The Case for $n-1$: Why We Use Bessel's Correction

Step 1: The Population Reality

Given a population of $N = 10$ values: $\{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$

$$\mu = \frac{\sum x_i}{N} = \frac{550}{10} = 55$$

The true population variance is calculated by dividing by N:

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N} = \frac{8250}{10} = 825$$

Step 2: The Sample Limitation

We take a sample of $n = 6$: $\{20, 30, 40, 60, 70, 90\}$. The sample mean is $\bar{x} \approx 51.67$. Note that $\bar{x} \neq \mu$.

Step 3: The Comparison

When we calculate the sum of squared deviations from the **sample mean** ($SS = 3484.43$), we face a choice:

1. **The Biased Method (Divide by n):**

$$s^2_{biased} = \frac{3484.43}{6} \approx 580.74 \quad \textbf{(Too Low!)}$$

2. **Bessel's Correction (Divide by $n-1$):**

$$s^2 = \frac{3484.43}{6-1} = \frac{3484.43}{5} \approx 696.89 \quad \textbf{(Closer to 825)}$$

Conclusion

Dividing by n underestimates the true variance because the sample data points are naturally ”closer” to their own average ($\bar{x}$) than they are to the true population average (μ). Dividing by $n-1$ mathematically compensates for this ”tightness” in the sample.