# Assignment 2 - Particle in a Box - II


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Assignment: Particle in a Box - II

Roll No. ⇒ 2020PHY1140

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Subject: Quantum Mechanics (Lab)

Date ..../..../........

1. (a) Particle of mass m trapped in infinite potential well

$$V(x) = \begin{cases} 0 & \text{for } |x| < L/2 \\ \infty & \infty \text{ otherwise} \end{cases}$$

The solution of Schrodinger wave equation in 1-D box for given boundary condition is:

$$\psi(x) = \sqrt{\frac{2}{L}}\left[\sin\left(\frac{2n\pi}{L}\right)x + \cos\left(\frac{(2n+1)\pi x}{L}\right)\right]$$

For particle in ground state, $n=0$, $\psi(x)$ becomes

$$\psi(x) = \sqrt{\frac{2}{L}} \cos\left(\frac{(2n+1)\pi x}{L}\right)$$

Probability of finding electron in $\left[-\frac{L}{4}, \frac{L}{4}\right]$

$$P = \int_{-L/4}^{L/4} \psi\psi^* dx = \frac{2}{L}\int_{-L/4}^{L/4} \cos^2\left(\frac{(2n+1)\pi x}{L}\right) dx$$

$$P = \frac{2}{2L}\int_{-L/4}^{L/4} \left(\cos\left(\frac{2\times(2n+1)\pi x}{L}\right) + 1\right) dx$$

$$\cos 2x = 2\cos^2 x - 1$$
$$\cos^2 x = \frac{\cos 2x + 1}{2}$$

$$P = \frac{1}{L}\left[\frac{\sin\left(\frac{2\times(2n+1)\pi x}{L}\right)}{\frac{2(2n+1)\pi}{L}} + x\right]_{-L/4}^{L/4}$$

$$P = \frac{2}{L}\left[\frac{\cancel{2}\sin\left(\frac{\cancel{2}\times(2n+1)\pi \times \cancel{L}}{\cancel{L}\ \cancel{4}\ 2}\right)}{\frac{2(2n+1)\pi}{L}} + \frac{L}{4}\right]$$

$$\sin\left(\frac{(2n+1)\pi}{2}\right) = 1 \qquad \text{at } n=0$$

$$P = \frac{2}{L}\left[\frac{L}{2\pi} + \frac{L}{4}\right] = \frac{2\times \cancel{L}}{\cancel{L}}\left[\frac{1}{2\pi} + \frac{1}{4}\right] = 0.8184$$

$$\boxed{P = 0.8184}$$

(b) Position - momentum uncertainty ~~principle~~ relation

Ans:

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

here $\Delta x$ and $\Delta p$ are uncertainty in position and momentum respectively.

(c) Compute expectation values $\langle \hat{x} \rangle$, $\langle \hat{p} \rangle$, $\langle \hat{x}^2 \rangle$, $\langle \hat{p}^2 \rangle$, $\Delta x\,(\sigma_x)$ & $\Delta p\,(\sigma_p)$ for particle in $n^{th}$ stationary state of infinite potential well

$$V(x) = \begin{cases} 0 & |x| < L/2 \\ \infty & \text{otherwise} \end{cases}$$

Ans:

$$\psi_e(x) = \sqrt{\frac{2}{L}} \sin(kx) \qquad k = \frac{2n\pi}{L} \qquad n = 1, 2, ---$$

$$\psi_o(x) = \sqrt{\frac{2}{L}} \cos(kx) \qquad k = \frac{(2n-1)\pi}{L}$$

$$\langle x \rangle = \int_{-L/2}^{L/2} \psi \, x \, \psi^* \, dx$$

To make integration easy, writing $\psi_e$ and $\psi_o$ in below form

$$\psi_o = \sqrt{\frac{2}{L}} \cos \frac{n\pi x}{L} \qquad n = 1, 3, 5, \ldots$$

$$\psi_e = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \qquad n = 2, 4, \ldots$$

As $\psi$ is real, so $\psi = \psi^*$

To calculate $\langle \hat{x} \rangle$

For even state

$$\langle \hat{x} \rangle = \int_{-L/2}^{L/2} \psi x \psi^* \, dx = \int_{-L/2}^{L/2} \psi^2 x \, dx$$

$$\frac{2}{L} \int_{-L/2}^{L/2} \underbrace{x \sin^2 \frac{n\pi x}{L}}_{\text{odd}} \, dx = 0$$

For odd state

$$\langle \hat{x} \rangle = \int_{-L/2}^{L/2} \underbrace{x \times \cos^2 \frac{n\pi x}{L}}_{\text{odd}} \, dx = 0$$

$\therefore \langle x \rangle = 0 \quad$ for $n = 1, 2, 3, \ldots$

To find $\langle \hat{x}^2 \rangle$

For even state

$$\langle \hat{x}^2 \rangle = \frac{2}{L}\int_{-L/2}^{L/2} x^2 \sin^2 \frac{n\pi x}{L}\, dx$$

$\cos 2x = 1 - 2\sin^2 x$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$$= \frac{1}{L}\int_{-L/2}^{L/2} x^2 \left(1 - \cos 2 \times \frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{L}\left[\int_{-L/2}^{L/2} x^2 - \int_{-L/2}^{L/2} x^2 \cos \frac{2n\pi x}{L}\, dx\right]$$

$$\frac{1}{L}\left[\left[\frac{x^3}{3}\right]_{-L/2}^{L/2} - \left[\frac{\left(\frac{4n^2\pi^2 x^2}{L^2} - 2\right)\sin \frac{2n\pi x}{L} + \frac{2n\pi x}{L}\cos \frac{2n\pi x}{L}}{\frac{8n^3\pi^3}{L^3}}\right]_{-L/2}^{L/2}\right]$$

$$= \frac{1}{L}\left[\left[\frac{1}{3}\left[\frac{L^3}{8} + \frac{L^3}{8}\right]\right] - \frac{4n\pi L^3}{8n^3\pi^3}\right]$$

$$= \frac{1}{L}\left[\frac{L^3}{12} - \frac{L^3}{2n^2\pi^2}\right] = \left[\frac{L^2}{12} - \frac{L^2}{2n^2\pi^2}\right]$$

$$\langle \hat{x}^2 \rangle = \left[\frac{L^2}{12} - \frac{L^2}{2n^2\pi^2}\right] \quad \text{for } n = 2, 4, 6, \ldots$$

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For Odd State,

$$\langle \hat{x}^2 \rangle = \int_{-L/2}^{L/2} \psi^2 x^2 \, dx = \frac{2}{L}\int_{-L/2}^{L/2} x^2 \cos^2 \frac{2n\pi x}{L}$$

$$= \frac{\cancel{2}}{\cancel{2}L}\int_{-L/2}^{L/2} x^2 \left(1 + \cos\frac{2n\pi x}{L}\right) dx$$

$$= \frac{1}{L}\int_{-L/2}^{L/2} x^2 + \int_{-L/2}^{L/2} x^2 \cos\frac{2n\pi x}{L} \, dx$$

$$= \frac{1}{L}\left[\left[\frac{x^3}{3}\right]_{-L/2}^{L/2} + \left[\frac{\left(\frac{4n^2\pi^2x^2}{L^2} - 2\right)\sin\frac{2n\pi x}{L} + \frac{4n\pi x}{L}\cos\frac{2n\pi x}{L}}{\frac{8n^3\pi^3}{L^3}}\right]_{-L/2}^{L/2}\right]$$

$$= \frac{1}{L}\left[\frac{L^3}{12} - \frac{L^3}{2n^2\pi^2}\right]$$

$$\langle \hat{x}^2 \rangle = \frac{L^2}{12} - \frac{L^2}{2n^2\pi^2}$$

So $\langle \hat{x}^2 \rangle = \frac{L^2}{12} - \frac{L^2}{2n^2\pi^2} \quad \forall n = 1, 2, 3, \ldots$

Now, $\sigma x = \langle \langle \hat{x}^2 \rangle - \langle x \rangle^2 \rangle$

$$\sigma x = \frac{L^2}{12} - \frac{L^2}{2n^2\pi^2}$$

For momentum, $\hat{p} = -i\hbar \frac{\partial}{\partial x}$

$$\langle \hat{p} \rangle = \int_{-L/2}^{L/2} \psi^* \hat{p}\, \psi \, dx = \int_{-L/2}^{L/2} \psi^* \left( -i\hbar \frac{\partial \psi}{\partial x} \right) dx$$

As $\psi$ is real, so $\psi = \psi^*$

$$= \int_{-L/2}^{L/2} \psi \left( -i\hbar \frac{\partial \psi}{\partial x} \right) dx$$

For even State

$$\langle \hat{p} \rangle = \int_{-L/2}^{L/2} \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \times -i\hbar \frac{\partial}{\partial x} \left( \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \right) dx$$

$$= \frac{-i\hbar \times 2}{L} \int_{-L/2}^{L/2} \sin \frac{n\pi x}{L} \times \cos \frac{n\pi x}{L} \times \frac{n\pi}{L} \, dx$$

$$= -i\hbar \times \frac{2 n\pi}{L^2} \int_{-L/2}^{L/2} \underbrace{\sin \frac{n\pi x}{L} \cos \frac{n\pi x}{L}}_{\text{orthogonal}} \, dx = 0$$

For odd state

$$\langle \hat{p} \rangle = \int_{-L/2}^{L/2} \sqrt{\frac{2}{L}} \cos \frac{n\pi x}{L} \times -i\hbar \frac{\partial}{\partial x} \sqrt{\frac{2}{L}} \cos \frac{n\pi x}{L} \, dx$$

$$= \frac{2 \times + i\hbar}{L} \times \frac{n\pi}{L} \int_{-L/2}^{L/2} \underbrace{\cos \frac{n\pi x}{L} \sin \frac{n\pi x}{L}}_{\text{orthogonal}} \, dx = 0$$

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So, $\langle \hat{p} \rangle = 0 \quad \forall n = 1, 2, 3, ----$

$$\langle \hat{p}^2 \rangle = \int_{-L/2}^{L/2} \psi \times (-i\hbar)^2 \frac{\partial^2}{\partial x^2} \psi \, dx = \left[ \int_{-L/2}^{L/2} \psi \frac{\partial^2 \psi}{\partial x^2} dx \right] - \hbar^2$$

For odd state

$$\langle \hat{p}^2 \rangle = -\hbar^2 \int_{-L/2}^{L/2} \sqrt{\frac{2}{L}} \cos\frac{n\pi x}{L} \times \sqrt{\frac{2}{L}} \times \frac{n^2\pi^2}{L^2} \left( -\cos\frac{n\pi x}{L} \right) dx$$

$$= \hbar^2 \times \frac{2}{L} \times \frac{n^2\pi^2}{L^2} \int_{-L/2}^{L/2} \cos^2\frac{n\pi x}{L} \, dx$$

$$= \frac{2\hbar^2 n^2\pi^2}{2L^3} \int_{-L/2}^{L/2} \left( 1 + \cancel{\sin} \cos\frac{2n\pi x}{L} \right) dx$$

$$= \frac{\cancel{2}\hbar^2 n^2\pi^2}{\cancel{2}L^3} \left[ x + \frac{\sin\frac{2n\pi x}{L}}{\frac{2n\pi}{L}} \right]_{-L/2}^{L/2}$$

$$= \frac{\hbar^2 n^2\pi^2}{L^3} \times L$$

$$\langle \hat{p}^2 \rangle = \frac{\hbar^2 n^2\pi^2}{L^2}$$

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For Even State

$$\langle \hat{p}^2 \rangle = \int_{-L/2}^{L/2} \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L} \times -\hbar^2 \times \frac{n^2\pi^2}{L^2}\left(-\sin\frac{n\pi x}{L}\right) dx$$

$$= \frac{\hbar^2 \times n^2\pi^2}{L^2} \times \frac{2}{L} \int_{-L/2}^{L/2} \sin^2 \frac{n\pi x}{L}\, dx$$

$\cos 2x = 1 - 2\sin^2 x$

$\frac{1-\cos 2x}{2}$

$$= \frac{\cancel{2}\hbar^2 n^2\pi^2}{\cancel{2}L^3} \int_{-L/2}^{L/2} \left(1 - \cos 2 \times \frac{n\pi x}{L}\right) dx$$

$$\frac{\hbar^2 n^2\pi^2}{L^3} \left[x - \frac{\sin\frac{2n\pi x}{L}}{\frac{2n\pi}{L}}\right]_{-L/2}^{L/2}$$

$$= \frac{\hbar^2 n^2\pi^2}{L^{\cancel{3}}} \times \cancel{L} = \frac{\hbar^2 n^2\pi^2}{L^2}$$

$$\langle \hat{p}^2 \rangle = \frac{\hbar^2 n^2\pi^2}{L^2}$$

So, $\langle \hat{p}^2 \rangle = \frac{\hbar^2 n^2\pi^2}{L^2}$ for $n = 1, 2, 3, ——$

$$\sigma_p = \langle\langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2 \rangle$$

$$\boxed{\sigma_p = \frac{\hbar^2 n^2\pi^2}{L^2}}$$

(iii) Uncertainty Principle in Dimensionless form:

Uncertainty principle relation is

$$\Delta P_x \, \Delta x \geq \frac{\hbar}{2} \quad -(1)$$

Let's take $\Delta P_x'$ and $\Delta x'$ are in dimensionless form.
We can write $\Delta P_x'$ and $\Delta x'$ as

$$\Delta P_x' = \frac{\Delta P_x}{P} \quad -(2) \qquad \Delta x' = \frac{\Delta x}{L} \quad -(3)$$

here P and L have dimensions of momentum and length respectively

eq^n (1) becomes

$$(\Delta P_x' \times P)(\Delta x' \times L) \geq \frac{\hbar}{2}$$

$$\Delta P_x' \, \Delta x' \geq \frac{\hbar}{PL}$$

here PL has dimensions of $\hbar$.

This is uncertainty product in dimensionless form.

## Programming

```python
import numpy as np
import matplotlib.pyplot as plt
import scipy.integrate as integrate
import pandas as pd
from scipy import optimize,stats
from IVP import RK_fourth_vec
def func_(x,x_vec,e):
    ans_vec = np.zeros((2))
    ans_vec[0] = x_vec[1]
    ans_vec[1] = (-e)*x_vec[0]
    return ans_vec
def analytical(x,n):
        if (i%2)== 0:
            return 2*(np.cos(n*np.pi*x))**2
        else:
            return 2*(np.sin(n*np.pi*x))**2
def analytical_1(x,n):
        return np.sin(n*np.pi*x)
E = np.linspace(0,400,100)
x  = np.linspace(-1/2,1/2,100)
initcond=[0,1]

t = []
for i in range(len(E)):
    e = E[i]
    y1 = RK_fourth_vec(x, initcond, func_,e).T[0]
    y=y1/np.sqrt(integrate.simps(np.power(y1,2),x))
    t.append(y[-1])
# plt.scatter(E,t)
# plt.xlabel("e")
# plt.ylabel("$u_{R}$")
# plt.title("u(\u03BE=1/2)")
# plt.grid(True)
# plt.show()

def f(e,x):
    y1 = RK_fourth_vec(x, initcond, func_,e).T[0]
    y2 = RK_fourth_vec(x, initcond, func_,e).T[1]
    y=y1/np.sqrt(integrate.simps(np.power(y1,2),x))
    y_=y2/np.sqrt(integrate.simps(np.power(y2,2),x))
    t.append(y[-1])
    return t[-1], y,x,y_

en_neg = [];en_pos=[];i_neg=[];i_pos=[]
def eigenvalues(t):
    for i in range(1,len(t)):
        if t[i-1]*t[i]<0:
            en_neg.append(t[i-1])
            i_neg.append(E[i-1])
            en_pos.append(t[i])
            i_pos.append(E[i])
    return en_neg,en_pos,i_pos,i_neg

phi_1,phi_2,s_1,s_0 = eigenvalues(t)
data = {
    "u_1":phi_1,
    "E1":s_0,
    "u_2":phi_2,
    "E_2":s_1
}
#print(pd.DataFrame(data))
def secant(s_1,s_0,iterations,x):
    sec_2=[]
    sec_2.append(s_0); sec_2.append(s_1)
    for i in range(1,iterations):
```

```
        sec_2.append(sec_2[i]-(((sec_2[i]-sec_2[i-1])*(f(sec_2[i],x)[0]))/((f(sec_2[i],x)[0])-(f(sec_2[i-1],x)[0]))))
        if abs(f(sec_2[-1],x)[0])<0.1e-12:
            return sec_2[-1], f(sec_2[-1],x)[0],f(sec_2[-1],x)[1],f(sec_2[-1],x)[2], f(sec_2[-1],x)[2]

E_n=[];u=[];v=[]

for i in range(0,len(s_0)):
    E_n_=secant(s_1[i],s_0[i],501,x)[0]
    u_ = secant(s_1[i],s_0[i],501,x)[1]
    ufull_ = secant(s_1[i],s_0[i],501,x)[2]
    ufull_prime = secant(s_1[i],s_0[i],501,x)[4]
    probability_density = secant(s_1[i],s_0[i],200,x)[3]
    x_ = secant(s_1[i],s_0[i],501,x)[3]
    # plt.scatter(x_,ufull_,label=f'n={i+1}')
    # # plt.plot(x_,analytical_1(x_,i+1),label=f'n={i+1}')
    # plt.xlabel("\u03BE")
    # plt.ylabel("u(\u03BE)")
    # plt.title("Normalised wave function for infinite square well")
    # plt.grid()
    # plt.legend()
    # plt.show()

    # plt.scatter(x_,ufull_*ufull_,label=f'n={i+1}')
    # plt.plot(x_,analytical(x_,i+1),label=f'n={i+1}(Analytical)')
    # plt.xlabel("\u03BE")
    # plt.ylabel("$(u(\u03BE))^2$")
    # plt.title("Probability Densities")
    # plt.grid()
    # plt.legend()
    # plt.show()

    E_n.append(E_n_)
    u.append(u_)
    v.append(secant(s_1[i],s_0[i],200,x)[2])

print(E_n);print(u)

data = {
    "N":[1,2,3,4,5,6],
    "Final Energy Eigen Value":E_n,
    "Corresponding u": u
}
print(pd.DataFrame(data))

slope, intercept, r, p, se = stats.linregress(np.array(E_n)/(np.pi)**2, E_n )
print("slope",slope)
# plt.scatter(np.array(E_n)/(np.pi)**2,E_n,label="Approximated")
# plt.plot(abs(np.array(E_n)/(np.pi)**2),(np.array(E_n)/(np.pi)**2)*(np.pi)**2,
    label="Analytical")
# plt.xlabel("$n^2$")
# plt.ylabel("$E_{n}$")
# plt.grid()
# plt.legend()
# plt.show()

def en_ev(E_n,h_cut,m_e,L):
    En_anal=[];prob_En_ev=[]
    for i in range(0,len(E_n)):
        prob_En_ev_ = ((h_cut**2)*np.array(E_n)[i])/(2*m_e*(L**2)*(1.609e-19))
        prob_En_ev.append(prob_En_ev_)
        E = (((i+1)**2)*((np.pi)**2)*(h_cut**2))/(2*m_e*(L**2)*(1.609e-19))
        En_anal.append(E)
    return prob_En_ev, En_anal
m_e = 9.1e-31; h_cut = 1.0545e-34;m_p=1.6e-27
print("Well of width = 5 Angstrom")
```

```
data = {
    "Approximated Eigen Values":en_ev(E_n, h_cut,m_e,5e-10)[0],
    "Analytical Eigen Values":en_ev(E_n, h_cut,m_e,5e-10)[1],
}
print(pd.DataFrame(data))
print("---------------------------------------------------------------------------------------------")
print("Well of width = 10 Angstrom")
data = {
    "Approximated Eigen Values":en_ev(E_n, h_cut,m_e,10e-10)[0],
    "Analytical Eigen Values":en_ev(E_n, h_cut,m_e,10e-10)[1],
}
print(pd.DataFrame(data))

print("---------------------------------------------------------------------------------------------")
print("Well of width = 5 Fermimeter for proton")
data = {
    "Approximated Eigen Values":en_ev(E_n, h_cut,m_p,5e-15)[0],
    "Analytical Eigen Values":en_ev(E_n, h_cut,m_p,5e-15)[1],
}
print(pd.DataFrame(data))

print("---------------------------------------------------------------------------------------------")
'''Uncertainty Principle'''
exp_x_2=np.power(ufull_,2)*(np.power(x_,2))
i1=integrate.simps(exp_x_2,x_)
exp=np.power(ufull_,2)*(x_)
i2=integrate.simps(exp,x_)
variance = i1-i2**2
st_dev = np.sqrt(variance)
print("Uncertainty in x is ", np.sqrt(variance))
exp_x_p1 = np.power(ufull_prime,2)*x_
ip_1=integrate.simps(exp_x_p1,x_)
exp_x_p2=np.power(ufull_prime,2)*np.power(x_,2)
ip_2=integrate.simps(exp_x_p2,x_)
variance1=ip_2-ip_1**2
print("uncertainty in momentum p is", np.sqrt(variance1))
print("sigma_x*sigma_p = ",np.sqrt(variance)*np.sqrt(variance1))
print("h_cut/2*pi=",h_cut/(2))
print("sigma_x*sigma_p >= h_cut/2*pi, Hence Uncertainty Principle Verified ")
```

## Result and Discussion

Here, We figure out the values of energy(e) and corresponding index $\xi$ where u($\xi$)) approaches zero.

```
PS C:\Users\adn19> & C:/Users/adn19/anaconda3/python.exe "d:/Sem 5/Quantum Mechanics/Lab/Assignments/Assignment 3/1140_Preetpal_qmLab-A3.py"
        u_1          E1       u_2         E_2
0  0.397263    8.080808 -0.494418   12.121212
1 -0.347030   36.363636  0.104070   40.404040
2  0.296276   84.848485 -0.004676   88.888889
3 -0.019051  157.575758  0.207527  161.616162
4  0.012515  246.464646 -0.168943  250.505051
5 -0.142226  351.515152  0.009081  355.555556
```

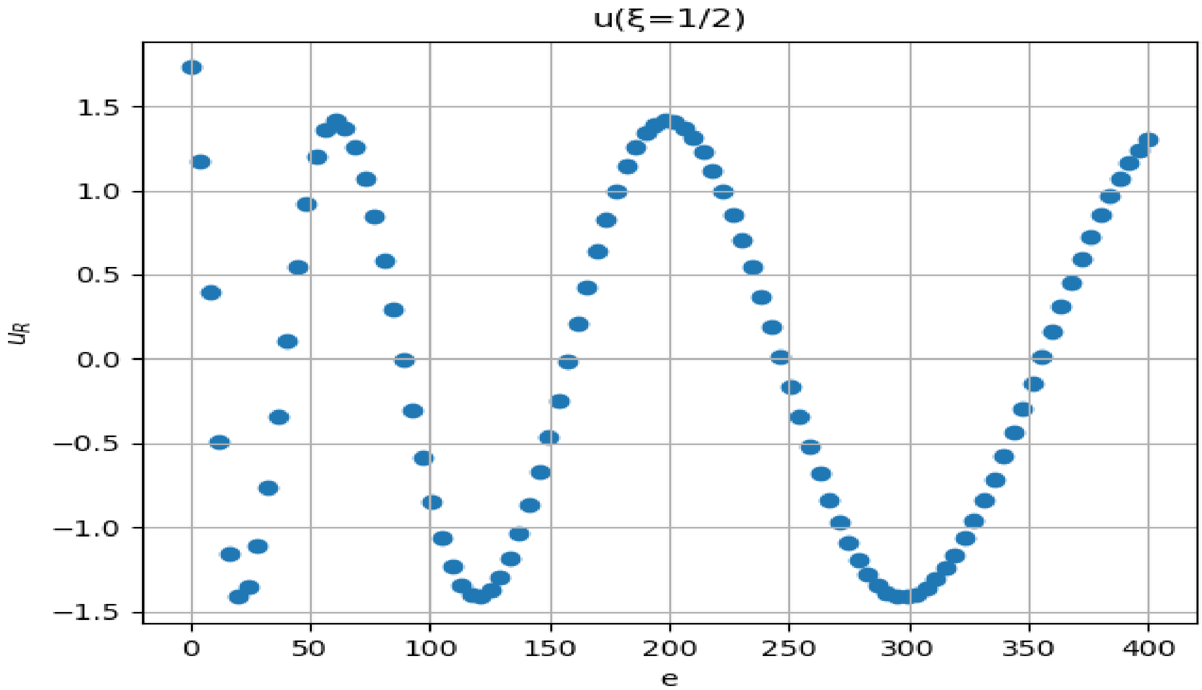
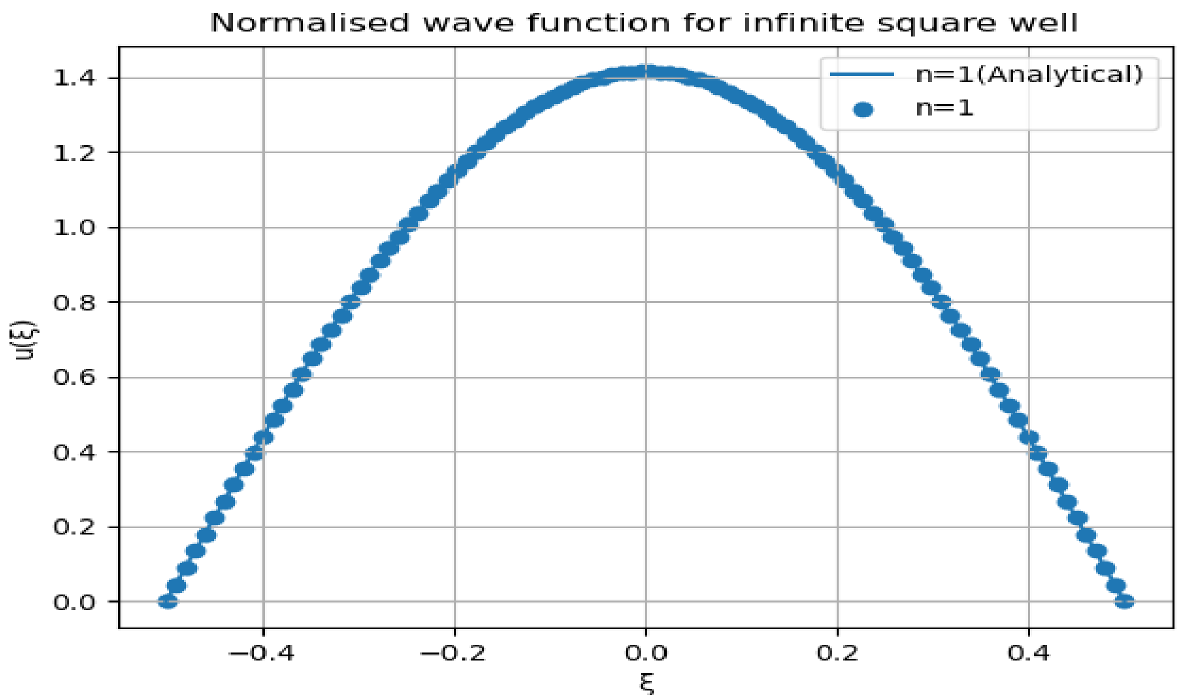
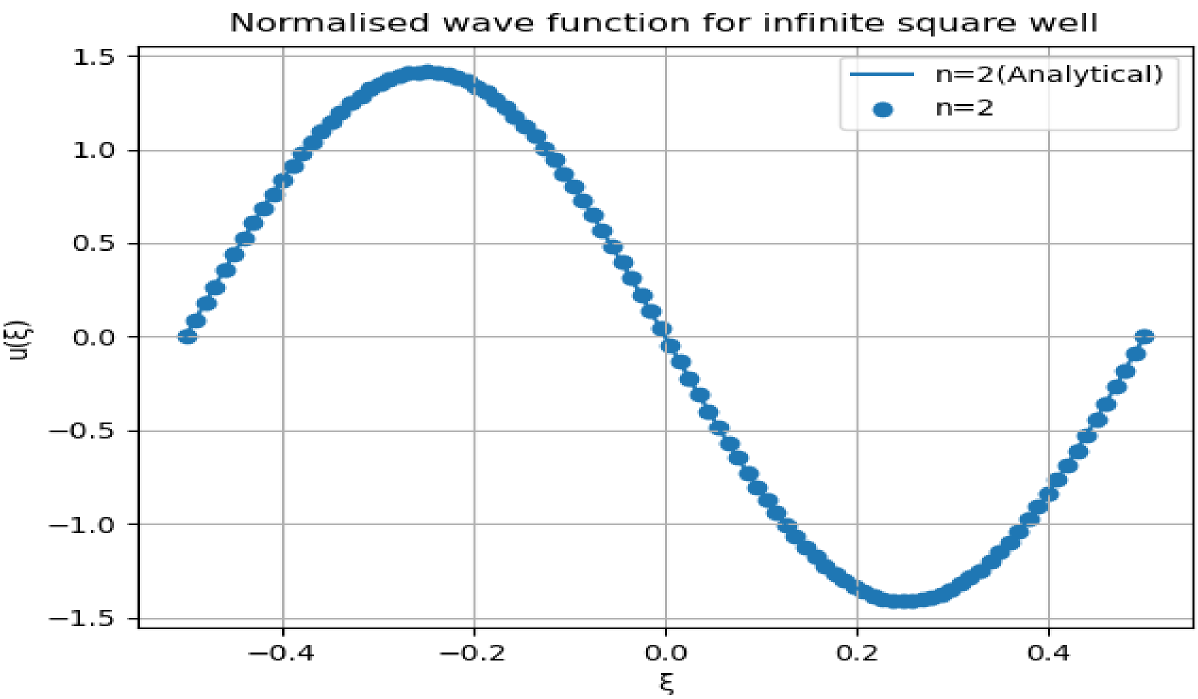


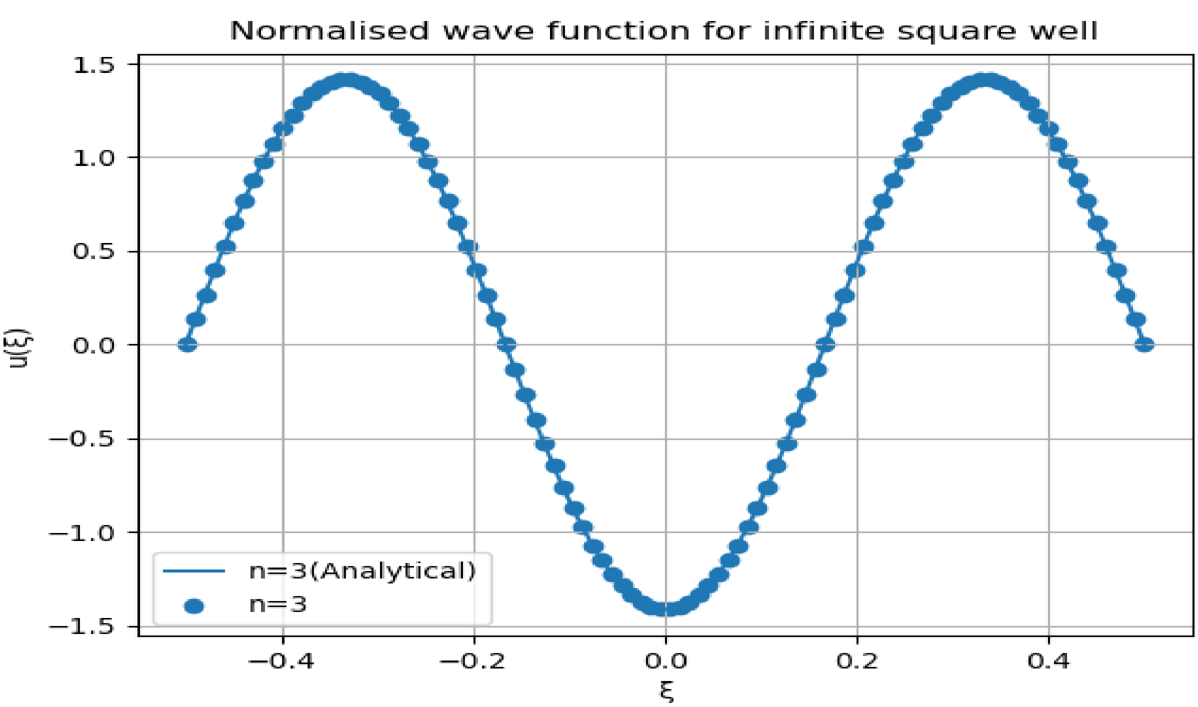

Figure 1: Plot between $u_R$ and e to check where $u_R$ approaches zero.

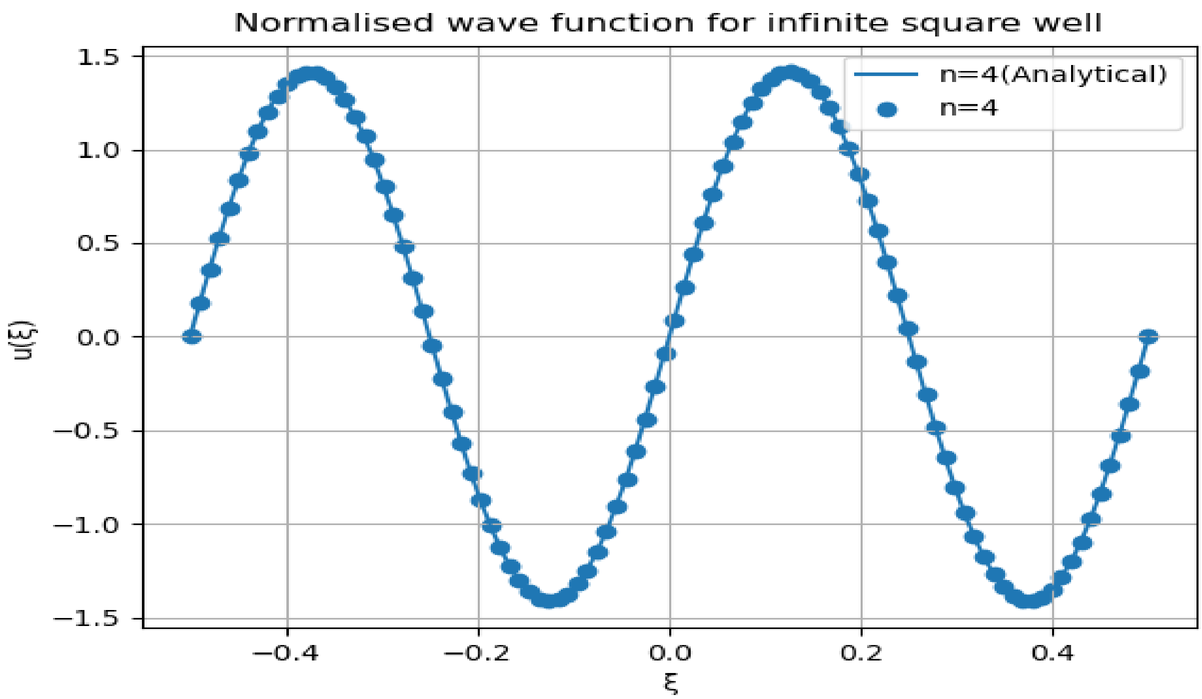
## Normalised wavefunction for Infinite potential Well

First, we predicted another values that closely approaches zero using shooting method to find energy eigen values. with this method, we find first 6 energy eigen values(Values of e where u($\xi$) corresponds to zero.Then using these eigen values, we plotted $u(\xi)$ vs $\xi$ for different energy levels.

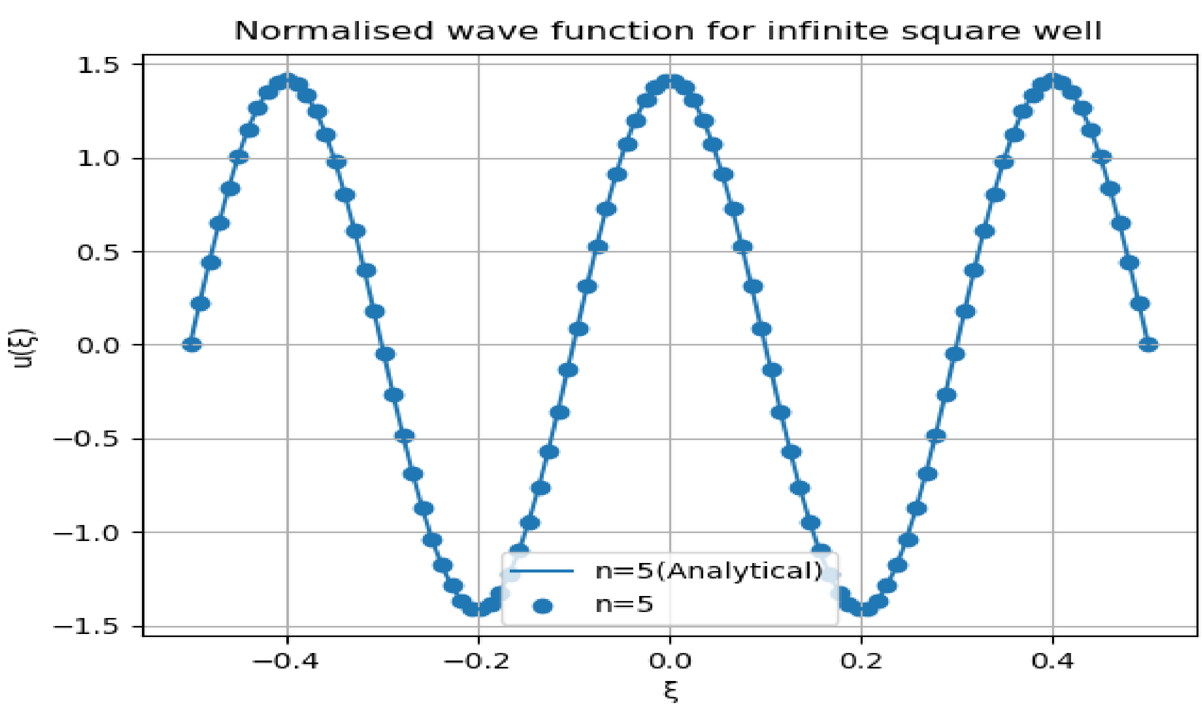



Normalised wave function for infinite square well
n=2(Analytical)
n=2
u(ξ)
ξ
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−0.4
−0.2
0.0
0.2
0.4

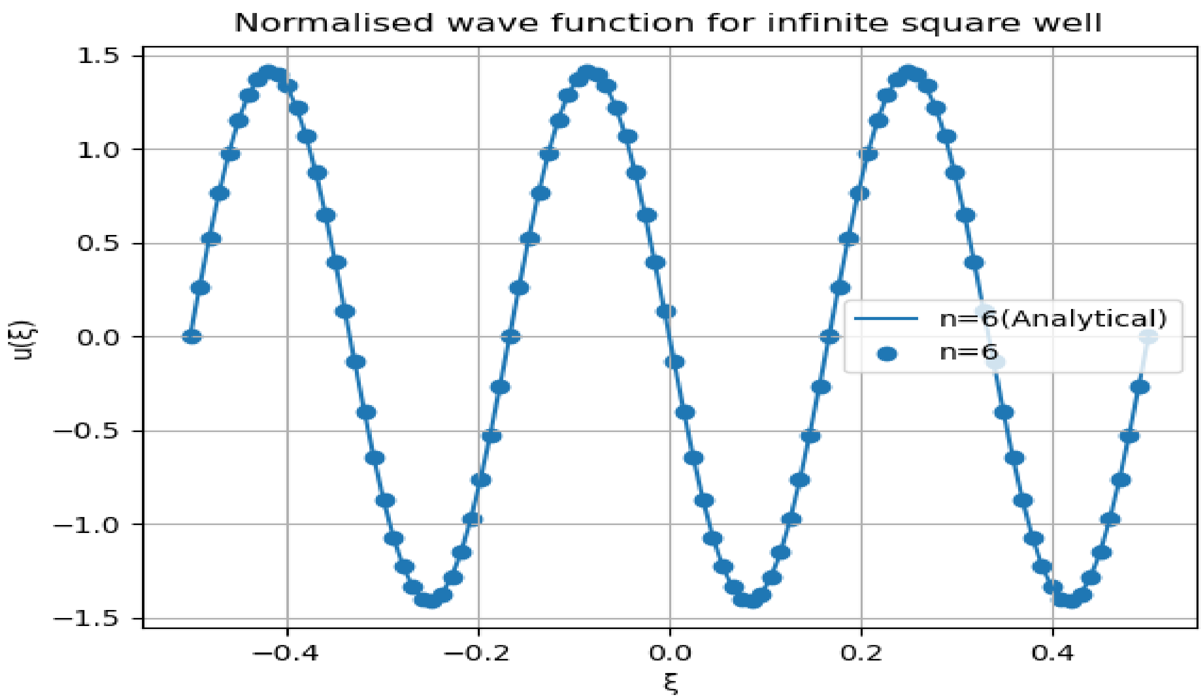

Normalised wave function for infinite square well
n=3(Analytical)
n=3
u(ξ)
ξ
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−0.4
−0.2
0.0
0.2
0.4



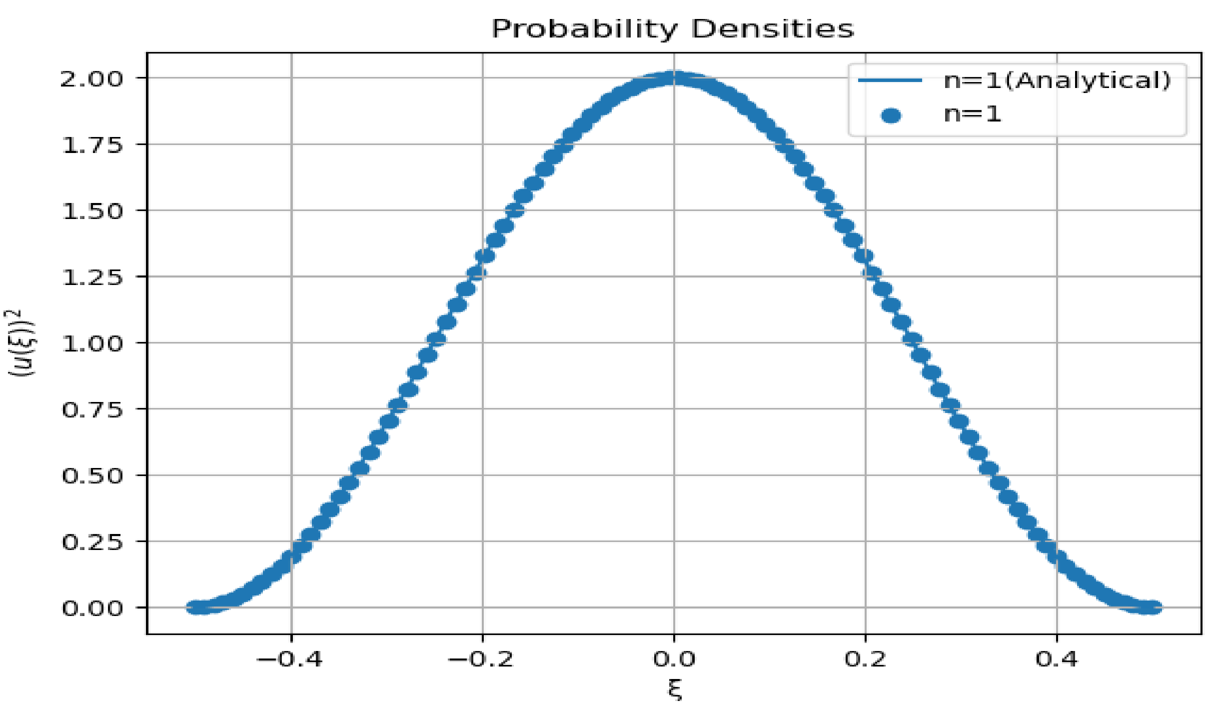
Normalised wave function for infinite square well
n=4(Analytical)
n=4
u(ξ)
ξ

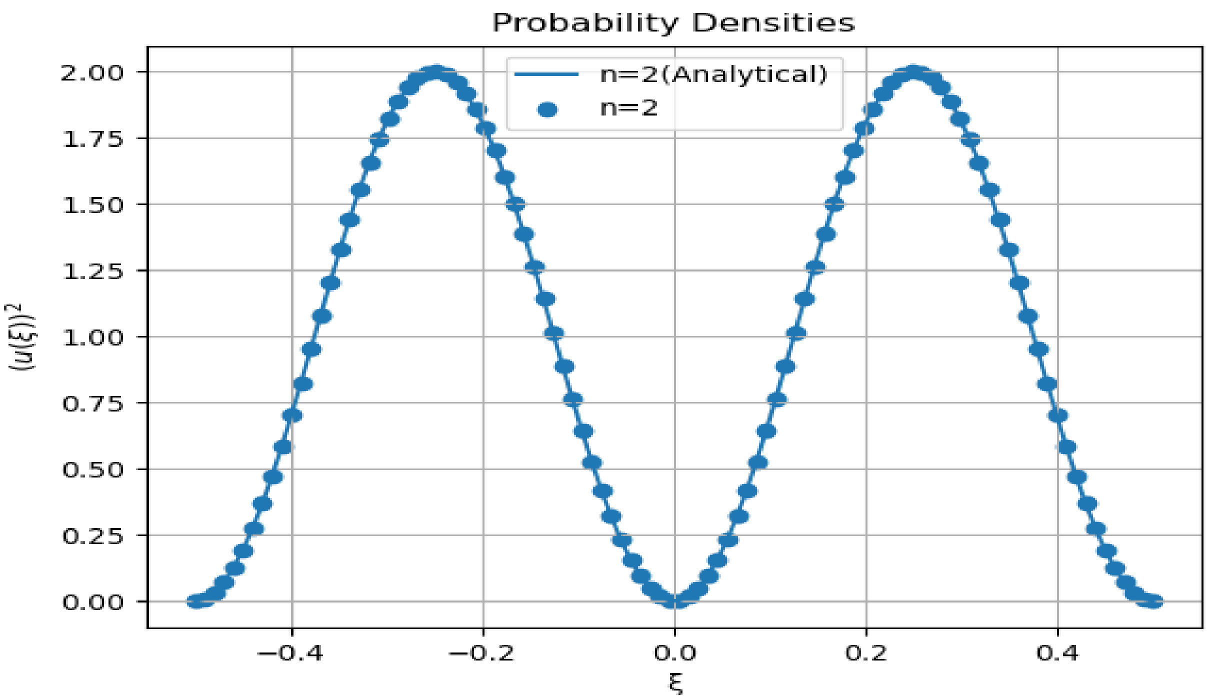



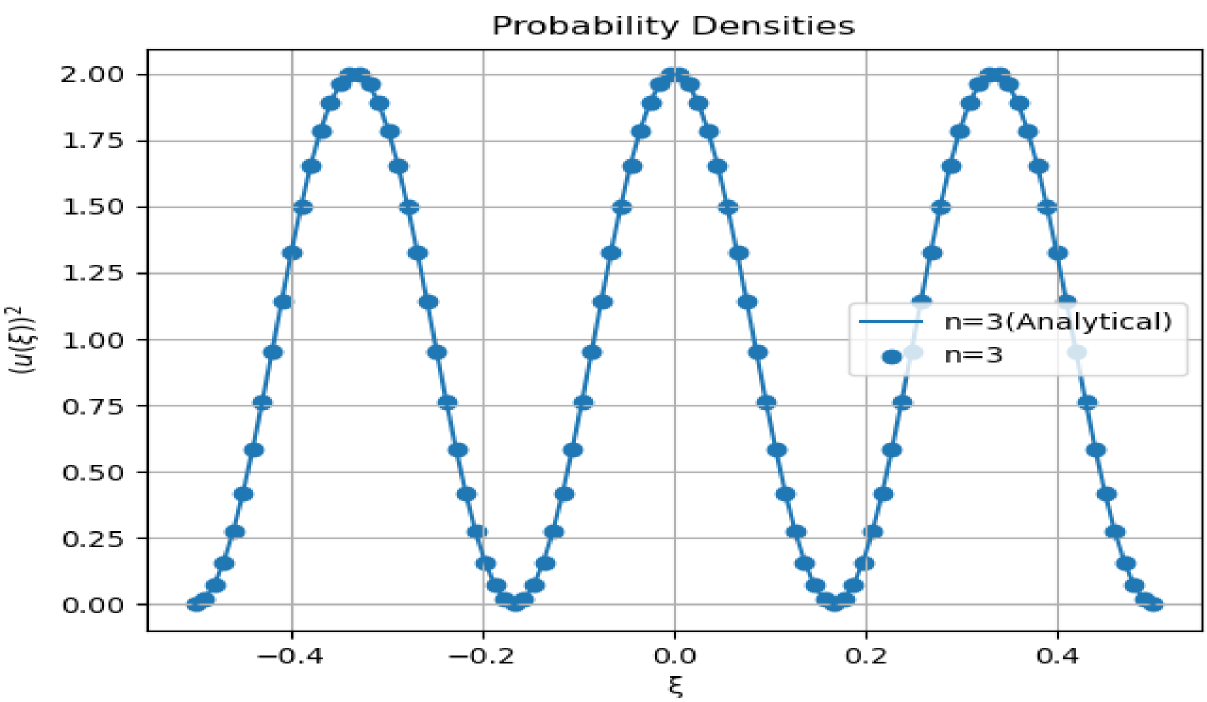
Normalised wave function for infinite square well
n=5(Analytical)
n=5
u(ξ)
ξ

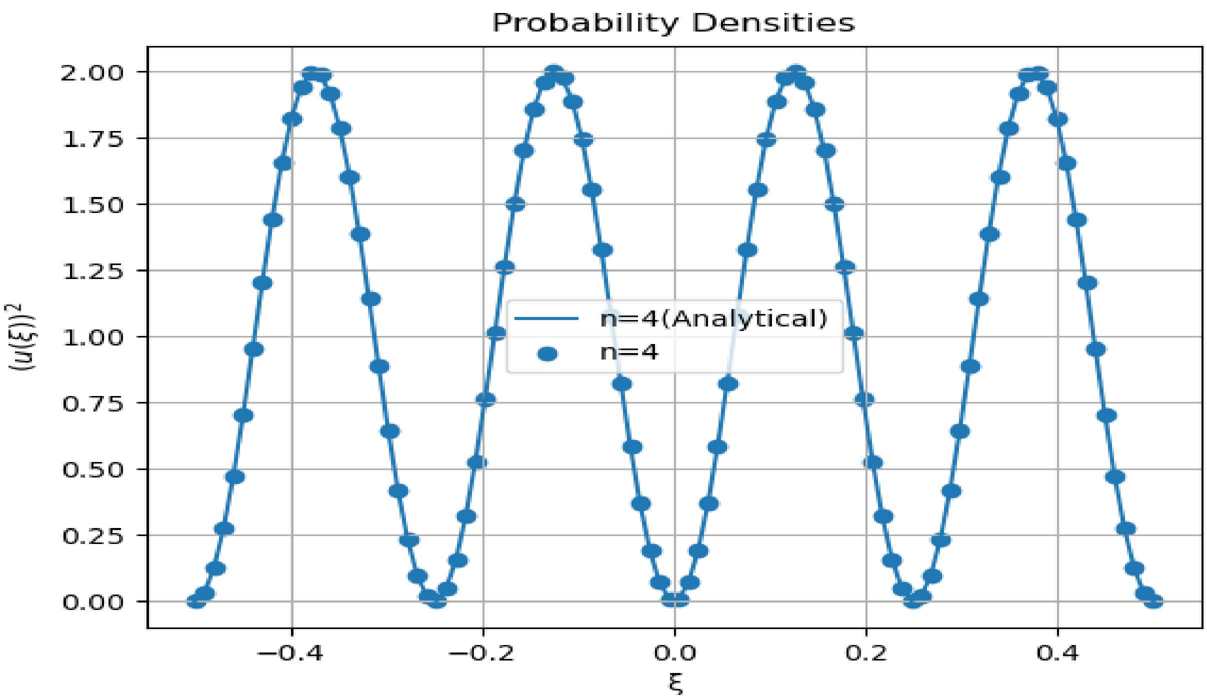


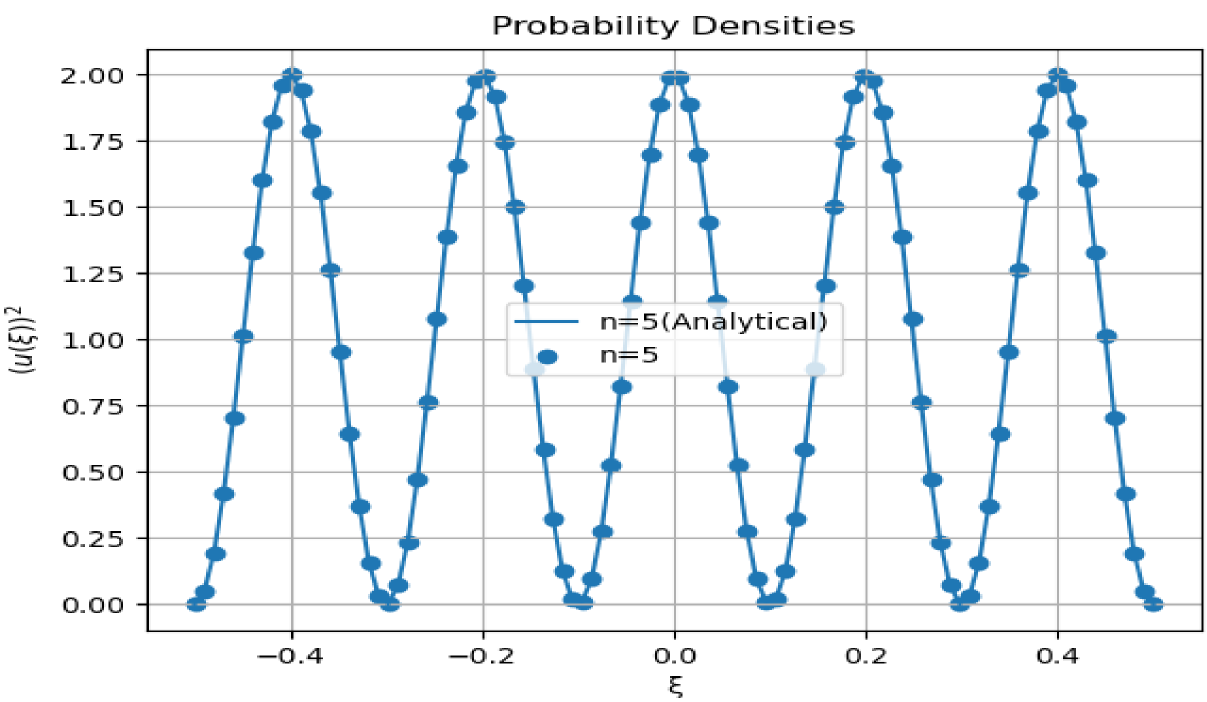

**Normalised Probability Density for infinite square well**

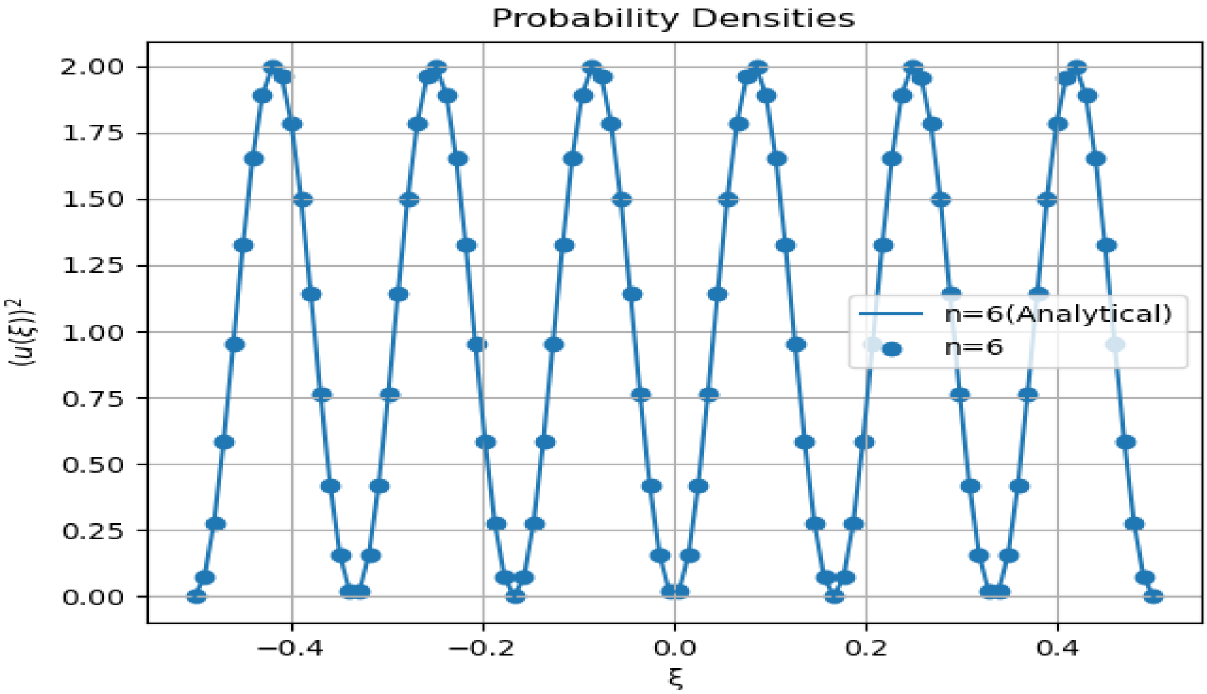



Probability Densities
n=2(Analytical)
n=2
$(u(\xi))^2$
ξ


Probability Densities
n=3(Analytical)
n=3
$(u(\xi))^2$
ξ


Probability Densities
n=4(Analytical)
n=4
$(u(\xi))^2$
$\xi$
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
−0.4
−0.2
0.0
0.2
0.4


Probability Densities
n=5(Analytical)
n=5
$(u(\xi))^2$
$\xi$
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
−0.4
−0.2
0.0
0.2
0.4



Probability Densities
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
$(u(\xi))^2$
−0.4
−0.2
0.0
0.2
0.4
ξ
n=6(Analytical)
n=6

### $E_n$ vs $n^2$ Graph

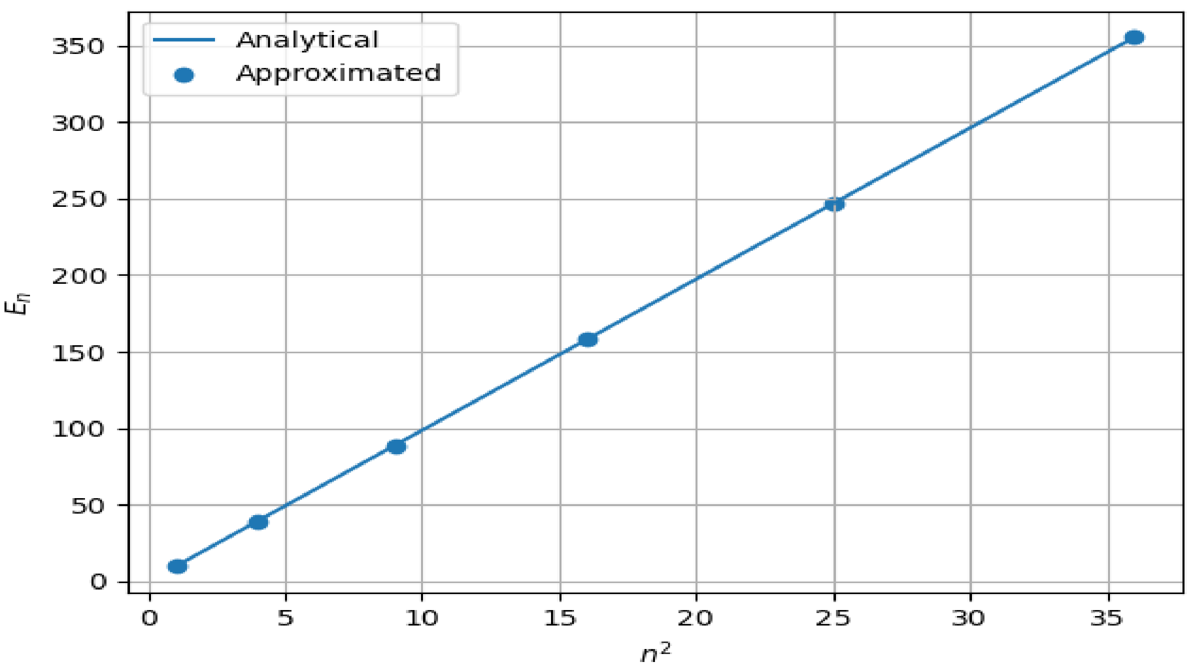



Figure 2: Here, Analytical solution and Numerical solutions matched. The Graph of $E_n$ vs $n^2$ is linear in nature because $E_n$=$-n^2$ $\pi^2$.

```
   N  Final Energy Eigen Value  Corresponding u
0  1                 9.869605     6.165728e-17
1  2                39.478428    -3.015026e-14
2  3                88.826561     6.011509e-16
3  4               157.914350    -7.398713e-16
4  5               246.742693    -2.312080e-16
5  6               355.313441     1.387245e-16
slope 9.86960440108936
Well of width = 5 Angstrom
   Approximated Eigen Values  Analytical Eigen Values
0                 1.499082                 1.499082
1                 5.996331                 5.996329
2                13.491759                13.491741
3                23.985420                23.985317
4                37.477450                37.477057
5                53.968129                53.966962
----------------------------------------------------------------------------------------------
Well of width = 10 Angstrom
   Approximated Eigen Values  Analytical Eigen Values
0                 0.374771                 0.374771
1                 1.499083                 1.499082
2                 3.372940                 3.372935
3                 5.996355                 5.996329
4                 9.369362                 9.369264
5                13.492032                13.491741
----------------------------------------------------------------------------------------------
Well of width = 5 Fermimeter for proton
   Approximated Eigen Values  Analytical Eigen Values
0             8.526031e+06             8.526031e+06
1             3.410413e+07             3.410412e+07
2             7.673438e+07             7.673427e+07
3             1.364171e+08             1.364165e+08
4             2.131530e+08             2.131508e+08
5             3.069437e+08             3.069371e+08
----------------------------------------------------------------------------------------------
Uncertainty in x is  0.28626042521917944
uncertainty in momentum p is 0.11180453413430508
sigma_x*sigma_p =  0.03200521348271843
h_cut/2*pi= 5.2725e-35
sigma_x*sigma_p >= h_cut/2*pi, Hence Uncertainty Principle Verified
```