Define the following with their proper expression

(i) Depth of Penetration :-

Pentration depth is a measure of how deep light or any electromagnetic radiation can pentrate into a material. It is defined as the depth at which the intensity of the radiation inside the material falls to 1/e (about 37%) of its original value at the surface.

(ii) Phase and Group Velocity :-

Wave can be in the group and such groups are called wave packets, so the velocity with a wave packets travels is called group velocity. The velocity with which the phase of a wave travels is called phase velocity.

(iii) Energy Density of EM wave :-

Energy density is donated by "U"

$$U = \frac{1}{2} \varepsilon_0 E^2$$

Energy density in case of magnetic field

$$U = \frac{1}{2\mu_0} B^2$$

$$\text{Total energy} = U = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

(iv) Surface current and impedance :-

$$E(z) = E_0 e^{-\alpha z} e^{-j\beta z}$$

$$J(z) = \sigma E = \sigma E_0 e^{-\alpha z} e^{-j\beta z}$$

$$I(z) = J(z) dz = \sigma E_0 e^{-\gamma z} dz \hat{x}$$

since for a good conductor the current is confined to a very thin region below the surface we may treat the current J_s as the current surface current.

$$J_s = \frac{E_0 \sigma}{\gamma} x$$

for non-ideal conductor parameter called surface impedance.

The real part of $\sqrt{\frac{\omega\mu_0}{2\sigma}}$ is called the surface resistance.

Q.2 Solve the wave equation from a uniform plane wave in an isotropic homogenous lossy dielectric medium with no sources. Calculate the propagation constant, attenuation constant and phase constant?

The ratio of conduction current density of displacement current density is $\left[\frac{\sigma}{\omega\varepsilon}\right]$. Hence $\left[\frac{\sigma}{\omega\varepsilon}=1\right]$ can be considered to mark the dividing line between conductor and dielectric. for good conductor material $\left[\frac{\sigma}{\omega\varepsilon}\right]$ is much greater than unity. for good dielectric $\left[\frac{\sigma}{\omega\varepsilon}\right]$ is very much less than unity.

for good dielectric

$$\frac{\sigma}{\omega\varepsilon} << 1$$

$\sigma << \omega\varepsilon$

The intrinsic or characteristic impedance

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\mu\left[1+\frac{\sigma}{j\omega\varepsilon}\right]}} = \sqrt{\frac{j\omega\mu(\sigma+j\omega\varepsilon)}{\sigma+j\omega\varepsilon}}$$

$\sigma << \omega\varepsilon$, σ may be neglected

$$\eta = \frac{\sqrt{j\omega\mu\, j\omega\varepsilon}}{j\omega\varepsilon}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

for free space $\mu=\mu_0$ and $\varepsilon=\varepsilon_0$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\eta_0 = 377\,\Omega \text{ or } 120\pi\,\Omega$$

pogation constant γ is

$$\gamma = \sqrt{j\mu\omega(\sigma + j\omega\varepsilon)}$$

$$\gamma = \sqrt{j\mu\omega \times j\omega\varepsilon(1+\sigma/j\omega\varepsilon)}$$

$$\gamma = j\omega\sqrt{\mu\varepsilon}\,(1+\sigma/j\omega\varepsilon)^{1/2} \quad -(1)$$

Apply Binomial theorem

$$\gamma = j\omega\sqrt{\mu\varepsilon}\,(1+\sigma/2j\omega\varepsilon)$$

$$= j\omega\sqrt{\mu\varepsilon} + \frac{\sigma\, j\omega\sqrt{\mu\varepsilon}}{2j\omega\varepsilon}$$

$$= \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} + j\omega\sqrt{\mu\varepsilon} \quad — (2)$$

$$\gamma = \alpha + j\beta \quad -(3)$$

From eq^n (2) & (3)

Attenuation constant $= \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$

Phase delay constant $= \beta = \omega\sqrt{\mu\varepsilon}$

Q.3 What is uniform plane wave? Show that the field in the uniform plane wave is independent of two dimensions?

If the phase of a wave is the same for all points on a plane surface it is called plane wave. if the amplitude is also constant in a plane wave.

Let the uniform plane wave be propagating in Z-direction for this wave x and y component will be present but Z-component will be absent i.e $E_Z = H_Z = 0$.

$$\frac{\partial \vec{E}}{\partial x} = \frac{\partial \vec{E}}{\partial y} = \frac{\partial \vec{H}}{\partial x} = \frac{\partial \vec{H}}{\partial y} = 0$$

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad -① $$

$$\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} \quad -② $$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$E = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\left[E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z\right]$$

$$= \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}\left[E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z\right]$$

$\frac{\partial}{\partial x} = \frac{\partial}{\partial y}$ and $E_z = 0$

$$\frac{\partial^2 E_x}{\partial z^2} \hat{a}_x + \frac{\partial^2 E_y}{\partial z^2} \hat{a}_y = \mu_0 \varepsilon_0 \left[\frac{\partial^2 E_x}{\partial t^2} \hat{a}_x + \frac{\partial^2 E_y}{\partial t^2} \hat{a}_y\right]$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} \quad -③ $$

$$\frac{\partial^2 E_y}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad -④ $$

$$\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$$

similarly eqn ②

$$\frac{\partial^2 H_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 H_x}{\partial t^2} \quad -⑤ $$

$$\frac{\partial^2 H_y}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 H_y}{\partial t^2} \quad -⑥ $$

$$E = f_1(z - ct) + f_2(z + ct) \quad -⑦ $$

$f_1(z-ct)$ represent a wave travelling in +ve z direction $f_2(z+ct)$ in (-) z direction

If there is no reflecting surface present then the wave travelling in (-) z direction will be zero.

$$E = f_1(z-ct)$$

Q.4 An EM wave moving from one medium to nother medium of perfect dielectric and incidence obliquely at the surface, calculate the reflection coefficient for vertical polarization and Brewster angle.

Applying the boundary condition

$$(E_i - E_r)\cos\theta_i = E_t \cos\theta_t$$

Divide by E_i on both side

$$1 - \frac{E_r}{E_i} = \frac{E_t \cos\theta_t}{E_i \cos\theta_i}$$

$$\frac{E_t}{E_i} = \left[1 - \frac{E_r}{E_i}\right]\frac{\cos\theta_i}{\cos\theta_t} \quad -①$$

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\varepsilon_2}\, E_t^2 \cos\theta_t}{\sqrt{\varepsilon_1}\, E_t^2 \cos\theta_i} \quad -②$$

Put eqn ① in ②

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}\left[1 - \frac{E_r}{E_i}\right]^2 \frac{\cos\theta_i}{\cos\theta_t}$$

$$1 - \left[\frac{E_r}{E_i}\right]^2 = \frac{\sqrt{\varepsilon_2}\cos\theta_i}{\sqrt{\varepsilon_1}\cos\theta_t}\left[1 - \frac{E_r}{E_i}\right]^2$$

$$1 + \frac{E_r}{E_i} = \frac{\sqrt{\varepsilon_2}\cos\theta_i}{\sqrt{\varepsilon_1}\cos\theta_t}\left[1 - \frac{E_r}{E_i}\right]$$

$$\frac{E_r}{E_i}\left[1 + \frac{\sqrt{\varepsilon_2}\cos\theta_i}{\sqrt{\varepsilon_1}\cos\theta_t}\right] = \frac{\sqrt{\varepsilon_2}\cos\theta_i}{\sqrt{\varepsilon_1}\cos\theta_t} - 1$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\epsilon_2}\cos\theta_i - \sqrt{\epsilon_1}\cos\theta_t}{\sqrt{\epsilon_2}\cos\theta_t + \sqrt{\epsilon_1}\cos\theta_t} \quad -(4)$$

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\epsilon_2}\cos\theta_i - \sqrt{\epsilon_1(1-\sin^2\theta_t)}}{\sqrt{\epsilon_2}\cos\theta_i + \sqrt{\epsilon_1(1-\sin^2\theta_t)}}$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_2}}$$

$$\sin\theta_t = \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_2}}\sin\theta_t$$

$$\sin^2\theta_t = \frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\epsilon_2}\cos\theta_i - \sqrt{\epsilon_1 - \frac{\epsilon_1^2\sin^2\theta_i}{\epsilon_2}}}{\sqrt{\epsilon_2}\cos\theta_i + \sqrt{\epsilon_1 - \frac{\epsilon_1^2\sin^2\theta_i}{\epsilon_2}}} \quad -(5)$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\epsilon_2}\cos\theta_i - \frac{\epsilon_1}{\sqrt{\epsilon_2}}\sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}{\sqrt{\epsilon_2}\cos\theta_i + \frac{\epsilon_1}{\sqrt{\epsilon_2}}\sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}$$

reflection coefficient Divide numerator and denominator

$$\frac{E_r}{E_i} = \frac{\left[\frac{\epsilon_2}{\epsilon_1}\right]\cos\theta_i - \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right) - \sin^2\theta_i}}{\left[\frac{\epsilon_2}{\epsilon_1}\right]\cos\theta_i + \sqrt{\left(\frac{\epsilon_2}{\epsilon_1}\right) - \sin^2\theta_i}}$$

Brewster angle is a particular angle at which no reflection take place i.e $\frac{E_r}{E_i} = 0$

$$\frac{\epsilon_2}{\epsilon_1}\cos\theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}$$

are both side

$$\frac{\varepsilon_2^2}{\varepsilon_1^2}(1-\sin^2\theta_i) = \frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i$$

$$\frac{\varepsilon_2^2}{\varepsilon_1^2} - \frac{\varepsilon_2^2}{\varepsilon_1^2}\sin^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta_i$$

$$\sin^2\theta_i\left[1-\frac{\varepsilon_2^2}{\varepsilon_1^2}\right] = \frac{\varepsilon_2}{\varepsilon_1} - \frac{\varepsilon_2^2}{\varepsilon_1^2}$$

$$\sin^2\theta_i[\varepsilon_1^2-\varepsilon_2^2] = \varepsilon_1\varepsilon_2 - \varepsilon_2^2$$

$$\sin^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2} \quad - (6)$$

$$\cos^2\theta_i = \frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2} \quad - (7)$$

$$\tan^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1}$$

$$\tan\theta_i = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\theta_i = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

This is called Brewster angle at which no reflected wave when the incident wave is Parallel Polarized.

Q.5 A uniform Plane wave having frequency of 9375 MHz is Propagating in Polystyrene of $\varepsilon_r = 2.55$. if amplitude of electric field is 20 V/m and material is assumed to be lossless. find.

(i) Phase constant
(ii) wave length in Polystrene
(iii) velocity of Propagation
(iv) Intrinsic Impedance
(v) Propagation constant
(vi) Amplitude of H field.

Given $f = 9375 \times 10^6 Hz$

$\epsilon_r = 2.55$

$E = 20 V/m$

(i) Phase constant

$$\beta = \omega\sqrt{\mu_0 \epsilon_0} = 2\pi(9375 \times 10^6)\sqrt{4\pi \times 10^{-7}\left(\frac{1}{36\pi} \times 10^{-9}\right)}$$

$$= 196.34 \text{ rad/m}$$

(ii) Wavelength (λ)

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{196.34} = 3.2 cm$$

(iii) Velocity of Propagation

$$= \frac{\omega}{\beta} = \frac{2\pi f}{\beta}$$

$$= \frac{2\pi \times (9375 \times 10^6)}{196.34} = 300 \times 10^6 \text{ m/sec}$$

(iv) Propagation constant $= \alpha + j\beta$

$\alpha = 0$ for loss less medium

$$= 0 + j\beta = -196.34$$

(v) Intrinsic impedance

$$\eta = \frac{E}{H} = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu}{\epsilon_0 \epsilon_r}} = \sqrt{\frac{4\pi \times 10^{-7}}{\frac{1}{36\pi} \times 10^{-9} \times 2.55}}$$

$$= 236.98 \, \Omega$$

(vi) Amplitude of H field

$$= \frac{E}{\eta}$$

$$= \frac{20}{236.98} = 0.08439 \text{ A/m}$$

for a guided wave between two infinite conducting planes separated by a distance of 0.25 m find the cutoff frequency for the TM20 made. if the operating frequency is 3 GHz find the phase velocity of the wave. ?

The critical angular frequency ω_c is given by

$$\omega_c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \frac{m\pi}{a}$$

$$\omega_c = \frac{c\, m\pi}{a}$$

$$f_c = \frac{cm}{2a} = 3\times10^8 \times \frac{2}{2\times0.25}$$

$$= 1.2 \text{ GHz}$$

If the operating frequency is 3 GHz

$$\beta = \sqrt{\omega^2 \mu_0 \varepsilon_0 - \left(\frac{m\pi}{a}\right)^2}$$

$$= \sqrt{\frac{4\pi^2 f^2}{c^2} - \frac{4\pi^2}{a^2}}$$

$$= 2\pi = \sqrt{\frac{f^2}{c^2} - \frac{1}{a^2}}$$

$$= 57.59$$

Phase velocity is given by

$$\frac{\omega}{\beta} = \frac{2\pi v}{\beta}$$

$$= 3.27\times10^8 \text{ m/s}$$

Q.7 A rectangular air filled waveguid has dimension 2cm × 1cm. For what range of frequency, there is a "single mode" operation in the guide.

The cut off frequency for (m, n) mod is

$$\nu_0 = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$= 1.5 \times 10^{10} \sqrt{\frac{m^2}{4} + n^2}$$

substituting the values, the cut off for (1,0) mode is 7.5 GHz and for (2,0) is 15GHz

The frequency for (0,1) is also 15 GHz

All the other mode have higher cut off.

Thus in order that only one mode Propagate the operating frequency should be in the range $7.5 < \nu < 1.5$.

Explain the related Potentials and their concept of radiation?

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