Unit 01:System of Forces

Mechanics: It can be defined as, "branch of physical science which deals with study of resultant effect of forces acting on the body, when the body is at rest or in motion."

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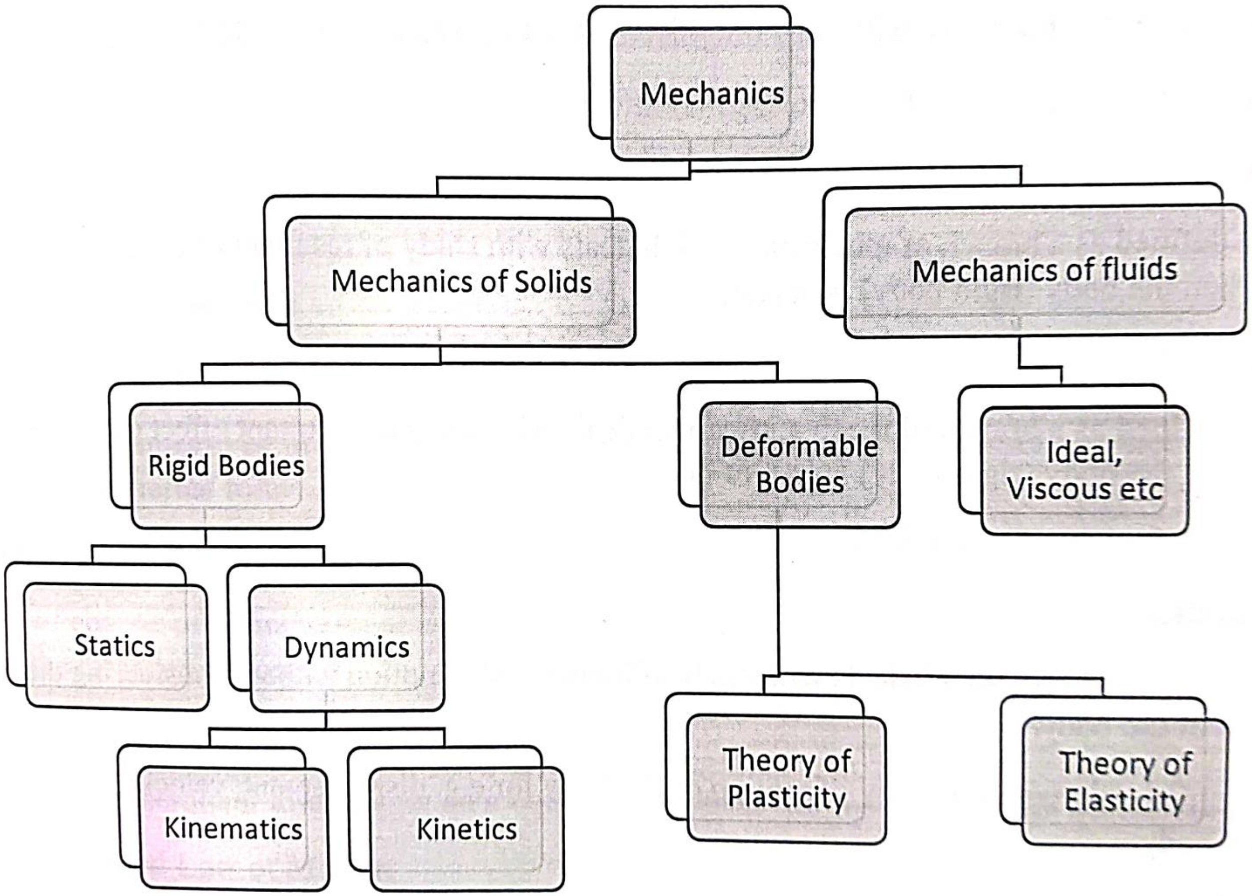
Classification of Mechanics:



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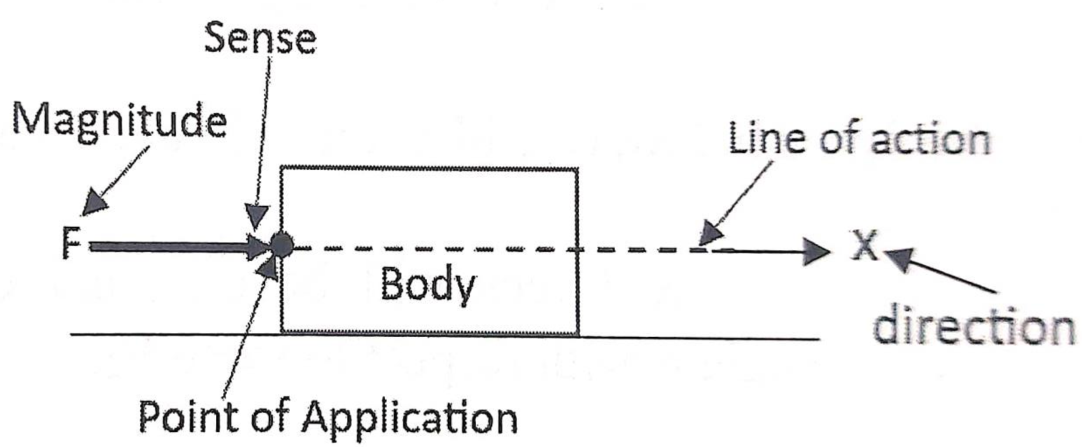
Force: It is defined as an external agency which changes or tends to change the state of rest or uniform motion of the body.

It is a vector quantity and is characterized by the following:

1. Magnitude
2. Direction
3. Sense
4. Point of Application



Unit of force: “N”, “kN”

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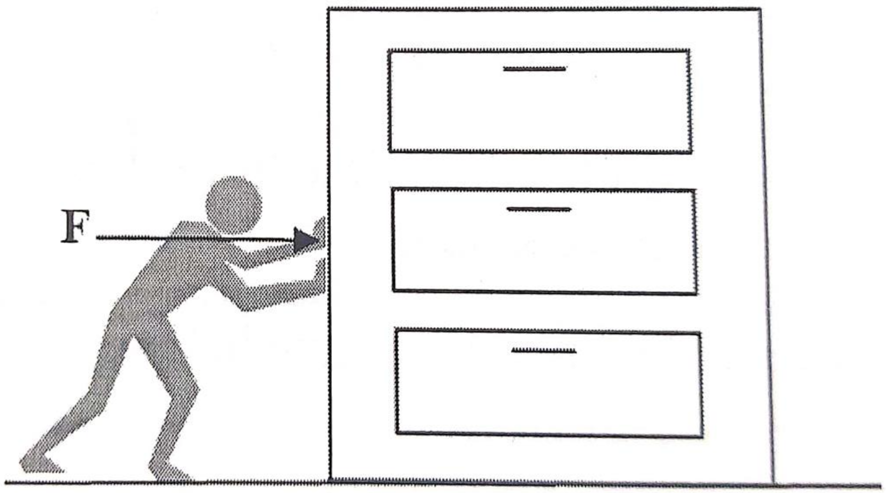
Force:



As shown in the figure above, a man trying to push a cupboard applies a force of magnitude F, whose direction is along x-axis (horizontal), sense is towards right and point of application is around middle of the cupboard.

Unit of Force: The unit of force is Newton (N).

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Fundamental principles

1. Newton's First Law of Motion

Every body continues to be in the state of rest or uniform motion along a straight line unless it is acted upon by some external force.

2. Newton's Second Law of Motion

The rate of change of momentum of a body is directly proportional to the force acting on it and take place in the direction of force.

Mathematically, $F = m * a$

Where F is the resultant force acting on a body having mass "m", moving with an acceleration "a"

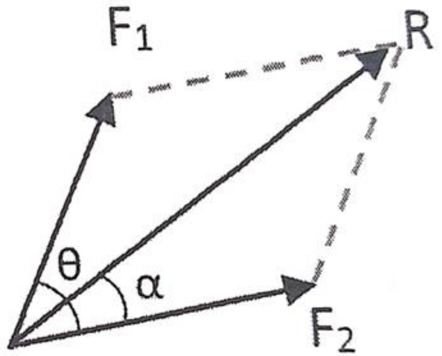
3. Newton's Third Law of Motion

For every action in nature (force) there has an equal and opposite reaction.

Parallelogram law of Forces

Parallelogram law of forces states that,

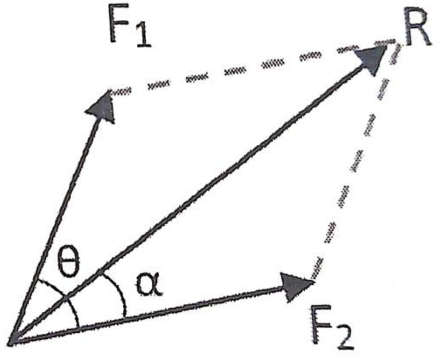
if two forces are acting simultaneously on a point and are represented in magnitude and direction by the adjacent sides of parallelogram, then the resultant of these two forces is represented in magnitude and direction by the diagonal of parallelogram passing through point of intersection of two sides representing the forces.



Parallelogram law of Forces

As shown in the figure below, consider forces F1 and F2 acting simultaneously at a point making angle Ө with each other.

The resultant R of these forces will be diagonal of parallelogram passing through point of intersection making angle α with respect to force F2.



The magnitude of the resultant is given by,

The magnitude of the resultant is given by,

$$R = \sqrt{F_1{}^2 + F_2{}^2 + 2\,F_1 F_2\,cos\theta}$$

Direction of the resultant is given by,

$$\tan\alpha = \frac{F_1\,sin\theta}{F_2 + F_1\,cos\theta}$$

Application of Engineering Mechanics in various engineering fields

Science and engineering are interconnected with respect to the course of engineering mechanics.

In civil engineering, mechanics concepts are applied to structural design and a variety of domains like structural, coastal, geotechnical, construction, and earthquake engineering.

In mechanical engineering, it is applied in domains like mechatronics and robotics, design and drafting, nanotechnology, machine elements, structural analysis etc.

In aerospace engineering, mechanics is used in domains like aerodynamics, aerospace structural mechanics and propulsion, aircraft design and flight mechanics

Application of Engineering Mechanics in various engineering fields

In materials engineering, mechanics concepts are used in thermo elasticity, elasticity theory, fracture and failure mechanisms, structural design optimization, fracture and fatigue, composites etc.

Research in mechanics is also applied in biomedical engineering areas of interest like orthopedics, biomechanics, human body motion analysis, soft tissue modelling of muscles, tendons, ligaments, and cartilage.

In Computer Science Engineering, simulation and modelling play a crucial role in designing and testing systems before implementation. Knowledge of engineering mechanics can help students develop more accurate and realistic simulations by considering physical constraints and behaviors.

Application of Engineering Mechanics in various engineering fields

In robotics and mechatronics, a strong foundation in engineering mechanics is essential. Understanding how forces, motion, and materials interact is crucial for designing and building robots, automated systems, and mechanical devices.

Engineering mechanics may not be a core subject in computer science and other allied branches but it provides students with valuable skills, knowledge, and perspectives that can enhance their understanding of complex systems and improve their problem-solving abilities in interdisciplinary projects.

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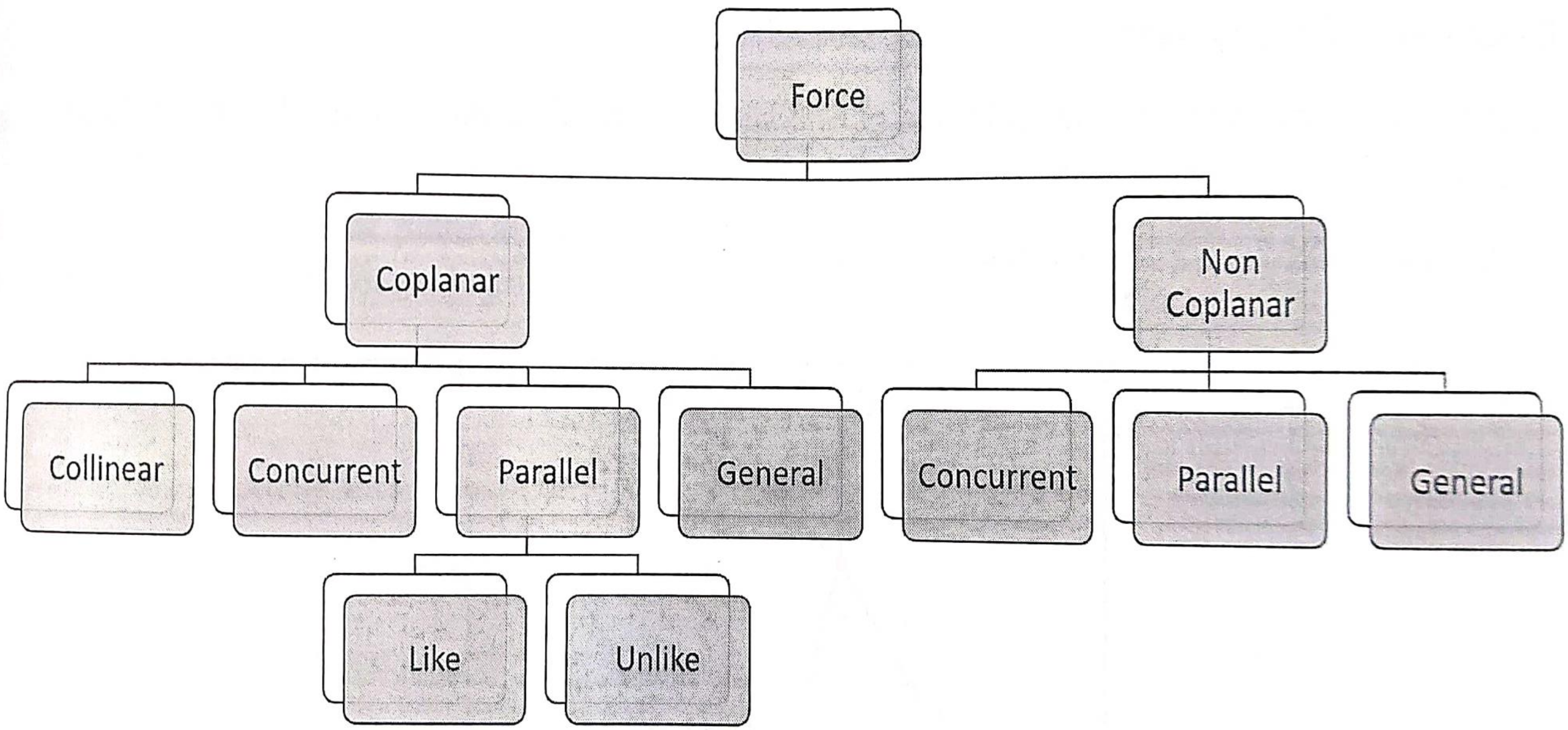
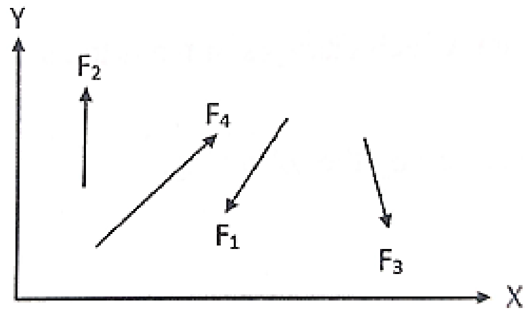
Classification of System of Force



A) Coplanar Force system

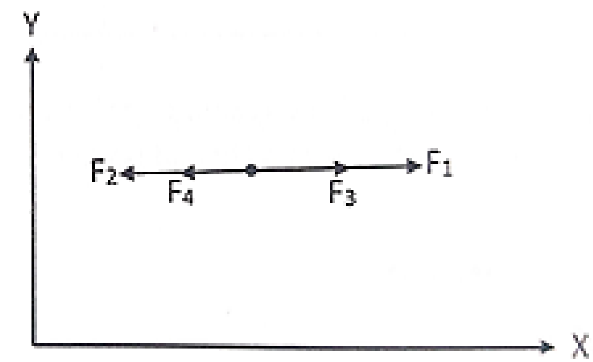
When the **forces lie in the same plane**, then the system of forces is known as coplanar force system



Coplanar force system is further divided into following force systems.

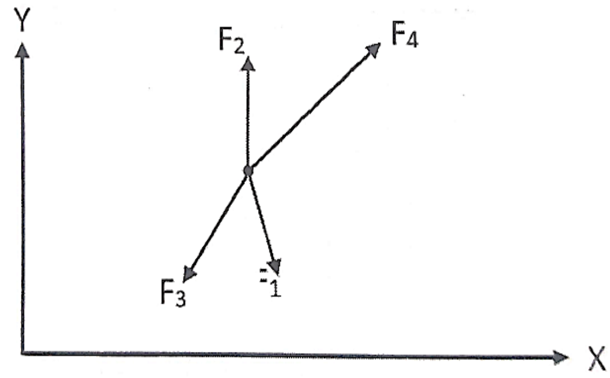
I) Collinear force system

When the forces lie in same plane and **line of action of the forces lies in the same line**, then the system of forces is known as collinear coplanar force system.



II) Concurrent force system

When the forces lie in same plane and **meet at one single point**, then the system of forces is known as concurrent coplanar force system.

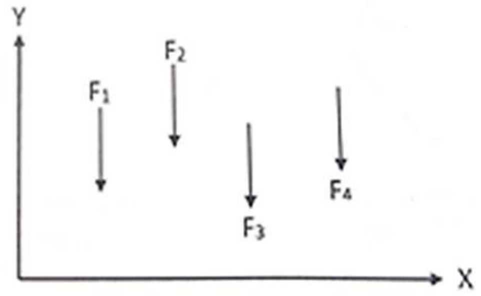


III) Parallel force system

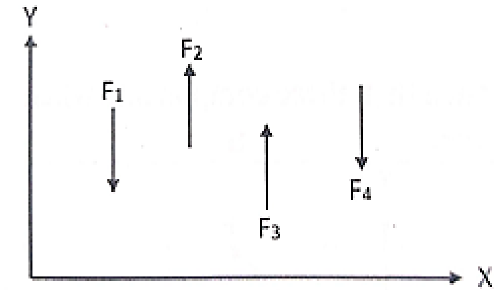
When the forces lie in same plane and **line of action are parallel to each other**, then the system of forces is known as parallel coplanar force system.

Parallel force system is further classified into following force systems.

i) Like parallel force system: When **line of action of all the forces are parallel to each other** and are **in same direction** then the system of forces is known as like parallel coplanar force system.



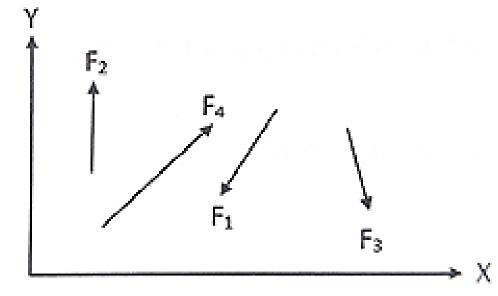
ii) Unlike parallel force system: When line of action of all the forces are parallel to each other and are **in opposite direction** then the system of forces is known as Unlike parallel coplanar force system.



IV) General force system

When the forces **lie in same plane** and are **neither parallel to each other** nor passing through a single point, then the system of forces is known as

General coplanar force system.

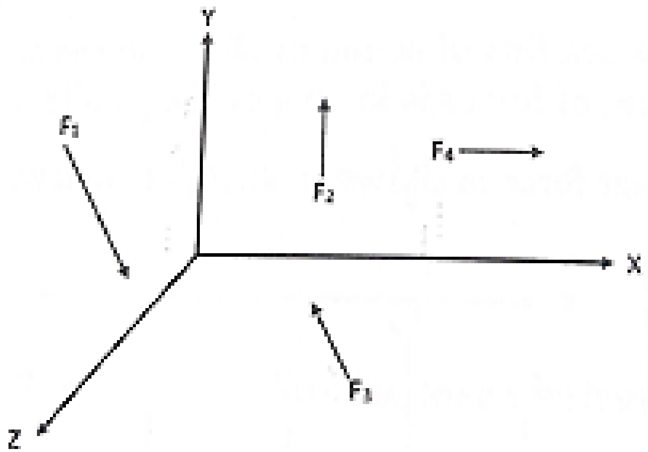


B) Non-Coplanar Force system

When the forces **do not lie in the same plane**, then the system of forces is known as non-coplanar force system.

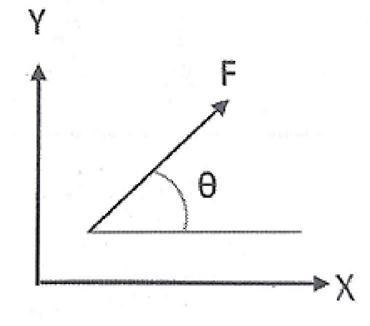
Non- Coplanar forces are also further classified into.

I) Concurrent force system
II) Parallel force system
III) General force system

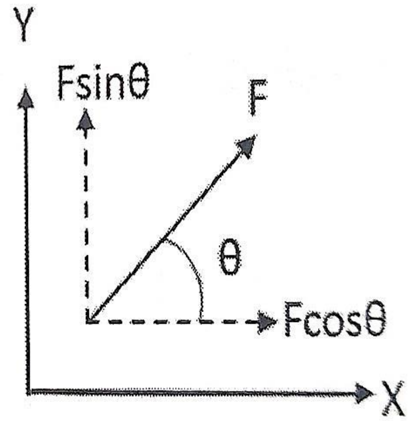


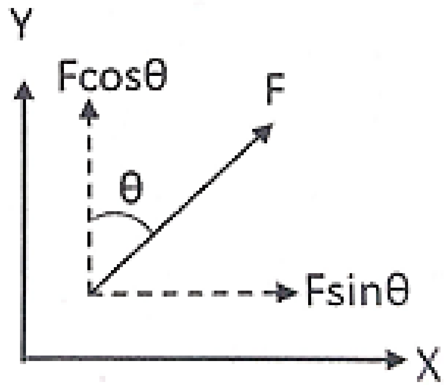
Resolution of Force

Splitting the force into components such that, these components when combined together would have the same effect as that of original force.



Consider a force of magnitude F, acting at an angle ϴ with respect to x-axis, this force can be resolved into two components **Fx** (along x-axis) and **Fy** (along y-axis).

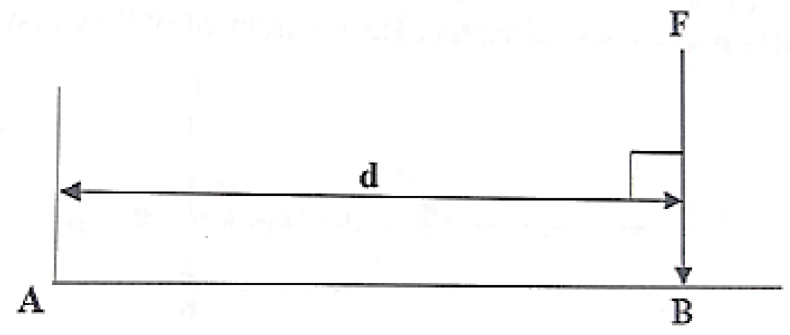




Moment of Force (M)

The rotational effect of the force on the body is known as moment.

- The point about which the moment is taken is termed as moment centre.
- The rotational effect of the force will vary from one moment centre to another moment centre.
- **The moment of a force is calculated by taking the product of the force and the perpendicular distance of the force** from the moment centre.
- This perpendicular distance from moment centre to the force is known as moment arm and is denoted by, "d"



Mathematically, **Moment = Force * Perpendicular distance from moment centre to the force.**

or **$M = F * d$**

Unit of Moment: The **unit** of Moment is **Nm** or **Nmm** or **kNm** or **kNmm**

For perpendicular distance: If the force is along x-axis (horizontal) than the perpendicular distance to be measured along y axis (vertical), If the force is along y-axis (vertical) than the perpendicular distance to be measured along x axis (horizontal).

Note: When the line of action of the force passes through moment centre than the moment of that force about that moment centre will be zero.

Classification of moment

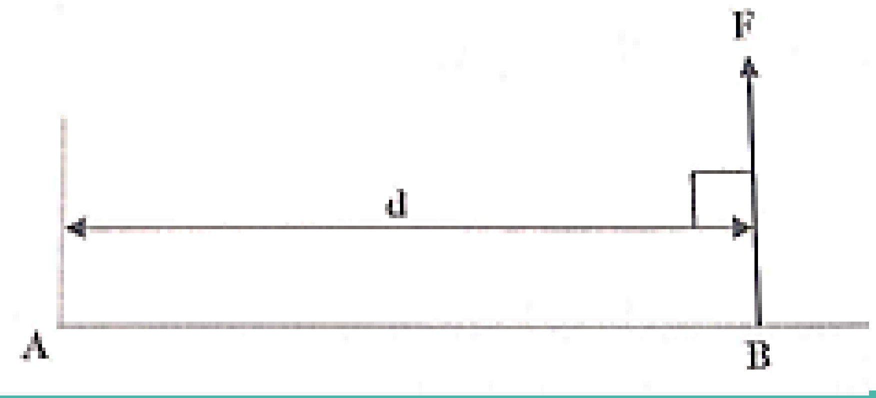
Based on the sense of rotation, moment is classified into following moments.

A) Anticlockwise Moment

When the force is having a tendency to rotate the body into anticlockwise direction, then it is considered to be as anticlockwise moment.

Anticlockwise moment will be considered as **positive.**

As shown in the figure below, if we consider point A as the moment centre, Force F is having the tendency to rotate the body in anticlockwise direction, hence moment will be considered positive.



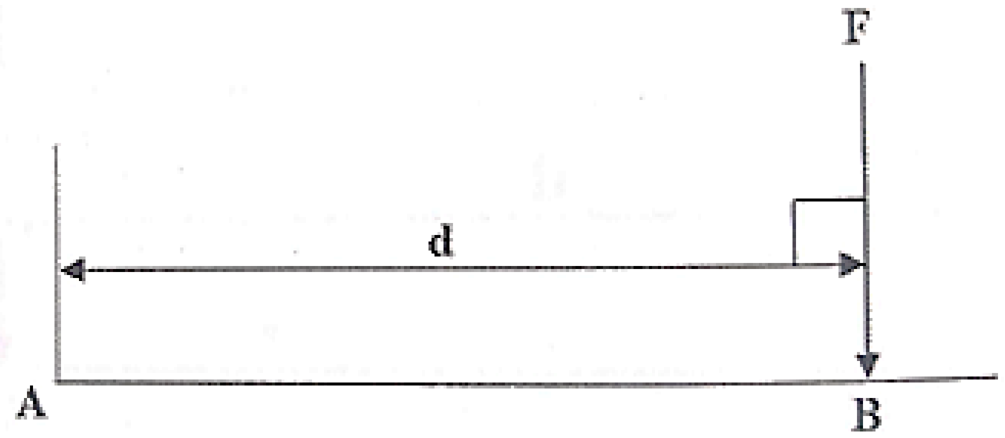
Classification of moment

B) Clockwise Moment

When the force is having a tendency to rotate the body into clockwise direction, then it is considered to be as clockwise moment.

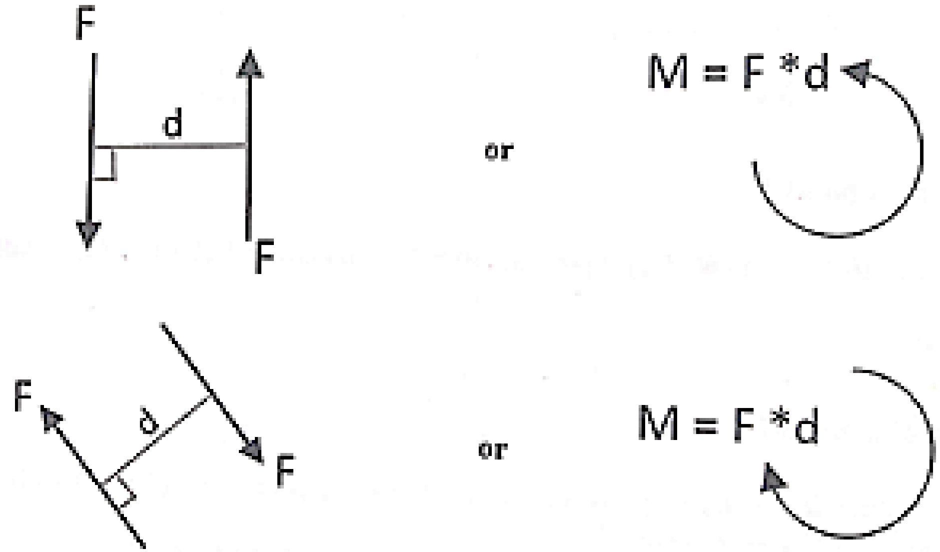
Clockwise moment will be considered as **negative.**

As shown in the figure below, if we consider point A as the moment centre, Force F is having the tendency to rotate the body in clockwise direction. Hence moment will be considered negative.



Couple

- Two parallel forces of equal magnitude, opposite in direction, separated by a distance form a couple.
- The effect of the couple is to rotate the body on which it is acting.
- Couple is a special case of a parallel force system.
- Figure below shows a couple formed by two forces which are equal in magnitude, opposite in direction and separated by perpendicular distance d (arm of couple).



Properties of Couple

1. Couple **causes rotation of body** about an axis perpendicular to the plane containing the two parallel forces.
2. The **magnitude** of rotation or moment caused by the couple is equal to **the product of magnitude of one of the forces** and **the arm** of the couple.
3. Couple is **a free vector** and can be moved anywhere on the body on without causing any effect.
4. The **resultant** of forces forming a couple is **zero**
5. A couple **cannot be balanced by a single force**, but can be balanced by a couple of opposite sense.
6. **Any number of coplanar couples** can be **replaced** by **single couple** of magnitude equal to algebraic sum of all couples.

Note: 1. A couple should be considered while calculating the moment even if it passes through a point at which the moment is being taken (moment centre).

2. Shifting of force from one point to another

To shift a force to a new position, a couple is required to be added to the system.

The magnitude of the couple added will be product of force and the distance by the which it is shifted.

Varignon’s Theorem

It states that, **the algebraic sum of moment of all forces in the system about a point is equal to the moment of the resultant force of the system about the same point.**

Mathematically, $\Sigma M_A^F = M_A^R$

considering A as the point about which the moment is taken.

Where F; are the forces acting on the body and

R; is the resultant force.

Steps to solve problems on resultant of concurrent force system

1. Find angle of inclined forces if not given, by geometry.
2. Resolve the inclined forces, if any.
3. Find $\sum$ Fx , $\sum$ Fy, R and Θ
4. Resultant is given by R = $\sqrt{\sum Fx^2 + \sum Fy^2}$
5. Θ = tan $^{-1}$ $\frac{\sum Fx^2}{\sum Fy^2}$

Steps to solve problems on resultant of parallel force system

1. Find $\sum$ Fx or $\sum$ Fy, and R.
2. Locate the resultant by applying varignon's theorem if asked in the question.
3. Find the parameters as required in the question.

Steps to solve problems on resultant of general force system

1. Find angle of inclined forces if not given, by geometry.
2. Resolve the inclined forces, if any.
3. Find $\sum Fx$, $\sum Fy$, R and Θ.
4. Locate the resultant by applying varignon's theorem.
5. Find the parameters as required in the question.

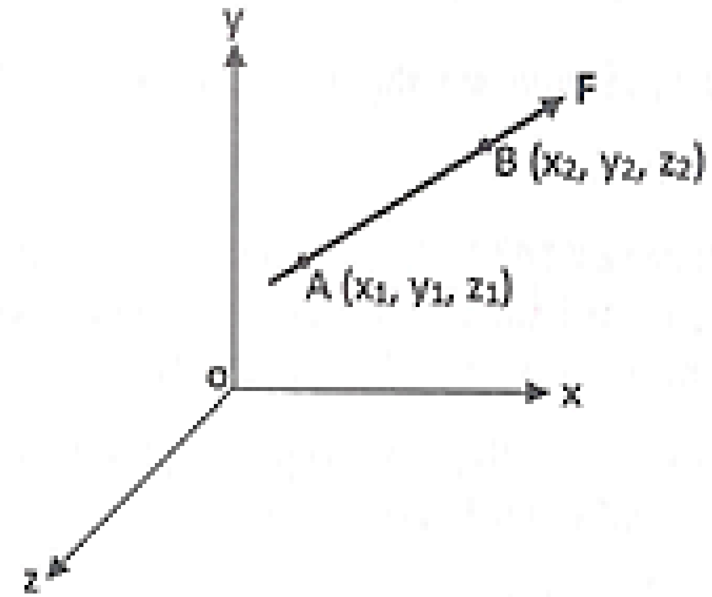
Forces in Space

A system of forces lying in different planes forming a three-dimensional force system are referred as forces in space.

We will be using vector-based approach to deal with the problems on forces in space.

Force in vector form:

Consider a force of magnitude F passing through points A (x_1, y_1, z_1) and B (x_2, y_2, z_2),



Then the force in vector form is given as

$$\overline{F} = F.\ \hat{e}_{ab}$$

Where F is the magnitude of force

$\hat{e}_{ab}$ is unit vector along AB

$\hat{e}_{ab}$ is given by, $\left[\dfrac{(x_2-x_1)\hat{\imath} + (y_2-y_1)\hat{\jmath} + (z_2-z_1)\hat{k}}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2 + (z_2-z_1)^2}}\right]$

$$\therefore\ \overline{F} = F.\left[\frac{(x_2-x_1)\hat{\imath} + (y_2-y_1)\hat{\jmath} + (z_2-z_1)\hat{k}}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2 + (z_2-z_1)^2}}\right]$$

or $\overline{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$, where $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ are unit vectors along x, y and z axis respectively.

From above

$$F_x = F.\left[\frac{(x_2-x_1)}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2 + (z_2-z_1)^2}}\right]$$

$$F_y = F.\left[\frac{(y_2-y_1)}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2 + (z_2-z_1)^2}}\right]$$

$$F_z = F.\left[\frac{(z_2-z_1)}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2 + (z_2-z_1)^2}}\right]$$

Magnitude and direction of a force

The magnitude of a force is given by $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

Also, $F_x = F.\cos\theta_x$

$F_y = F.\cos\theta_y$

$F_z = F.\cos\theta_z$

Where θ_x , θ_y and θ_z are known as direction of forces.

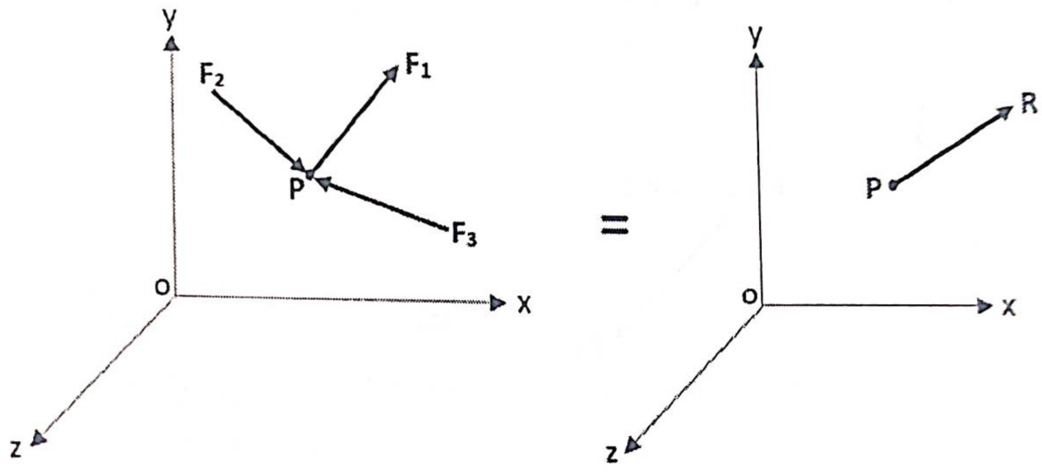
The identity which relates θ_x , θ_y and θ_z is given by

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

Note: If a force acts parallel to an axis, its vector form is obtained by attaching the unit vector of the axis to magnitude of the force with a proper sign convention.

- If the force acts along the direction of the axis it is to be taken as positive, whereas be taken negative. if it is in the direction opposite to the axis than it should be taken negative
- For e.g. Let us say that a force of F = 25N is acting along z axis, then its vector form will be $\bar{F}$ =25 $\hat{k}$

Resultant of Concurrent Space force System



Resultant of concurrent space force system is a single force $\bar{R}$, which acts through the point of concurrence.

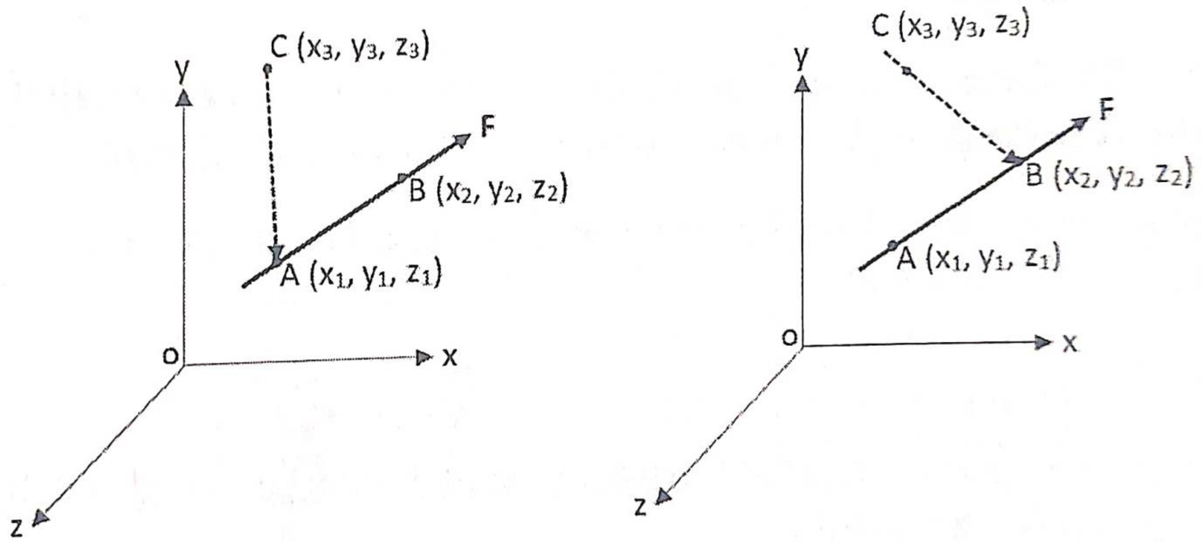
Figure above, shows a concurrent system of three forces passing through point P.

The resultant of this system is given by

$$\overline{R} = \overline{F_1} + \overline{F_2} + \overline{F_3}$$

Moment of a system about a point

Figure below shows the force F in space passing through points A (x_1, y_1, z_1) and B (x_2, y_2, z_2). Let C (x_3, y_3, z_3) be the moment centre (the point about which the moment is to be found out).



Steps to find out moment of a force about a given point

1. Convert the force into vector form, i.e. **or** $\overline{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$,
2. Find the position vector extending **from the moment center** to **any point on the force**, i.e. $\overline{r} = r_x\,\hat{\imath} + r_y\,\hat{\jmath} + r_z\,\hat{k}$

   Where $r_x = (x_2 - x_1)$

   $r_y = (y_2 - y_1)$

   $r_z = (z_2 - z_1)$

   Important note: In above first point will always be the point about which moment is to be taken and second point will be any point on the force whose moment needs to be found out.
3. Perform the cross product of position vector and force vector to get moment vector.

$$\overline{M}^{F}{}_{\text{point}} = \overline{r} \times \overline{F}$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Date :

Unit 02 Centroid

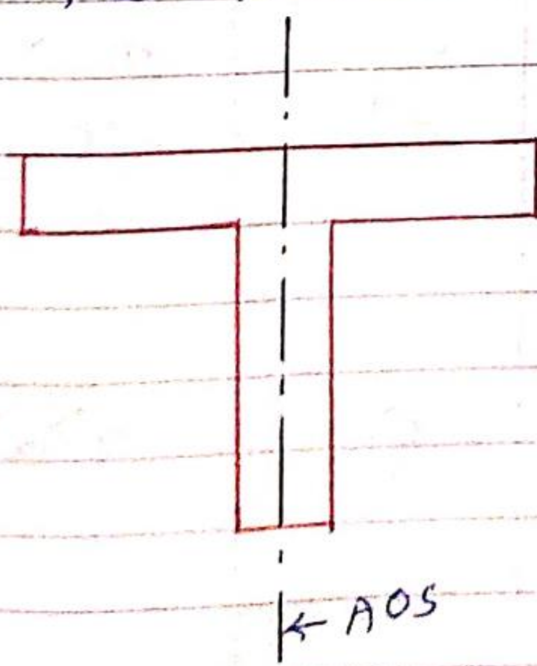
* **Center of Gravity**: It is defined as a point through which the whole weight of the body is assumed to act. It is denoted by '<u>G</u>'.

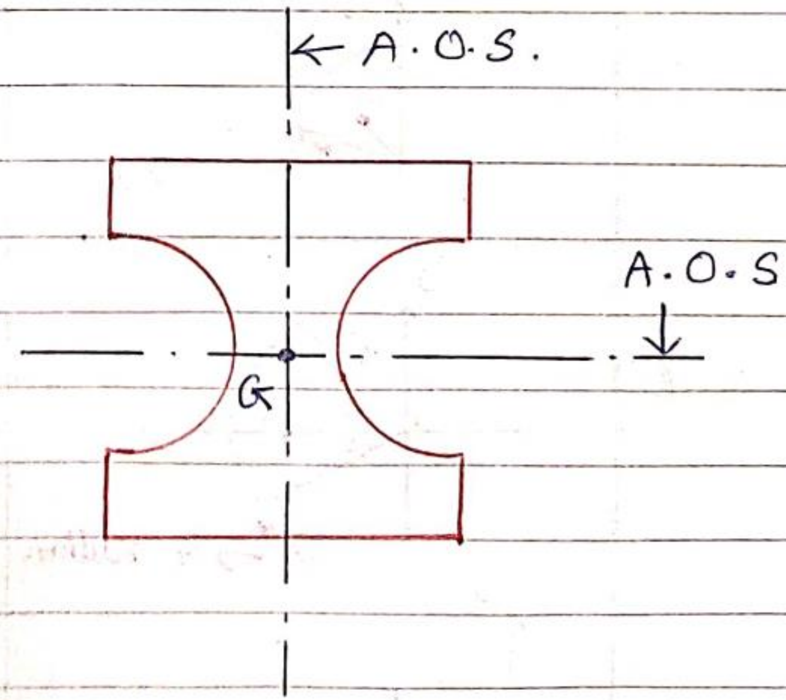
* **Centroid**: It is a single point, at which the total area of a plane or Lamina is assumed to be concentrated is called the centroid of that area.

[<u>ex</u>. Rectangle, Circle, Square, Semi-circle, Quarter Circle etc.]

* Axis of Symmetry (AOS) : It is defined as the line which divides the figure into two equal parts.

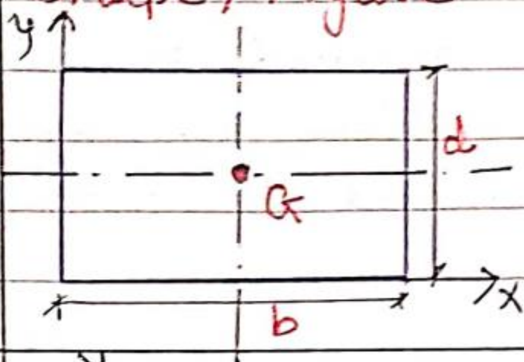
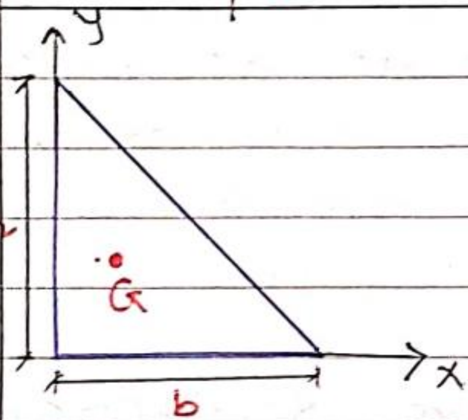
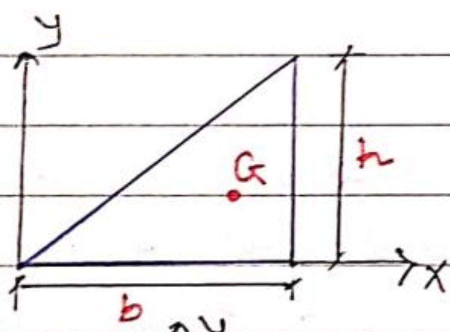
- Centroid lies on the axis of symmetry.
- If any figure has more than one (AOS) then centroid will lie on the intersection of the AOS.





* x_i & y_i are coordinates of center of gravity G.

Date :

	Shape / Figure	$\bar{X}$ or x_i	$\bar{Y}$ or y_i	Area (A_i)
1.		$\frac{b}{2}$	$\frac{d}{2}$	bd
2.		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{1}{2}bh$
3.		$\frac{2}{3}b$	$\frac{h}{3}$	$\frac{1}{2}bh$

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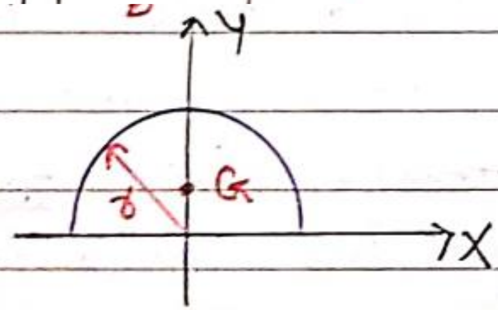
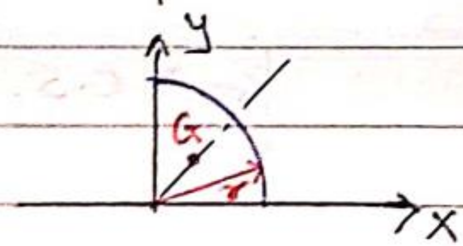
Date :

	Shape / Figure	$\bar{X}$ or x_i	$\bar{Y}$ or y_i	Area (A_i)
4.		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
5.		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$

Centroid of composite Area :

(more than one)

Step 1 : Divide composite area into the regular area.

Step 2 : Mark the centroids of each regular Areas, based on coordinates taken

Step 3 : Prepare a table for Area (A), x_i, y_i, A_ix_i & A_iy_i

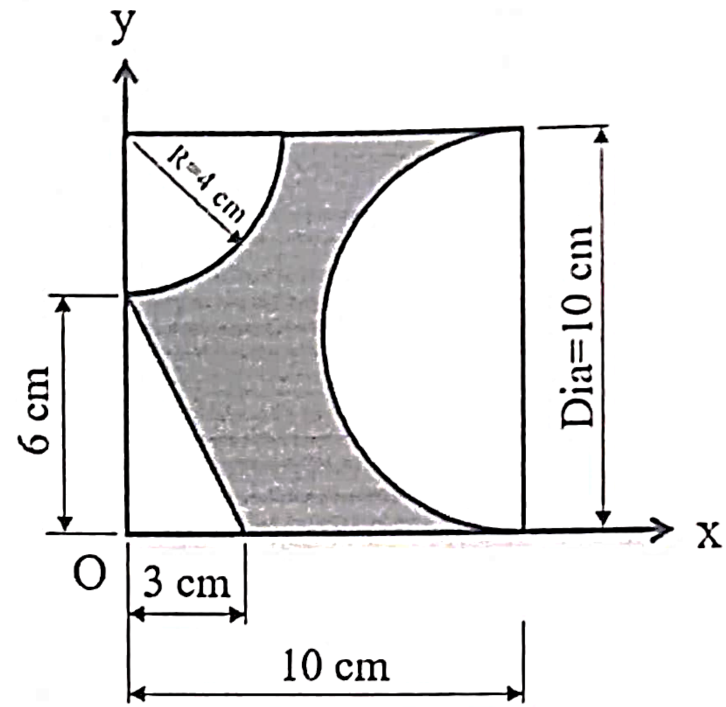
* Note : put **-ve sign** if any portion is removed or cut.

Step 4 : Find ΣA_i, ΣA_ix_i & ΣA_iy_i

Step 5 : Use formula $\bar{x} = \dfrac{\Sigma A_ix_i}{\Sigma A_i}$

$$\bar{Y} = \frac{\Sigma A_iy_i}{\Sigma A_i}$$

1. Determine the "x" and "y" coordinates of the centroid for the shaded area shown. (May-24, 6 marks), (May-23).



Solution:

Step 1: Check whether the given lamina is symmetric or not.

The given lamina is not symmetric based on origin given. Determine both $\bar{x}$ and $\bar{y}$.

Step 2: Select reference point.

As the origin (x-y axis) is given, select origin as reference point.

Step 3: Divide the given lamina into minimum number of standard shapes (parts).

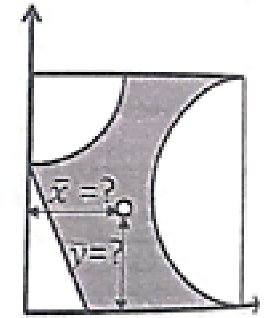
The above lamina can be divided into four parts

a) a square of size 10 cm * 10cm

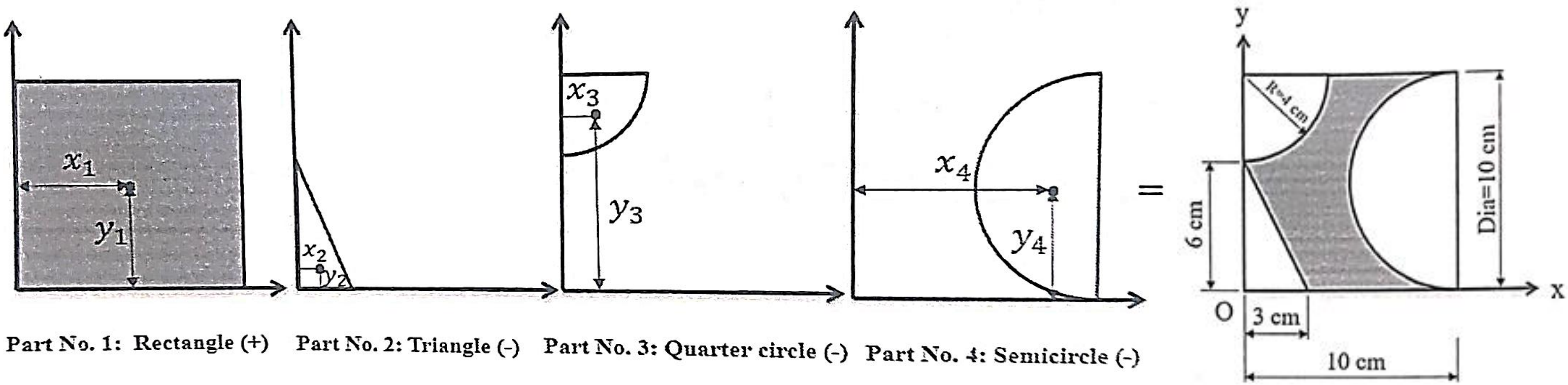
b) a triangle of base 3cm and height 6cm (to be subtracted as it is not a part of shaded region)

c) a quarter circle of radius 4 cm (to be subtracted as it is not a part of shaded region)

d)a semicircle of diameter 10 cm (to be subtracted as it is not a part of shaded region)



Required Result



Part No. 1: Rectangle (+) Part No. 2: Triangle (-) Part No. 3: Quarter circle (-) Part No. 4: Semicircle (-)

Part No.	Area (cm^2)	x_i (cm)	y_i (cm)	$A_i x_i$ (cm^3)	$A_i y_i$ (cm^3)
1.	s^2 $= 10^2$ $= 100$	$\frac{s}{2}$ $= \frac{10}{2}$ $= 5$	$\frac{s}{2}$ $= \frac{10}{2}$ $= 5$	$100 * 5$ $= 500$	$100 * 5$ $= 500$
2.	$-\frac{1}{2} * b * h$ $= -\frac{1}{2} * 3 * 6$ $= -9$	$\frac{b}{3}$ $= \frac{3}{3}$ $= 1$	$\frac{h}{3}$ $= \frac{6}{3}$ $= 2$	$-9 * 1$ $= -9$	$-9 * 2$ $= -18$
3.	$-\frac{\pi * r^2}{4}$ $-\frac{\pi * 4^2}{4}$ $= -12.566$	$\left(\frac{4 * r}{3\pi}\right)$ $= \left(\frac{4 * 4}{3\pi}\right)$ $= 1.697$	$10 - \left(\frac{4 * r}{3\pi}\right)$ $= 10 - \left(\frac{4 * 4}{3\pi}\right)$ $= 8.302$	$-12.566 * 1.697$ $= -21.324$	$-12.566 * 8.302$ $= -104.322$
4.	$-\frac{\pi * r^2}{2}$ $= -\frac{\pi * 5^2}{2}$ $= -39.269$	$= 10 - \left(\frac{4 * r}{3\pi}\right)$ $= 10 - \left(\frac{4 * 5}{3\pi}\right)$ $= 7.877$	r $= 5$	$-39.269 * 7.877$ -309.35	$-39.269 * 5$ $= -196.345$
	$\sum A = 39.165$			$\sum A_i x_i = 160.32$	$\sum A_i y_i = 181.33$

Part No.	Area (cm^2)	x_i (cm)	y_i (cm)	$A_i x_i$ (cm^3)	$A_i y_i$ (cm^3)
1.	s^2 $= 10^2$ $= 100$	$\frac{s}{2}$ $= \frac{10}{2}$ $= 5$	$\frac{s}{2}$ $= \frac{10}{2}$ $= 5$	$100 * 5$ $= 500$	$100 * 5$ $= 500$
2.	$-\frac{1}{2} * b * h$ $= -\frac{1}{2} * 3 * 6$ $= -9$	$\frac{b}{3}$ $= \frac{3}{3}$ $= 1$	$\frac{h}{3}$ $= \frac{6}{3}$ $= 2$	$-9 * 1$ $= -9$	$-9 * 2$ $= -18$

3.	$-\frac{\pi * r^2}{4}$ $-\frac{\pi * 4^2}{4}$ $= -12.566$	$\left(\frac{4 * r}{3\pi}\right)$ $= \left(\frac{4 * 4}{3\pi}\right)$ $= 1.697$	$10 - \left(\frac{4 * r}{3\pi}\right)$ $= 10 - \left(\frac{4 * 4}{3\pi}\right)$ $= 8.302$	$-12.566 * 1.697$ $= -21.324$	$-12.566 * 8.302$ $= -104.322$
4.	$-\frac{\pi * r^2}{2}$ $= -\frac{\pi * 5^2}{2}$ $= -39.269$	$= 10 - \left(\frac{4 * r}{3\pi}\right)$ $= 10 - \left(\frac{4 * 5}{3\pi}\right)$ $= 7.877$	r $= 5$	$- 39.269 * 7.877$ $- 309.35$	$- 39.269 * 5$ $= -196.345$
	$\sum A = 39.165$			$\sum A_i x_i = 160.32$	$\sum A_i y_i = 181.33$

Step 5: Find centroid using the formula.

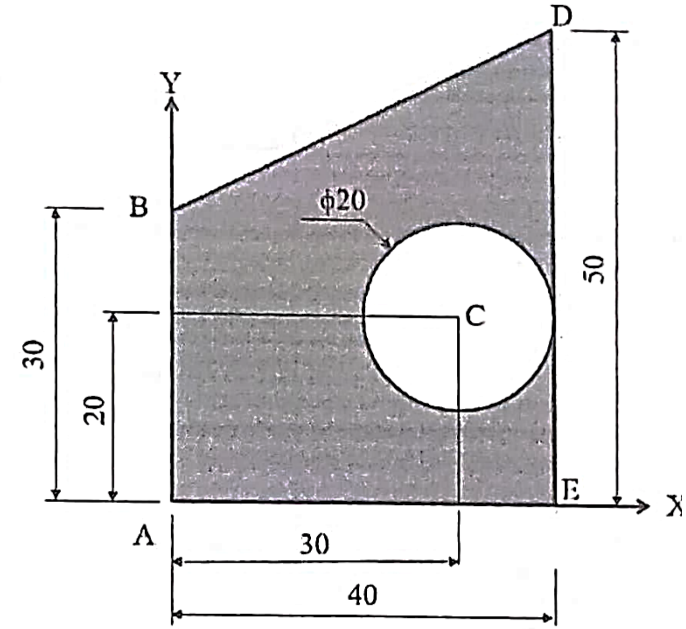
$$\bar{x} = \frac{\sum A_i x_i}{A_i} = \frac{160.32}{39.165} = 4.093 \text{ cm.}$$

$$\bar{y} = \frac{\sum A_i y_i}{A_i} = \frac{181.33}{39.165} = 4.629 \text{ cm.}$$

Step 6: Final answer

Centroid of shaded region is at (4.093, 4.629) cm from the origin.

2. **Determine the centroid about given x & y axes. (Dec-21 - 5 Marks, May-21 - 5 Marks)**



Solution:

Step 1: Check whether the given lamina is symmetric or not.

The given lamina is not symmetric based on origin given. Determine both $\bar{x}$ and $\bar{y}$.

Step 2: Select reference point.

As the origin (x-y axis) is given, select origin as reference point.

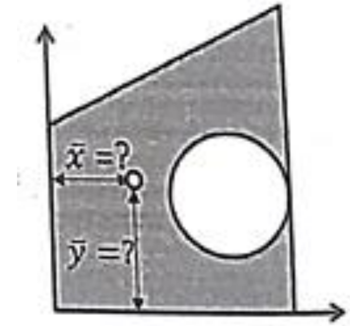
Step 3: Divide the given lamina into minimum number of standard shapes (parts).

The above lamina can be divided into two parts

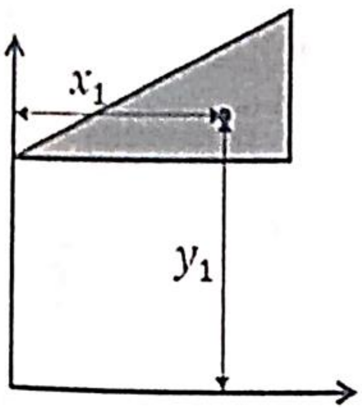
a) a triangle of base 40 units and height 20 units

b) a rectangle of 40 units * 30 units

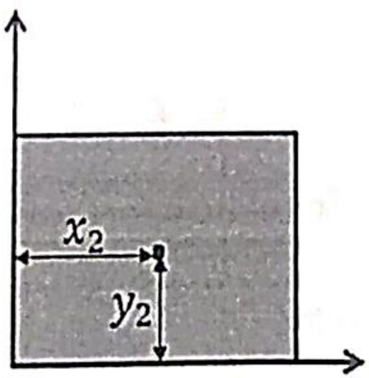
c) a circle of diameter 20 units (to be subtracted as it is not a part of shaded region)



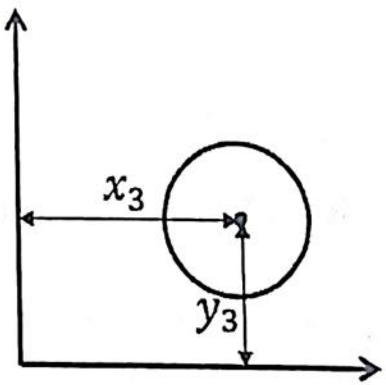
Required Result



Part No. 1: Triangle (+)

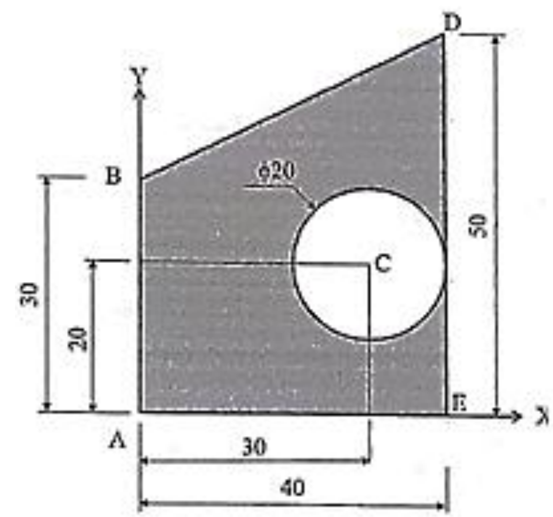


Part No. 2: Rectangle (+)



Part No. 3: Circle (-)

=



Step 4: Calculation table

Part No.	Area ($unit^2$)	x_i (unit)	y_i (unit)	A_ix_i ($unit^3$)	A_iy_i ($unit^3$)
1.	$\frac{1}{2} * b * h$ $= \frac{1}{2} * 40 * 20$ $= 400$	$40 - \left(\frac{b}{3}\right)$ $= 40 - \left(\frac{40}{3}\right)$ $= 26.67$	$30 + \left(\frac{h}{3}\right)$ $= 30 + \left(\frac{20}{3}\right)$ $= 36.67$	400*26.67 = 10668	400 *36.67 = 14668
2.	$b * h$ $= 40 * 30$ $= 1200$	$\frac{b}{2}$ $= \frac{40}{2}$ $= 20$	$\frac{h}{2}$ $= \frac{30}{2}$ $= 15$	1200*20 = 24000	1200*15 = 18000
3.	$-\pi * r^2$ $= -\pi * 10^2$ $= -314.159$	30	20	$-314.159 * 30$ $= -9424.77$	$-314.159 * 20$ $= -6283.18$
	$\sum A = 1285.841$			$\sum A_ix_i = 25243.23$	$\sum A_iy_i = 26384.82$

Step 4: Calculation table

Part No.	Area ($unit^2$)	x_i (unit)	y_i (unit)	$A_i x_i$ ($unit^3$)	$A_i y_i$ ($unit^3$)
1.	$\frac{1}{2} * b * h$ $= \frac{1}{2} * 40 * 20$ $= 400$	$40 - \left(\frac{b}{3}\right)$ $= 40 - \left(\frac{40}{3}\right)$ $= 26.67$	$30 + \left(\frac{h}{3}\right)$ $= 30 + \left(\frac{20}{3}\right)$ $= 36.67$	400*26.67 = 10668	400 *36.67 = 14668
2.	$b * h$ $= 40 * 30$ $= 1200$	$\frac{b}{2}$ $= \frac{40}{2}$ $= 20$	$\frac{h}{2}$ $= \frac{30}{2}$ $= 15$	1200*20 = 24000	1200*15 = 18000

3.	$-\pi * r^2$ $= -\pi * 10^2$ $= -314.159$	30	20	$-314.159 * 30$ $= -9424.77$	$-314.159 * 20$ $= -6283.18$
	$\sum A$ $= 1285.841$			$\sum A_i x_i$ $= 25243.23$	$\sum A_i y_i$ $= 26384.82$

Step 5: Find centroid using the formula.

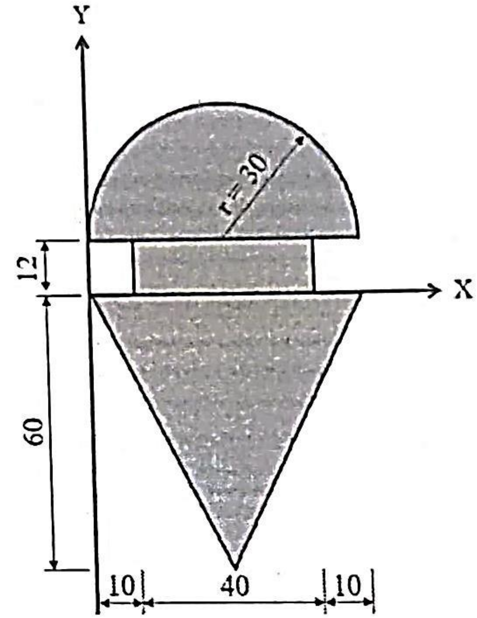
$$\bar{x} = \frac{\sum A_i x_i}{A_i} = \frac{25243.23}{1285.841} = 19.63 \text{ units.}$$

$$\bar{y} = \frac{\sum A_i y_i}{A_i} = \frac{26384.82}{1485.841} = 20.51 \text{ units.}$$

Step 6: Final answer

Centroid of shaded region is at (19.63, 20.51) units, from the origin.

3. Find the centroid of the shaded area as shown in figure with respect to given reference axes x & y. (May-22, 10 Marks)



Solution:

Step 1: Check whether the given lamina is symmetric or not.

The given lamina is not symmetric based on origin given. Determine both $\bar{x}$ and $\bar{y}$.

Step 2: Select reference point.

As the origin (x-y axis) is given, select origin as reference point.

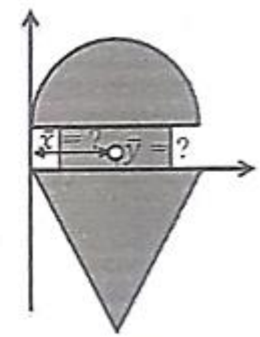
Step 3: Divide the given lamina into minimum number of standard shapes (parts).

The above lamina can be divided into two parts

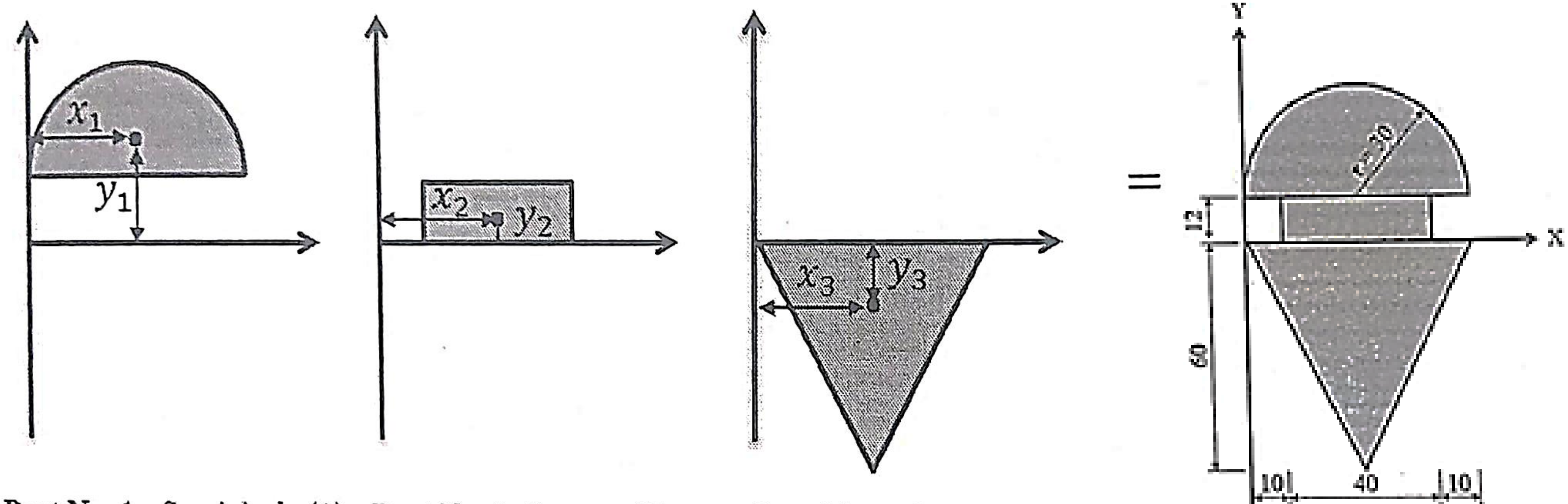
a) a semi-circle of radius 30 units

b) a rectangle of 12 units* 40 units

c) a triangle of base 60 units and height 60 units



Required Result



Part No. 1: Semicircle (+) Part No. 2: Square (+) Part No. 3: Triangle (+)

Step 4: Calculation table

Part No.	Area ($unit^2$)	x_i (unit)	y_i (unit)	A_ix_i ($unit^3$)	A_iy_i ($unit^3$)
1.	$\frac{\pi * r^2}{2}$ $= \frac{\pi * 30^2}{2}$ $= 1413.716$	30	$12 + \left(\frac{4 * r}{3\pi}\right)$ $= 12 + \left(\frac{4 * 30}{3\pi}\right)$ $= 24.732$	$1413.716 * 30$ $= 42411.48$	$1413.71 * 24.732$ $= 34964.024$
2.	$b * h$ $= 40 * 12$ $= 480$	$10 + \left(\frac{b}{2}\right)$ $= 10 + \left(\frac{40}{2}\right)$ $= 30$	$\frac{h}{2}$ $= \frac{12}{2}$ $= 6$	$480 * 30$ $= 14400$	$480 * 6$ $= 2880$
3.	$\frac{1}{2} * b * h$ $= \frac{1}{2} * 60 * 60$ $= 1800$	$\frac{b}{2}$ $= \frac{60}{2}$ $= 30$	$-h/3$ $= -60/3$ $= -20$	$1800 * 30$ $= 54000$	$1800 * -20$ $= -36000$
	$\sum A = 3693.716$			$\sum A_ix_i$ $= 110811.48$	$\sum A_iy_i$ $= 1844.024$

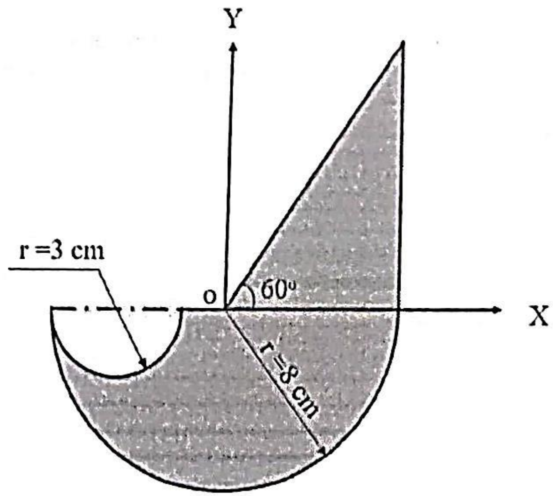
Step 5: Find centroid using the formula.

$$\bar{x} = \frac{\sum A_i x_i}{A_i} = \frac{110811.48}{3693.716} = 30 \text{ units.}$$

$$\bar{y} = \frac{\sum A_i y_i}{A_i} = \frac{1844.024}{3693.716} = 0.5 \text{ units.}$$

Step 6: Final answer: Centroid of shaded region is at (30, 0.5) units from the origin.

4. **Find the centroid of the shaded area as shown in the figure. (May-23, 7 Marks)**



Step 1: Check whether the given lamina is symmetric or not.

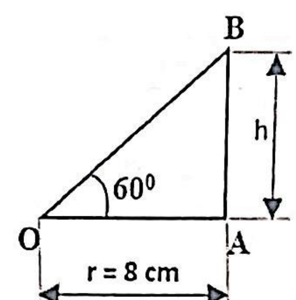
The given lamina is not symmetric based on origin given. Determine both $\bar{x}$ and $\bar{y}$.

Step 2: Select reference point.

As the origin (x-y axis) is given, select origin as reference point.

Step 3: Divide the given lamina into minimum number of standard shapes (parts).

In triangle, OAB



$$\tan\theta = \frac{AB}{OA}$$

$$\tan 60 = \frac{h}{8}$$

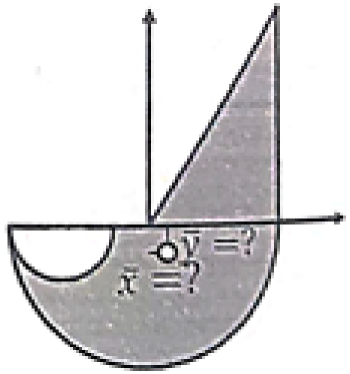
$\therefore$ h = 13.86 cm

The above lamina can be divided into three parts

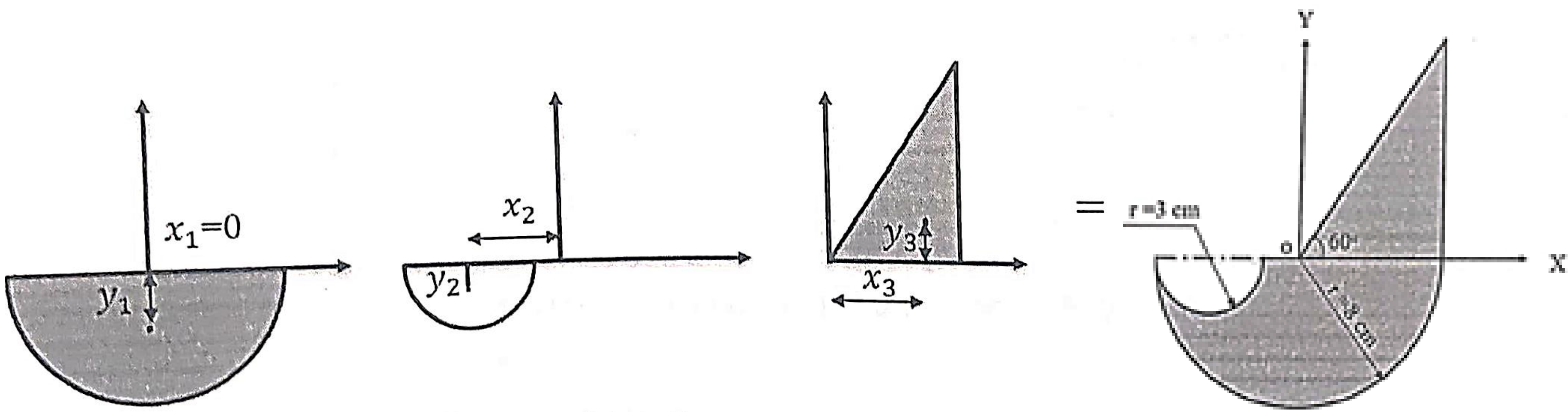
a) a semi-circle of radius 8 cm

b) a semi-circle of radius 3 cm (to be subtracted as it is not a part of shaded region)

c) a triangle of base 8cm and height 13.86 cm



Required Result



Part No. 1: Semicircle (+) Part No. 2: Semicircle (-) Part No. 3: Triangle (+)

Step 4: Calculation table

Part No.	Area (cm^2)	x_i (cm)	y_i (cm)	A_ix_i (cm^3)	A_iy_i (cm^3)
1.	$\frac{\pi * r^2}{2}$ $= \frac{\pi * 8^2}{2}$ $= 100.53$	0	$-\left(\frac{4 * r}{3\pi}\right)$ $= -\left(\frac{4 * 8}{3\pi}\right)$ $= -3.395$	0	$100.53 * -3.395$ $= -341.299$
2.	$-\frac{\pi * r^2}{2}$ $-\frac{\pi * 3^2}{2}$ $= -14.137$	- (8-3) -5	$-\left(\frac{4 * r}{3\pi}\right)$ $= -\left(\frac{4 * 3}{3\pi}\right)$ $= -1.273$	$-14.137 * -5$ $= 70.685$	$-14.137 * -1.273$ $= 17.996$
3.	$\frac{1}{2} * b * h$ $= \frac{1}{2} * 8 * 13.86$ $= 55.44$	$(8 - b/3)$ $= 8 - \left(\frac{8}{3}\right)$ $= 5.333$	$h/3$ $= \frac{13.86}{3}$ $= 4.62$	$= 55.44 * 5.333$ $= 295.66$	$= 55.44 * 4.62$ $= 256.132$
	$\sum A = 141.833$			$\sum A_ix_i$ $= 366.345$	$\sum A_iy_i$ $= -67.171$

Step 5: Find centroid using the formula.

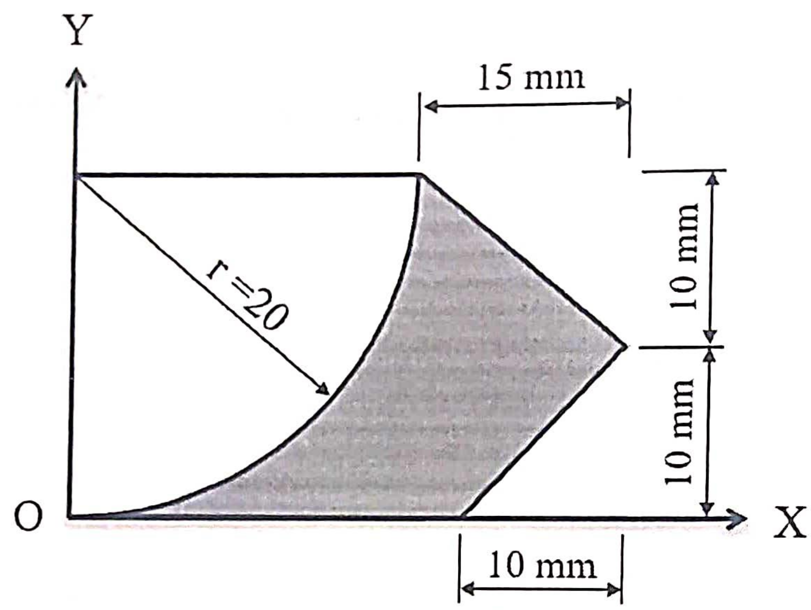
$$\bar{x} = \frac{\sum A_i x_i}{A_i} = \frac{366.345}{141.833} = 2.583 \text{ cm.}$$

$$\bar{y} = \frac{\sum A_i y_i}{A_i} = \frac{-67.171}{141.833} = -0.4735 \text{ cm.}$$

Step 6: Final answer

Centroid of shaded region is at (2.583, - 0.4735) cm from the origin.

5. **Find the centroid of the shaded area with reference to x and y axis. (Dec-19, 6 Marks)**



Solution:

Step 1: Check whether the given lamina is symmetric or not.

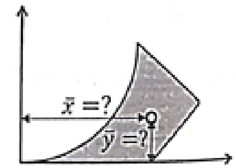
The given lamina is not symmetric based on origin given. Determine both $\bar{x}$ and $\bar{y}$.

Step 2: Select reference point.

As the origin (x-y axis) is given, select origin as reference point.

Step 3: Divide the given lamina into minimum number of standard shapes (parts).

The above lamina can be divided into four parts



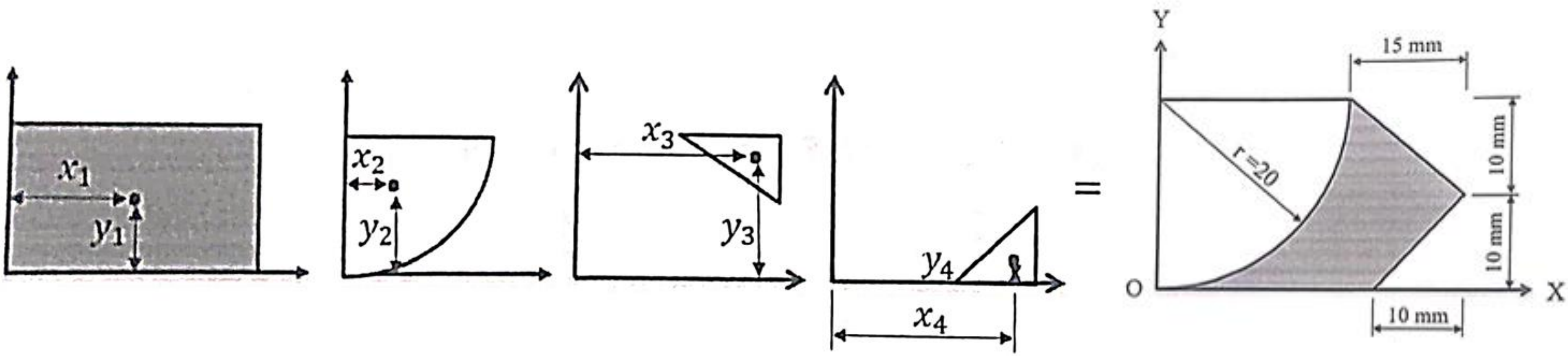
Required Result

a) a rectangle of size 35 mm * 20mm

b) a quarter circle of radius 20 mm (to be subtracted as it is not a part of shaded region)

c) a triangle of base 15 mm and height 10 mm (to be subtracted as it is not a part of shaded region)

d) a triangle of base 10 mm and height 10 mm (to be subtracted as it is not a part of shaded region)



Part No. 1: Rectangle (+) **Part No. 2: Quarter Circle (-)** **Part No. 2: Triangle (-)** **Part No. 4: Triangle (-)**

Step 4: Calculation table

Part No.	Area (mm^2)	x_i (mm)	y_i (mm)	A_ix_i (mm^3)	A_iy_i (mm^3)
1.	$b * h$ $= 35 * 20$ $= 700$	$\frac{35}{2}$ $=17.5$	$\frac{20}{2}$ $=10$	700*17.5 =12250	700*10 =7000
2.	$-\frac{\pi * r^2}{4}$ $-\frac{\pi * 20^2}{4}$ $= -314.159$	$\left(\frac{4 * r}{3\pi}\right)$ $= \left(\frac{4 * 20}{3\pi}\right)$ $=8.488$	$20 - \left(\frac{4 * r}{3\pi}\right)$ $= 20 - \left(\frac{4 * 20}{3\pi}\right)$ $=11.51$	$-314.159 * 8.488$ $= -2666.581$	$-314.159 * 11.51$ $= -3615.97$
3.	$-\frac{1}{2} * b * h$ $= -\frac{1}{2} * 15 * 10$ $= -75$	$35 - \left(\frac{b}{3}\right)$ $= 35 - \left(\frac{15}{3}\right)$ $= 30$	$20 - \left(\frac{h}{3}\right)$ $= 20 - \left(\frac{10}{3}\right)$ $=16.67$	$-75 * 30$ $= -2250$	$-75 * 16.67$ $= -1250.25$
4.	$-\frac{1}{2} * b * h$ $= -\frac{1}{2} * 10 * 10$ $= -50$	$35 - \left(\frac{b}{3}\right)$ $= 35 - \left(\frac{10}{3}\right)$ $= 31.67$	$h/3$ $= \frac{10}{3}$ $= 3.33$	- 50*31.67 = -1583.5	- 50*3.33 = -166.5
	$\sum A = 260.841$			$\sum A_ix_i$ $= 5749.919$	$\sum A_iy_i$ $= 1967.28$

Step 5: Find centroid using the formula.

$$\bar{x} = \frac{\sum A_i x_i}{A_i} = \frac{5749.919}{260.841} = 22.043 \text{ mm.}$$

$$\bar{y} = \frac{\sum A_i y_i}{A_i} = \frac{1967.28}{260.841} = 7.542 \text{ mm.}$$

Step 6: Final answer

Centroid of shaded region is at (22.043, 7.542) mm from the origin.

Unit 03:Equilibrium of Force system and friction

Introduction

If the **resultant** of force system **is zero** than the system is said to be in state of **equilibrium.**

e.g. 1. A lamp hanging from the ceiling (Concurrent forces in equilibrium)

2. Students sitting on a bench in classroom (Parallel forces in equilibrium)

3. Structures such as buildings, dams etc (General forces in equilibrium)

Conditions Of Equilibrium

A body is said to be in equilibrium if algebraic sum of forces acting on it is zero and algebraic sum of moments acting on it is zero.

Mathematically, $\sum \bar{F} = 0$ and $\sum \bar{M} = 0$

This implies,

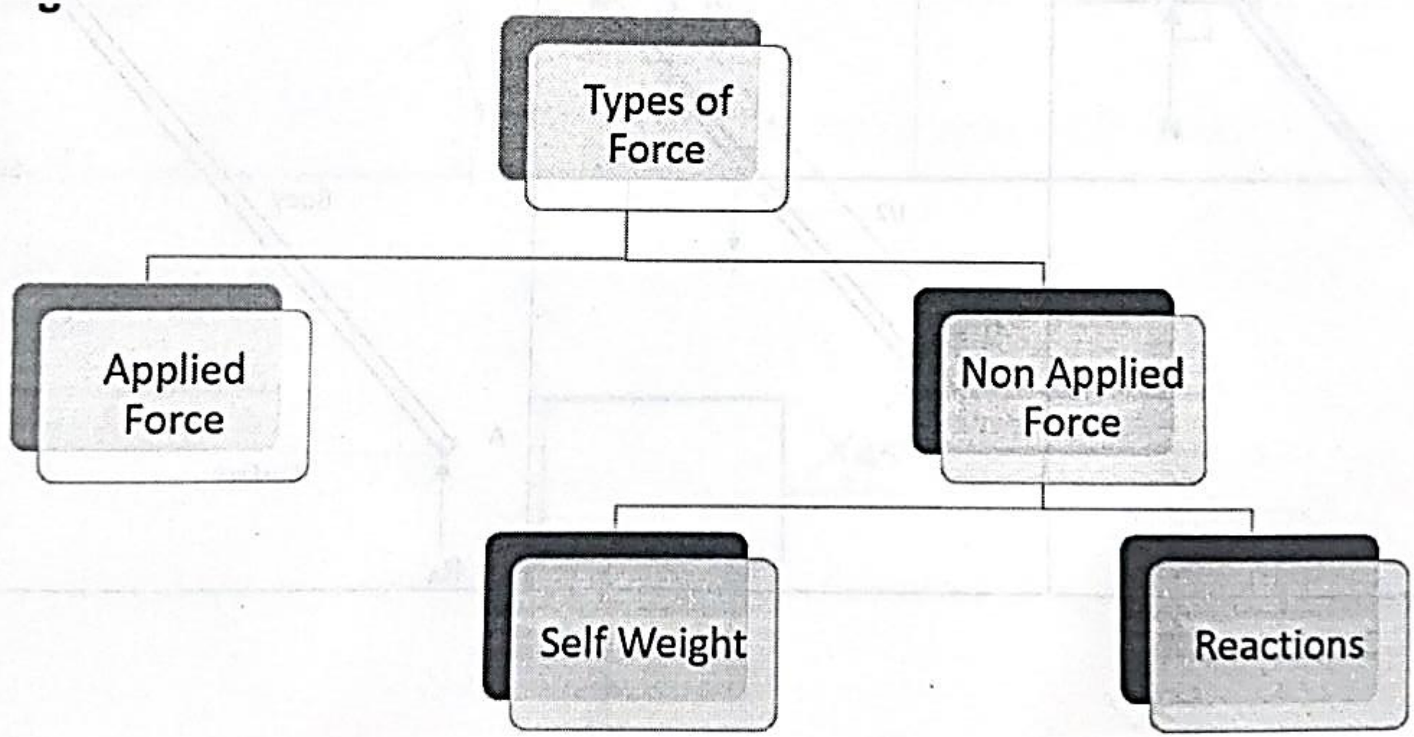
$\sum \boldsymbol{F_x} = \mathbf{0}$ …. sum of all forces in x direction is zero

$\sum \boldsymbol{F_y} = \mathbf{0}$ …. sum of all forces in y direction is zero

$\sum \boldsymbol{M} = \mathbf{0}$ …. sum of all moment of all forces about any point is zero

Free Body Diagram (F.B.D.)

When the body is isolated from the surroundings and all the forces acting on it are shown, then diagram so formed is known as Free Body Diagram (F.B.D). This F.B.D is required to be drawn to analyse the forces acting on the body.



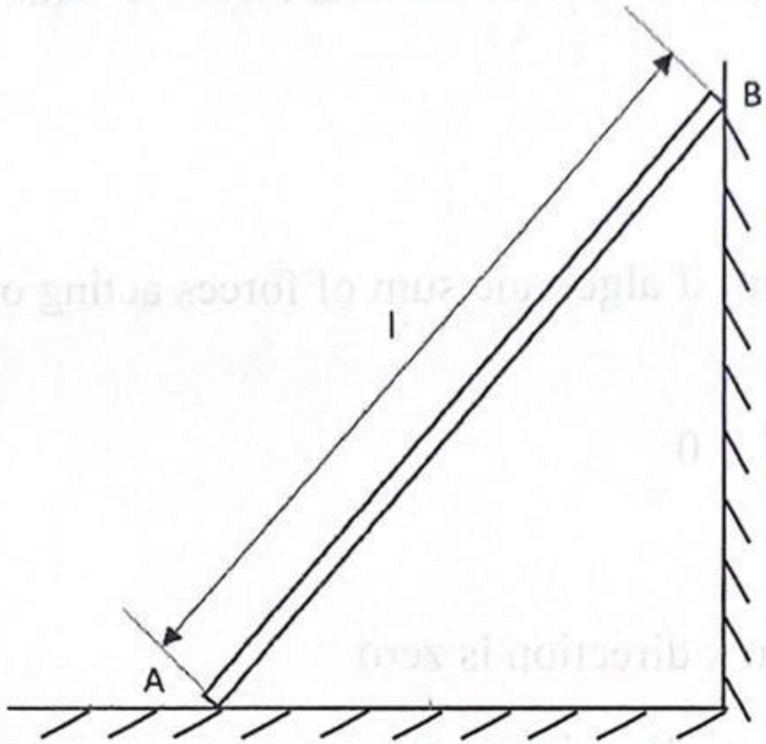
Free Body Diagram (F.B.D.)

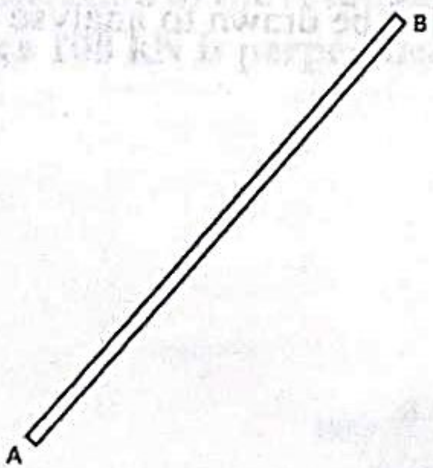
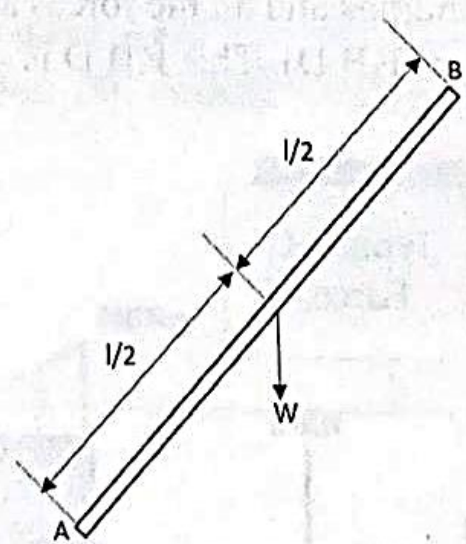
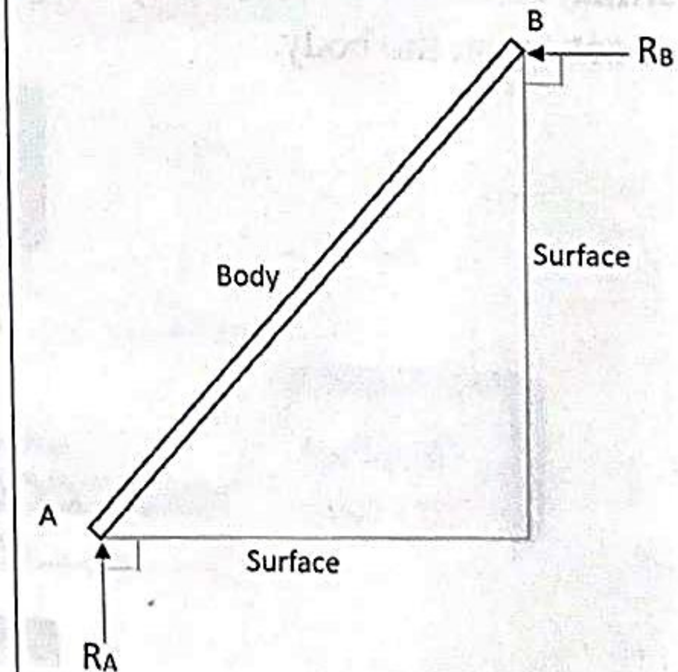
Forces to consider while drawing free body diagram (F.B.D)

1. **Applied forces,** to be shown as it is shown in the given question,
   Note: If it's a rope/ cable support than, tension in the rope/ cable has to be shown directed away from the body.
2. **Self-weight,** to be shown in vertically downward direction and at the centre of the body
3. **Reaction** from contact surface, to be shown perpendicular to surface and direction should be from surface towards the body.

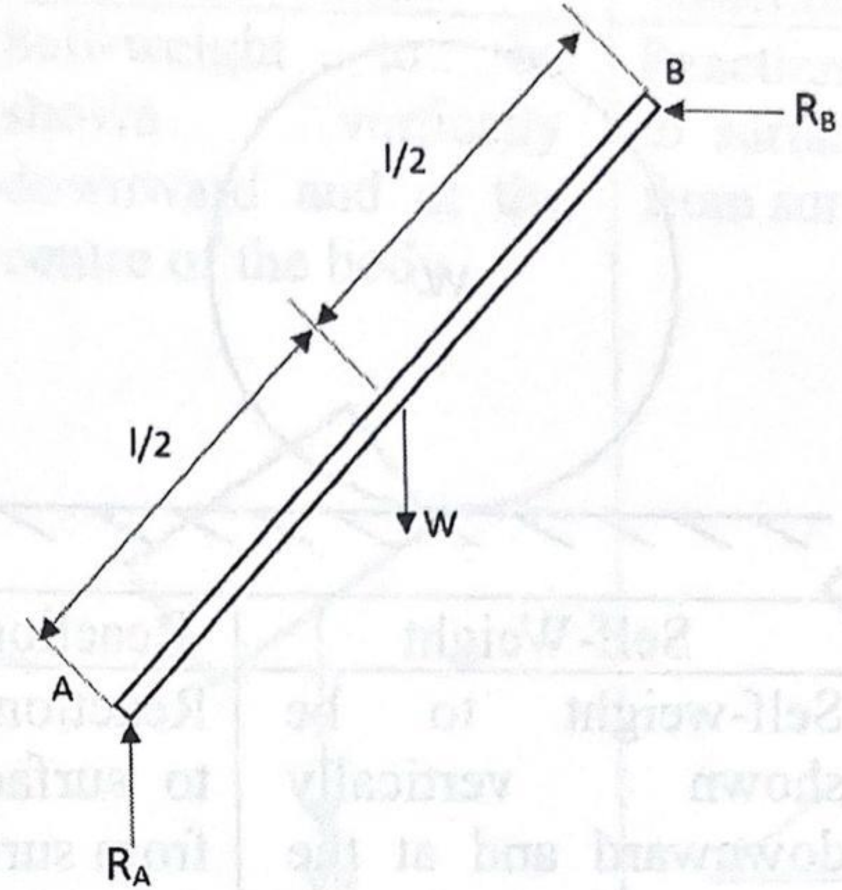
Examples of Free Body Diagram

1. A ladder AB of weight ‘W’ resting against a smooth vertical wall and horizontal floor

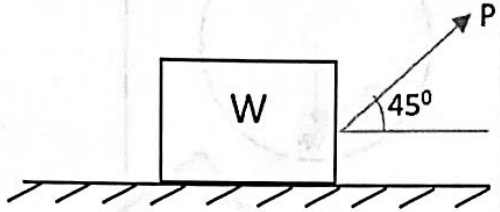


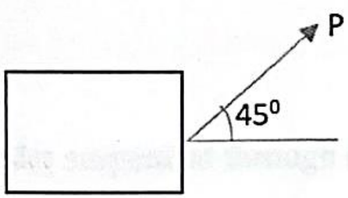
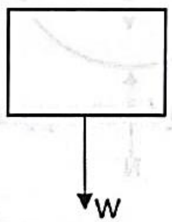
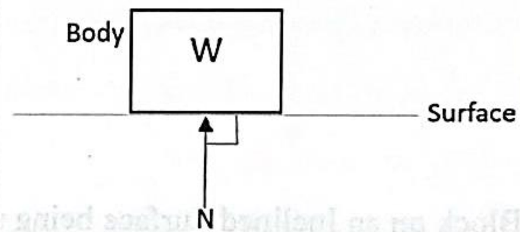
Applied Forces	Self-Weight	Reaction from contact surfaces
As there are no applied forces given in the question, nothing will be shown	Self-weight to be shown vertically downward and at the centre of the body	Reaction to be shown perpendicular to surface and direction should be from surface towards the body.
		

Complete F.B.D of ladder

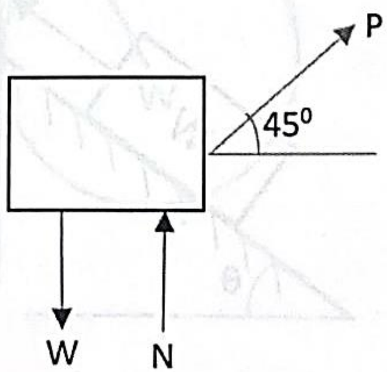


2. A block of weight 'W' resting on a horizontal floor

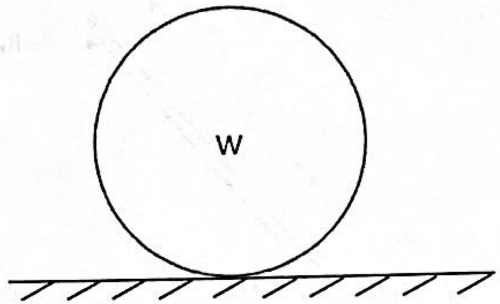


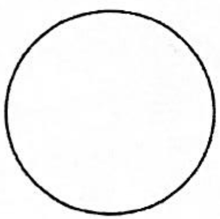
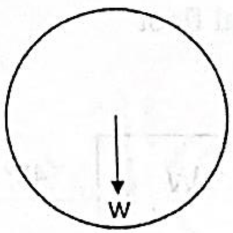
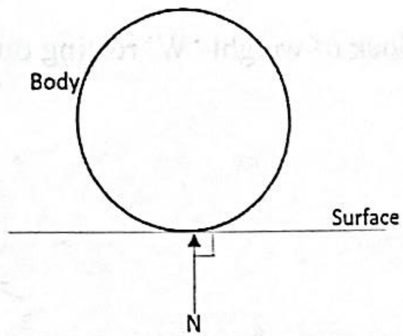
Applied Forces	Self-Weight	Reaction from contact surfaces
Applied force to be shown as it is shown in the diagram 	Self-weight to be shown vertically downwards and at the centre of the body 	Reaction to be shown perpendicular to surface and direction should be from surface towards the body. 

Complete F.B.D of block

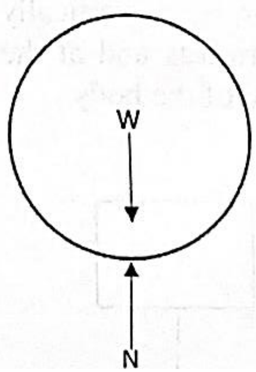


3. A cylinder of weight 'W' resting on a horizontal floor

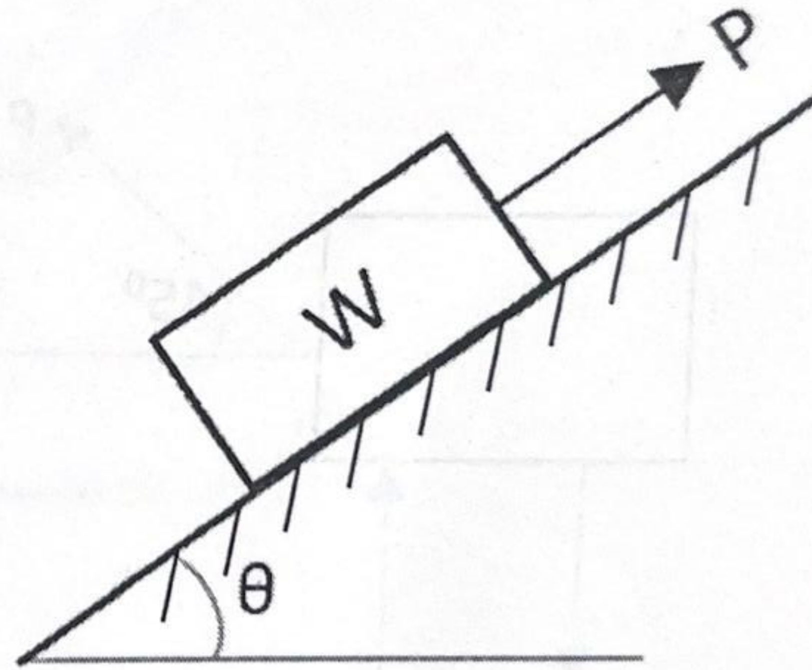


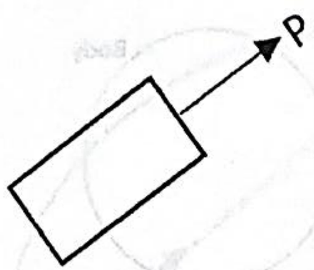
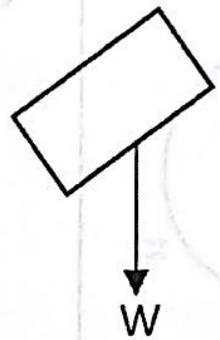
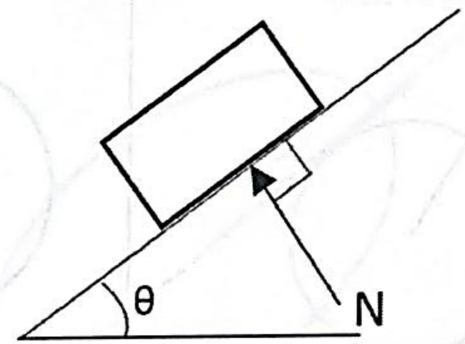
Applied Forces	Self-Weight	Reaction from contact surfaces
As there are no applied forces given in the question, nothing will be shown 	Self-weight to be shown vertically downward and at the centre of the body 	Reaction to be shown perpendicular to surface and direction should be from surface towards the body. 

Complete F.B.D of cylinder

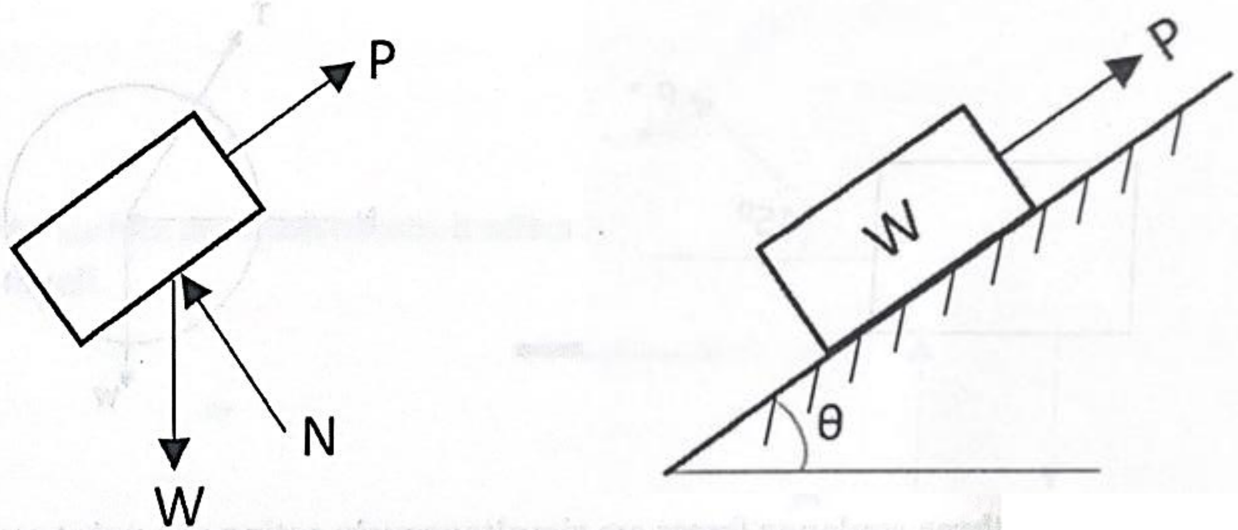


4. Block on an Inclined Surface being pulled by a force ‘P’

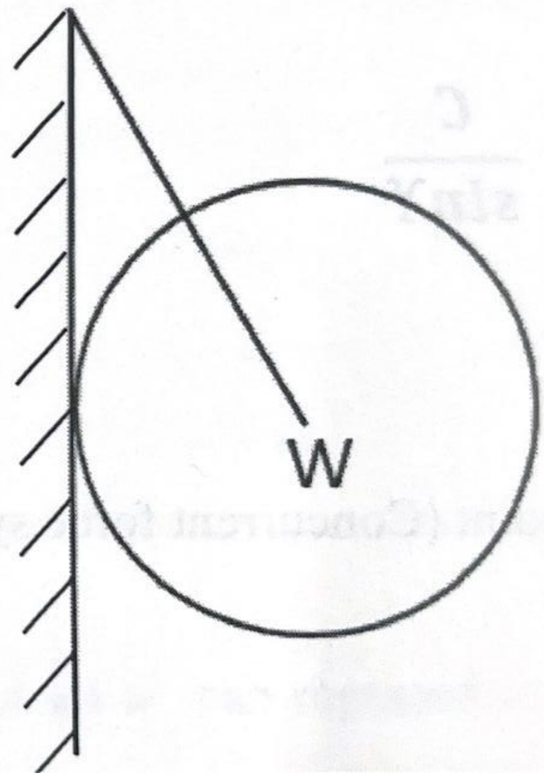


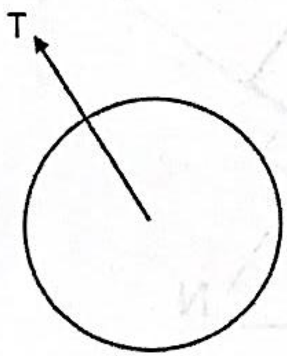
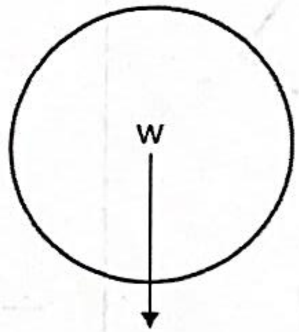
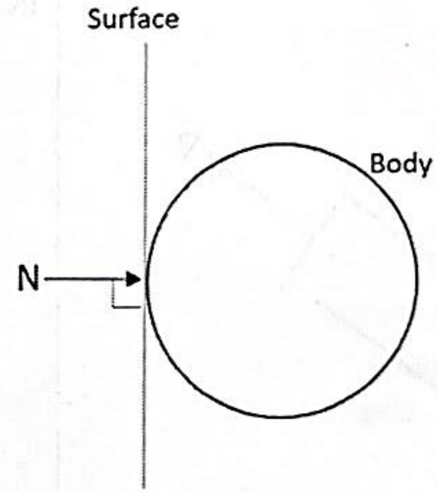
Applied Forces	Self-Weight	Reaction from contact surfaces
Applied force to be shown as it is shown in the diagram	Self-weight to be shown vertically downward and at the centre of the body	Reaction to be shown perpendicular to surface and direction should be from surface towards the body.
		

Complete F.B.D of block

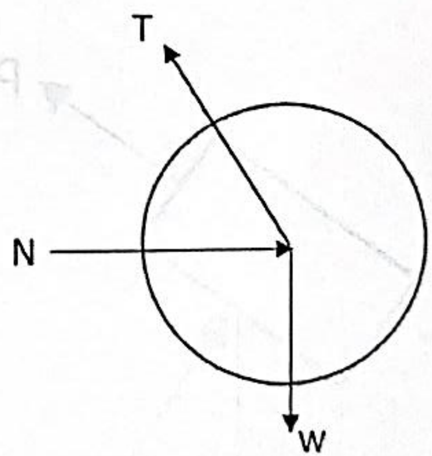


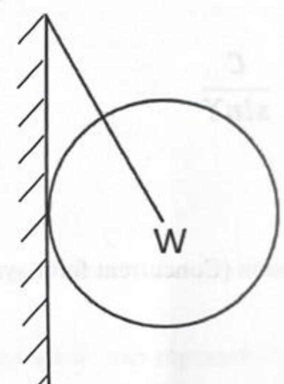
5. Cylinder suspended through rope and supported by a wall



Applied Forces	Self-Weight	Reaction from contact surfaces
Applied force to be shown as it is shown in the diagram. As cylinder is connected through a rope, tension in the rope will be shown acting away from the body.	Self-weight to be shown vertically downward and at the centre of the body.	Reaction to be shown perpendicular to surface and direction should be from surface towards the body.
		

Complete F.B.D of cylinder

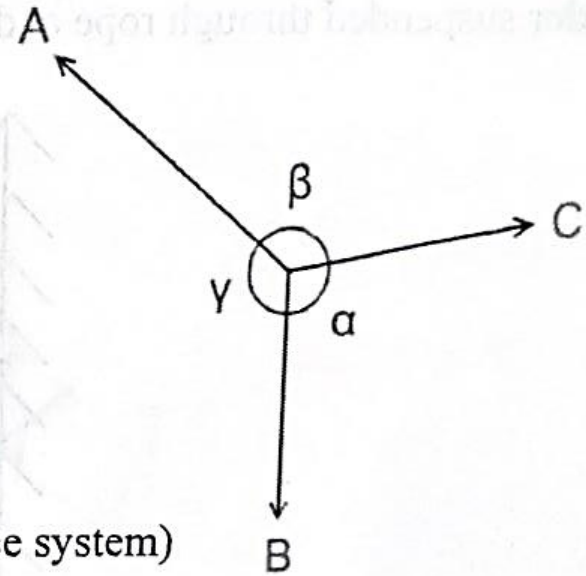




Lami's Theorem

If **three coplanar** forces are **simultaneously acting** at **a point** are in **equilibrium**, then each force is proportional to the sine of the angle between other two forces.

Consider the forces A, B and C as shown in the figure



According to Lamis theorem,

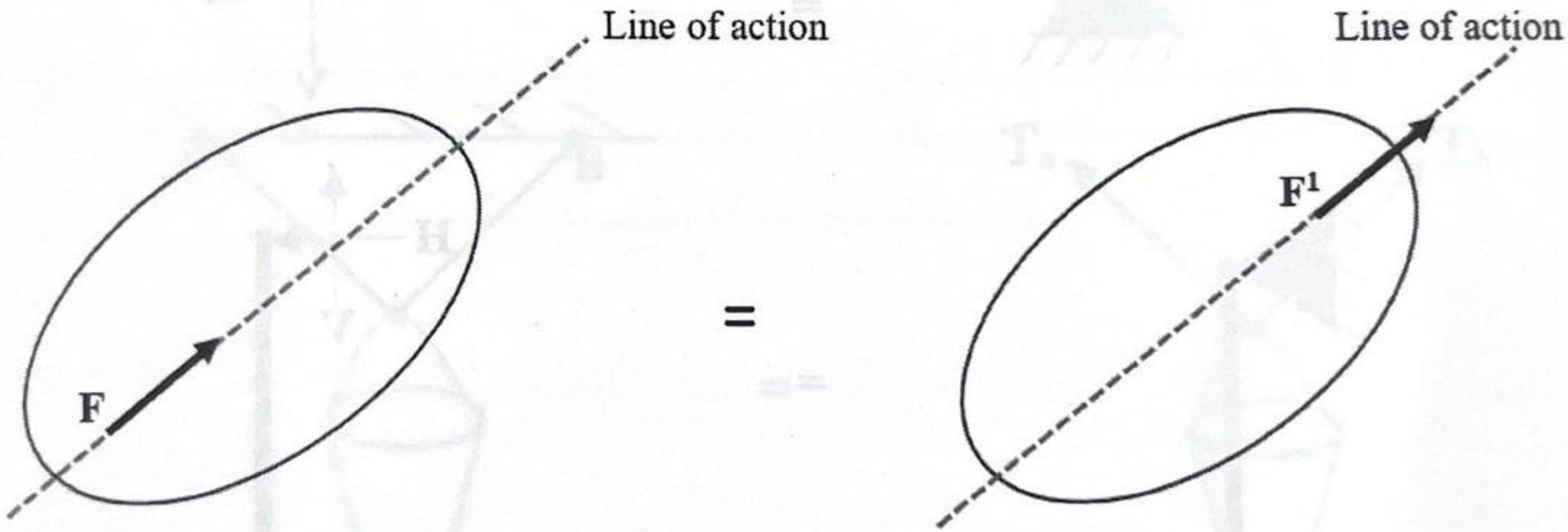
$$\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \Upsilon}$$

Conditions to apply Lamis Theorem

1. Forces must act simultaneously at a point (Concurrent force system)
2. There should be three forces
3. Forces should be coplanar
4. The system should be in equilibrium
5. All the forces should be either away or towards the point of application.

Law of Transmissibility

The state of rest or motion of the body remains unchanged if a force acting on a body is replaced by another force of same magnitude and direction along the line of action of replaced force.



In the figure shown above if $F^1 = F$, Then the effect of force acting on the body remains same.

Types of Supports

A) Roller Support

A support which is free to roll on the surface on which it rests, it offers a reactive force perpendicular to the surface on which it is free to roll.

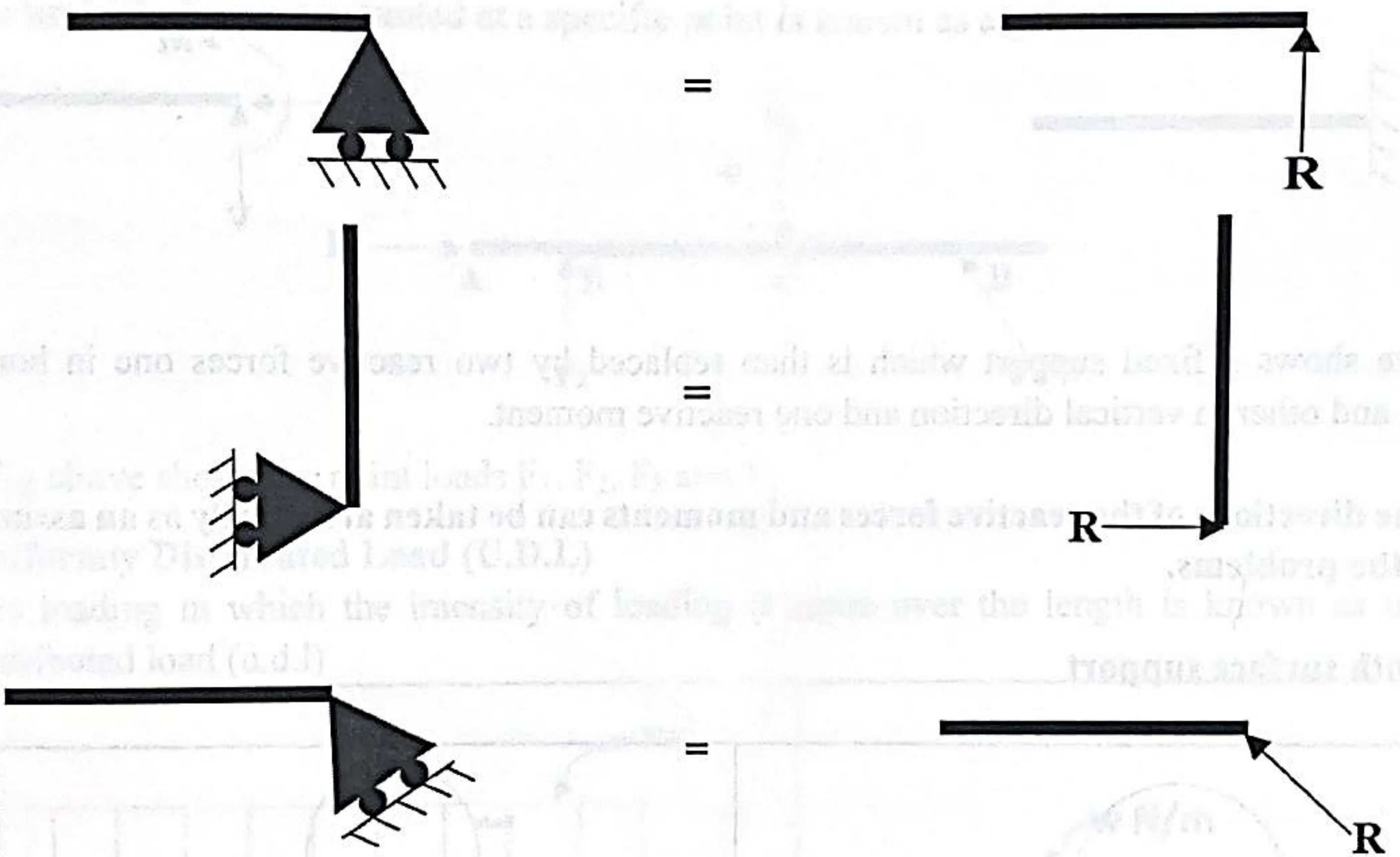


Fig above shows a roller support which is than replaced by one reactive force perpendicular to the surface.

B) Hinge Support

A support which allows the free rotation of the body but does not allow the body to have any linear motion. It therefore offers a reaction in horizontal and vertical directions.

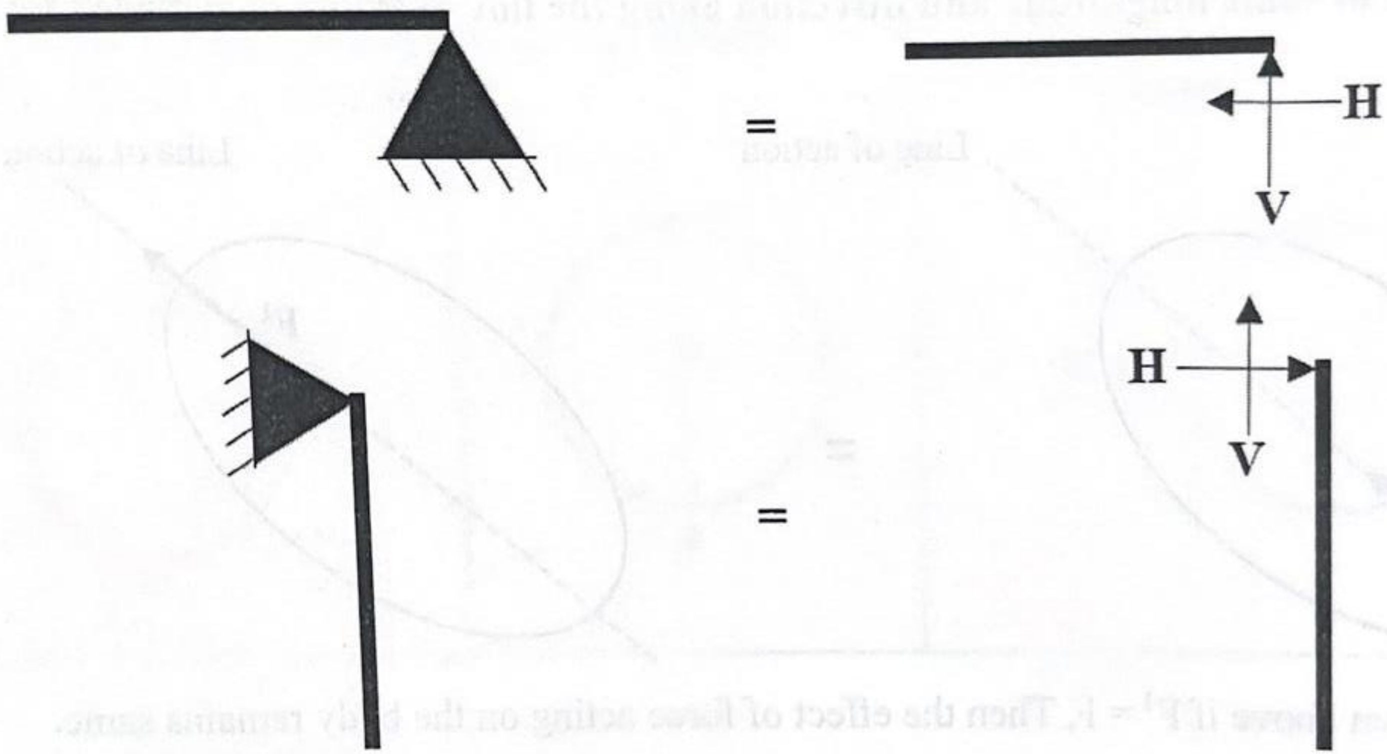


Fig above shows a hinge support which is than replaced by two reactive forces one in horizontal direction and other in vertical direction.

C) Fixed Support

A support which neither allows any linear motion nor allows any rotation. It, therefore offers a reaction in horizontal and vertical directions and also the moment reaction.

Fig above shows a fixed support which is than replaced by two reactive forces one in horizontal direction and other in vertical direction and one reactive moment.

Note: The directions of the reactive forces and moments can be taken arbitrarily as an assumption to solve the problems.

D) Smooth surface support

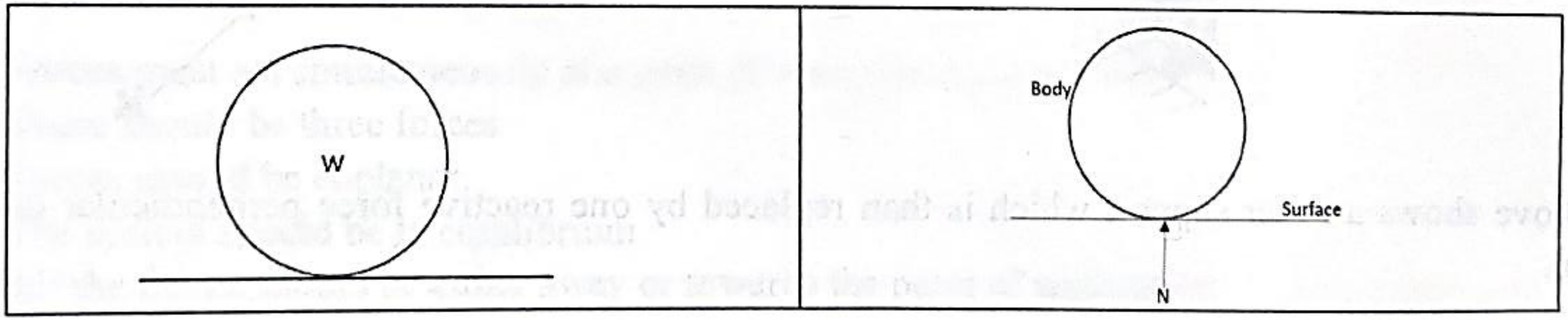


Fig above shows a smooth surface support which is than replaced by one reactive force in the direction perpendicular to the direction of support and from support to the body.

E) Rope/ String / Cable support

This type of support offers a pull force in the direction away from the body.

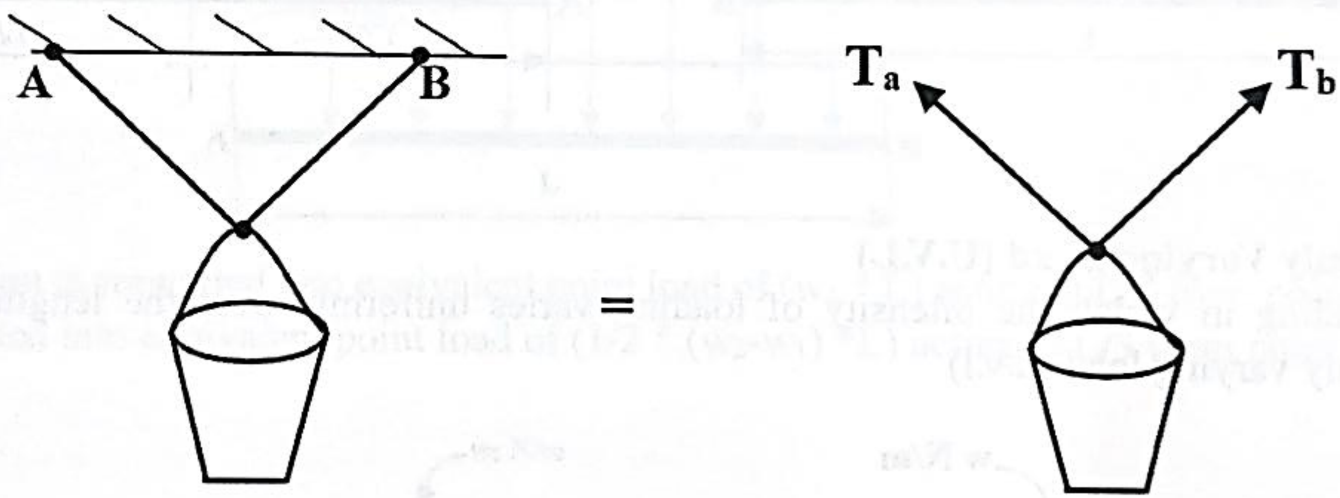


Fig above shows a bucket being supported by means of two cables at point A and B , which is than replaced by two reactive force in the direction away from the body.

Types of Loads

A) Point Load

The load which is concentrated at a specific point is known as a point load.

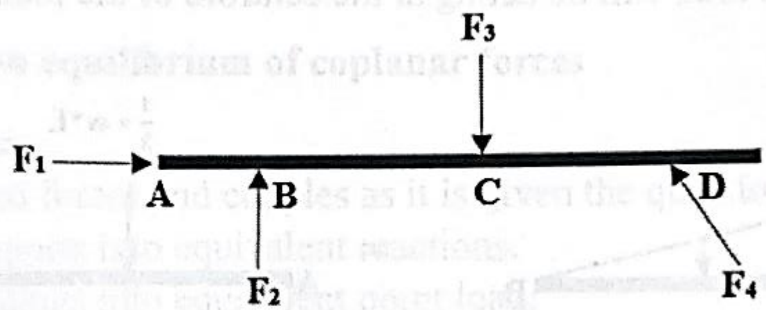
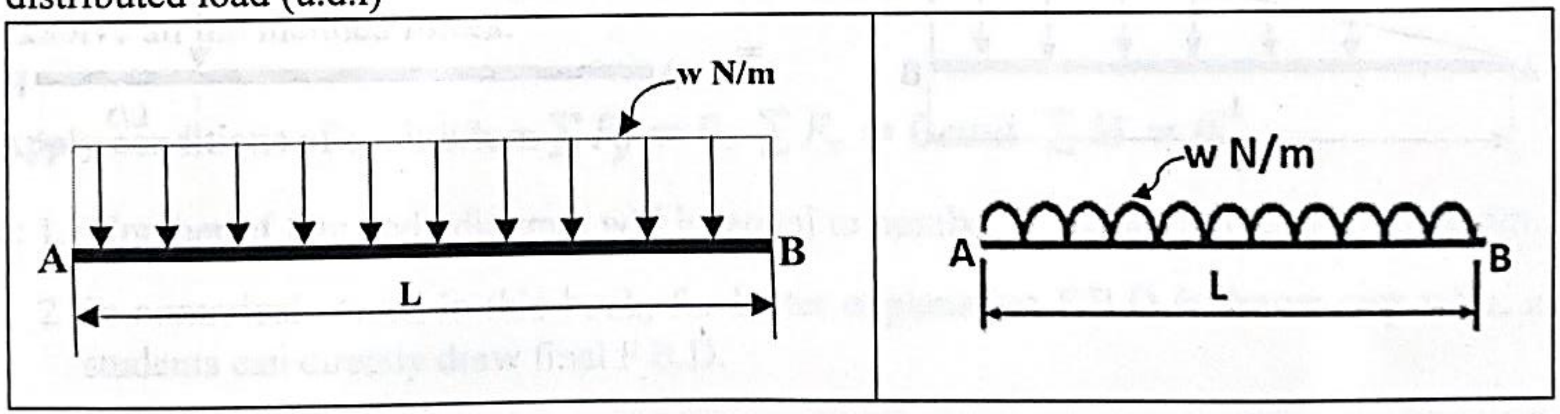


Fig above shows the point loads F_1, F_2, F_3 and F_4

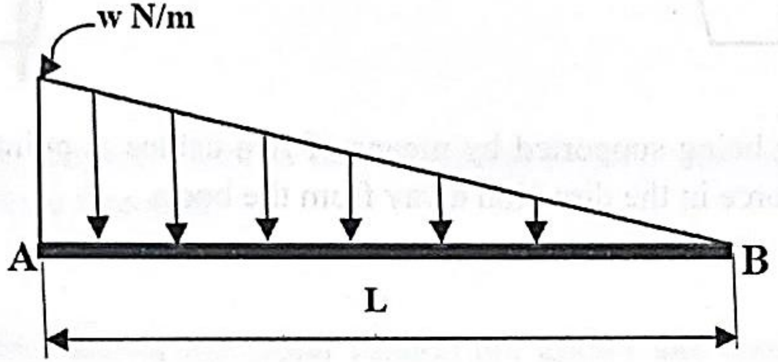
B) Uniformly Distributed Load (U.D.L)

The loading in which the intensity of loading is same over the length is known as uniformly distributed load (u.d.l)

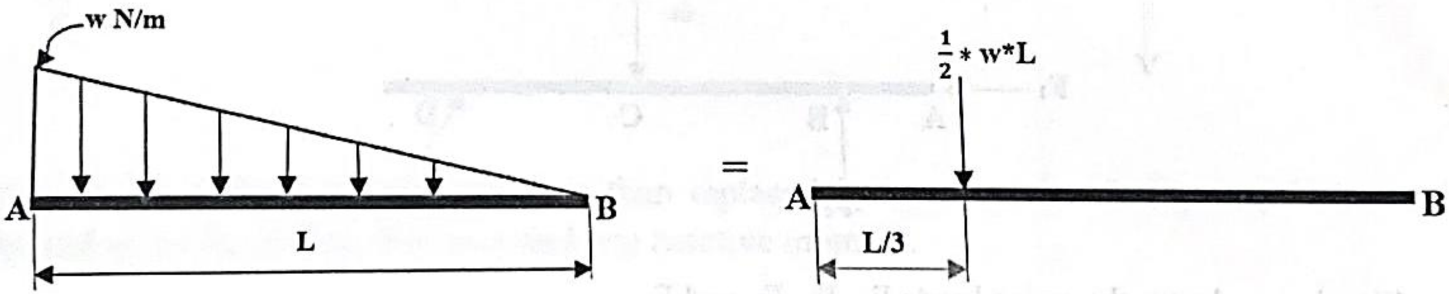


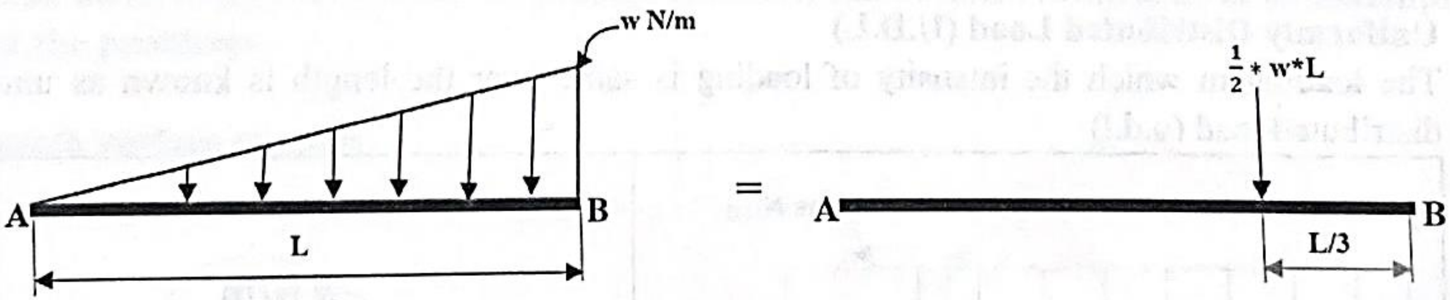
C) **Uniformly Varying Load (U.V.L)**

The loading in which the intensity of loading varies uniformly over the length is known as uniformly varying load (u.v.l)



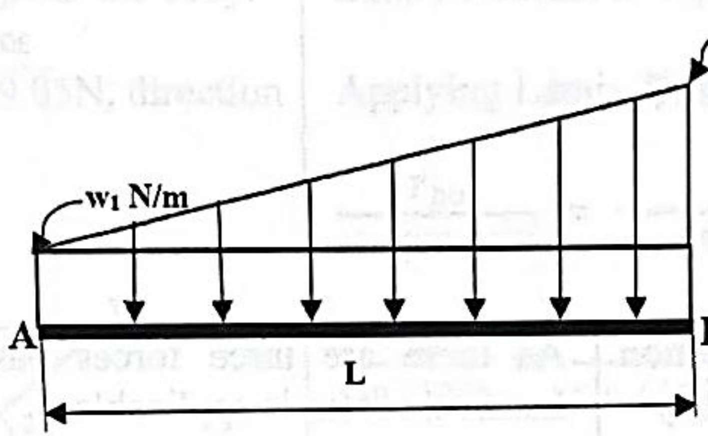
U.V.L. can be converted into equivalent point load which is equal to area under load diagram (area of triangle). This equivalent point load will be acting at the centroid of the load diagram (triangle).



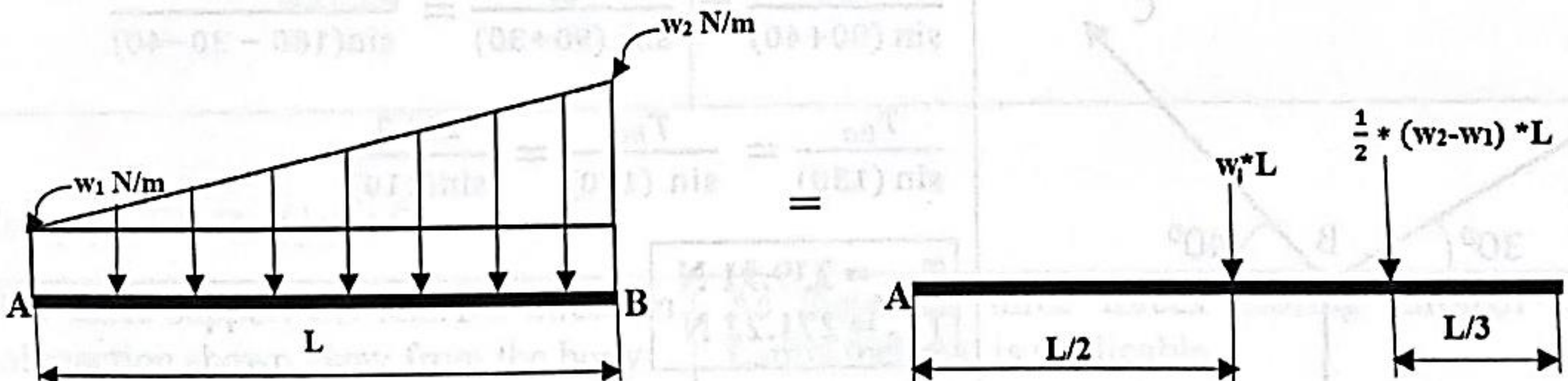


D) Trapezoidal Load (U.D.L + U.V.L)

The loading in which the intensity of loading varies uniformly with lower intensity to higher intensity over the length is known as trapezoidal load. It is a combination of u.d.l and u.v.l.



The u.d.l portion is converted into equivalent point load of (w_1 * L) acting at L/2 from point A, whereas u.v.l is converted into equivalent point load of (1/2 * (w_2-w_1) *L) acting at L/3 from point B.



Note: In above while calculating point load of u.v.l we need height of triangle only, hence height is taken as (w_2-w_1), where w_2 is the total intensity and w_1 is intensity of u.v.l.

Steps to solve problems on equilibrium of coplanar forces

1. Draw free body diagram,
   a) Show all the applied forces and couples as it is given the question.
   b) Convert all the supports into equivalent reactions.
   c) Convert all the loadings into equivalent point load.
   d) Show the self-weight (self-weight, to be shown in vertically downward direction and at the centre of the body).
   e) Show the reactions from contact surface (reactions, to be shown perpendicular to the surface and direction should be from surface towards the body).

2. Resolve all the inclined forces.

3. Apply conditions of equilibrium $\sum F_y = 0$, $\sum F_x = 0$ and $\sum M = 0$.

Note: 1. Number of free body diagram will be equal to number of bodies in the given diagram.

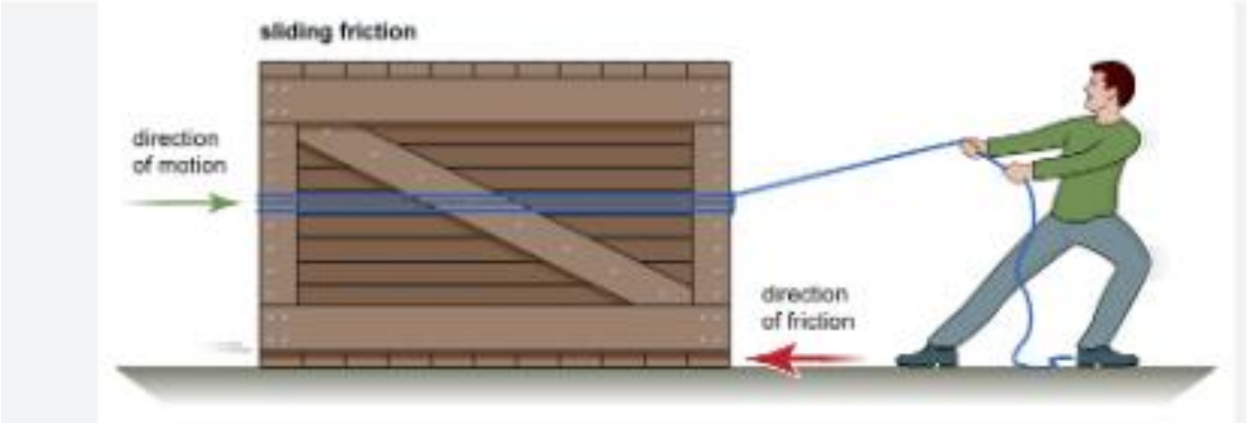
2. In numerical solved in this book, for better explanation F.B.D is drawn step wise, in exam students can directly draw final F.B.D.

UNIT 2:

Part 3: Friction

Friction: Definition

- When a body moves or rolls over a rough surface the body experience resistance, this resistance offered by the surface is called as **friction**.
- The resistive tangential force, which acts in the direction opposite to the direction of motion of the body is called **frictional force**.

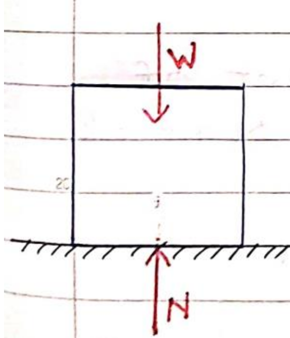


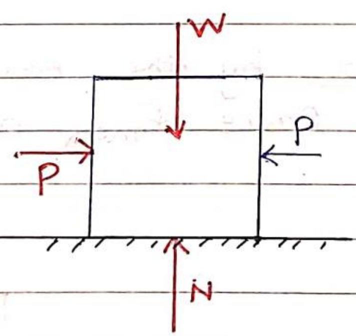
Friction :

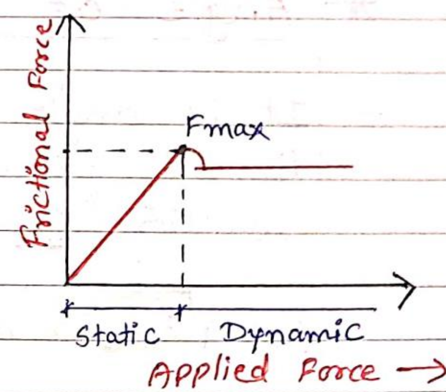
- Smooth surface offers → single Reaction
- Rough surface offers → Reaction + friction [additional reaction]
- Friction is seen in → blocks, Ladder, wedges etc.

Friction

- Dry friction (also called Coulomb friction)
  - Static
  - Kinetic
- Fluid friction.







* Always remember Frictional force is generated in the opposite direction of applied force.

Coefficient of friction : Ratio of frictional force and normal reaction

There are two co-efficient of friction

(1) Coefficient of static friction (μ_s)

$$\mu_s = \frac{F_{max}}{N}$$

F_{max} = Limiting Friction Force

(2) Co-efficient of Dynamic / Kinetic friction. (μ_k)

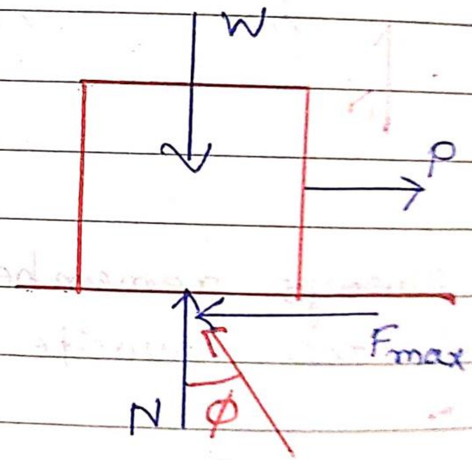
$$\mu_k = \frac{F_k}{N} = \frac{\text{Kinetic frictional Force}}{\text{Normal Reaction}}$$

* Angle of Friction :

- It is angle made by the Resultant of limiting frictional force F_{max} and Normal reaction "N"

$R = \sqrt{F_{max}^2 + N^2}$

$= \sqrt{(\mu_s N)^2 + N^2}$



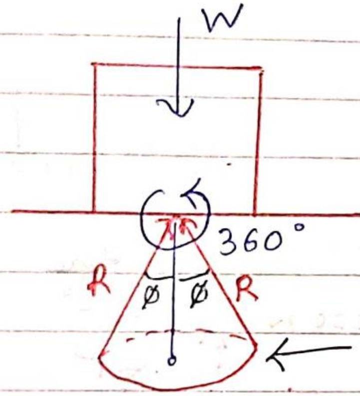
& $\tan\phi = \frac{F_{max}}{N} = \frac{\mu_s \cancel{N}}{\cancel{N}}$

$\boxed{\therefore \ \phi = \tan^{-1}\mu_s}$ ⇐ VVIMP angle of friction

Remember this

Cone of friction :

When we rotate the whole arrangement then accordingly resultant rotates & forms a "Cone" This cone is known as cone of friction.

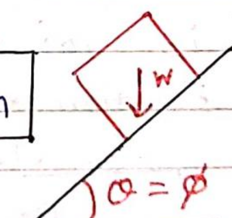


Angle of Repose :

It is the angle for which body just slide down without any Force.

$$\boxed{\text{Angle of Repose} = \text{Angle of Friction}}$$

↑
VVIMP



Laws of Friction

1. The frictional force is always **tangentia**l to the contact surface

2. It acts in the direction **opposite** to the direction of impending motion

3. The value of frictional force Fr increases with the increase in the disturbing force ‘P’.till it reaches a limiting value 'F_{max}' .At this the body is said to be on the verge of motion known as impending motion

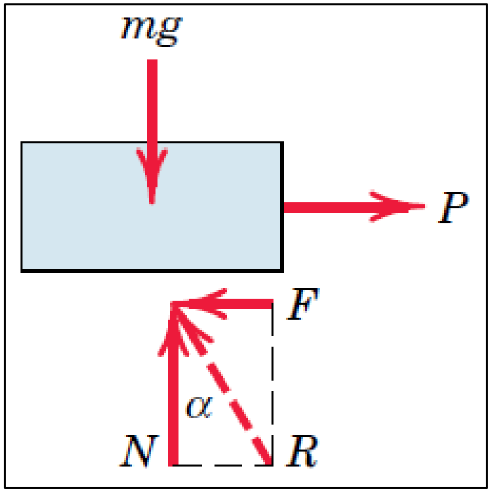
4. The ratio of limiting frictional force 'F_{max}' and normal reaction ‘N’ is a constant and is known as coefficient of static friction (μ_s).

5. The ratio of limiting frictional force ‘Fk’ and the normal reaction ‘N’ is a constant and is known as coefficient of dynamic friction (μ_k)

6. The frictional force generated is **independent** of the area of contact.

Angles in Friction

- The direction of the resultant R measured from the direction of N is specified by $\tan\alpha = F/N$
- When the friction force reaches its limiting static value F_{max}, the angle α reaches a maximum value φ_s.
- Thus $\tan\varphi_s = \mu_s$
- When slippage is occurring, the angle α has a value φ_k corresponding to the kinetic friction force.
- In like manner $\tan\varphi_k = \mu_k$
- $\mu s = \frac{F_{max}}{N}$ $\mu k = \frac{F_k}{N}$



Angles in Friction

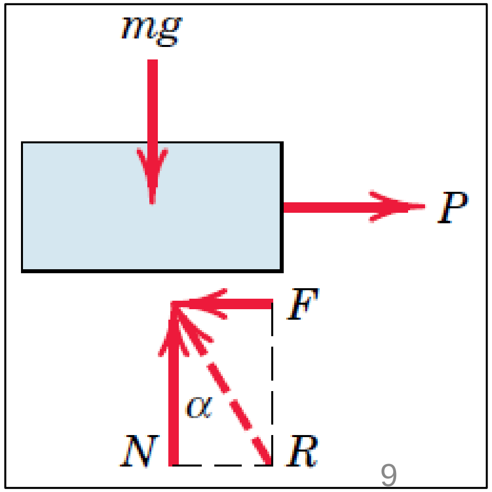
- In practice we often see the expression

$$\tan \varphi = \mu$$

in which the coefficient of friction may refer to either the static or the kinetic case, depending on the particular problem.

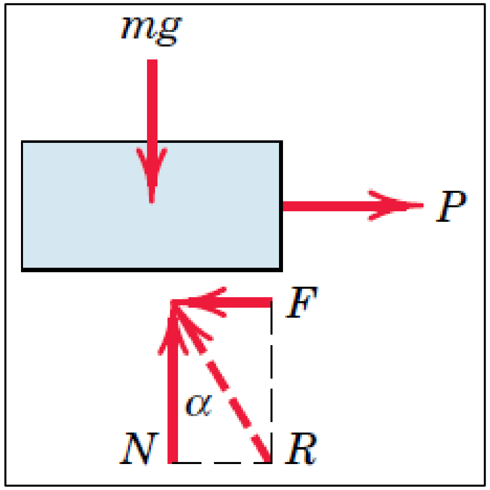
- The angle φ_s is called the angle of static friction, and

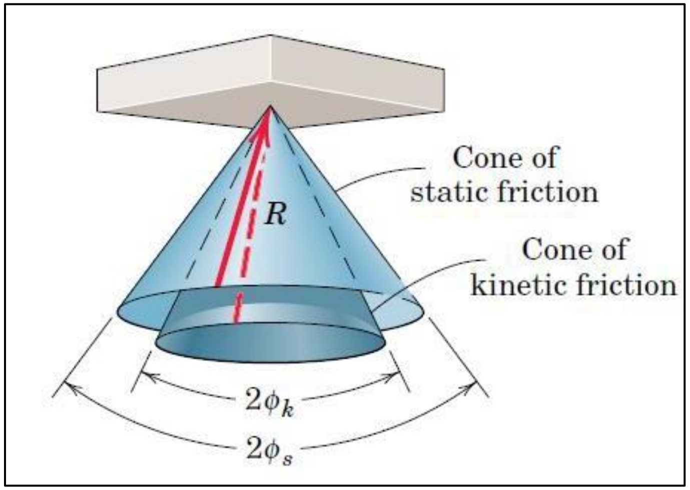
- The angle φ_k is called the angle of kinetic friction.



Cone of Friction

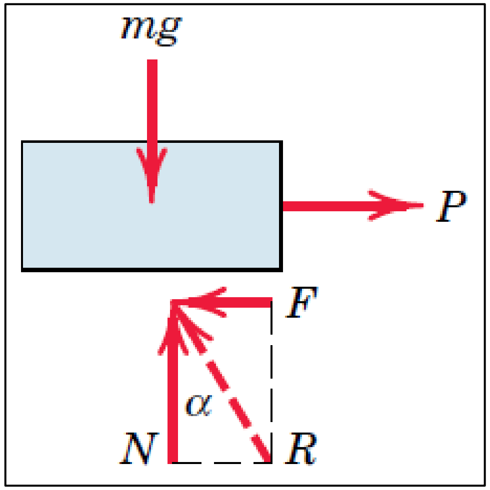
- The friction angle for each case clearly defines the limiting direction of the total reaction R between two contacting surfaces.
- If motion is impending, R must be one element of a right-circular cone of vertex angle $2\varphi_s$.
- If motion is not impending, R is within the cone.

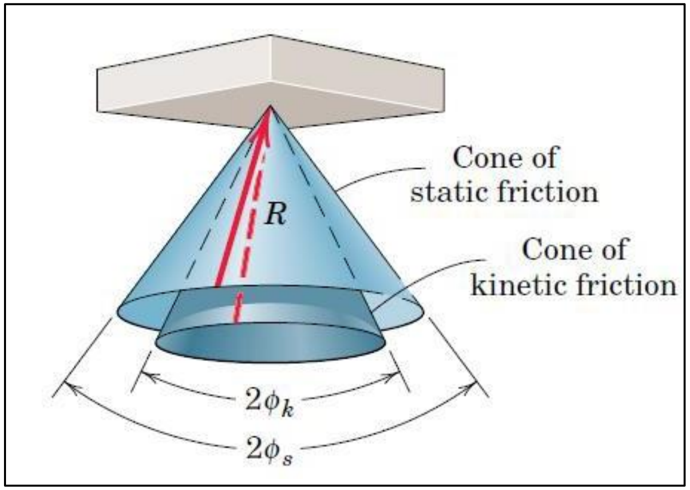




Cone of Friction

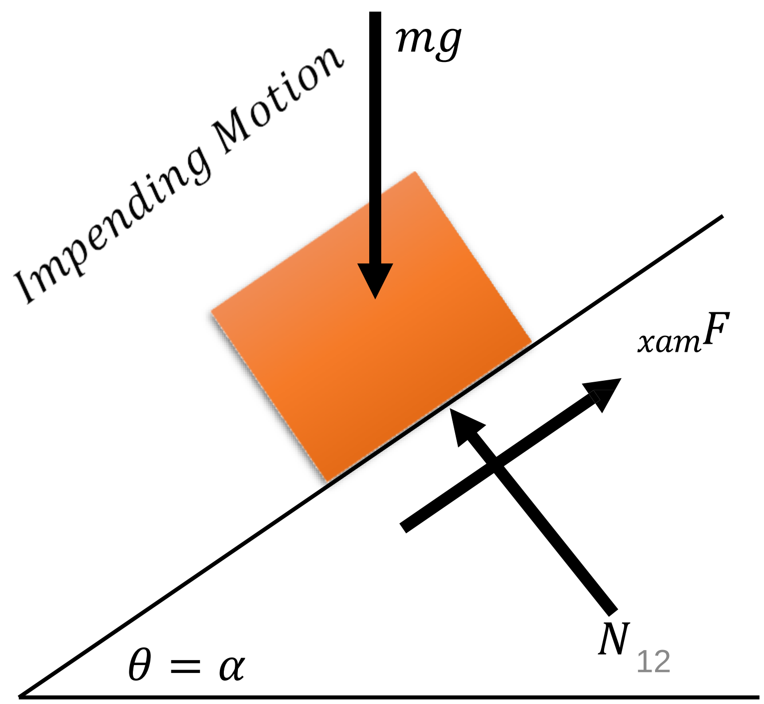
- This cone of vertex angle $2\varphi_s$ is called the **<u>cone of static friction</u>** and represents the locus of possible directions for the reaction R at impending motion.
- If motion occurs, the angle of kinetic friction applies, and the reaction must lie on the surface of a slightly different cone of vertex angle $2\varphi_k$. This cone is the **<u>cone of kinetic friction</u>**.





Angle of Repose

- It is defined as the minimum angle of inclination of a plane with the horizontal for which a body kept on it will just slide down in it without the application of any external force.
- Consider a block of mass m resting on a rough horizontal plane. The plane is slowly titled till the block is just on the verge of sliding down the plane.
- The angle of inclination of the plane at this position is known as angle or repose, denoted by α



Unit 04:Kinematics of Particle and Rigid bodies

Mechanics of rigid bodies is divided into two parts statics and dynamics.

1) Statics

branch of mechanics which deals with the study of resultant effect of forces acting on the body when the body (rigid body) is at rest.

II) Dynamics

branch of mechanics which deals the with study of resultant effect of forces acting on the body when the body (rigid body) is in motion.

Dynamics is further classified into

a) Kinematics

branch of dynamics which deals with the study of bodies under motion without considering the cause of motion of the body.

It is concerned only with the study of motion of the body for e.g. displacement, velocity, acceleration of the body.

b) Kinetics

branch of dynamics which deals with the study of bodies under motion considering the cause of motion of the body i.e. forces.

Unit 04:Kinematics of Particle and Rigid bodies

In this chapter we will focus on motion analysis of particles without considering the cause of the motion of the body i.e. kinematics.

Study of motion of the particle will enable us to find out the terms like position, displacement, acceleration, velocity and time.

The following motion will be discussed in this chapter

1. Rectilinear motion.
2. Motion under gravity.
3. Projectile motion.
4. Curvilinear motion.
5. Motion with variable acceleration.
6. Motion curves.

Unit 04:Kinematics of Particle and Rigid bodies

Rectilinear motion

When the motion of the particle is along a straight line than it is termed as rectilinear motion.

Some of the examples of rectilinear motion are

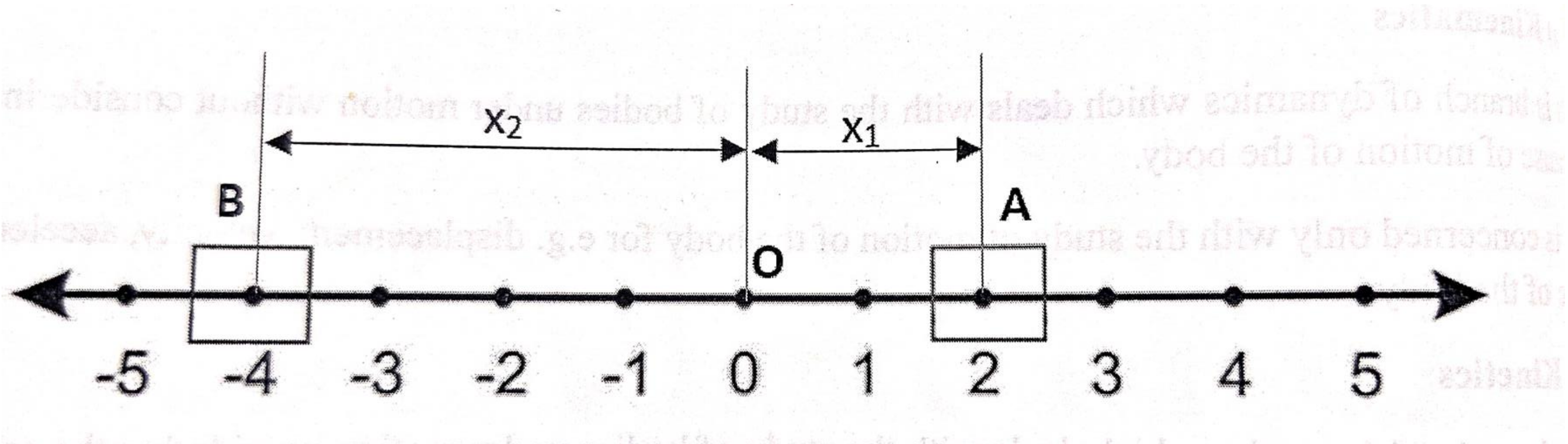
- Beverage falling from a machine nozzle to the cup kept below
- Lift moving along the vertical guideways
- Automobiles moving on a straight road

Unit 04:Kinematics of Particle and Rigid bodies

Position, displacement and distance

For motion analysis of the particle its position at various instants is the crucial parameter.

Position means the location of a particle with respect to a fixed datum (reference) point. This datum point can also be referred to as origin.



In the figure above particle A is at position $x_1 = +2$ or 2 as it is towards right of origin 'O',

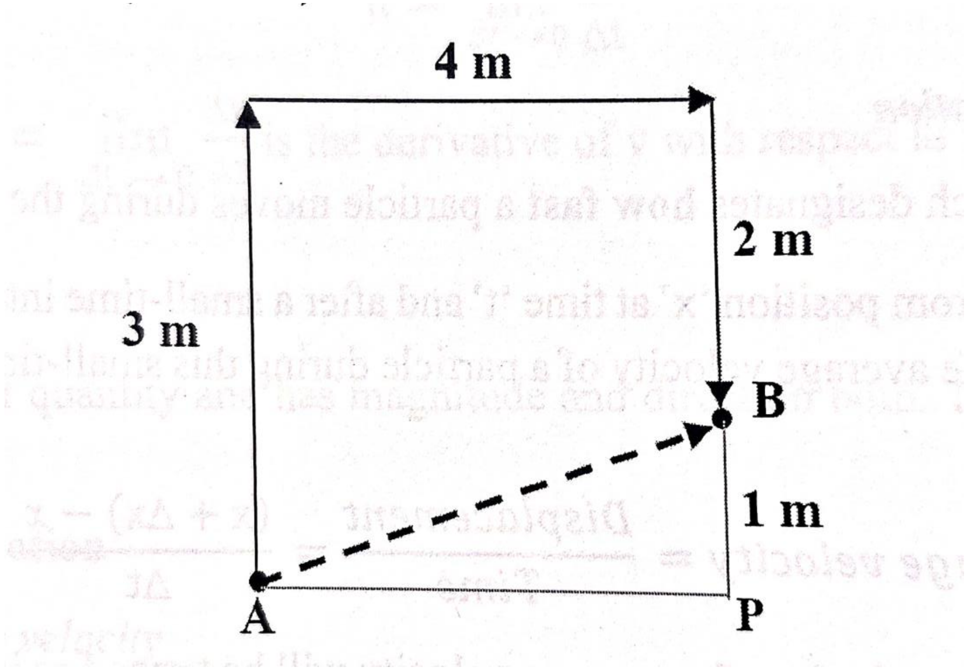
whereas particle B is at position $x_2 = -4$ as it is towards left of origin.

Unit 04:Kinematics of Particle and Rigid bodies

Distance vs Displacement

Displacement is the shortest distance between initial and the final position of a particle undergoing the motion.
It is a vector quantity and has magnitude and direction both.

Distance is the actual path length cover by the object or a body in a given interval of time. It is a scalar quantity.



Unit 04:Kinematics of Particle and Rigid bodies

Velocity, speed and acceleration

Velocity is the quantity which designates how fast a particle moves during the motion.
Velocity is a vector quantity and has magnitude and direction both.
The S.I unit of velocity is m/s.
The magnitude of velocity is termed as speed of the particle.

$$v = \frac{dx}{dt}$$

Acceleration is the rate of change of velocity with respect to time.

$$a = \frac{dv}{dt}$$

Unit 04:Kinematics of Particle and Rigid bodies

Types of rectilinear motion

a. Motion with uniform velocity

When a particle undergoes same amount of displacement in equal intervals of time the particle is said to be moving with uniform velocity, for e.g. velocity of sound, a vehicle moving at constant speed for some interval of time, conveyer belts etc.

For motion with uniform velocity, **$v = s/t$**

Where **s** is the displacement of particle and

t is the time during which particle was in motion.

Unit 04:Kinematics of Particle and Rigid bodies

b. Motion with uniform acceleration

When the velocity of a particle changes at uniform rate than it is said that particle is undergoing motion with uniform acceleration.

- For motion with uniform acceleration the following equations are used.

$$s = ut + 1/2\, at^2$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

Where,

s is the displacement of particle

t is the time during which particle was in motion.

u is the initial velocity of the particle.

v is the final velocity of the particle.

a is the acceleration of the particle.

Unit 04:Kinematics of Particle and Rigid bodies

c. Motion with variable acceleration

If the rate of change of velocity of the particle is not uniform than it is said that particle is undergoing motion with variable acceleration.

Variable acceleration is usually defined by writing acceleration as a function of time, velocity or position.
To calculate the parameters for a particle undergoing variable acceleration use the basic differential equation of velocity and acceleration which are given as follows

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} \quad \text{or} \quad a = \frac{v.dv}{dx}$$

Unit 04:Kinematics of Particle and Rigid bodies

Curvilinear motion

The motion of an object (particle) along a circular path is called as curvilinear motion.
Movement of car along curved road, planet revolving around sun, whirling of ball by a person are examples of curvilinear motion.

There are two methods to analyse this type of motion.
a) Curvilinear motion by rectangular component system.
b) Curvilinear motion by tangential and normal component.

Curvilinear motion by rectangular component system.

When a particle moves along a curved path, it's motion can be split into x, y and z direction as independently performing rectilinear motion.

Unit 04:Kinematics of Particle and Rigid bodies

The displacement, velocity and acceleration are given by

$$\text{Position:} \quad \bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$|\bar{r}| = \sqrt{x^2 + y^2 + z^2}$$

$$\text{Velocity:} \quad \bar{v} = \frac{d\bar{r}}{dt} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$$

$$|\bar{v}| = \sqrt{v_x{}^2 + v_y{}^2 + v_z{}^2}$$

$$\text{Acceleration:} \quad \bar{a} = \frac{d\bar{v}}{dt} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$|\bar{a}| = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$$

For co-planer motion, the particle can be considered as moving in x-y plane.
Hence eliminate the z terms in the above equations.

Unit 04:Kinematics of Particle and Rigid bodies

Directions are given by

$$\cos\alpha = \frac{x}{r} = \frac{v_x}{v} = \frac{a_x}{a}$$

$$\cos\beta = \frac{y}{r} = \frac{v_y}{v} = \frac{a_z}{a}$$

$$\cos\gamma = \frac{z}{r} = \frac{v_z}{v} = \frac{a_z}{a}$$

For co-planer motion, the particle can be considered as moving in x-y plane.
Hence eliminate the z terms in the above equations.

Unit 04:Kinematics of Particle and Rigid bodies

Curvilinear motion by tangential and normal component system

In curvilinear motion the acceleration can be splitted into two components, one along tangential direction (a_t) and another in normal direction (a_n).

Hence acceleration can be written as

$$\bar{a} = a_t.\bar{e}_t + a_n.\bar{e}_n$$

Magnitude of acceleration is given by

$$a = \sqrt{a_t^2 + a_n^2}$$

Unit 04:Kinematics of Particle and Rigid bodies

1) **Tangential component of acceleration ($\bar{a}_t$)**

This component of acceleration arises due to rate of change of speed of a particle. The direction of $\bar{a}_t$ is along the velocity if speed increases and opposite to the direction of velocity if speed decreases. As the direction of velocity always remains tangential to the path so does the direction of $\bar{a}_t$.

For constant speed in curvilinear motion.

$$a_t = \frac{dv}{dt} = 0$$

Following equations of motion can be used.

$$v = u + a_t * t$$

$$s = u * t + \frac{1}{2} * a_t * t^2$$

$$v^2 = u^2 + 2 * a_t * s$$

Where, s is the distance covered along the path.

u and v are initial and final velocity respectively.

a_t tangential component of acceleration.

2) Normal component of acceleration ($\overline{a}_n$)

This component of acceleration arises due to the change in direction of motion and is always directed towards the centre of curvature of the path. This acceleration is also called as centripetal acceleration.

$$a_n = \frac{v^2}{\rho}$$

Where, v is speed at the given instant and ρ is the radius of curvature.

Unit 04:Kinematics of Particle and Rigid bodies

Radius of curvature: It is the distance from the centre of curvature to the point on the curvilinear path. It is given by the formula:

$$\rho = \left|\frac{(v_x^2 + v_y^2)^{\frac{3}{2}}}{v_x a_y - v_y a_x}\right| = \left|\frac{v^3}{v_x a_y - v_y a_x}\right|$$

If given curved path is defined by function $y = f(x)$ then radius of curvature can be determined using following equation.

$$\rho = \frac{[1+\left(\frac{dy}{dx}\right)^2]^{3/2}}{\frac{d^2y}{dx^2}}$$

slope will be given by

$$\frac{dy}{dx} = tan\theta$$

And velocity is given by

$$v_x = v * cos\theta \qquad v_y = v * sin\theta$$

7.5 Motion Curves

A Graphical representation of displacement, velocity and acceleration with respect to time is known as motion curve.

Consider the following displacement equation

s *or* $x = 2t^3 + 5t^2 - 6t$ .... (1)

Rate of change of displacement with respect to time is velocity, differentiating eqn (1) with respect to time we get,

$v = 6t^2 + 10t - 6$ .... (2)

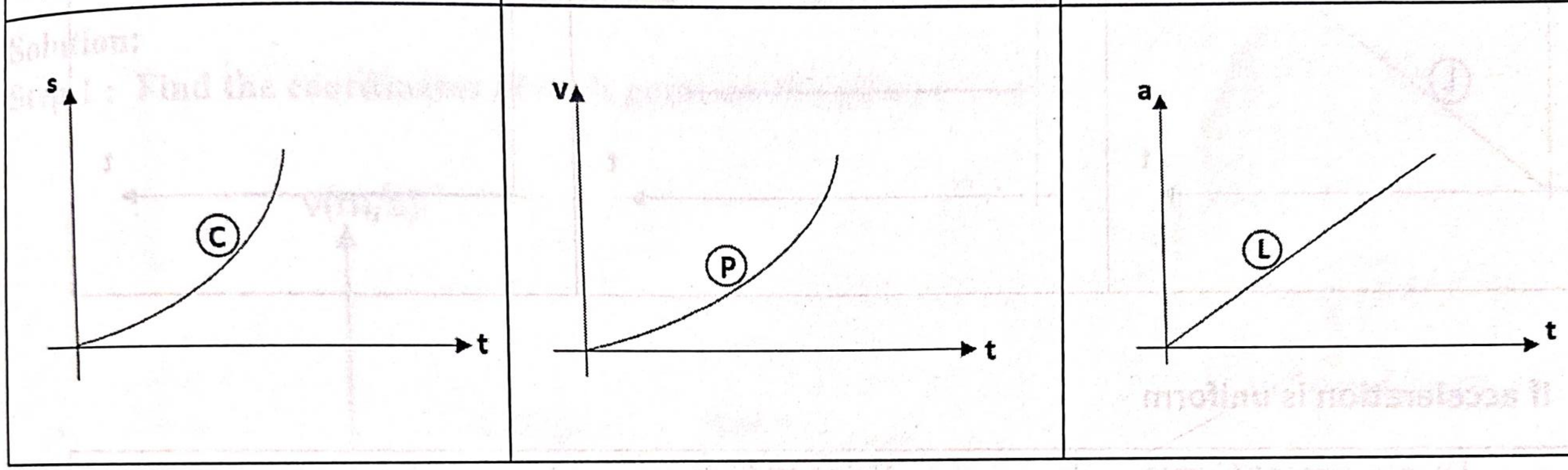
Rate of change of velocity with respect to time is acceleration, differentiating eqn (2) with respect to time we get,

$a = 12t + 10$ .... (3)

From above equations (1), (2) and (3) it can be observed that, the highest degree of eqn (1) is 3, Hence displacement - time curve will be a cubic curve, Similarly the highest degree of eqn (2) is 2, Hence velocity - time curve will be a parabolic curve and the highest degree of eqn (3) is 1, Hence acceleration-time curve will be linear. The curves in the graphical form are shown below.

Unit 04:Kinematics of Particle and Rigid bodies

Displacement Curve	Velocity Curve	Acceleration Curve



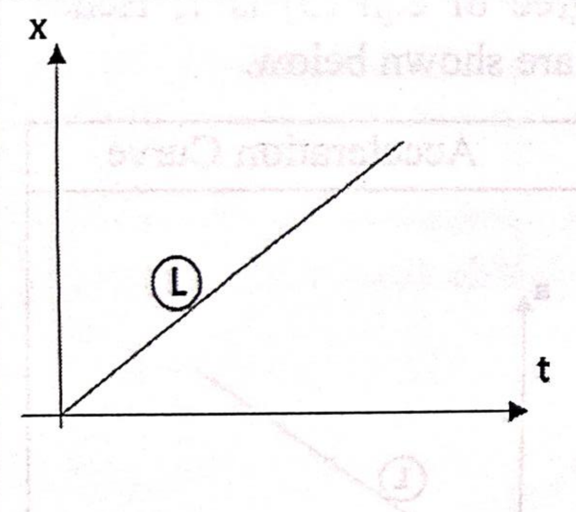
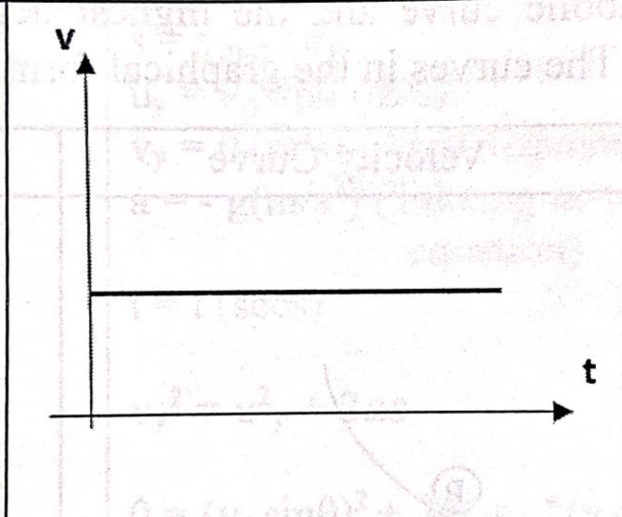
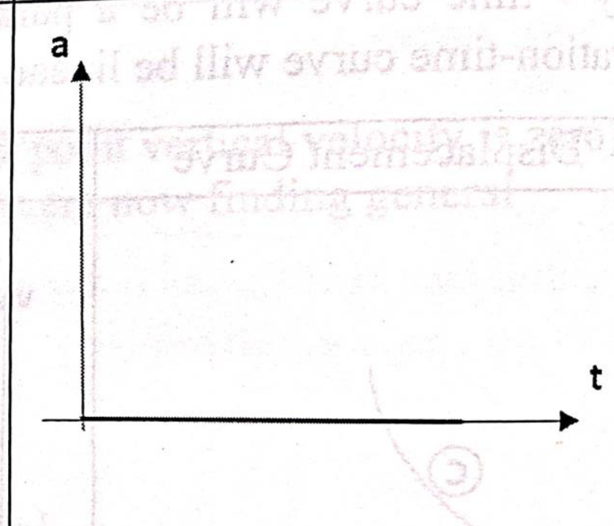
From above, it is worth noting that

1. As we come down the line from displacement to velocity and velocity to acceleration, We need to take derivative (i.e Slope of the curve) and the **degree** of curve **decrease.**
2. As we go up the line from acceleration to velocity and velocity to displacement, We need to take integration (i.e Area under the curve) and the **degree** of curve **increase.**

Degree of curve	↓	Degree of curve
Decreases	**Displacement**	**Increase**
	Velocity	
(Take derivative **i.e. Slope of the curve)**	**Acceleration**	(Take integration **i.e. Area under the curve)**

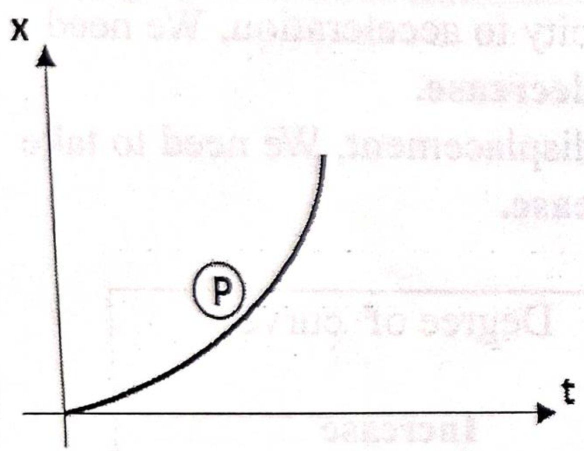
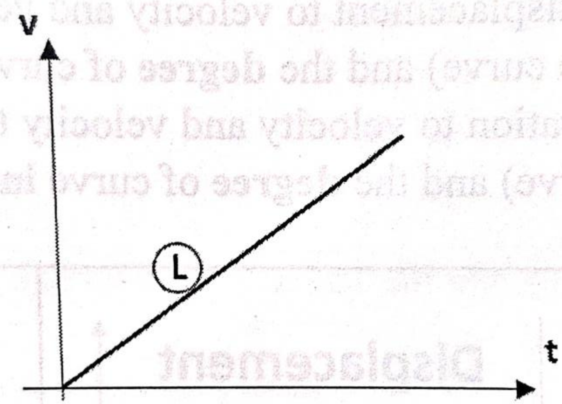
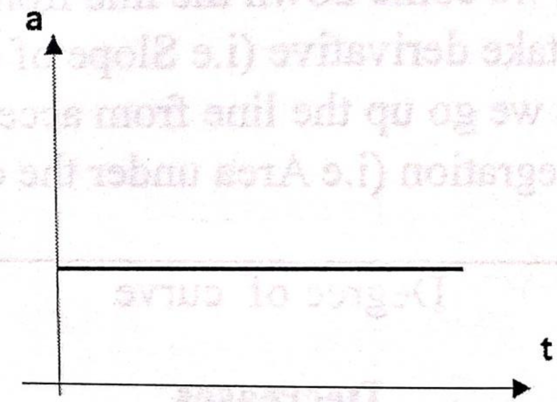
- **Standard Motion Curves**

a) If acceleration is zero i.e. Uniform velocity

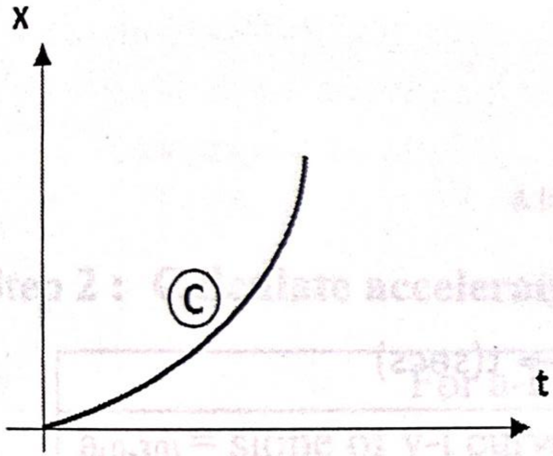
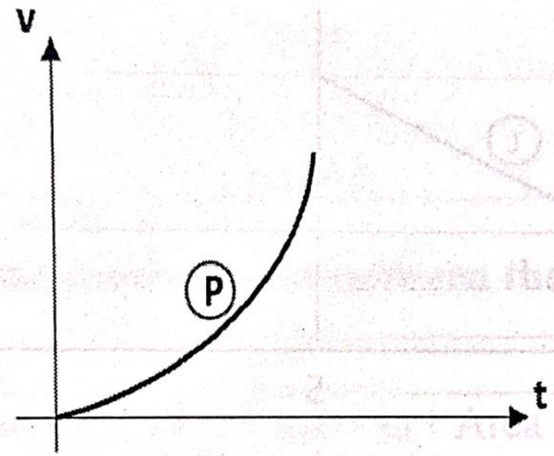
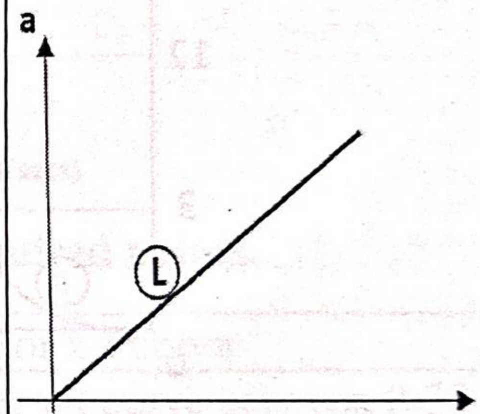
Displacement Curve	Velocity Curve	Acceleration Curve
		

Unit 04:Kinematics of Particle and Rigid bodies

b) If acceleration is uniform

Displacement Curve	Velocity Curve	Acceleration Curve
		

c) **If acceleration is variable (Linearly Varying)**

Displacement Curve	Velocity Curve	Acceleration Curve
		

L – Linear, P – Parabolic, C - Cubic

Unit 04:Kinematics of Particle and Rigid bodies

The following table shows the uses of various motion curves

Motion Curve	Use	Formula
s-t or x-t	Slope of s-t or x-t curve gives velocity	v = Slope of (x-t or s-t) curve
v-t	a) Slope of v-t curve gives acceleration b) Area under v-t curve gives the change in position and hence the new position	a = Slope of (v-t) curve $x_f = x_i$ + Area under (v-t) curve
a-t	Area under a-t curve gives the change in velocity and hence the new velocity	$v_f = v_i$ + Area under (a-t) curve
v-x	Slope of v-x curve helps in finding the particle acceleration.	a = v * slope of (v-x) curve

Unit 04:Kinematics of Rigid bodies

- Relation between displacement, velocity and acceleration of various points **on the body** under Consideration.

Difference between a body and a particle

Particles are bodies where the entire mass is concentrated at a single point.
Rigid bodies are those which have their mass distributed throughout a finite volume.
Particles have non-deforming mass whereas **rigid bodies** have non-deforming mass **along with shape & size.**
Rigid body analyses are required when **length or size of the object must be considered**, including rotation and torque.

Unit 04:Kinematics of Rigid bodies

Example:

For an **aeroplane** it is considered as particle if it flies in the air at high speed,

whereas the same aeroplane will be considered **as rigid body** if we need to analyze the **rotation of the plane while making a turn, its size and deflection of wings to remain in air, the size of the wheels while it is taxing on a runway** etc.

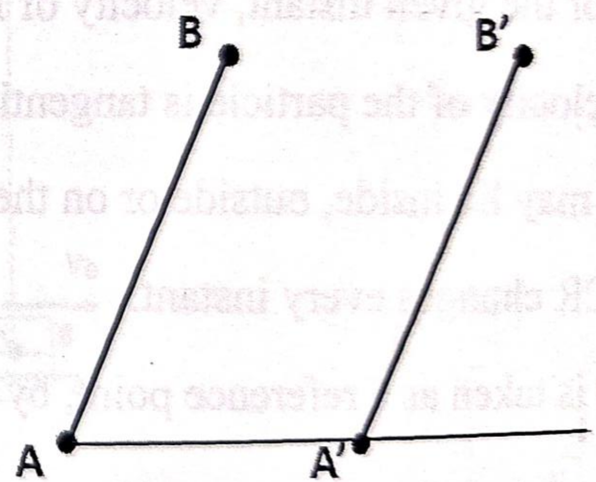
In **baseball**, if we are interested **only in how far the ball travels**, then we **consider it as a particle** and do the analysis, as the speed is much greater than the size of the ball.

On the other hand, **a rigid body analysis** would also involve **how the bat swings to hit the ball**, as the length of the bat will change how far the ball travels.

8.2 Types of motion

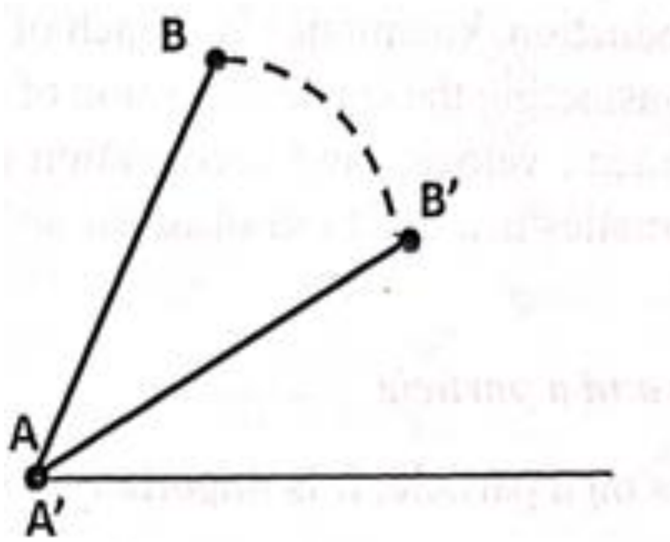
1. Translation motion.

If all points of a moving body move uniformly in the same line or direction, then the motion is termed as translation motion.



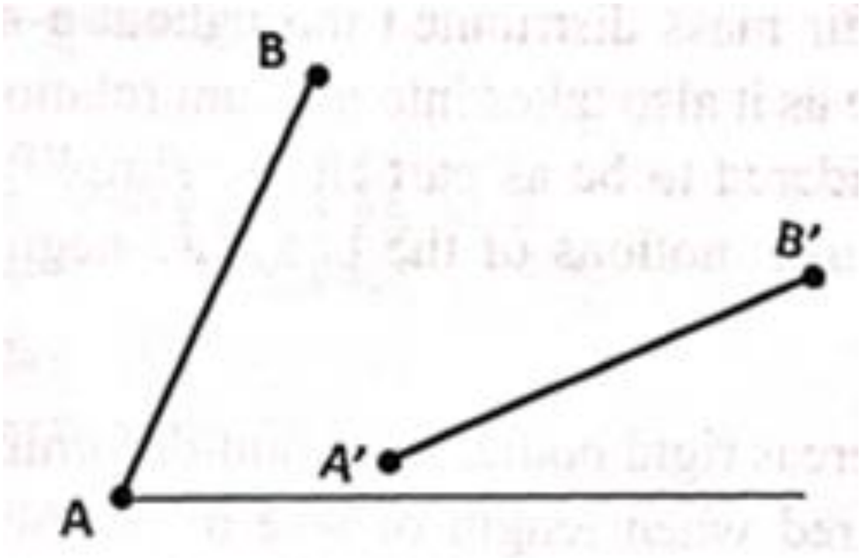
2. Rotational motion.

Rotational motion is the motion of an object around a fixed axis of rotation. In this type of motion an object is rotating about a point, rather than moving in a straight line.



3. General plane motion.

Any plane motion that is neither rotation nor translation is termed as general plane motion. (G.P.M).
e.g. Tyre being rolled along the ground, slipping of a ladder, crank and piston mechanism.



In general plane motion which is combination of rotation and translation motion, there exist a imaginary point about which the body is assumed to perform pure rotation motion.
This imaginary point is known as Instantaneous Centre of Rotation (I.C.R).

- To locate ICR of the body, we need to draw perpendiculars to resultant velocity of the points on the body. The point at which these perpendiculars of the resultant velocities meet is Instantaneous Centre of Rotation (I.C.R).

- It is denoted by “ I ”

Properties of ICR

1. For the given instant, velocity of ICR is zero.

2. Velocity of the particle is tangential to the line joining the particle and ICR.

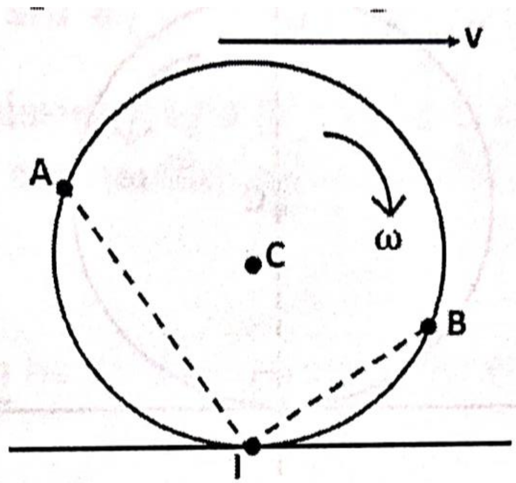
3. It may lie inside, outside or on the body.

4. ICR changes every instant.

5. It is taken as a reference point, by which the velocities of other points on the links are determined

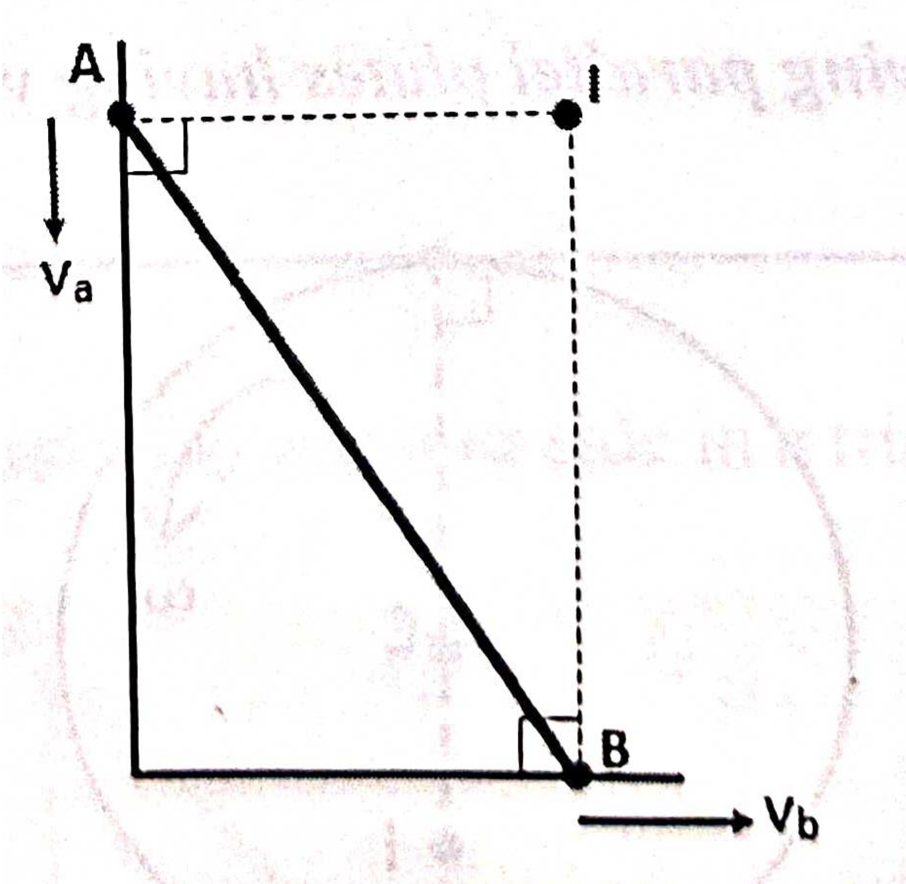
LOCATION OF INSTANTANEOUS CENTER OF ROTATION (ICR)

Case a) Body Rotating and translating on a fixed surface.

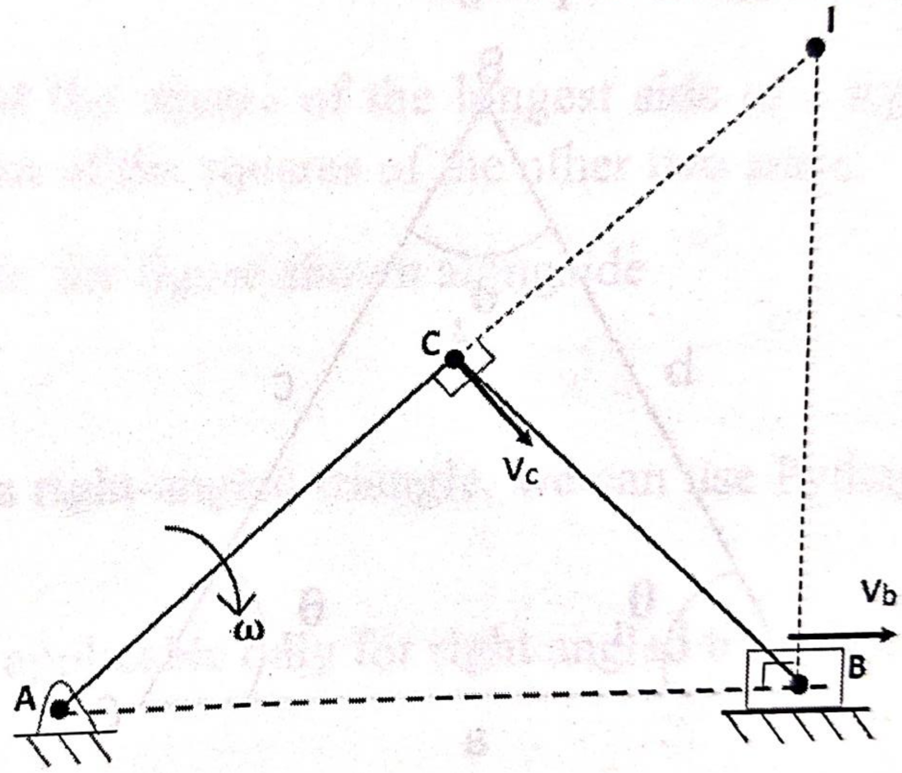


As shown in the figure above, consider a cylinder rotating with angular velocity 'ω' and linear velocity 'v' simultaneously, on a fixed surface. In this case, ICR (I) will be the point of intersection of body with the ground.

Case b) ladder leaning against a wall

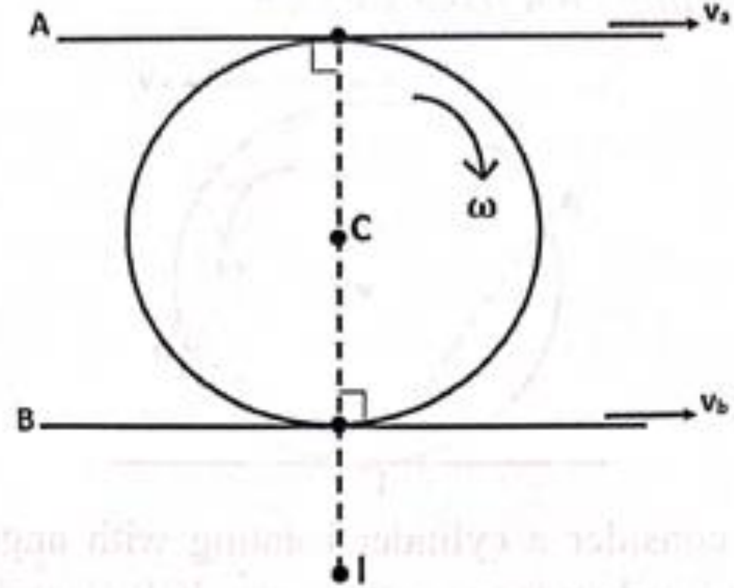


Case c) Crank and Slider mechanism



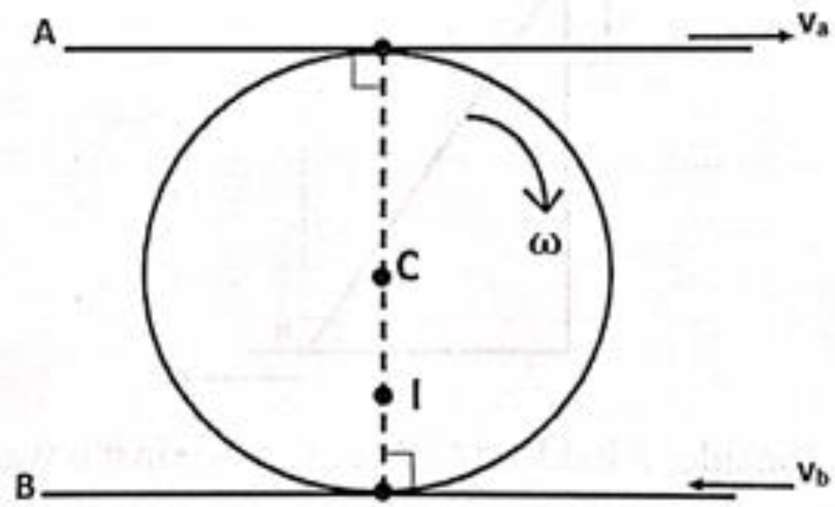
As shown in the figure above, consider a crank and slider mechanism in which when crank rotates with angular velocity 'ω' and thereby the slider 'B' slides with velocity 'v_b'. In this case ICR (I) will be the point of intersection of perpendiculars drawn from the velocities of point C and slider B.

Case d) Body in between two moving parallel plates having velocity in same direction

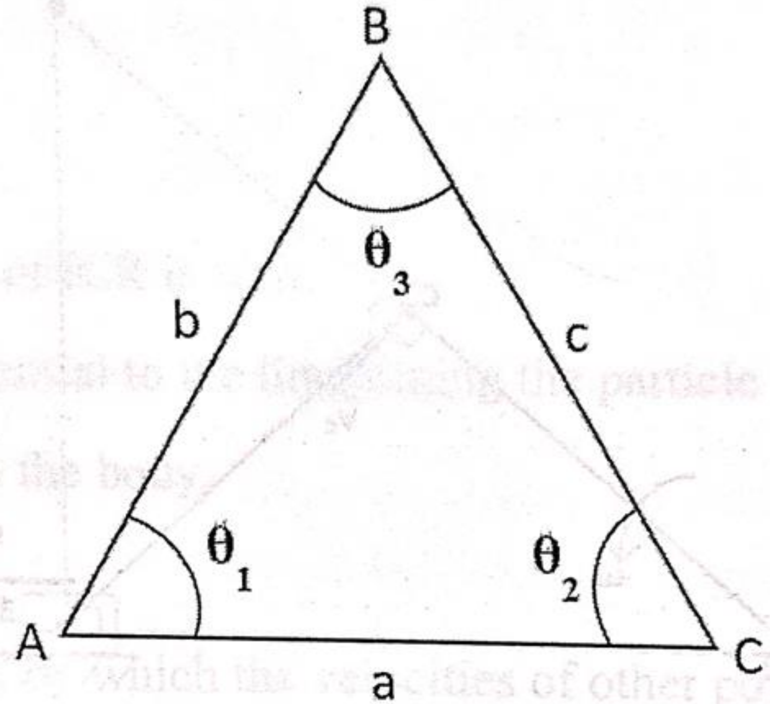


As shown in the figure above, consider a body rotating with angular velocity '**ω**' in between two plates linear velocity '**v_a**'and '**v_b**' (same direction). In this case, ICR (I) will lie outside the body on the diameter line perpendicular to velocities of plates.

Case e) Body in between two moving parallel plates having velocity in opposite direction



As shown in the figure above, consider a body rotating with angular velocity '**ω**' in between two plates linear velocity '**v_a**'and '**v_b**' (opposite direction). In this case, ICR (I) will lie inside the body on the diameter line perpendicular to velocities of plates.



Sine Rule

$$\frac{a}{\sin\,\theta_3} = \frac{b}{\sin\,\theta_2} = \frac{c}{\sin\,\theta_1}$$

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\,\cos\,\theta_3$$

$$b^2 = a^2 + c^2 - 2ac\,\cos\,\theta_2$$

$$c^2 = a^2 + b^2 - 2ab\,\cos\,\theta_1$$

Steps to solve problems by ICR method

1. Locate ICR(s)
2. Write the name of the link, its corresponding ICR, name of points on that link and the formula for velocity (**$v = r \times \omega$**) for all the points on the link.

Note:

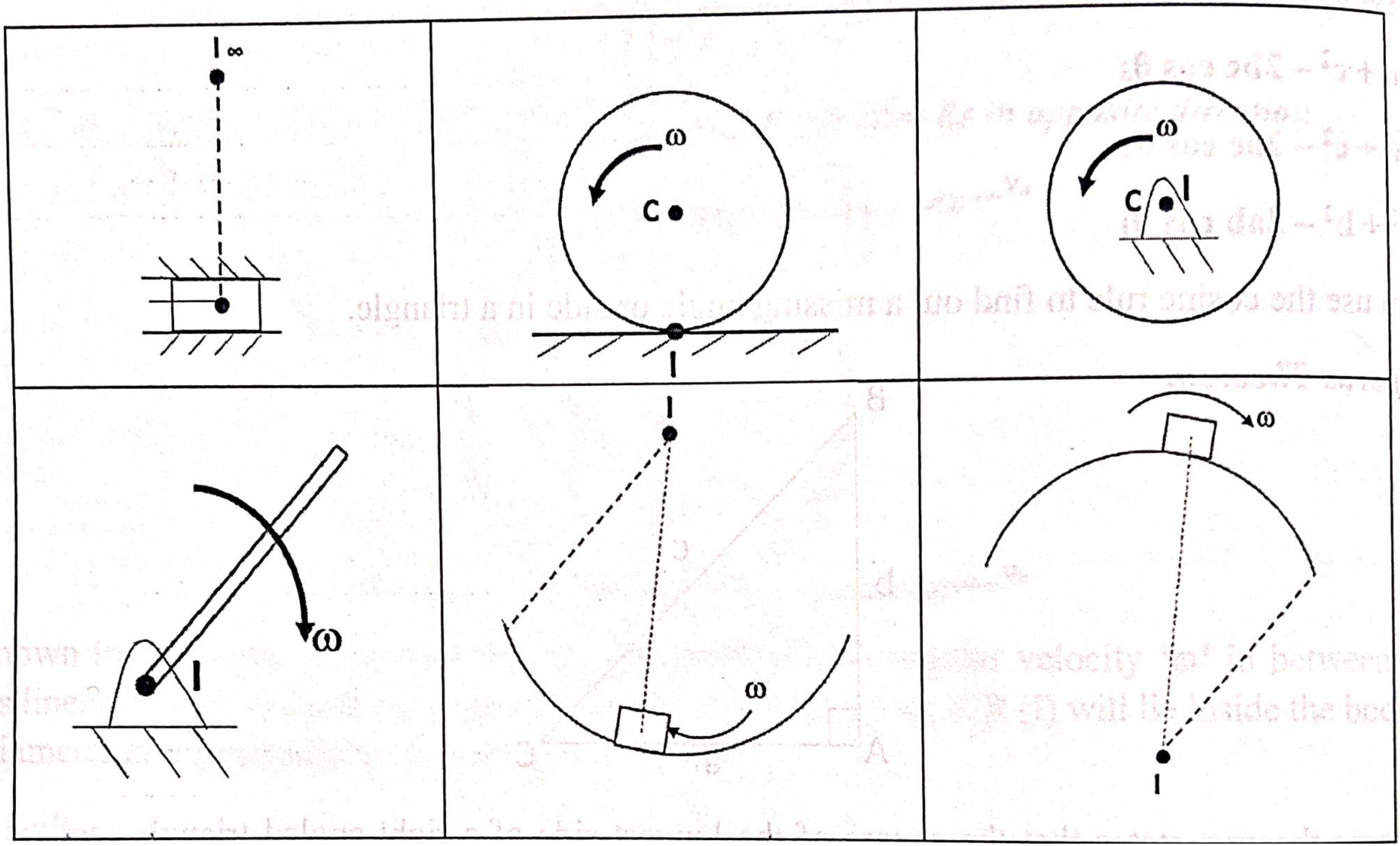
I) **r** ; is the distance from ICR of that link to the corresponding point, for e.g. if point A is under consideration whose ICR is I, then **r** will be distance IA from I to A

II) ω ; is the angular velocity of the link under consideration, for e.g. if point A is under consideration is on link OA, then **ω** will be $\boldsymbol{\omega_{OA}}$

3. Find the required distance and angle by sine rule, cosine rule, Pythagoras theorem or any other Geometrical method
4. Substitute all the values in the equation formed at step 2 and find the required answers.

Hint: Start with the equation whose data is given in question.

Some examples for location of ICR



Theory Questions:

Q1. Define general plane motion and ICR. What are properties of ICR? (Dec-22 – 4 Marks, Dec-23- 5 Marks, May-24 – 5 Marks).

Q2. Explain the following with example-

i) General Plane motion.

ii) Instantaneous centre of rotation (May – 22 – 5 Marks).

Unit 05:Kinetics of Particles

Kinetics is branch of dynamics which deals with study of bodies under motion considering the cause of motion of the body i.e. forces.

Kinetics of Particles: It is the study of motion of the particles considering the forces acting on the body.

Motion of the particles under kinetics can be studied by three different methods

a) D'Alembert Principle (Dynamic Equilibrium)

b) Work - Energy Principle

c) Impulse – Momentum

Unit 05:Kinetics of Particles

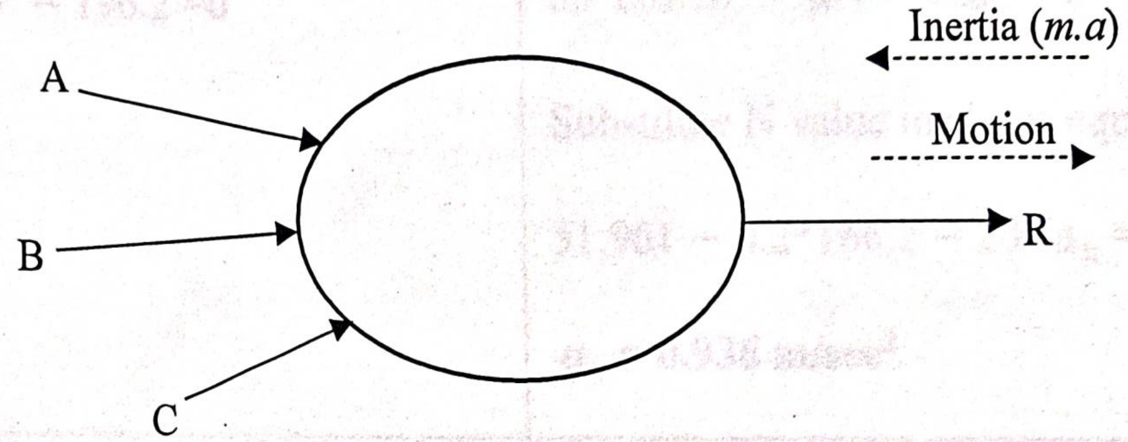
D'Alembert's Principle (Dynamic Equilibrium)

D'Alembert's Principle allows to treat a given dynamic problem into equivalent static problem.

Its concept is to introduce the inertia forces in addition to the resultant force that cause motion of the body, then such body can be treated as in dynamic equilibrium. This allows to use equation of static equilibrium to analyze the motion.

D'Alembert's Principle (Dynamic Equilibrium)

Let's consider a body of mass 'm' acted by forces A, B, C that puts body in motion. If the net resultant force is ∑F that accelerates the body by acceleration 'a' and I is the inertia force.



According to D'Alembert's Principle,

$\sum F + \mathrm{I} = 0$

Since the inertia force acting on the body is in the opposite direction of accelerated motion, $\mathrm{I} = -ma$

$\sum F - ma = 0$

By rearranging above equation,

$\sum F = ma$ (According to Newton's Law)

D'Alembert's Principle (Dynamic Equilibrium)

This can also be written as three independent equations,

$$\sum F_x - ma_x = 0$$

$$\sum F_y - ma_y = 0$$

$$\sum F_z - ma_z = 0$$

Unit 05:Kinetics of Particles

D'Alembert's Principle (Dynamic Equilibrium)

For motion with uniform acceleration the following equations are used.

$s = ut + \frac{1}{2} at^2$

$v = u + at$

$v^2 = u^2 + 2as$

Where, s is the displacement of particle

t is the time during which particle was in motion.

u is the initial velocity of the particle.

v is the final velocity of the particle.

a is the acceleration of the particle .

Unit 05:Kinetics of Particles

Steps to solve problem using D'Alembert's Principle

1) Write all the given data and parameters required to be determined.

2) Draw the Free Body Diagram.

3) Apply D'Alembert's principle (inertia force should be included only in direction of motion).

4) Determine the required parameters as asked in the question.

Note:
1. If more than one element is involved in a problem then find the kinematic relation between them i.e. relation in terms of displacement, velocity and acceleration and thereafter follow from step 2.

2. If particles are interconnected by means of thread, then introduce the tension in each thread (chord) followed by step 2 and step 3.

Impulse - Momentum

Impulse

- ✓ Impulse of a force is the cause of changes to motion and thereby the momentum of the body.
- ✓ The magnitude of this impulse force is the product of the force and the duration for which it acts.
- ✓ If the force F is constant, during the time (t) for which it acts than,

Impulse = F * t

If the force F is variable, then the impulse between time interval 't_1' and 't_2' is,

Impulse = F dt...... (A)

It is a vector quantity; unit of impulse is: **N.s**

Impulsive Force

- **A large amount of force acting for a very small amount of time causing a considerable change in the particle momentum is called as impulsive force.**
- For example, let us say a particle collides with another moving particle, the duration of collision is very small, but the particles after collision have different magnitudes of velocities i.e. change in momentum of the body.
- Some other examples of impulsive forces are, kick starting a bike, striker hitting a coin on a carrom board, batsman hitting a ball while playing cricket, spring loaded gun releasing the bullet, a martial artist hitting the object.

Impulsive Momentum Equation

According to Newtons second law,

$$\sum F = m\,a$$

$$\text{or } \sum F = m\frac{dv}{dt} \quad \ldots\ldots \text{ since } a = \frac{dv}{dt}$$

$$\text{or } \sum F = \frac{d(mv)}{dt} \quad \ldots\ldots \text{ as m is constant}$$

the term mv (i.e. the product of mass and velocity) is known as the linear momentum of the particle and is defined as the amount of motion possessed by a moving body.

It is a vector quantity and S.I. units are kg m/s or N.s.

$\sum F \, dt = d(mv)$

Integrating the above equation within the limits t_1 to t_2 during which velocity of particle changes from v1 to v2.

$$\int_{t_1}^{t_2} \sum F \, dt = \int_{v_1}^{v_2} d\,(mv) = mv_2 - mv_1$$

$$mv_1 + \int_{t_1}^{t_2} \sum F \, dt = mv_2$$

here, $\int_{t_1}^{t_2} \sum F \, dt$ is the impulse on the particle, therefore

$$mv_1 + \boldsymbol{Impulse_{1-2}} = mv_2 \quad \ldots\ldots \text{(B)}$$

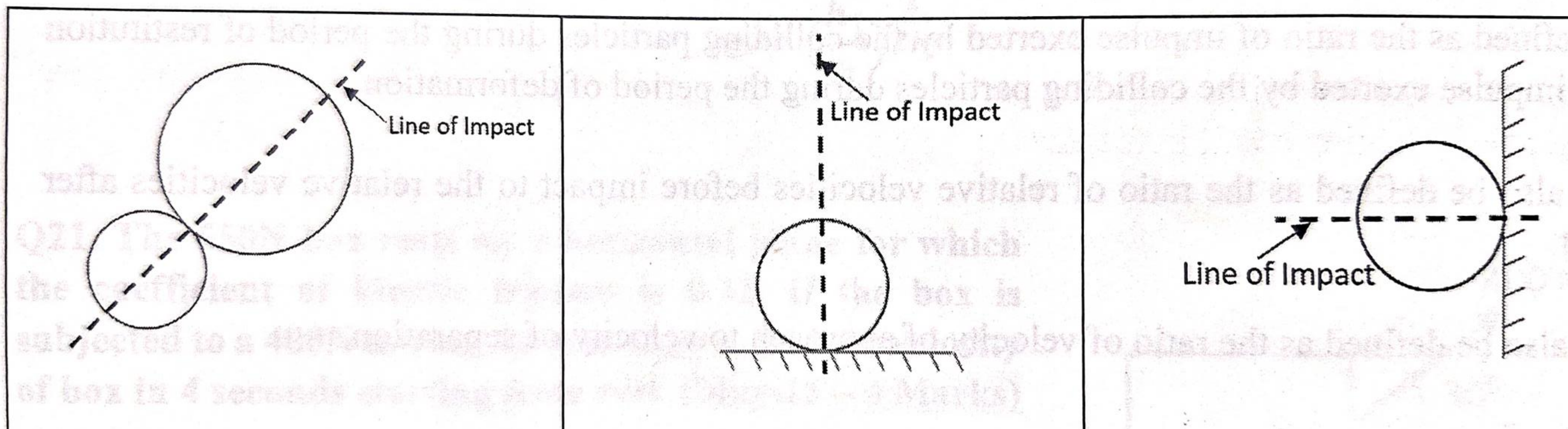
The above equation (B) is known as Impulse-Momentum equation, which states that "For a particle or a system of particle acted upon by the forces during a given time interval, the total impulse acting on the system is equal to the difference between the final momentum and the initial momentum.

Impact

A collision of two bodies which take place during a small amount of time, during which the the bodies which are colliding exert relatively large amount of force on each other is known as impact.

Line of Impact

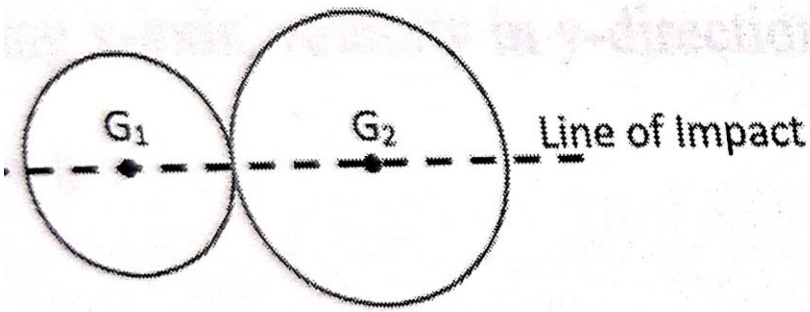
When two bodies collide with each other, the line joining the common normal of these bodies is known as line of impact.



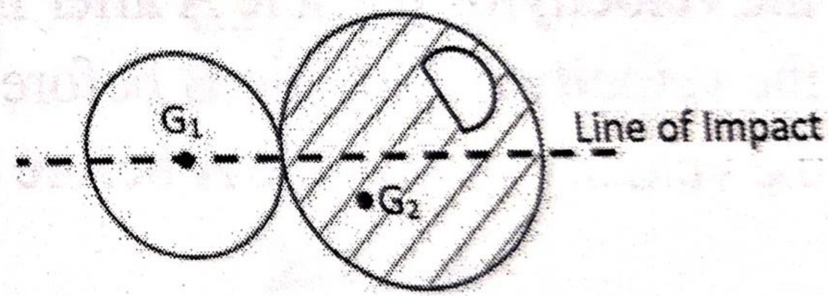
(a) Two ball colliding | (b) Ball colliding with ground | (c) Ball colliding with wall

Types of Impact

Basically, impacts are classified into two categories **namely Central Impact and Eccentric Impact**. If the mass centres of colliding particles lie on the same line of impact, then collision is referred to as Central impact, whereas when mass centres of colliding particles are not on the line of impact than it is referred to as Eccentric Impact.



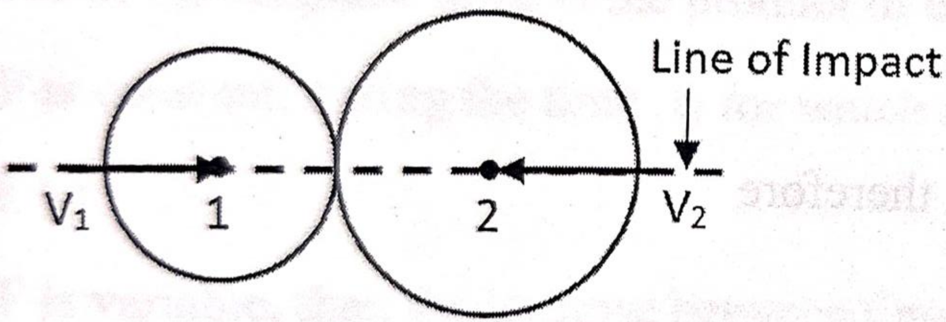
Central Impact



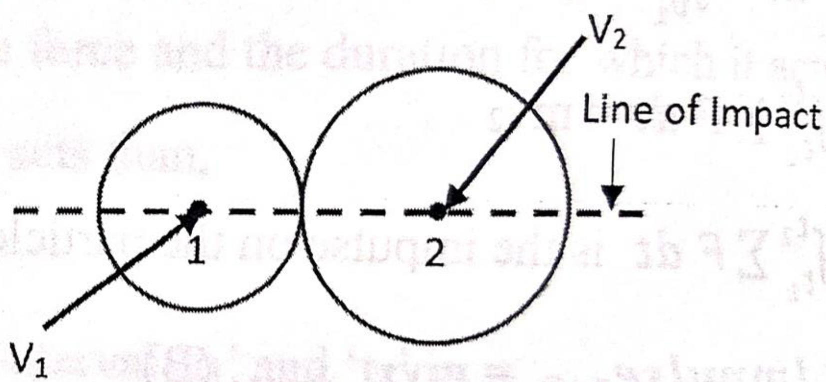
Eccentric Impact

Central impact is further classified into **Direct Impact** and **Oblique Impact.**

When the colliding particles travels along the line of impact i.e. their velocities are along the line of impact then they are referred as **Direct Impact**, whereas if the velocities are not along the line of impact, it is referred to as **Oblique Impact**.



Direct Central Impact



Oblique Central Impact

Note:

1. **In oblique impact**, the **velocities before and after impact remains same**, along the **direction perpendicular to** direction of **line of impact**.
2. If the line of impact is along x-axis, then velocities after impact in y - direction for both particles will remain same, similarly if the line of impact is along y-axis, then velocities after impact in x - direction for both particles will remain same

Coefficient of Restitution

- ✓ It is defined **as the ratio of impulse exerted by the colliding particles during the period of restitution to the impulse exerted by the colliding particles during the period of deformation.**
- ✓ It can **also** be defined as **the ratio of relative velocities before impact to the relative velocities after impact.**
- ✓ It can also be defined as the ratio of velocity of approach to velocity of separation.
- ✓ It is denoted by letter 'e' As it is ratio of two similar quantities it is unit less.

Coefficient of Restitution

Mathematically it is given by,

$$e = \frac{v_b - v_a}{u_a - u_b}$$

Where, v_b is the velocity of particle B after impact.
v_a is the velocity of particle A after impact.
u_b is the velocity of particle B before impact.
u_a is the velocity of particle A before impact.

The value of coefficient of restitution lies between '0' and '1'.
It depends on the nature of the particles of collision.
For e.g. a rubber ball and a tennis ball will have different coefficient of restitution when they fall freely on the surface from same height.

Special cases for Coefficient of Restitution.

1. If **e = 0**, This type of impact is known as **perfectly plastic impact**. In this case the particles move with the **same velocity after impact**. The **momentum is conserved** but there **is a loss of kinetic energy**.
2. If **e = 1**, This type of impact is known as **perfectly elastic impact**. In this case the **momentum is conserved** but there **is no loss of kinetic energy i.e**. energy is also conserved.
3. If, **0 < e < 1**, This type of impact is known as **neither perfectly elastic nor perfectly plastic impact**. In this case the **momentum is conserved** but there **is loss of kinetic energy.**

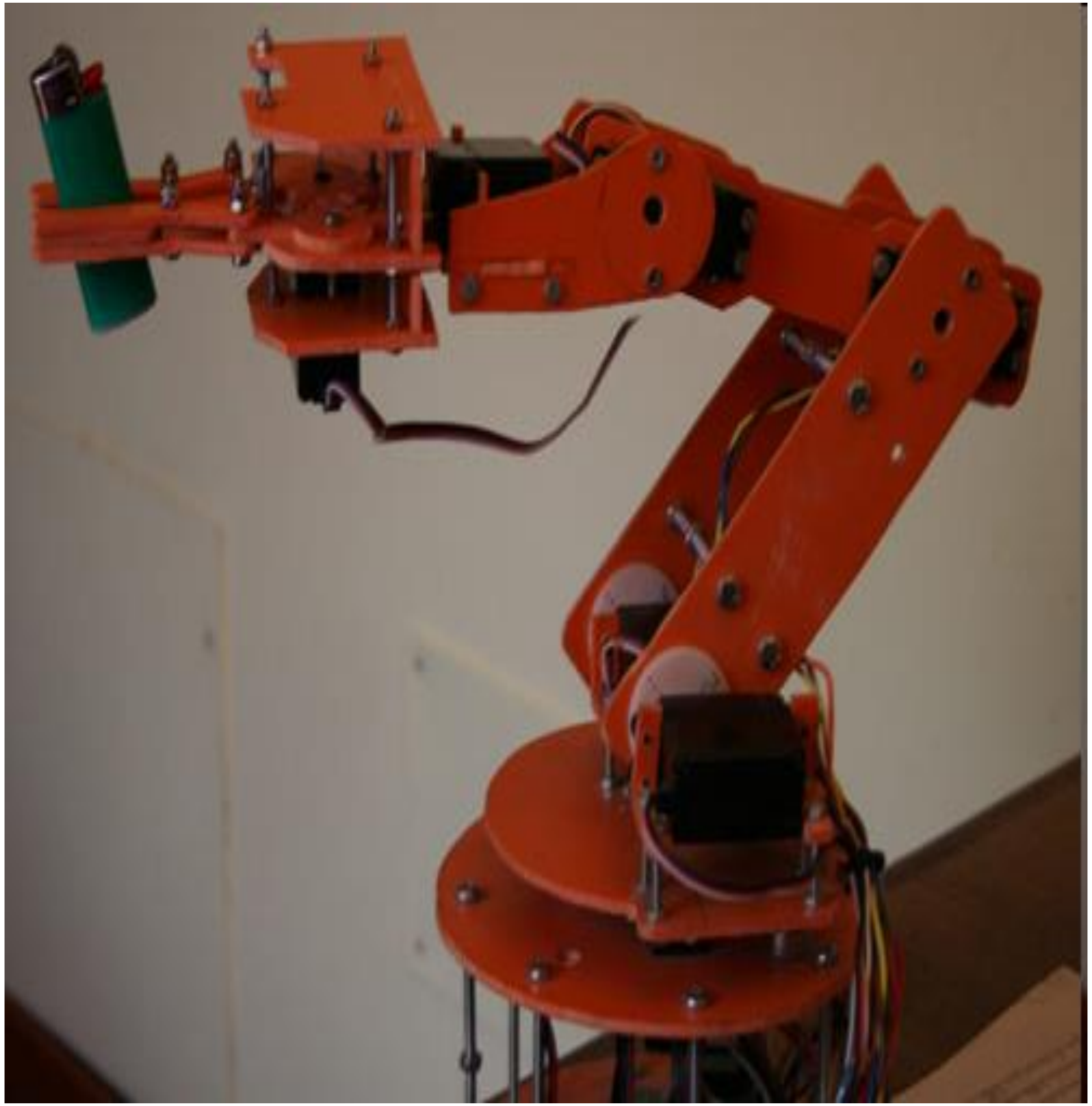
Relation between 'e' and height of bounce

Let the particle be dropped from a height 'h' on the ground and let h_1 be the height of rebound after 'n' bounces. If 'e' is the coefficient of restitution between the ground and the particle than 'e' is related to h_1 as

$$\mathbf{e} = \left(\frac{h_1}{h}\right)^{\frac{1}{2n}}$$

Introduction to Robot Kinematics

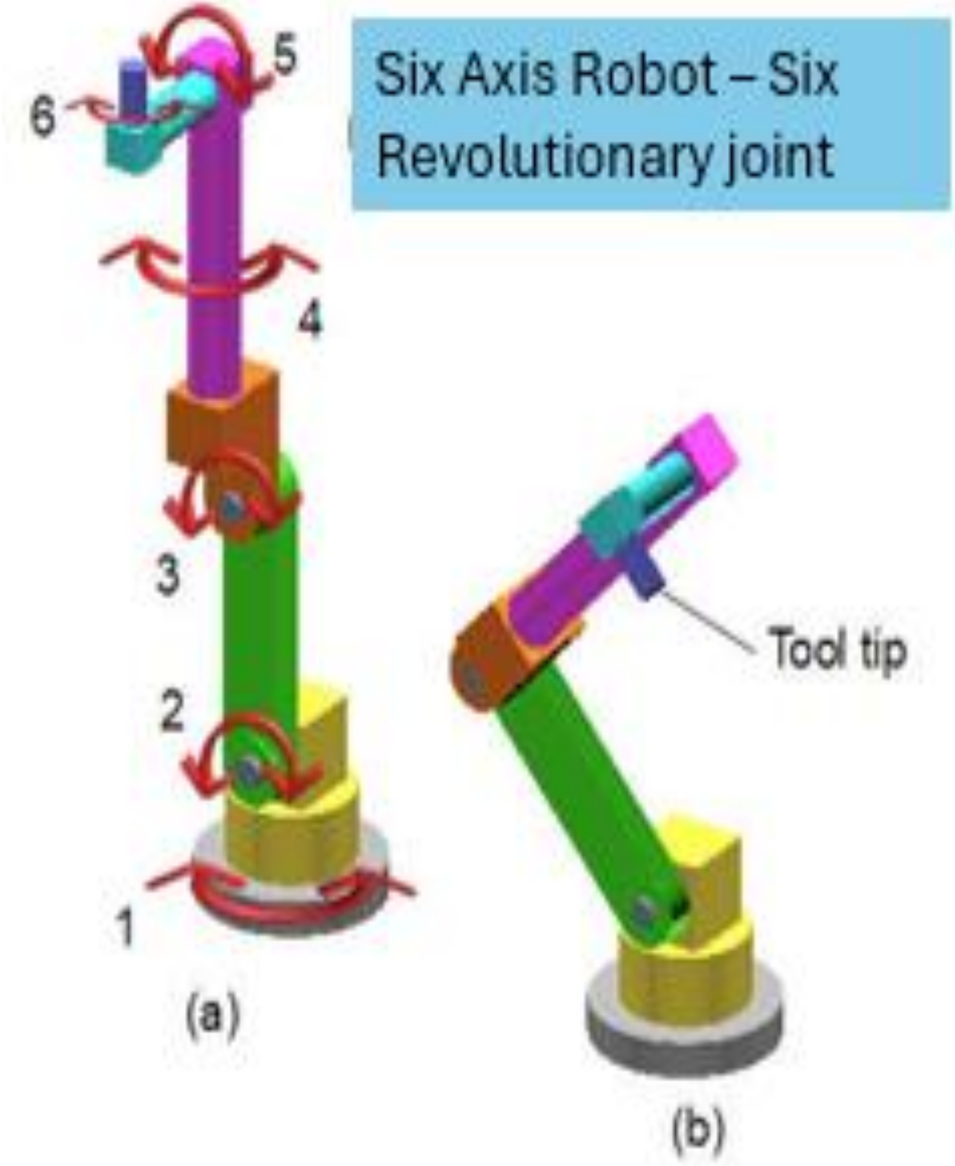
TSEC



Contents

Introduction

- A robot is a machine that mimics human actions, is devoid of any emotions, and works with the highest precision and efficiency n number of times without getting tired.
- Robots were designed to work in factories as industrial robots to perform repetitive work to increase production and product quality.
- Today robots are designed to carry out nearly all tasks to get exact quality work with high efficiency, saving time and replacing human work.
- Robots are employed in the electronic industry,



Robot Mechanics

It deals with robot systems' design, analysis, and control.

It has three main sub-divisions:

Robot Kinematics –
Robot design begins with robot kinematics. It works with geometric analysis of motion, finding out the robot's orientation, position, velocity, and acceleration of different moving parts. Forces and torques involved in the process are not

Robot Dynamics –
It deals with forces, torques, and other inputs like voltage, moment of inertia of different moving parts, and the physical factors needed to actuate the desired motion.

Robot Control Systems –
is used to control the behavior of robots. Its two main components are controller and sensor. The control system tracks the motion of different parts of a robot by signals received by the sensors and directs the controller to send

- Main Parts of a Robot

1. Manipulator –

It makes up the main structure of the robot.

It consists of links and joints and gives the shape and form of a robot.

Joints in a robot are mainly of two kinds viz. Revolute and Prismatic.

A revolute joint allows only rotation and does not allow any linear motion at the joint.

Whereas a Prismatic joint allows linear motion at the joint and does not facilitate any rotation.

2. End effector –

It is the last extreme part of a robot.

It is designed to interact with the environment.

It is finally the end effector which performs the task for which the robot is designed.

The type of end effector depends on the task to be performed.

For example, for a robot designed to pick an object from one

- Main Parts of a Robot

3. Actuators - are devices that provide motion to the joints and links.

Actuators convert either electrical, pneumatic, or hydraulic energy into mechanical energy.

Actuators may be servo motors or pneumatic or hydraulic actuators.

4. Sensors -

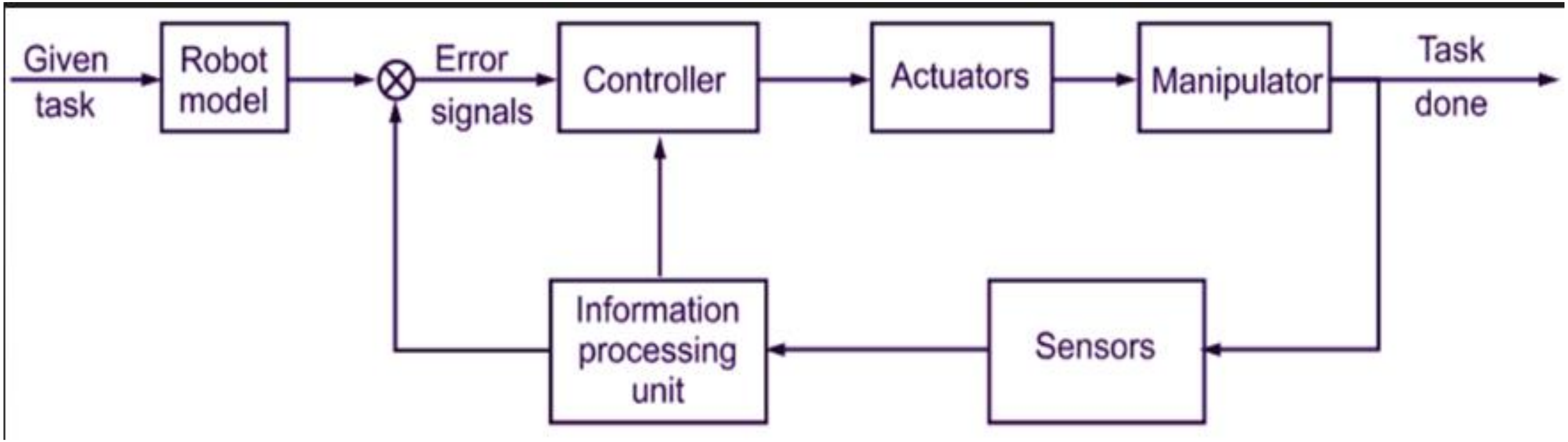
Sensors monitor the robot's internal and external environments.

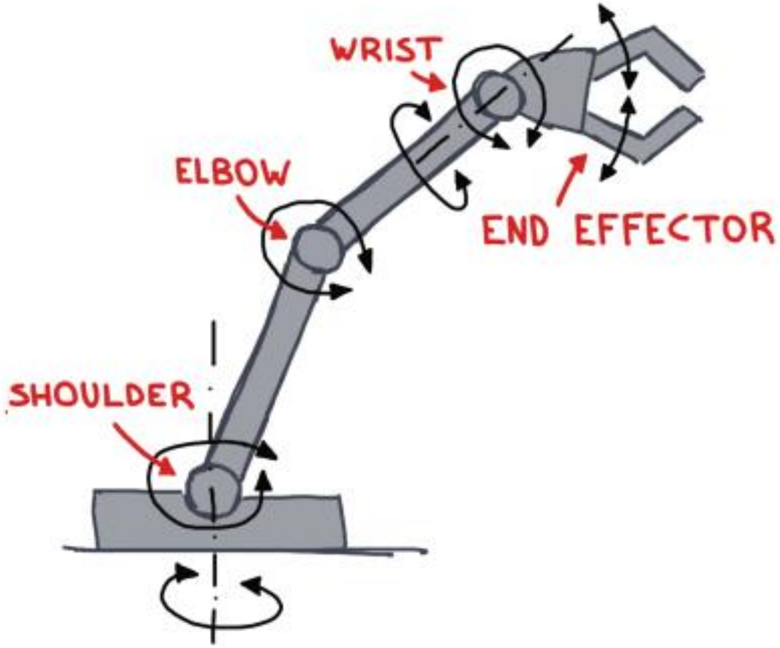
Sensors track the position, orientation, speed, acceleration, and other changes and send the same input signal to the controller to adjust the robot's movement.

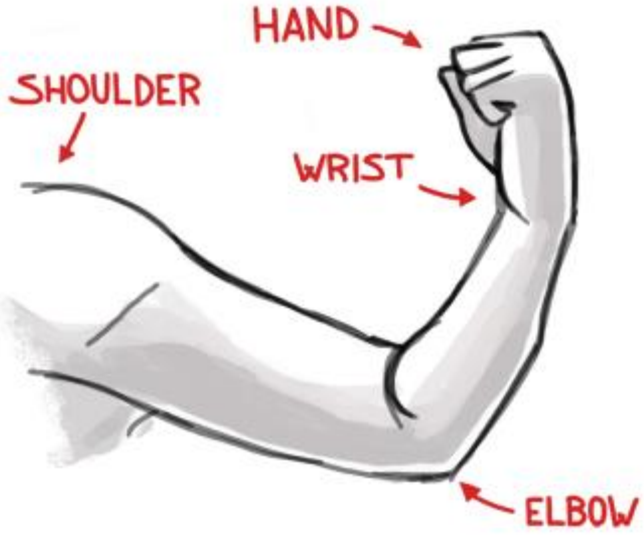
5. Controller -

It is the brain of a robot.

It receives command inputs from the computer and signals from the sensors. On these bases, it directs and

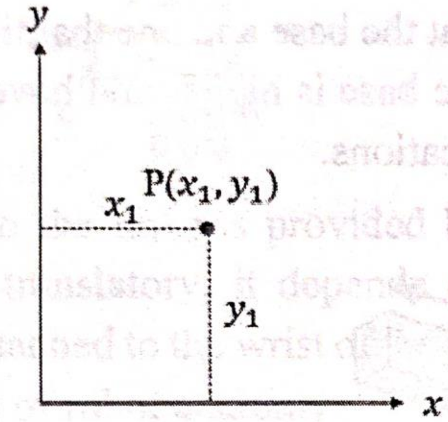

Given task
Robot model
Error signals
Controller
Actuators
Manipulator
Task done
Information processing unit
Sensors


WRIST
ELBOW
END EFFECTOR
SHOULDER

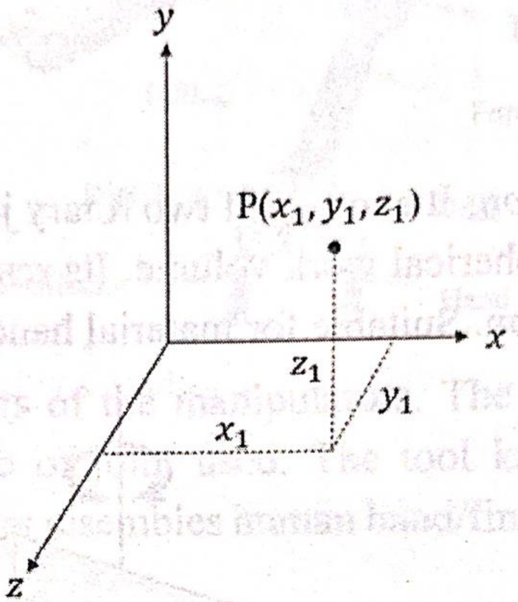

HAND
SHOULDER
WRIST
ELBOW

Degree of Freedom

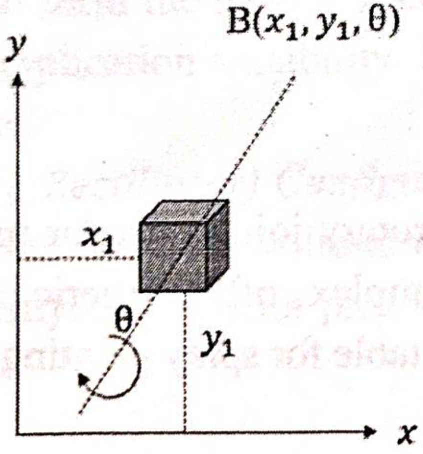
- **In general degree of freedom is defined as the minimum number of independent parameters / variables / coordinates needed to describe a system completely.**
- **For mechanism like robot/manipulator, the degree of freedom is the number of independent inputs needed to completely specify the configuration of the robot.**
- **Every geometric axis that a joint can rotate around or allow linear motion constitutes a degree of freedom.**



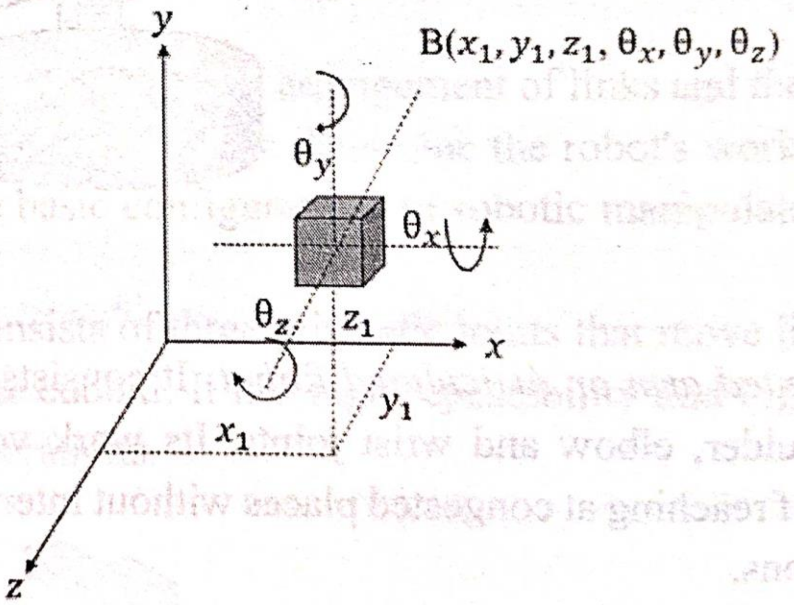
Point P in two dimensional space



Point P in three dimensional space



Body B in two dimensional space



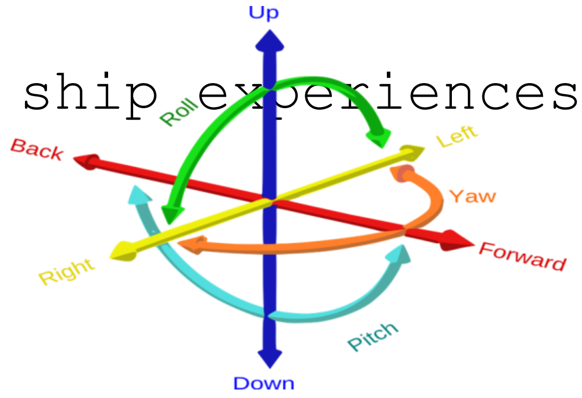
Body B in three dimensional space

Examples (concept of DOF)

- An elbow joint in humans has one DOF because it allows rotation motion around a single axis.
- Insertion robots used in electronic industry work are examples of two DOF robots. These robots pick up a component from one location on a flat PCB and place it at some other location on the PCB.
- The joint of the mini-drafter which carries the right-angled scale is a joint having 3 DOF, since translation motion along X-axis and Y-axis and rotation about Z axis are possible.
- The shoulder and even the wrist of the human arm both have 3 DOF each. The shoulder or wrist allows rotation around all three geometric axes, but they cannot provide any linear motion.
- A rotating chair allowing height adjustment and fitted with wheels has 4 DOF. The chair can translate linearly along all three axes and can also rotate about the vertical axis.
- A ship in the sea has 6 DOF. In a rough sea, a ship experiences all 6 possible motions.
- These are

a. Forward or backward,

b. Up or down, c. Left or right

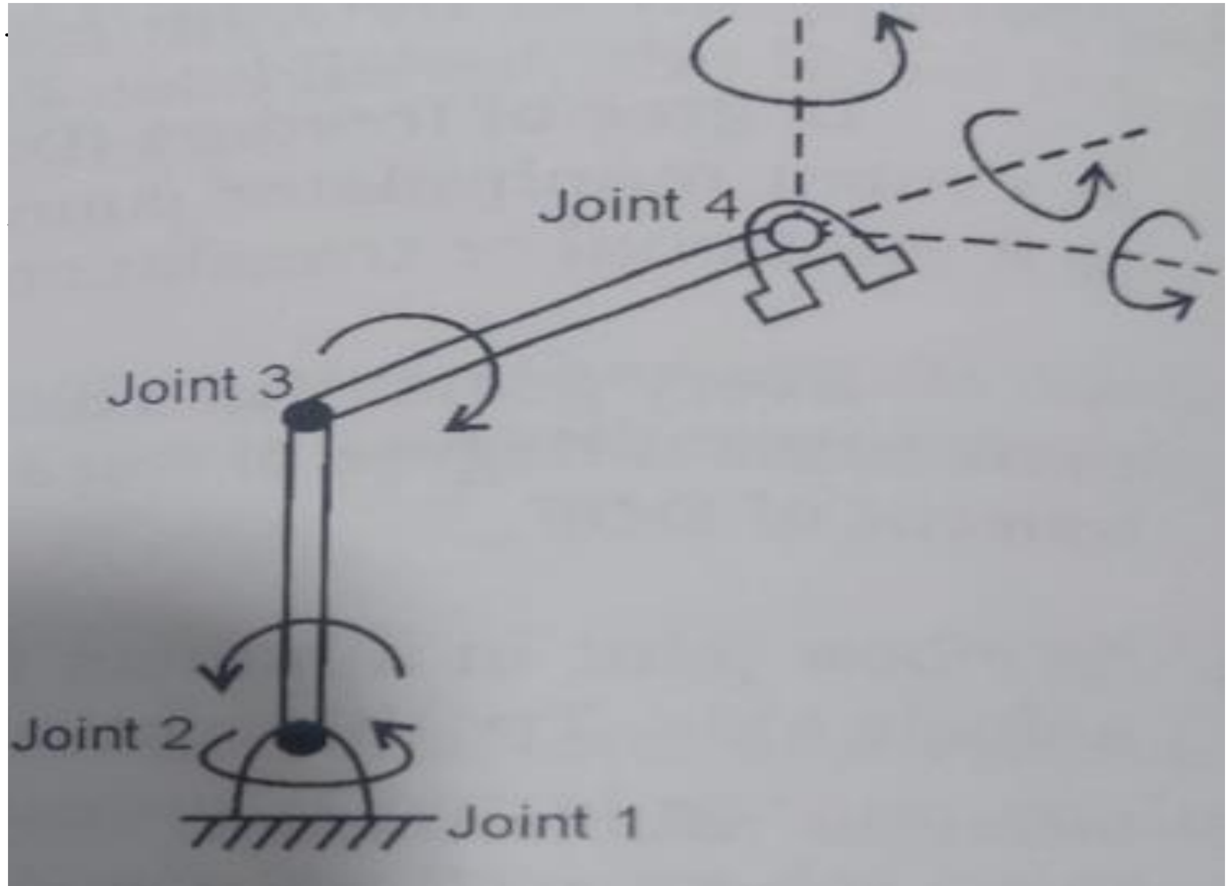


6 DOF (Six Degrees of Freedom):

- This is considered full articulation for most industrial robots.
- They can achieve all three linear movements (forward/back, lef
  rotational movements
  rotating the arm itse

Robot Manipulator

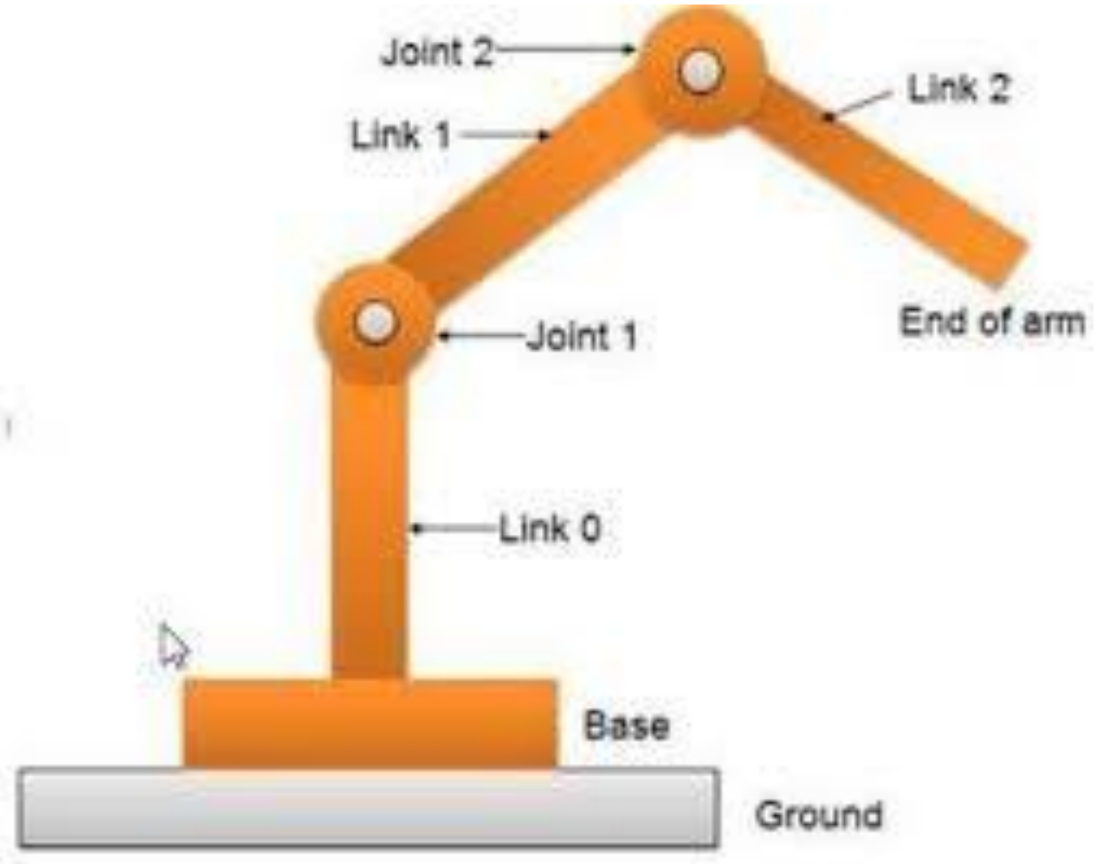
A robot manipulator is a simple robot that resembles a human arm. It is designed to create all motions which a human arm can perform. robot manipulator shown in figure has 6 DOF and



6 DOF Robot Manipulator

- **Joints and Links:**
  - The manipulator of an industrial robot consists of **a series of joints and links**.
  - Robot anatomy deals with the study of different joints and links and other aspects of the manipulator's physical construction.
  - A robotic joint provides relative motion between two links of the robot.
  - Each joint, or axis, provides a certain degree of freedom (dof) of motion.
  - In most of the cases, only one degree of freedom is associated with each joint.
  - Therefore, the robot's complexity can be

- **Each joint is connected to two links, an input link and an output link.**
- Joint provides controlled relative movement between the input link and output link.
- A robotic link is the rigid component of the robot manipulator.
- Most of the robots are mounted upon a stationary base, such as the floor. From this base, a joint-link numbering scheme may be recognized as shown in Figure.
- The robotic base and its connection to the first joint are termed as link-0. The first joint in the sequence is joint-1. Link-0 is the input link for joint-1, while the output link from joint-1 is link-1 which leads to joint-2.
- Thus link 1 is, simultaneously, the output link for joint-1 and the input link for joint-2. This joint-link-numbering scheme is further followed for all joints and links in the robotic systems.



Robotic Joints

- Robotic motion is possible because of its powered joints. The individual joint motions possible are called degree of freedom.
- Robotic motion involves either linear or rotational motion. These motions are achieved using linear or rotating joints.

Linear joint/Prismatic joint [L]: (Sliding motion/Translation motion)

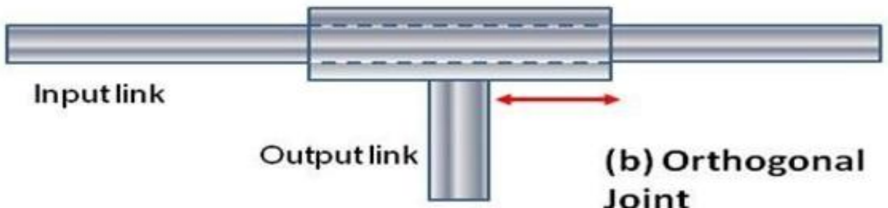
The relative movement between the input link and the output link is a translational sliding motion with axes of both the link colinear.

This motion is achieved by a piston, a telescoping mechanism and a relative motion along a linear track or rail.

The degree of freedom is one.

Orthogonal joint (type U joint)

This is also a translational sliding motion, but the input and output links are perpendicular to each other during the movement.

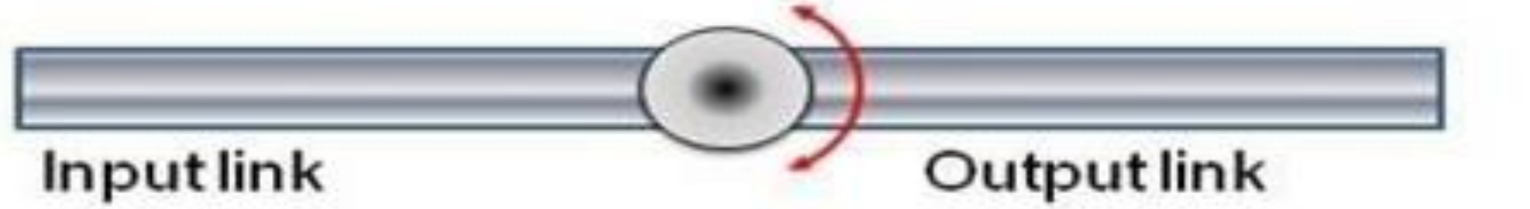


Robotic Joints

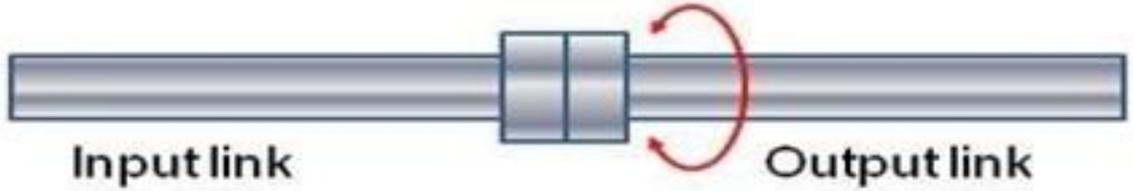
Rotating joint: There are three types of rotating joint

Rotational Joint [R]: Axis of rotation is perpendicular to the axes of two connecting links.

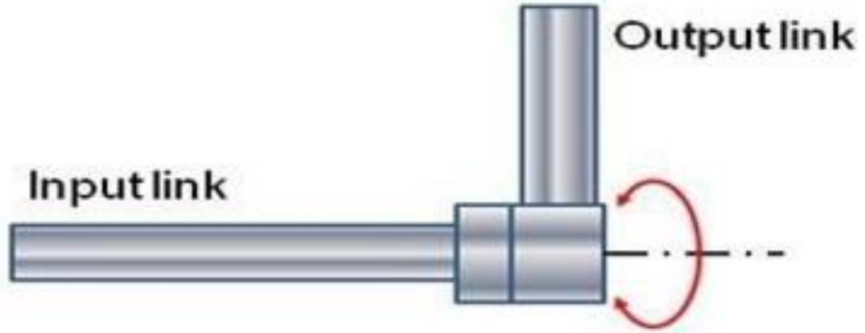
The degree of freedom is one.



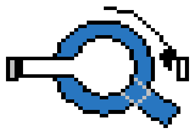
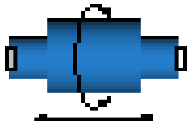
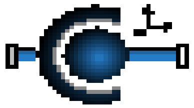
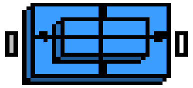
Twisting joint [T]: Axis of rotation of twisting joint is parallel to the axis of both links. The degree of freedom is one.



Revolving joint [V]: Input link is parallel to the axis of rotation and the output link is perpendicular to the axis of rotation. The degree of freedom is one.



Types of joints

Name of joint	Representation	Description
Revolute		Allows relative rotation about one axis.
Cylindrical		Allows relative rotation and translation about one axis.
Prismatic		Allows relative translation about one axis.
Spherical		Allows three degrees of rotational freedom about the center of the joint. Also known as a ball-and-socket joint.
Planar		Allows relative translation on a plane and relative rotation about an axis perpendicular to the plane.

- Degree of Freedom
- The degrees of freedom of a mechanism are defined as the number of independent variables required to identify its configuration in space completely.
- The number of degrees of freedom for a manipulator can be calculated as

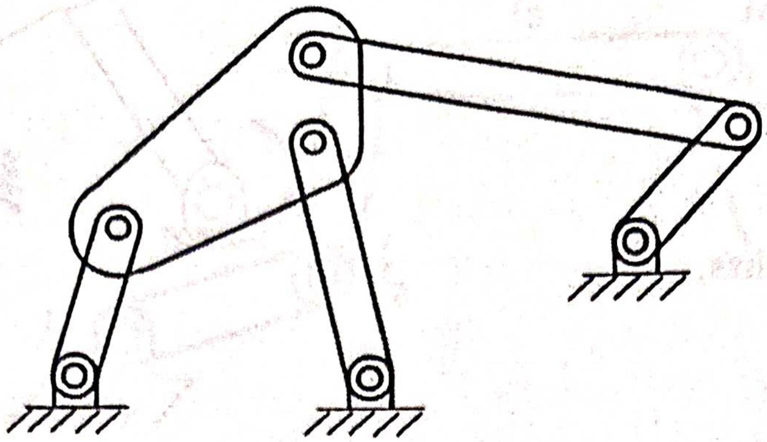
- DOF = λ (n-j-1) + $\sum_{i=1}^{j} fi$

where,

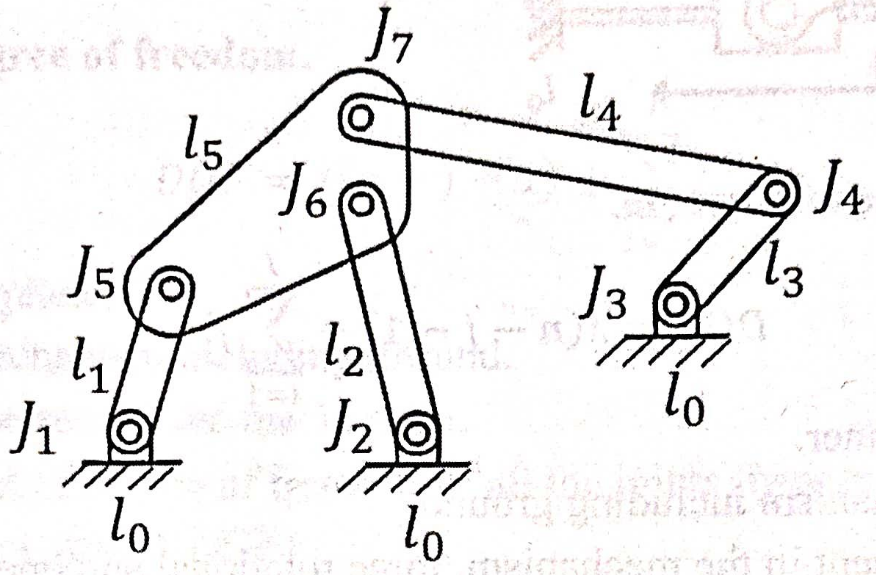
- λ = Degree of freedom of the space
- n = number of links (this includes the ground/ base link)
- J = number of joints in the mechanism
- fi = degrees of freedom of each joint

Numerical on Degree of Freedom

Q1. Determine the degree of freedom of the following mechanism.



Step1: Label the links and joints.



Step2: Determine the degree of freedom.

$$DOF = \lambda(n - j - 1) + \sum_{i=1}^{j} f_i$$

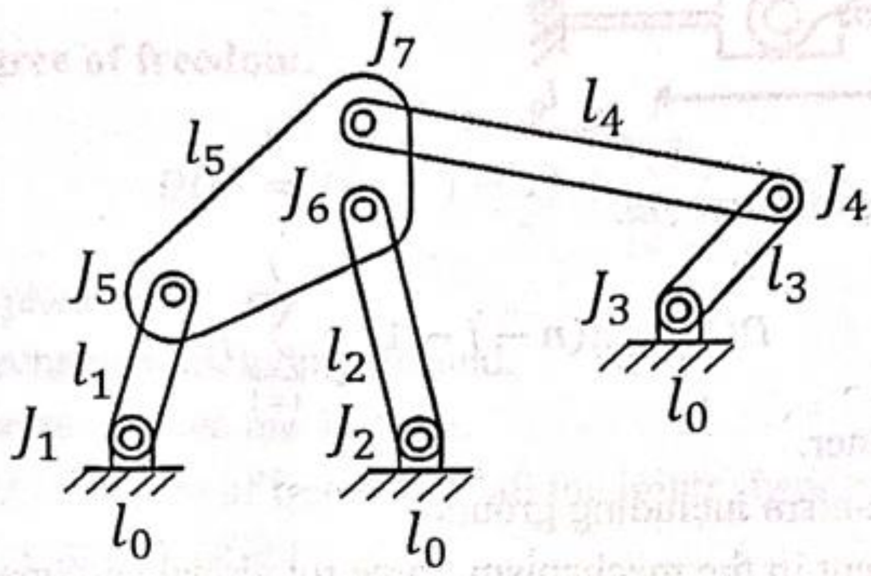
$\lambda = 3$, as the mechanism is planer.

$n = 6$, total six links in mechanism including ground (ground link is counted only once irrespective of the number of times it appears in mechanism)

$j = 7$, total seven joints are present in the mechanism

$\sum_{i=1}^{7} f_7 = 7$, it is the addition of degree of freedom of all the joints (here total 7 joints are present and each joint is rotational joint with 1 DOF).

$$DOF = 3 * (6 - 7 - 1) + \sum_{i=1}^{7} f_7$$



$= 3 * (6 - 7 - 1) + (1 + 1 + 1 + 1 + 1 + 1 + 1)$

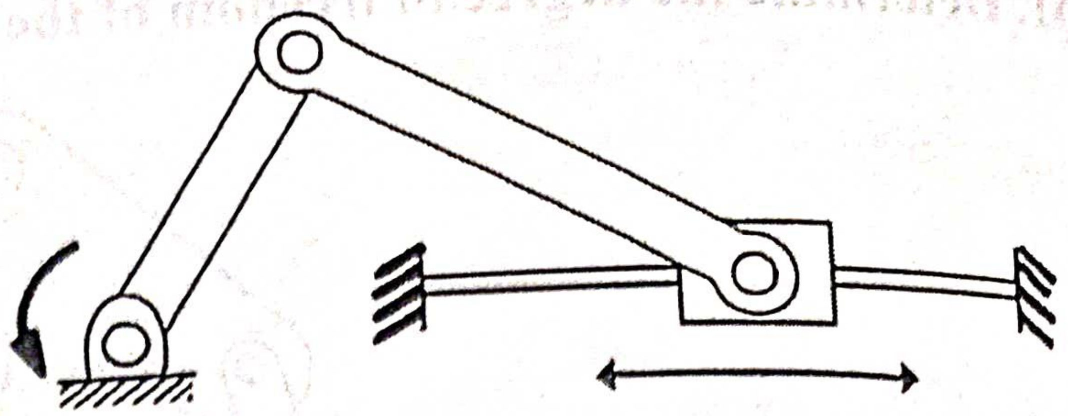
$= 3*(-2) + 7$

$= -6 + 7$

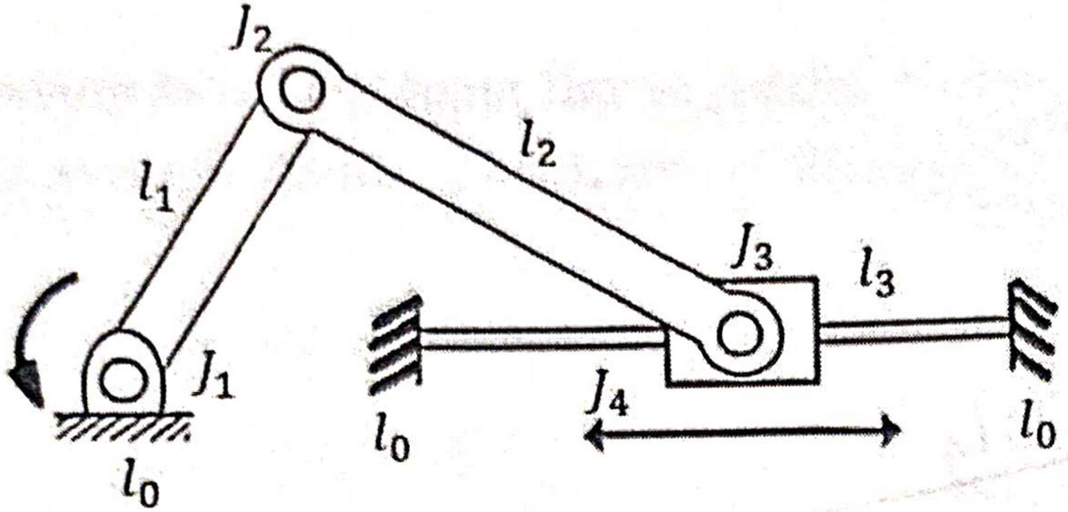
$= 1$

The degree of freedom of given mechanism is 1.

Q2. **Determine the degree of freedom of the following mechanism.**



Step1: Label the links and joints.



Step2: Determine the degree of freedom.

$$DOF = \lambda(n - j - 1) + \sum_{i=1}^{j} f_i$$

$\lambda = 3$ as the mechanism is planer.

$n = 4$ total four links in mechanism including ground.

$j = 4$ total four joints are present in the mechanism, three rotational and one linear.

$\sum_{i=1}^{4} f_4 = 4$ it is the addition of degree of freedom of all the joints (here total 4 joints are present, three are rotational joint and one linear joint each with 1 DOF)

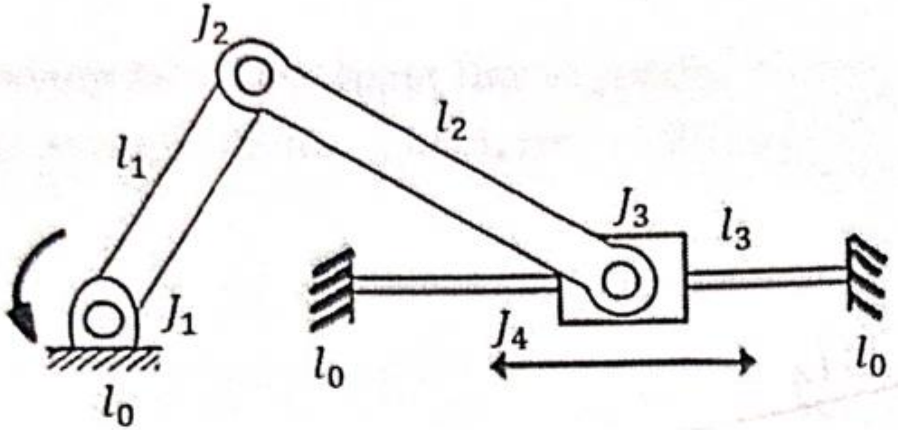
$$DOF = 3 * (4 - 4 - 1) + \sum_{i=1}^{4} f_4$$

$= 3 * (4 - 4 - 1) + (1 + 1 + 1 + 1)$

$= 3*(-1) + 4$

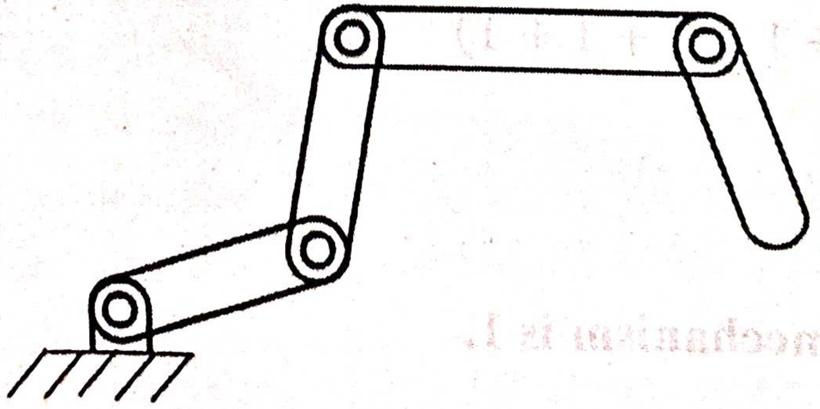
$= -3 + 4$

$= 1$

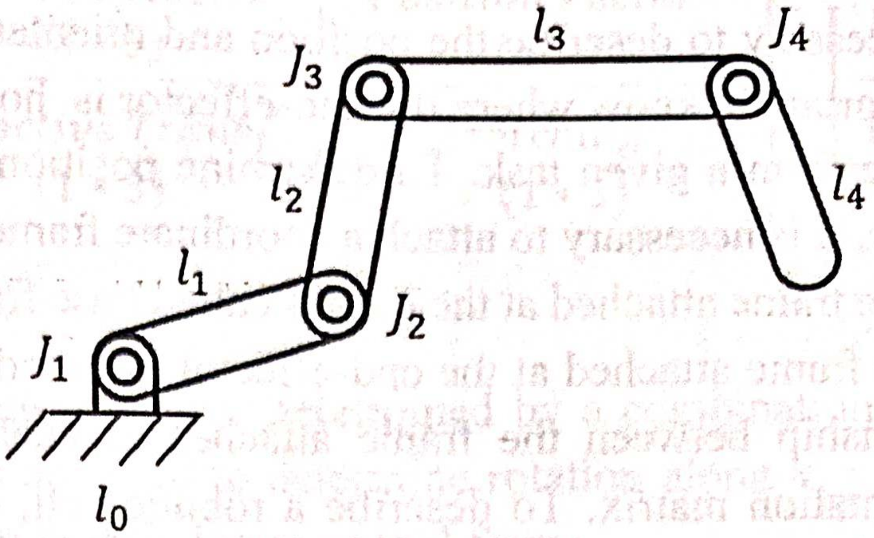


The degree of freedom of given mechanism is 1.

Q3. Determine the degree of freedom of the following serial manipulator.



Step1: Label the links and joints.



Step2: Determine the degree of freedom.

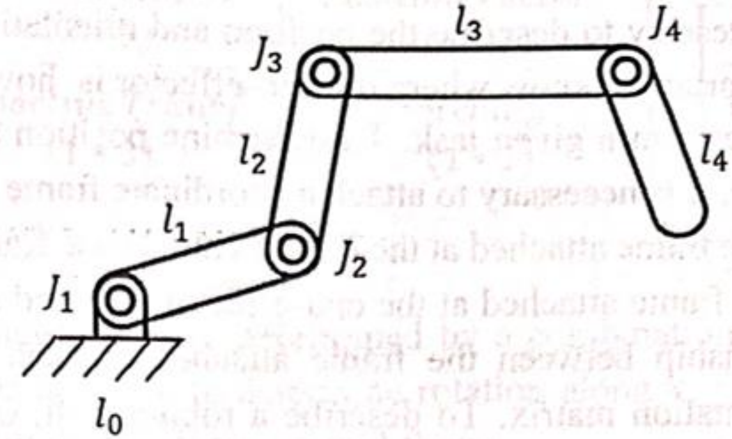
$$DOF = \lambda(n - j - 1) + \sum_{i=1}^{j} f_i$$

$\lambda = 3$ as the mechanism is planer.
$n = 5$ total four links in mechanism including ground.
$j = 4$ total four joints are present in the mechanism.
$\sum_{i=1}^{4} f_4 = 4$ it is the addition of degree of freedom of all the joints (here total 4 joints are present, and each joint is rotational joint with 1 DOF)

$$DOF = 3 * (5 - 4 - 1) + \sum_{i=1}^{4} f_4$$

$= 3 * (5 - 4 - 1) + (1 + 1 + 1 + 1)$
$= 0 + 4$
$= 4$

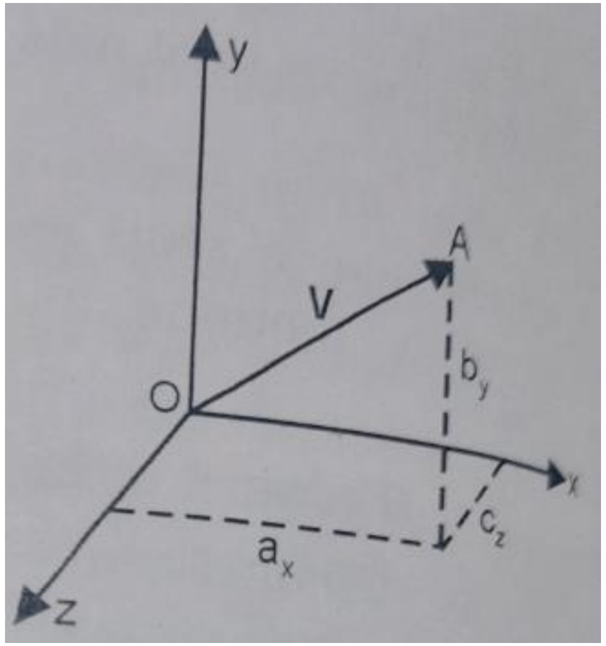
The degree of freedom of given serial manipulator is 4.



Robot Kinematics

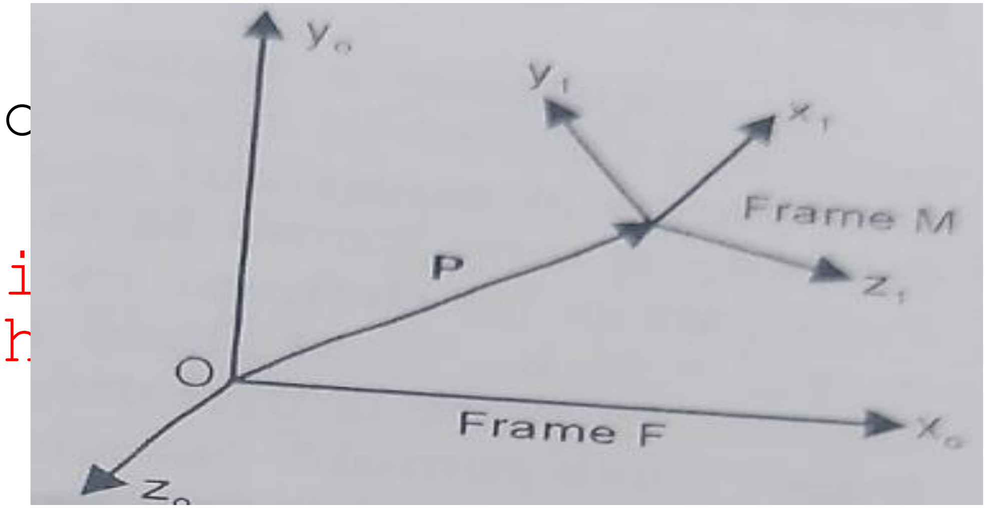
- Representation of vector in space
- Matrix representation of a vector
- Matrix representation of a vector with a scaling factor
- Fixed frame and moving frame

- Representation of vector in space
- Consider a vector V in space, originating from O to a point A in frame Fx,y,z.
- If a_x, b_y, c_z are the components of the vector along the x, y, and z axes resp. and i, j and k represent the unit vectors along the x, y, and z axes resp., then vector v is
- V = axi + byj + czk



- Matrix representation of a vector
- $V = \begin{bmatrix} ax \\ by \\ cz \end{bmatrix}$
- Matrix representation of a vector with factor
- $V = \begin{bmatrix} ax \\ by \\ cz \\ 1 \end{bmatrix}$

- Fixed frame and moving frame
- Consider a frame $F_{x0\ y0\ z0}$ which is fixed at O. Consider another frame $M_{\ x1\ y1\ z1}$ located at a point P.
- Point P is located from Point O by a position vector P = Pxi + Pyj + Pzk.
- Let vectors xi, yi and zi of the moving frame be of unit length.
- Let X_{1x}, X_{1y}, X_{1z} represents the components of x_1 on x_0, y_0, z_0 axis.
- Let Y_{1x}, Y_{1y}, Y_{1z} represents the components of y_1 on x_0, y_0, z_0 axis.
- Let Z_{1x}, Z_{1y}, Z_{1z} represents the co x_0, y_0, z_0 axis.
- Moving frame can be represented i orientation by a 4×4 matrix as sh



$$\begin{bmatrix} X_{1x} & Y_{1x} & Z_{1x} & P_x \\ X & Y & Z & P \end{bmatrix}$$

- Homogeneous Matrices

1. They are square form matrices usually a 4×4 matrix.
2. A scale factor of 1 may be introduced to make the matrix square.
3. A 3×3 homogeneous matrix represents only orientation.
4. A 4×4 homogeneous matrix represents both position and orientation.

- Homogeneous Transformation

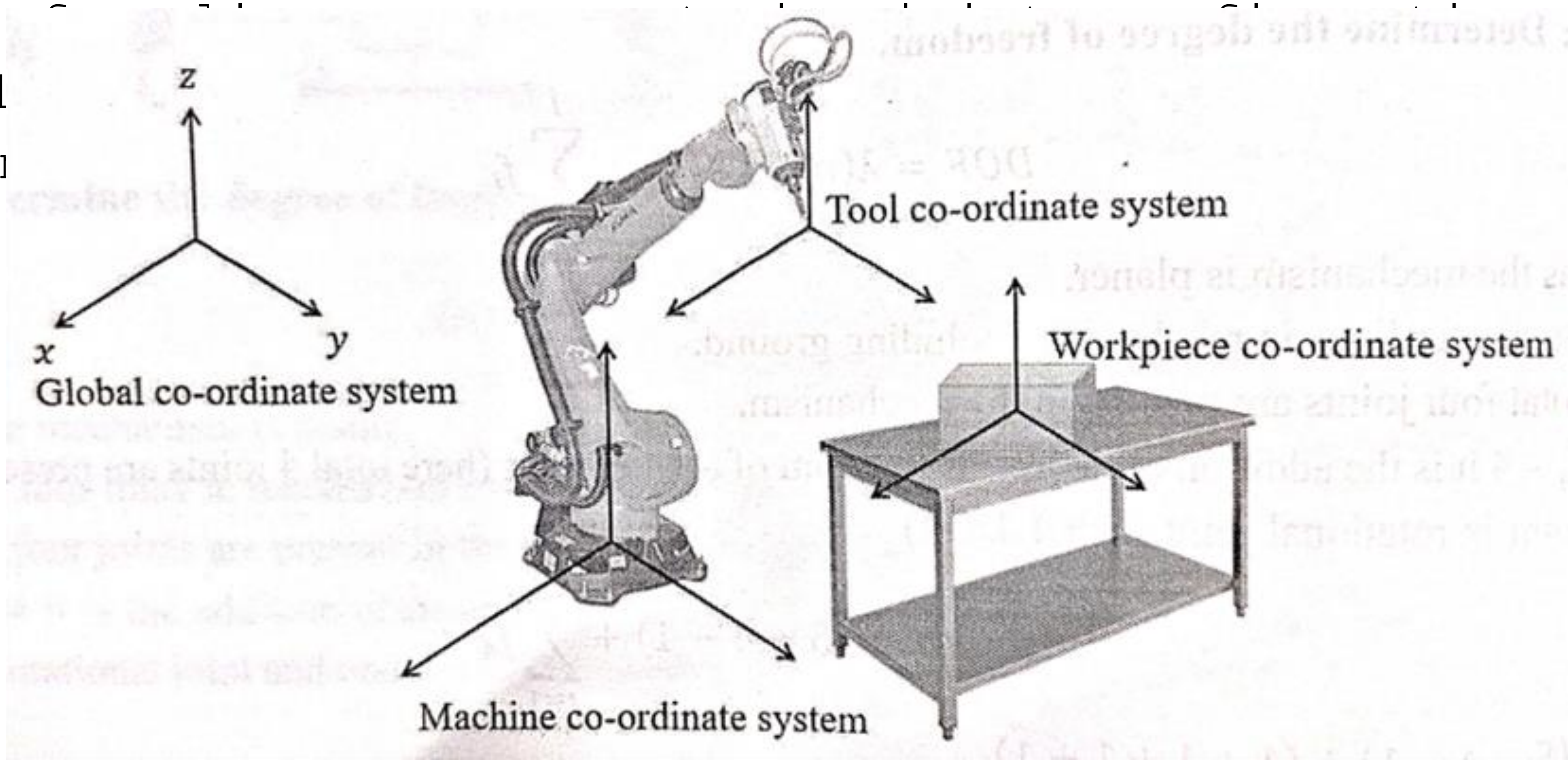
- In robotics, it is always necessary to describe the position orientation of the end-effector with respect to the base,

(i) to know where the end effector is, how much it needs to move, and how it needs to orient to perform a given task.

(ii) to attach a coordinate frame along the joint axis of each link in the manipulator.

- The frame attached at the base is called base frame,

- The relationship between the frame attached at each joint axis of the links is determined by the transformation matrix.
- To describe a robotic cell, other coordinate frames are necessary, such as the global coordinated system and the workpiece coordinate system.
- A robotic manipulator consists of joints with rotational and translatory movements. A normal rotation matrix is 3x3 in size and does not account . To accommodate transl ional matrix into 4x4 ho



- The transformation matrix that represents the relationship between two frames is represented by the following sub-transformations.
- Rotation matrix: A rotation matrix in robotics represents the orientation of an object or a coordinate frame in three-dimensional space.
- Translation or position vector: The translation vector or position vector represents the displacement of an object or a coordinate frame in three-dimensional space.
- Perspective transformation: A transformation that involves the projection of three-dimensional points onto a two-dimensional plane, mostly used in computer graphics and not in robotics.

$$T = \left[\begin{array}{c|c} \begin{matrix} \text{Rotation Matrix} \\ (3*3) \end{matrix} & \begin{matrix} \text{Position Vector} \\ (3*1) \end{matrix} \\ \hline \begin{matrix} \text{Perspective Transf.} \\ (1*3) \end{matrix} & \begin{matrix} \text{Stretching} \\ (1*1) \end{matrix} \end{array}\right] = \left[\begin{array}{c|c} R & P \\ \hline 0 & 1 \end{array}\right]$$

refers to

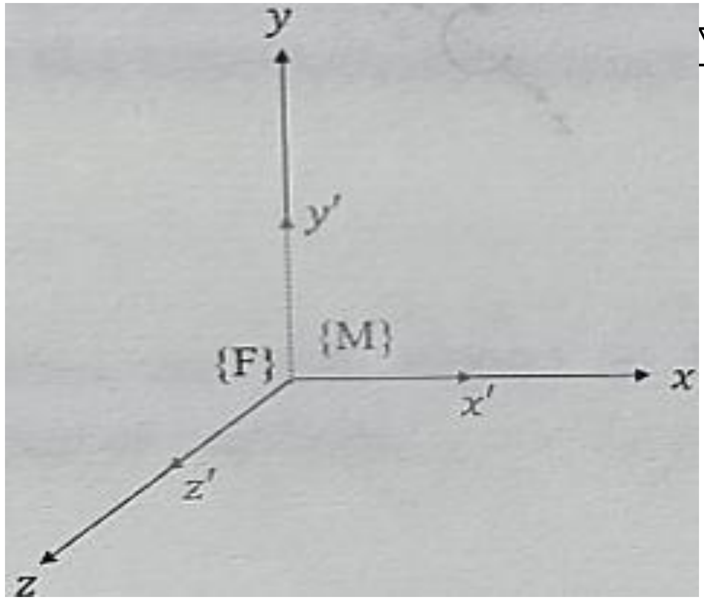
ojects or

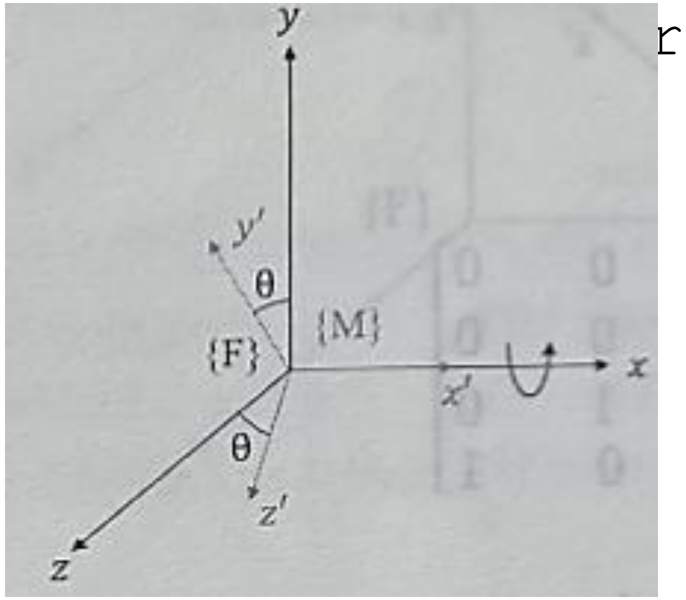
Rotation Matrix

- A robot's end-effector orientation can be determined by a combination of rotations along x, y, z directions. The following matrices are used to determine rotation along the x, y, and z-axis. (F) represents fixed frame and {M} represents mobile frame or tool frame.

➤Rotation about x-axis

- The following rotation matrix represents the rotation of y respect to the x-axis of

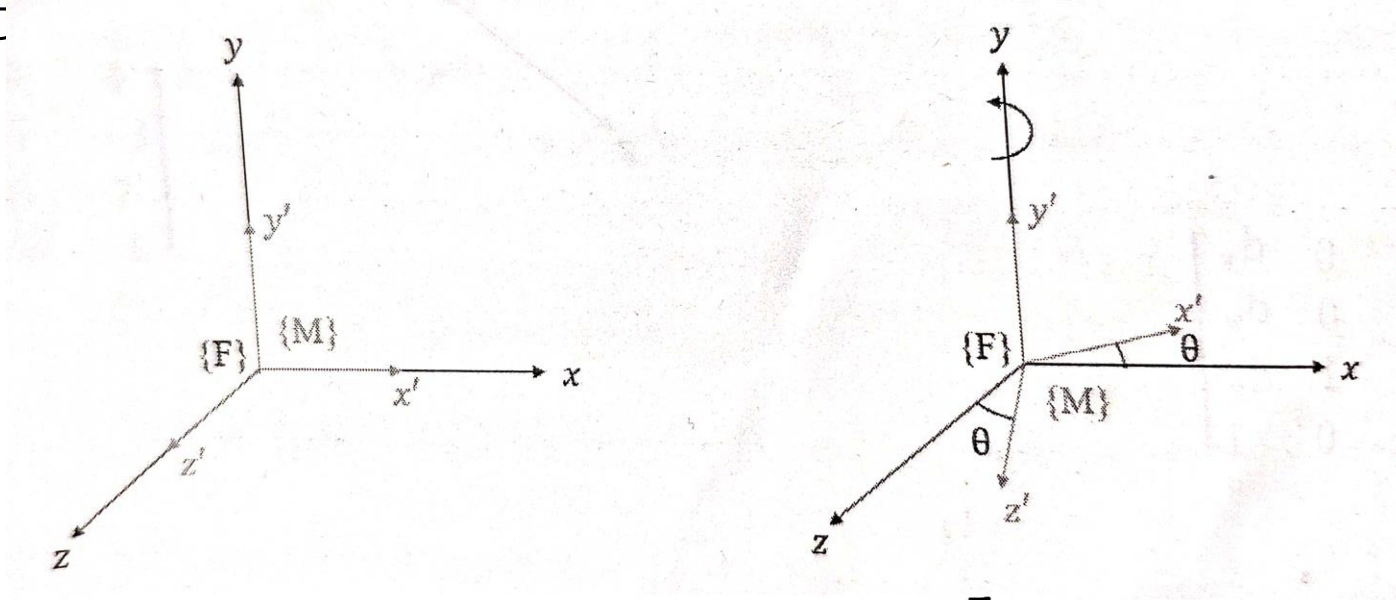




$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\theta & -sin\theta & 0 \\ 0 & sin\theta & cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

➢Rotation about y-axis

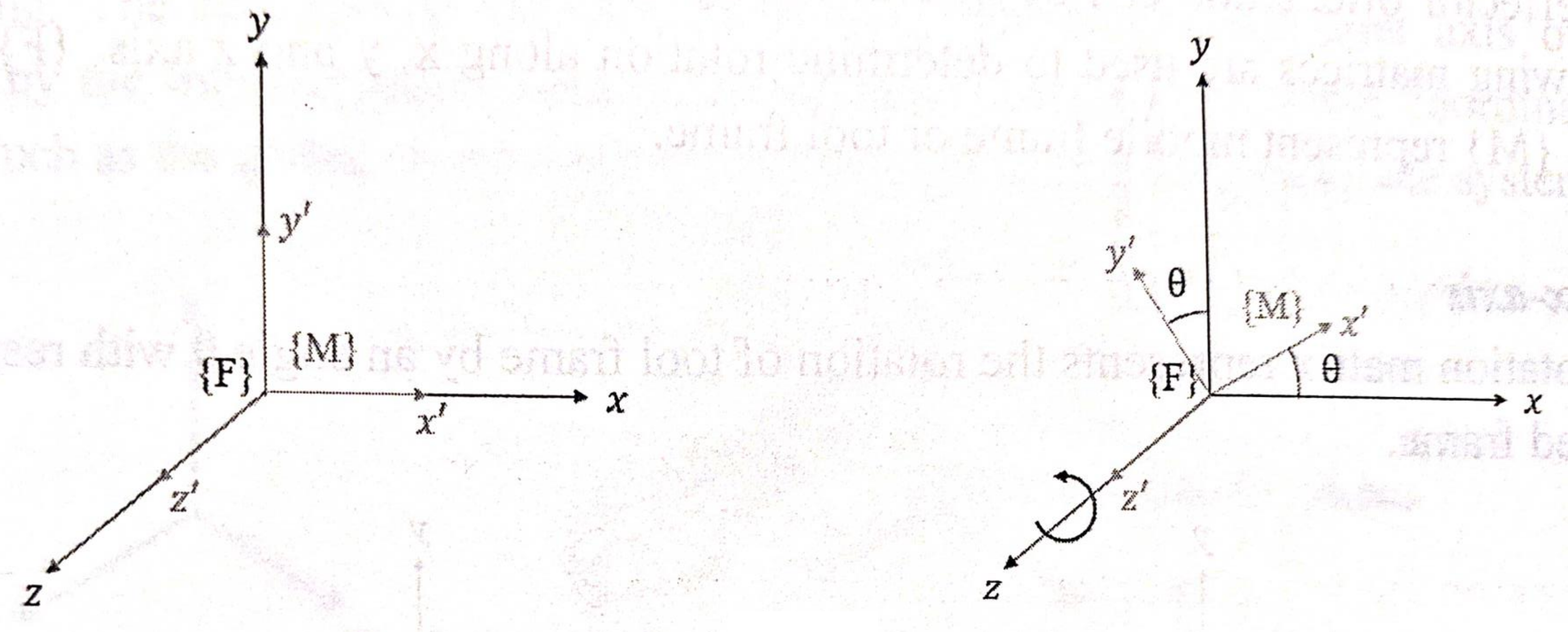
- The following rotation matrix represents the rotation of the tool frame by an angle θ with respect to t



$$R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

➢Rotation about z-axis

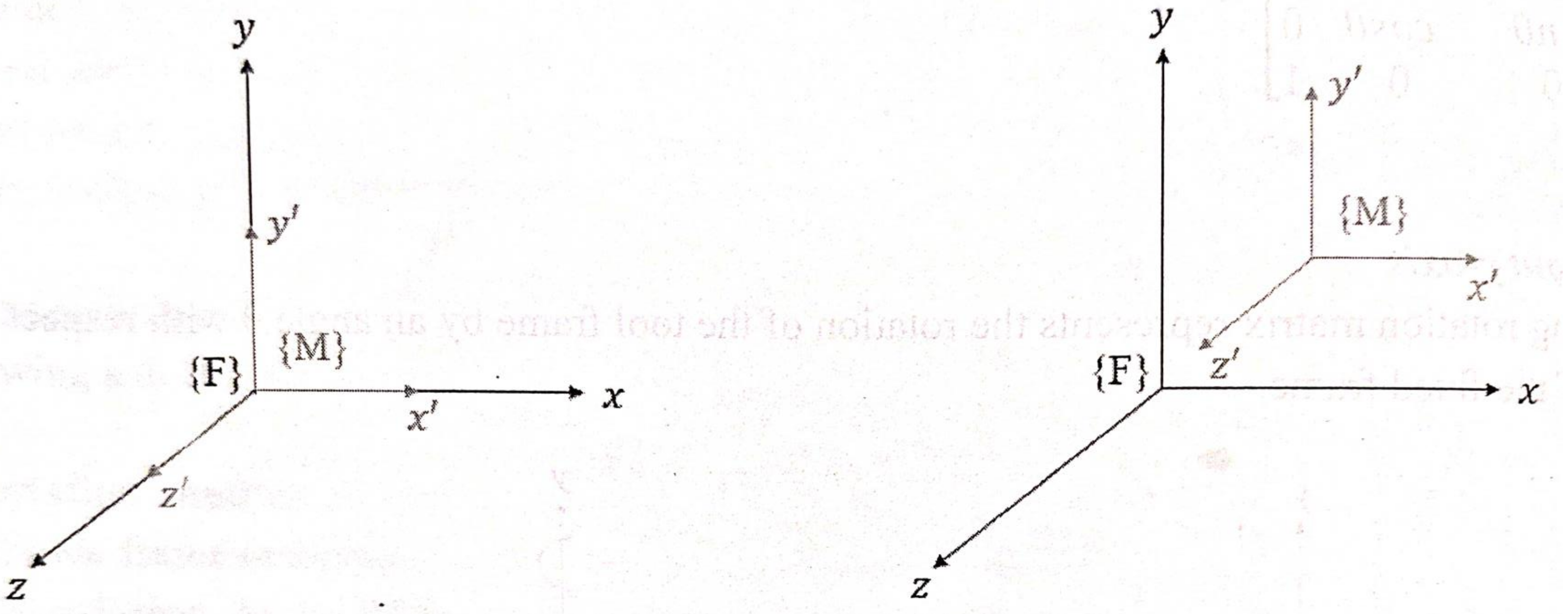
- The following rotation matrix represents the rotation of the tool frame by an angle θ with respect to the z-axis of the fixed



$$R_z = \begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation Matrix

The following translational matrix represents the movement of the tool frame by amount dx, dy, and dz with respect to the fixed frame



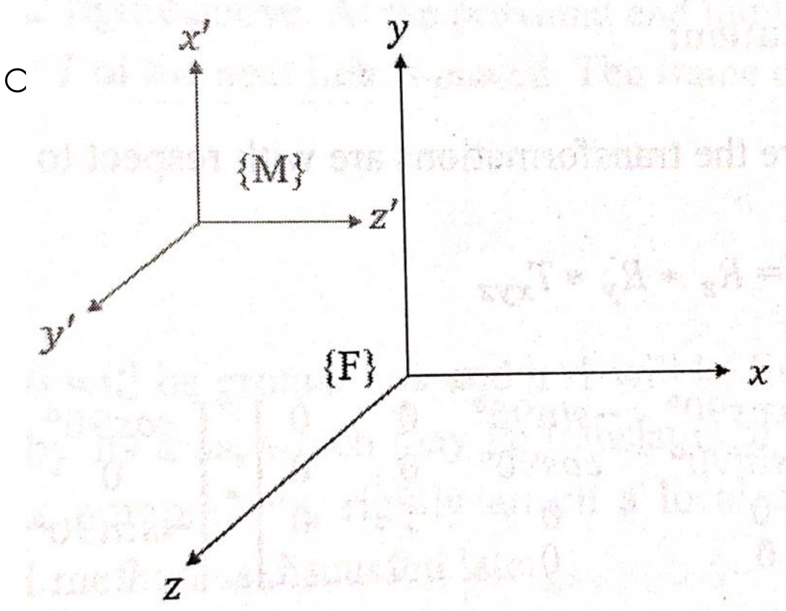
$$T_{xyz} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The position and orientation of the end effector are achieved by combined rotation and translation of the links about respective joints.
- It requires a composite homogenous transformation matrix.
- Matrix multiplication is non-commutative, care should be taken in the composite transformation.

The following rules can be used:

1. At the start, both fixed and mobile coordinate frames are coincident.
2. If mobile frame rotates/translate with respect to one of the axes of fixed frame, pre-multiply the previous resultant matrix with current matrix.

Q-4.Frame {M} is rotated by 90° about the z-axis. followed by a rotation of 90° about the y-axis, then it is translated by 4 unit along x-axis, -3 unit along y-axis and 7 units along z-axis with respect to fixed frame {F}. Determine the



$$R_M^F = T_{xyz} * R_y * R_z$$

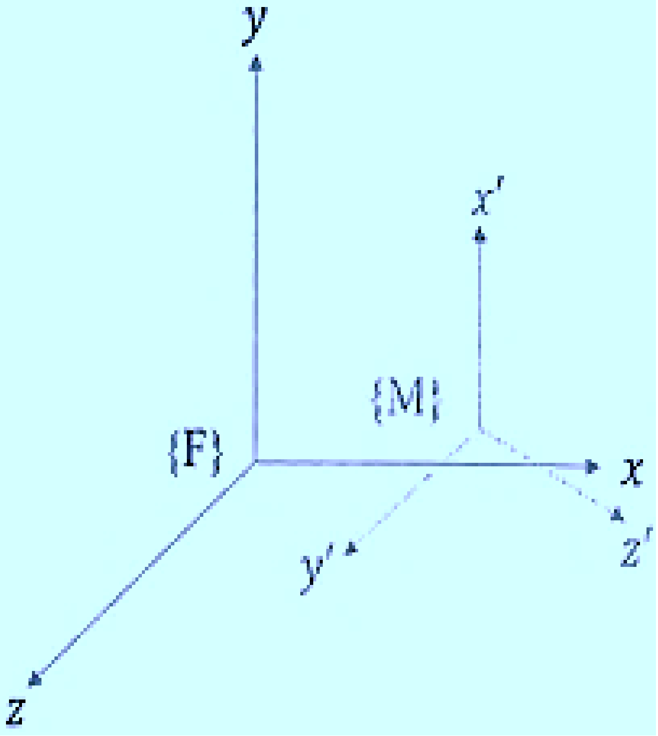
$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} cos90^o & 0 & sin90^o & 0 \\ 0 & 1 & 0 & 0 \\ -sin90^o & 0 & cos90^o & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} cos90^o & -sin90^o & 0 & 0 \\ sin90^o & cos90^o & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q 5. Frame {M} is rotated by 90^0 about the z'-axis, followed by a rotation of 90^0 about the y- axis, then it is translated by 4 units along x'-axis, -3 units along y′-axis, and 7 units along z with -axis with respect to itself. Determine the transformation matrix.

Here the transformations are with respect to mobile frame, post-multiplication of



$$R_M^F = R_z * R_y * T_{xyz}$$

$$= \begin{bmatrix} cos90^o & -sin90^o & 0 & 0 \\ sin90^o & cos90^o & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} cos90^o & 0 & sin90^o & 0 \\ 0 & 1 & 0 & 0 \\ -sin90^o & 0 & cos90^o & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 7 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

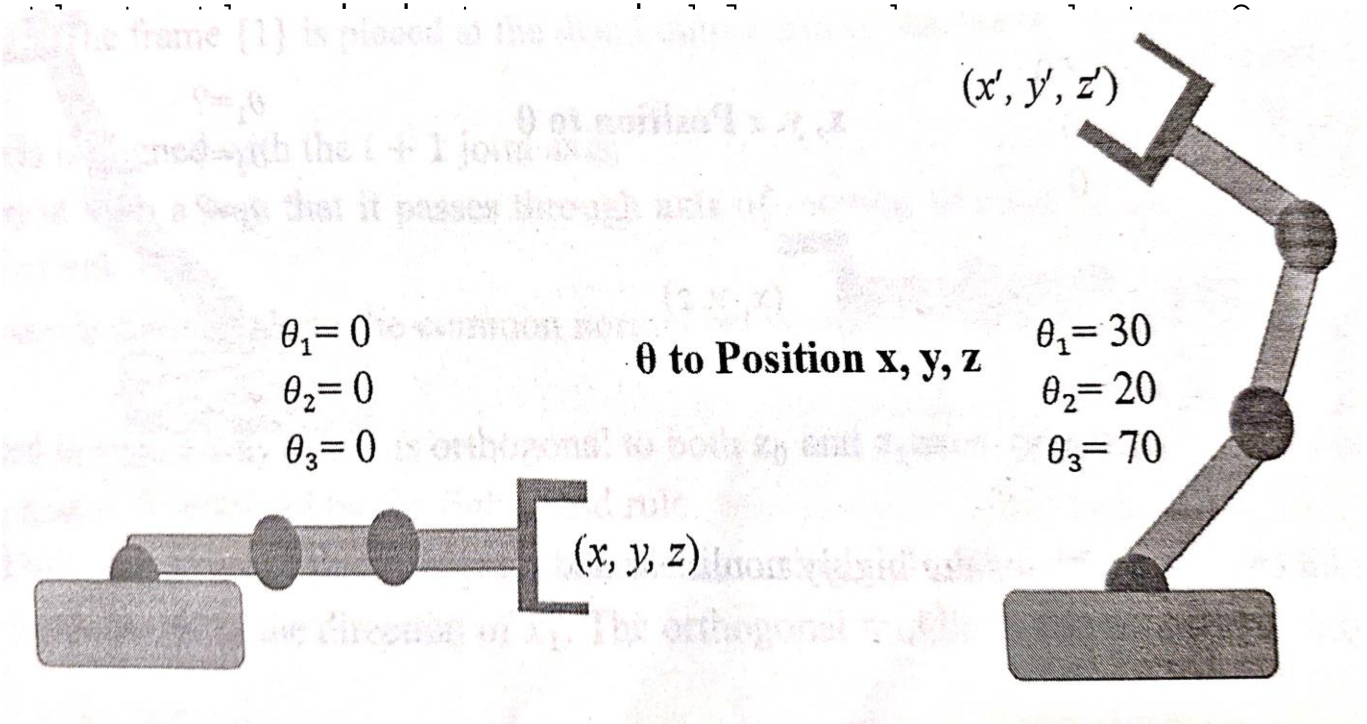
Robot Kinematics

- Robot kinematics mainly deals with forward kinematics and inverse kinematics.
- Kinematic analysis involves examining the position, velocity and acceleration of robots.
- It establishes the kinematic relationships between the end-effector and the various links of the robot.

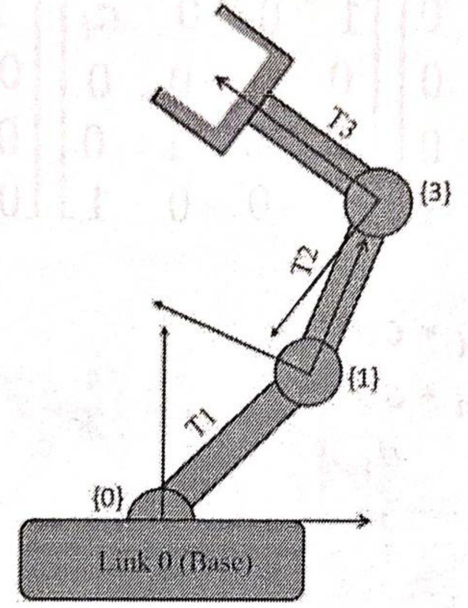
Forward Kinematics

- Forward kinematics is determining the position and orientation of the end effector, given that joint variable of the robot is known.
- Joint variables are the angles between two links when joint is revolute or rotational and it will be link extension in case of prismatic or sliding joint.
- In the following diagram, the joint variables θ_1, θ_2 and θ_3 are zero, the end- effector current position is x, y, z.
- The forward kinematics problem here is to find the position of the end-effector i.e. x', y', z';given

30^0 $\theta_2 = 20^0$ $\theta_3 = 70^0$.



- In forward kinematics, a set of equations that relate to the particular configuration of a robot is developed. By substituting the joint variables in these equations, the position and orientation of the end-effector is determined.
- These equations are derived by assigning frames at each joint and then moving from one frame to another frame gives the transformation equation between the joints.
- At the end by combin n a set of equation that det of end effector is determine



The total transformation (Forward Kinematics) is given by

[T] =[T1] [T2] [T3] ………[Tn]

Inverse Kinematics

Inverse kinematics is determining the sets of joint variables which will satisfy the position and orientation of the end-effector, given that position and orientation of the tool and the geometric link parameters are known.

In the following diagram, the joint variables θ_1, θ_2 and θ_3 are zero, the end- effector current position is x = 50 y = 0 z = 0

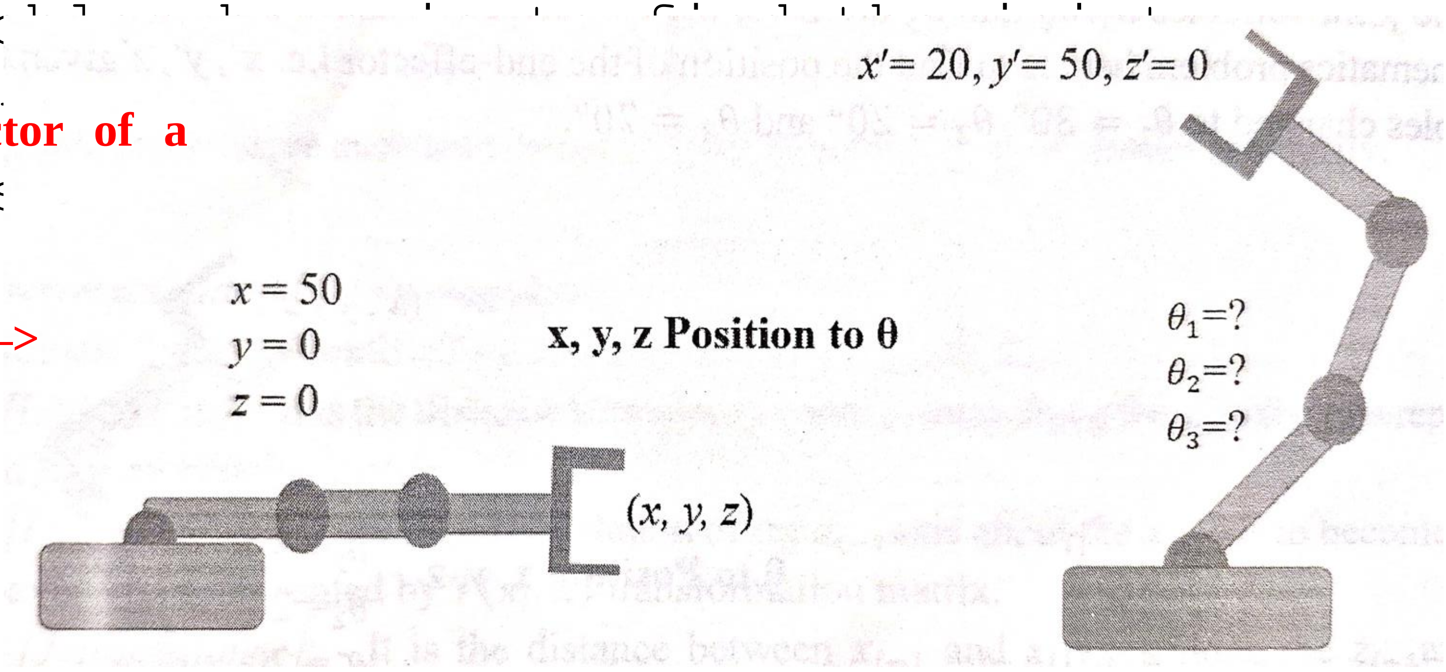
The inverse kinematics pr

variables θ_1, θ_2 and θ_3 gi

effector x', y', z' = 0 is

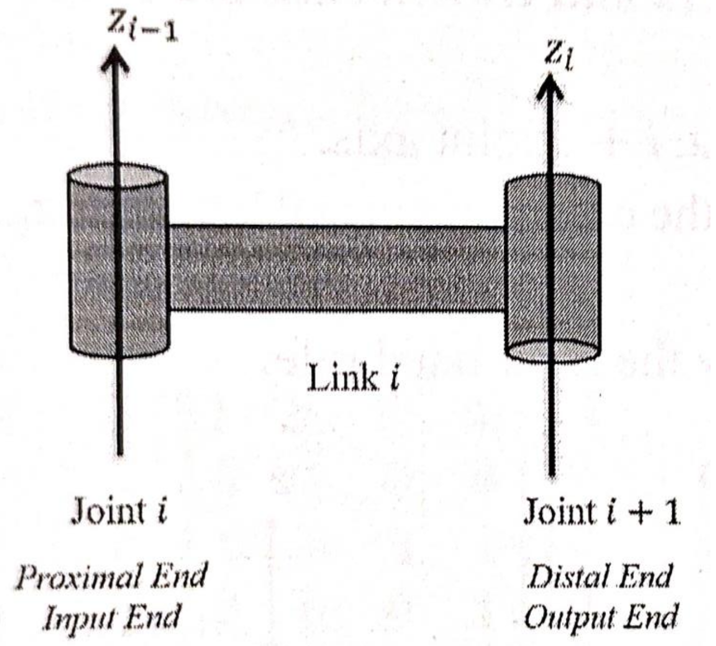
- **Joint Variables ——> Pose of the End effector of a Robotic Arm = Forward Kinematics**

- **Pose of the End effector of a Robotic Arm ——> Joint Variables = Inverse Kinematics**



D-H Parameters

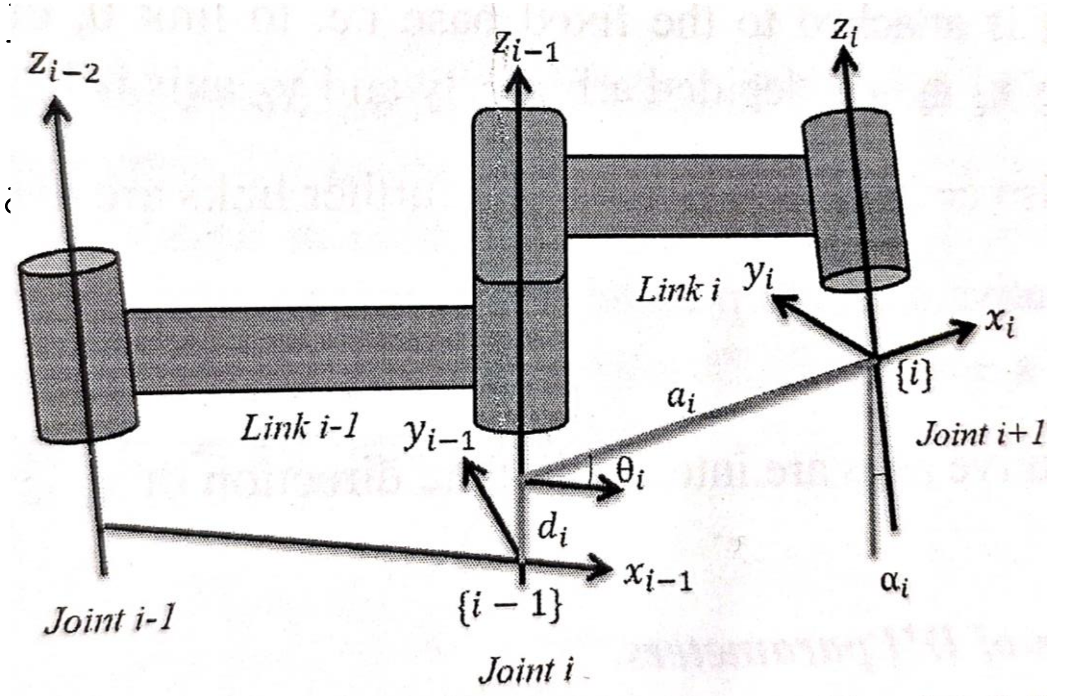
- Motion transformation between any pairs of frames involves six motion parameters in space, three for translation and three for rotation e.g. x_1, y_1, z_1, θ_x, θ_y, θ_z.
- A simple method exists that requires only four motion parameters to represent the transformation between two frames and gives relative posture between two consecutive links called **DH parameters or Denavit-Hartenberg notation.**
- Links are solid members between joints, the link has a proximal end/input end close to the base and a distal end/out end-effector as shown in the figure above.
- At the proximal end joint i of the current link i distal end joint i + 1 of the next link is plac current link is attached at the distal end.



Denavit - Hartenberg Notation (DH Parameters)

- For a serial manipulator with n joint and n + 1 link. Link 0 will be the ground link and n + 1 will be the link where the end-effector is attached.
- Every joint is indicated by its axis, which may be translational or rotational.
- To relate the kinematic information of robot components, rigidly attach a local co-ordinate frame {i} to each link i at joint i + I based on the DH method as discussed
- This transformation between frame (i) determined by the following rotations a sequence.

1) Translate along z_{i-1} by d_i.
2) Rotation about z_{i-1} by angle θi.
3) Translate along x_i by a_i.
4) Rotation about x_i by α_i.



- The overall transformation matrix called as arm matrix is given by

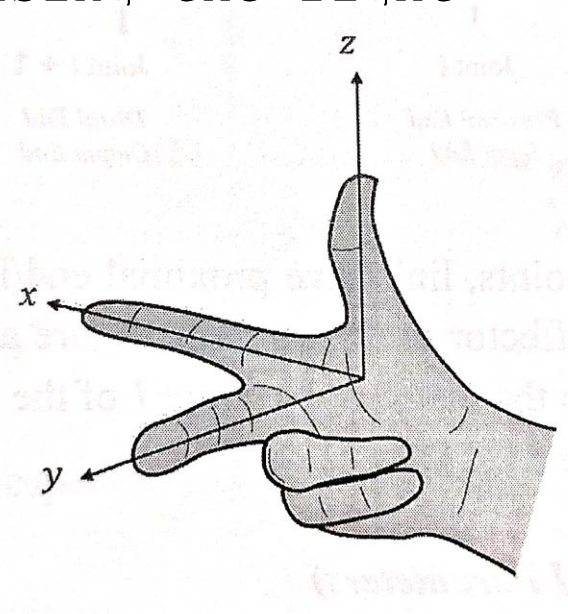
- **Steps to determine DH parameters and transformation between two frames.**

Step 1: Frame assignment

- The zi axis is aligned with the i + 1 joint axis.
- The xi axis is defined along the common normal between z_{i-1} and z_i axes, pointing from z i – 1 to zi.
- The yi axis is determined by the right-hand rule.

The y axis is placed according to the right-hand rule i.e. using the right hand, place the thumb in the direction of z and the index direction of x. The middle figure will show the direction of

The x, y and z axes will be orthogonal to each other.



- **Note:**

i) For frame (0) that is attached to the fixed base i.e. to the direction of

z_0 axis is specified. Direction of x_0 axis is decided arbitra

y_0 axis is based on the right-hand rule.

ii) The last frame is also decided arbitrarily as no further links are available.

iii) When two consecutive axes are parallel, the common normal between

Step 2: Determination of DH parameters.

DH co-ordinate frame is identified by four parameters a_i , α_i, d_i, θ_i.

1) $\mathbf{a_i}$ (Link length); - It is the distance between z_{i-1} and z_i axes along the x_i axis. It is represented by the T($\mathbf{x_i}$, **a**) transformation matrix.

2) $\boldsymbol{\alpha_i}$ (Link twist): - it is the required rotation of the z_{i-1} axis about x_i axis to become parallel to the z_i axis.

It is represented by the T(**xi**, **α**) transformation matrix.

3) $\mathbf{d_i}$ (Joint distance): -It is the distance between the x_{i-1} and x_i axes along the z_{i-1} axis. It is represented by the T($\mathbf{z_{i-1}}$, **d**) transformation matrix.

Step 3: Determine the transformation.

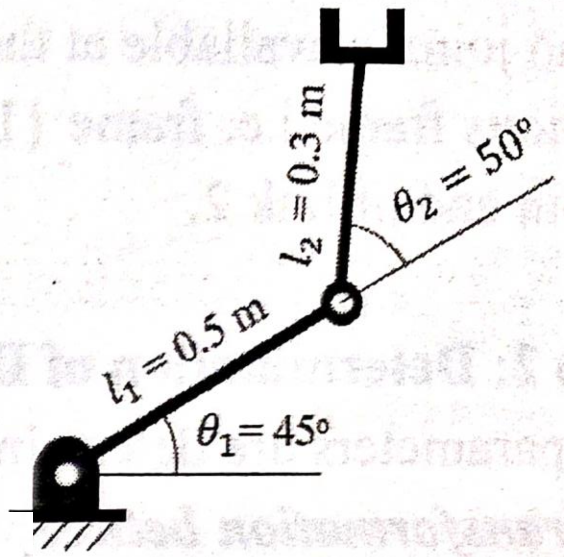
- Following equation gives transformation between frame {i} to frame {i-1}.
- $\mathbf{T}_i^{\,i-1} = \mathbf{T}(\mathbf{z}_{i-1}, \mathbf{d}) * \mathbf{T}(\mathbf{z}_{i-1}, \theta) * \mathbf{T}(\mathbf{x}_i, \mathbf{a}) * \mathbf{T}(\mathbf{x}_i, \alpha)$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i * c\alpha_i & s\theta_i * s\alpha_i & a_i * c\theta_i \\ s\theta_i & c\theta_i * c\alpha_i & -c\theta_i * s\alpha_i & a_i * s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

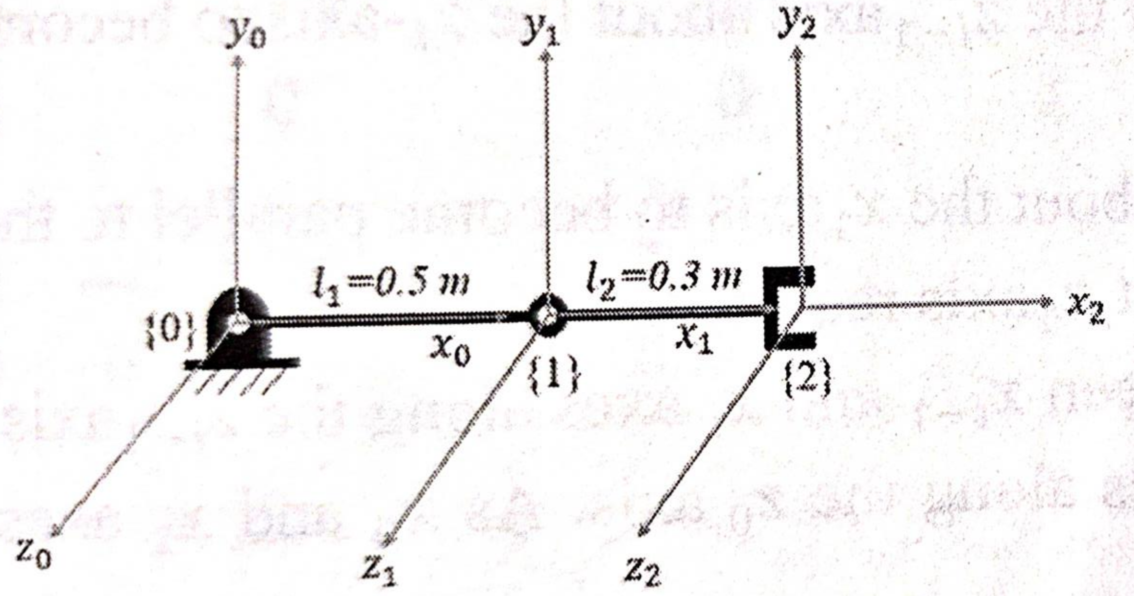
Where, $\mathbf{c\theta_i = \cos\ \theta_i}$, $\mathbf{s\theta_i = \sin\ \theta_i}$, $\mathbf{c\alpha_i = \cos\ \alpha_i}$, $\mathbf{s\alpha_i = \sin\ \alpha_i}$

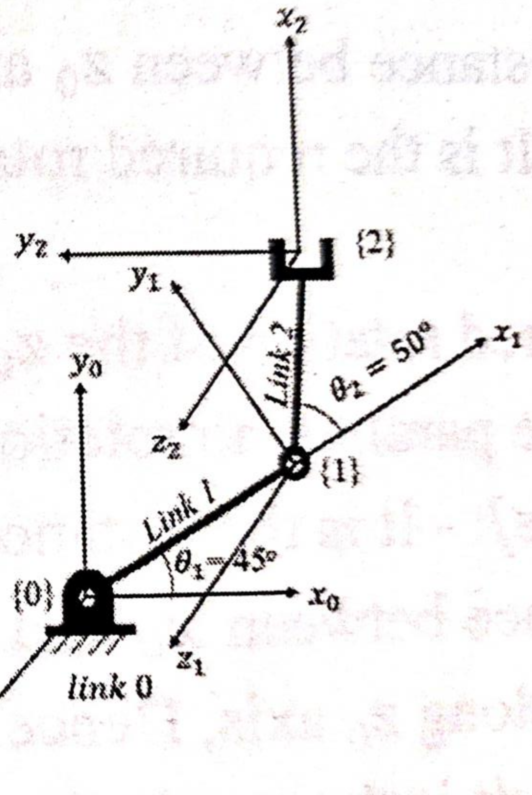
Q6. Determine the DH parameters of the following two DOF serial robotic manipulator and T_2^0 transformation matrix.



Solution : The manipulator consists of two rotational joints.

Step 1: Frame assignment.





Link 0 (*i=0*): Fixed link is treated as link 0, frame {0} is assigned to link 0.

i) The z_i axis is aligned with the $i+1$ joint axis.

z_0 is placed in such a way that it passes through axis of rotation of joint 1 which is placed at link 0.

ii) The x_i axis is defined along the common normal between z_{i-1} and z_i axes, pointing from z_{i-1} to z_i.

x_0 is placed arbitrarily as no previous frame available, here it is placed along horizontal intersecting z_0.

iii) The y_i axis is determined by the right-hand rule.

The y_0 axis is placed according to right hand rule i.e. using right hand, place thumb in the direction of z_0 and index finger in the direction of x_0. The orthogonal middle figure gives the direction of y_0.

Link 1 (*i=1*): The frame {1} is placed at the distal/output end of the link 1.

i) The z_i axis is aligned with the $i+1$ joint axis.

z_1 is placed in such a way that it passes through axis of rotation of joint 2 which is placed at the output end of link 1.

ii) The x_i axis is defined along the common normal between z_{i-1} and z_i axes, pointing from z_{i-1} to z_i.

x_1 is placed in such a way that it is orthogonal to both z_0 and z_1 axes, pointing from z_0 and z_1.

iii) The y_i axis is determined by the right-hand rule.

The y_1 axis is placed according to right hand rule i.e. using right hand, place thumb in the direction of z_1 and index finger in the direction of x_1. The orthogonal middle figure gives the direction of y_1.

Link 2 (i=2): The frame {2} is placed at the distal/output end of the link 2, here it is end-effector. As no joint is available at the output end of the link 2, frame {2} can be placed arbitrarily. Here the previous frame i.e. frame {1} orientation is taken as the orientation of frame {2} and placed at the output end of link 2.

Step 2: Determination of DH parameters.

DH parameters are determined by the transformation between two frames.

a) Transformation between frame {0} and frame {1} i.e. between link 0 and link 1

i) a_i *[Link length]*: - It is the distance between z_{i-1} and z_i axes along the x_i axis.

a_1= 0.5m, it is the distance between z_0 and $z_1$axes along the $x_1$axis

ii) α_i *[Link twist]*: - It is the required rotation of the z_{i-1}axis about the x_i-axis to become parallel to the z_i-axis.

α_1= 0^0, it is the required rotation of the z_0 axis about the $x_1$axis to become parallel to the z_1 axis. As both axis z_0 and $z_1$are parallel, no rotation about $x_1$axis required.

iii) d_i *[Joint distance]*: - It is the distance between x_{i-1} and x_i axes along the z_{i-1}axis.

d_1= 0, it is the distance between x_0 and x_1 axes along the z_0 axis. As x_0 and x_1 axes lie in same plane with no offset along z_0 axis, Hence $d_1$is 0 .

iv)θ_i *[Joint angle]*: - It is the required rotation of x_{i-1} axis about the z_{i-1} axis to become parallel to the x_i axis.

θ_1= 45^0, it is the required rotation of x_0 axis about the z_0 axis to become parallel to the x_1 axis.

b) Transformation between frame {1} and frame {2} i.e. between link 1 and link 2

i) a_i *[Link length]*: - It is the distance between z_{i-1} and z_i axes along the x_i axis.

$a_2 = 0.3$m, it is the distance between z_1 and z_2 axes along the x_2 axis.

ii) α_i *[Link twist]*: - It is the required rotation of the z_{i-1}axis about the x_i-axis to become parallel to the z_i-axis.

$\alpha_2 = 0^0$, it is the required rotation of the z_1 axis about the x_2 axis to become parallel to the z_2 axis. As both axis z_1 and z_2 are parallel, no rotation about x_2 axis required.

iii) d_i *[Joint distance]*: - It is the distance between x_{i-1} and x_i axes along the z_{i-1}axis.

$d_2 = 0$, it is the distance between x_1 and x_2 axes along the z_1 axis. As x_1 and x_2 axes lie in same plan with no offset along z_1 axis.

iv) θ_i *[Joint angle]*: - It is the required rotation of x_{i-1} axis about the z_{i-1} axis to become parallel to the x_i axis.

$\theta_2 = 50^0$, it is the required rotation of x_1 axis about the z_1 axis to become parallel to the x_2 axis.

DH Parameters Table

Link *i*	a_i	α_i	d_i	θ_i
Link 1	0.5	0^0	0	45^0
Link 2	0.3	0^0	0	50^0

Step 3 : Determine the transformation.

$$T_i^{i-1} = T(z_{i-1}, d) * T(z_{i-1}, \theta) * T(x_i, a) * T(x_i, \alpha)$$

$$T_1^0 = T(z_0, d) * T(z_0, \theta) * T(x_1, a) * T(x_1, \alpha)$$

$$T_1^0 = \begin{bmatrix} c\theta_1 & -s\theta_1 * c\alpha_1 & s\theta_1 * s\alpha_1 & a_1 * c\theta_1 \\ s\theta_1 & c\theta_1 * c\alpha_1 & -c\theta_1 * s\alpha_1 & a_1 * s\theta_1 \\ 0 & s\alpha_1 & c\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c45^0 & -s45^0 * c0^0 & s45^0 * s0^o & 0.5 * c45^0 \\ s45^0 & c45^0 * c0^0 & -c45^{0o} * s0^o & 0.5 * s45^0 \\ 0 & s0^0 & c0^0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7070 & -0.7070 & 0 & 0.3535 \\ 0.7070 & 0.7070 & 0 & 0.3535 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = T(z_1, d) * T(z_1, \theta) * T(x_2, a) * T(x_2, \alpha)$$

$$= \begin{bmatrix} c\theta_2 & -s\theta_2 * c\alpha_2 & s\theta_2 * s\alpha_2 & a_2 * c\theta_2 \\ s\theta_2 & c\theta_2 * c\alpha_2 & -c\theta_2 * s\alpha_2 & a_2 * s\theta_2 \\ 0 & s\alpha_2 & c\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c50^0 & -s50^0 * c0^0 & s50^0 * s0^o & 0.3 * c50^0 \\ s50^0 & c50^0 * c0^0 & -c50^0 * s0^o & 0.3 * s50^0 \\ 0 & s0^0 & c0^0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

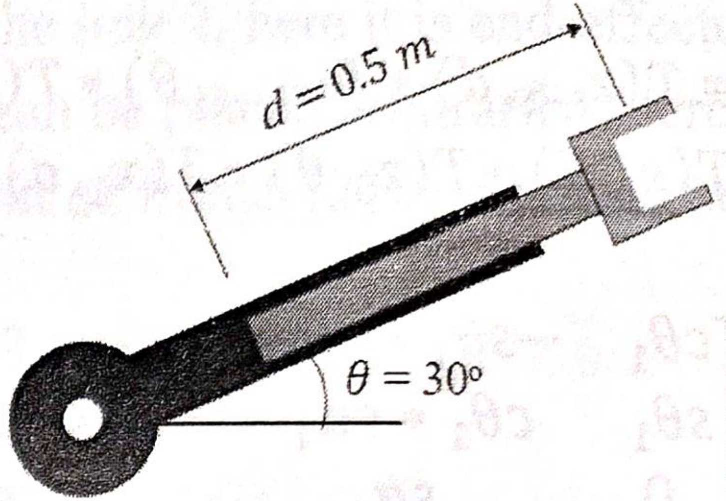
$$= \begin{bmatrix} 0.6427 & -0.7660 & 0 & 0.1928 \\ 0.7660 & 0.6427 & 0 & 0.2298 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = [T_1^0]\,[T_2^1]$$

$$= \begin{bmatrix} 0.7070 & -0.7070 & 0 & 0.3535 \\ 0.7070 & 0.7070 & 0 & 0.3535 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.6427 & -0.7660 & 0 & 0.1928 \\ 0.7660 & 0.6427 & 0 & 0.2298 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

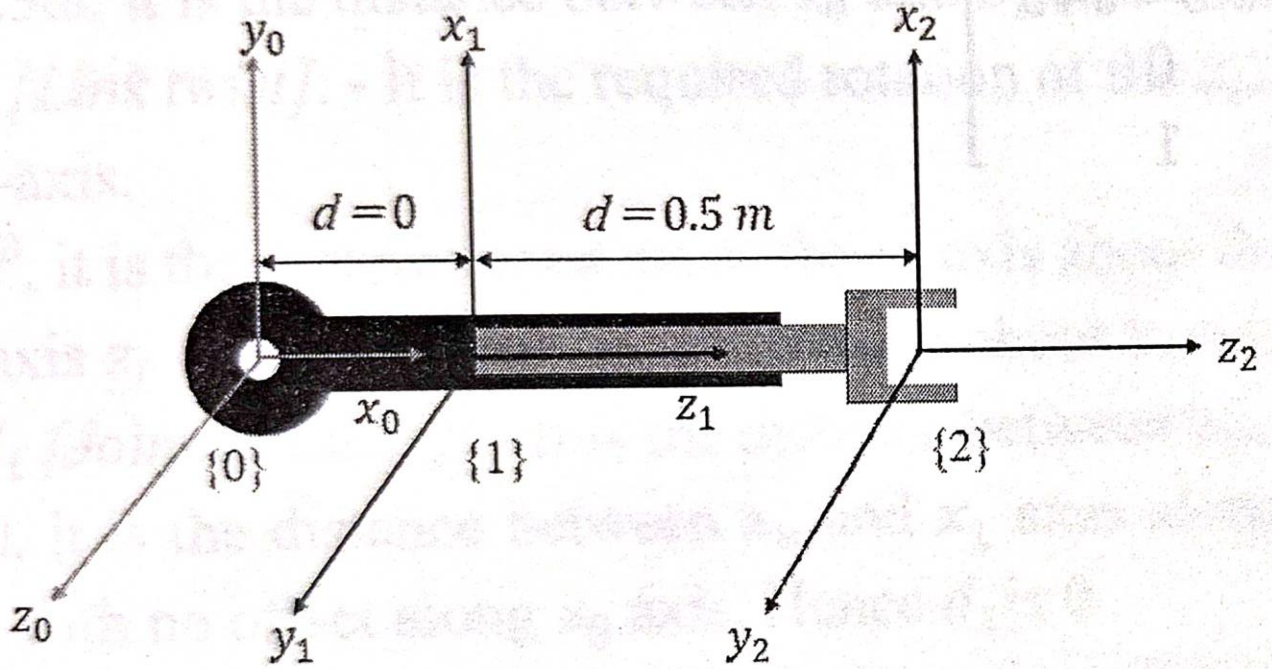
$$T_2^0 = \begin{bmatrix} -0.0871 & -0.9959 & 0 & 0.3273 \\ 0.9959 & -0.0871 & 0 & 0.6522 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

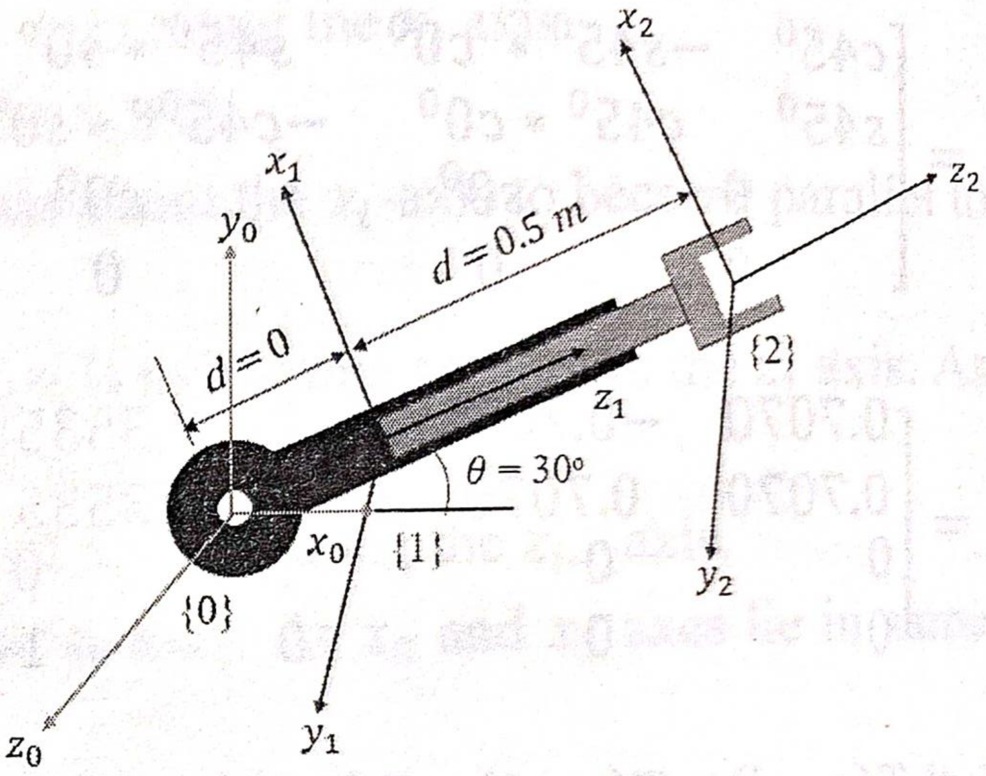
Q7. Determine the DH parameters of the following two DOF serial robotic manipulator and T_2^0 transformation matrix.



Solution: The manipulator consists of one rotational joint and one prismatic joint.

Step 1: Frame assignment.





Link 0 (*i*=*0*): Fixed link is treated as link 0, frame {0} is assigned to link 0.

i) The z_i axis is aligned with the $i + 1$ joint axis.
z_0 is placed in such a way that it passes through axis of rotation of joint 1 which is placed at link 0.
ii) The x_i axis is defined along the common normal between z_{i-1} and z_i axes, pointing from z_{i-1} to z_i.
x_0 is placed arbitrarily as no previous frame available.
iii) The y_i axis is determined by the right-hand rule.
The y_0 axis is placed according to right hand rule i.e. using right hand, place thumb in the direction of z_0 and index finger in the direction of x_0. The orthogonal middle figure gives the direction of y_0.

Link 1 (*i*=*1*): The frame {1} is placed at the distal/output end of the link 1.

i) The z_i axis is aligned with the $i + 1$ joint axis.
z_1 is placed in such a way that it passes through linear movement of joint 2 which is placed at the output end of link 1.
ii) The x_i axis is defined along the common normal between z_{i-1} and z_i axes, pointing from z_{i-1} to z_i.
x_1 is placed in such a way that it is orthogonal to both z_0 and z_1 axes.
iii) The y_i axis is determined by the right-hand rule.
The y_1 axis is placed according to right hand rule i.e. using right hand, place thumb in the direction of z_1 and index finger in the direction of x_1. The orthogonal middle figure gives the direction of y_1.

Link 2 (*i*=*2*): The frame {2} is placed at the distal/output end of the link 2, here it is end-effector. As no joint is available at the output end of the link 2, frame {2} can be placed arbitrarily. Here the previous frame i.e. frame {1} orientation is taken as the orientation of frame {2} and placed at the output end of link 2.

Step 2 : Determination of DH parameters.

DH parameters are determined by the transformation between two frames.

a) Transformation between frame {0} and frame {1} i.e. between link 0 and link 1

i) a_i *[Link length]*: - It is the distance between z_{i-1} and z_i axes along the x_i axis.

a_1= 0 m, it is the distance between z_0 and z_1 axes along the x_1 axis. Here z_0 and z_1 axes are orthogonal, hence distance between them is zero.

ii) α_i *[Link twist]*: - It is the required rotation of the z_{i-1} axis about the x_i-axis to become parallel to the z_i-axis.

α_1= 90°, it is the required rotation of the z_0 axis about the x_1 axis to become parallel to the z_1 axis. Here z_0 is rotated by 90° about x_1 to become parallel with z_1 axis.

iii) d_i *[Joint distance]*: - It is the distance between x_{i-1} and x_i axes along the z_{i-1} axis.

d_1= 0, it is the distance between x_0 and x_1 axes along the z_0 axis. As x_0 and x_1 axes lie in same plane with no offset along z_0 axis, hence d_1 is 0.

iv) θ_i *[Joint angle]*: - It is the required rotation of x_{i-1} axis about the z_{i-1} axis to become parallel to the x_i axis.

θ_1= 30^0, as it joint variable

b) Transformation between frame {1} and frame {2} i.e. between link 1 and link 2

i) a_i *[Link length]*: - It is the distance between z_{i-1} and z_i axes along the x_i axis.

$a_2 = 0$ m, it is the distance between z_1 and z_2 axes along the x_2 axis. As z_1 and z_2 axes are collinear, hence a_2 is 0 m.

ii) α_i *[Link twist]*: - It is the required rotation of the z_{i-1}axis about the x_i-axis to become parallel to the z_i-axis.

$\alpha_2 = 0^0$, It is the required rotation of the z_1 axis about the x_2 axis to become parallel to the z_2 axis. As both axis z_1 and z_2 are collinear, no rotation about x_2 axis required.

iii) d_i *[Joint distance]*: - It is the distance between x_{i-1} and x_i axes along the z_{i-1}axis.

$d_2 = 0.5$ m, it is the distance between x_1 and x_2 axes along the z_1 axis.

iv) θ_i *[Joint angle]*: - It is the required rotation of x_{i-1} axis about the z_{i-1} axis to become parallel to the x_i axis.

$\theta_2 = 0^0$, as joint 2 is prismatic joint.

DH Parameters Table

Link *i*	a_i	α_i	d_i	θ_i
Link 1	**0**	**90^0**	**0**	**30^0**
Link 2	**0**	**0^0**	**0.5**	**0^0**

Step 3: Determine the transformation.

$$T_i^{i-1} = T(z_{i-1}, d) * T(z_{i-1}, \theta) * T(x_i, a) * T(x_i, \alpha)$$

$$T_1^0 = T(z_0, d) * T(z_0, \theta) * T(x_1, a) * T(x_1, \alpha)$$

$$T_1^0 = \begin{bmatrix} c\theta_1 & -s\theta_1 * c\alpha_1 & s\theta_1 * s\alpha_1 & a_1 * c\theta_1 \\ s\theta_1 & c\theta_1 * c\alpha_1 & -c\theta_1 * s\alpha_1 & a_1 * s\theta_1 \\ 0 & s\alpha_1 & c\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c30^0 & -s30^0 * c90^0 & s30^0 * s90^o & 0 * c30^0 \\ s30^0 & c30^0 * c90^0 & -c30^o * s90^o & 0 * s30^0 \\ 0 & s90^0 & c90^0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & 0 & 0.5 & 0 \\ 0.5 & 0 & -0.866 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = T(z_1, d) * T(z_1, \theta) * T(x_2, a) * T(x_2, \alpha)$$

$$= \begin{bmatrix} c\theta_2 & -s\theta_2 * c\alpha_2 & s\theta_2 * s\alpha_2 & a_2 * c\theta_2 \\ s\theta_2 & c\theta_2 * c\alpha_2 & -c\theta_2 * s\alpha_2 & a_2 * s\theta_2 \\ 0 & s\alpha_2 & c\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c0^0 & -s0^0 * c0^0 & s0^0 * s0^o & 0 * c0^0 \\ s0^0 & c0^0 * c0^0 & -c0^o * s0^o & 0 * s0^0 \\ 0 & s0^0 & c0^0 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

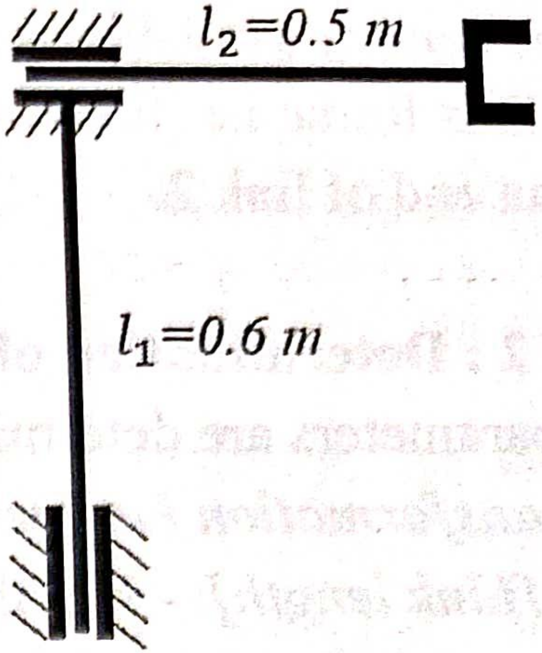
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = [T_1^0]\,[T_2^1]$$

$$= \begin{bmatrix} 0.866 & 0 & 0.5 & 0 \\ 0.5 & 0 & -0.866 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

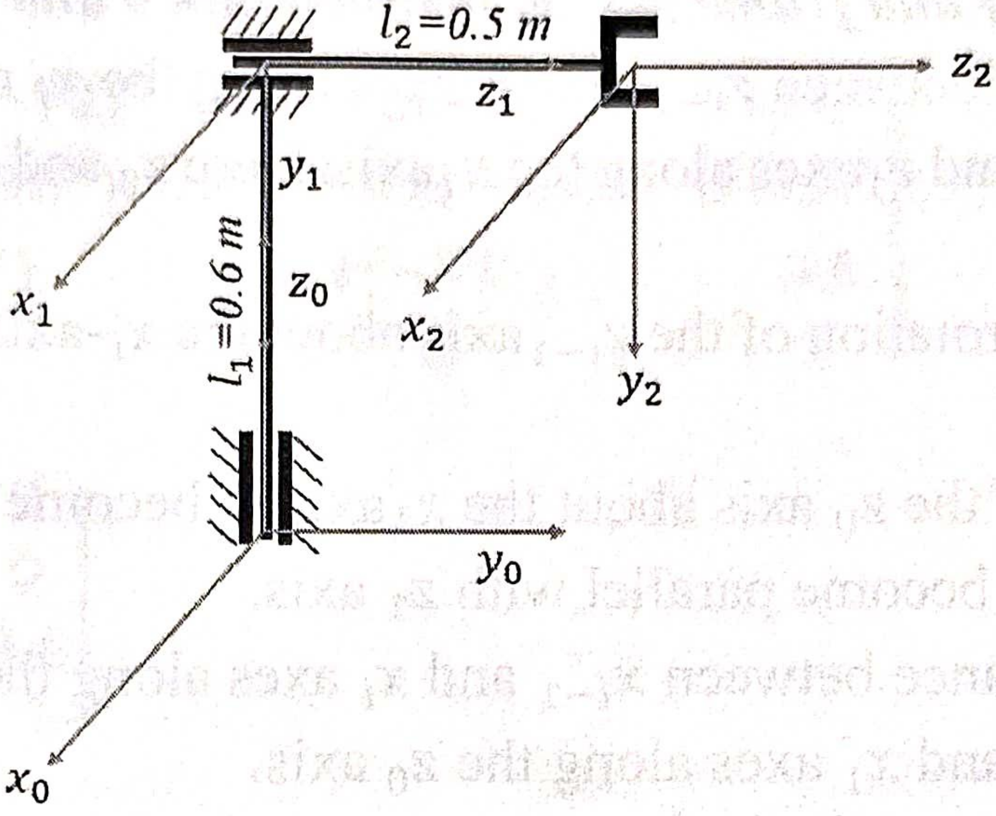
$$T_2^0 = \begin{bmatrix} 0.866 & 0 & 0.5 & 0.25 \\ 0.5 & 0 & -0.866 & -0.433 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q8. Determine the DH parameters of the following two DOF serial robotic manipulator and T_2^0 transformation matrix.



Solution: The manipulator consists of two prismatic joints.

Step 1: Frame assignment.



Link 0 (*i*=*0*): Fixed link is treated as link 0, frame {0} is assigned to link 0.

i) The z_i axis is aligned with the $i + 1$ joint axis.
z_0 is placed in such a way that it passes through linear movement of joint 1 which is placed at link 0 as the joint is prismatic joint.
ii) The x_i axis is defined along the common normal between z_{i-1} and z_i axes, pointing from z_{i-1} to z_i.
x_0 is placed arbitrarily as no previous frame available.
iii) The y_i axis is determined by the right-hand rule.
The y_0 axis is placed according to right hand rule i.e. using right hand, place thumb in the direction of z_0 and index finger in the direction of x_0. The orthogonal middle figure gives the direction of y_0.

Link 1 (i=1): The frame {1} is placed at the distal/output end of the link 1.

i) The z_i axis is aligned with the $i + 1$ joint axis.
z_1 is placed in such a way that it passes through linear movement of joint 2 which is placed at output end of link 1 as the joint is prismatic joint.
ii) The x_i axis is defined along the common normal between z_{i-1} and z_i axes, pointing from z_{i-1} to z_i.
x_1 is placed in such a way that it is orthogonal to both z_0 and z_1 axes.
iii) The y_i axis is determined by the right-hand rule.
The y_1 axis is placed according to right hand rule i.e. using right hand, place thumb in the direction of z_1 and index finger in the direction of x_1. The orthogonal middle figure gives the direction of y_1.

Link 2 (i=2): The frame {2} is placed at the distal/output end of the link 2, here it is end-effector. As no joint is available at the output end of the link 2, frame {2} can be placed arbitrarily. Here the previous frame i.e. frame {1} orientation is taken as the orientation of frame {2} and placed at the output end of link 2.

Step 2 : Determination of DH parameters.

DH parameters are determined by the transformation between two frames.

a) Transformation between frame {0} and frame {1} i.e. between link 0 and link 1

i) a_i *[Link length]*: - It is the distance between z_{i-1} and z_i axes along the x_i axis.

a_1= 0 m, it is the distance between z_0 and z_1 axes along the x_1 axis. Here z_0 and z_1 axes are orthogonal, hence distance between them is zero.

ii) α_i *[Link twist]*: - It is the required rotation of the z_{i-1} axis about the x_i-axis to become parallel to the z_i-axis.

α_1= - 90^0, it is the required rotation of the z_0 axis about the x_1 axis to become parallel to the z_1 axis. Here z_0 is rotated by - 90^0 about x_1 to become parallel with z_1 axis.

iii) d_i *[Joint distance]*: - It is the distance between x_{i-1} and x_i axes along the z_{i-1} axis.

d_1= 0.6, it is the distance between x_0 and x_1 axes along the z_0 axis.

iv) θ_i *[Joint angle]*: - It is the required rotation of x_{i-1} axis about the z_{i-1} axis to become parallel to the x_i axis.

θ_1= 0^0 as joint 1 is prismatic joint.

b) Transformation between frame {1} and frame {2} i.e. between link 1 and link 2

i) a_i *[Link length]*: - It is the distance between z_{i-1} and z_i axes along the x_i axis.

$a_2 = 0$ m, it is the distance between z_1 and z_2 axes along the x_2 axis. As z_1 and z_2 axes are collinear hence a_2 is 0 m.

ii) α_i *[Link twist]*: - It is the required rotation of the z_{i-1}axis about the x_i-axis to become parallel to the z_i-axis.

$\alpha_2 = 0^0$, It is the required rotation of the z_1 axis about the x_2 axis to become parallel to the z_2 axis. As both axis z_1 and z_2 are collinear, no rotation about x_2 axis required.

iii) d_i *[Joint distance]*: - It is the distance between x_{i-1} and x_i axes along the z_{i-1}axis.

$d_2 = 0.5$ m, it is the distance between x_1 and x_2 axes along the z_1 axis.

iv) θ_i *[Joint angle]*: - It is the required rotation of x_{i-1} axis about the z_{i-1} axis to become parallel to the x_i axis.

$\theta_2 = 0^0$, as joint 2 is prismatic joint.

DH Parameters Table

Link i	a_i	α_i	d_i	θ_i
Link 1	0	-90^0	0.6	0^0
Link 2	0	0^0	0.5	0^0

Step 3: Determine the transformation.

$$T_i^{i-1} = T(z_{i-1}, d) * T(z_{i-1}, \theta) * T(x_i, a) * T(x_i, \alpha)$$
$$T_1^0 = T(z_0, d) * T(z_0, \theta) * T(x_1, a) * T(x_1, \alpha)$$

$$T_1^0 = \begin{bmatrix} c\theta_1 & -s\theta_1 * c\alpha_1 & s\theta_1 * s\alpha_1 & a_1 * c\theta_1 \\ s\theta_1 & c\theta_1 * c\alpha_1 & -c\theta_1 * s\alpha_1 & a_1 * s\theta_1 \\ 0 & s\alpha_1 & c\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c0^0 & -s0^0 * c(-90)^0 & s0^0 * s(-90)^\circ & 0 * c0^0 \\ s0^0 & c0^0 * c(-90^0) & -c0^\circ * s(-90)^\circ & 0 * s0^0 \\ 0 & s(-90^0) & c(-90^0) & 0.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = T(z_1, d) * T(z_1, \theta) * T(x_2, a) * T(x_2, \alpha)$$

$$= \begin{bmatrix} c\theta_2 & -s\theta_2 * c\alpha_2 & s\theta_2 * s\alpha_2 & a_2 * c\theta_2 \\ s\theta_2 & c\theta_2 * c\alpha_2 & -c\theta_2 * s\alpha_2 & a_2 * s\theta_2 \\ 0 & s\alpha_2 & c\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c0^0 & -s0^0 * c0^0 & s0^0 * s0^o & 0 * c0^0 \\ s0^0 & c0^0 * c0^0 & -c0^o * s0^o & 0 * s0^0 \\ 0 & s0^0 & c0^0 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

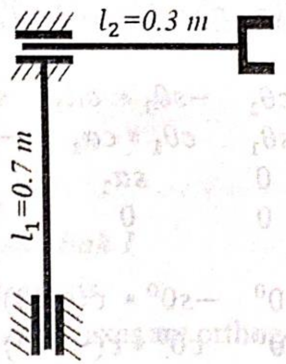
$$T_2^0 = [T_1^0]\,[T_2^1]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0.6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{T_2^0} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0.5} \\ \mathbf{0} & \mathbf{-1} & \mathbf{0} & \mathbf{0.6} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

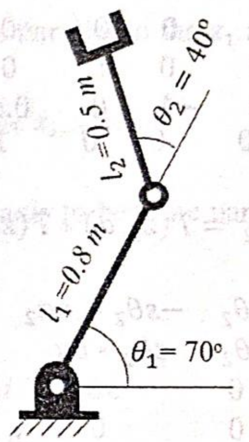
Questions for Practice

Q1. Determine the DH parameters of the following Prismatic-Prismatic planer two DOF serial robotic manipulator and T_2^0 transformation matrix.



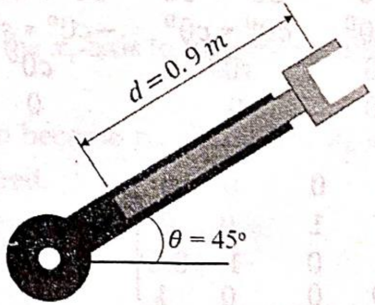
Ans: $T_2^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.3 \\ 0 & -1 & 0 & 0.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Q2. Determine the DH parameters of the following two DOF serial robotic manipulator and T_2^0 transformation matrix.



Ans: $T_2^0 = \begin{bmatrix} -0.342 & -0.9396 & 0 & 0.1026 \\ 0.9396 & -0.3420 & 0 & 1.2216 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Q3. Determine the DH parameters of the following two DOF serial robotic manipulator and T_2^0 transformation matrix.



Ans: $T_2^0 = \begin{bmatrix} 0.7071 & 0 & 0.7071 & 0.6363 \\ 0.7071 & 0 & -0.7071 & -0.6363 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

End of Unit