Conditional Probability and Independence Checkpoint 2

Question (1)

Dogs are inbred for such desirable characteristics as blue eye color; but an unfortunate by-product of such inbreeding can be the emergence of characteristics such as deafness. A 1992 study of Dalmatians (by Strain and others, as reported in *The Dalmatians Dilemma*) found the following:

(i)	31% of all Dalmatians have blue eyes.
(ii)	38% of all Dalmatians are deaf.
(iii)	42% of blue-eyed Dalmatians are deaf.

What is the probability that a randomly chosen Dalmatian is blue-eyed *and* deaf?

A: .31 * .38 = .1178
B: .31 * .42 = .1302
C: .38 * .42 = .1596
D: .31 / .38 = .8158
E: .31 / .42 = .7381
F: .38 / .42 = .9048

Feedback

A : 0

✗ This is not quite right. The question implies that "having blue eyes" and "being deaf " are dependent events. As a result, you cannot apply the Multiplication Rule for Independent Events. Consider the remaining options. (B) is the right answer.

B : 10

Good job! We need to find P(B and D). Using the General Multiplication Rule, P(B and D) = P(B) * P(D | B) = .31 * .42 = .1302.

C : 0

This is not quite right. Recall that the General Multiplication Rule states the following: P(B and D) = P(B) * P(D | B) = P(D) * P(B | D). Consider the remaining options. (B) is the right answer.

D : 0

This is not quite right. Recall that the General Multiplication Rule states the following: P(B and D) = P(B) * P(D | B) = P(D) * P(B | D). Consider the remaining options. (B) is the right answer.

E : 0

This is not quite right. Recall that the General Multiplication Rule states the following: P(B and D) = P(B) * P(D | B) = P(D) * P (B | D). Consider the remaining options. (B) is the right answer.

F : 0

This is not quite right. Recall that the General Multiplication Rule states the following: P(B and D) = P(B) * P(D | B) = P(D) * P (B | D). Consider the remaining options. (B) is the right answer.

The next four questions refer to the following information:

Two methods, A and B, are available for teaching a certain industrial skill. There is an 80% chance of successfully learning the skill if method A is used, and a 95% chance of success if method B is used. However, method B is substantially more expensive and is therefore used only 25% of the time (method A is used the other 75% of the time). The following notations are suggested:

- ***A***—method ***A*** is used
- ***B***—method ***B*** is used
- ***L***—the skill was ***L***earned successfully

Question (2)

Which of the following is the correct representation of the information that is provided to us?

A: P(A) = .75, P(B) = .25, P(L | A) = .80, P(L | B) = .95

B: P(A) = .75, P(B) = .25, P(A | L) = .80, P(B | L) = .95

C: P(A) = .75, P(B) = .25, P(A and L) = .80, P(B and L) = .95

D: P(A | L) = .75, P(B | L) = .25, P(L | A) = .80, P(L | B) = .95

E: P(A and L) = .75, P(B and L) = .25, P(L | A) = .80, P(L | B) = .95

Feedback

A : 10

Good job! There is an 80% chance of learning the skill if method ***A*** is used â†’ P(***L***|***A***) = .80. There is a 95% chance of learning the skill if method ***B*** is used â†’ P(***L***|***B***) = .95. Method ***B*** is used 25% of the time â†’ P(B) = .25 â†’ P(not B) = P(***A***) = .75.

B : 0

This is not quite right. Recall that if method ***A*** is used, then there is an 80% chance of successfully learning the skill and if method ***B*** is used, then there is a 95% probability of learning the skill. In other words, given that you have used method ***A*** or ***B***, you will learn the skill with probability 80% or 95%, respectively. Consider the remaining options. (A) is the right answer.

C : 0

This is not quite right. P(***A*** and ***L***) represents the probability that a randomly chosen person has both learned the skill and used method ***A***. P(***B*** and ***L***) represents the probability that a randomly chosen person has both learned the skill and used method ***B***. Recall that given method ***A*** is used, the probability of successfully learning the skill is 80%. Also, given that method ***B*** is used, the probability of successfully learning the skill is 95%. Consider the remaining options. (A) is the right answer.

D : 0

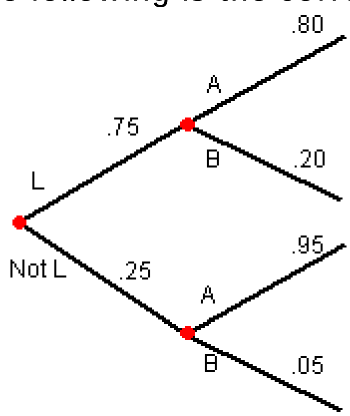
This is not quite right. P(***A*** | ***L***) represents the probability that a randomly chosen person that learned the skill successfully has learned the skill using method ***A***. Similarly, P(***B*** | ***L***) represents the probability that a randomly chosen person that learned the skill successfully has learned the skill using method ***B***.Recall that method ***A*** is used 75% of the time because it costs less than using method ***B***, which is used 25% of the time. Consider the remaining options. (A) is the right answer.

E : 0

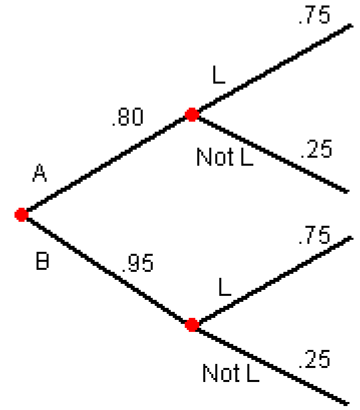
This is not quite right. P(***A*** and ***L***) represents the probability that a randomly chosen person has both learned the skill and used method ***A***. P(***B*** and ***L***) represents the probability that a randomly chosen person has both learned the skill and used method ***B***. Recall that methods ***A*** and ***B*** are used 75% and 25% of the time, respectively. Consider the remaining options. (A) is the right answer.

Question (3)

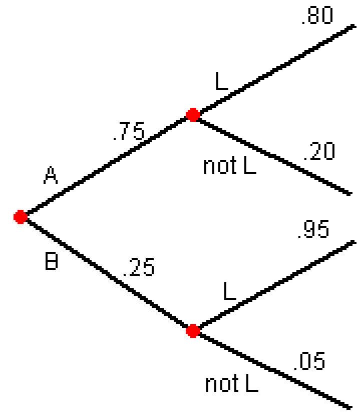
Which of the following is the correct probability tree for this problem?



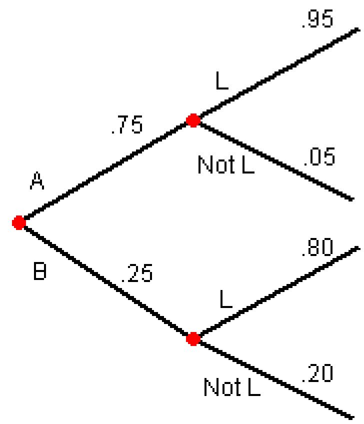
A:



B:



C:



D:

Feedback

A : 0

✗ This is not quite right. In this case, the events occur in stages: first you must choose which method to use, second you must see whether the person successfully learns the skill or not. This ordering should be exemplified in the probability tree by putting the first stage before the second. Consider the remaining options. (C) is the right answer.

B : 0

✗ This is not quite right. Remember that probability trees must have the following property: the sum of the probabilities of each branch coming from a common point must be 1. This means that if you have two branches coming out of a single point, the probability of branch 1 plus the probability of branch 2 must be 1. Consider the remaining options. (C) is the right answer.

C : 10

Good job! The first stage is choosing the method, ***A*** or ***B***. Only in options (C) and (D) does the tree start with these branches and the right probabilities. The second stage is either learning or not learning the skill. Note that only in option (C) are the probabilities correct for this stage. (In option (D), P(***L*** | ***A***) and P (***L*** | ***B***) are switched).

D : 0

This is not quite right. Remember that the second set of branches in the tree must be have conditional probabilities (from top to bottom) of: P(***L*** | ***A***), P(not ***L*** | ***A***), P (***L*** | ***B***), and P (not ***L*** | ***B***). Consider the remaining options. (C) is the right answer.

Question (4)

What is the probability that a worker will learn the skill successfully?

A: P(L) = .75 * .80 = .60

B: P(L) = .25 * .95 = .2375

C: P(L) = .75 * .25 + .80 * .95 = .9475

D: P(L) = .75 * .95 + .25 * .80 = .9125

E: P(L) = .75 * .80 + .25 * .95 = .8375

Feedback

A : 0

This is not quite right. It seems that you have calculated P(***A*** and ***L***). However, P(***L***) = P(***A*** and ***L***) + P (not ***A*** and ***L***) = P(***A*** and ***L***) + P(***B*** and ***L***). Recall that you have four pieces of information: P(***A***) = .75, P(***B***) = .25, P (***L*** | ***A***) = .80, P(***L*** | ***B***) = .95. Using these, you can apply the Law of Total Probability to find P (***L***). Consider the remaining options. (E) is the right answer.

B : 0

This is not quite right. It seems that you have calculated P(***B*** and ***L***). However, P(***L***) = P(***B*** and ***L***) + P(not ***B*** and ***L***) = P(***B*** and ***L***) + P(***A*** and ***L***). Recall that you have four pieces of information: P(***A***) = .75, P(***B***) = .25, P(***L*** | ***A***) = .80, P(***L*** | ***B***) = .95. Using these, you can apply the Law of Total Probability to find P (***L***). Consider the remaining options. (E) is the right answer.

C : 0

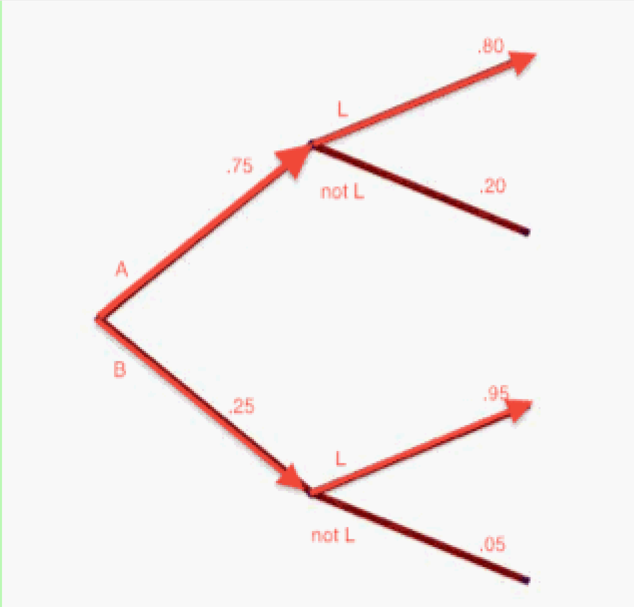
This is not quite right. Recall that you have four pieces of information: P(***A***) = .75, P(***B***) = .25, P(***L*** | ***A***) = .80, P(***L*** | ***B***) = .95. Using these, you can apply the Law of Total Probability to find P(***L***). Consider the remaining options. (E) is the right answer.

D : 0

This is not quite right. Recall that you have four pieces of information: P(***A***) = .75, P(***B***) = .25, P(***L*** |***A***) = .80, P(***L*** | ***B***) = .95. Using these, you can apply the law of total probability to find P(***L***). Consider the remaining options. (E) is the right answer.

E : 10

Good job! We need to find P(***L***). Using the tree, we need to identify the "paths" in the tree to â€˜***L***â€™, multiply the probabilities along each of those paths, and then add the resulting probabilities. In this case P(***L***) = .75 * .80 + .25 * .95 = .60 + .2375 = .8375.



Question (5)

A worker learned the skill successfully. What is the probability that he was taught by method A?

A: .75 * .80 = .60

B: .80

C: $\frac{.25*.95}{.75*.80+.25*.95} = .2836$

D: $\frac{.75*.80}{.75*.80+.25*.95} = .7164$

E: $\frac{.75*.80}{.8+.95} = .3429$

Feedback

A : 0

This is not quite right. It seems that you have calculated P(***A*** and ***L***). Remember that you are interested in finding P(***A*** | ***L***). You may need to apply the definition of conditional probability more than once to find the correct answer. Consider the remaining options. (D) is the right answer.

B : 0

✗ This is not quite right. It seems that you have found P (***L*** | ***A***) rather than P (***A*** | ***L***). You may need to apply the definition of conditional probability more than once to find the correct answer. Consider the remaining options. (D) is the right answer.

C : 0

✗ This is not quite right. It seems that you have calculated P (***B*** | ***L***) rather than P (***A*** | ***L***). Consider the remaining options. (D) is the right answer.

D : 10

✓ Good job! We need to find P (***A*** |***L***). By definition,

$$P(\mathbf{A}|\mathbf{L}) = \frac{P(\mathbf{A} \text{ and } \mathbf{L})}{P(\mathbf{L})}.$$

Now,

- $P(\mathbf{A}$ and $\mathbf{L})$ is obtained by multiplying the probabilities along the "**A**-**L**" path.
- $P(\mathbf{L})$ was found in the previous problem.

Putting it all together, we get:

$$P(\mathbf{A}|\mathbf{L}) = \frac{P(\mathbf{A} \text{ and } \mathbf{L})}{P(\mathbf{L})} = \frac{.75 \cdot .80}{.75 \cdot .80 + .25 \cdot .95} = .7164$$

E : 0

✗ This is not quite right. This calculation has no real meaning. Remember that you are interested in finding P (***A*** | ***L***). You may need to apply the definition of conditional probability more than once to find the correct answer. Consider the remaining options. (D) is the right answer.