> MATRICES

i) Symmetric Matrix: $\boxed{A = A^T} = \frac{1}{2}(A + A^T)$

ii) Skew Symmetric Matrix: $\boxed{A = -A^T} = \frac{1}{2}(A - A^T)$

iii) Hermitian Matrix: $\boxed{A = A^\theta} = \frac{1}{2}(A + A^\theta)$

$$A^\theta = (\bar{A})'$$

iv) Skew Hermitian Matrix: $\boxed{A = -A^\theta} = \frac{1}{2}(A - A^\theta)$

v) Orthogonal Matrix: $\boxed{A A^T = I}$

$$A^{-1} = A^T$$

vi) Unitary Matrix: $\boxed{A A^\theta = I}$

$$A^\theta = A^{-1}$$

> GAMMA FUNCTION

i) $\int_0^\infty e^{-x} x^{n-1} dx = \Gamma n$

$$\boxed{\text{ii) } \int_0^\infty e^{-x} x^n dx = \Gamma n+1}$$

$\Gamma 0 = \infty$ $\boxed{\Gamma n = (n-1)!}$ $\Rightarrow$ If n is +ve

$\Gamma 1 = 1$

$\boxed{\Gamma n = \frac{\Gamma n+1}{n}}$ $\Rightarrow$ If n is -ve.

$\Gamma \frac{1}{2} = \sqrt{\pi}$

GAMMA FUNCTION

i) TYPE - 1

$$\int_0^\infty e^{-ax^n}\,dx = \frac{1}{n\sqrt[n]{a}}\,\Gamma\frac{1}{n}$$

put $x^n = t$, $\boxed{x = t^{1/n}}$ and solve using $\boxed{\int_0^\infty e^{-x}x^n\,dx = \Gamma n+1}$

ii) TYPE - 2

$$\int_0^\infty x^m e^{-ax^n}\,dx = \frac{1}{n}\,\frac{1}{a^{\frac{m+1}{n}}}\,\Gamma\frac{m+1}{n}$$

put $x^n = t$, $\boxed{x = t^{1/n}}$ and solve using $\boxed{\int_0^\infty e^{-x}x^n\,dx = \Gamma n+1}$

iii) TYPE - 3

$$\int_0^\infty x^m(\log x)^n\,dx = \frac{(-1)^n}{(m+1)^{n+1}}\,\Gamma n+1$$

put $\log x = -t$, $\boxed{x = e^{-t}}$ and solve using $\boxed{\int_0^\infty e^{-x}x^n\,dx = \Gamma n+1}$

iv) TYPE - 4

$$\int_0^\infty \frac{x^a}{a^x}\,dx = \frac{1}{(\log a)^{a+1}}\,\Gamma a+1$$

put $a^x = e^t$, $\boxed{x\log a = t}$ and solve using $\boxed{\int_0^\infty e^{-x}x^n\,dx = \Gamma n+1}$

> BETA FUNCTION

$$\boxed{\int_0^1 x^m (1-x)^n \, dx = B(m+1, n+1)}$$

- Properties:

i) $B(m,n) = B(n,m)$

ii) $B(m,n) = \dfrac{\Gamma m \, \Gamma n}{\Gamma m+n} = \dfrac{(m-1)!(n-1)!}{(m+n-1)!}$

iii) $\Gamma p \, \Gamma 1-p = \dfrac{\pi}{\sin p\pi}$

$$\boxed{(v) \int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{1}{2} B\left(\frac{p+1}{2}, \frac{q+1}{2}\right)}$$

iv) Duplication formula: $\boxed{\Gamma m \, \Gamma m+\frac{1}{2} = \dfrac{\sqrt{\pi} \, \Gamma 2m}{2^{2m-1}}}$

Ex: $\Gamma \frac{1}{4} \, \Gamma \frac{3}{4} = \sqrt{2}\pi$

$\Gamma \frac{5}{4} \, \Gamma \frac{7}{4} = \dfrac{3\sqrt{2}\pi}{16}$

$\Gamma \frac{3}{4} \, \Gamma \frac{5}{4} = \dfrac{\pi}{2\sqrt{2}}$

- BETA FUNCTION TYPES:

1] TYPE 1: $\int_0^a x^m (a-x)^n \, dx = a^{m+n+1} B(m+1, n+1)$

put $x = at$

and solve using $\boxed{\int_0^1 x^m (1-x)^n \, dx = B(m+1, n+1)}$

TYPE 2: $\int_0^1 x^m (1-x^n)^p \, dx = \frac{1}{n} B\left(\frac{m+1}{n}, p+1\right)$

put $x^n = t$

$x = t^{1/n}$ and solve using

$$\boxed{\int_0^1 x^m (1-x)^n \, dx = B(m+1, n+1)}$$

> TYPE - III

$$\int_0^1 (1-\sqrt[n]{x})^m dx = \frac{m!\,n!}{(m+n)!}$$

put $\sqrt[n]{x} = t$

$x = t^{1/n}$ and solve using $\int_0^1 x^m (1-x)^n dx = B(m+1, n+1)$

* SINGLE INTEGRATION

i) forward difference operator Δ : $\boxed{\Delta f(x) = f(x+h) - f(x)}$

ii) Backward difference operator ∇ : $\boxed{\nabla f(x) = f(x) - f(x-h)}$

iii) Rectangular Rule :

$$I = \int_a^b f(x)\,dx = h\left[f(x_1)^* + f(x_2)^* + f(x_3)^* + \cdots f(x_n)^*\right]$$

$f(x_1)^*$ — midpoint of sub-intervals.

$$h = \frac{b-a}{n}$$

iv) Trapezoidal Rule :

$$\int_a^b y\,dx = \frac{h}{2}[\underbrace{X}_{\text{Extreme ordinates}} + 2\underbrace{R}_{\text{Remaining ordinates}}]$$

$$= \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 \cdots y_{n-1})\right]$$

v) Simpson's $\frac{1}{3}$rd Rule : $\int_a^b y\,dx = \frac{h}{3}[\underbrace{X}_{\text{Extreme}} + 2\underbrace{E}_{\text{Even}} + 4\underbrace{O}_{\text{Odd}}]$

vi) Simpson's $\frac{3}{8}$th Rule : $\int_a^b y\,dx = \frac{3h}{8}[X + 2\underbrace{T}_{\text{Sum of multiple of 3}} + 3\underbrace{R}_{\text{Remaining ordinates.}}]$