Digital filter Structure :-

1. Direct Form Structure :-

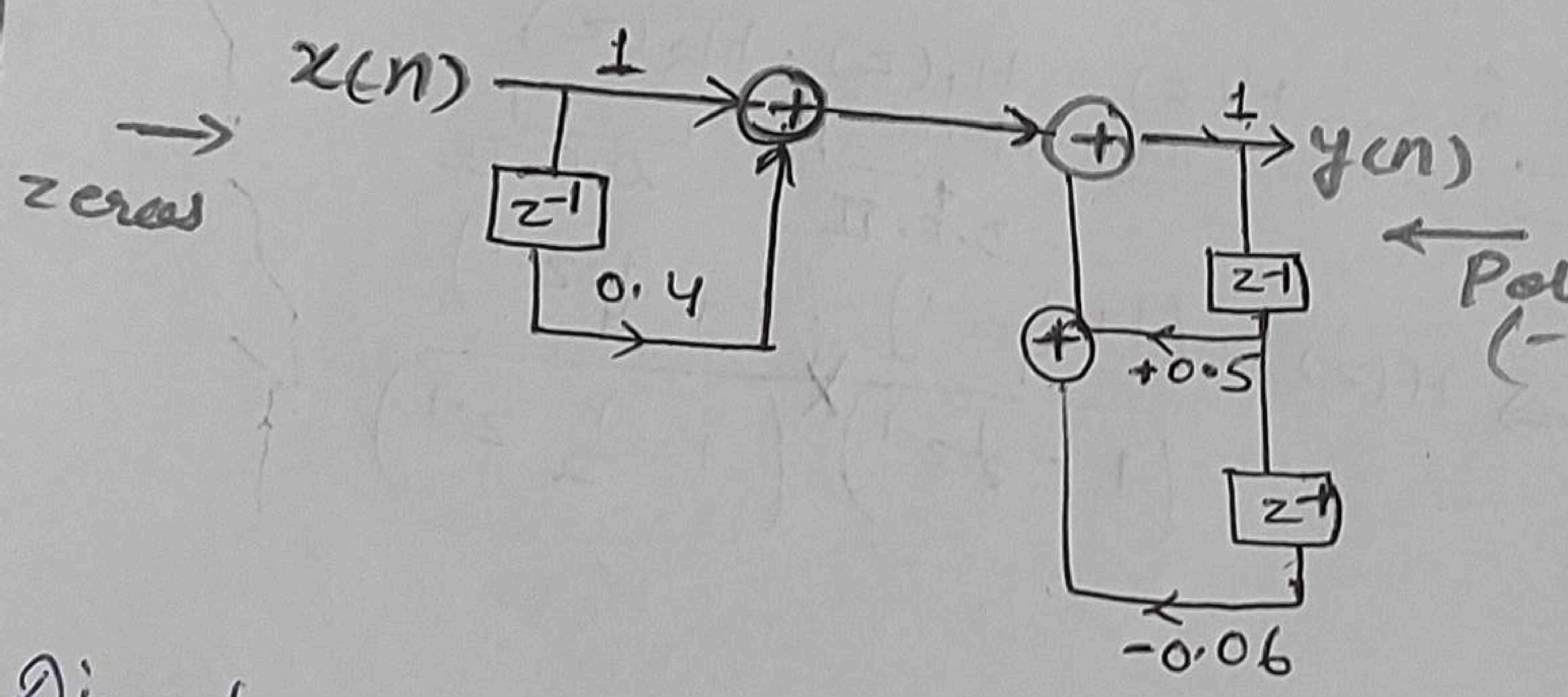
(a) Direct form - I

(b) Direct form - II

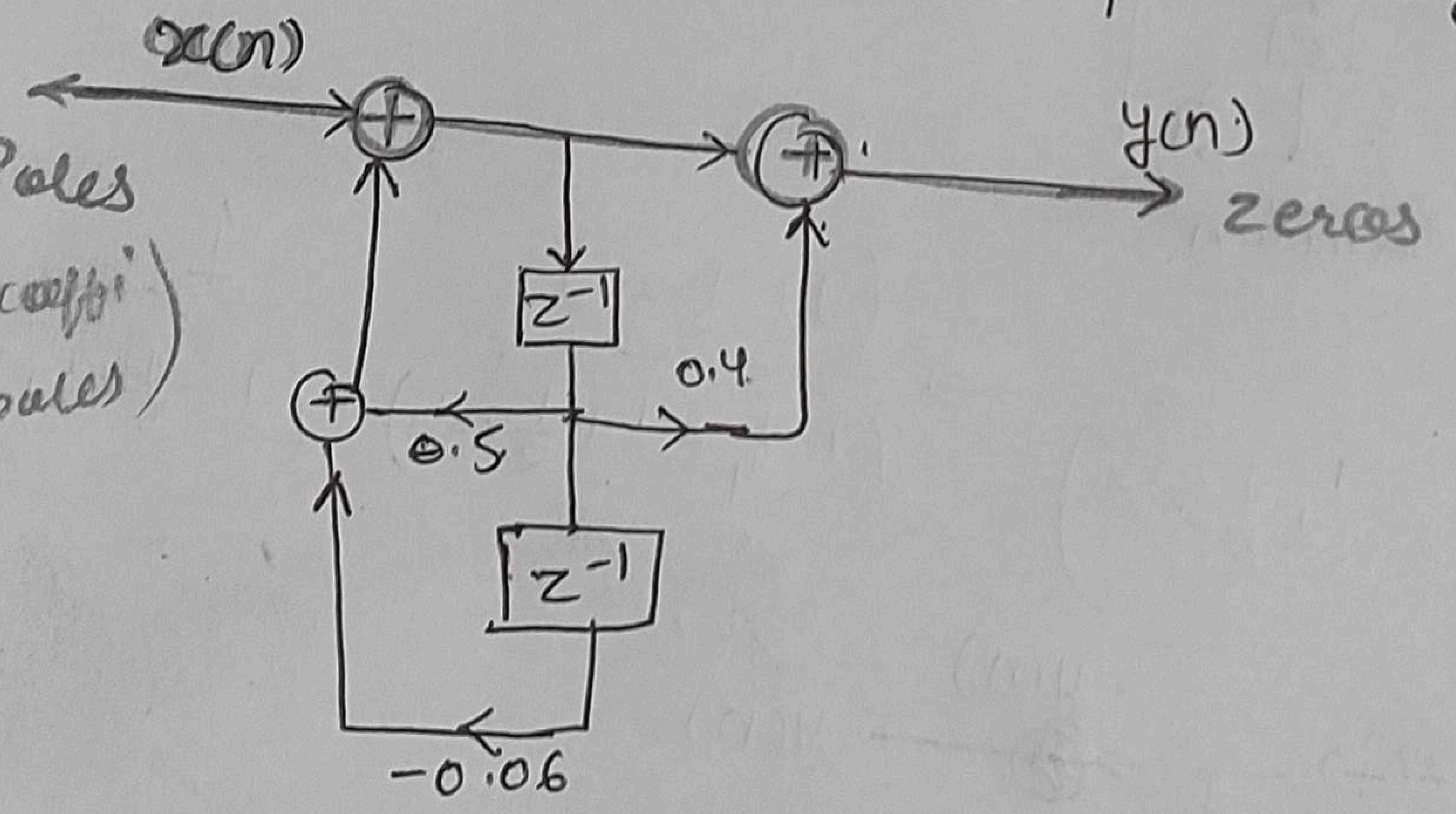
(1) $H(z) = \dfrac{1 + 0.4z^{-1}}{1 - 0.5z^{-1} + 0.06z^{-2}}$ coeff. of zeros / poles

$z^{-n} \rightarrow$ delay. z^{-1} (one step delay)

Direct form 1 :



Direct form II :- (Instead of seperate delay, we use common delay. (z^{-n}))

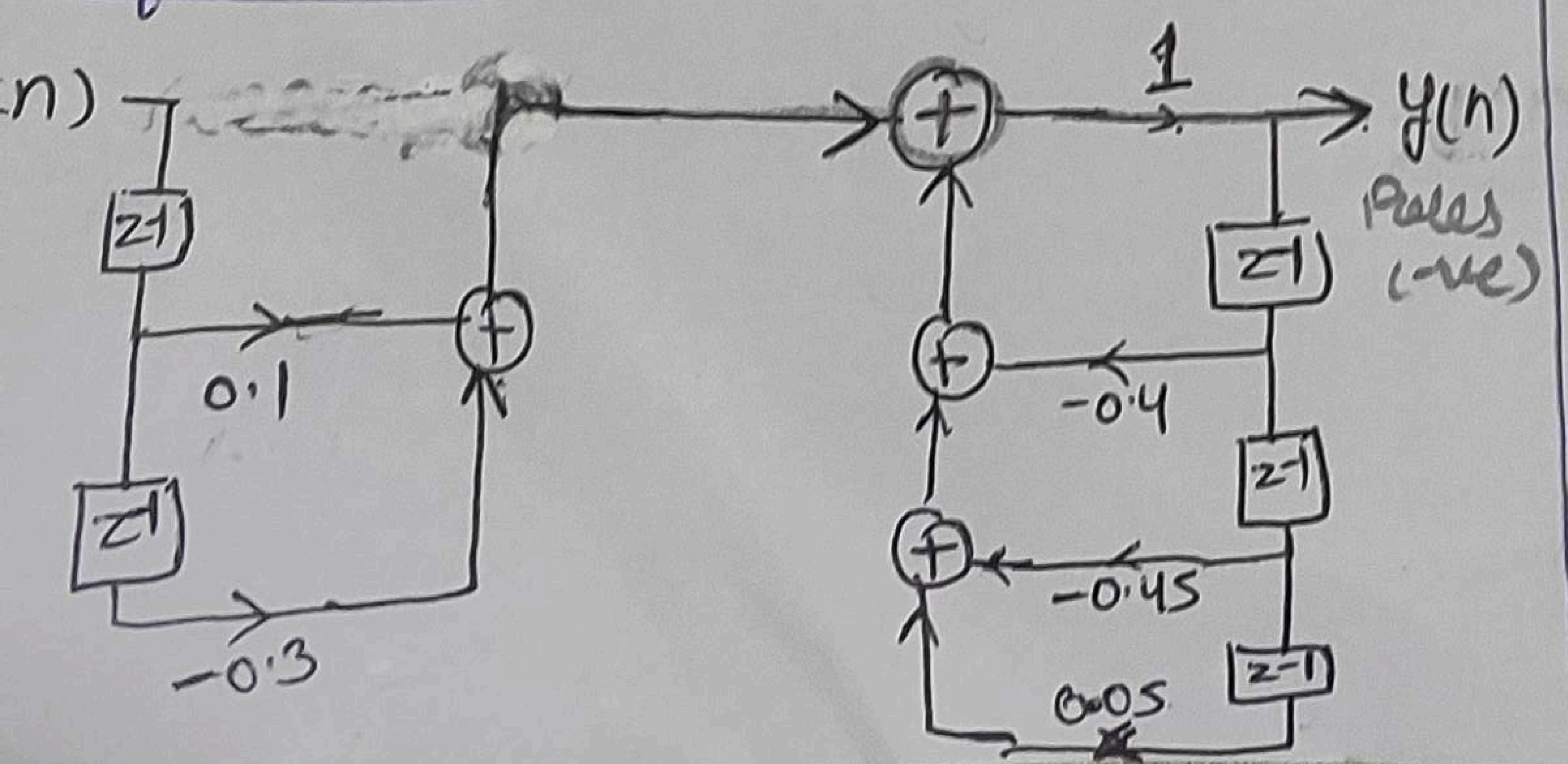


$H(z) = \dfrac{z^{-1} - 3z^{-2}}{(10 - z^{-1})(1 + 0.5z^{-1} + 0.5z^{-2})}$

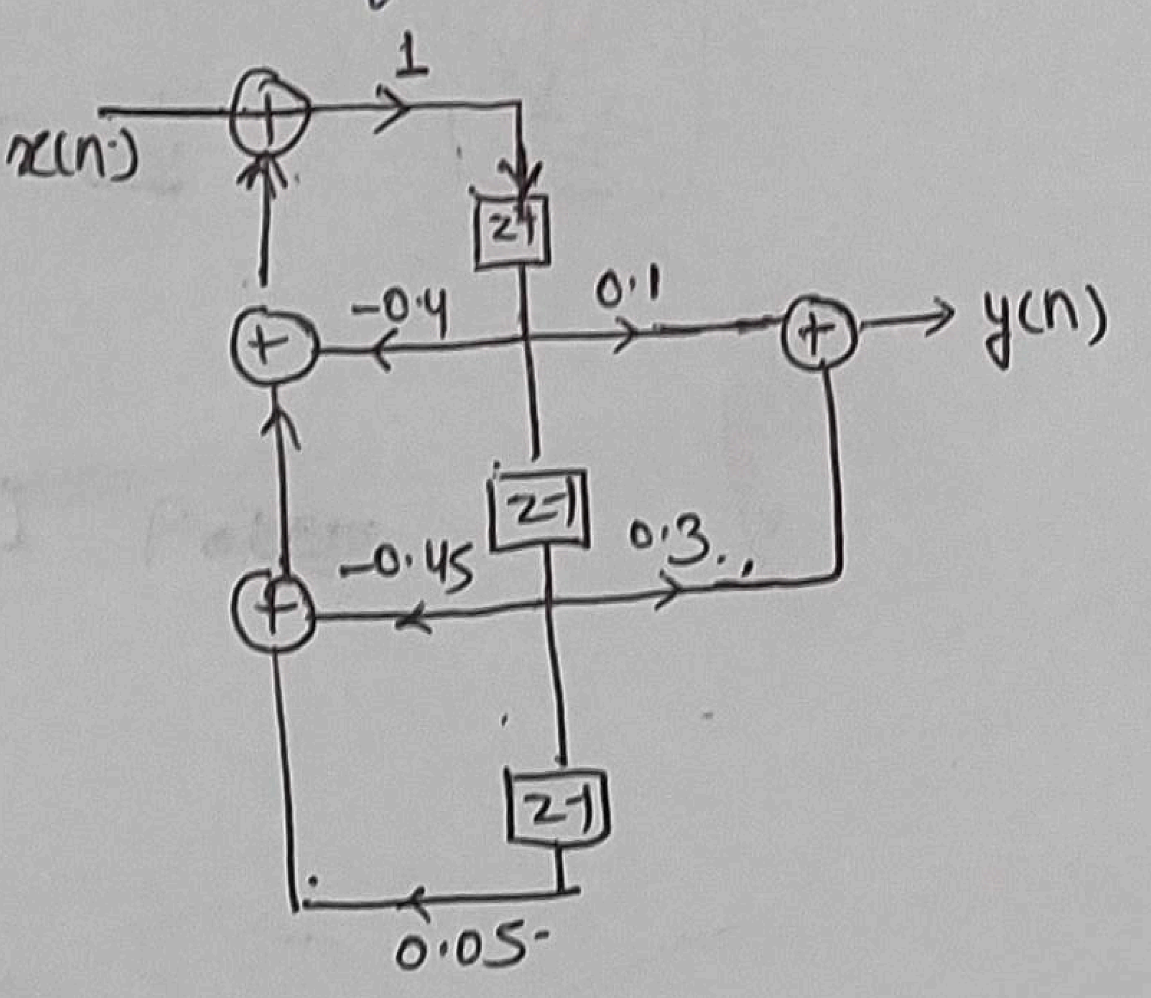
$\Rightarrow H(z) = \dfrac{z^{-1} - 3z^{-2}}{10 + 4z^{-1} + 4.5z^{-2} - 0.5z^{-3}}$

$\boxed{H(z) = \dfrac{0.1z^{-1} - 0.3z^{-2}}{1 + 0.4z^{-1} + 0.45z^{-2} - 0.05z^{-3}}}$ zeros / Poles

Direct form I :-



Direct form - II :-



III **Num 3:** $y(n) - \frac{1}{4} y(n-1) + \frac{1}{8} y(n-2) = x(n) + \frac{1}{2} x(n-2)$

z-transform in both sides,

$$Y(z) - \frac{1}{4} z^{-1} Y(z) + \frac{1}{8} z^{-2} Y(z) = X(z) + \frac{1}{2} z^{-2} X(z)$$

$$H(z) \Rightarrow \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2} z^{-2}}{1 - \frac{1}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

II. Cascade form structure :-

$$H(z) = H_1(z) \cdot H_2(z)$$

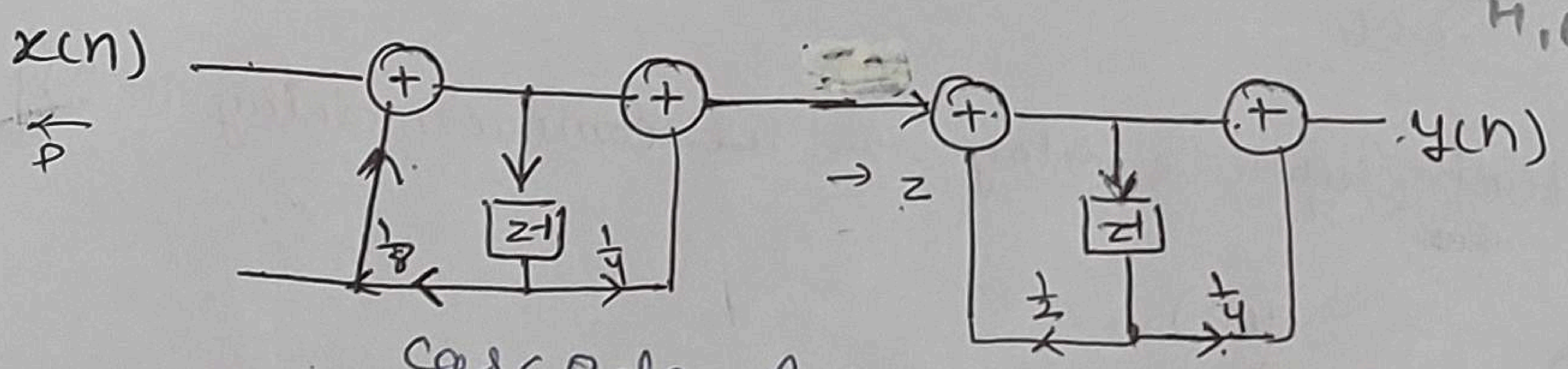
larger into smaller

D.F. II   D.F-II

(1.) $$H(z) = \frac{1 + \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}{1 - \frac{5}{8} z^{-1} + \frac{1}{16} z^{-2}} \leftarrow -\frac{1}{4}, -\frac{1}{2}$$

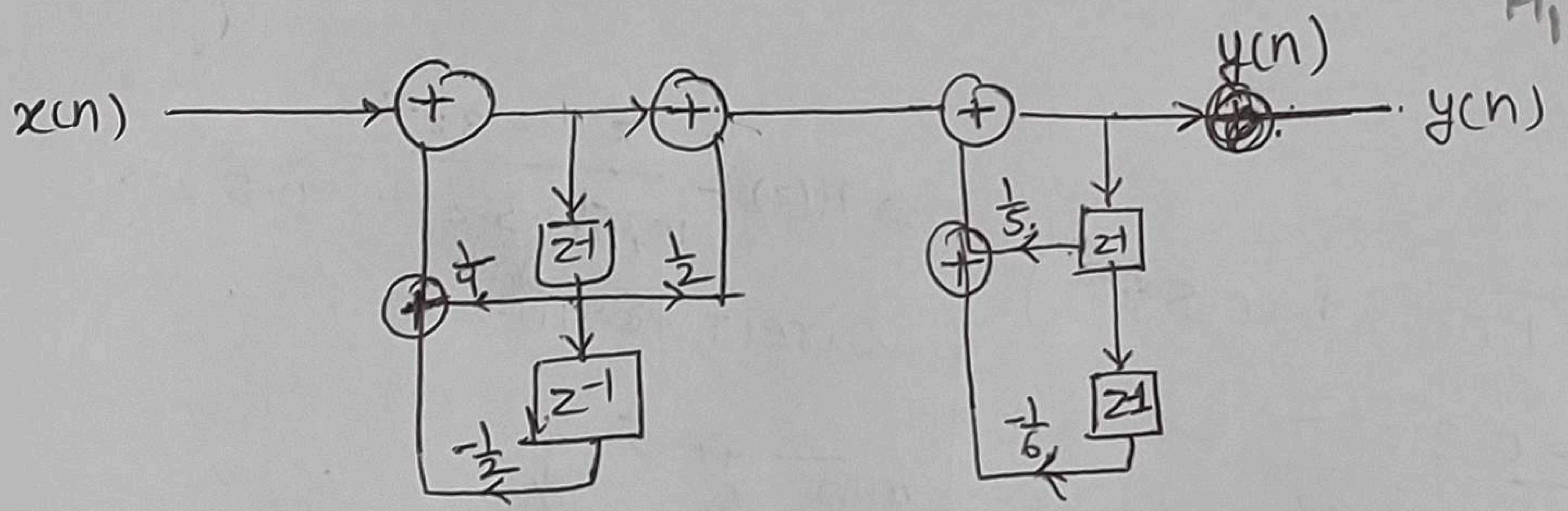
$\approx \frac{1}{8}, \frac{1}{2}$

$$\Rightarrow H(z) = \underbrace{\frac{(1 + \frac{1}{4} z^{-1})}{(1 - \frac{1}{8} z^{-1})}}_{H_1(z)} \underbrace{\frac{(1 + \frac{1}{4} z^{-1})}{(1 - \frac{1}{2} z^{-1})}}_{H_2(z)}$$



cascade Representation

(2) $$H(z) = \frac{(1 + \frac{1}{2} z^{-1})}{(1 - \frac{1}{4} z^{-1} + \frac{1}{2} z^{-2})(1 - \frac{1}{5} z^{-1} + \frac{1}{6} z^{-2})} \Rightarrow \underbrace{\frac{(1 + \frac{1}{2} z^{-1})}{(1 - \frac{1}{4} z^{-1} + \frac{1}{2} z^{-2})}}_{H_1(z)} \cdot \underbrace{\frac{1}{(1 - \frac{1}{5} z^{-1} + \frac{1}{6} z^{-2})}}_{H_2(z)}$$



III. Latice Structure :-

(1) $H(z) = \dfrac{1}{1 + \frac{2}{5}z^{-1} + \frac{3}{4}z^{-2} + \frac{1}{3}z^{-3}}$

$a_3(0) \quad a_3(1) \quad a_3(2) \quad a_3(3)$ 3rd order

$\therefore$ m = 3 (3rd order filter) $k_0 = a_3(0) = 1$

$a_3(1) = \frac{2}{5}$

$a_3(2) = \frac{3}{4}$

$K_3 = a_3(3) = \frac{1}{3}$

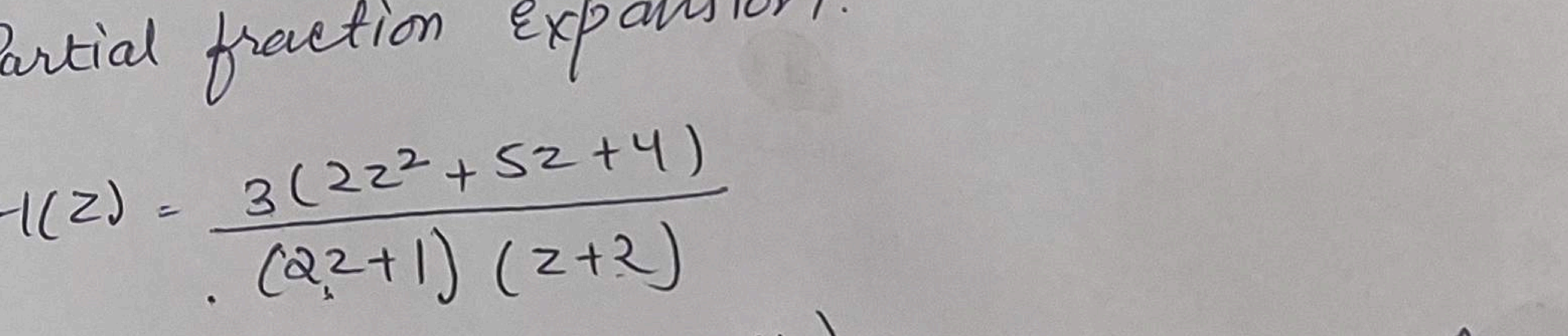
m=3

$i = 1$, General formula, $a_2(1) = \dfrac{a_3(1) - K_3 a_3(2)}{1 - K_3^2} = \dfrac{\frac{2}{5} - \frac{1}{3}\cdot\frac{3}{4}}{1 - \frac{1}{9}} = 0.16875$

$i = 2$, $a_2(2) = \dfrac{a_3(2) - K_3 a_3(1)}{1 - K_3^2} = \dfrac{\frac{3}{4} - \frac{1}{3}\cdot\frac{2}{5}}{1 - \frac{1}{9}} = 0.69375 = k_2$

m=2

$i = 1$, $a_1(1) = \dfrac{a_2(1) - K_2 a_2(1)}{1 - k_2^2} = \dfrac{0.16875 - (0.16875)}{1 - (0.69375)^2} \Rightarrow K_1 = 0.0996$



II Parallel Form Realization :-

Partial fraction expansion.

(a) $H(z) = \dfrac{3(2z^2 + 5z + 4)}{(2z+1)(z+2)}$

$F(z) = \dfrac{H(z)}{z} = \dfrac{\frac{3}{2}(2z^2 + 5z + 4)}{z\cdot(z + \frac{1}{2})(z+2)} \Rightarrow \dfrac{A_1}{z} + \dfrac{A_2}{(z + \frac{1}{2})} + \dfrac{A_3}{(z+2)}$

$A_1 = z\cdot F(z)\Big|_{z=0} = \dfrac{\frac{3}{2}(2z^2 + 5z + 4)}{z\cdot(z + \frac{1}{2})(z+2)}\Bigg|_{z=0} = 6$

$$A_2 = \left(z+\frac{1}{2}\right)F(z)\Big|_{z=-\frac{1}{2}} = \frac{\frac{3}{2}(2z^2+5z+4)}{z(z+2)}\Big|_{z=-\frac{1}{2}} = -4$$

$$A_3 = (z+2)\,F(z)\Big|_{z=-2} = \frac{\frac{3}{2}(2z^2+5z+4)}{z(z+2)}\Big|_{z=-2} = 1$$

Therefore, $\frac{H(z)}{z} = \frac{6}{z} - \frac{4}{z+\frac{1}{2}} + \frac{1}{z+2}$

Hence, $H(z) = 6 - \frac{4z}{z+\frac{1}{2}} + \frac{z}{z+2} \Rightarrow 6 - \frac{4}{(1+\frac{1}{2}z^{-1})} + \frac{1}{1+2z^{-1}}$