Derivative Rules: Practice Questions with Step-by-Step Solutions

Questions

1. Constant Rule

1. $\dfrac{d}{dx}(7)$
2. $\dfrac{d}{dx}(-12)$
3. $\dfrac{d}{dx}\left(\dfrac{5}{2}\right)$

2. Constant Multiple Rule

1. $\dfrac{d}{dx}(6x^3)$
2. $\dfrac{d}{dx}(4\sin x)$
3. $\dfrac{d}{dx}(-3\ln x)$

3. Power Rule

1. $\dfrac{d}{dx}(x^5)$
2. $\dfrac{d}{dx}(x^{10})$
3. $\dfrac{d}{dx}(\sqrt{x})$
4. $\dfrac{d}{dx}\left(\dfrac{1}{x^3}\right)$

4. Sum Rule

1. $\dfrac{d}{dx}(3x^2 + 5x)$
2. $\dfrac{d}{dx}(\sin x + x^2)$
3. $\dfrac{d}{dx}(e^x + \ln x)$

5. Difference Rule

1. $\frac{d}{dx}(x^4 - x^2)$
2. $\frac{d}{dx}(\tan x - \sec x)$
3. $\frac{d}{dx}(x^3 - 7)$

6. Product Rule

1. $\frac{d}{dx}(x^2 \sin x)$
2. $\frac{d}{dx}(xe^x)$
3. $\frac{d}{dx}((3x + 1)x^2)$

7. Quotient Rule

1. $\frac{d}{dx}\left(\frac{x^3}{x+1}\right)$
2. $\frac{d}{dx}\left(\frac{\sin x}{x}\right)$
3. $\frac{d}{dx}\left(\frac{e^x}{\ln x}\right)$

8. Chain Rule

1. $\frac{d}{dx}(\sin(x^2))$
2. $\frac{d}{dx}(e^{3x})$
3. $\frac{d}{dx}((5x + 1)^7)$
4. $\frac{d}{dx}(\sqrt{4x^2 + 1})$

Mixed Practice

1. $\frac{d}{dx}(3x^4 - 5x + 7)$
2. $\frac{d}{dx}(x^2 e^{x^3})$
3. $\frac{d}{dx}\left(\frac{x^2+1}{x^3}\right)$
4. $\frac{d}{dx}(\ln(x^2 + 5))$
5. $\frac{d}{dx}((x^3 + 1)\sin(2x))$
6. $\frac{d}{dx}\left(\frac{5x^4 - 3}{\sqrt{x}}\right)$

Step-by-Step Solutions

1. Constant Rule

1. $\dfrac{d}{dx}(7)$

$$\frac{d}{dx}(7) = 0 \qquad \text{(derivative of a constant is 0)}$$

2. $\dfrac{d}{dx}(-12)$

$$\frac{d}{dx}(-12) = 0 \qquad \text{(constant rule)}$$

3. $\dfrac{d}{dx}\left(\dfrac{5}{2}\right)$

$$\frac{d}{dx}\left(\frac{5}{2}\right) = 0 \qquad \text{(constant rule)}$$

2. Constant Multiple Rule

1. $\dfrac{d}{dx}(6x^3)$

$$\begin{aligned}
\frac{d}{dx}(6x^3) &= 6 \cdot \frac{d}{dx}(x^3) && \text{(constant multiple)} \\
&= 6 \cdot 3x^2 && \text{(power rule)} \\
&= 18x^2
\end{aligned}$$

2. $\dfrac{d}{dx}(4\sin x)$

$$\begin{aligned}
\frac{d}{dx}(4\sin x) &= 4 \cdot \frac{d}{dx}(\sin x) && \text{(constant multiple)} \\
&= 4 \cdot \cos x && \text{(derivative of } \sin x\text{)} \\
&= 4\cos x
\end{aligned}$$

3. $\dfrac{d}{dx}(-3\ln x)$

$$\begin{aligned}
\frac{d}{dx}(-3\ln x) &= -3 \cdot \frac{d}{dx}(\ln x) && \text{(constant multiple)} \\
&= -3 \cdot \frac{1}{x} && \text{(derivative of } \ln x\text{)} \\
&= -\frac{3}{x}
\end{aligned}$$

3. Power Rule

1. $\frac{d}{dx}(x^5)$

$$\begin{aligned}\frac{d}{dx}(x^5) &= 5x^{5-1} && \text{(power rule)}\\ &= 5x^4\end{aligned}$$

2. $\frac{d}{dx}(x^{10})$

$$\begin{aligned}\frac{d}{dx}(x^{10}) &= 10x^{10-1} && \text{(power rule)}\\ &= 10x^9\end{aligned}$$

3. $\frac{d}{dx}(\sqrt{x})$

$$\begin{aligned}\sqrt{x} &= x^{1/2}\\ \frac{d}{dx}(x^{1/2}) &= \frac{1}{2}x^{1/2-1} && \text{(power rule)}\\ &= \frac{1}{2}x^{-1/2}\\ &= \frac{1}{2\sqrt{x}}\end{aligned}$$

4. $\frac{d}{dx}\left(\frac{1}{x^3}\right)$

$$\begin{aligned}\frac{1}{x^3} &= x^{-3}\\ \frac{d}{dx}(x^{-3}) &= -3x^{-4} && \text{(power rule)}\end{aligned}$$

4. Sum Rule

1. $\frac{d}{dx}(3x^2 + 5x)$

$$\begin{aligned}\frac{d}{dx}(3x^2 + 5x) &= \frac{d}{dx}(3x^2) + \frac{d}{dx}(5x) && \text{(sum rule)}\\ &= 3 \cdot 2x + 5 \cdot 1 && \text{(power rule)}\\ &= 6x + 5\end{aligned}$$

2. $\frac{d}{dx}(\sin x + x^2)$

$$\begin{aligned}\frac{d}{dx}(\sin x + x^2) &= \frac{d}{dx}(\sin x) + \frac{d}{dx}(x^2)\\ &= \cos x + 2x\end{aligned}$$

3. $\dfrac{d}{dx}(e^x + \ln x)$

$$\begin{aligned}\frac{d}{dx}(e^x + \ln x) &= \frac{d}{dx}(e^x) + \frac{d}{dx}(\ln x)\\ &= e^x + \frac{1}{x}\end{aligned}$$

5. Difference Rule

1. $\dfrac{d}{dx}(x^4 - x^2)$

$$\begin{aligned}\frac{d}{dx}(x^4 - x^2) &= \frac{d}{dx}(x^4) - \frac{d}{dx}(x^2)\\ &= 4x^3 - 2x\end{aligned}$$

2. $\dfrac{d}{dx}(\tan x - \sec x)$

$$\begin{aligned}\frac{d}{dx}(\tan x - \sec x) &= \frac{d}{dx}(\tan x) - \frac{d}{dx}(\sec x)\\ &= \sec^2 x - \sec x \tan x\end{aligned}$$

3. $\dfrac{d}{dx}(x^3 - 7)$

$$\begin{aligned}\frac{d}{dx}(x^3 - 7) &= \frac{d}{dx}(x^3) - \frac{d}{dx}(7)\\ &= 3x^2 - 0\\ &= 3x^2\end{aligned}$$

6. Product Rule

1. $\dfrac{d}{dx}(x^2 \sin x)$

   Let $f(x) = x^2$, $g(x) = \sin x$. Then $f'(x) = 2x$, $g'(x) = \cos x$.

$$\begin{aligned}\frac{d}{dx}(x^2 \sin x) &= f'(x)g(x) + f(x)g'(x) && \text{(product rule)}\\ &= 2x \sin x + x^2 \cos x\end{aligned}$$

2. $\dfrac{d}{dx}(xe^x)$

   Let $f(x) = x$, $g(x) = e^x$. Then $f'(x) = 1$, $g'(x) = e^x$.

$$\begin{aligned}\frac{d}{dx}(xe^x) &= f'(x)g(x) + f(x)g'(x)\\ &= 1 \cdot e^x + x \cdot e^x\\ &= e^x(1 + x)\end{aligned}$$

3. $\dfrac{d}{dx}((3x+1)x^2)$

Let $f(x) = 3x+1,\ g(x) = x^2$. Then $f'(x) = 3,\ g'(x) = 2x$.

$$\begin{aligned}\frac{d}{dx}((3x+1)x^2) &= f'(x)g(x) + f(x)g'(x)\\ &= 3x^2 + (3x+1)\cdot 2x\\ &= 3x^2 + 6x^2 + 2x\\ &= 9x^2 + 2x\end{aligned}$$

7. Quotient Rule

1. $\dfrac{d}{dx}\left(\dfrac{x^3}{x+1}\right)$

Let $f(x) = x^3,\ g(x) = x+1$. Then $f'(x) = 3x^2,\ g'(x) = 1$.

$$\begin{aligned}\frac{d}{dx}\left(\frac{x^3}{x+1}\right) &= \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} && \text{(quotient rule)}\\ &= \frac{3x^2(x+1) - x^3\cdot 1}{(x+1)^2}\end{aligned}$$

2. $\dfrac{d}{dx}\left(\dfrac{\sin x}{x}\right)$

Let $f(x) = \sin x,\ g(x) = x$. Then $f'(x) = \cos x,\ g'(x) = 1$.

$$\begin{aligned}\frac{d}{dx}\left(\frac{\sin x}{x}\right) &= \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}\\ &= \frac{\cos x\cdot x - \sin x\cdot 1}{x^2}\\ &= \frac{x\cos x - \sin x}{x^2}\end{aligned}$$

3. $\dfrac{d}{dx}\left(\dfrac{e^x}{\ln x}\right)$

Let $f(x) = e^x,\ g(x) = \ln x$. Then $f'(x) = e^x,\ g'(x) = \dfrac{1}{x}$.

$$\begin{aligned}\frac{d}{dx}\left(\frac{e^x}{\ln x}\right) &= \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}\\ &= \frac{e^x\ln x - e^x\cdot \frac{1}{x}}{(\ln x)^2}\end{aligned}$$

8. Chain Rule

1. $\dfrac{d}{dx}(\sin(x^2))$

Let $u = x^2$, so $y = \sin u$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du}\cdot\frac{du}{dx} && \text{(chain rule)}\\ &= \cos u\cdot 2x\\ &= 2x\cos(x^2)\end{aligned}$$

2. $\dfrac{d}{dx}(e^{3x})$

Let $u = 3x$, so $y = e^u$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du}\cdot\frac{du}{dx}\\ &= e^u \cdot 3\\ &= 3e^{3x}\end{aligned}$$

3. $\dfrac{d}{dx}((5x+1)^7)$

Let $u = 5x+1$, so $y = u^7$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du}\cdot\frac{du}{dx}\\ &= 7u^6 \cdot 5\\ &= 35(5x+1)^6\end{aligned}$$

4. $\dfrac{d}{dx}(\sqrt{4x^2+1})$

Write $\sqrt{4x^2+1} = (4x^2+1)^{1/2}$. Let $u = 4x^2+1$.

$$\begin{aligned}\frac{d}{dx}(u^{1/2}) &= \frac{1}{2}u^{-1/2}\cdot\frac{du}{dx}\\ \frac{du}{dx} &= 8x\\ \Rightarrow \frac{d}{dx}\sqrt{4x^2+1} &= \frac{1}{2}(4x^2+1)^{-1/2}\cdot 8x\\ &= \frac{8x}{2\sqrt{4x^2+1}}\\ &= \frac{4x}{\sqrt{4x^2+1}}\end{aligned}$$

Mixed Practice

1. $\dfrac{d}{dx}(3x^4 - 5x + 7)$

$$\begin{aligned}\frac{d}{dx}(3x^4 - 5x + 7) &= \frac{d}{dx}(3x^4) - \frac{d}{dx}(5x) + \frac{d}{dx}(7)\\ &= 12x^3 - 5 + 0\\ &= 12x^3 - 5\end{aligned}$$

2. $\dfrac{d}{dx}(x^2 e^{x^3})$

Let $f(x) = x^2$, $g(x) = e^{x^3}$.

$$f'(x) = 2x, \quad g'(x) = e^{x^3}\cdot 3x^2 \quad \text{(chain rule)}$$

$$\begin{aligned}
\frac{d}{dx}(x^2 e^{x^3}) &= f'(x)g(x) + f(x)g'(x) \\
&= 2xe^{x^3} + x^2(3x^2 e^{x^3}) \\
&= 2xe^{x^3} + 3x^4 e^{x^3}
\end{aligned}$$

3. $\frac{d}{dx}\left(\frac{x^2+1}{x^3}\right)$

   Let $f(x) = x^2 + 1$, $g(x) = x^3$. Then $f'(x) = 2x$, $g'(x) = 3x^2$.

$$\begin{aligned}
\frac{d}{dx}\left(\frac{x^2+1}{x^3}\right) &= \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \\
&= \frac{2x \cdot x^3 - (x^2+1) \cdot 3x^2}{x^6} \\
&= \frac{2x^4 - 3x^4 - 3x^2}{x^6} \\
&= \frac{-x^4 - 3x^2}{x^6} \\
&= -\frac{x^2+3}{x^4}
\end{aligned}$$

4. $\frac{d}{dx}(\ln(x^2+5))$

   Let $u = x^2 + 5$, so $y = \ln u$.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{1}{u} \cdot \frac{du}{dx} \\
\frac{du}{dx} &= 2x \\
\Rightarrow \frac{dy}{dx} &= \frac{2x}{x^2+5}
\end{aligned}$$

5. $\frac{d}{dx}((x^3+1)\sin(2x))$

   Let $f(x) = x^3 + 1$, $g(x) = \sin(2x)$.

$$f'(x) = 3x^2, \quad g'(x) = \cos(2x) \cdot 2 \quad \text{(chain rule)}$$

$$\begin{aligned}
\frac{d}{dx}((x^3+1)\sin(2x)) &= f'(x)g(x) + f(x)g'(x) \\
&= 3x^2 \sin(2x) + (x^3+1) \cdot 2\cos(2x)
\end{aligned}$$

6. $\frac{d}{dx}\left(\frac{5x^4-3}{\sqrt{x}}\right)$

   First rewrite using exponents:

$$\frac{5x^4-3}{\sqrt{x}} = (5x^4-3)x^{-1/2}$$

Let $f(x) = 5x^4 - 3$, $g(x) = x^{-1/2}$.

$$f'(x) = 20x^3, \quad g'(x) = -\frac{1}{2}x^{-3/2}$$

$$\begin{aligned}
\frac{d}{dx}\left(\frac{5x^4 - 3}{\sqrt{x}}\right) &= f'(x)g(x) + f(x)g'(x) \\
&= 20x^3 \cdot x^{-1/2} + (5x^4 - 3)\left(-\frac{1}{2}x^{-3/2}\right) \\
&= 20x^{3-\frac{1}{2}} - \frac{1}{2}(5x^4 - 3)x^{-3/2} \\
&= 20x^{5/2} - \frac{5x^4 - 3}{2x^{3/2}}
\end{aligned}$$

(You may leave it in this form or simplify further if desired.)