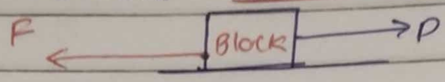
MODULE - 3

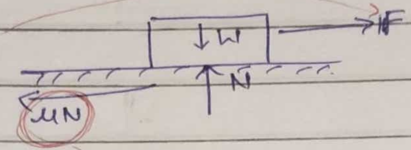
~~Fricition~~

Friction

Friction → opposes the relative motion

→ friction not have fixed direction





Limiting Friction Force :-

It is the maximum frictional force exerted at the time of impending motion.

i.e [when the motion is about to begin.]

$$\boxed{F_{limiting} = \mu_s N}$$

Limiting Friction is directly proportional to Normal reaction b/w surface in contact

and

coefficient of static friction μ_s

Kinetic Friction :-

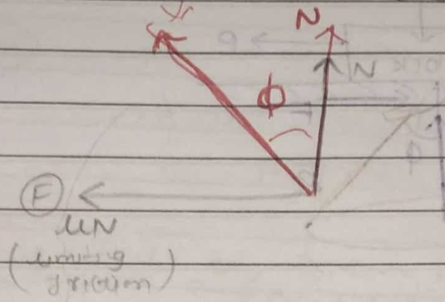
It is friction experience during the sliding b/w the surface in contact after attaining limiting friction under the action external force

kinetic friction is directly proportional to Normal reaction b/w surface in contact

$$\boxed{F_{kinetics} = \mu_K N}$$

angle of friction ϕ :-

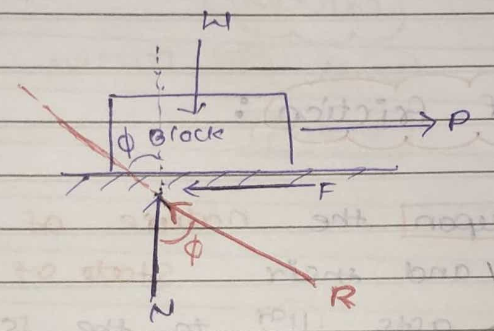
It is an angle b/w Normal reaction force. and Resultant of {Normal reaction to limiting frictional force}.

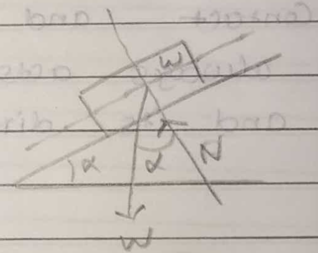


$$\tan\phi = \frac{F}{N} = \frac{\mu \cancel{N}}{\cancel{N}} = \mu$$

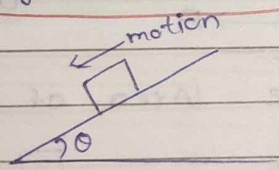
$$\tan\phi = \mu$$

$$\boxed{\phi = \tan^{-1}(\mu)}$$





angle of repose (θ) :

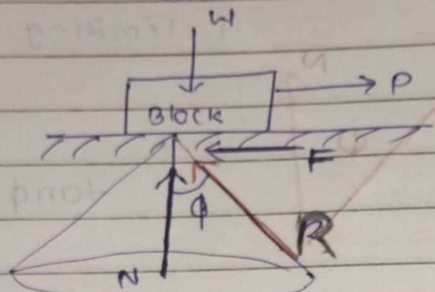


- when a body is about to slide down on inclined plane due to its own weight, then the angle made by the plane with the Horizontal is known as angle of repose.

Static friction → friction that exist b/w stationary object and surface on which it's rest

Cone of friction :

If the resultant (R) reaction is rotated about Normal reaction force, it will form a cone which is known as cone of friction.

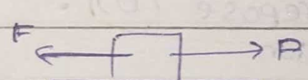


friction → opposes the relative motion
→ friction does not have a fixed direction

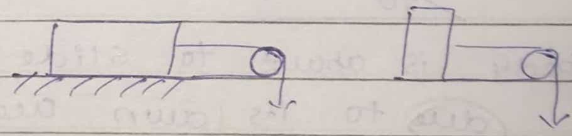
Law of friction :

(1) It depend upon the nature of two surface in contact and their state of roughess.

(2) It always acts $\parallel^{ar}$ to the surface in contact and its direction is opposite to the motion and attempted motion



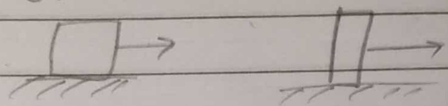
(3) It is independent to the Area of contact of two surface



(4) The magnitude of frictional force is directly proportional Normal to surface in contact

(1) depend → Nature of surface in contact

(2) Independence → Area of contact two surface



(3) direction → It alway act $\parallel$ar to surface of contact

→ its direction is opposite to the motion or attempted motion

(4) The magnitude of force is directly proportional to Normal reaction at the surface of contact

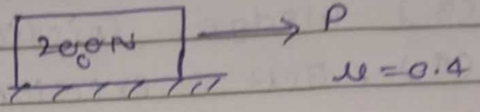
$$F = \mu N$$

$$\boxed{F \propto N}$$

Date______
Page______

problem No.: I

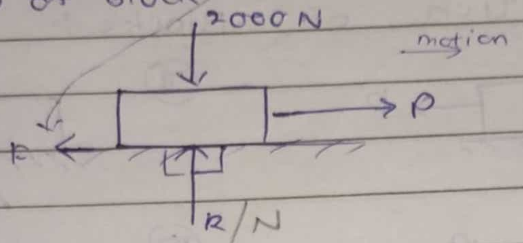
(1) A Body of weight 2000 N rest on a horizontal plane. If the coefficient of friction is 0.4, Find the horizontal force P required to move the body.



(R=(N))

⇒

FBD of Block



frictional force (F) = μR

$\Sigma f_y = 0 \quad +(\uparrow) = 0$

$R - 2000 = 0$

$\boxed{R = 2000N}$

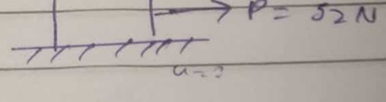
$F = \mu R/N$

$= 0.4 \times 2000 \qquad \boxed{F = 800N}$

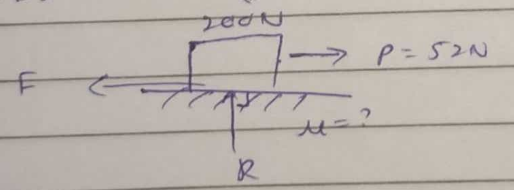
$(+\rightarrow)\ \Sigma f_x = 0 \qquad P - F = 0$

$P - 800 = 0 \qquad \boxed{P = 800N}$

(2) A Block of weight 200N is just on the point of moving horizontally by a force of 52N. what is Coefficient of friction



=> FBD of Block



$\Sigma f_x = 0 \qquad \Sigma F_y = 0$

$P - F = 0 \qquad R - 200 = 0$

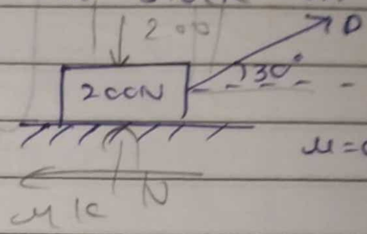
$\boxed{F = P} = 52N \qquad R = 200N$

$F = \mu\ 200 = 52$

$\frac{F}{\mu} = 200 \qquad \mu = \frac{52}{200}$

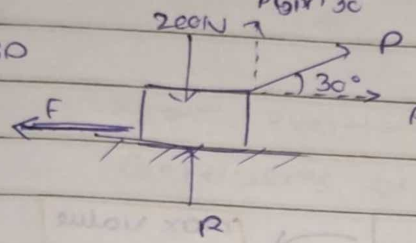
$\boxed{\mu = 0.26}$

(3) A block of weight 200N rest on a rough horizontal surface. Find the magnitude of the force to be applied at an ~~the~~ angle of 30 degree to the horizontal in order to move the block on the surface.



Assume $\mu = 0.30$

⇒ FBD



$\sum F_y = 0 \quad (+\uparrow)$

$$\boxed{-200 + \frac{P}{2} + R = 0} \quad \text{-(2)}$$

$$\boxed{\frac{P}{2} + R = 200}$$

$\sum F_x = 0 \quad (+\rightarrow)$

$P\cos 30° - F = 0$

$P\cos 30° - \mu R = 0$

$P\cos 30° - R(0.30) = 0$

$$\frac{P\sqrt{3}}{2} - (0.30)R = 0 \quad \text{-(1)}$$

$(0.866)P - (0.3)R = 0$

~~Add~~ eq (1) and (2)

Solve simultaneously in Calculator

$\cancel{\left(\frac{P}{2} + R = 200\right) \times \sqrt{3}}$

$\cancel{-\frac{P\sqrt{3}}{2} + (0.30)R = 0}$

$\cancel{\sqrt{3}R(0.30) = 200 \times \sqrt{3}}$

$\cancel{R = \frac{200}{0.30}}$?

$\cancel{3\overline{)2000}}$ 6, 18, 2

$$\boxed{P = 59.05N}$$

$R = 170.4$

$P - \mu N = 0$

$P = \mu N$

$52 = \mu N$

$\frac{52}{200}$

$F_y = R - 200N$

$N = 200$

$\boxed{\text{Types of fricition}}$

- $\boxed{\text{static friction}}$
- $\boxed{\text{Kinetic friction}}$
- $\boxed{\text{Rolling friction}}$

Static friction

Static friction → Jab motion start hone wala hai

Static friction → $\boxed{\text{limiting friction}}$ → $\boxed{\text{max value is fixed}}$

Static friction → not fixed value

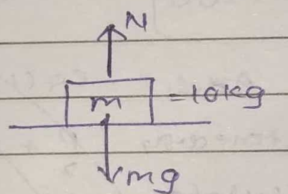
$\boxed{F_{limiting} = \mu_s N}$

μ_s → coefficient of static friction

$\mu = 0.2$

e.g

$F = \mu_s N$



$\Sigma F_y = 0$

$N = mg = 10 \times 10 = 100N$

$\boxed{N = 100N}$

$F = 0.2 \times 100$

$\boxed{F = 20 N}$ → limiting value of friction

Block 20 N force rok ne ke ghomta hai

or

resist the Block 20 N

Imp part

$\mu_s > \mu_k$ $\mu_s > \mu_k$ $\mu_s > \mu_k$

$\boxed{u_s > u_k}$

Coefficient of static friction

Coefficient of kinetic friction

agar question me given ho

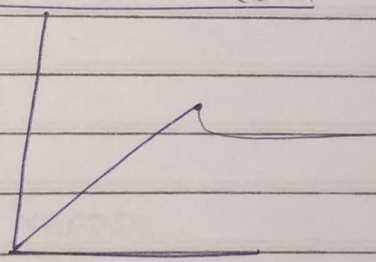
Coefficients of friction are 0.2 and 0.4

big wale value u_s hoge (0.4)

aur

small wale value u_k hoge (0.2)

- graphical Nature of friction

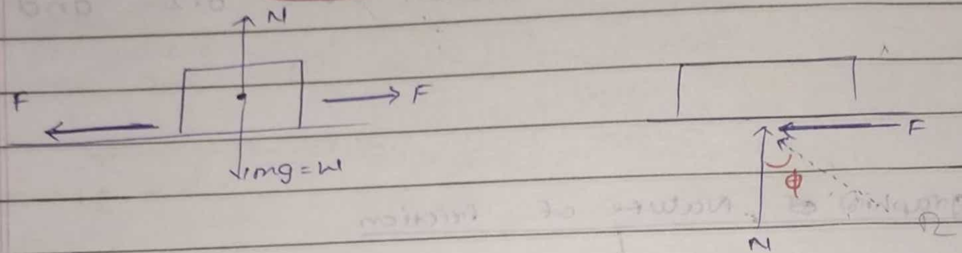


In friction → surface area does not matter
→ mass does not matter
→ angle is matter

Angle of Friction :

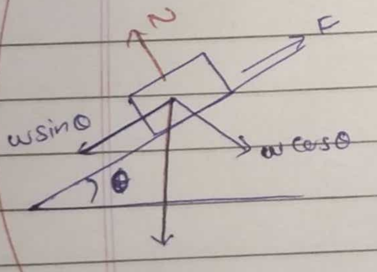
ϕ

It's the angle made by the resultant of the limiting force and the Normal reaction



angle of repose :

It is defines as the minimum angle of inclination of a plane with the horizontal For which a body kept on it will just slide down it without the application of external force



$\Sigma F_x = 0$

$F - w\sin\theta = 0$

$\mu_s N - w\sin\theta = 0$

$\mu_s N = w\sin\theta \quad \cdots (1)$

$\Sigma F_y = 0$

$N - w\cos\theta = 0$

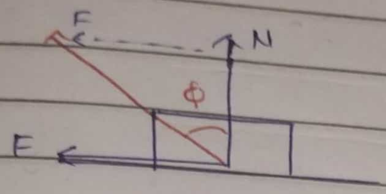
$N = w\cos\theta \quad \cdots (2)$

divide (1) and (2)

$\frac{\mu_s \cancel{N}}{\cancel{N}} = \frac{\cancel{w}\sin\theta}{\cancel{w}\cos\theta}$ → $\boxed{\tan\theta = \mu_s}$ ***

also find μ_s when angle of repose is given

Imp terms



$R = \sqrt{(F_{max})^2 + N^2}$

$R = \sqrt{(\mu_s N)^2 + N^2}$

$\tan\phi = \frac{opp}{Adj} = \frac{F}{N} = \frac{\mu_s \cancel{N}}{\cancel{N}}$

☆ ☆ ☆ $\boxed{\tan\phi = \mu_s}$

angle of friction

$\tan\phi = \mu_s$

$\tan\phi = \tan\theta$

$\boxed{\phi = \theta}$

angle of repose

$\tan\theta = \mu_s$

Numerical

(1) $F_k \leftarrow$ [4kg] $\rightarrow$ 10N   find F_k (friction) and aceleration (a)

$\mu_K = 0.2$

$\boxed{N = mg = 4 \times 10 = 40N}$

$F_k = \mu_k N$

$F_k = 0.2 \times 40 = 8N$

a = ?

$\cancel{F_k = ma}$

$\cancel{8 = 4 \times a}$

$\cancel{a = \frac{8}{2}}$

$F = ma$

$10 = 4a$ $\boxed{a = 2.5 m/s^2}$

8N ← • → 10N   • → Resultant forc 2N

10 − 8

$\Sigma F = ma$

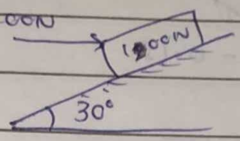
$10 - 8 = 4 \times a$

$\cancel{2} = 4 \times a$

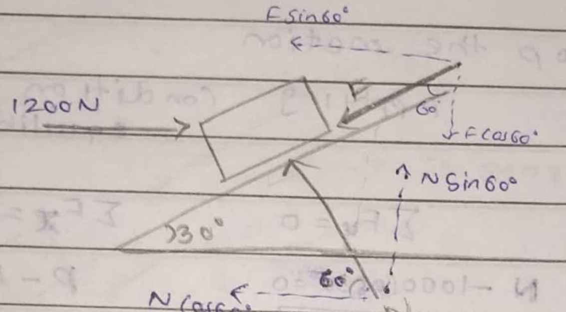
$\boxed{a = \frac{1}{2} = 0.5 m/s^2}$

(1) If a horizontal force of 1200N is applied horizontally on a block weighting 1000N then what will be direction of motion of the block

· Take $\mu = 0.3$



=>



$F = \mu \times N$

Apply condition of equilibrium on block

$\Sigma F_x = 0$, $(+ \rightarrow)$

$1200 - F\sin 60° - N\cos 60° = 0$

$1200 - F\frac{\sqrt{3}}{2} - \frac{N}{2} = 0$

$1200 - \mu N\frac{\sqrt{3}}{2} - \frac{N}{2} = 0$

$1200 - \frac{0.3 \times \sqrt{3} N}{2} - \frac{N}{2}$

$1200 - 0.25N - 0.5N$

$1200 - 0.75N = 0$

$N = \frac{1200}{0.7595}$

$\boxed{N = 1579.98}$

$\Sigma F_y = 0$ $(+\uparrow)$

$-F\cos 60° + N\sin 60° = 0$

$\frac{1}{2}F = \frac{N\sqrt{3}}{2}$

$\cancel{\boxed{\frac{F}{\sqrt{3}} = N}}$

$\cancel{F = \sqrt{3} \times 1579.98}$

$F = \mu \times 1579.36$

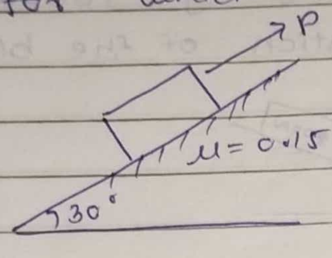
$F = 0.3 \times 1579.36$

$\boxed{F = 473.808\ N}$

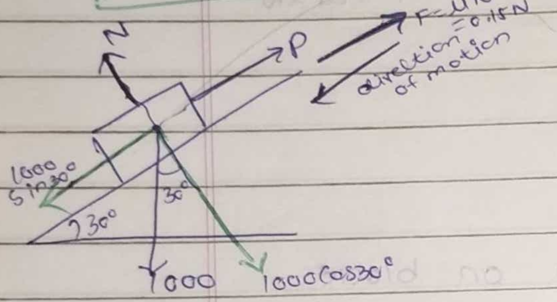
Since applied force $P = 1200N > F = 473.8\ N$,

Hence block will move in upward direction

(1) A Block of weight 1000N is keept on a raugh inclined surface. Find out range of P for which the block will be equilibrium.



Case (i) P → To stop the motion



Applying condition of equilibrium :

$\Sigma F_y = 0$

$N - 1000 \cos 30° = 0$

$\boxed{N = 866.02}$

$\Sigma F_x = 0$

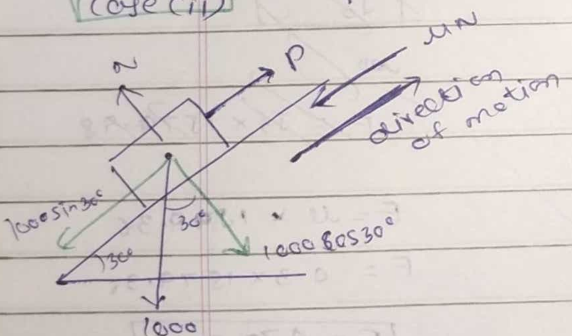
$P - 1000 \sin 30° + 0.15 N = 0$

$P = 1000 \sin 30° - 0.15 \times 866.02$

$P = 500 - 129.908$

$\boxed{P = 370.097 N}$

Case (ii) P → To start the motion



Applying condition of equilibrium :

$\Sigma F_y = 0$

$N = 1000 \cos 30°$

$\boxed{N = 866.02}$

$\Sigma F_x = 0$

$P - 1000 \sin 30° - 0.15 N = 0$

$P = 1000 \sin 30° + 0.15 \times 866.02$

$P = 500 + 129.908$

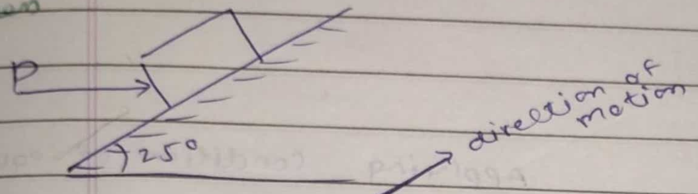
$\boxed{P = 629.908 N}$

Range of P for equilibrium

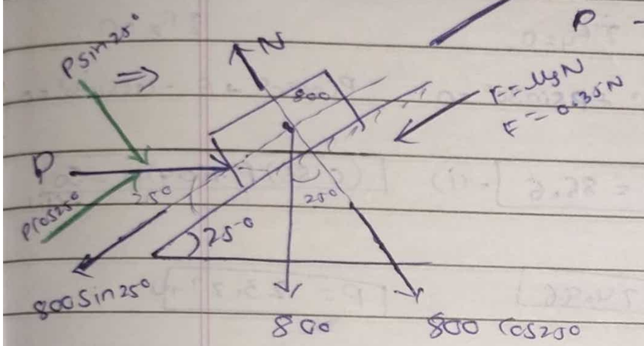
$370.097 < P < 629.908$

(2) A block of weight 800N is acted upon by a horizontal force P as shown in figure. If the coefficient of friction b/w the block and inclined surface are $\mu_s = 0.35$ and $\mu_k = 0.25$. determine the value of P for impending motion up the plane

do not take extra data given



P → To start the motion



Applying ~~equation~~ condition of equilibrium

$\Sigma F_y = 0$

$N - P\sin 25° - 800\cos 25° = 0$

$\boxed{N - P(0.42) = 725.04}$ --(1)

$N - P(0.42) = 725.04$

$\boxed{N = 1055.15}$

$\Sigma F_x = 0$

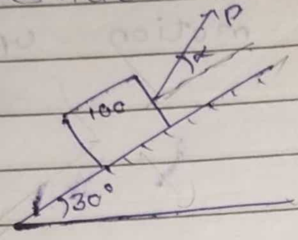
$P\cos 25° - 800\sin 25° - 0.35N = 0$

$P(0.9) - (0.35)N = 338.09$

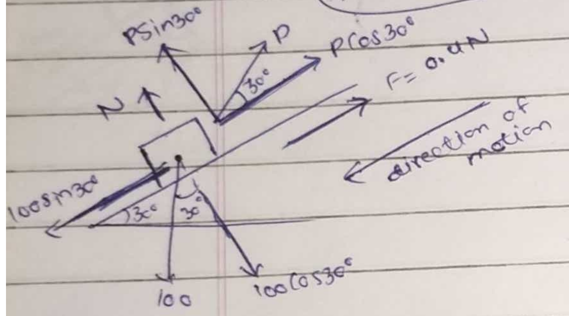
$(-0.35)N + P(0.9) = 338.09$

$\boxed{P = 785.99\ N}$

(4) A wooden block rest on a horizontal plane as shown in figure. determine the force 'P' required to (a) pull it (b) push it. Assume the weight of block as 100N and the coefficient of friction $\mu = 0.4$



→ (a) pull it



Applying condition of equilibrium

$\Sigma F_y = 0$

$N - 100\cos 30° + P\sin 30° = 0$

$\boxed{N + (0.5)P = 86.6}$ - (i)

$\boxed{N = 74.96}$

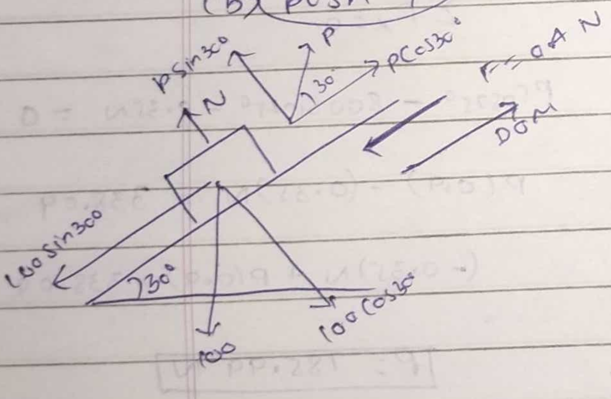
$\Sigma F_x = 0$

$P\cos 30° + F - 100\sin 30° = 0$

$\boxed{(0.86)P + (0.4)N = 50}$ - (ii)

$\boxed{P = 23.27\,N}$

(b) push it



Applying condition of equilibrium

$\Sigma F_y = 0$

$N - 100\cos 30° + P\sin 30° = 0$

$\boxed{N + (0.5)P = 86.6}$

$\boxed{N = 46.67}$

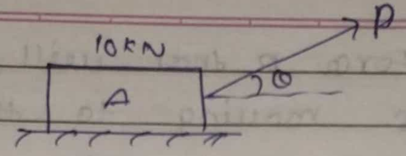
$\Sigma F_x = 0$

$P\cos 30° - F - 100\sin 30° = 0$

$\boxed{(0.86)P - (0.4)N = 50}$

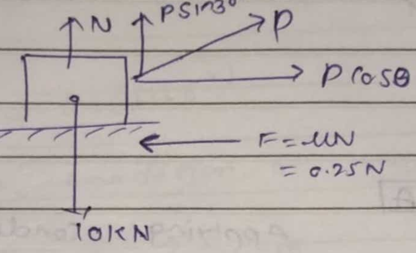
$\boxed{P = 79.84\,N}$

(1)



Find the $\boxed{\text{minimum}}$ force P required to move, the Block A weight 10KN ? $\mu = 0.25$ Also find the value of θ.

$\Rightarrow$



Applying condition of equilibrium

$\Sigma F_y = 0$

$\cancel{N = 10}$

$N + P\sin\theta = 10$

$\boxed{N = 10 - P\sin\theta}$

$\Sigma F_x = 0$

$P\cos\theta = 0.25 N$

$P\cos\theta = 0.25\,(10 - P\sin\theta)$

$P\cos\theta = 2.5 - 0.25 P\sin\theta$

$P(\cos\theta + 0.25\sin\theta) = 2.5$

$$P = \frac{2.5}{\cos\theta + 0.25\sin\theta}$$

$\cancel{(\cos\theta + 0.25\sin\theta)}$

To get minimum value of P

the denominator should be maximum (For P to be minimum)

$\frac{d}{d\theta}(\text{Dr}) = 0$

$\frac{d}{d\theta}(\cos\theta + 0.25\sin\theta) = 0$

$\boxed{-\sin\theta + 0.2\cos\theta = 0}$

$\frac{\sin\theta}{\cos\theta} = 0.25$

$\theta = \tan^{-1}(0.25)$

$\boxed{\theta = 14°}$

put

$$P = \frac{2.5}{\cos(14.036°) + 0.25\cos(14.036°)}$$

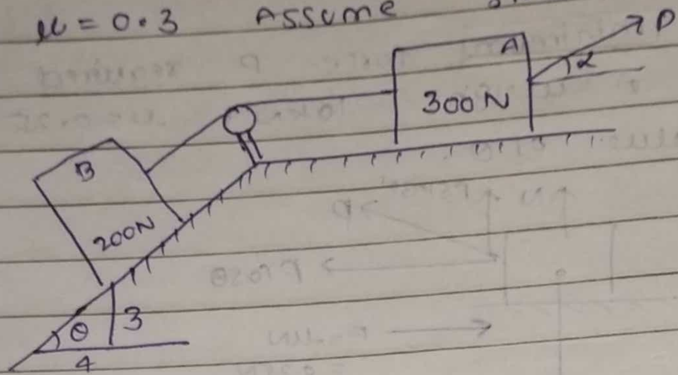
$$P = \frac{2.5}{1.21}$$

$P = 2.066\,N$

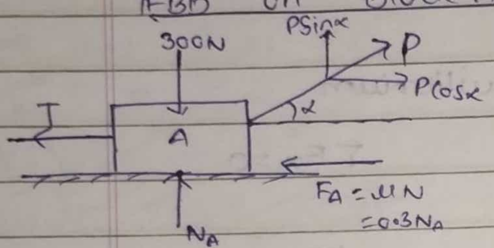
most Imp → Same of page 98

minimum force

(1) Find the least Force P that will just start the System of Blocks moving to the right. Take $\mu = 0.3$ Assume smooth pulley



FBD on Block A



Applying condition of equilibrium

$\Sigma F_y = 0$

$+N_A + 300 + P\sin\alpha = 0$

~~$N_A = 300$~~

$\boxed{P\sin\alpha + N_A = 300}$

$\boxed{N_A = 300 - P\sin\alpha}$

$\Sigma F_x = 0$

$-0.3N_A + P\cos\alpha - T = 0$

$\boxed{P\cos\alpha - 0.3N_A = T}$

$P\cos\alpha - T = 0.3N_A$

$P\cos\alpha - T = 0.3(300 - P\sin\alpha)$

$P\cos\alpha - T = 90 - 0.3P\sin\alpha$

$P(\cos\alpha + 0.3\sin\alpha) = 90 + T$

$$\boxed{P = \frac{90 + T}{(\cos\alpha + 0.3\sin\alpha)}} \quad \text{--- (1)}$$

put eq (2) in eq (1)

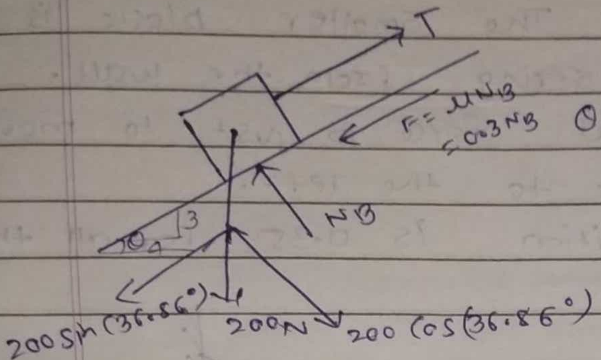
$$P = \frac{90 + 167.97}{(\cos\alpha + 0.3\sin\alpha)}$$

$$P = \frac{257.97}{(\cos\alpha + 0.3\sin\alpha)}$$

To Find the least value of force P

maximize the denominator

FBD of Block B



$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.86°$

Applying condition of equilibrium

$\Sigma F_x = 0$ ① $\qquad \Sigma F_y = 0$

$T - 0.3 N_B - 200 \sin(36.86)° = 0$

$N_B = 200 \cos(36.86°)$

$\boxed{N_B = 160 N}$

$T = 0.3 \times 160 + 200 \sin(36.86°)$

$T = 48 + 119.97$

$\boxed{T = 167.97}$ --- (3)

For that, differentiate the denomentor

$\frac{d}{dx}(D^r) = 0 \qquad \frac{d}{dx}(\cos\alpha + 0.3 \sin\alpha) = 0$

$-\sin\alpha + 0.3 \cos\alpha = 0$

put in eq(1)

$\sin x = 0.3 \cos x$

$\tan\alpha = 0.3$

$\alpha = \tan^{-1}(0.3)$

$\boxed{\alpha = 16.69°}$

$P = \frac{90 + 167.97}{\cos(16.69°) + 0.3 \sin(16.69°)}$

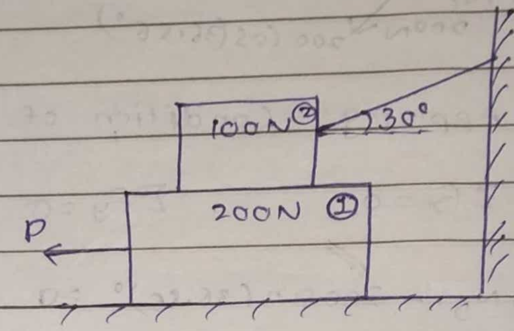
$= \frac{257.97}{1.044}$

$P = 247.097 \text{ N}$

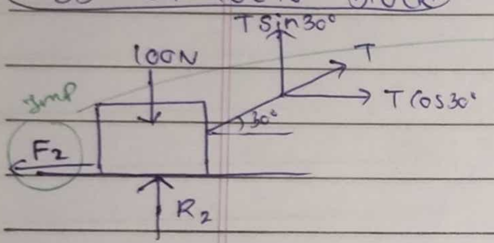
Page

Imp

(1) A Block of mass 200N resting on a horizontal Surface supports another block of 100N as shown in figure. The Smaller block is attach to the string from the wall. Find the horizontal force P just to move the 200N Block to the left. Coefficient of fricition is 0.35 for all the rubbing surface.

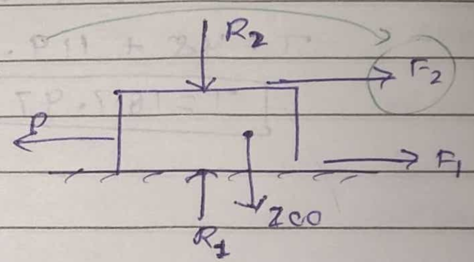


FBD of 100N Block



Both force apply equal and opposite force

FBD of 200N Block



$\Sigma F_x = 0$

$-F_2 + T\cos 30° = 0$

$\boxed{T\cos 30° = F_2}$

$T\cos 30° - \mu R_2 = 0$

$\boxed{T\cos 30° - (0.35)R_2 = 0}$ ---(i)

$\Sigma F_y = 0$

$\boxed{T\sin 30° + R_2 = 100}$ ---(ii)

$\boxed{T = 33.6N}$

$\boxed{R_2 = 83.18N}$

$F_2 = \mu R_2$

$F_2 = 0.35 \times 83.18$

$\boxed{F_2 = 29.113N}$

$\Sigma F_x = 0$

$F_2 + F_1 - P = 0$

$\boxed{P - F_1 = 29.11}$ ---(iii)

$\boxed{P - (0.3)R_1 = 29.11}$ ---(iii)

$\Sigma F_y = 0$

$-R_2 + R_1 = 200$

$R_1 = 283.18N$

put in eq (iii)

$P - 0.3(283.18) = 29.11$

$P = 29.11 + 84.954$

$\boxed{P = 143.174 N}$

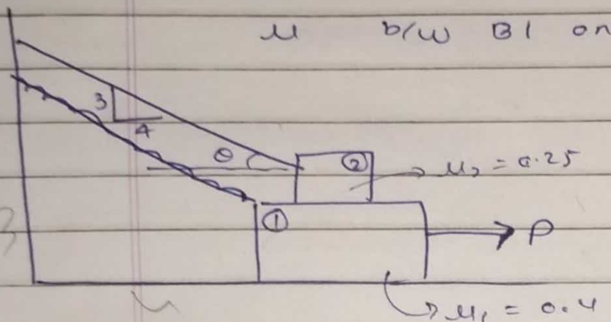
Imp

resource Book page-97

(2) A Block of weight $W_1 = 1290N$ rests on a horizontal surface and support another Block of weight $W_2 = 570N$ on top of it's as shown in fig. Block of weight W_2 is attached to a vertical wall by an inclined string AB. Find the force P applied to the lower Block, that will be necessory to cause the slipping to impend.

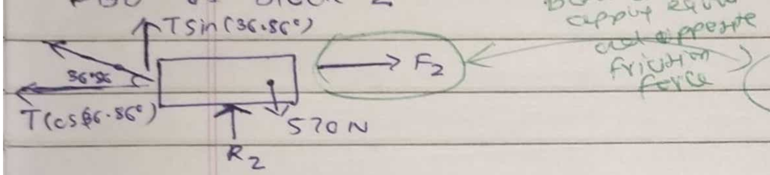
μ b/w B1 and B2 $= 0.25$

μ b/w B1 and Horizontal surface $= 0.4$

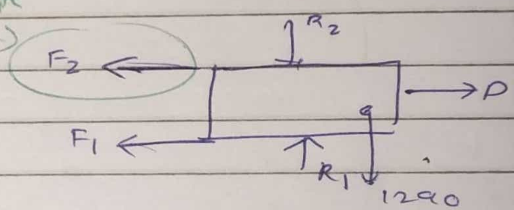


$\theta = \tan^{-1}(3/4) = 36.86°$

FBD of Block 2



FBD of Block 1



$\Sigma F_x = 0$

$F_2 = T\cos(36.86)°$

$\mu R_2 - T\cos(36.86°) = 0$

$\boxed{(0.25)R_2 - T(0.8) = 0}$ ---(1)

$\Sigma F_y = 0$

$T\sin(36.86°) + R_2 = 570$

$\boxed{R_2 + T(0.6) = 570}$ --(2)

$R_2 = 480\ N$

$T = 150N$

$F_2 = \mu_2 R_2 = 0.25 \times 480$

$F_2 = 120N$

$\Sigma F_x = 0$

$-F_2 - F_1 + P = 0$

$P - F_1 = 120$

$P - \mu_1 R_1 = 120$

$\boxed{P - (0.4)R_1 = 120}$

$P - (0.4) \times (1770) = 120$

$P - 708 = 120$

$P = 120 + 708$

$P = 828$

$\Sigma F_y = 0$

$R_1 - R_2 = 1290$

$R_1 = 1290 + R_2$

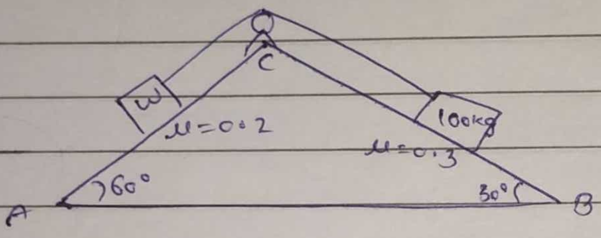
$R_1 = 1290 + 480$

$R_1 = 1770N$

Try ladder wala
and
wedge wala

page 97

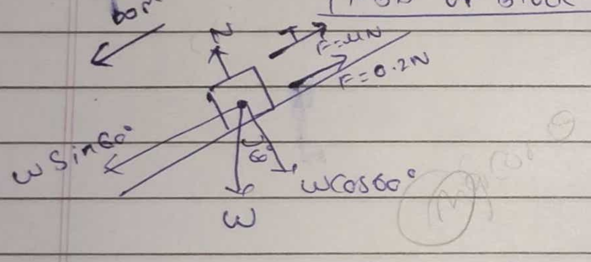
(3) Two inclined plane AC and BC inclined at 60° and 30° to the horizontal meet at a ridge C. A mass of 100 kg rests on the inclined plane BC and is tied to a rope which passes over smooth pulley at the ridge, the other end of the rope being connected to a Block of W kg mass resting on the plane AC shown in fig. ~~that~~. determine the least and greatest value of W for the the equilibrium of the whole system.



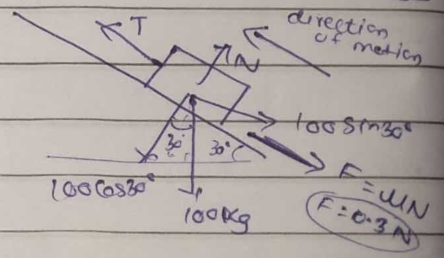
case(1) $\boxed{W > 100kg}$

∴ Block move down on inclined AC plane

$\boxed{\text{FBD of Block W kg}}$



$\boxed{\text{FBD of Block 100kg}}$



$\Sigma F_x = 0$ (→+ve)

$T + 0.2N - W\sin 60° = 0$

$\Sigma F_y = 0$ (↑+ve)

$\boxed{N = W\cos 60°}$

$T + 0.2(W\cos 60°) - W\sin 60° = 0$

$T = (0.86)W - (0.1)W$

$\boxed{T = (0.766)W}$

$\frac{75.98}{0.766} = W$

$\boxed{W = 99.19N}$

$\Sigma F_x = 0$

$0.3N + \frac{100}{2}^{5} = T$

$\boxed{T = 50 + 0.3N}$

$\Sigma F_y = 0$

$N = 100\cos 30°$

$\boxed{N = 86.6N}$

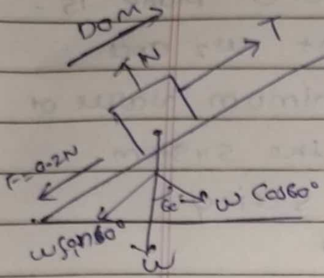
~~T = 50 + 0.3×86.6~~

$\boxed{T = 75.98}$

Case(ii) $\boxed{W < 100kg}$

Block move ~~down~~ upword on inclined AC plane

FBD of Block W kg



$\Sigma F_y = 0$

$\boxed{N = W\cos 60^\circ}$

$\Sigma F_x = 0$

$\boxed{T - 0.2N - W\sin 60^\circ = 0}$

$24.01 - 0.2(W\cos 60^\circ) - W\sin 60^\circ = 0$

$24.01 = W\,(0.2\cos 60^\circ + \sin 60^\circ)$

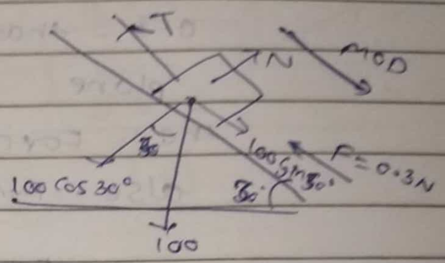
$24.01 = W\,(0.1 + 0.866)$

$= W\,(0.966)$

$\frac{24.01}{0.966} = W$

$\boxed{W = 24.8\ N}$

FBD of Block 100kg



$\Sigma F_x = 0$

$100\sin 30^\circ - F - T = 0$

$\frac{100}{2} - 0.3N = T$

$\boxed{50 - 0.3N = T}$

$\Sigma F_y = 0$

$N = 100\cos 30^\circ$

$\boxed{N = 86.6}$

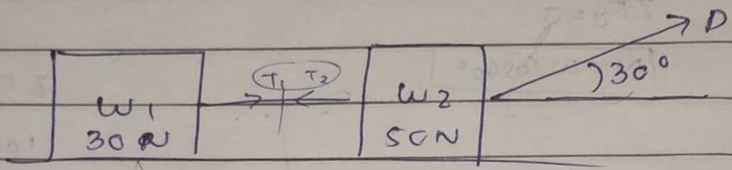
$50 - 25.98 = T$

$\boxed{T = 24.01}$

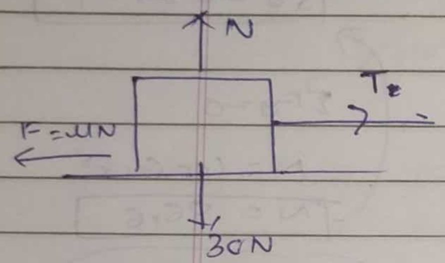
least value of $W = 24.8N$

greatest value of $W = 99.19N$

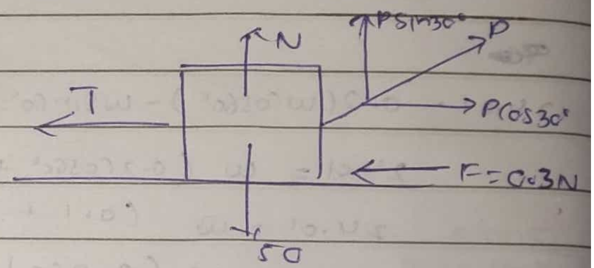
(4) In the figure below, the two Blocks ($W_1 = 30N$ and $W_2 = 50N$) are placed on rough Horizontal plane. Coefficient of fricition of b/w the Block of weight W_1 and plane is 0.3 that b/w Block of weight W_2 and plane is 0.2. Find the minimum value of the Force 'P' to just move the system. also Find the tension in the string



⇒ FBD Block W_1



FBD of Block W_2



$\Sigma F_x = 0$ $\Sigma F_y = 0$

$T_2 = \mu N$ $\boxed{N = 30}$

$\boxed{T_2 = 0.3N}$

$\boxed{T_2 = 9N}$

$\Sigma F_x = 0$

$\boxed{P\cos 30° = T + 0.2N}$

$\boxed{P\cos 30° - 0.2N = 9}$

$\boxed{(0.86)P - 0.2N = 9}$ — (1)

$\Sigma F_y = 0$

$N + \frac{P}{2} = 50$

$\boxed{0.5P + N = 50}$ — (2)

~~$\Sigma F_y = 0$~~

~~$\boxed{N = 50}$~~

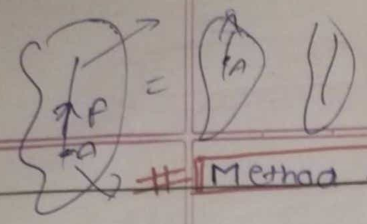
~~$P\cos 30° = T + 0.3 \times 50$~~

~~$= 9 + 15$~~

~~$P = \frac{9+15}{\cos 30°}$~~

solve simultaneously

$\boxed{P = 19.79}$ //



the point of application of fce
Can be transmite anywhere
belong the line of
action
but within

Method to Find ICR

Basic formulae :-

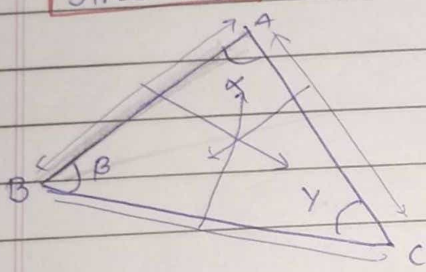
distance b/w fixed point and moving point

(1) $\boxed{V = r \times \omega}$ → angular velocity (rad/sec)

velocity

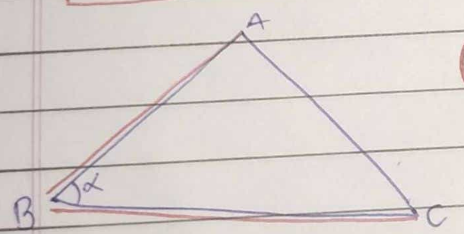
(2) $\boxed{\omega = \frac{2\pi N}{60}}$ → number of revolution (rpm)

*** (3) Sine rule :



$$\frac{AB}{\sin\gamma} = \frac{AC}{\sin\beta} = \frac{BC}{\sin\alpha}$$

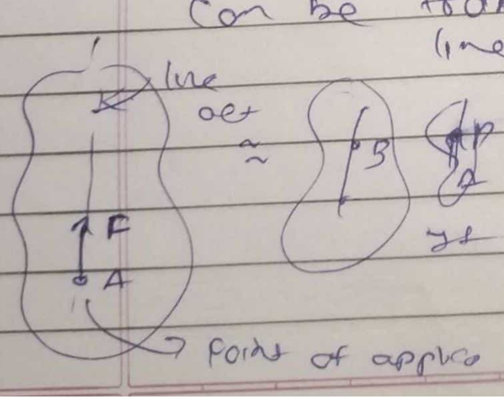
*** (4) Cosine rule :



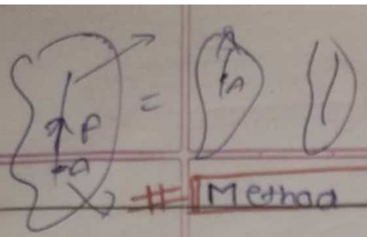
$$AC = \sqrt{(AB)^2 + (BC)^2 - 2(AB)\times(BC)\cos\alpha}$$

when two side and one angle are given

The point of application of a force can be transmitted anywhere along its line of action



but within a body
It only applicable for rigid body



the point of application of force can be transmitted anywhere belong the line of action but within

Method to Find ICR

Basic formulae :-

distance b/w fixed point and moving point

(1) $\boxed{V = r \times \omega}$

velocity

angular velocity (rad/sec)

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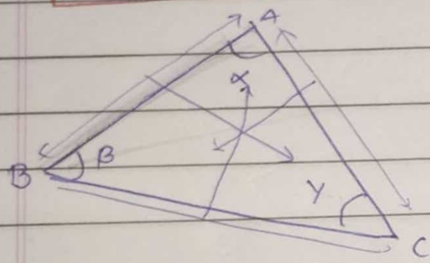
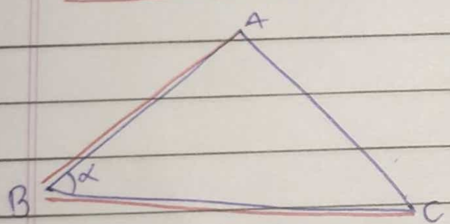
★★★ (3) Sine rule :



$$\frac{AB}{\sin\gamma} = \frac{AC}{\sin\beta} = \frac{BC}{\sin\alpha}$$

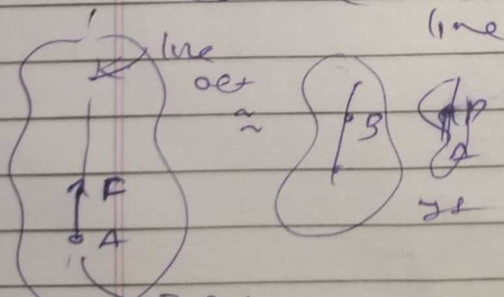
★★★ (4) Cosine rule :



$$AC = \sqrt{(AB)^2 + (BC)^2 - 2(AB)\times(BC)\cos\alpha}$$

When two side and one angle are given

The point of application of a force can be transmitted anywhere along its line of action



line of act

but within a body

It only applicable for rigid body

Point of applica

is known as centre of rotation

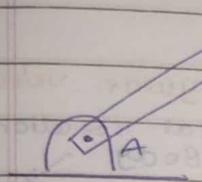
An imaginary instantaneous point about which a link is assumed to rotate.

(ICR) Instantaneous Centre of rotation :

- It is fixed point i.e (zero velocity point)
- It is an imaginary point.
- It may be lies within the body or outside the body
- It is a point of pure rotation

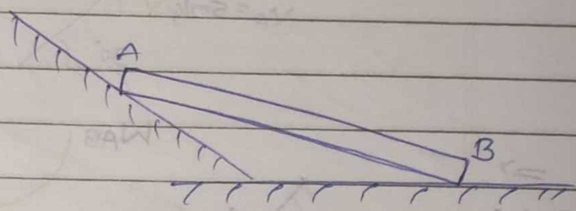
Type of link :

(1) Rotating link (fixed link)



Note : ICR lies inside the body

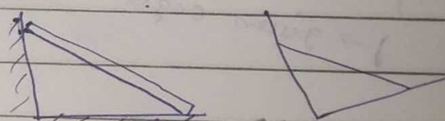
(2) Sliding link



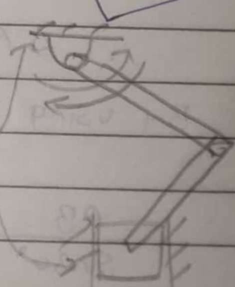
Note : ICR lies outside ~~the body~~

method of Find ICR

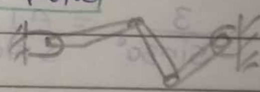
(1) M-1 : when body slides on two surface



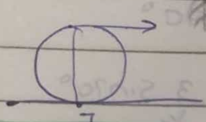
(2) M-2 : when one part of body slide and another part of body rotates about a hinge point



(3) M-3 : when two link of the system rotates about seperate hinge point



(4) M-4 : when body rolls on Fixed surface



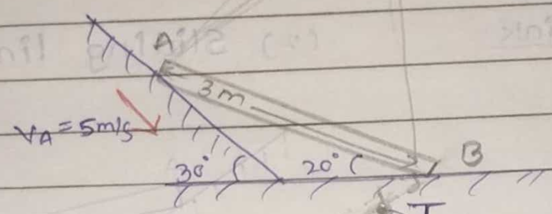
(5) M-5 : when body lies b/w two moving surface

Notes: Number of bodies = Number of ICR = number of angular velocity

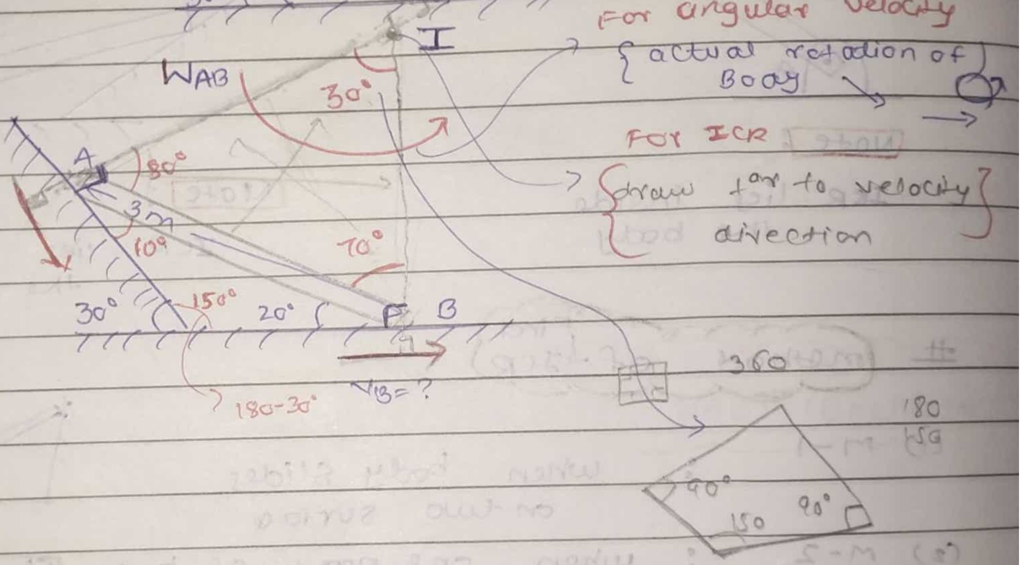
Types of problem:

(1) One sliding link

Imp ★★★ (Q1) Rod AB of length 3m is kept on smooth planes as shown in figure. The velocity of the end A is 5 m/s along the inclined. Locate the ICR and find the velocity of the end B.



Sol :-



) → determine angle

) → given angle

ΔAIB

By using sine rule

$$\frac{AB}{\sin 30^\circ} = \frac{AI}{\sin 70^\circ} = \frac{BI}{\sin 80^\circ}$$

$$\frac{3}{\sin 30^\circ} = \frac{AI}{\sin 70^\circ} \qquad \frac{3}{\sin 30^\circ} = \frac{BI}{\sin 80^\circ}$$

$$AI = \frac{3 \sin 70^\circ}{1/2} \qquad \sin 80^\circ\, 6 = BI$$

$$\boxed{AI = 5.638\,m} \qquad \boxed{BI = 5.9\,m}$$

$V = r \times \omega$

(a) $\boxed{\text{link } AB}$

Point I is on ICR

$V_A = AI \times \omega_{AB}$

$5 m/s = 5.638 \times \omega_{AB}$

$\boxed{\omega_{AB} = 0.886}$ rad/s

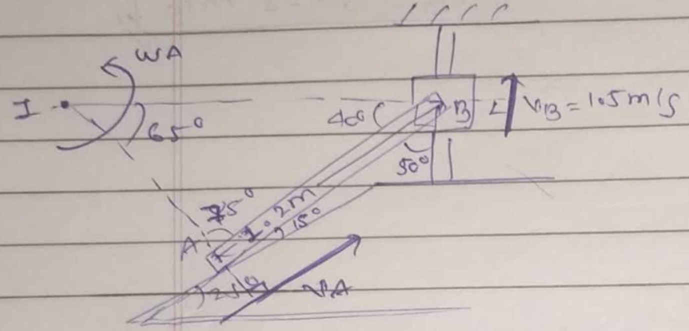
$V_B = BI \times \omega_{AB}$

$= 5.9 \times 0.886$

$\boxed{V_B = 5.28 \text{ m/s}}$

m-5
page -186

(4) Figure shows a collar B which move upwards with a constant velocity of 1.5 m/s. At instant when $\theta = 50°$ determine
(i) the angular velocity of rod AB which is pinned at B and freely resting at A against 25° slopping ground.
(ii) the velocity of end A of the rod.



$$\begin{array}{r} 40 \\ +15 \\ \hline 55 \end{array} \qquad \begin{array}{r} 75 \\ 15 \\ \hline 90 \end{array}$$

ΔAIB

By using sine rule

$$\frac{AB}{\sin 65°} = \frac{AI}{\sin 40°} = \frac{BI}{\sin 75°}$$

$$\frac{1.2}{\sin 65°} =$$

$$1.32 = \frac{AI}{\sin 40°}$$

$AI = 0.84$

$BI = 1.275$

$\boxed{\text{link AB}}$

$V_A = AI \times \omega_{AB}$

$V_B = BI \times \omega_{AB}$

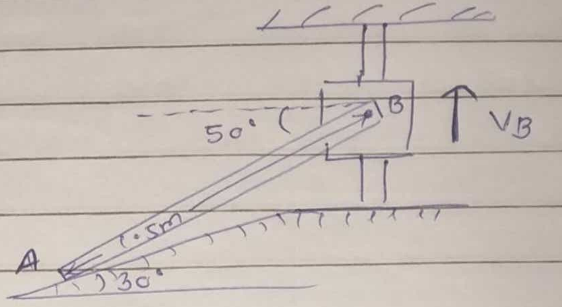
$1.5 = 1.275 \times \omega_{AB}$

$\omega_{AB} = 1.176$

$= 0.84 \times 1.176$

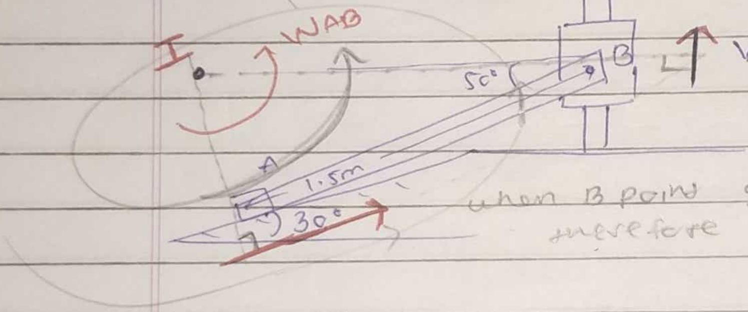
$\boxed{V_A = 0.988 \text{ m/s}}$

(2) End B move up with constant velocity of 2m/s. Find out the angular velocity of rod AB and velocity of end A. where the length of rod AB = 1.5m

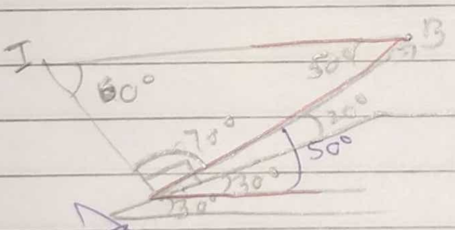


Sol:

determined By How body actually rotating



when B point goes on up direction therefore velocity A direction shown in diagram



ΔAIB

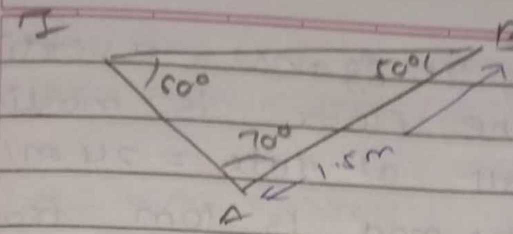
By using sine rule

$$\frac{AB}{\sin 40^\circ} = \frac{AI}{\sin 50} = \frac{BI}{\sin 90^\circ}$$

$$\frac{AB}{\sin 40^\circ} = \frac{BI}{(1)} \qquad \frac{AI}{\sin 50} = \frac{BI}{(1)}$$

$$\frac{1.5}{0.642} = BI \qquad AI = 2.336 \times \sin 50$$

$$\boxed{BI = 2.336 \text{ m}} \qquad \boxed{AI = 1.789 \text{ m}}$$



ΔAIB

BI using sine rule

$$\frac{AB}{\sin 60°} = \frac{AI}{\sin 50°} = \frac{BI}{\sin 70°}$$

$$\boxed{\frac{1.5 \times 2}{\sqrt{3}} = \frac{AI}{\sin 50}}$$

$$\frac{1.5 \times 2}{\sqrt{3}} = \frac{BI}{\sin 70°}$$

$$\boxed{AI = 1.732 \times \sin 50}$$

$$\boxed{BI = 1.732 \times \sin 70°}$$

$$\boxed{BI = 1.627\ m}$$

$$\boxed{AI = 1.326\ m}$$

link AB

point I is on ICR

$$V_B = BI \times W_{AB}$$

$$2 = 1.627 \times W_{AB}$$

$$\frac{2}{1.627} = W_{AB}$$

$$W_{AB} = 1.229\ rad/s$$

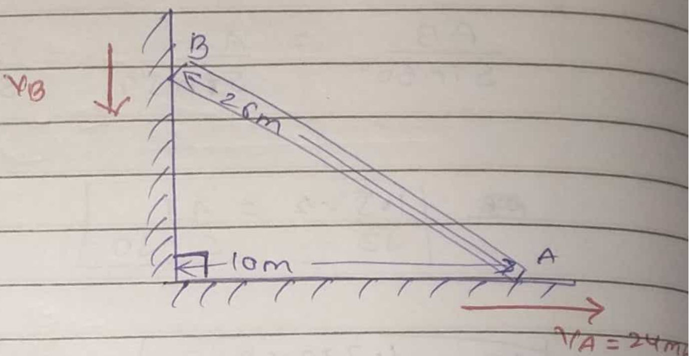
$$V_A = AI \times W_{AB}$$

$$= 1.326 \times 1.229$$

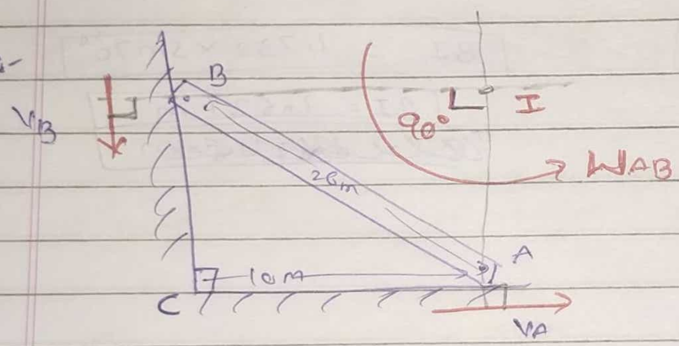
$$\boxed{V_A = 1.629\ rad/s}$$

(3) A rod AB 26m long leans against a vertical wall. The end A on the floor is moving away from the wall at rate = 24 m/s. When the end A of the rod is 10m from the wall, determine the velocity of end B sliding down vertically and the angular velocity of the rod AB.

=>



Sol :-



Hint :- only one angle is given therefore we used cosine rule

$\triangle ABC$

By using pythagoras theorem

$AB^2 = AC^2 + BC^2$

$BC^2 = AB^2 - AC^2$

$= (26)^2 - (10)^2$

$= 676 - 100$

$BC = \sqrt{576}$

$\boxed{BC = 24\,m}$

$\boxed{AC = BI = 10m}$

$\boxed{BC = AI = 24m}$

link AB

point I is an ICR

$V_A = AI \times \omega_{AB}$

$24 = 24\,\omega_{AB}$

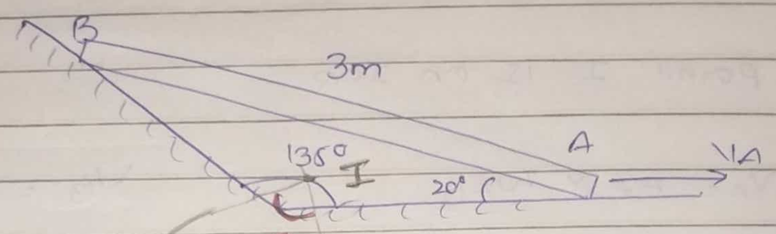
$\boxed{\omega_{AB} = 1\ rad/s}$

$V_B = BI \times \omega_{AB}$

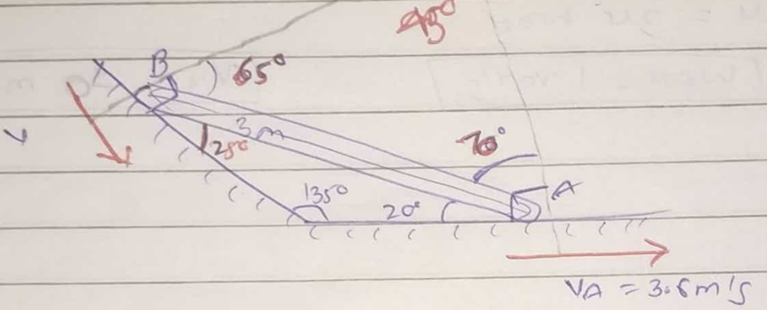
$\boxed{V_B = 10\ m/s}$

Imp

(4) A Bar, 3m long slides down the plane shown in fig. The velocity of end A is 3.6 m/s to the right. determine the angular velocity of AB and velocity of end B the instant shown



sol =>



ΔAIB

By using sine rule

$$\frac{AB}{\sin 45^\circ} = \frac{AI}{\sin 65^\circ} = \frac{BI}{\sin 70^\circ}$$

$$\frac{AB}{\frac{1}{\sqrt{2}}} = \frac{BI}{\sin 70^\circ} \qquad \frac{AB}{1/\sqrt{2}} = \frac{AI}{\sin 65^\circ}$$

$$3\sqrt{2}\sin 70^\circ = BI \qquad 3\sqrt{2}\sin 65 = AI$$

$$\boxed{BI = 3.9867m} \qquad \boxed{AI = 3.845\ m}$$

~~Consider~~ link AB

point I is on ICR

$v_A = AI\ \omega_{AB}$

$3.6 = 3.845\ \omega_{AB}$

$\boxed{\omega_{AB} = 0.936 \text{ rad/s}}$

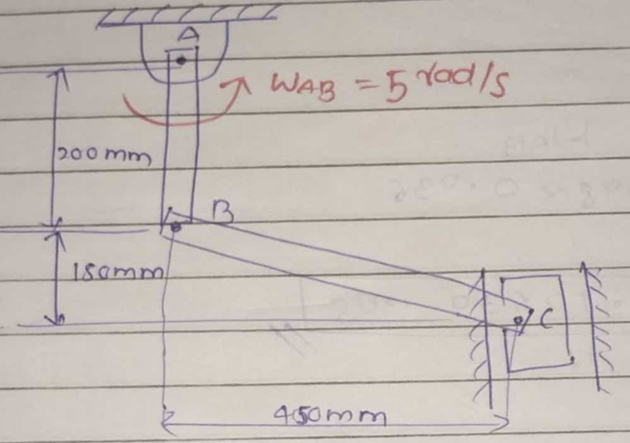
$v_B = BI\ \omega_{AB}$

$= 3.98 \times 0.936$

$\boxed{v_B = 3.72539 \text{ m/s}}$ //

(2) One sliding link and one rotating link

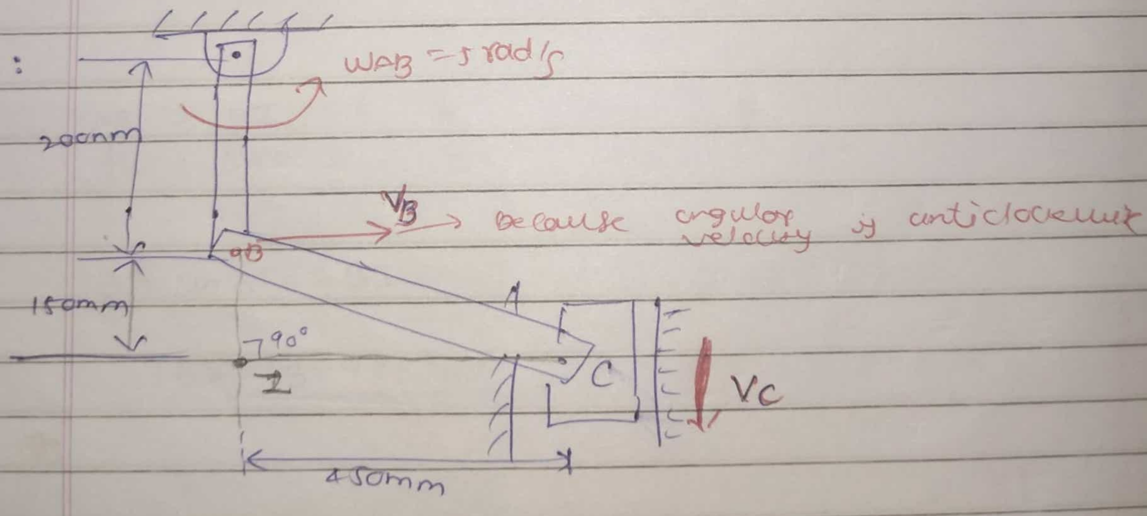
Q(1) In the mechanism shown the angular velocity of link AB is 5 rad/sec anticlockwise. At the instant shown, determine the angular velocity of link BC and velocity of piston C.

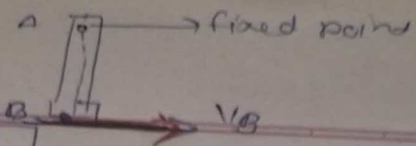


Note :-

no. of Bodies = no. of ICR = no. of angular velocity

Sol :





(a) Link AB — point A is on ICR $\boxed{V_A = 0}$

$V_B = AB \times \omega_{AB}$

$V_B = 200 \times 5$

$\boxed{V_B = 1000 \text{ mm/sec}}$

(b) Link BC

point I is an ICR

$\boxed{BI = 150 \text{ mm}}$

$\boxed{CI = 450 \text{ mm}}$

$V_B = BI \times \omega_{BC}$

$100\cancel{0} = 15\cancel{0}\, \omega_{BC}$

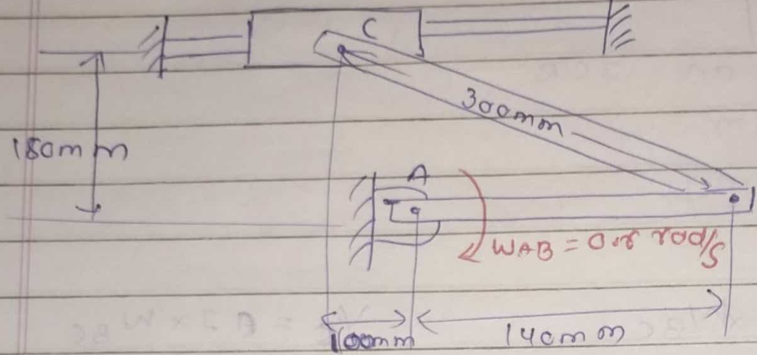
$\boxed{\omega_{BC} = 6.67 \text{ rad/s}}$

$V_C = CI \times \omega_{BC}$

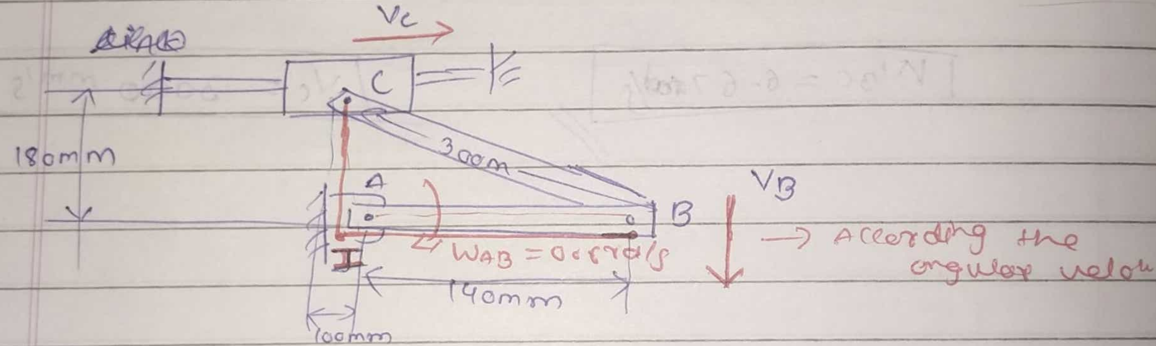
$= 450 \times 6.67$

$\boxed{V_C = 3000 \text{ mm/s}}$

(3) In figures collar C slides on a horizontal rod. In the position shown rod AB is horizontal and has angular velocity of 0.6 rad/sec clockwise. Determine angular velocity of link BC and velocity of collar C



Sol..



Link AB

point A is on ICR

$\boxed{V_A = 0}$

$V_B = AB \times W_{AB}$

$V_B = 140mm \times 0.6$

$\boxed{V_B = 84 \text{ mm/s}}$

Link BC

Point I is an ICR

$V_B = BI \times W_{BC}$

$84 = 240 \times W_{BC}$

$W_{BC} = \frac{84}{240}$

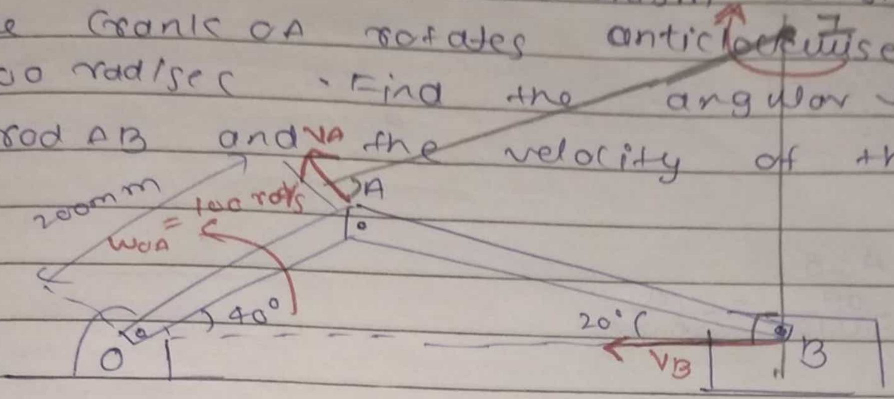
$\boxed{W_{BC} = 0.35 \text{ rad/s}}$

$V_C = CI \times W$

$V_C = 180 \times 0.35$

$\boxed{V_C = 63 \text{ mm/sec}}$

(4) A slider crank mechanism is shown in figure. The crank OA rotates anticlockwise at 100 rad/sec. Find the angular velocity rod AB and the velocity of the slider at B



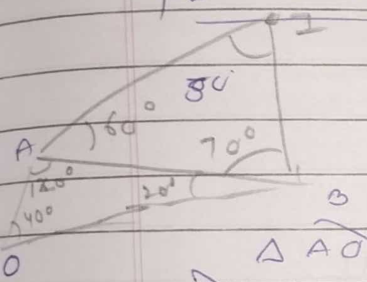
① Link OA

point O is on ICR $\boxed{V_O = 0}$

$V_A = OA \; W_{OA}$

$= 200 \times 100$

$\boxed{V_A = 20000 \text{ mm/s}}$



ΔAOB

~~$\frac{AB}{\sin 50} = \frac{OB}{\sin 60°} = \frac{OA}{\sin 70°}$~~

~~$\frac{AB}{\sin 50} = \frac{OB}{1/2}$~~

~~$\frac{AB}{\sin 50} = 2$~~

Link AB

point I is on ICR

$V_A = AI \times W_{AB}$

$\frac{20000}{AI} = W_{AB}$

$W_{AB} = \underline{20000}$

~~ΔACB~~

~~By using sine rule~~

~~$\frac{AB}{\sin 40} = \frac{AO}{\sin 20°} = \frac{BO}{\sin 60°}$~~

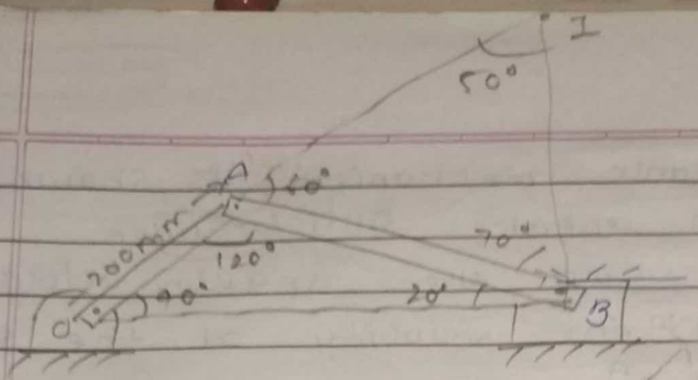
~~$AO = \frac{BC \times \sin 20}{\sin 60°}$~~

OR

$\frac{\sin 60°}{\sin 20°} 200 = BO$

$\frac{173.20}{\sin 20} = BO$ (0.342)

$5064.32 = BO$



ΔAOB

$$\frac{AB}{\sin 40°} = \frac{OB}{\sin 120} = \frac{OA}{\sin 20°}$$

$$\frac{OB}{\sin 120°} = \frac{200}{\sin 20°} \qquad \frac{AB}{\sin 40} = \frac{200}{\sin 20}$$

$$OB = \frac{173.32}{\sin 20°} \qquad AB = \frac{128.55}{\sin 20}$$

$$\boxed{OB = 506.41 m} \qquad \boxed{AB = 375.877 m}$$

ΔABI

$$\frac{AB}{\sin 50°} = \frac{AI}{\sin 70°} = \frac{BI}{\sin 60°}$$

$$\frac{AB}{\sin 50°} = \frac{AI}{\sin 70°} \qquad \frac{AB}{\sin 50°} = \frac{BI}{\sin 60°}$$

$$\frac{375.87 \times \sin 70°}{\sin 50} = AI \qquad \frac{375.87 \times \sin 60°}{\sin 50°} = BI$$

$$AI = \frac{353.20}{\sin 50} \qquad \frac{325.55}{\sin 50} = BI$$

~~AI = 424.929 mm~~ $\qquad$ $\boxed{BI = 424.92 mm}$

$\boxed{AI = 461.08 m}$

Link AB

$V_A = AI \; W_{AB}$ $\qquad$ $V_B = BI \; W_{AB}$

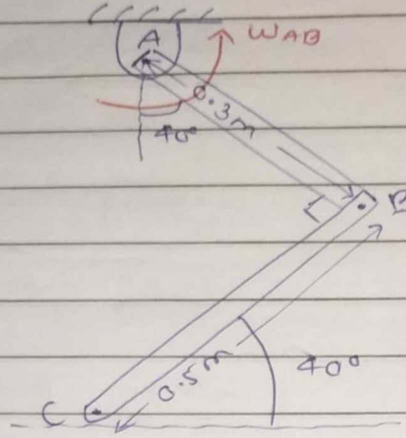
$20000 = 461.08 \; W_{AB}$

$\boxed{W_{AB} = 43.37 \text{ rad/s}}$ $\qquad$ $V_B = 424.92 \times 43.37$

$\boxed{V_B = 18431.50 \text{ mm/s}}$

Imp

(2) A rod AB has an angular velocity 2 rad/sec counter clockwise as shown. End C of rod BC is free to move on a horizontal surface. Determine (i) angular velocity of BC and (ii) velocity of C



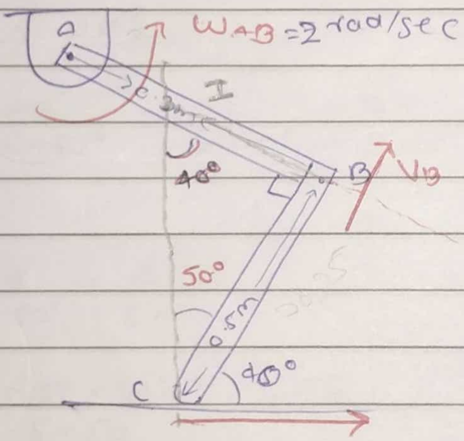
~~ΔCBI~~

~~$V_B = BI \times W_{BC}$~~

~~$0.6 = BI \times W_{BC}$~~

~~$V_C = CI \times W_{BC}$ ---(1)~~

Sol: =>



By using sine rule

$$\frac{CB}{\sin 40°} = \frac{BI}{\sin 50°} = \frac{CI}{\sin 90°}$$

$\boxed{\frac{0.5}{\sin 40°} = CI}$ $\qquad$ $\boxed{\frac{BI}{\sin 50} = CI}$

$\boxed{CI = 0.777 m}$ $\qquad$ $BI = 0.77 \times \sin 50$

$\boxed{BI = 0.598 m}$

link AB

Point A is on ICR ... {Fixed link}

$\boxed{V_A = 0}$

$V_B = AB \times W_{AB}$

$V_B = 0.3 \times 2$

$\boxed{V_B = 0.6 \ m/s}$

link BC

$V_B = BI \times W_{BC}$

$0.6 = 0.598 \times W_{BC}$

$0.6 = 0.6 \ W_{BC}$

$\boxed{W_{BC} = 1 \ rad/s}$

$V_C = CI \times W_{BC}$

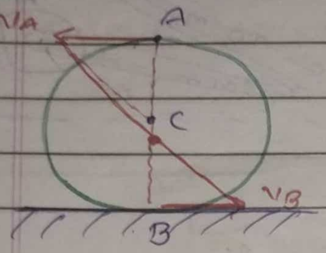
$V_C = 0.777 \times 1$

$V_C = 0.777 \ m/s$

③ Circular Body

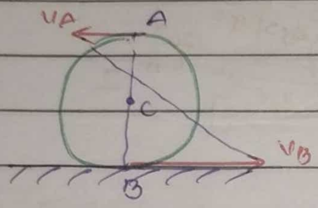
Case(1) Circular Body moving with slipping

Note :- ICR is lies within the Body



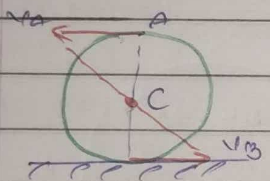
$V_A > V_B$

↳ ICR lies below the circle of centre



$V_A < V_B$

↳ ICR lies above the centre of circle

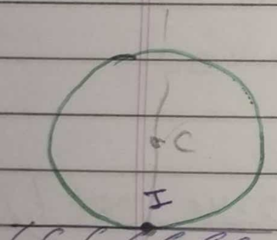


$V_A = V_B$

↳ ICR lies on centre of circle

perfectly ~~rotating~~ or

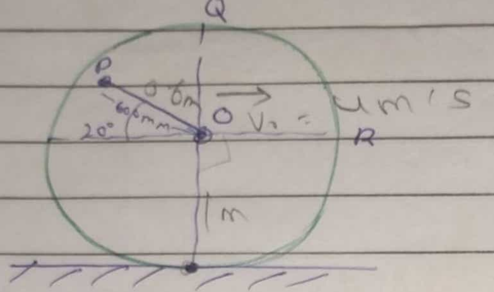
Case 2 Circular Body moving without slipping



Note: ICR lies in point of contact

Date ____
Page ____

(Q1) A wheel of 2m diameter rolls without slipping on a flat surface. The centre of the wheel is moving with a velocity 4m/s towards the right. determine the angular velocity of the wheel and velocity of point P, Q and R on the wheel

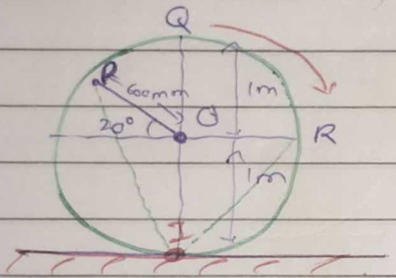


$\omega = ?$

$V_P \quad V_Q \quad V_R$

⇒ given $V_0 = 4m/s$, $D = 2m$ radius $(r) = 1$

∴ point of contact is on ICR



distance b/w fixed and moving point

$V_O = IO \times W$

$4 = 1 \times W$

$\boxed{W = 4 \ rad/s}$

hypotenuse theorem

$IR^2 = CR^2 + (OI)^2$

$= 1+1$

$IR = \sqrt{2}$

$V_Q = IQ \ W$

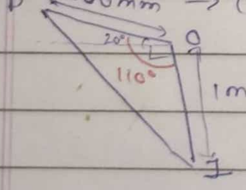
$V_Q = 2 \times 4$

$\boxed{V_Q = 8 \ m/s}$

$V_R = IR \times W$

$= \sqrt{2} \times 4$

$\boxed{V_R = 5.656 \ m/s}$



600mm = 0.6m → convert into m → 0.6m

By using cosine rule

$PI = \sqrt{(0.6)^2 + (1)^2 - 2 \times 0.6 \times 1 \times \cos 110°}$

$\boxed{PI = 1.3305 m}$

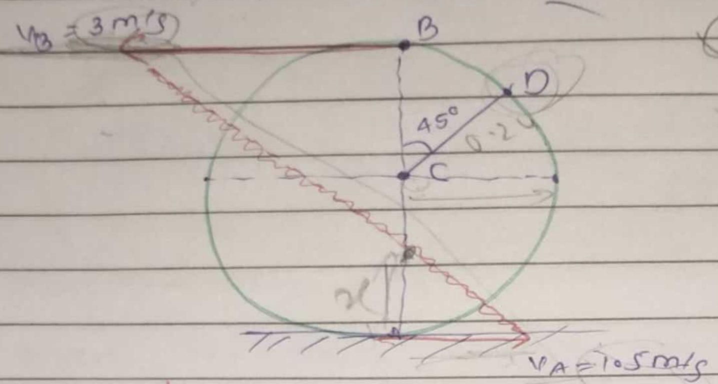
$V_P = PI \times W$

$V_P = 1.3305 \times 4$

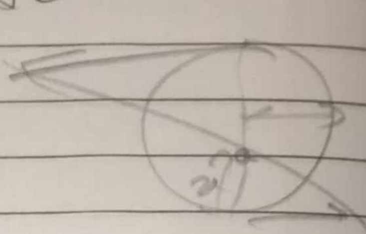
$\boxed{V_P = 5.322 \ m/s}$

Imp ★★★

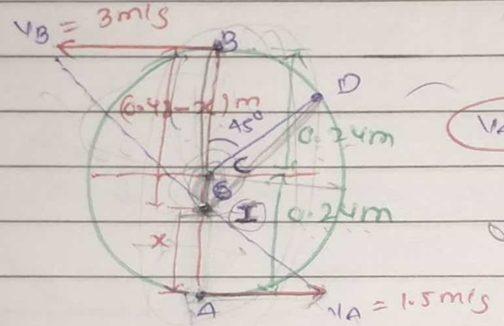
(Q2) due to slipping point A and B on the rim of the disk have the velocity shown in figure. The velocities of the center point C and point D on the rim at this instant. Take radius of disk 0.24m



$v_C = ?$ $v_D = ?$



Sol^n ⟹



$v_A < v_B$ → { ICR lies below the centre of circle }

0.48 − 0.16 = 0.32

0.24 − 0.16 = 0.08

$v_A = AI \times w$

$\boxed{1.5 = x \times w}$ --(1)

$v_B = BI \times w$

$v_B = (0.48 - x) \times w$

$\boxed{3 = (0.48 - x) \times w}$ --(2)

dividing eq(1) and eq(2)

$$\frac{3}{1.5} = \frac{(0.48 - x) \times \cancel{w}}{x \times \cancel{w}}$$

$3x = 0.72 - 1.5x$

$4.5x = 0.72$

$\boxed{x = 0.16 m}$

put the value of x in eq(1)

$1.5 = 0.16 w$

$\boxed{w = 9.375 \ rad/s}$

$v_C = CI \times w$

$v_C = 0.08 \times 9.375$

{CA = CI + IA}

{CI = CA − IA}

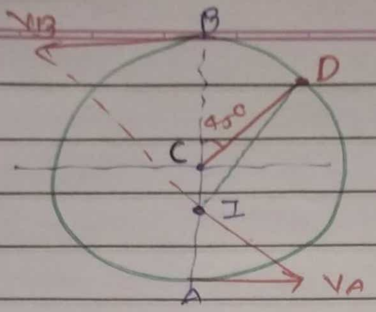
CI = 0.24 − x

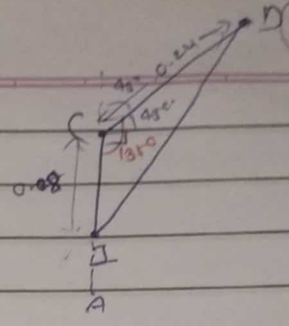
0.24 − 0.16

= 0.08

$\boxed{v_C = 0.75 m/s}$

$v_C = 0.08 \times 9.375$





$\boxed{V_D = DI \times W}$ ... (3)

By using cosine rule

$DI = \sqrt{(0.24)^2 + (0.08)^2 - 2 \times 0.08 \times 0.24 \cos 135^0}$

$\boxed{DI = 0.301 m}$

$V_C = CI \times \omega$

Put DI value in equation (3)

$v_c =$

$V_D = 0.301 \times 9.3075$

$\boxed{V_D = 2.81 m/s}$

$V_A = AI \times \omega = 1.5 = x \times \omega$

$V_B = BI \times \omega \quad (0.48 - x) \times \omega = 3$

$\frac{1.5}{3} = \frac{x \cancel{\times \omega}}{0.48 - x \cancel{\times \omega}}$

0.16
0.32

$1.5(0.48 - x) = 3x$

$0.72 - 1.5x = 3x$

$0.72 = 4.5x$

$x = 0.16m$

$1.5 = 0.16 \times \omega$

$\omega = 9.375$.