Import Libraries and Data

```
# Import Libraries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from google.colab import drive
import gdown
gdown.download('https://drive.google.com/uc?id=10B_GMPQwlIGFpn6KudUTf-Sig85cWU4c',
               'train.csv')
gdown.download('https://drive.google.com/uc?id=1euXRCik-3y-Ip9Z0E06WIK7gKBFexBAL',
               'test.csv')
/usr/local/lib/python3.10/dist-packages/dask/dataframe/ init .py:42: Future
    Dask dataframe query planning is disabled because dask-expr is not installed.
    You can install it with `pip install dask[dataframe]` or `conda install dask`.
    This will raise in a future version.
      warnings.warn(msg, FutureWarning)
    Downloading...
    From: https://drive.google.com/uc?id=10B GMPOwlIGFpn6KudUTf-Sig85cWU4c
    To: /content/train.csv
    100% 461k/461k [00:00<00:00, 51.1MB/s]
    Downloading...
    From: https://drive.google.com/uc?id=1euXRCik-3y-Ip9Z0E06WIK7gKBFexBAL
    To: /content/test.csv
    100% 451k/451k [00:00<00:00, 47.4MB/s]
    'test.csv'
# Load datasets
train_df = pd.read_csv("/content/train.csv", delimiter=',')
test_df = pd.read_csv("/content/test.csv", delimiter=',')
np.random.seed(42)
```

Cleaning

```
# Understand missing values

train_df.describe(include="all")

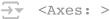
na_counts = train_df.isnull().sum()

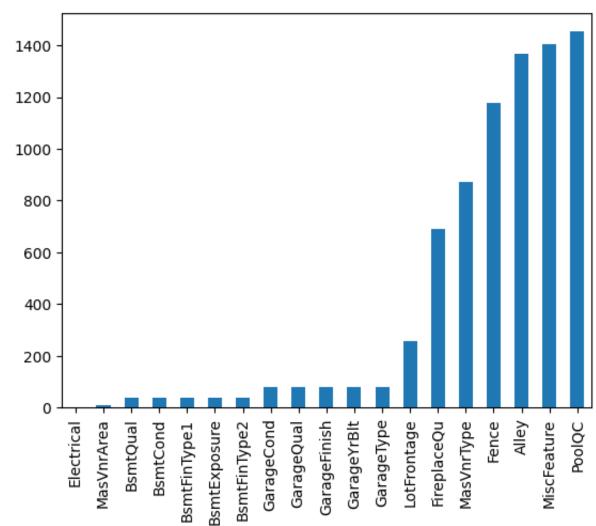
missing = train_df.isnull().sum()

missing = missing[missing > 0]

missing.sort_values(inplace=True)

missing.plot.bar()
```





######################################

```
def data cleaner(dt):
 dt['SalePrice'] = np.log(dt['SalePrice'])
 dt = dt.drop('Id', axis=1)
 ### Lot Frontage
 dt["LotFrontage"] = dt.groupby(["Street","Neighborhood"])["LotFrontage"].transf
 ### Allev
 dt["Alley"] = dt["Alley"].fillna("None")
 ### MasVnrType & MasVnrArea
 dt["MasVnrArea"] = dt.groupby(["Exterior1st", "Exterior2nd"])["MasVnrArea"].tra
  dt["MasVnrType"] = np.where(dt["MasVnrArea"] == 0, "None",dt["MasVnrType"])
 dt["MasVnrType"] = dt.groupby(["MasVnrArea", "Exterior1st", "Exterior2nd"])["Mas
 ### BsmtQual, BsmtCond, BsmtExposure, BsmtFinType1, BsmtFinType2
 dt[["BsmtQual", "BsmtCond", "BsmtExposure", "BsmtFinType1", "BsmtFinType2"]]=dt
 ### Electrical
 dt["Electrical"] = dt["Electrical"].transform(lambda x: x.fillna(x.mode()[0] if
 # filling missing fireplace column with None
  dt['FireplaceQu'] = dt['FireplaceQu'].fillna('None')
 ### GarageType, GarageYrBlt, GarageFinish, GarageQual, GarageCond
 dt[["GarageType", "GarageYrBlt", "GarageFinish", "GarageQual", "GarageCond"]] =
 ### PoolOC
 dt["PoolQC"] = dt["PoolQC"].fillna("None")
 ### Fence
 dt["Fence"] = dt["Fence"].fillna("None")
 ### MiscFeature
  dt["MiscFeature"] = dt["MiscFeature"].fillna("None")
 ## Convert Time variables to Integer Age
 dt["Age"] = 2010 - dt["YearBuilt"]
 dt = dt.drop(columns=['YearBuilt'])
 #dt['Age'] = dt.fillna(0)
 dt["YearsSinceRemodeled"] = 2010 - dt["YearRemodAdd"]
  dt = dt.drop(columns=['YearRemodAdd'])
 #dt["YearsSinceRemodeled"] = dt["YearsSinceRemodeled"].fillna(0)
 dt["GarageYrBlt"] = pd.to numeric(dt["GarageYrBlt"], errors='coerce')
 dt["GarageAge"] = 2010 - dt["GarageYrBlt"]
 dt = dt.drop(columns=['GarageYrBlt'])
  dt['GarageAge'] = dt['GarageAge'].fillna(0)
 dt["TimeSinceSold"] = 2010 - dt["YrSold"] + ((12 - dt["MoSold"]) / 12)
```

cleaned_train

\Rightarrow		MSSubClass	LotFrontage	LotArea	OverallQual	OverallCond	MasVnrArea	B:
	0	60	65.0	8450	7	5	196.0	
	1	20	80.0	9600	6	8	0.0	
	2	60	68.0	11250	7	5	162.0	
	3	70	60.0	9550	7	5	0.0	
	4	60	84.0	14260	8	5	350.0	
	1455	60	62.0	7917	6	5	0.0	
	1456	20	85.0	13175	6	6	119.0	
	1457	70	66.0	9042	7	9	0.0	
	1458	20	68.0	9717	5	6	0.0	
	1459	20	75.0	9937	5	6	0.0	

1460 rows × 302 columns

Testing Feature Selection Algorithms

In this analysis, we will evaluate the effectiveness of various feature selection algorithms in two distinct scenarios based on the relationship between the number of rows (n) and columns (m) in our dataset. Specifically, we will assess:

- 1. Evaluating Feature Selection Algorithms when (n > m)
- 2. Evaluating Feature Selection Algorithms when (m > n)

Where:

- (n) is the number of rows (observations).
- (m) is the number of columns (features).

Objective

The primary goal is to evaluate the **Root Mean Squared Error (RMSE)** of a standard linear regression model across three different feature selection algorithms, utilizing **5-fold cross-validation** and **Booststrapping** to ensure robust results.

Feature Selection Algorithms Under Evaluation

1. Select K Best

 Uses mutual information regression to select the top k features based on their individual performance in predicting the target variable.

2. Variance Threshold

 Eliminates features with low variance, retaining only those that contribute meaningful variability to the dataset.

3. Recursive Feature Elimination (RFE) with Lasso Regression

• This method recursively removes features and builds a model on the remaining attributes, leveraging Lasso regression to promote sparsity in feature selection.

Methodology

To create a scenario where (m > n), we will utilize a random number generator to obtain a consistent set of random index numbers, with a length of (m - 25). This method ensures that the random indices remain constant across each feature selection algorithm during the experiment, thus minimizing variance and bias in the results. For the scenerio where (n > m), we utilize the entire provided training dataset. We use bootstrapping in a few ways:

- 1. To calculate robust RMSE values and confidence intervals for the baseline model.
- 2. To calculate robust RMSE values and confidence intervals for the full data partition.
- 3. To calculate robust RMSE and Percent Features in Common measures and confidence intervals for the subset data partition - not necessarily because of variability in crossvalidation fold randomization, but because of the variation in features selected from the high-dimensionality scenerio.

```
# ----- Testing Feature Selection Algorithms -----
from sklearn.linear_model import LinearRegression, Lasso
from sklearn.tree import DecisionTreeRegressor
from sklearn.preprocessing import Normalizer
from sklearn.feature selection import SelectKBest, RFE, VarianceThreshold, Select
from sklearn.pipeline import Pipeline
from sklearn.metrics import root_mean_squared_error, make_scorer
from sklearn.model_selection import cross_val_predict, KFold
feature_selection_algos = [
   ('SelectKBest', SelectKBest(mutual info regression,
                             k = 25), # mutual info regression, r regression,
   ('RFE', RFE(estimator = Lasso(),
              n_features_to_select = 25,
              step = 1)),
   ('VarianceThreshold', VarianceThreshold(threshold = 1.9)), # about 25 feature
   ('SelectFromModel_Lasso', SelectFromModel(estimator = Lasso(),
                                    max_features = 25)),
    ('SelectFromModel Tree', SelectFromModel(estimator = DecisionTreeRegressor()
                                    max features = 25))
1
model = LinearRegression()
cv = KFold(n_splits=5, shuffle=True)
y, X = cleaned_train['SalePrice'], cleaned_train.drop('SalePrice', axis=1)
##########################
## Running All Tests
#############################
# BOOTSTRAP CONTROL:
```

```
n_bootstrap = 100
###############################
# BASELINE MODEL
lasso rmse = []
print("Running control model")
for iter in range(0, n bootstrap):
  lasso_pred = lasso_pred = cross_val_predict(Lasso(), X, y, cv=cv)
  lasso_rmse.append(np.sqrt(np.mean((np.exp(y) - np.exp(lasso_pred)) ** 2)))
lasso_cv_rmse = np.mean(lasso_rmse)
# Bootstraping Begins
bootstrap_rmse_results = {name: [] for name, _ in feature_selection_algos}
bootstrap_common_feature_percentages = {name: [] for name, _ in feature_selection
###############################
# FULL PARTITION
full_rmse_results = {}
full_rmse_ci = {}
full rmse std = {}
selected_features_full = {}
for name, selector in feature_selection_algos:
    print(f'Running "full" partition for {name}')
    selector.fit(X, y)
    X full transformed = selector.transform(X)
    # Store selected features for 'full'
    if hasattr(selector, 'get_support'):
        selected features full[name] = X.columns[selector.get support()]
    elif hasattr(selector, 'get_feature_names_out'):
        selected_features_full[name] = selector.get_feature_names_out(input_featu
    else:
        selected features full[name] = []
    temp_rmse_results = []
    for iter in range(0, n_bootstrap):
      y_pred = cross_val_predict(model, X_full_transformed, y, cv=cv)
      temp_rmse_results.append(np.sqrt(np.mean((np.exp(y) - np.exp(y_pred)) ** 2)
    full_rmse_results[name] = np.mean(temp_rmse_results)
    full_rmse_std[name] = np.std(temp_rmse_results)
    ci_low, ci_high = np.percentile(temp_rmse_results, [2.5, 97.5])
```

```
full_rmse_ci[name] = (ci_low, ci_high)
##########################
# SUBSET PARTITION
for bootstrap_iteration in range(n_bootstrap):
    print(f'Running bootstrap sample {bootstrap_iteration + 1}/{n_bootstrap}')
   # Create the random subset for the bootstrap sample
    subset_indices = np.random.randint(0, len(cleaned_train), size=len(cleaned_train)
    for name, selector in feature selection algos:
        # Fit selector to the 'subset'
        X_subset = X.iloc[subset_indices]
        y_subset = y.iloc[subset_indices]
        selector.fit(X subset, y subset)
        X_subset_transformed = selector.transform(X)
        # Calculate RMSE for the 'subset'
        subset_y_pred = cross_val_predict(model, X_subset_transformed, y, cv=cv)
        bootstrap_rmse_results[name].append(np.sqrt(np.mean((np.exp(y) - np.exp(s
        # Percentage of feature commonality
        common_feat_list = [
            feature for feature in selected features full[name]
            if feature in X.columns[selector.get support()]
        common_percentage = len(common_feat_list) / len(selected_features_full[nail]
        # Store the common feature percentage
        bootstrap_common_feature_percentages[name].append(common_percentage)
# Calculate confidence intervals for percentage of feature commonality for each a
for name in bootstrap_rmse_results:
    common_feat_array = np.array(bootstrap_common_feature_percentages[name])
    # Common feature percentage confidence intervals
    common_ci_low, common_ci_high = np.percentile(common_feat_array, [2.5, 97.5])
→ Running control model
    Running "full" partition for SelectKBest
    Running "full" partition for RFE
    Running "full" partition for VarianceThreshold
    Running "full" partition for SelectFromModel_Lasso
    Running "full" partition for SelectFromModel_Tree
    Running bootstrap sample 1/100
    Running bootstrap sample 2/100
    Running bootstrap sample 3/100
```

```
Running bootstrap sample 4/100
Running bootstrap sample 5/100
Running bootstrap sample 6/100
Running bootstrap sample 7/100
Running bootstrap sample 8/100
Running bootstrap sample 9/100
Running bootstrap sample 10/100
Running bootstrap sample 11/100
Running bootstrap sample 12/100
Running bootstrap sample 13/100
Running bootstrap sample 14/100
Running bootstrap sample 15/100
Running bootstrap sample 16/100
Running bootstrap sample 17/100
Running bootstrap sample 18/100
Running bootstrap sample 19/100
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Running bootstrap sample 40/100
Running bootstrap sample 41/100
Running bootstrap sample 42/100
Running bootstrap sample 43/100
Running bootstrap sample 44/100
Running bootstrap sample 45/100
Running bootstrap sample 46/100
Running bootstrap sample 47/100
Running bootstrap sample 48/100
Running bootstrap sample 49/100
Running bootstrap sample 50/100
Running bootstrap sample 51/100
Running bootstrap sample 52/100
Running bootstrap sample 53/100
```

```
# Create a dictionary to store the formatted RMSE results
formatted_rmse_results = {
```

```
'Full': [],
    'Full 95% CI Lower': [],
    'Full 95% CI Upper': [],
    'Subset': [],
    'Subset 95% CI Lower': [],
    'Subset 95% CI Upper': []
}
for name in bootstrap_rmse_results:
   # Get the full RMSE value
    full rmse = full rmse results[name]
   # Confidence Int for full subset
   full_rmse_ci_low, full_rmse_ci_high = full_rmse_ci[name]
   # Calculate the mean and confidence interval for the subset RMSE
    subset_rmse_array = np.array(bootstrap_rmse_results[name])
    subset rmse mean = subset rmse array.mean()
    subset_rmse_ci_low, subset_rmse_ci_high = np.percentile(subset_rmse_array, [2
    formatted_rmse_results['Full'].append(full_rmse)
    formatted_rmse_results['Full 95% CI Lower'].append(full_rmse_ci_low)
    formatted rmse results['Full 95% CI Upper'].append(full rmse ci high)
    formatted_rmse_results['Subset'].append(subset_rmse_mean)
    formatted_rmse_results['Subset 95% CI Lower'].append(subset_rmse_ci_low)
    formatted_rmse_results['Subset 95% CI Upper'].append(subset_rmse_ci_high)
rmse_df = pd.DataFrame(formatted_rmse_results, index=[name for name, _ in feature]
rmse_df = np.round(rmse_df, 2)
control_lasso_rmse = np.round(lasso_cv_rmse, 2)
lasso rmse ci low, lasso rmse ci high = np.percentile(lasso rmse, [2.5, 97.5])
# Append the Control Lasso row to the DataFrame
control_row = pd.DataFrame({
    'Full': [control lasso rmse],
    'Full 95% CI Lower': [lasso_rmse_ci_low],
    'Full 95% CI Upper': [lasso_rmse_ci_high],
    'Subset': [' '],
    'Subset 95% CI Lower': [' '],
    'Subset 95% CI Upper': [' ']
}, index=['Control Lasso'])
# Combine the original DataFrame with the control row
rmse_df = pd.concat([rmse_df, control_row])
```

 $\overline{2}$

```
Subset
                                 Full 95% CI
                                               Full 95% CI
                           Full
                                                               Subset
                                                                        95% CI
                                       Lower
                                                      Upper
                                                                         Lower
     SelectKBest
                       69838.90
                                  62898.870000
                                                84738.730000
                                                              75437.46 63035.86
        RFE
                      107954.10 85511.670000 134999.420000 302574.49 77774.09 28
  VarianceThreshold
                       83253.75
                                72062.080000 100890.790000
                                                              97356.36 69991.35
SelectFromModel Lasso 116037.21 106580.530000 136815.180000
                                                              118236.3 82876.55
SelectFromModel Tree
                       90812.73 80617.430000 109767.010000
                                                              81865.78 62075.79
```

```
# Standard Deviation between Full and Subset RMSE Values
rmse_stdevs = {
    'Full': [],
    'Subset': [],
}

for name in bootstrap_rmse_results:
    # FULL RMSE STD
    full_std = full_rmse_std[name]

# SUBSET RMSE STD
    subset_rmse_std = np.std(bootstrap_rmse_results[name])

    rmse_stdevs['Full'].append(full_std)
    rmse_stdevs['Subset'].append(subset_rmse_std)

rmse_std_df = pd.DataFrame(rmse_stdevs, index=[name for name, _ in feature_select
np.round(rmse_std_df, 2)
```

₹

	Full	Subset
SelectKBest	6416.26	8096.52
RFE	12330.15	1156413.07
VarianceThreshold	8885.44	20898.27
SelectFromModel_Lasso	8535.57	16537.02
SelectFromModel_Tree	9275.00	12992.92

```
formatted_percentage_results = {
   'Percentage': [],
   'Percentage 95% CI Lower': [],
   'Percentage 95% CI Upper': []
}
# Iterate over each algorithm to fill in the DataFrame
for name in bootstrap_common_feature_percentages:
   # Get the array of common feature percentages
   percentage_array = np.array(bootstrap_common_feature_percentages[name])
   # Calculate the mean and confidence interval for the percentages
   percentage_mean = percentage_array.mean()
   percentage_ci_low, percentage_ci_high = np.percentile(percentage_array, [2.5,
   # Add the values to the dictionary
   formatted_percentage_results['Percentage'].append(percentage_mean)
   formatted_percentage_results['Percentage 95% CI Lower'].append(percentage_ci_
   formatted_percentage_results['Percentage 95% CI Upper'].append(percentage_ci_
```

np.round(percentage_df, 3)

→		Percentage	Percentage 95% CI Lower	Percentage 95% CI Upper	
	SelectKBest	0.862	0.80	0.96	
	RFE	0.890	0.72	0.96	
	VarianceThreshold	0.970	0.92	1.00	
	SelectFromModel_Lasso	0.854	0.70	1.00	
	SelectFromModel_Tree	0.561	0.45	0.75	

```
# What are the common features across full subset?
select_algorithms = ['SelectKBest', 'RFE', 'VarianceThreshold', 'SelectFromModel_
# FULL
common features full = set(selected features full[select algorithms[0]])
for algo in select_algorithms[1:]:
  features_algo = set(selected_features_full[algo])
  common features full &= features algo
common_feat_list_full = list(common_features_full)
from scipy.stats import pearsonr
unsure_features = []
for feature in common_feat_list_full:
    coef, p_value = pearsonr(cleaned_train['SalePrice'], cleaned_train[feature])
    if coef > 0.1 and p_value < 0.05:
        print(f"{feature} increases house prices [corr_coef: {coef:.2f}, p-value:
    elif coef < -0.1 and p_value < 0.05:
        print(f"{feature} decreases house prices [corr_coef: {coef:.2f}, p-value:
    else:
        unsure_features.append(feature)
        print(f"**Unsure about {feature}.** Correlation coefficient: {coef:.2f}, |
→ YearsSinceRemodeled decreases house prices [corr_coef: -0.57, p-value: 0.00].
    TotalBsmtSF increases house prices [corr_coef: 0.61, p-value: 0.00].
    GrLivArea increases house prices [corr_coef: 0.70, p-value: 0.00].
    GarageArea increases house prices [corr coef: 0.65, p-value: 0.00].
    Age decreases house prices [corr_coef: -0.59, p-value: 0.00].
    2ndFlrSF increases house prices [corr_coef: 0.32, p-value: 0.00].
```

Lasso Comparison

from sklearn.linear_model import LassoCV, Lasso

```
from sklearn.preprocessing import StandardScaler
import numpy as np
# Standardize features
scaler = StandardScaler()
X_scaled = scaler.fit_transform(X)
alpha = 0.01
lasso = Lasso(alpha=alpha).fit(X_scaled, y)
# Count non-zero coefficients
non_zero_count = np.sum(lasso.coef_ != 0)
print(f'Number of non-zero coefficients: {non_zero_count}')
# If not within the desired range, adjust alpha
while non_zero_count < 10 or non_zero_count > 15:
    if non_zero_count > 15:
        alpha *= 1.1 # Increase alpha to increase regularization (fewer features
    elif non_zero_count < 10:</pre>
        alpha /= 1.1 # Decrease alpha to decrease regularization (more features)
    lasso = Lasso(alpha=alpha).fit(X_scaled, y)
    non zero_count = np.sum(lasso.coef_ != 0)
    print(f'Adjusted alpha: {alpha}, Non-zero coefficients: {non_zero_count}')
selected_features = np.array(X.columns)[lasso.coef_ != 0]
selected_coefficients = lasso.coef_[lasso.coef_ != 0]
# Create a DataFrame to display features and coefficients
ls_mod_features = pd.DataFrame({
    'Feature': selected features,
    'Coefficient': selected coefficients
}).sort_values(by='Coefficient', ascending=False).reset_index(drop=True)
```

```
Number of non-zero coefficients: 57
    Adjusted alpha: 0.01100000000000001, Non-zero coefficients: 53
    Adjusted alpha: 0.01210000000000001, Non-zero coefficients: 51
    Adjusted alpha: 0.01331000000000002, Non-zero coefficients: 48
   Adjusted alpha: 0.014641000000000003, Non-zero coefficients: 44
    Adjusted alpha: 0.016105100000000004, Non-zero coefficients: 39
    Adjusted alpha: 0.017715610000000007, Non-zero coefficients: 36
    Adjusted alpha: 0.019487171000000008, Non-zero coefficients: 31
    Adjusted alpha: 0.021435888100000012, Non-zero coefficients: 30
   Adjusted alpha: 0.023579476910000015, Non-zero coefficients: 28
    Adjusted alpha: 0.025937424601000018, Non-zero coefficients: 25
    Adjusted alpha: 0.02853116706110002, Non-zero coefficients: 23
    Adjusted alpha: 0.031384283767210024, Non-zero coefficients: 20
    Adjusted alpha: 0.03452271214393103, Non-zero coefficients: 21
    Adjusted alpha: 0.03797498335832414, Non-zero coefficients: 18
    Adjusted alpha: 0.04177248169415655, Non-zero coefficients: 17
    Adjusted alpha: 0.04594972986357221, Non-zero coefficients: 15
```

np.round(ls mod features, 4).set index('Feature')



Coefficient

Feature

OverallQual	0.1404
GrLivArea	0.0888
GarageCars	0.0504
TotalBsmtSF	0.0267
Fireplaces	0.0111
BsmtFinSF1	0.0063
1stFlrSF	0.0034
MSZoning_RL	0.0030
GarageArea	0.0029
CentralAir_Y	0.0000
FireplaceQu_None	-0.0039
CentralAir_N	-0.0062
MSZoning_RM	-0.0115
Age	-0.0234
YearsSinceRemodeled	-0.0257