Random Variables Checkpoint 4

Color-blindness is any abnormality of the color vision system that causes a person to see colors differently than most people or to have difficulty distinguishing among certain colors (www.visionrx.xom).

Color-blindness is gender-based, with the majority of sufferers being males.

Roughly 8% of white males have some form of color-blindness, while the incidence among white females is only 1%.

A random sample of 20 white males and 40 white females was chosen.

Let ***X*** be the number of males (out of the 20) who are color-blind.

Let ***Y*** be the number of females (out of the 40) who are color-blind.

Let ***Z*** be the total number of color-blind individuals in the sample (males and females together).

Question (1)

Which of the following is true regarding the random variables X and Y?

A: Both X and Y can be well-approximated by normal random variables.

B: Only X can be well-approximated by a normal random variable.

C: Only Y can be well-approximated by a normal random variable.

D: Neither X nor Y can be well-approximated by a normal random variable.

Feedback

A : 0

X This is not quite right. Remember that a binomial random variable with parameters n and p can be well-approximated by a normal random variable if both n * p ≥ 10 and n * (1 - p) ≥ 10. Consider the remaining options. (D) is the right answer.

B : 0

X This is not quite right. Remember that a binomial random variable with parameters n and p can be well-approximated by a normal random variable if both n * p ≥ 10 and n * (1 - p) ≥ 10. Consider the remaining options. (D) is the right answer.

C : 0

X This is not quite right. Remember that a binomial random variable with parameters n and p can be well-approximated by a normal random variable if both n * p ≥ 10 and n * (1 - p) ≥ 10. Consider the remaining options. (D) is the right answer.

D : 10

✓ Good job! A binomial random variable will be well-approximated by a normal random variable if the following two conditions hold: $n * p \geq 10$ and $n * (1 - p) \geq 10$. In our case, for ***X***, $n = 20$, $p = 0.08$, and $20 * 0.08 < 10$ → cannot be well-approximated by a normal random variable. For ***Y***, $n = 40$, and $p = 0.01$, and $40 * 0.01 < 10$ → cannot be well-approximated by a normal random variable.

The remaining questions refer to the following information:

Suppose the scores on an exam are normally distributed with a mean $\mu = 75$ points, and standard deviation $\sigma = 8$ points.

Question (2)

The instructor wanted to "pass" anyone who scored above 69.

What proportion of exams will have passing scores?

A: .25
B: .75
C: .2266
D: .7734
E: -.75

Feedback

A : 0

This is not quite right. You want to find the proportion of exams that have a score above 69 points. This is equivalent to finding the probability that a given exam has a score of at least 69 points. Consider the remaining options. (D) is the right answer.

B : 0

This is not quite right. You want to find the proportion of exams that have a score above 69 points. This is equivalent to finding the probability that a given exam has a score of at least 69 points. Consider the remaining options. (D) is the right answer.

C : 0

This is not quite right. It seems that you have found the proportion of exams that do not have a score above 69 points. You want to find the proportion of exams that have scores above 69 points. Consider the remaining options. (D) is the right answer.

D : 10

Good job! Let the random variable ***X*** represent the score on the exam. We are given that ***X*** is normal with a mean of 75 and standard deviation of 8, and need to find P(X < 69).

$$P(\mathbf{X} > 69) = P(\mathbf{Z} > \frac{69-75}{8}) = P(\mathbf{Z} > -0.75) = 1 - P(\mathbf{Z} < -0.75)$$

$$= (\text{table entry for } -0.75 \text{ is } .2266) = 1 - .2266 = .7734$$

E : 0

This is not quite right. The z-score for 69 points is -0.75.

✗ However, this is not the proportion of exams that passed. Remember that a proportion canâ€™t be negative. You should find P(***Z*** > -0.75). Consider the remaining options. (D) is the right answer.

Question (3)

What is the exam score for an exam whose z-score is 1.25?

A: 65

B: 75

C: 85

D: .8944

E: .1056

Feedback

A : 0

✗ This is not quite right. Since the z-score is positive, the exam score associated with this z-score should be greater than the mean exam score. Consider the remaining options. (C) is the right answer.

B : 0

✗ This is not quite right. Since the z-score is positive, the exam score associated with this z-score should be greater than the mean exam score. Consider the remaining options. (C) is the right answer.

C : 10

✓ Good job! A z-score of 1.25 means that the actual exam score is 1.25 standard deviations above the mean, and therefore the exam score we are looking for is: mean + 1.25 * SD = 75 + 1.25 * 8 = 85.

D : 0

✗ This is not quite right. It seems that you have found the proportion of exams that have an associated z-score of

X at most 1.25. However, you should find the exam score associated with a z-score of 1.25. Consider the remaining options. (C) is the right answer.

E : 0

X This is not quite right. It seems that you have found the proportion of exams that have an associated z-score of at least 1.25. However, you should find the exam score associated with a z-score of 1.25. Consider the remaining options. (C) is the right answer.

Question (4)

Suppose that the top 4% of the exams will be given an $\boldsymbol{A^+}$. In order to be given an $\boldsymbol{A^+}$, an exam must earn at least what score?

A: 61

B: 73

C: .516

D: 77

E: 89

Feedback

A : 0

X This is not quite right. The top 4% of exam scores should have a minimum exam score of at least the mean of all exam scores. Consider the remaining options. (E) is the right answer.

B : 0

X This is not quite right. The top 4% of exam scores should have a minimum exam score of at least the mean of all exam scores. Consider the remaining options. (E) is the right answer.

C : 0

X This is not quite right. It seems that you have found the proportion of exams that have a score of at most 79 points. However, you should find the minimum exam score that would allow an exam to earn an A^+. Consider the remaining options. (E) is the right answer.

D : 0

This is not quite right. It seems that you have found the minimum exam score in order for an exam to be in the top 40% of all exam scores. However, you should find the minimum exam core that would allow an exam to earn an A^+. Consider the remaining options. (E) is the right answer.

E : 10

Good job! We need to find the exam score such that the probability of getting a score above it is 0.04. Equivalently (and more practical, given the way our table works) we need to find the exam score such that the probability of getting a score below it is 1 - 0.04 = 0.96. Looking in the body of the table for the table entry that is closest to 0.96 (which is 0.9599) we learn that the exam score that we are looking for has a z-score of 1.75. This means that the exam score that we are looking for is

1.75 * SD above the mean, and therefore is: 75 + 1.75 * SD = 75 + 14 = 89.