Conditional Probability and Independence Checkpoint 1

The first three questions refer to the following information:

Suppose a basketball team had a season of games with the following characteristics:

- 60% of all the games were ***at-home*** games. Denote this by ***H*** (the remaining were ***away*** games).
- 25% of all games were ***wins***. Denote this by ***W*** (the remaining were ***losses***).
- 20% of all games were at-home wins.

Question (1)

Of the at-home games, we are interested in finding what proportion were wins. In order to figure this out, we need to find:

A: P(H)
B: P(W)
C: P(H and W)
D: P(H | W)
E: P(W | H)

Feedback

A : 0

✗ This is not quite right. P (***H***) represents the proportion of games which were played at home. You should find the proportion of home games which were wins. In other words, if you look only at the home games, what proportion of them were wins? Consider the remaining options. (E) is the right answer.

B : 0

✗ This is not quite right. P (***W***) represents the proportion of games which were wins. You should find the proportion of home games which were wins. In other words, if you look only at the home games, what proportion of them were wins? Consider the remaining options. (E) is the right answer.

C : 0

This is not quite right. P (***H*** and ***W***) represents the proportion of games which were played at home and were wins. You should find the proportion of home games which were wins. In other words, if you look only at the home games, what proportion of them were wins? Consider the remaining options. (E) is the right answer.

D : 0

This is not quite right. P (***H*** |***W***) represents the proportion of wins which were played at home. You should find the proportion of home games which were wins. In other words, if you look only at the home games, what proportion of them were wins? (E) is the right answer.

E : 10

Good job! We are limiting our interest to the home games only, so we condition on ***H***. Out of the home games, we are interested in the probability of winning (***W***), therefore we are interested in P (***W*** | ***H***).

Question (2)

Of the at-home games, what proportion of games were wins? (Note: Some answers are rounded to two decimal places.)

A: .12
B: .15
C: .20
D: .33
E: .42

Feedback

A : 0

✗ This is not quite right. In general, P (***H*** and ***W***) * P (***H***) has no useful meaning. Recall that you should be finding P (***W*** | ***H***), the proportion of home games which were wins. Consider the remaining options. (D) is the right answer.

B : 0

✗ This is not quite right. It seems that you have incorrectly assumed that ***H*** and ***W*** are independent events and calculated P(***H*** and ***W***) rather than calculating the proportion of home games which were wins, P (***W*** | ***H***). Consider the remaining options. (D) is the right answer.

C : 0

✗ This is not quite right. You have found P (***H*** and ***W***). Recall that you should find P(***W*** | ***H***), the proportion of home games which were wins. Consider the remaining options. (D) is the right answer.

D : 10

✓ Good job! Now we actually want to find P(***W*** |***H***). We apply the definition of conditional probability:

$$P(\mathrm{W}|\mathrm{H}) = \frac{P(\mathrm{H\ and\ W})}{P(\mathrm{H})} = \frac{.20}{.60} = .33.$$

E : 0

✗ This is not quite right. Recall that you should calculate P(***W*** |***H***), the proportion of home games which were wins. Consider the remaining options. (D) is the right answer.

Question (3)

If the team won a game, how likely is it that this was a home game? (Note: Some answers are rounded to 2 decimal places.)

A: .05

B: .12

C: .15

D: .42

E: .80

Feedback

A : 0

✗ This is not quite right. In general, P(***H*** and ***W***) * P(***W***) has no useful meaning. Recall that you want to find P (***H*** | ***W***), the probability that a win was a home game. Consider the remaining options. (E) is the correct answer.

B : 0

✗ This is not quite right. In general, P(***H*** and ***W***) * P(***H***) has no useful meaning. Recall that you should be finding P (***H*** | ***W***), the probability that a win was a home game. Consider the remaining options. (E) is the correct answer.

C : 0

✗ This is not quite right. It seems that you have incorrectly assumed that ***H*** and ***W*** are independent events and calculated P(***H*** and ***W***) rather than calculating the proportion of wins which were home games, P(***H*** |***W***). Consider the remaining options. (E) is the correct answer.

D : 0

✗ This is not quite right. Recall that you should calculate the probability that a win was a home game, P(***H*** | ***W***). Consider the remaining options. (E) is the correct answer.

E : 10

✓ Good job! Given that the team won the game (***W***), how likely is it that this was a home game (***H***)? Therefore, we are interested in P(***H*** | ***W***). Again, using the definition of conditional probability: $P(\mathrm{H}|\mathrm{W}) = \frac{P(\mathrm{H\ and\ W})}{P(\mathrm{W})} = \frac{.20}{.25} = .80.$

Question (4)

Let A and B be two independent events. If P(A) = .5, what can you say about P(A | B)?

A: Cannot find it since P(B) is not known.

B: Cannot find it since P(A and B) is not known.

C: Cannot find it since both P(B) and P(A and B) are not known.

D: It is equal to .5.

E: It is equal to .25.

Feedback

A : 0

This is not quite right. Recall that if A and B are independent, then P(A and B) = P(A) * P(B). Although you don't know P (B), you can still use the definition of conditional probability to find the correct answer. Consider the remaining options. (D) is the right answer.

B : 0

This is not quite right. Recall that if A and B events are independent, then P(A and B) = P(A) * P(B). Using the definition of conditional probability, you should be able to find the correct answer. Consider the remaining options. (D) is the right answer.

C : 0

This is not quite right. Recall the definition of conditional probability: P(A | B) = P(A and B)/P(B) . When A and B are independent, this simplifies very nicely, because P (A and B) = P(A) * P(B). Using these facts you should be able to find the correct answer. Consider the remaining options. (D) is the right answer.

D : 10

Good job! If two events are independent, then P(A | B) = P(A) [knowing that B occurs has no impact on the probability that A occurs]. Therefore, if we are given P(A) = .5, and that A and B are independent, then it must also be true that P(A | B) = .5.

E : 0

✗ This is not quite right. Recall the definition of conditional probability and consider the remaining options. (D) is the right answer.

Question (5)

Dogs are inbred for such desirable characteristics as blue eye color; but an unfortunate by-product of such inbreeding can be the emergence of characteristics such as deafness. A 1992 study of Dalmatians (by Strain and others, as reported in *The Dalmatians Dilemma*) found the following:

(i)	31% of all Dalmatians have blue eyes.
(ii)	38% of all Dalmatians are deaf.
(iii)	42% ***of blue-eyed Dalmatians*** are deaf.

Based on the results of this study is "having blue eyes" independent of "being deaf"?

A: No, since .31 * .38 is not equal to .42.
B: No, since .38 is not equal to .42.
C: No, since .31 is not equal to .42.
D: Yes, since .31 * .38 is not equal to .42.
E: Yes, since .38 is not equal to .42.

Feedback

A : 0

✗ This is not quite right. P(blue eyes and deaf) = P(deaf | blue eyes) doesn't tell us anything useful. Recall that having blue eyes and being deaf are independent if knowing a Dalmatian has blue eyes doesn't change the probability that it is deaf. Consider the remaining options. (B) is the right answer.

B : 10

✓ Good job! Using B for blue-eyed and D for deaf , the information that was given is: P(B) = .31; P(D) = .38; and P(D | B) = .42. Using this information, we can say that the events B and D are not independent, since P(D) and P(D | B are not equal. In other words, knowing the Dalmatian has blue eyes changes the probability that the Dalmatian is deaf (from .38 to .42).

C : 0

✗ This is not quite right. Knowing that P(blue eyes) ≠ P (deaf | blue eyes) doesn't tell us anything. Recall that having blue eyes and being deaf are independent if knowing a Dalmatian has blue eyes doesn't change the probability that it is deaf. Consider the remaining options. (B) is the right answer.

D : 0

✗ This is not quite right. It does not make sense to compare P(blue eyes and deaf) to P(deaf | blue eyes). Recall that having blue eyes and being deaf are independent if knowing a Dalmatian has blue eyes doesn't change the probability that it is deaf. Consider the remaining options. (B) is the right answer.

E : 0

✗ This is not quite right. Knowing that a Dalmatian has blue eyes changes the probability that it is deaf. As a result, the two events cannot be independent. Consider the remaining options. (B) is the right answer.