Random Variables Checkpoint 1

The first three questions refer to the following information:

The random variable X, representing the number of accidents in a certain intersection in a week, has the following probability distribution:

x	0	1	2	3	4	5
P(X=x)	0.20	0.30	0.20	0.15	0.10	0.05

Question (1)

What is the probability that in a given week there will be at most 3 accidents?

A: 0.70

B: 0.85

C: 0.35

D: 0.15

E: 1.00

Feedback

A : 0

This is not quite right. It seems that you have found the probability that in a given week there will be less than 3 accidents. Recall that you want to find the probability that in a given week there will be at most 3 accidents. In other words, what is the probability that in a given week there will be less than 4 accidents? Consider the remaining options. (B) is the right answer.

B : 10

Good job! $P(X \leq 3) = .20 + .30 + .20 + .15 = .85$ or $P(X \leq 3) = 1 - P(X \leq 4) = 1 - (.10 + .05) = 1 - .15 = .85$

C : 0

✗ This is not quite right. It seems that you have found the probability that in a given week there will be either 2 or 3 accidents. Recall that you want to find the probability that in a given week there will be at most 3 accidents. As a result, you must take into account the probability that in a given week there will be no accidents, 1 accident, 2 accidents, and 3 accidents. Consider the remaining options. (B) is the right answer.

D : 0

✗ This is not quite right. It seems that you have found the probability of the complementary event, that in a given week there will be more than 3 accidents. Recall that you want to find the probability that in a given week there will be at most 3 accidents. Consider the remaining options. (B) is the right answer.

E : 0

✗ This is not quite right. Remember that if an event has probability 1 of occurring, then it must contain all outcomes in the sample space that have non-zero

X probability of occurring. Notice that the probability that in a given week there are more than 3 accidents is non-zero. This means that the probability that in a given week there are at most 3 accidents must not be equal to 1. Consider the remaining options. (B) is the right answer.

Question (2)

By the third day of a particular week, 2 accidents have already occurred in the intersection. What is the probability that there will be less than a total of 4 accidents during that week?

A: 1.00
B: 0.90
C: 0.85
D: 0.70
E: 0.50

Feedback

A : 0

✗ This is not quite right. The fact that 2 accidents have occurred over the course of the first 3 days in a week does not imply that these are the only accidents that will occur in the intersection for that entire week. However, this information does imply that there must be at least 2 accidents over the course of that week, because 2 accidents have already occurred. Consider the remaining options. (D) is the right answer.

B : 0

✗ This is not quite right. It seems that you have found: given that 2 accidents have already occurred, the probability that less than 5 accidents will occur in that week. Recall that you want to find: given that 2 accidents have already occurred, the probability that less than 4 accidents will occur in the entire week. Consider the remaining options. (D) is the right answer.

C : 0

This is not quite right. It seems that you have found the probability that in a given week there are at most 3 accidents. Recall that you are given the fact that two accidents have occurred in the first 3 days and you want to find the probability that there are less than 4 accidents in the week. You should be conditioning in some way on the fact that 2 accidents have already occurred during the week. Consider the remaining options. (D) is the right answer.

D : 10

Good job! We are given that 2 accidents have already happened. In other words, we are given Xâ‰¥2 and we need to find how likely X is to be less than 4.

$$P(X < 4 \mid X \geq 2) = \frac{P(X < 4 \text{ and } X \geq 2)}{P(X \geq 2)} = \frac{P(X = 2 \text{ or } X = 3)}{P(X \geq 2)} = \frac{.35}{.50} = .70$$

E : 0

This is not quite right. It seems that you have found the probability that in a given week there are at least 2 accidents. Recall that you want to find, given that 2 accidents have already occurred, the probability that less than 4 accidents will occur in the entire week. Consider the remaining options. (D) is the right answer.

Question (3)

On average, how many accidents are there in the intersection in a week?

A: 5.3
B: 2.5
C: 1.8
D: 0.30
E: 0.1667

Feedback

A : 0

This is not quite right. Notice that the maximum number of accidents allowed in a week in this distribution is 5. This implies that the average number of accidents in a week must be at most 5. Consider the remaining options. (C) is the right answer.

B : 0

This is not quite right. It seems that you have weighted each of the possible numbers of accidents in a week equally. Remember that the average, or mean, of a distribution is a weighted sum of the possible outcomes, in which each weight is equal to the probability of that outcome occurring. Consider the remaining options. (C) is the right answer.

C : 10

Good job! We need to find the mean of X, μ_x. $\mu_x = 0 * .20 + 1 * .30 + 2 * .20 + 3 * .15 + 4 * .10 + 5 * .05 = 1.8$

D : 0

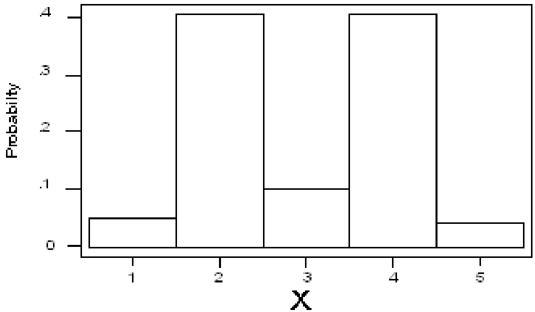
This is not quite right. The average of a distribution is generally not the same as the most likely outcome of that distribution. Remember that the average, or mean, of a distribution is a weighted sum of the possible outcomes, where each weight is equal to the probability of that outcome occurring. Consider the remaining options. (C) is the right answer.

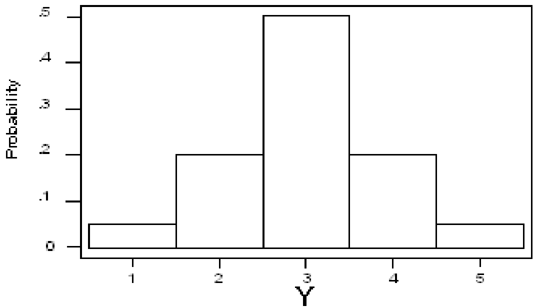
E : 0

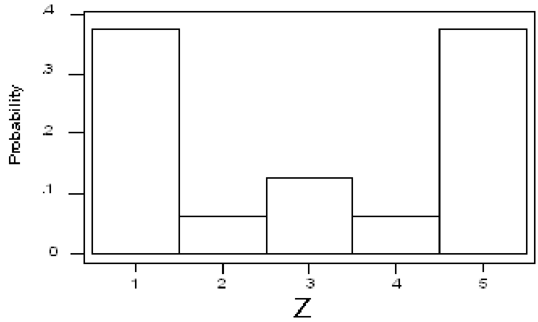
This is not quite right. The average of a distribution is generally not the same as 1/|S|, where S is the sample space and |S| represents the number of outcomes in the sample space. Remember that the average, or mean, of a distribution is a weighted sum of the possible outcomes where each weight is equal to the probability of that outcome occurring. Consider the remaining options. (C) is the right answer.

Question (4)

The following three histograms represent the probability distributions of the three random variables X, Y, and Z.







Which of the three random variables has the largest standard deviation?

A: X

B: Y

C: Z

D: All three random variables have the same standard deviation.

E: It is impossible to tell from the histograms.

Feedback

A : 0

✗ This is not quite right. All of these variables take on the same values, but the one with the largest standard deviation will take on values further from the mean much more often than values closer to the mean. Consider the remaining options. (C) is the right answer.

B : 0

✗ This is not quite right. All of these variables take on the same values, but the one with the largest standard deviation will take on values further from the mean much more often than values closer to the mean. Consider the remaining options. (C) is the right answer.

C : 10

✓ Good job! All three random variables take the values 1, 2, 3, 4, 5, and it is pretty easy to see (by symmetry) that the mean of all three random variables is 3. The random variables are different, though, with respect to how likely they are to have values that are "far" from the mean. We see that out of the three random variables, random variable Z is the most likely to have values 1 and 5

✓ (which are the furthest from the mean), and therefore Z has the largest standard deviation.

D : 0

✗ This is not quite right. Although the random variables take on the same values, they do not have equal standard deviations. For example, random variable X will have a larger standard deviation than random variable Y because it takes on values 2 and 4 more often. Consider the remaining options. (C) is the right answer.

E : 0

✗ This is not quite right. Remember that the standard deviation of a random variable is a measure of how far from the mean common values of the random variable are located. Since the three random variables do not take on values with the same probability, you should be able to get a sense of which random variable has the highest standard deviation by looking at the histograms. Consider the remaining options. (C) is the right answer.