Set Theory, Permutations and Combinations

Mathematics for Artificial Intelligence

1 Set Theory

1.1 Definition of a Set

A **set** is a well-defined collection of distinct objects, called elements. If x is an element of A, we write $x \in A$; if not, $x \notin A$.

Examples:

$$A = \{1, 2, 3\}, \quad B = \{\text{apple}, \text{banana}\}$$

1.2 Types of Sets

- **Finite set:** $A = \{1, 2, 3\}$
- **Infinite set:** $\mathbb{N}, \mathbb{R}$
- **Subset:** $A \subseteq B$
- **Proper subset:** $A \subset B$
- **Universal set:** U
- **Empty set:** $\emptyset$

1.3 Set Operations

Union:

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Intersection:

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Difference:

$$A - B = \{x : x \in A, x \notin B\}$$

Complement:

$$A' = U - A$$

1.4 Important Laws of Sets

Commutative:

$$A \cup B = B \cup A, \qquad A \cap B = B \cap A$$

Associative:

$$A \cup (B \cup C) = (A \cup B) \cup C$$

Distributive:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

These principles form the foundation for probability operations such as:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

2 Permutations

2.1 Definition

A **permutation** is an arrangement of objects where **order matters**.

The number of permutations of selecting r objects from n objects is:

$$^nP_r = \frac{n!}{(n-r)!}$$

2.2 Example

How many 3-digit numbers can be formed using the digits 1,2,3,4 without repetition?

$$^4P_3 = \frac{4!}{1!} = 24$$

In AI, permutations appear in ordered hyperparameter sequences for grid or random search.

3 Combinations

3.1 Definition

A **combination** is a selection of objects where **order does not matter**.

The number of combinations of selecting r objects from n objects is:

$$^nC_r = \frac{n!}{r!(n-r)!}$$

3.2 Example

How many teams of 3 can be formed from 8 players?

$$^8C_3 = \frac{8!}{3!5!} = 56$$

In machine learning, combinations describe selecting feature subsets for a model.

4 Permutation vs Combination

Scenario	**Order Matters?**	**Formula**
Permutation	Yes	$^nP_r = \frac{n!}{(n-r)!}$
Combination	No	$^nC_r = \frac{n!}{r!(n-r)!}$

5 Connection to Probability

Counting outcomes in sample spaces is done using permutations and combinations. The probability of an event A is:

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total possible outcomes}}$$

Example: Probability of exactly 2 heads in 4 tosses:

$${}^4C_2 = 6$$