Multirate Digital Signal Processing

- systems that employ multiple sampling rates in the processing of digital signals are called multirate digital signal processing systems.
- Multirate systems are sometimes used for sampling-rate conversion

Multirate Digital Signal Processing

In most applications multirate systems are used to improve the performance, or for increased computational efficiency.

Multirate Digital Signal Processing

- The basic Sampling operations in a multirate system are:

Decimation

Decreasing the sampling rate of signal

Interpolation

Sampling Rate Reduction by Integer Factor D

- Decimation by a factor of D, where D is a positive integer, can be performed as a two-step process, consisting of an anti-aliasing filtering followed by an operation known as downsampling

$$Y(n)=v(nD)$$

$$=\sum_{k=0}^{\infty} h(k)x(nD-k)$$

∞

Sampling Rate Reduction by Integer Factor D

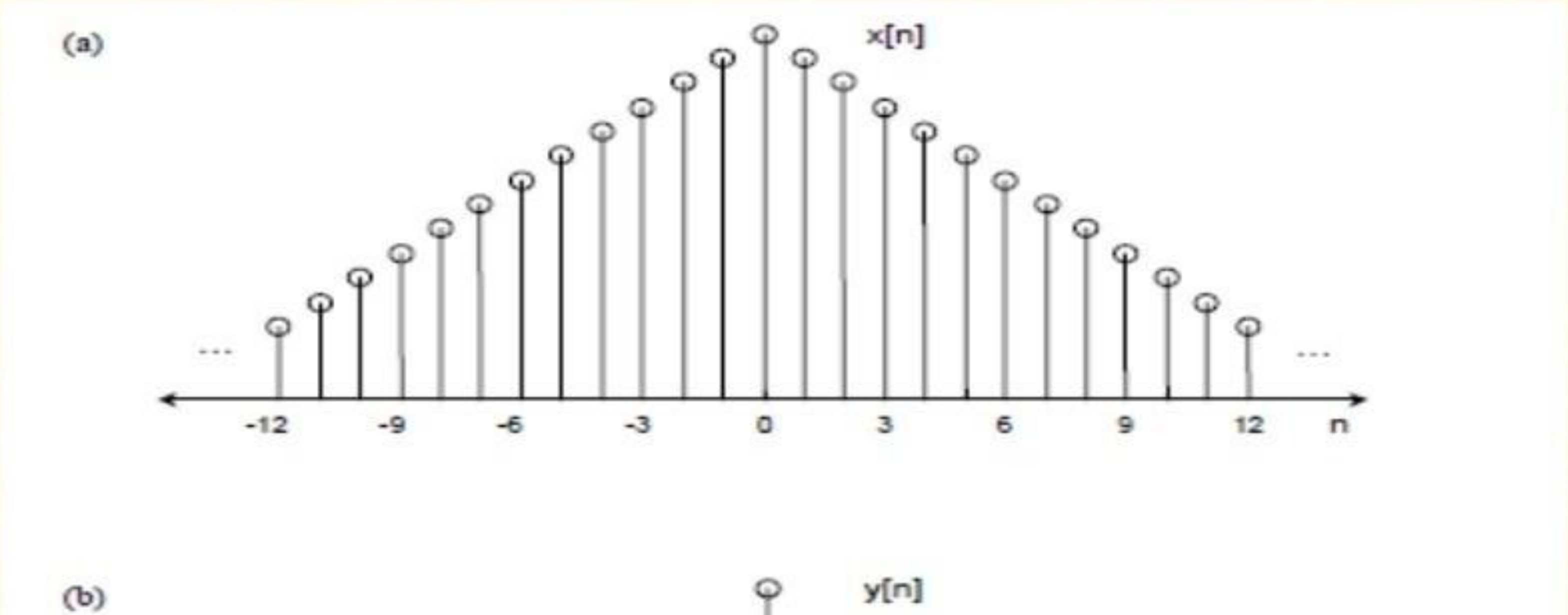
In decimation, the sampling rate is reduced from F_x to F_x /D by discarding D– 1 samples for every D samples in the original sequence

$$H_{D=}\begin{cases}1, & |W| \le \pi/D \\ 0 & otherwise\end{cases}$$

Digital anti-aliasing   Sampling-rate

Sampling Rate Reduction by Integer Factor D

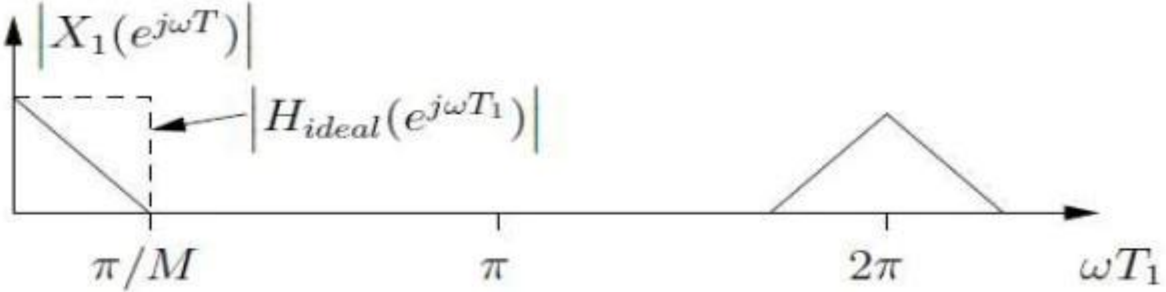
M=3



Sampling Rate Reduction by Integer Factor D

The frequency domain representation of downsampling can be found by taking the z-transform to both sides of (1.5) as

$$Y(e^{j\omega T}) = \sum_{-\infty}^{+\infty} x(mM)e^{-j\omega Tm} = \frac{1}{M}\sum_{k=0}^{M-1} X(e^{j(\omega T - 2\pi k)/M}). \qquad (1.6)$$



Sampling Rate Reduction by Integer Factor

Example 8.2

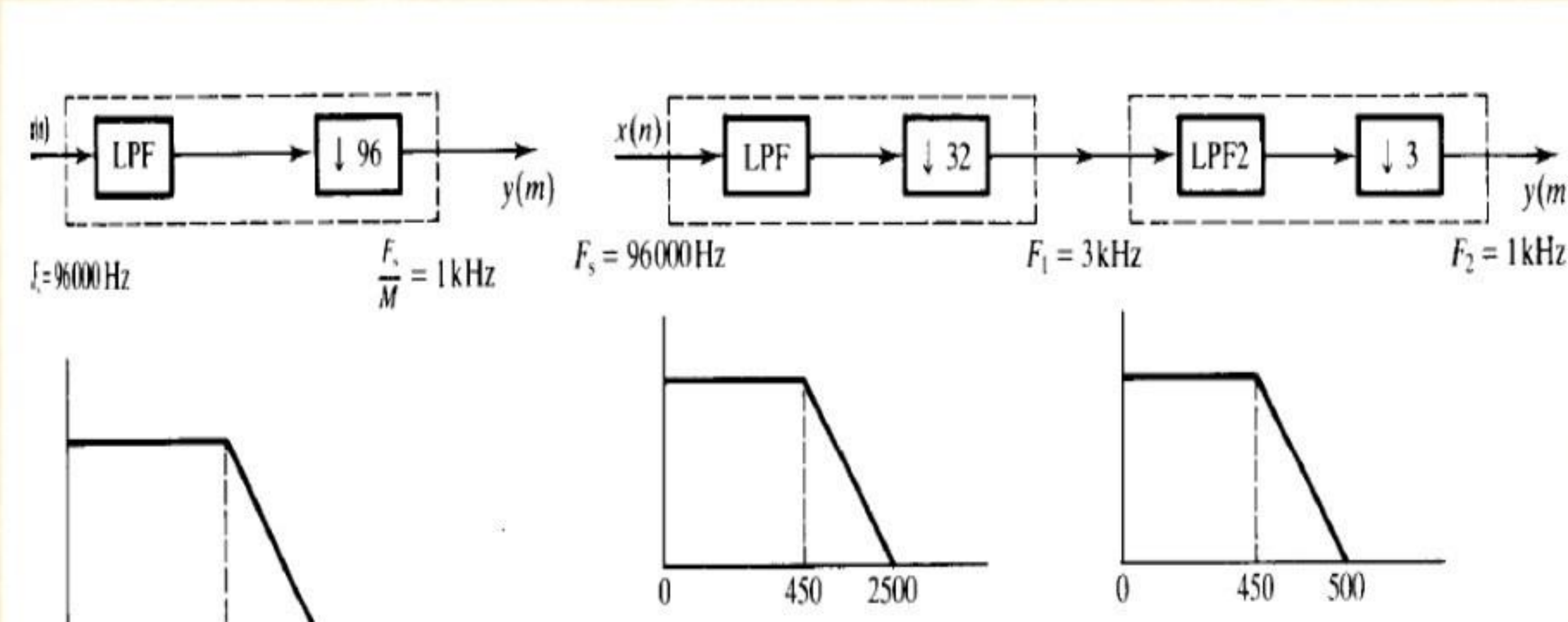
The sampling rate of a signal $x(n)$ is to be reduced, by decimation, from 96 kHz to 1 kHz. The highest frequency of interest after decimation is 450 Hz. Assume that an optimal FIR filter is to be used, with an overall passband ripple, $\delta_p = 0.01$, and passband deviation, $\delta_s = 0.001$. Design an efficient decimator.

Solution

We will start by finding the most efficient design for each value of I, $I = 1, 2, 3, 4$. We will then compare these designs and select the best.

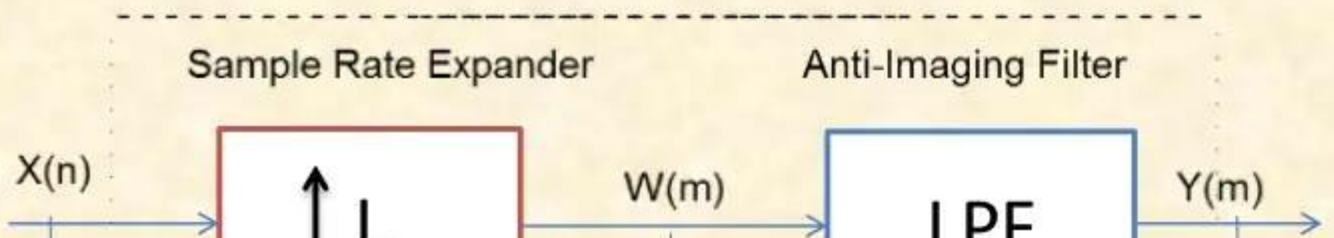
(1) First let us consider a one-stage design ($I = 1$). The block diagram and filter specifications for the stage are given in Figure 8.10(a).

Sampling Rate Reduction by Integer Factor D



Sampling Rate Increase by Integer Factor I

- Interpolation by a factor of L, where L is a positive integer, can be realized as a two-step process of upsampling followed by an anti-imaging filtering.



Sampling Rate Increase by Integer Factor I

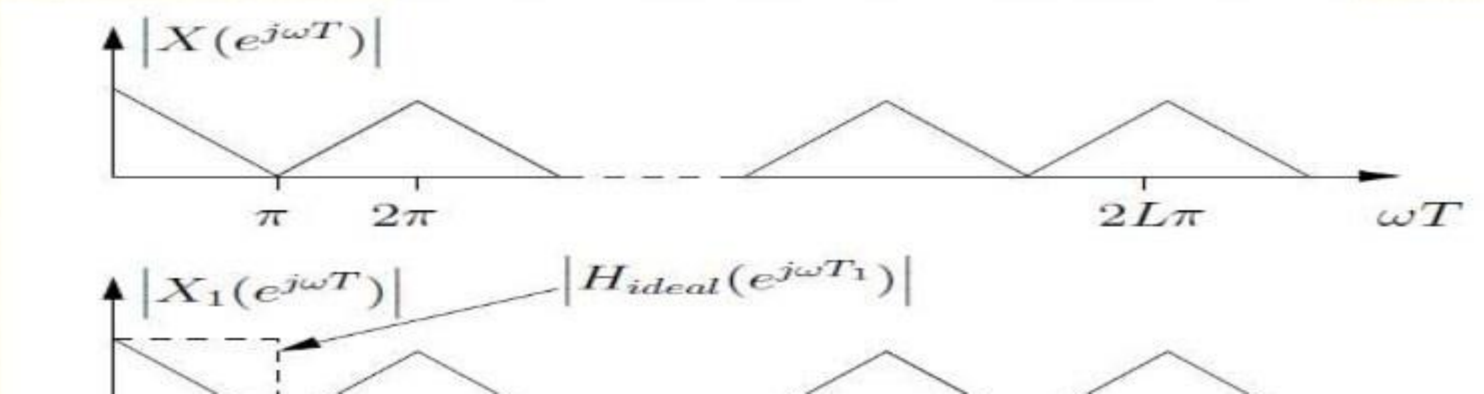
- An upsampling operation to a discrete-time signal x(n) produces an upsampled signal y(m) according to

$$\mathrm{y}(m) = \begin{cases} x\left(\frac{n}{L}\right), & n = 0, \pm L, \pm 2L, \ldots, \\ 0, & otherwise \end{cases}$$

Sampling Rate Increase by Integer Factor I

- The frequency domain representation of upsampling can be found by taking the z-transform of both sides

$$Y(e^{j\omega T}) = \sum_{-\infty}^{+\infty} y(m) e^{-j\omega T m} = X(e^{j\omega T L}).$$

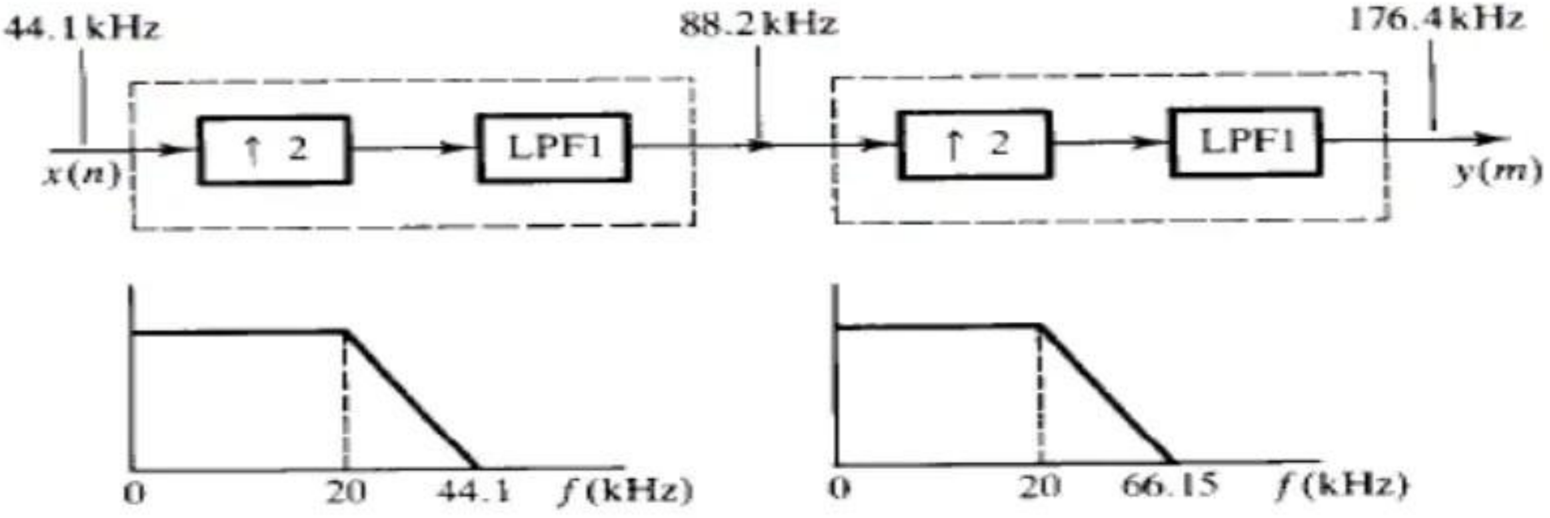


Example 8.3

A digital audio system exploits oversampling techniques to relax the requirements of the analogue anti-imaging filter. The overall filter specifications for the system is given below:

baseband	0 to 20 kHz
input sampling frequency F_s	44.1 kHz
output sampling frequency	176.4 kHz
stopband attenuation	50 dB
passband ripple	0.5 dB
transition width	2 kHz
stopband edge frequency	22.05 kHz

Design a suitable interpolator.



$\Delta f_1 \quad \frac{44.1-20}{88.2}$

$= 0.02324$

$\delta_{P2} = 0.0296; \delta_{S2} = 0.00316$

$N_2 = 83$

$\Delta f_1 = \frac{66.15 \quad 20}{176.4}$

$= 0.26162$

$\delta_{P1} = 0.0296; \delta_{S1} = 0.00316$

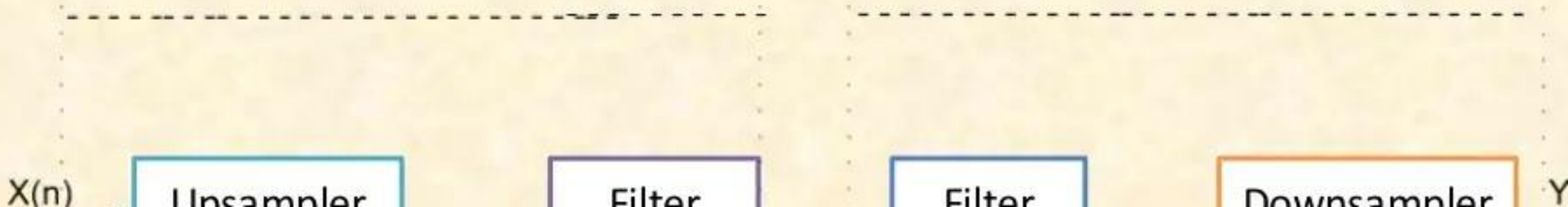
$N_1 = 6$

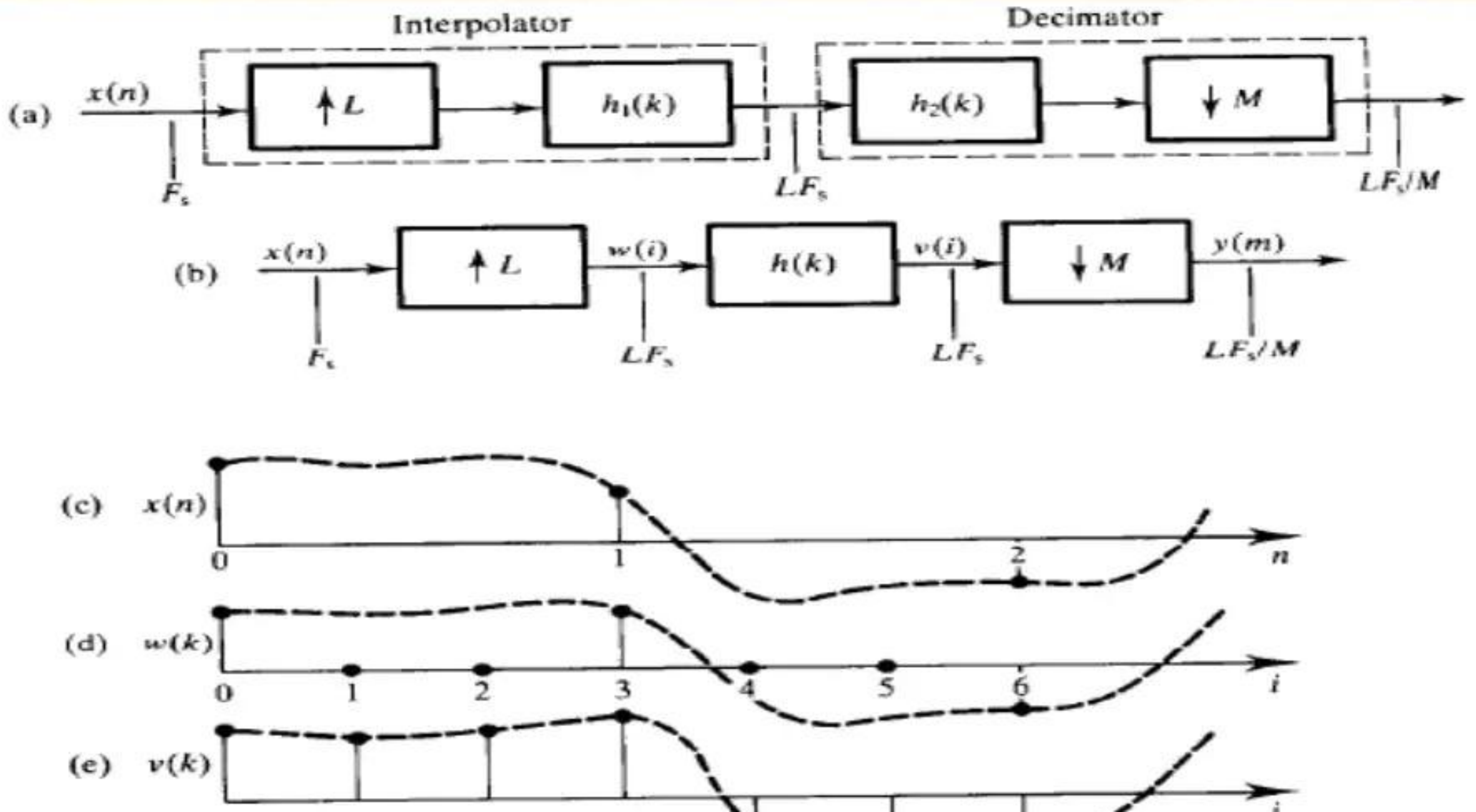
Figure 8.20 A two-stage interpolator for Example 8.3.

Sampling Rate conversion by Integer Rational Factor L/D

Sampling rate conversion by a rational factor ‘L/D’ can be achieved by first performing interpolation by the factor ‘L’ and then decimation the interpolator o/p by a factor ‘D’ .

In this process both the interpolation and decimator are cascaded as shown in the figure below:





Interpolator
Decimator
(a)
x(n)
↑L
h1(k)
h2(k)
↓M
Fs
LFs
LFs/M
(b)
x(n)
↑L
w(i)
h(k)
v(i)
↓M
y(m)
Fs
LFs
LFs
LFs/M
(c) x(n)
0
1
2
n
(d) w(k)
0
1
2
3
4
5
6
i
(e) v(k)
i

- Example:

Consider a multirate signal processing problem:

i. State with the aid of block diagrams the process of changing sampling rate by a non-integer factor.

ii. Develop an expression for the output y[n] and g[n] as a function of input x[n] for the multirate structure of fig .

- Answer:

i. .

1. We perform the upsampling process by a factor L following of an interpolation filter h1(l).
2. We continue filtering the output from the the interpolation filter via anti-aliasing filter h2(l) and finally operate downsampling.

Sampling Rate conversion by Integer Rational Factor L/D

ii.

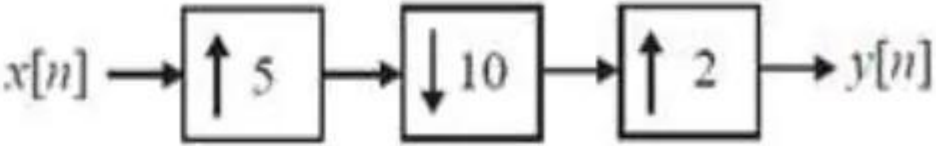
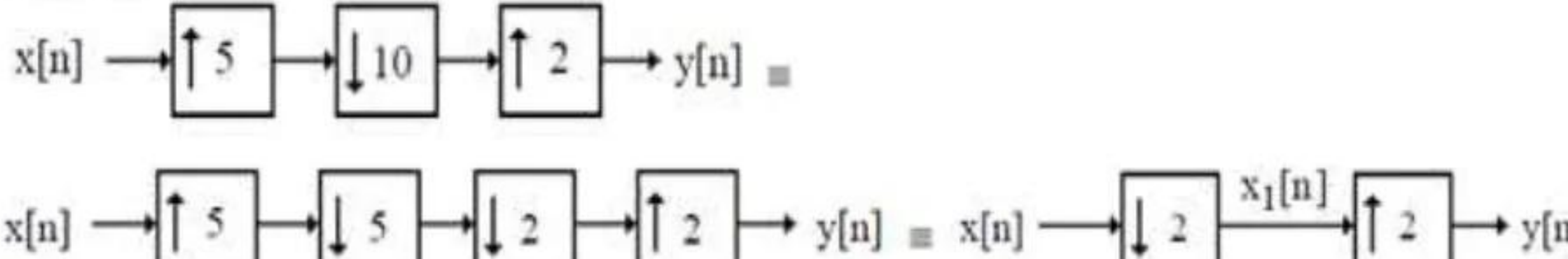


Figure E13.1

Answer:

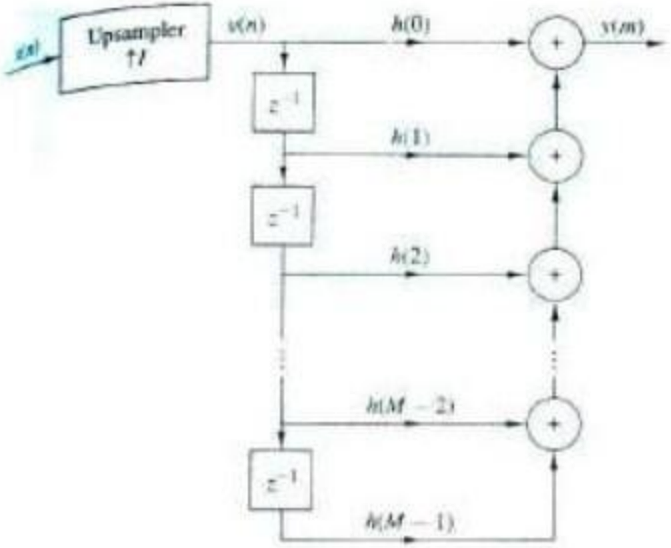


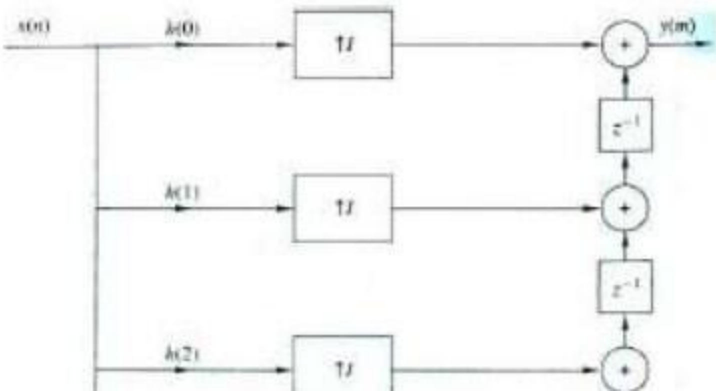
Polyphase filters

- Polyphase filters A very useful tool in multirate signal processing is the so-called poly phase representation of signals and systems facilitates considerable simplifications of theoretical results as well as efficient implementation of multirate systems.
- To formally define it, an LTI system is considered with a transfer function

$$+\infty$$

- The major problem in this realization is that the filter computations are performed at high sampling rate ***I*** f_x.
- This problem is solved by using transposed form of FIR filter and embedding the up sampler within the filter as shown in the figure.
- So all multiplications are

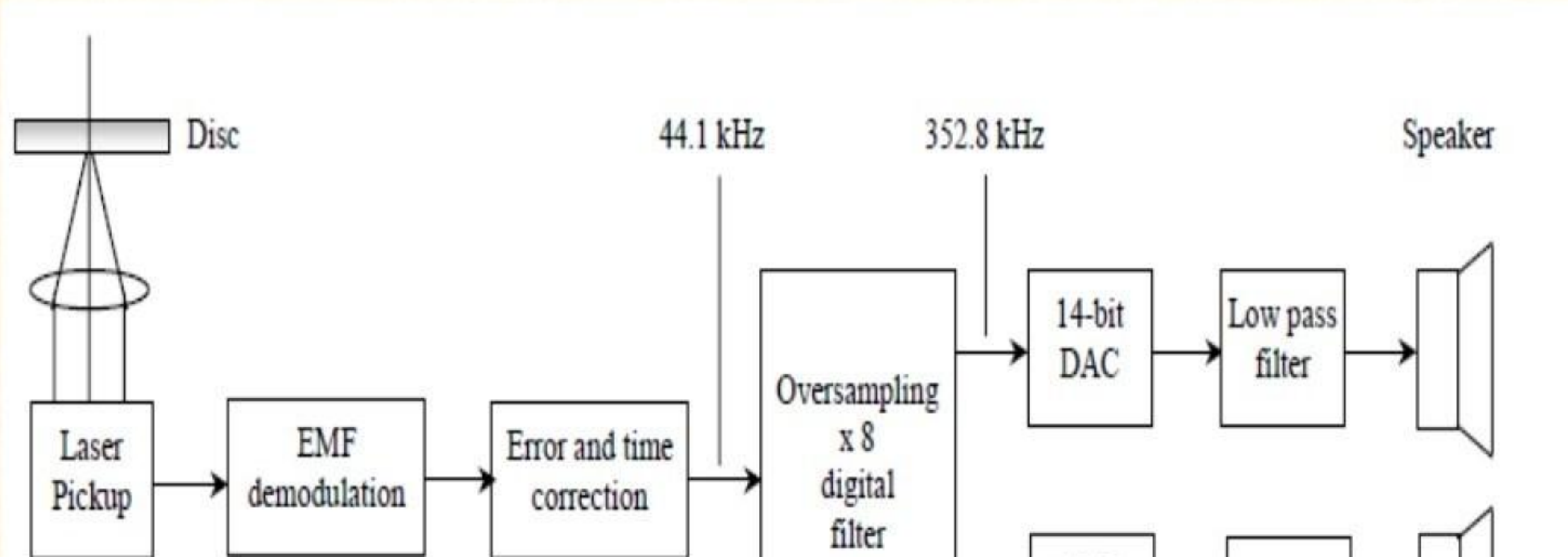




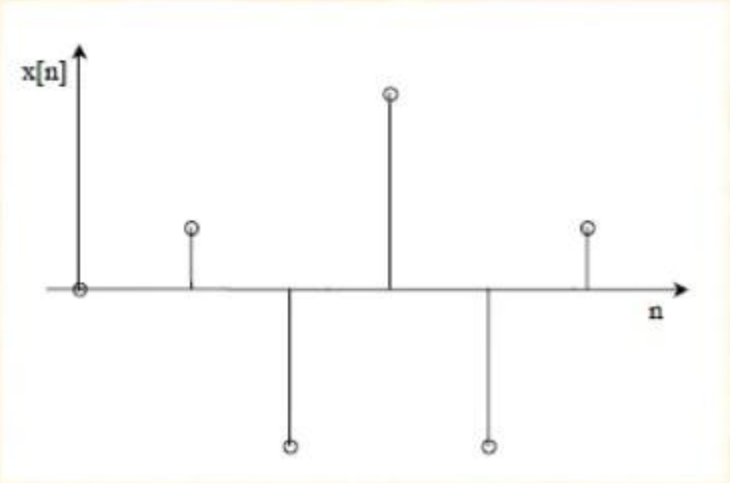
Applications of Multirate DSP

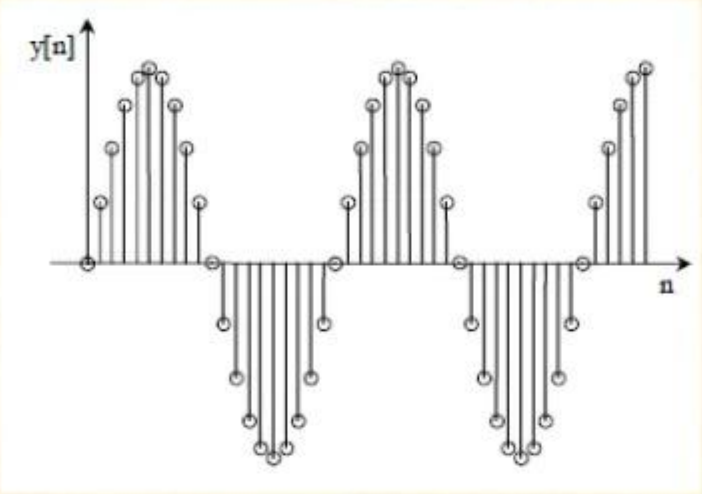
- Multirate systems are used in a CD player when the music signal is converted from digital into analog (DAC).

Applications of Multirate DSP



Applications of Multirate DSP



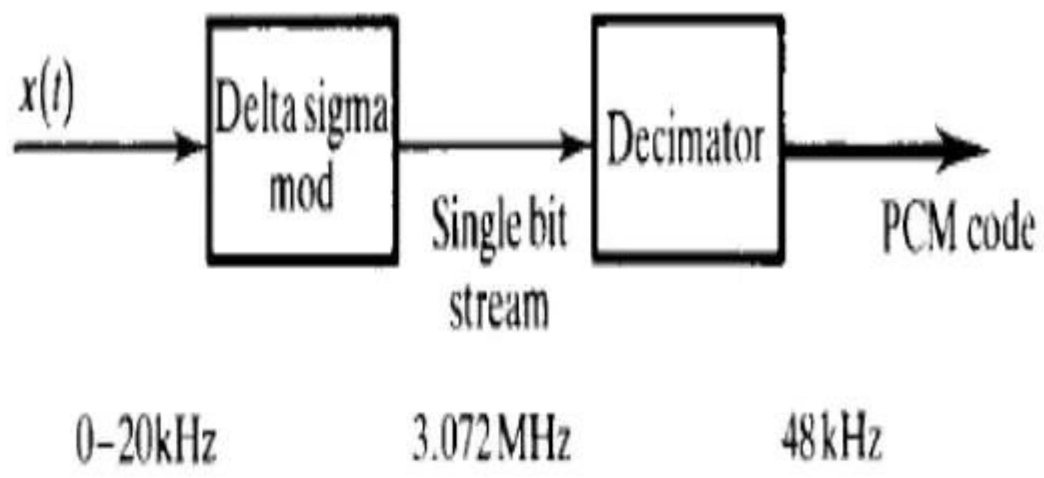


(c)

Applications of Multirate DSP

- ❑ The effect of oversampling also has some other desirable features:
  - Firstly, it causes the image frequencies to be much higher and therefore easier to filter out.
  - Secondly reducing the noise power spectral density, by spreading the noise power over a larger bandwidth.

High quality Analog to Digital conversion for digital audio



Thanks>>>>