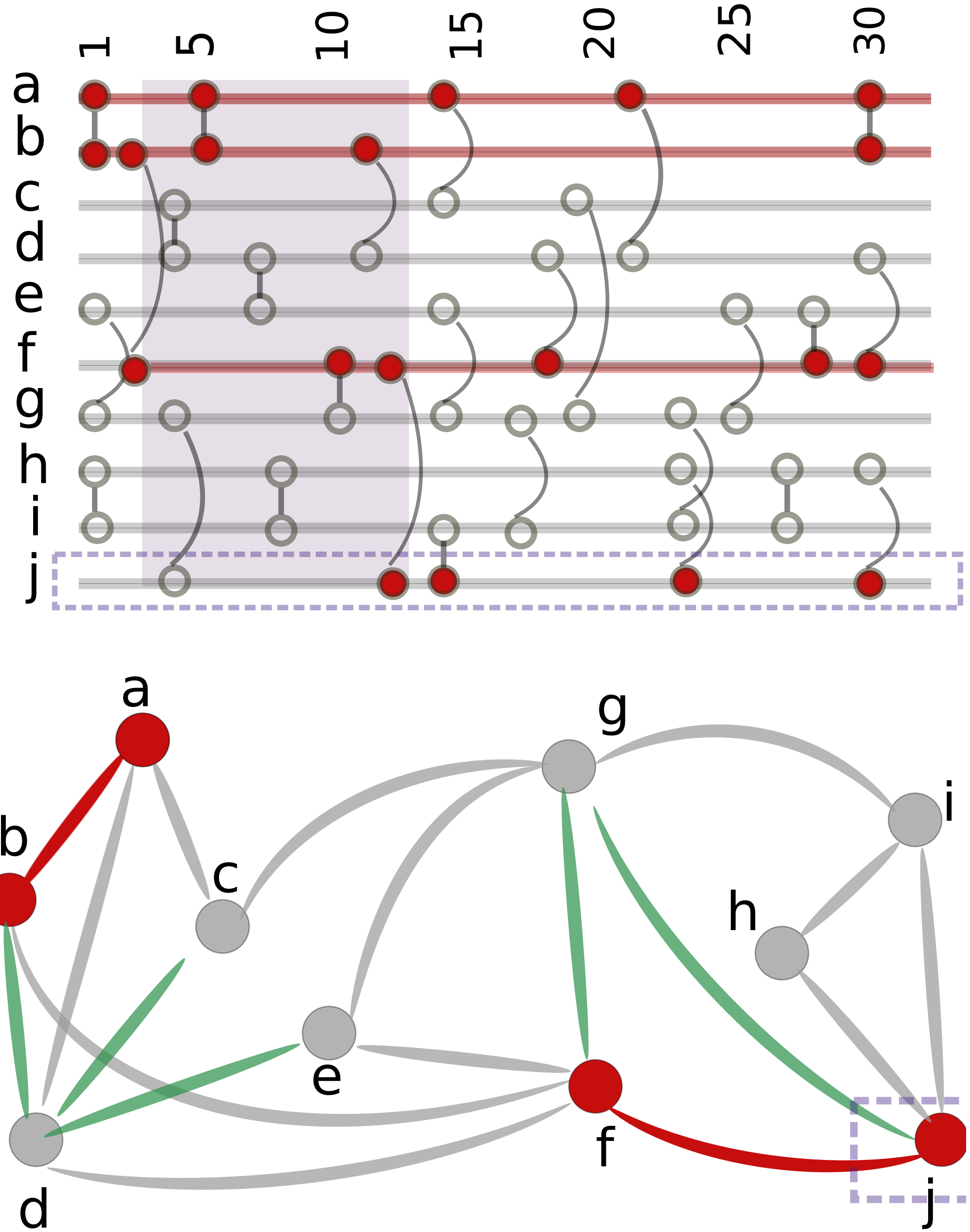

Threshold = 0.5

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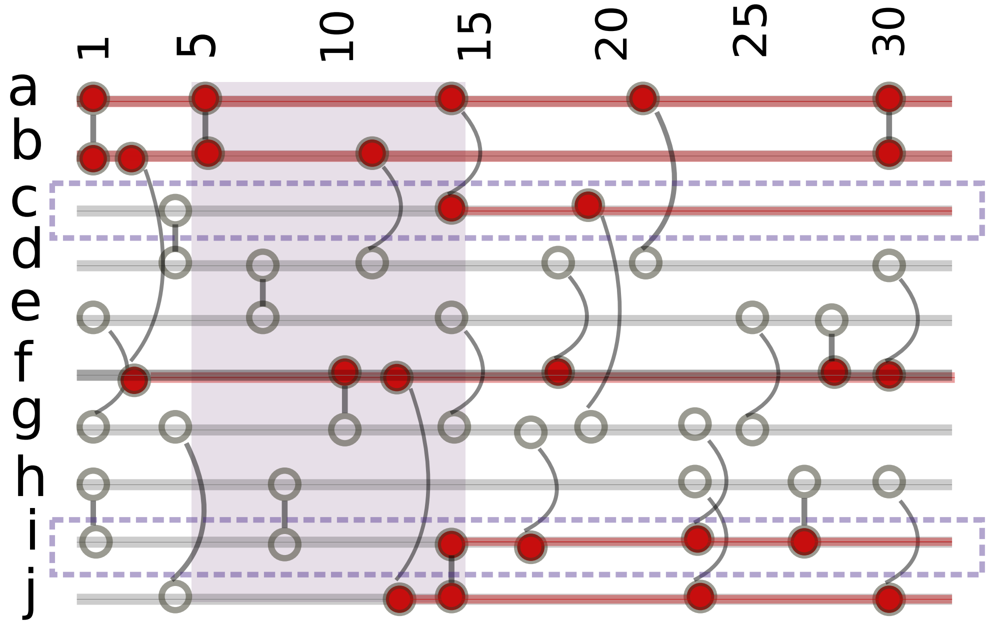

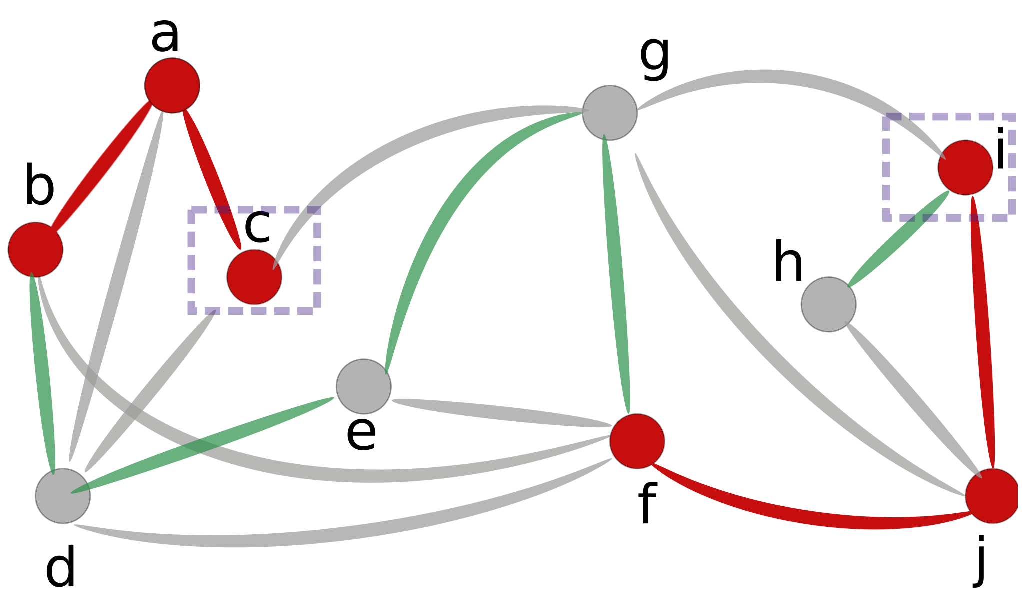

Threshold = 0.5

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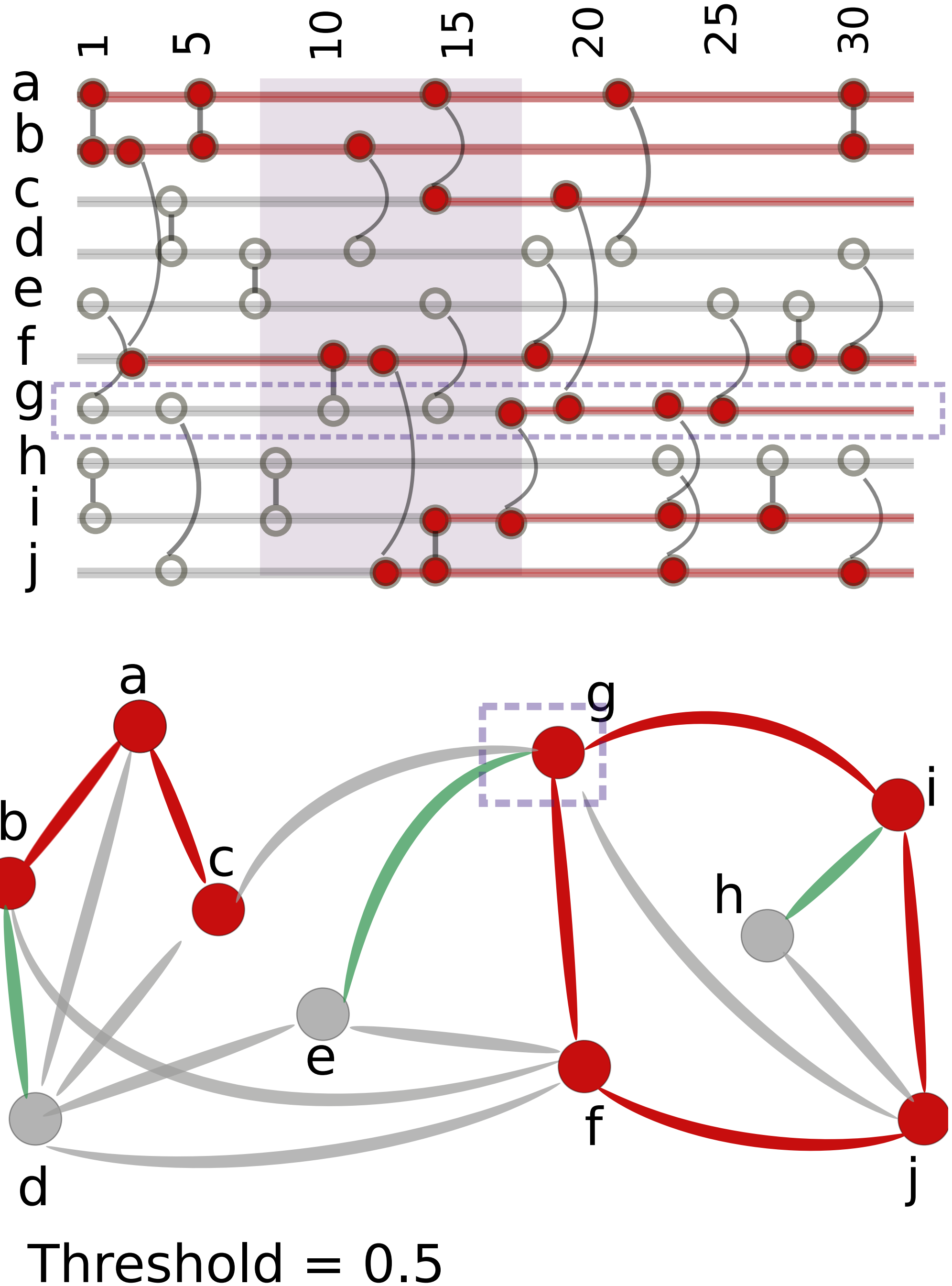

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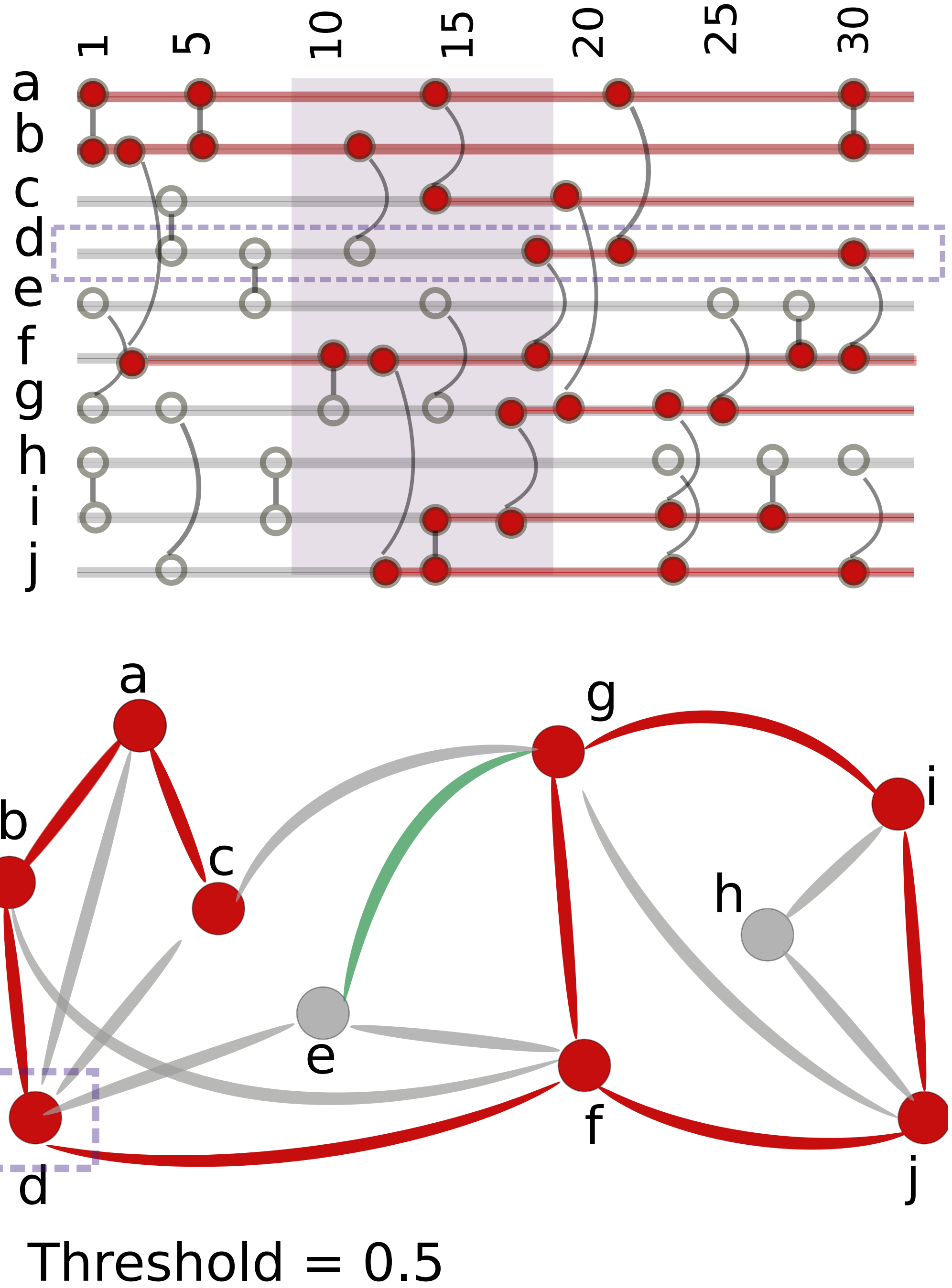

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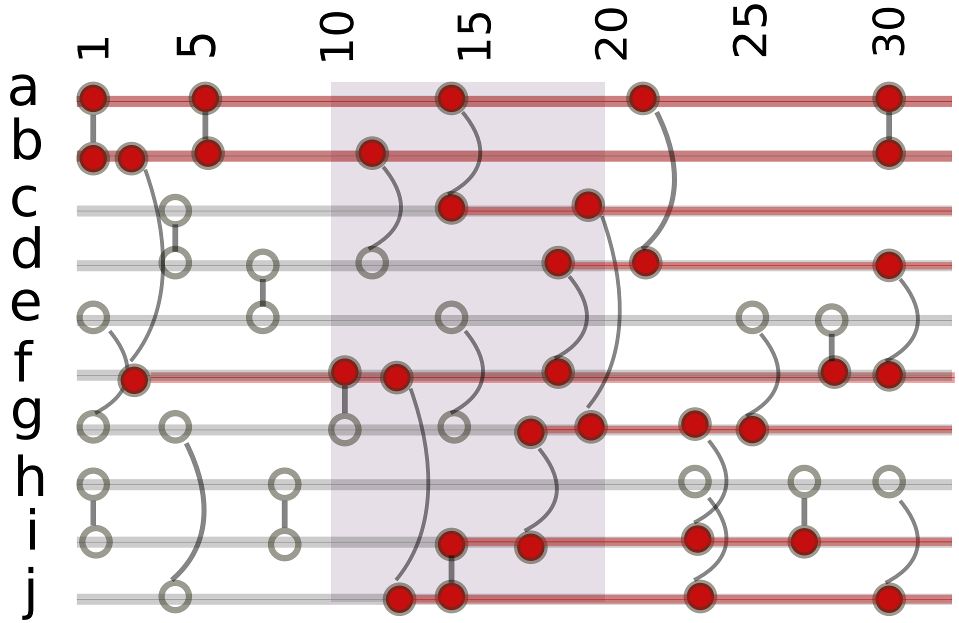

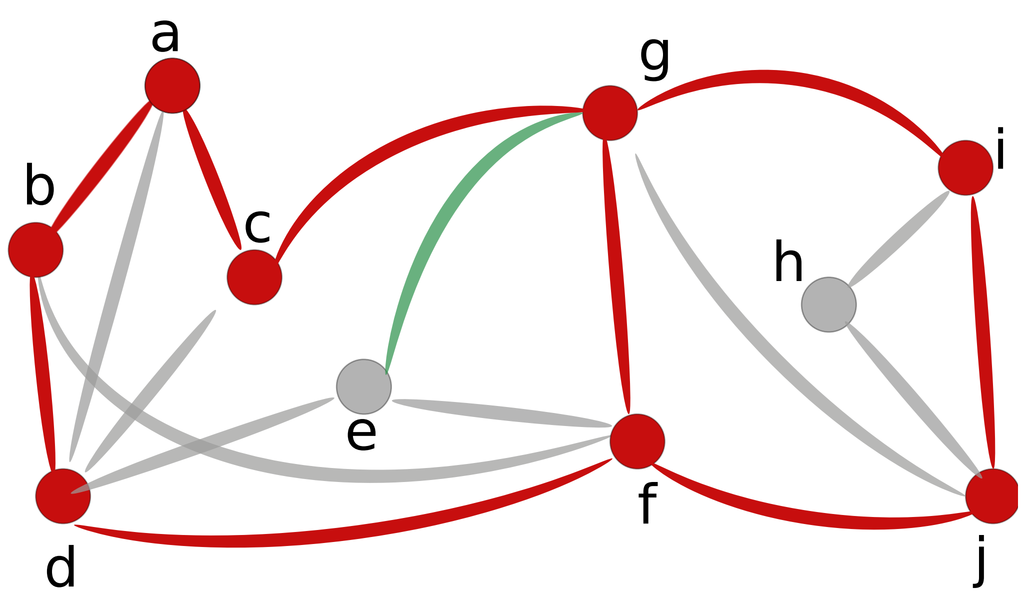

Threshold = 0.5

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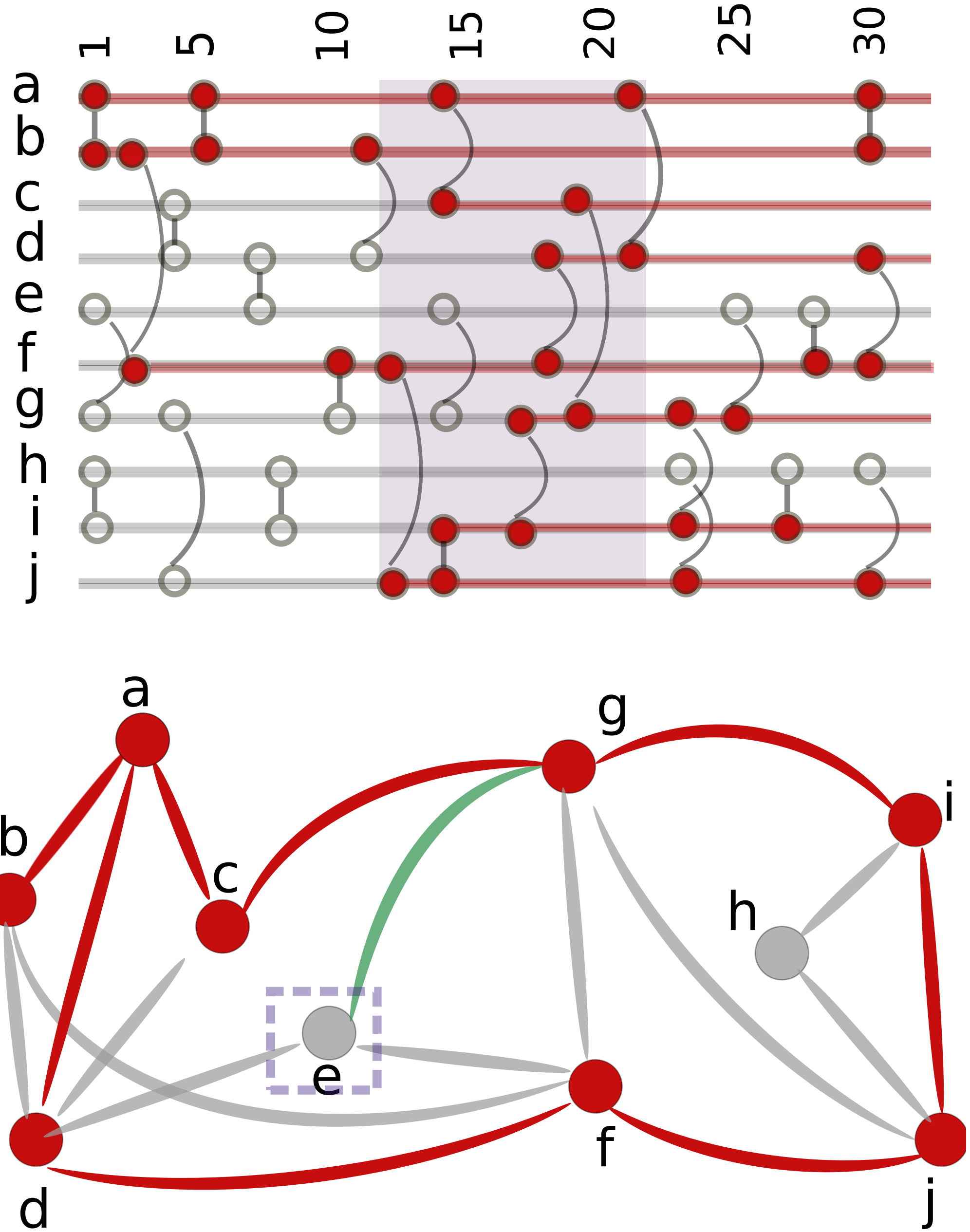

Threshold = 0.5

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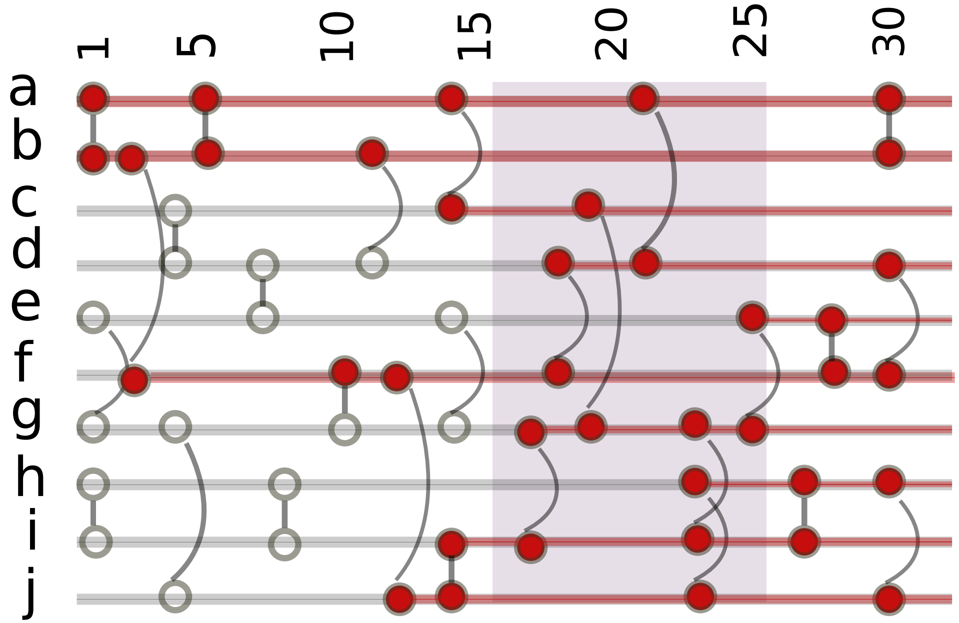

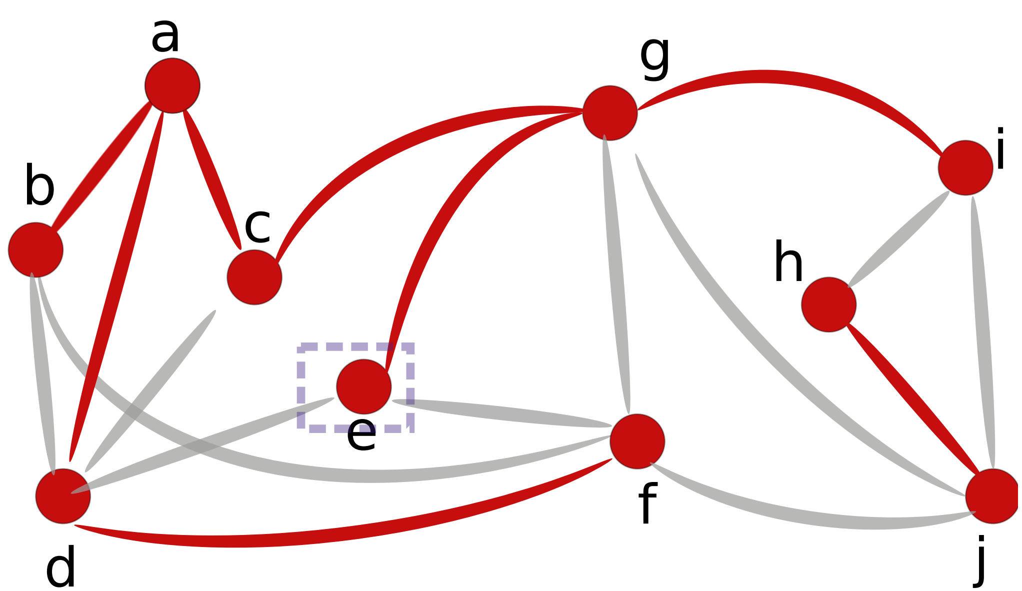

Threshold = 0.5

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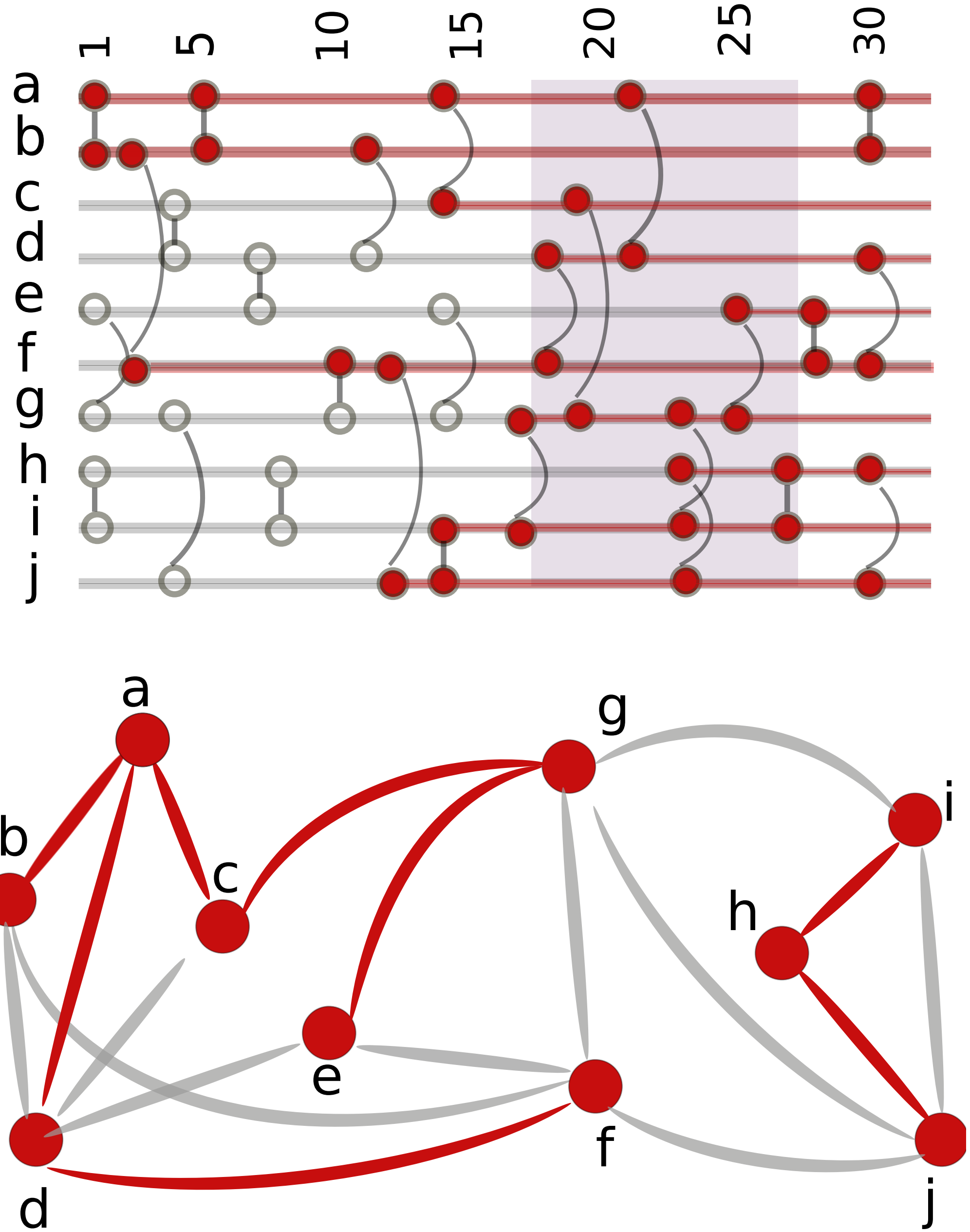

Threshold = 0.5

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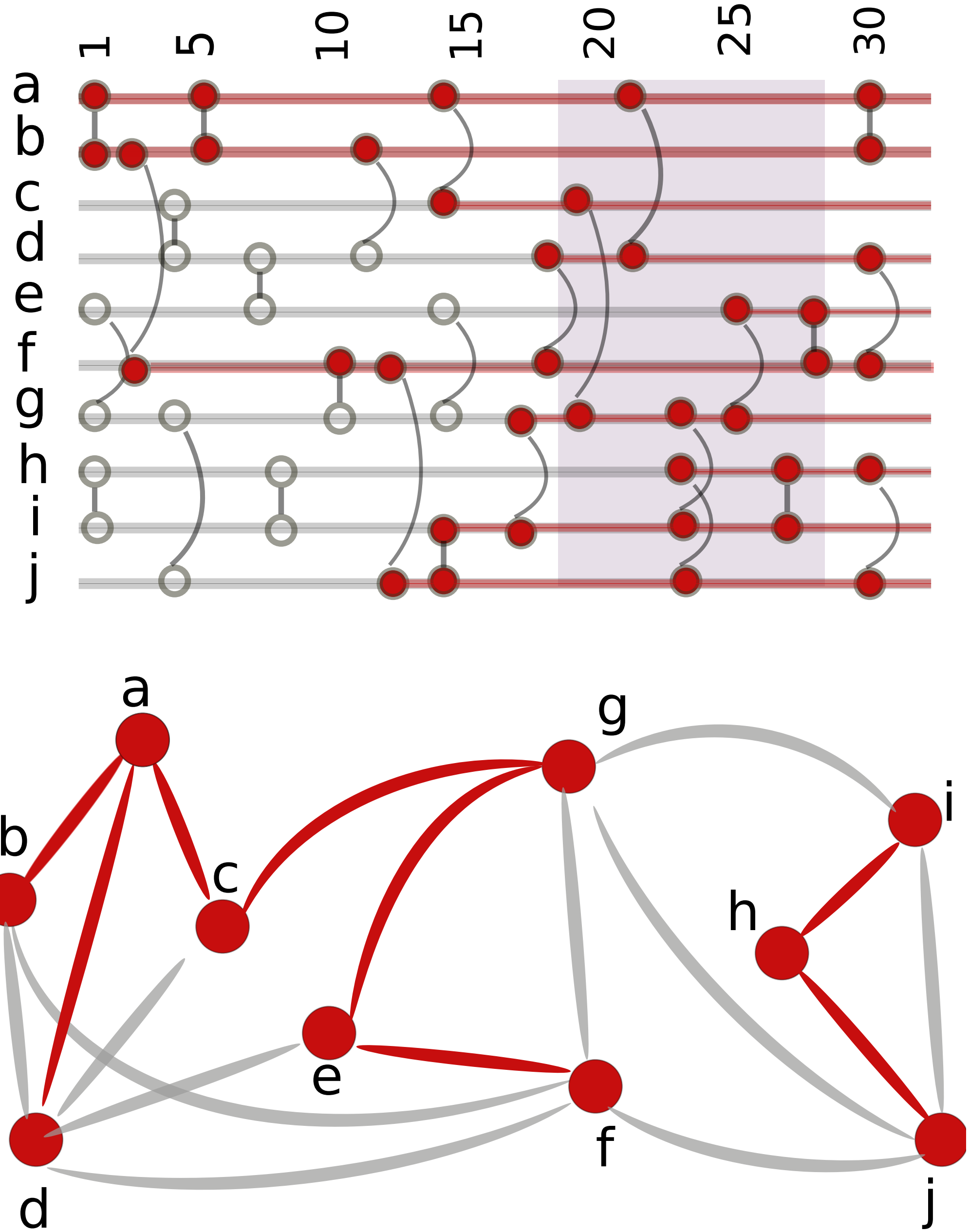

Threshold = 0.5

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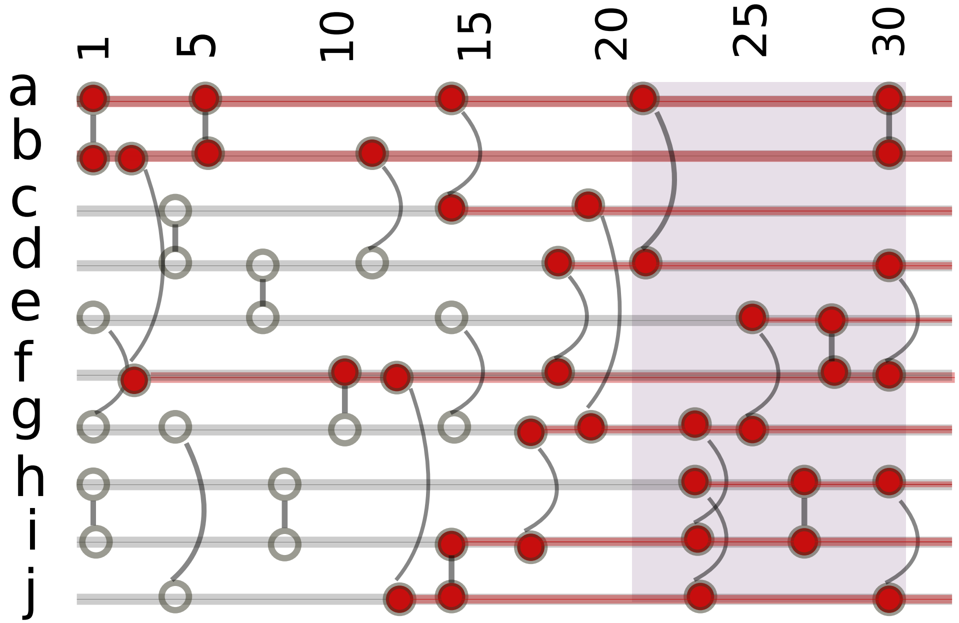

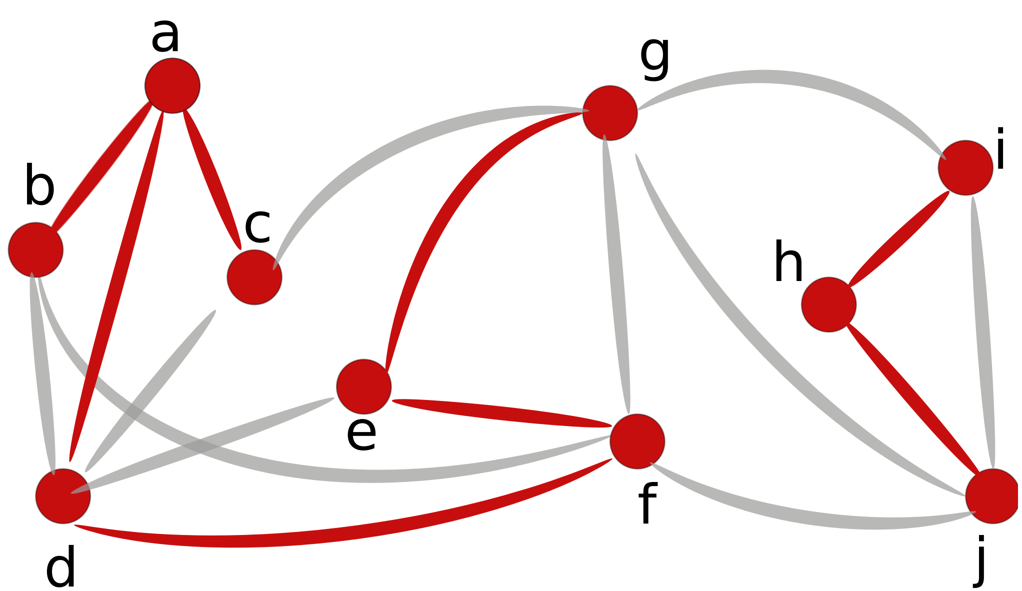

Threshold = 0.5

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Ideas and quiz

- Which of these is complex and which is complicated?
  - An airplane
  - The Web
  - A deep neural network

What is the actual threshold of people?

Idea: we have a minority and majority group. What is the transition point in the Granovetter model?

Overview

1. Voter model
2. Bounded confidence
3. Modeling polarization in the digital society

Opinion dynamics outcomes

- **Consensus**:
  - Distribution of opinions in which the vast majority of agents agree on the same opinion
- **Chaos:**
  - Distribution of opinions in a population in which no apparent agreement group or structure can be identified (fully random)
- **Polarization:**
  - Bimodal distribution of opinions in a population towards the extremes of an opinion spectrum
- **Fragmentation:**
  - Multimodal distribution of opinions in a population into groups of high internal agreement and high external disagreement (multi-polarization)

Opinion dynamics modelling principles

- Agent-based modeling approach:
  - internal state includes (at least) the agent's opinion
  - interaction rules between agents are specified
  - these interactions cause emergent system properties

- No central authority:
  - Collective opinions, consensus, polarization, and cultural traits emerge without central coordination

- Reactive rather than reflective agents:
  - No advanced cognitive process, only adaptation to the environment (neighborhood) by following simple rules

Voter models of opinion dynamics

- Rate to change opinion depends on other agents:
  - neighbors (networks, spatial models)
  - randomly chosen agents (also called mean-field interaction)

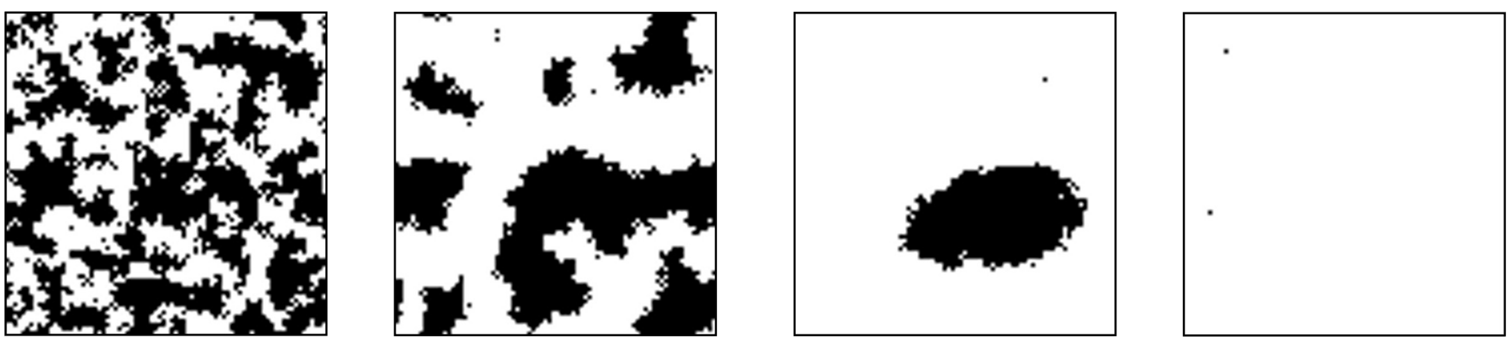
- Principle: *frequency* dependent dynamics. Opinion change based on:

$$P(1-\theta_i|\theta_i) = \kappa(f)\ f_i^{1-\theta_i}$$

- θ_i: opinion of agent i
- $0 \leq f_i^{1-\theta_i} \leq 1$: frequency of agents with *opposite* opinions in "neighborhood" of agent i
- $\kappa(f)$: response function to frequency of other opinions
- Analysis question: How does the outcome depend on $\kappa(f)$?

Linear voter model

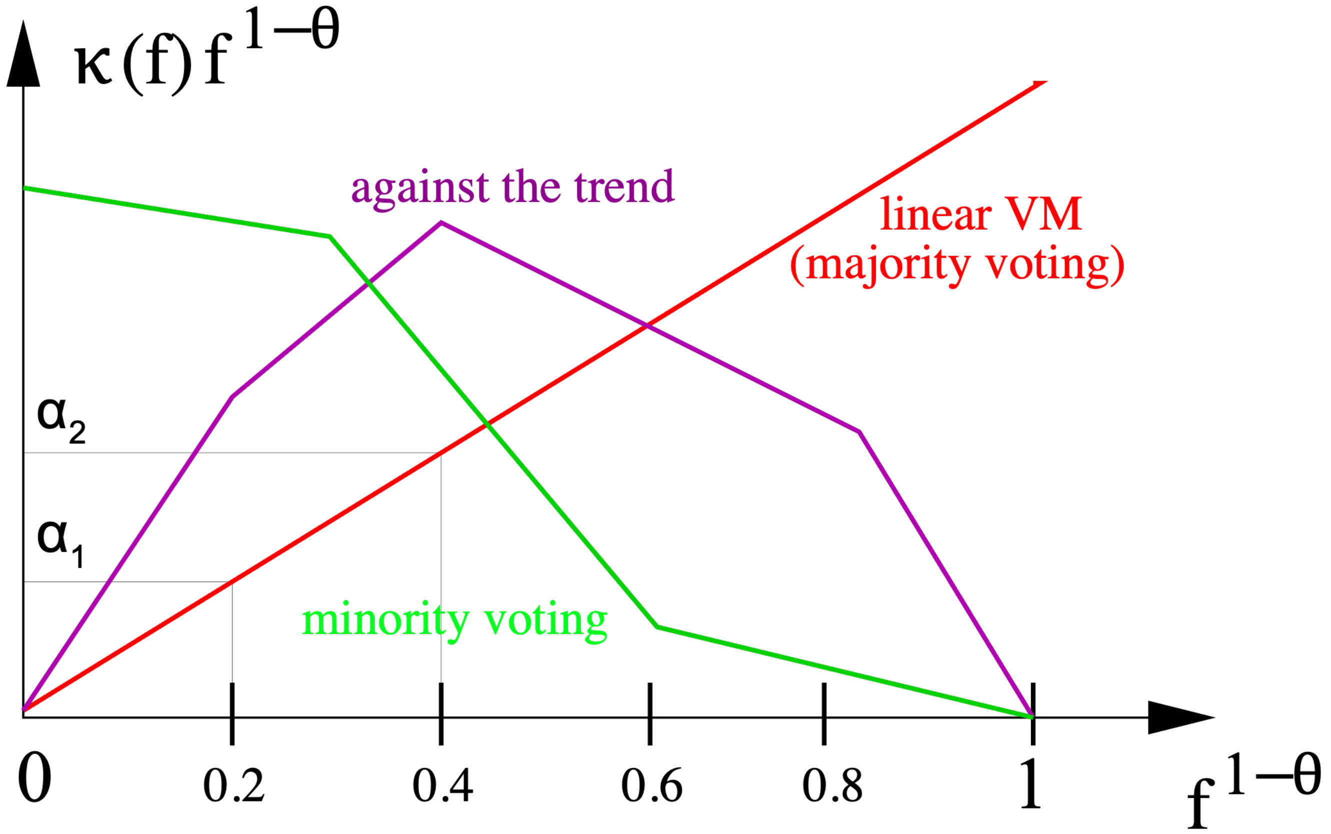
- Dynamics: $P(1-\theta|\theta)=f^{1-\theta}$
- Stochastic simulation agents update at random on a grid
- Initially $f=0.5$, random distribution
- Results:
    - coordination of decisions on medium time scales
    - outcome: consensus as an equilibrium
    - How long does it take to reach consensus?
    - Simulation at: $t=10^1, 10^2, 10^3, 10^4$



Observations on linear voter model

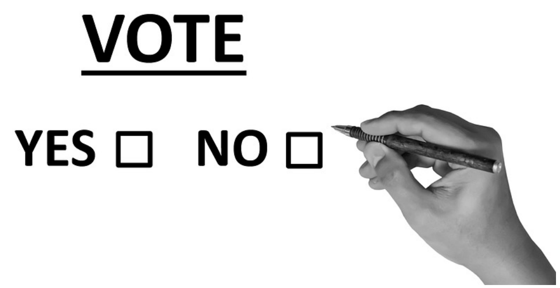
- In the time limit, always only one opinion exists
  - Consensus always appears
  - There are two "absorbing" states: all agents are either 0 or 1
  - The probability to reach an all 0 or all 1 consensus equals initial frequency f(0)
- Model limitations/drawbacks
  - very limited social/biological interpretation (remember Social Impact Theory)
  - what about coexistence of opinions? The reality is not always consensus
- Some interesting features for analysis:
  - Time to reach consensus (TTC)
  - Intermediate dynamics or dependence on grid topology/network

Nonlinear voter model functions

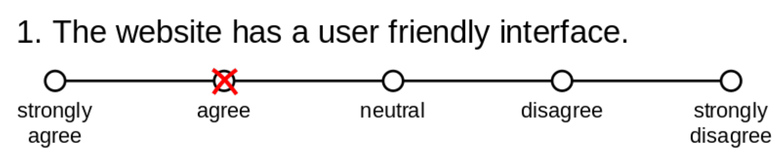


Discrete versus continuous opinion models

- **Voter models:**
  - agents are characterized by a discrete opinion (e.g. binary)



- **Bounded confidence models:**
  - *continuous opinions* x_i (e.g. real number between 0 and 1)



The baseline model: pairwise bounded confidence

- Consider a population of N agents i with continuous opinions x_i
- At each time step any two randomly chosen agents meet
- Re-adjust opinion if absolute opinion difference is smaller than a threshold ϵ
  - In other words: agents i and j with opinions x_i and x_j interact if:

$$|x_i - x_j| < \epsilon$$

- New opinions are adjusted according to

$$x_i(t+1) = x_i(t) + \zeta \cdot (x_j(t) - x_i(t))$$

$$x_j(t+1) = x_j(t) + \zeta \cdot (x_i(t) - x_j(t))$$

- ζ is the convergence parameter: measures speed of opinions approaching

Simulation exmples

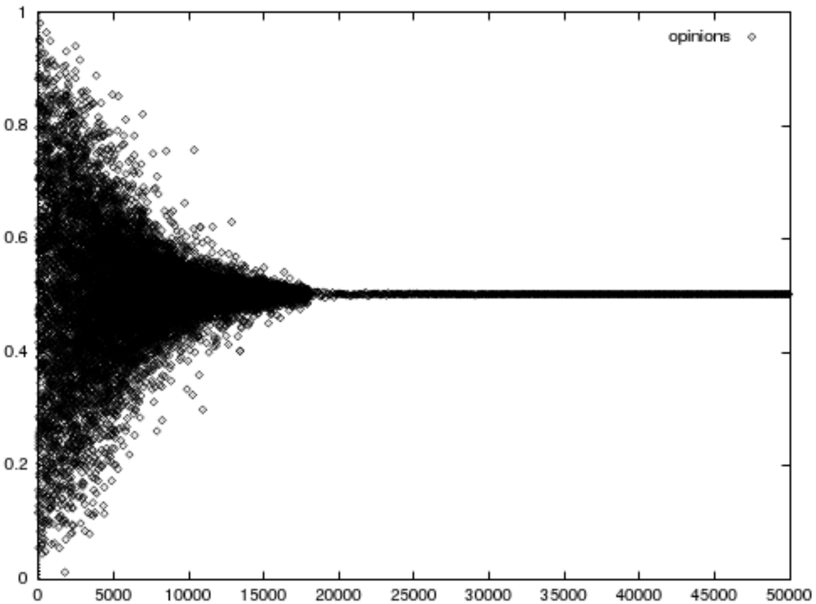
$\epsilon = 0.5, \zeta = 0.5, N = 2000$

$\epsilon = 0.2, \zeta = 0.5, N = 1000$

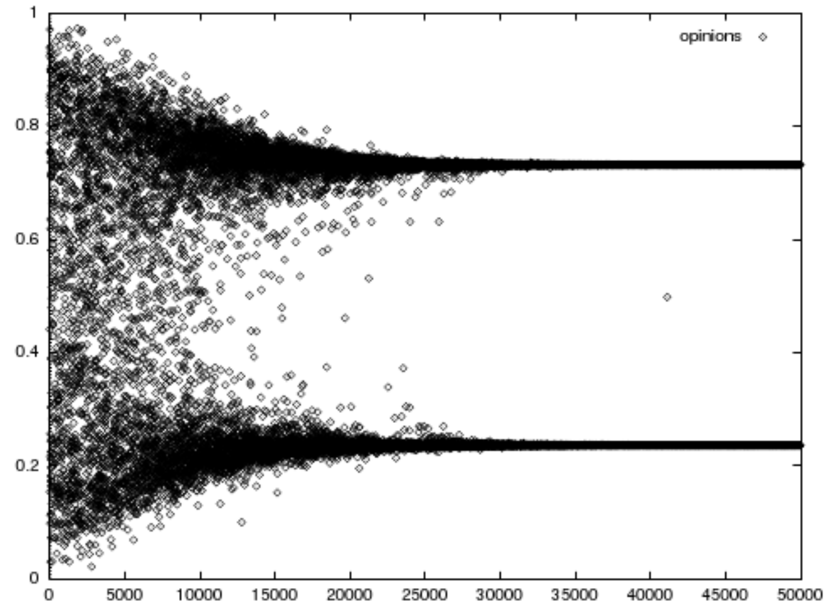
Consensus, polarization, and fragmentation are possible outcomes

[Mixing beliefs among interacting agents. Guillaume Deffuant, David Neau, Frederic Amblard and Gerard Weisbuch. Advances in Complex Systems (2000)]

Simulation exmples



$$\epsilon = 0.5, \zeta = 0.5, N = 2000$$



$$\epsilon = 0.2, \zeta = 0.5, N = 1000$$

Consensus, polarization, and fragmentation are possible outcomes
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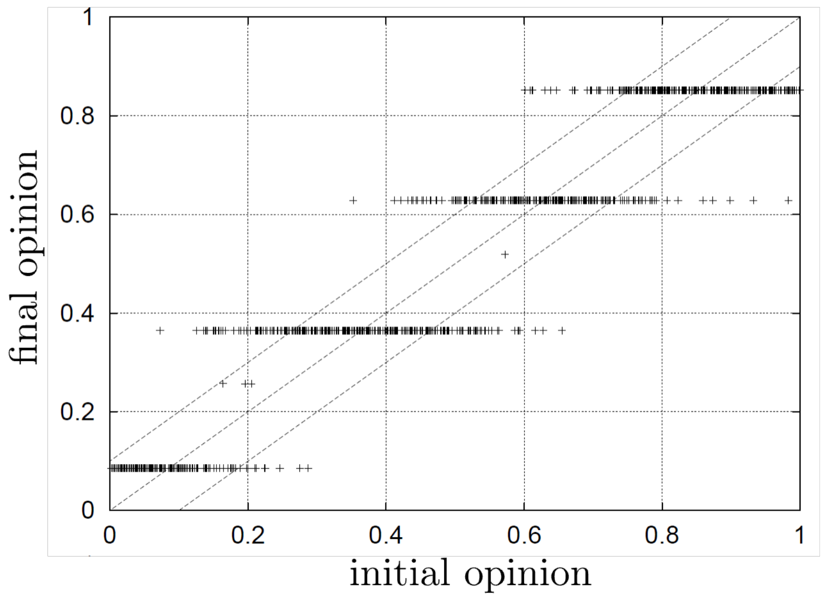
Final opinion vs initial opinions



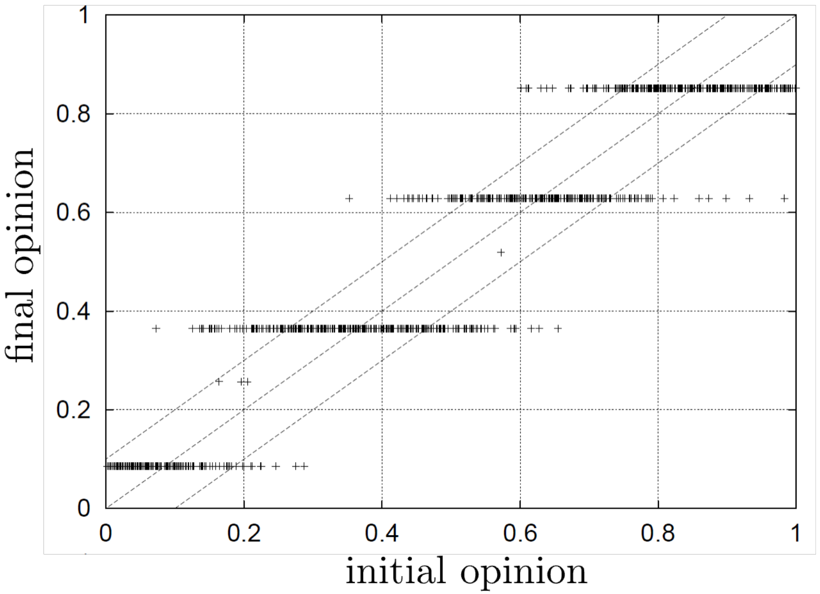
- qualitative dynamics/number of opinion groups depend on??

- What influence convergence time and width of the distribution of final opinions?

- Sometimes you can get "wings" of agents at the edges or stuck between major groups

Deffuant et al. (2000)

Final opinion vs initial opinions



Deffuant et al. (2000)

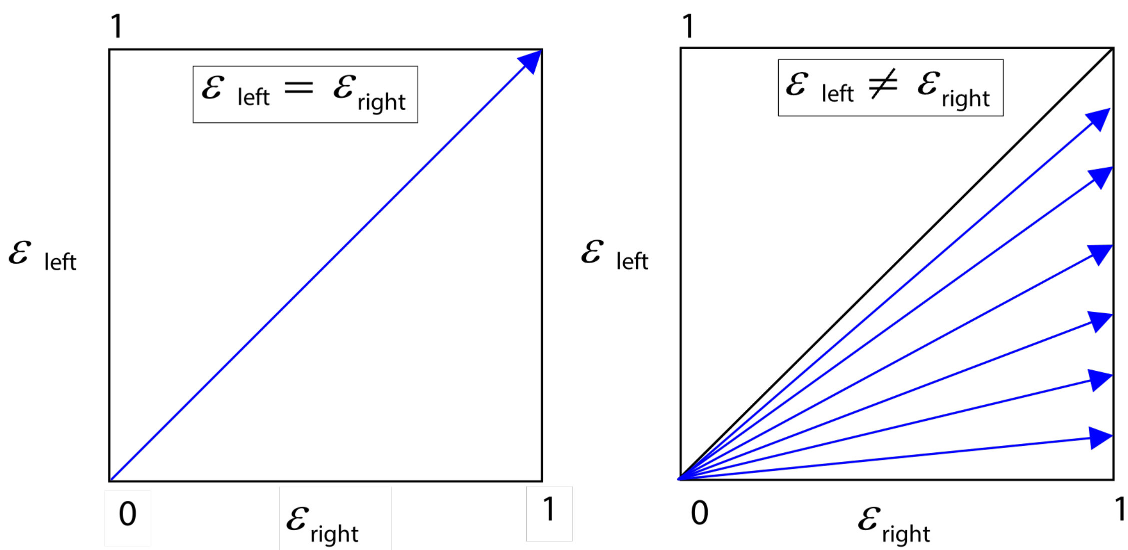
- qualitative dynamics mostly depend on the threshold ϵ:
  - controls the number of peaks of the final distribution of opinions
  - **The final expected number of groups is 1/2ϵ**

- ζ and N only influence convergence time and width of the distribution of final opinions

- Sometimes you can get "wings" of agents at the edges or stuck between major groups

Asymmetric confidence model

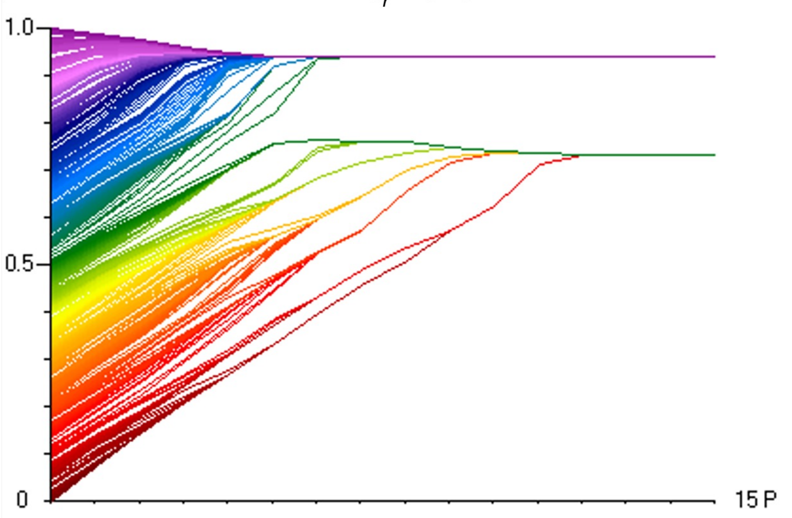
- **symmetric:** $|x_i - x_j| < \epsilon$
- **asymmetric:** $-\epsilon_l < x_i - x_j < \epsilon_r$



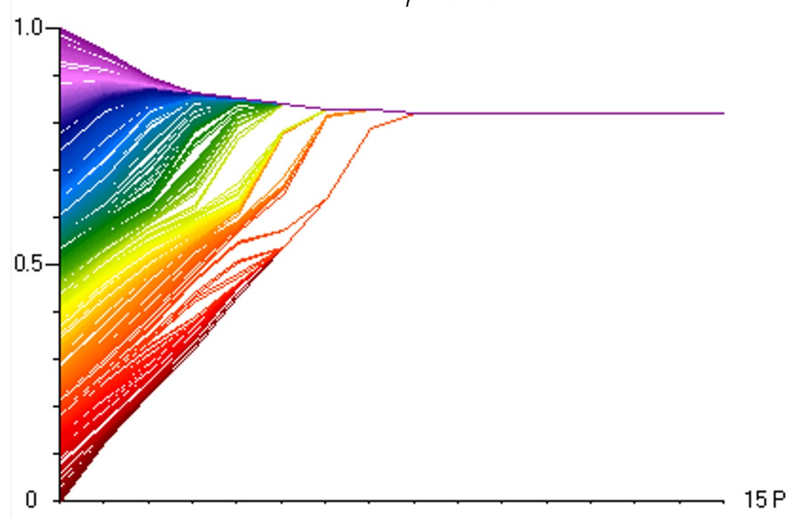
Confidence can extend further on one side than on the other

Opinion- independent asymmetry

$\varepsilon_l = 0.03$
$\varepsilon_r = 0.15$

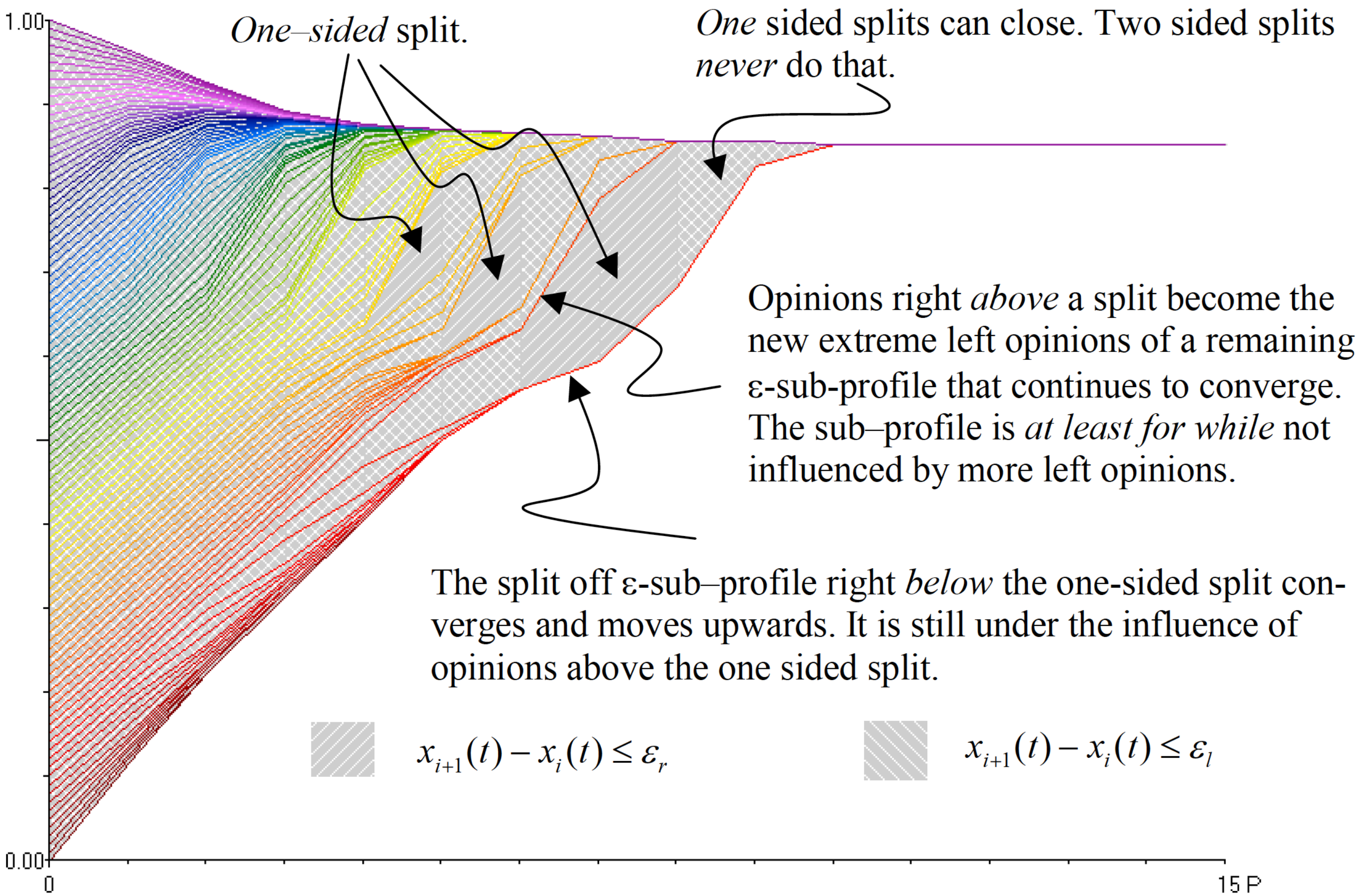


$\varepsilon_l = 0.10$
$\varepsilon_r = 0.25$



Collective opinion drifts in the direction favoured by the asymmetry

One-sided splits



Asymmetric bounded confidence generates temporary fragmentation!

Modeling polarization in the digital society

Could more links create polarization? (see Axelrod's conjecture)

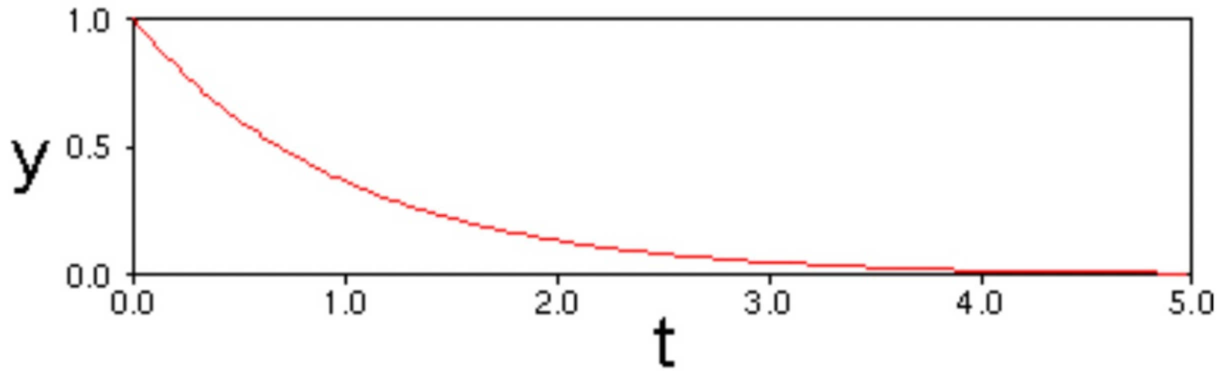
[Tipping diffusivity in information accumulation systems: more links, less consensus. Jae K Shin and Jan Lorenz Journal of Statistical Mechanics: Theory and Experiment (2010)]

Information accumulation model

- n agents with continuous opinions y_i and the following opinion dynamics:

$$y_i^{t+1} = (1 - \Delta) y_i^t + \sum_{j \in \Gamma_i} \omega y_j^t (1 - |y_i^t|)$$

- Δ: measures how much agents opinions relax over time
  - In absence of interaction $y \rightarrow 0$



The information accumulation model

- n agents with continuous opinions y_i and the following opinion dynamics:

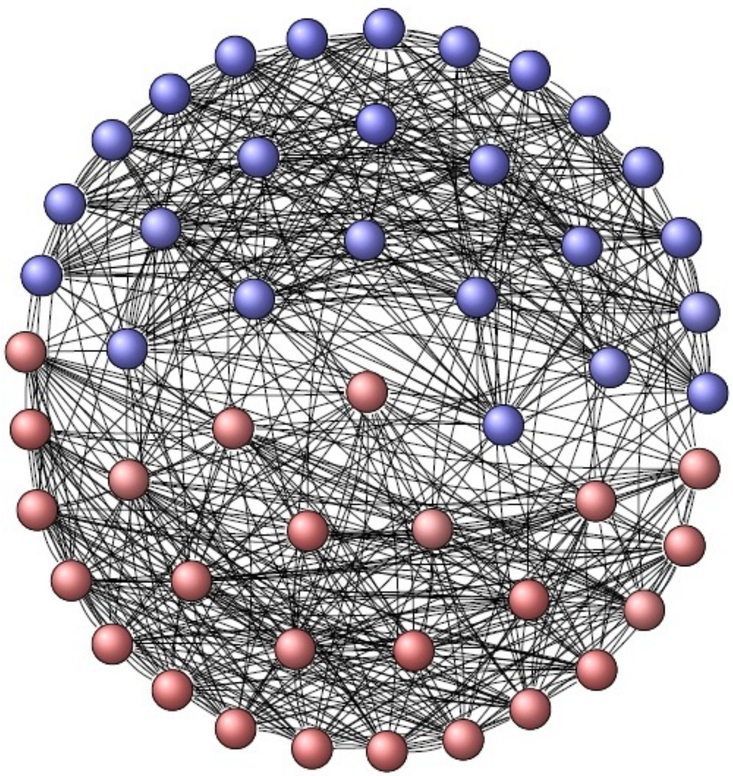
$$y_i^{t+1} = (1 - \Delta) y_i^t + \sum_{j \in \Gamma_i} \omega y_j^t (1 - |y_i^t|)$$

- Δ: measures how much agents opinions relax over time
  - In absence of interaction $y \to 0$

- Diffusivity ω: coupling of opinions between agents
  - Agents approach the opinions of others in their neighborhood

- Saturation $1 - |y_i^t|$: limits opinions to interval $(-1, 1)$

Interaction in echo chambers



- Neighborhood Γ_i contains:
    - m_O connections to neighbors in same community
    - m_X connections to neighbors in the other community
- Weak inter-community interaction (filter bubble effects):

$$m_O > m_X$$

- Intra-community diffusivity: $\Omega_O = m_o \omega$
- Inter-community diffusivity: $\Omega_x = m_x \omega$

Community dynamics

Agent dynamics simplifies to community-level variables:

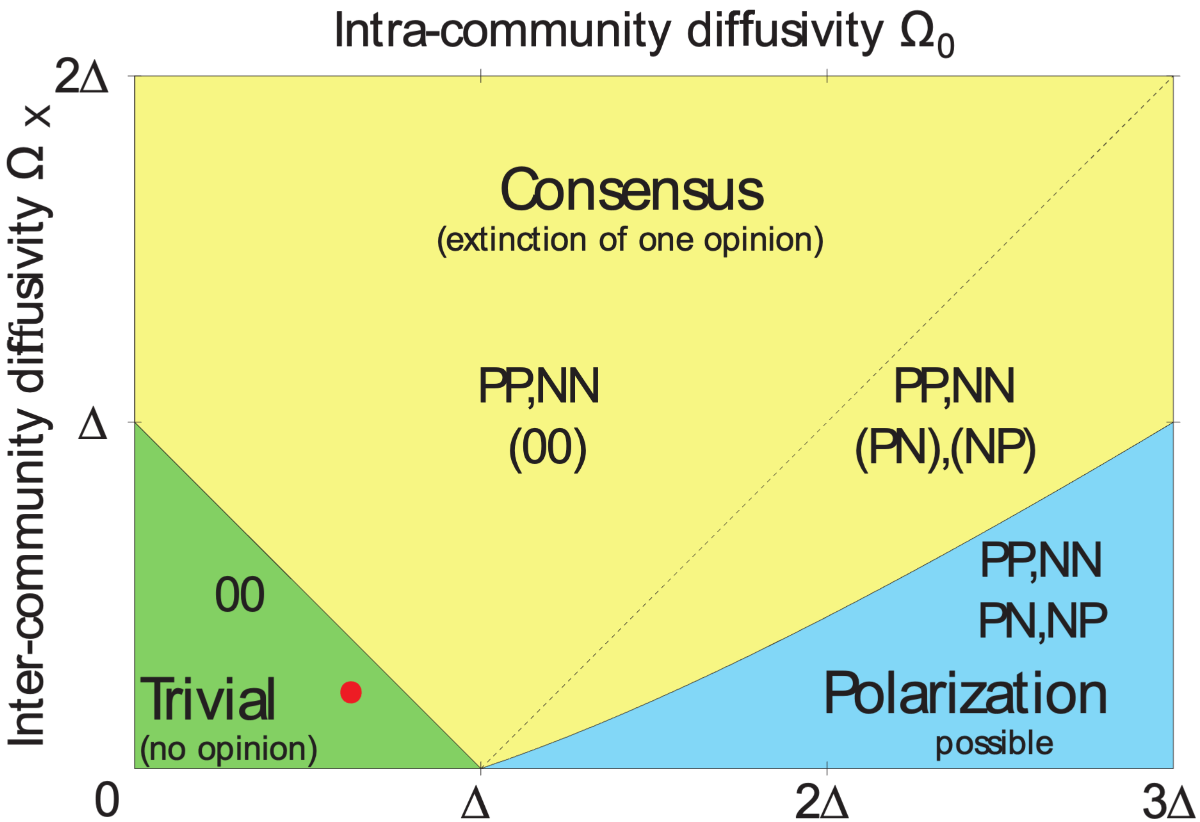
$$y_1^{t+1} = (1-\Delta)y_1^t + (\Omega_0 y_1^t + \Omega_X y_2^t)(1-|y_1^t|)$$
$$y_2^{t+1} = (1-\Delta)y_2^t + (\Omega_X y_1^t + \Omega_0 y_2^t)(1-|y_2^t|).$$

- **00**: no opinion in both communities
- **PP**, **NN**: consensus (both same sign)
- **PN**, **NP**: polarization (differn signs)

When $t \to \infty$ in **PN** mode, opinions follow this system of equations:

$$Y_1 = (1-\Delta)Y_1 + (\Omega_0 Y_1 + \Omega_X Y_2)(1-Y_1)$$
$$Y_2 = (1-\Delta)Y_2 + (\Omega_X Y_1 + \Omega_0 Y_2)(1+Y_2).$$

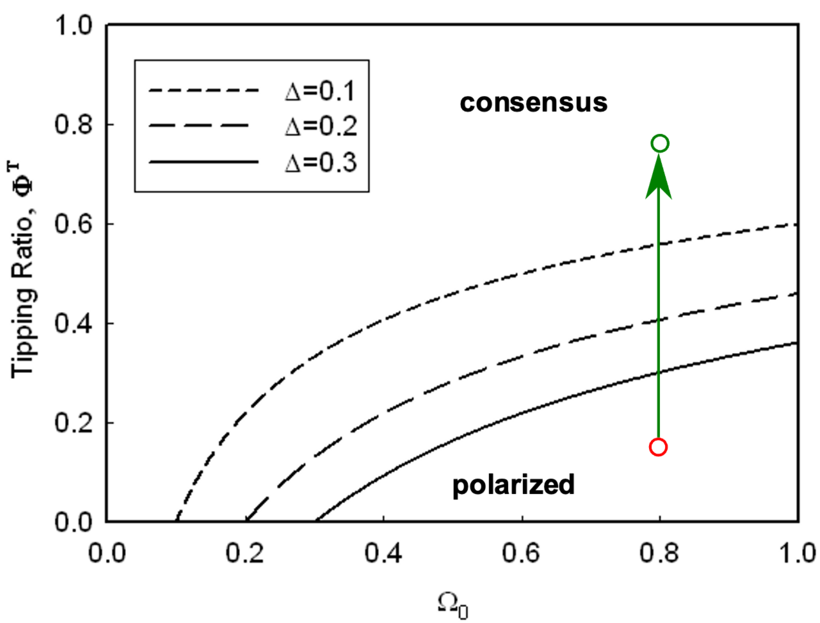
Opinion parameter states



Tipping diffusivity ratio

Tipping diffusivity ratio: Fraction of inter- and intra-diffusivity above which two polarized communities would reach a consensus

$\Phi = \Omega_X / \Omega_0$



Green arrow: fostering consensus by increasing inter-community diffusivity (creating links between groups)

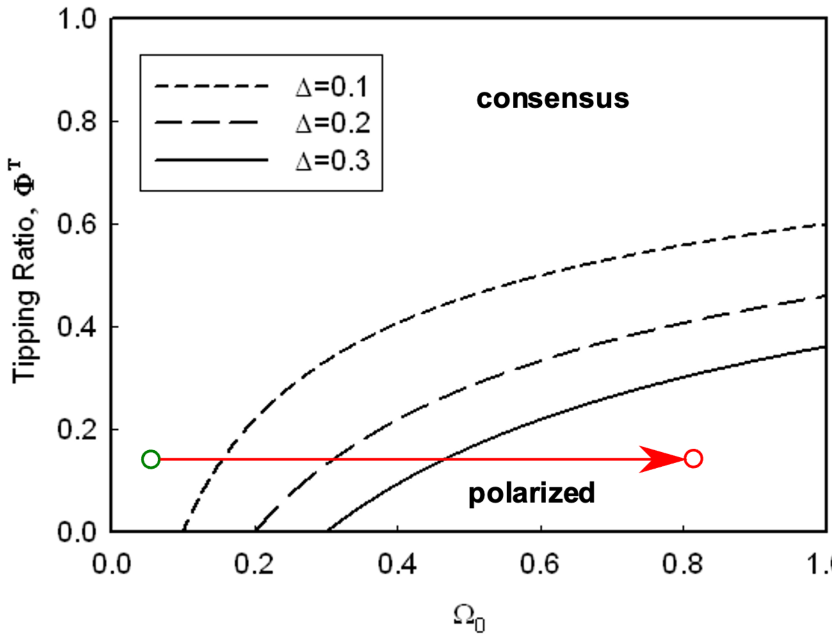
Connectivity can increase polarization

More links, less consensus effect:

For low Ω_X/Ω_0 there is a level of total diffusivity that creates polarization

Red arrow: echo chamber effect is constant (fixed ΩXΩ0), increase in total connectivity (thus increase in Ω0): **Polarization appears!**

Only for low ΩXΩ0: weakening echo chamber effect fosters consensus above a threshold



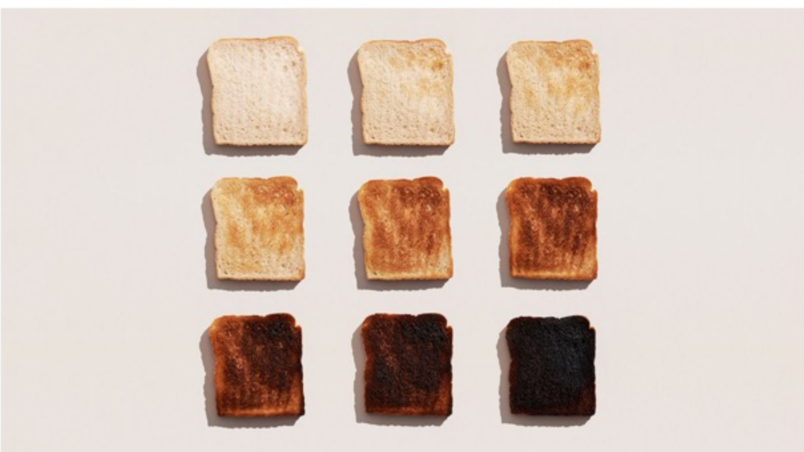
Summary

- The voter model: binary opinions model
  - Probability to change opinion based on opinions of neighbors
  - Linear voter model: consensus. Nonlinear model can have coexistence
  - Adding reluctance to change can speed up consensus
- Bounded confidence: continuous opinions
  - Interaction only when opinions are close enough
  - Generates consensus, polarization, and fragmentation
  - Asymmetry of thresholds creates one-sided splits
- Information accumulation systems
  - Interaction in echo chambers with relaxation towards zero
  - Polarization depends on intra- and inter-community diffusivity
  - Tipping ratio shows that more links can generate polarization

Why “Many-Model Thinkers” Make Better Decisions

by Scott E. Page

November 19, 2018



“To be wise you must arrange your experiences on a lattice of models.”

— Charlie Munger

Summary. Organizations are awash in data — from geocoded transactional data to real-time website traffic to semantic quantifications of corporate annual reports. All these data and data sources only add value if put to use. And that typically means that the data is... **more**

Quiz

- If ϵ=0.2, how many opinion groups can you expect in the bounded confidence model?

- In our online society, do you expect Ω_X/Ω_O to be above or below 1?