Project: Superconductors with non-unitary triplet pairing in external magnetic field

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What this project is about?

- Motivation: Interesting experiment

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Nonunitary Triplet Pairing in the Centrosymmetric Superconductor $LaNiGa_2$

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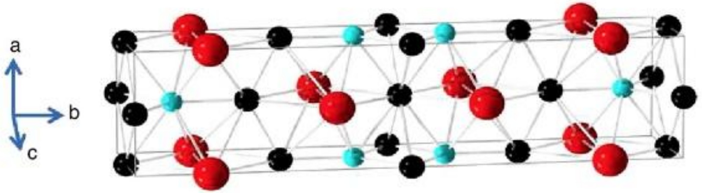


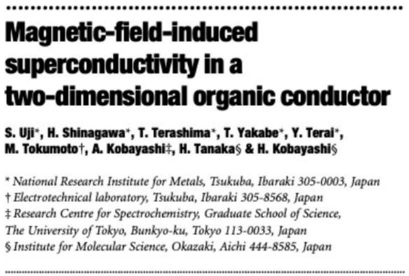
$SO(3) \times D_{2h}$	Gap function (unitary)	Gap function (nonunitary)
${}^1A_1$	$\Delta(\mathbf{k}) = 1$	...
${}^1B_1$	$\Delta(\mathbf{k}) = XY$	...
${}^1B_2$	$\Delta(\mathbf{k}) = XZ$	...
${}^1B_3$	$\Delta(\mathbf{k}) = YZ$	...
${}^3A_1$	$\mathbf{d}(\mathbf{k}) = (0, 0, 1)XYZ$	$\mathbf{d}(\mathbf{k}) = (1, i, 0)XYZ$
${}^3B_1$	$\mathbf{d}(\mathbf{k}) = (0, 0, 1)Z$	$\mathbf{d}(\mathbf{k}) = (1, i, 0)Z$
${}^3B_2$	$\mathbf{d}(\mathbf{k}) = (0, 0, 1)Y$	$\mathbf{d}(\mathbf{k}) = (1, i, 0)Y$
${}^3B_3$	$\mathbf{d}(\mathbf{k}) = (0, 0, 1)X$	$\mathbf{d}(\mathbf{k}) = (1, i, 0)X$

D_{2h}	Gap function with strong SOC
A_1	$\mathbf{d}(\mathbf{k}) = (AX, BY, CZ)$
B_1	$\mathbf{d}(\mathbf{k}) = (AY, BX, CXYZ)$
B_2	$\mathbf{d}(\mathbf{k}) = (AZ, BXYZ, CX)$
B_3	$\mathbf{d}(\mathbf{k}) = (AXYZ, BZ, CY)$

Question: What are experimental signatures of the nonunitary triplet superconductivity in thermodynamics, transport and various nonlinear responses?

What this project is about?

- The problem of triplet pairing under external magnetic field has also been motivated by earlier experimental and theoretical studies



Magnetic-field-induced superconductivity in a two-dimensional organic conductor

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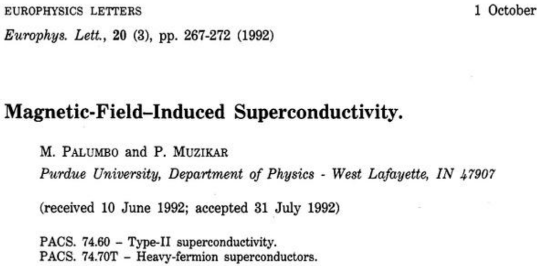

EUROPHYSICS LETTERS 1 October
Europhys. Lett., 20 (3), pp. 267-272 (1992)

Magnetic-Field–Induced Superconductivity.

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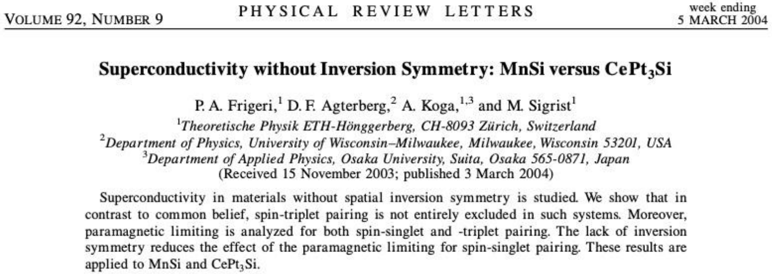
(received 10 June 1992; accepted 31 July 1992)

PACS. 74.60 – Type-II superconductivity.
PACS. 74.70T – Heavy-fermion superconductors.



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Superconductivity without Inversion Symmetry: MnSi versus $CePt_3Si$

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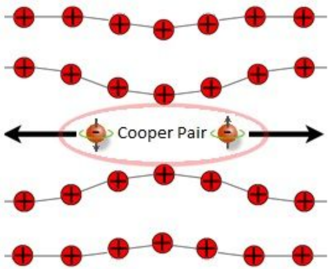
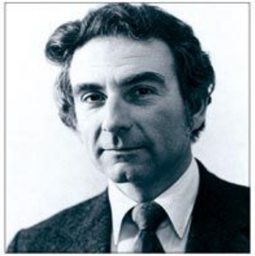
Superconductivity in materials without spatial inversion symmetry is studied. We show that in contrast to common belief, spin-triplet pairing is not entirely excluded in such systems. Moreover, paramagnetic limiting is analyzed for both spin-singlet and -triplet pairing. The lack of inversion symmetry reduces the effect of the paramagnetic limiting for spin-singlet pairing. These results are applied to MnSi and $CePt_3Si$.

- The problem can be studied within the framework of the Bardeen-Cooper-Schrieffer (BCS) model of superconductivity



Schematic illustration of the Cooper pairing interaction in BCS superconductors

John Bardeen



Leon Cooper

John Schrieffer

What this project is about?

- Superconductivity with a triplet order parameter

Consider a hamiltonian of the BCS model with a magnetic field term:

$$H = \sum_{\mathbf{k}} \epsilon(k) a_{\mathbf{k}s} a^{\dagger}_{\mathbf{k}s} + \frac{1}{2} \sum_{\mathbf{k}} \left(\Delta_{s_1 s_2}(\mathbf{k}) a^{\dagger}_{\mathbf{k},s_1} a^{\dagger}_{-\mathbf{k},s_2} - \Delta^{*}_{s_1 s_2}(-\mathbf{k}) a_{-\mathbf{k},s_1} a_{\mathbf{k},s_2} \right) - \sum_{\mathbf{k}} a^{\dagger}_{\mathbf{k}} (\sigma H) a_{\mathbf{k}},$$

$\Delta(\mathbf{k})$ is an order parameter.
U(1) symmetry is broken - $< a_{\mathbf{k}\alpha} a_{\mathbf{k}\beta} > \neq 0$.
Singlet pairing ($S = 0$):

$$\Delta(\mathbf{k}) = i g(\mathbf{k}) \sigma_y,$$

Triplet pairing ($S = 1$):

$$\Delta(\mathbf{k}) = i(\mathbf{d}(\mathbf{k})\sigma)\sigma_y.$$

$\Delta(\mathbf{k})\Delta^{\dagger}(\mathbf{k}) = |\mathbf{d}|^2 \sigma_0 + \mathbf{q}_{\mathbf{k}} \sigma, \;\; \mathbf{q}_{\mathbf{k}} = i[\mathbf{d}, \mathbf{d}^*].$
$\mathbf{q}_{\mathbf{k}} = 0$ - unitary pairing.
Our goals: diagonalize H, compute some thermodynamical properties.

Main technical tasks

- Goal: diagonalize mean-field Hamiltonian for a triplet order parameter in the presence of Zeeman coupling [or spin-orbit coupling]

$$\hat{A}_{\mathbf{k}i} \equiv \begin{pmatrix} a_{\mathbf{k}\alpha} \\ a^{\dagger}_{-\mathbf{k}\alpha} \end{pmatrix} \equiv \begin{pmatrix} a_{\mathbf{k}\uparrow} \\ a_{\mathbf{k}\downarrow} \\ a^{\dagger}_{-\mathbf{k}\uparrow} \\ a^{\dagger}_{-\mathbf{k}\downarrow} \end{pmatrix}.$$

In these notations, Hamiltonian takes the form

$$\hat{H} = \frac{1}{2}\sum_{\mathbf{k}} A^{\dagger}_{\mathbf{k}i} H_{\mathrm{BdG},\mathbf{k}ij} A_{\mathbf{k}j}, \qquad H_{\mathrm{BdG},\mathbf{k}ij} = \begin{pmatrix} \xi_{\mathbf{k}}\delta_{\alpha\beta} - \boldsymbol{\sigma}_{\alpha\beta}\mathbf{H}_{\mathbf{k}} & \Delta_{\mathbf{k}\alpha\beta} \\ \Delta^{\dagger}_{\mathbf{k}\alpha\beta} & -\xi_{\mathbf{k}}\delta_{\alpha\beta} + \boldsymbol{\sigma}^{*}_{\alpha\beta}\mathbf{H}_{\mathbf{k}} \end{pmatrix}, \qquad \boldsymbol{\sigma}^{*} = (\sigma_x, -\sigma_y, \sigma_z).$$

In order to diagonalize the Hamiltonian, we introduce unitary transformation matrix $\hat{U}$:

$$\hat{U}_{\mathbf{k}} = \begin{pmatrix} u_{\mathbf{k}} & v_{\mathbf{k}} \\ v^{*}_{-\mathbf{k}} & u^{*}_{-\mathbf{k}} \end{pmatrix}, \qquad \hat{U}^{\dagger}\hat{U} = 1, \qquad \hat{U}^{\dagger} H_{\mathrm{BdG}} \hat{U} = \begin{pmatrix} \hat{E}_{\mathbf{k}} & 0 \\ 0 & -\hat{E}_{\mathbf{k}} \end{pmatrix}, \qquad \hat{E}_{\mathbf{k}} = \mathrm{diag}\{E_{\mathbf{k}+}, E_{\mathbf{k}-}\}$$

Main technical tasks: what has been accomplished

- Single-particle spectrum and Bogoliubov amplitudes for the case of singlet pairing

$$
\begin{cases}
E_\pm = \sqrt{\xi^2 + |g|^2} \pm |H|, \\
u_{\mathbf{k}} = X_1(-H\sigma_0 + \boldsymbol{\sigma}\mathbf{H})(\sigma_0 + \sigma_z) + X_2(H\sigma_0 + \boldsymbol{\sigma}\mathbf{H})(\sigma_0 - \sigma_z), \\
v_{\mathbf{k}} = -\frac{iX_1}{g^*}(H + \xi - E_+)(H\sigma_0 + \boldsymbol{\sigma}\mathbf{H})\sigma_y(\sigma_0 + \sigma_z) - \frac{X_2}{g*}(H + E_- - \xi)(H\sigma_0 - \boldsymbol{\sigma}\mathbf{H})\sigma_y(\sigma_0 - \sigma_z)
\end{cases}
$$

$$
X_1^2 = X_2^2 = \frac{1}{16(H - H_z)h}\left(1 + \frac{\xi}{\sqrt{\xi^2 + |g|^2}}\right)
$$

Main technical tasks: what has been accomplished

- Single-particle spectrum and Bogoliubov amplitudes for the case of triplet pairing

$$E^2_{\mathbf{k}\pm} = \xi^2 + |\mathbf{d}|^2 + H^2 \pm \sqrt{(\mathbf{q} - 2\xi\mathbf{H})^2 + 4(\mathbf{dH})(\mathbf{d}^*\mathbf{H})}.$$

$$u = X_+ \left\{\zeta_0\sigma_0 + (\boldsymbol{\zeta}, \boldsymbol{\sigma})\right\}(\sigma_0 + \sigma_z) + X_- \left\{\eta_0\sigma_0 + (\boldsymbol{\eta}, \boldsymbol{\sigma})\right\}(\sigma_0 - \sigma_z)$$

$$v_{\mathbf{k}} = X_+ \left(\beta_0^*\sigma_0 + \boldsymbol{\beta}^*\boldsymbol{\sigma}\right)\sigma_y(\sigma_0 + \sigma_z) + X_- \left(\gamma_0^*\sigma_0 + \boldsymbol{\gamma}^*\boldsymbol{\sigma}\right)\sigma_y(\sigma_0 - \sigma_z)$$

$$\begin{aligned}
\beta_0^* &= \frac{i}{d^{*2}}\left(\mathbf{d}^*, (E_+ - \xi)\boldsymbol{\zeta}^* + \zeta_0^*\mathbf{H} - i\left[\mathbf{H}\times\boldsymbol{\zeta}^*\right]\right)\\
\boldsymbol{\beta}^* &= -\frac{i}{d^{*2}}\left\{\left[(E_+ - \xi)\zeta_0^* + (\boldsymbol{\zeta}^*\mathbf{H})\right]\mathbf{d}^* - i\left[\mathbf{d}^*\times\left((E_+ - \xi)\boldsymbol{\zeta}^* + \zeta_0^*\mathbf{H} - i\left[\mathbf{H}\times\boldsymbol{\zeta}^*\right]\right)\right]\right\}\\
\gamma_0^* &= \frac{i}{d^{*2}}\left(\mathbf{d}^*, (E_- - \xi)\boldsymbol{\eta}^* + \eta_0^*\mathbf{H} - i\left[\mathbf{H}\times\boldsymbol{\eta}^*\right]\right)\\
\boldsymbol{\eta}^* &= -\frac{i}{d^{*2}}\left\{\left[(E_- - \xi)\eta_0^* + (\boldsymbol{\eta}^*\mathbf{H})\right]\mathbf{d}^* - i\left[\mathbf{d}^*\times\left((E_- - \xi)\boldsymbol{\eta}^* + \eta_0^*\mathbf{H} - i\left[\mathbf{H}\times\boldsymbol{\eta}^*\right]\right)\right]\right\}
\end{aligned}$$

$$\alpha = 2\frac{(\mathbf{dH})}{d^2}, \qquad d^2 = (\mathbf{d}, \mathbf{d}), \qquad a_0 = \xi^2 + |\mathbf{d}|^2 + H^2 - \alpha(\mathbf{dH}), \qquad \mathbf{a} = \mathbf{q} - 2\xi\mathbf{H} + \alpha\xi\mathbf{d} - i\alpha\left[\mathbf{d}\times\mathbf{H}\right]$$

Main technical tasks: what remains to be calculated

- Magnetization

$$\mathbf{M} = \mu_B \sum_{\mathbf{k}} \langle \hat{c}^\dagger_{\mathbf{k}\alpha} \boldsymbol{\sigma}_{\alpha\beta} \hat{c}_{\mathbf{k}\beta} \rangle$$

Question: Is there a contribution to the components of the magnetization which would remain finite even in zero magnetic field in the case of the nonunitary pairing?