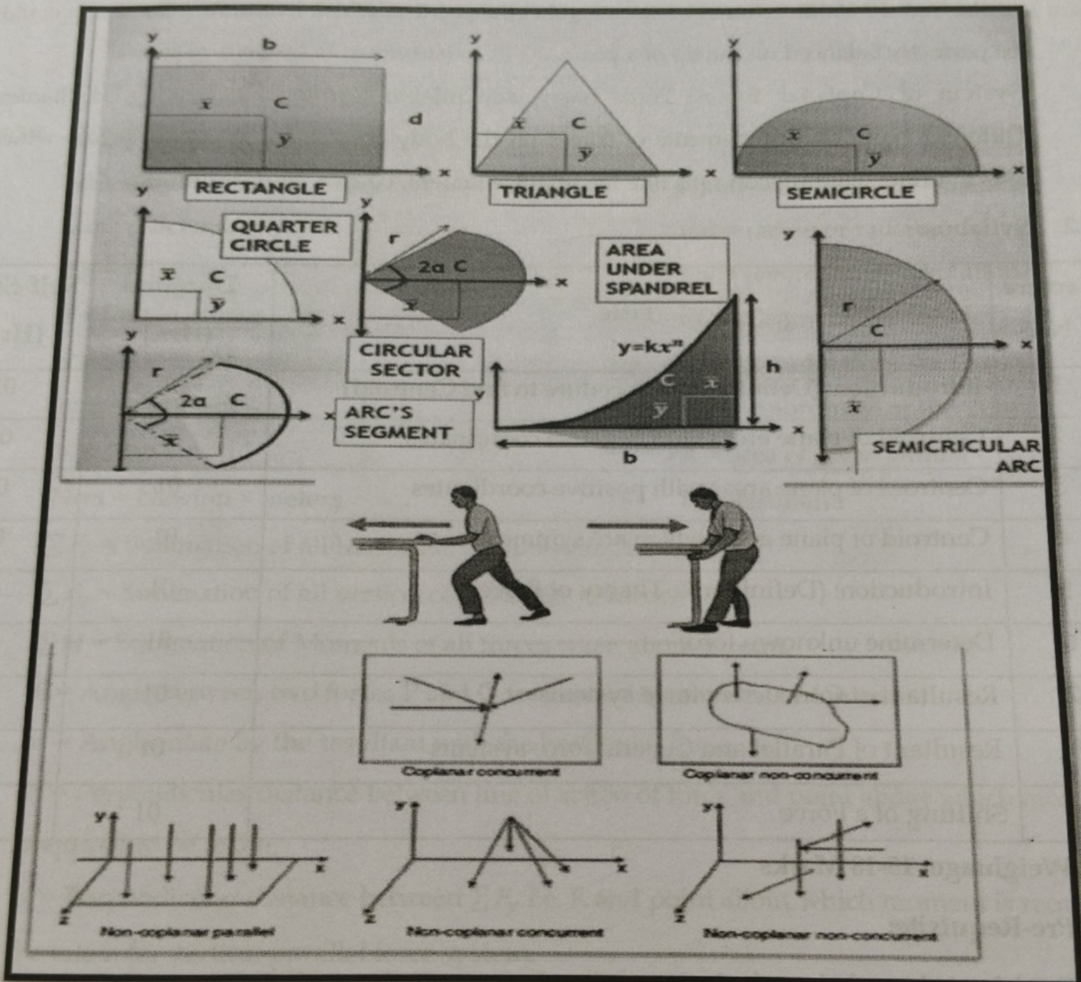
Module 1: Introduction to Co-Planar System of Forces

Infographics



Lecture: 1

1.1 *Introduction to Centroid of Plane Laminas*

1.1.1 Motivation:

Centroid: In mathematics and physics, the centroid or geometric center of a plane figure is the arithmetic mean (average) position of all the points in the shape. The definition extends to any object in n-dimensional space: its centroid is the mean position of all the points in all of the coordinate directions. Informally, it is the point at which a cutout of the shape could be perfectly balanced on the tip of a pin.

System of Coplanar forces: Force has a key role in learning Engineering Mechanics. Different types of arrangements of forces on the body constitutes System of forces which aids in understanding concepts like Resultant, Moment, couple, equilibrium etc.

1.1.2 Syllabus:

Lecture No.	Title	Duration (Hrs.)	Self-Study (Hrs.)
L	Introduction: (Definition & Procedure to find Centroid)	01	02
2	Centroid of plane areas with negative coordinates	01	02
3	Centroid of plane areas with positive coordinates	01	02
4	Centroid of plane areas which are symmetric	01	02
5	Introduction: (Definition & Theory of Force)	01	02
6	Determine unknown force	01	02
7	Resultant of Concurrent force systems	01	02
8	Resultant of Parallel and General force systems	01	02
9	Shifting of a Force	01	02

1.1.3 Weightage: 15-18 Marks

1.1.4 Pre-Requisite:

1) A basic knowledge of a 2- Dimensional geometrical co-ordinate system is required.
2) Knowledge of formulas for calculating areas of basic shapes like circle, rectangle, triangle, semicircle, quarter circle is required.
3) Knowledge of location of centroid of the above-mentioned shapes is required.
4) Knowledge of fundamentals of physics (forces) and mathematical formulation learnt at higher secondary level of education (trigonometry).

1.1.5 Learning Objectives: Learners shall be able to

1) Locate and place forces such that the body remains balanced.
2) Identify all the standard shapes of the given composite shape and find the individual centroids.
3) Calculate the position of centroid of the complete composite body using the Centroid formula for areas.
4) Understand various systems of forces; Calculate and find the effect forces exerted on them.
5) Find resultant of two forces by Parallelogram law of Forces& Resultant of three of more forces by method of resolution
6) Locate of resultant by Varignon's Theorem

1.1.6 Key Notations:

m = meter
km = kilometer
kg = kilogram
t = for ton or tons
s = for second
min = minute
N = Newton
N-m = Newton × meters

kN-m = kilo-Newton × meters
rad = radian or radians
rev = revolution or revolutions
CG = Centre of gravity
$\bar{X}$= x-co-ordinate of the CG
$\bar{Y}$ = y-co-ordinate of the CG
A = area of given lamina
R = Resultant.

$\sum F_x$ = Summation of all horizontal components of forces.
$\sum F_y$ = Summation of all vertical components of forces.
$\sum M$ = Summation of Moments of all forces taken about a point.
θ = Angle between two forces P and Q for Parallelogram Law of forces.
α = Angle made by the resultant with the horizontal force.
d = Perpendicular distance between line of action of force and point about which moment is required to be taken.
x = Perpendicular distance between $\sum F_y$ i.e. R and point about which moment is required to be taken for vertical parallel force system.
y = Perpendicular distance between $\sum F_x$ i.e. R and point about which moment is required to be taken for horizontal parallel force system.

1.1.7 Theoretical Background:

<u>Locating the centroid:</u>

➢ **Plumb line method** - The centroid of a uniform planar lamina, such as (a) below, may be determined, experimentally, by using a plumb line and a pin to find the center of mass of a thin body of uniform density having the same shape. The body is held by the pin inserted at a point near the body's perimeter, in such a way that it can freely rotate around the pin; and the plumb line is dropped from the pin (b). The position of the plumb line is traced on the body. The experiment is repeated with the pin inserted at a different point of the object. The intersection of the two lines is the centroid of the figure (c). This method can be extended (in theory) to concave shapes where the centroid lies outside the shape, and to solids (of uniform density), but the positions of the plumb lines need to be recorded by means other than drawing.

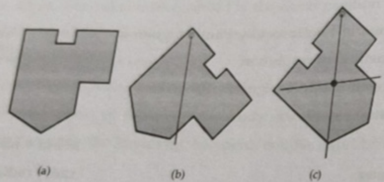


Fig. 1.1

➤ **Balancing method -** For convex two-dimensional shapes, the centroid can be found by balancing the shape on a smaller shape, such as the top of a narrow cylinder. The centroid occurs somewhere within the range of contact between the two shapes. In principle, progressively narrower cylinders can be used to find the centroid to arbitrary precision. In practice air currents make this unfeasible. However, by marking the overlap range from multiple balances, one can achieve a considerable level of accuracy.

1.1.8 Formulae:

1. Centroid for areas,

$$\bar{X} = \frac{\sum A_i X_{Gi}}{\sum A_i};\ \bar{Y} = \frac{\sum A_i Y_{Gi}}{\sum A_i};$$

2. Parallelogram law of forces,

Magnitude:

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

Direction:

$$\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

3. Lami's Theorem:

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

4. Resolution of forces,

Magnitude:

ΣF_x= Forces along X-direction,

ΣF_y= Forces along Y-direction

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

Direction: $\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right)$

5. Moment: $M_O = F \times d$ (Force x Perpendicular distance)

1.1.9 ***Introduction: (General-Mechanics, Definition & Procedure to find Centroid)***

Learning Objective: Learner will be able to understand the key concepts related to Engineering Mechanics

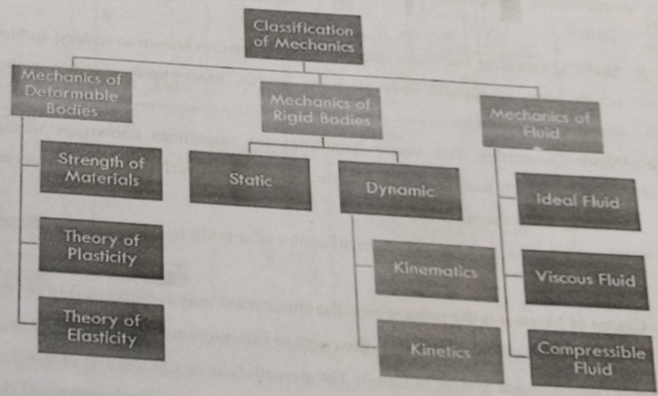
1.1.10 Theory:

➤ **Newtonian mechanics:** A branch of mechanics that deals with concepts of Newton's law of motion as distance, time, and mass in a period of time are known as Newtonian mechanics. This Newtonian mechanics describes the motion of objects in routine life affected by the forces. Newtonian mechanics is a very straightforward formulation theory that deals with Newton's second law of motion.

Generally, as for the theoretical approach, mechanics is split into three parts Newtonian, Lagrangian and Hamiltonian mechanics, but our routine work deals in Newtonian mechanics.

➤ **Engineering Mechanics:** The subject of Engineering Mechanics is that branch of Applied Science, which deals with the laws and principles of Mechanics, along with their applications to engineering problems. As a matter of fact, knowledge of Engineering Mechanics is very essential for an engineer in planning, designing and construction of his various types of structures and machines. To take up the job more skillfully, an engineer must pursue the study of Engineering Mechanics in a most systematic and scientific manner.

➤ **Divisions of Engineering Mechanics:** The subject of Engineering Mechanics may be divided into the following groups:



➤ **Fundamental Units:** The measurement of physical quantities is one of the most important operations in engineering. Every quantity is measured in terms of some arbitrary, but internationally accepted units, called fundamental units. All the physical quantities, met with in Engineering Mechanics, are expressed in terms of three fundamental quantities, i.e. Length, Mass, Time.

- ***METRE (Length):*** *The international 'metre' may be defined as the shortest distance (at 0°C) between two parallel lines engraved upon the polished surface of the Platinum-Iridium bar, kept at the International Bureau of Weights & Measures at Sevres near Paris.*
- ***KILOGRAM (Mass):*** *The international kilogram may be defined as the mass of the Platinum-Iridium cylinder, which is also kept at the International Bureau of Weights and Measures at Sevres near Paris.*
- ***SECOND (Time):*** *The fundamental unit of time for all the four systems is second, which is 1/ (24 × 60 × 60) = 1/86 400th of the mean solar day. A solar day may be defined as the interval of time between the instants at which the sun crosses the meridian on two consecutive days. This value varies throughout the year. The average of all the solar days, of one year, is called the mean solar day.*

➢ ***Derived Units:*** *Sometimes, the units are also expressed in other units (which are derived from fundamental units) known as derived units e.g. units of area, velocity, acceleration, pressure etc.*

➢ ***Systems of Units:*** *There are only four systems of units, which are commonly used and universally recognized. These are known as: S.I. units, M.K.S. units, C.G.S. units, sssF.P.S. units*

➢ ***Scalar Quantities:*** *The scalar quantities (or sometimes known as scalars) are those quantities which have magnitude only such as length, mass, time, distance, volume, density, temperature, speed etc.*

➢ ***Vector Quantities:*** *The vector quantities (or sometimes known as vectors) are those quantities which have both magnitude and direction such as force, displacement, velocity, acceleration, momentum etc.*

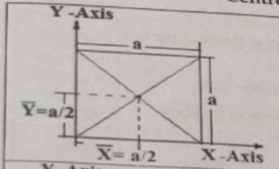
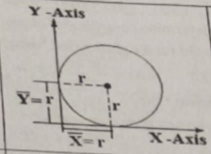
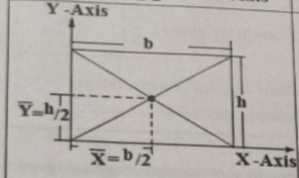
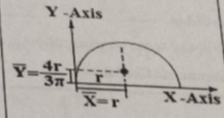
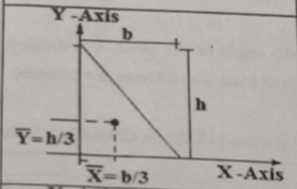
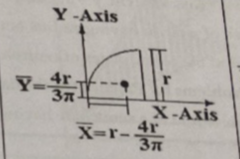
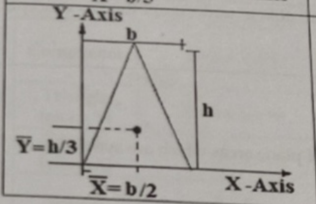
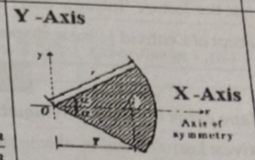
➢ ***Centroid:*** *It is defined as geometrical center of a body (e.g., center of a rectangle, center of triangle etc.).*

➢ ***Centre of Mass:*** *It is the point where the entire mass may be supposed to be concentrated.*

➢ ***Center of Gravity:*** *It is defined as the point of intersection of all the gravity axes of the body.*

S.I. Units (International System of Units): *The eleventh General Conference* of Weights and Measures has recommended a unified and systematically constituted system of fundamental and derived units for international use. This system of units is now being used in many countries. In India, the Standards of Weights and Measures Act of 1956 (vide which we switched over to M.K.S. units) has been revised to recognize all the S.I. units in industry and commerce. In this system of units, the fundamental units are meter (m), kilogram (kg) and second (s) respectively. But there is a slight variation in their derived units. Density (Mass density) - kg/m³, Force - N (Newton), Pressure - N/mm² or N/m², Work done (in joules) J = N-m, Power in watts W = J/s*

Centroid of Standard Shapes

	**Square:** $\bar{X} = \frac{a}{2}$ $\bar{Y} = \frac{a}{2}$		**Circle** $\bar{X} = r$ $\bar{Y} = r$
	**Rectangle** $\bar{X} = \frac{b}{2}$ $\bar{Y} = \frac{h}{2}$		**Semi - Circle** $\bar{X} = r$ $\bar{Y} = \frac{4r}{3\pi}$
	**Right Angle Triangle** $\bar{X} = \frac{b}{3}$ $\bar{Y} = \frac{h}{3}$		**Quarter - Circle** $\bar{X} = r - \frac{4r}{3\pi}$ $\bar{Y} = \frac{4r}{3\pi}$
	**Equilateral Triangle** $\bar{X} = \frac{b}{2}$ $\bar{Y} = \frac{h}{3}$		**Sector of Circle** $\bar{X} =$ 2rsinα / 3α

- **Procedure to solve problems of centroid of a given respective figure:**
  a) From a given composite figure, consider each figure separately in the form of triangle, circle, semicircle, etc.
  b) Specify the reference axis as x-axis and y-axis, if not specified.
  c) Determine the area of each figure as A_1, A_2, A_3, A_4, etc. and find the addition of all areas considering the shape to be subtracted.
  d) Determine x_1, x_2, x_3, x_4, etc. i.e. distance between centroid of the figure and references y-axis.
  e) Similarly, y_1, y_2, y_3, y_4, etc. i.e. distance between centroid of the figure and references x-axis.
  f) Adding the product of area and distance ($A_i x_i$) for plane figure whereas for hollow figure required figure is to be added and remaining part is to be deducted.
  g) By using formula,

$\bar{X} = (A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4) / (A_1 + A_2 + A_3 + A_4)$

$\bar{Y} = (A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4) / (A_1 + A_2 + A_3 + A_4)$

we can determine co-ordinates of centroid with respect to the reference axis.

1. *The Centroid for a right-angled triangle is*
   - i) h/3   ii) 4r/3π
   - iii) 2rsinα / 3α   iv) None of the above
2. *The Centroid of a Sector of a circle*
   - i) h/3   ii) 4r/3π
   - iii) 2rsinα / 3α   iv) None of the above
3. *The Centroid is*
   - i) Point of mass concentration   ii) Point of weight concentration
   - iii) Geometric Center   iv) None of the above

Exercise:

1) Determine the centroid of a right-angled triangle with height 4cm (along y-axis) and breadth 3cm (along x-axis). (**Ans: x = 1cm, y = 1.33cm**)

2) The center of a circle having radius 6mm is placed at the origin of X-Y plane. It is divided in IV quadrants. Find the co-ordinates of centroid of the II quadrant? (**Ans: x = -2.55mm, y = 2.55mm**)

Questions/Problems for Practice for the day:

1) Determine centroid of a semicircle having dia. 25mm along x-axis? (**Ans: x = 12.5mm, y = 5.30mm**)

Learning from the lecture 'Introduction': Learner will be able to understand the key concepts and apply the concept of Centroid to basic shapes

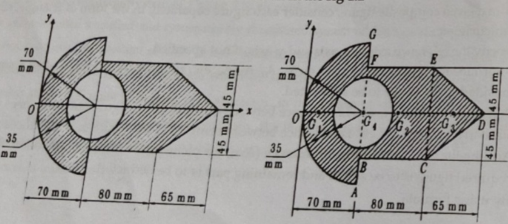
Lecture: 2

Centroid of plane areas which are Symmetric

Learning Objectives: Learners will be able to find centroid of plane areas which are symmetric in shape

Solved Problems:

1) **Find the centroid of the shaded area shown in the fig 1.2**



g 1.2

Solution:

Component	Area A_i (mm²)	Co-ordinates X_i(mm)	A_iX_i (mm³)
Triangle	$\frac{1}{2} \times 90 \times 65 = 2925$	$150 + \frac{b}{3} = 171.67$	502.13 x 10³
Semi-circle	$\frac{\Pi}{2}r^2 = \frac{\Pi}{2}[70^2] = 7697$	$70 - \frac{4r}{3\pi} = 70 - \frac{4(70)}{3\pi} = 40.3$	310.115 x 10³
Rectangle	80 x 90 = 7200	$70 + \frac{b}{2} = 110$	792 x 10³
Circle	$-\Pi r^2 = -\Pi[35^2] = -3848.45$	35 + r = 70	269.4 x 10³
	$\sum A_i = 13.97 \times 10^3$		$\sum A_iX_{Gi} = 1334.85 \times 10^3$

$\bar{X} = \frac{\sum A_iX_i}{\sum A_i} = \frac{1334.85 \times 10^3}{13.97 \times 10^3} = 95.53$mm, Centroid $[\bar{X}, \bar{Y}]$ = [95.53, 0] mm

2) **Find the centroid of the shaded area shown in the given figure 1.3**

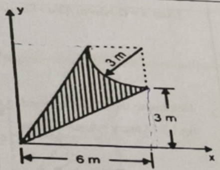


Fig 1.3

Component	Area A_i (m²)	Co-ordinates X_i(m)	Co-ordinates Y_i(m)	A_iX_i (m³)	A_iY_i (m³)
Triangle (along Y)	$-\frac{1}{2} \times 3 \times 6 = -9$	$\frac{b}{3} = 1$	$\frac{2h}{3} = 4$	-9	-36
Triangle (along X)	$-\frac{1}{2} \times 6 \times 3 = -9$	$= \frac{2b}{3} = 4$	$\frac{h}{3} = 1$	-36	-9
Square	6 x 6 = 36	$\frac{b}{2} = 3$	$\frac{b}{2} = 3$	108	108
Quarter circle	$-\frac{\Pi}{4}r^2 = -\frac{\Pi}{4}[3^2] = -7.07$	4r/3 Π = 4[3]/3 Π = 4.727	4r/3 Π = 4[3]/3 Π = 4.727	-33.42	-33.42
$\sum$	$\sum A_i = 10.93$			$\sum A_iX_{Gi} = 29.58$	$\sum A_iY_{Gi} = 29.58$

$\bar{X} = \frac{\sum A_iX_{Gi}}{\sum A_i} = \frac{29.58}{10.93} = 2.706m$, $\bar{Y} = \frac{\sum A_iY_{Gi}}{\sum A_i} = \frac{29.58}{10.93} = 2.706m$ *Centroid* $[\bar{X}, \bar{Y}]$ = [2.706, 2.706]m

1. If the given section is symmetrical about y-y axis, then we must calculate for
   i) X coordinate ii) Y coordinate
   iii) both X & Y coordinates iv) None of the above
2. If the given section is symmetrical about x-x axis, then we must calculate for
   i) X coordinate ii) Y coordinate
   iii) both X & Y coordinates iv) None of the above

Exercise:

1. Determine the X-co-ordinate of the centroid of the portion of a circular segment in terms of radius r and angle a (for shaded area only) figure. 1.4 **[Ans. x = (2 rsin³α) /(3(α - Sin α. Cos α)]**

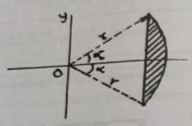
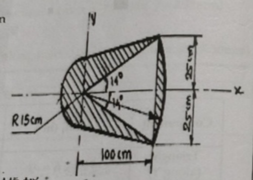


Fig 1.4

2. Determine the CG of the shaded area as shown in fig 1.5. **[Ans.: y = 0, x = 29.6cm]**



Questions/Problems for Practice for the day:

1. An isosceles triangle is cut from a square plate as shown in figure. The plate remains in the equilibrium in any position when suspended from point E (apex of the triangle). Determine height of the removed portion of the triangle for fig 1.6. **[Ans: h= 0.634m]**

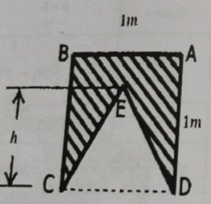


Fig 1.6

2. Centroid of T-section shown in figure 1.7 is on line AB. Find depth 'h' of the web. **[Ans: h=58.14mm]**

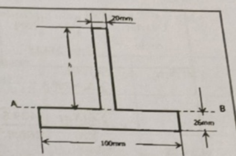


Fig 1.7

Learning from the lecture 'Centroid for Symmetric Areas': Learner will be able to Apply the formulae for basics shapes which are symmetric in shape

Lecture: 3

Centroid of plane areas with positive coordinates

Learning Objectives: Learners will be able to find centroid of plane areas with positive co-ordinates

Solved Problems:

3) **Find centroid of plane area for given fig 1.8.**

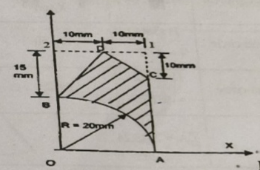


Fig 1.8

Solution:

Component	Area A_i (m^2)	Co-ordinates X_i(m)	Co-ordinates Y_i(m)	A_iX_i (m^3)	A_iY_i (m^3)
Triangle B2D	$\frac{1}{2} \times 10 \times 15 = -75$	$\frac{b}{3} = 3.33$	$20 + \frac{2h}{3} = 30$	- 249.7	- 2250
Rectangle OA12	$20 \times 35 = 700$	$\frac{b}{2} = 10$	$\frac{h}{2} = 17.5$	7000	2250
Triangle C1D	$\frac{1}{2} \times 10 \times 10 = -50$	$20 + \frac{2b}{3} = 16.67$	$25 + \frac{2h}{3} = 31.67$	- 833.5	- 1583.5

Quarter circle	$-\frac{\Pi}{4}r^2 = -\frac{\Pi}{4}[20^2]$ = - 314.15	$4r/3\Pi$ = 4[20]/3 Π = 8.5	$4r/3\Pi$ = 4[20]/3 Π = 8.5	- 2667.2	- 2667.221
	$\sum A_i$ = 260.8			$\sum A_iX_{Gi}$ = 3429.5	$\sum A_iY_{Gi}$ = 5749.3

$$\bar{X} = \frac{\sum A_i X_{Gi}}{\sum A_i} = \frac{3429.5}{260.8} = 12.45mm, \quad \bar{Y} = \frac{\sum A_i Y_{Gi}}{\sum A_i} = \frac{5749.3}{260.8} = 22.04mm,$$

Centroid [X̄, Ȳ] = [12.45, 22.04] mm

4) Find the centroid of shaded area of the semicircle of dia. 100cm for fig 1.9.

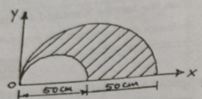


Fig 1.9

Solution:

Component	Area A_i (m^2)	Co-ordinates X_i(m)	Co-ordinates Y_i(m)	A_iX_i (m^3)	A_iY_i (m^3)
Semi-circle	Π x50²/2 = 3927	R = 50	=4R/3Π = 21.22	196.35 x 10^3	83.33 x 10^3
Semi-circle	Πx25²/2 = -981.75	r = 25	=4r/3Π = 10.61	-24.5 x 10^3	-10.41 x 10^3
	$\sum A_i$ = 2945.24			$\sum A_iX_{Gi}$ = 171.8 x 10^3	$\sum A_iY_{Gi}$ = 72.91 x 10^3

$$\bar{X} = \frac{\sum A_i X_{Gi}}{\sum A_i} = \frac{171.8 \times 10^3}{2945.24} = 58.33cm, \quad \bar{Y} = \frac{\sum A_i Y_{Gi}}{\sum A_i} = \frac{72.91 \times 10^3}{2945.24} = 24.76cm$$

Centroid [X̄, Ȳ] = [58.33, 24.76] cm

5) Determine the coordinates Xc and Yc of the center of a 100 mm diameter circular hole cut in a thin plate so that this point will be the centroid of the remaining shaded area shown in Fig 1.10. (All dimensions are in mm)

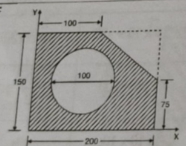


Fig. 1.10

Solution: If Xc and Yc are the coordinates of the center of the circle, centroid also must have the coordinates Xc and Yc as per the condition laid down in the problem. The shaded area may be

considered as a rectangle of size 200 mm × 150 mm minus a triangle of sides 100 mm × 75 mm and a circle of diameter 100 mm.

Component	Area A_i (m^2)	Co-ordinates X_i(m)	Co-ordinates Y_i(m)	A_iX_i (m^3)	A_iY_i (m^3)
Circle	Π x50² =- 7855	Xc	Yc	-7855* Xc	-7855* Yc
Rectangle	b*d = 30000	100	75	3 x 10[illegible]	22.5 x 10[illegible]
Triangle	-100*75/2=-3750	200-(100/3) =166.67	150-25 =125	-62.5 x 10[illegible]	46.875 x 10[illegible]
	$\sum A_i$ = 18393			$\sum A_iX_{Gi}$ = -7855* Xc +3 x 10[illegible]-62.5 x 10[illegible]	$\sum A_iY_{Gi}$ = -7855* Yc+22.5 x 10[illegible]-46.875 x 10[illegible]

$$\bar{X} = Xc = \frac{\sum A_i X_{Gi}}{\sum A_i} = 90.48mm, \quad \bar{Y} = Yc = \frac{\sum A_i Y_{Gi}}{\sum A_i} = 67.86mm,$$ **Centroid [X̄, Ȳ] = [90.4867.86]sssss**

1. Where will the C.G of this plane area will lie

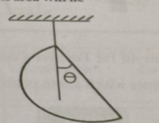


Fig 1.11

i) On the circumference of circle
ii) On the diametrical line of the circle
iii) On the vertical line passing through the point of suspension
iv) Outside the Semicircle

2. For a Semi-Circle having its center on the origin and diameter horizontal along it, its x-coordinate will be ____________

i) On the circumference of circle
ii) R
iii) 0
iv) -r

Exercise:

1. Determine the co-ordinates of centroid of the shaded portion as shown in figure 1.12 [Ans: x= 53.21mm, y= 38.54mm]

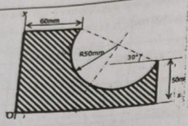


Fig 1.12

Questions/Problems for Practice for the day:

1. Find the distance 'y' so that C.G. of given area in the figure 1.13 has co-ordinates (25, 20) mm [Ans: y= 25.625mm]

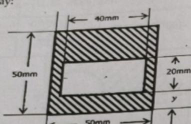


Fig 1.13

2. A plane lamina is hung freely from point D. Find the angle made by BD with the vertical for given fig 1.14. [Ans.: θ=29.62°]

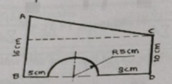


Fig 1.14

Learning from the lecture 'Centroid for Positive Coordinates': Learner will be able to Apply the formulae for basics shapes which are in positive coordinates.

Lecture: 4

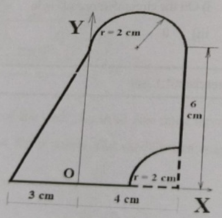
Centroid of plane areas with Negative coordinates

Learning Objectives: Learners will be able to find centroid of plane areas with positive co-ordinates

Solved Problems

6) Find the centroid of the given shape for a given fig. 1.15

Solution:



Component	Area 'A_i' (cm^2)	Co-ordinates 'X_{Gi}'(cm)	Co-ordinates 'Y_{Gi}'(cm)	'A_iX_{Gi}' (cm^3)	'A_iY_{Gi}' (cm^3)
Semi-circle	$\Pi r^2/2 = 6.28$	$r = 2$	$= 6 + 4r/3\Pi = 6.84$	12.56	42.95
Rectangle	$6 \times 4 = 24$	$\frac{b}{2} = 2$	$\frac{h}{2} = 3$	48	72
Triangle	$\frac{1}{2} \times 3 \times 6 = 9$	$-\frac{b}{3} = -1$	$\frac{h}{3} = 2$	-9	18
Quarter circle	$-\frac{\Pi}{4}r^2 = -3.14$	$4 - 4r/3\Pi = 3.15$	$4r/3\Pi = 0.85$	-9.89	-2.67
	$\sum A_i = 36.14$			$\sum A_iX_{Gi} = 41.67$	$\sum A_iY_{Gi} = 130.28$

$$\bar{X} = \frac{\sum A_i X_{Gi}}{\sum A_i} = \frac{41.67}{36.14} = 1.15\ cm,\ \bar{Y} = \frac{\sum A_i Y_{Gi}}{\sum A_i} = \frac{130.28}{36.14} = 3.60\ cm$$

Centroid $[\bar{X}, \bar{Y}] = [1.15, 3.60]\ cm$

7) Determine the location of the centroid of the plane area shown in fig. 1.16 shaded on sketch.

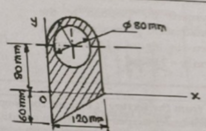


Fig 1.16

Solution:

Component	Area 'A_i' (mm^2)	Co-ordinates 'X_{Gi}'(mm)	Co-ordinates 'Y_{Gi}'(mm)	'A_iX_{Gi}' (mm^3)	'A_iY_{Gi}' (mm^3)
Semi-circle	$\frac{\pi r^2}{2} = \frac{\pi(60)^2}{2} = 5.65 \times 10^3$	$r = 60$	$= 80 + \frac{4r}{3\pi}$ $= 80 + \frac{4(60)}{3\pi} = 105.46$	576×10^3	384×10^3
Rectangle	$120 \times 80 = 9.6 \times 10^3$	$\frac{b}{2} = 60$	$\frac{h}{2} = 40$	144×10^3	-72×10^3
Triangle	$\frac{1}{2} \times 120 \times 60 = 3.6 \times 10^3$	$\frac{b}{3} = 40$	$-\frac{h}{3} = -20$	339.3×10^3	596.4×10^3
Circle	$-\Pi r^2 = -\Pi [40^2] = -5.65 \times 10^3$	$20 + r = 60$	$40 + r = 80$	-301.6×10^3	-402.2×10^3
	$\sum A_i = 13.828 \times 10^3$			$\sum \sum A_iX_{Gi} = 757.7 \times 10^3$	$\sum A_iY_{Gi} = 506.2 \times 10^3$

$$\bar{X} = \frac{\sum A_i X_{Gi}}{\sum A_i} = \frac{757.7 \times 10^3}{13.828 \times 10^3} = 54.8\ mm,\ \bar{Y} = \frac{\sum A_i Y_{Gi}}{\sum A_i} = \frac{506.2 \times 10^3}{13.828 \times 10^3} = 36.6\ mm$$

Centroid $[\bar{X}, \bar{Y}] = [54.8, 36.6]\ mm$

1. What should be taken as a negative value for a shape not to be included in centroid

calculations?

i) X-coordinate   ii) Y-coordinate
iii) Area   iv) None of the above

2. **What does the negative sign indicate?**

i) Removal of area   ii) Negative area
iii) Removal of negative coordinate   iv) None of the above

Exercise:

1. Determine the centroid of the following plane area shown in fig. 1.17. **[Ans.: 54.867mm, 18.454mm]**

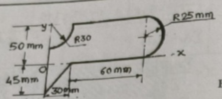


Fig 1.17

Questions/Problems for Practice for the day:

2. Determine the position of the centroid of the plane-shaded area shown in figure. **[Ans.: x = 1.59cm, y = 2.08cm]**

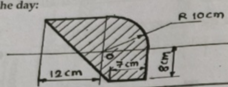


Fig 1.18

Learning from the lecture 'Centroid for Negative Coordinates': Learner will able to Apply the formulae for basics shapes which are in negative coordinates.

Lecture: 5

1.2 *System of Coplanar Forces*

- **Force:** It is defined as an external agency which produces or tends to produce, destroys or tends to destroy the motion. It is characterized by Magnitude, Direction, Sense and Point of application. It is a vector quantity and S.I unit is Newton (N).1 Newton force is defined as force required to produce unit acceleration on unit mass. Therefore, 1 Kg = 9.81 N
- **System of forces:** There are mainly seven types of system of forces:
  - **Co-planar forces:** The forces which are acting in the same plane are known as co-planer forces. **(Fig 1.19)**

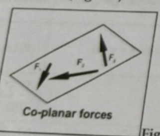


Fig 1.19

  - **Non-Coplanar forces:** The force system in which the forces acting in the different planes is called as non-coplanar forces. **(Fig 1.20)**

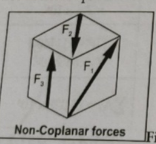


Fig 1.20

  - **Collinear forces:** The forces which are acting along the same straight line are called as collinear forces. **(Fig 1.21)**

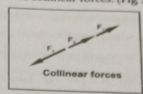


Fig 1.21

  - **Non-collinear forces:** The forces which are not acting along the straight line are called as Non-collinear forces. **(Fig 1.22)**

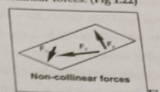


Fig 1.22

  - **Concurrent forces:** The forces which are passing through a common point are called concurrent forces. **(Fig 1.23)**

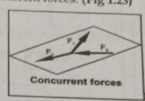


Fig 1.23

  - **Non-concurrent forces:** The forces which are not passing through a common point are called as non-concurrent forces. **(Fig 1.24)**

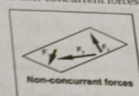


Fig 1.24

  - **Parallel forces:** The forces whose lines of action are parallel to each other are known as parallel forces. **(Fig 1.25)**

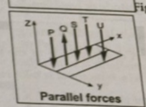


Fig 1.25

- **Resultant:** A single force producing the same effect that as produced by number of forces when acting together. It is denoted by 'R'

Methods of composition: (to find R): -

- **Resultant of two concurrent forces: (Law of parallelogram of forces):** It states that "if two forces simultaneously acting at a point be represented in magnitude and direction by two adjacent sides of a parallelogram, the diagonal will represent resultant in magnitude and direction, but passing through the point of intersection of two forces"

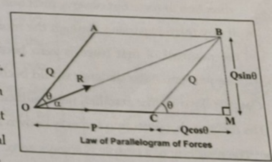


Fig 1.26

Consider two forces P and Q acting at a point represented by two sides OA and OC of a parallelogram OABC. Let θ be the angle between two forces P and Q, α be the angle between P and R. Draw perpendicular BM and produce QC.

In triangle CMB, BM= $Q\sin\theta$ and CM = $Q\cos\theta$

➢ **Magnitude of R:**

In triangle OMB, $OB^2 = OM^2 + BM^2$

$OB^2 = (OC + CM)^2 + BM^2$

$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$

$R^2 = (P^2 + 2PQ\cos\theta + Q^2\cos^2\theta) + (Q^2\sin^2\theta)$

$R^2 = P^2 + 2PQ\cos\theta + Q^2(\sin^2\theta + \cos^2\theta)$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

➢ **Direction of R:**

In triangle OMB, $\tan\alpha = BM/OM$

$\tan\alpha = BM/(OC + CM)$

$\tan\alpha = Q\sin\theta / (P + Q\cos\theta)$

If $\theta = 90°$, $R = \sqrt{P^2 + Q^2}$, $\tan\alpha = \frac{Q}{P}$

- **Resultant of two or more forces: (Method of resolution):** When two or more coplanar concurrent or non-concurrent forces acting on a body the resultant can be found out by using resolution procedure.

Direction: $\theta = \tan^{-1}(\frac{F_y}{F_x})$

Where, ΣF_x = Forces along X-direction,

ΣF_y = Forces along Y-direction

θ = Angle of 'R' with x-axis

Magnitude of resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

➢ **Moment:** It is the turning effect produced by a force. of a force about any point is the product of magnitude of the force and perpendicular distance about that point. The point about which moment is taken is called as moment center.

Moment about O, $M_O = F \times d$. S.I unit: N-m.

- While taking moment of any force do not observe direction of force but observe direction of rotation.
- If any force is passing through the moment center, the moment of that force is zero because for the case perpendicular distance would become zero

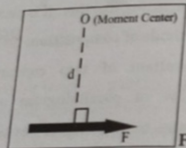


Fig 1.27

➢ **Couple:** Two unlike parallel, non-collinear forces having same magnitude form a Couple. The distance between two forces is known as arm or lever of the couple.

- The resultant of a couple is always zero.
- The moment of a couple is product of forces and lever arm of the couple. Therefore, M= F X d.
- A couple cannot be balanced by a single force.It can be balanced only by another couple of opposite nature.
- The moment of couple is independent of the moment center.

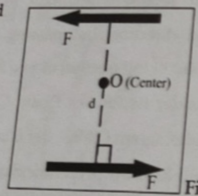


Fig 1.28

➢ **Composition of forces:** The process of addition of forces is called as composition of forces.

➢ **Resolution of forces:** It is the procedure of splitting up a single force into number of components without changing the effects of the same.

➢ **Principle of transmissibility:** The point of application of a force can be transmitted anywhere along its line of action, but within the body. It is only applicable to rigid bodies. The principle is neither applicable from the point of view of internal resistances nor internal forces developed in the body nor to deformable bodies under any circumstances.

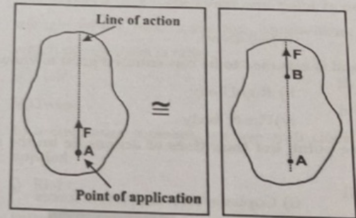


Fig 1.29 Principle of Transmissibility

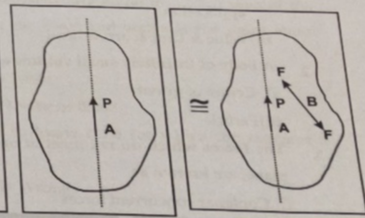


Fig 1.30 Principle of Superposition

➢ **Principle of Superposition:** The effect of a force on a body remains unaltered if we add or subtract any system which is in equilibrium.

➢ **Varignon's theorem:** The sum of the moment of all the forces about a point is equal to the moment of their resultant about the same point. $\Sigma M_o^F = M_o^R$

Consider a force F acting at a point A and having component F_1 and F_2 in ant two directions. Let us choose any point O, lying in the plane of the forces, as a moment center. Attach at A two rectangular axes such that the y-axis is along the line AO and the x-axis is perpendicular to it, as shown in the figure 1.31.

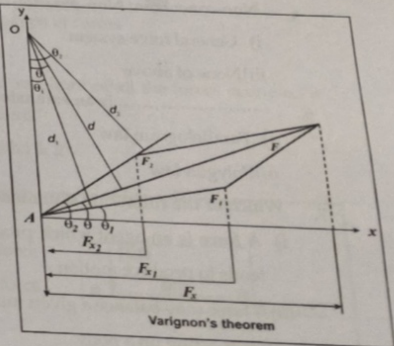


Fig 1.31

Moment of the force F about O

$F \times d = F \times OA \cos\theta = OA \times F\cos\theta$

$F \times d = OA \times F_x$ .... (1)

Moment of the force F_1 about O,

$F_1 \times d_1 = F_1 \times OA \cos\theta_1 = OA \times F_1\cos\theta_1$

$F_1 \times d_1 = OA \times F_{x1}$ .... (2)

Moment of the force F_2 about O,

$F_2 \times d_2 = F_2 \times OA \cos\theta_2 = OA \times F_2\cos\theta_2$

$F_2 \times d_2 = OA \times F_{x2}$ .... (3)

$F_1 \times d_1 + F_2 \times d_2 = OA \times F_x$......... [Sum of the x-components of the forces F_1 and F_2 = x-components of the resultant force F; $F_x = F_{x1} + F_{x2}$]

$F_1 \times d_1 + F_2 \times d_2 = F \times d$ .... from (1) &

Adding (2) and (3) (4)

$F_1 \times d_1 + F_2 \times d_2 = OA \times (F_{x1} + F_{x2})$ .... (4) i.e. $\Sigma M_o^F = M_o^R$

1. **Force can be completely defined by it's**
   - i) Magnitude, Direction & point of application
   - ii) Unit & sense
   - iii) Value & Unit & arrow head
   - iv) All above
2. **A body of infinitely small volume and is assumed to be concentrated point is known as**
   - i) Centre of gravity
   - ii) Rigid body
   - iii) Particle
   - iv) Plastic body
3. **The forces which do not meet at one point, but their lines of actions lie on the same plane, are known as**
   - i) Coplanar concurrent forces
   - ii) Coplanar Non-concurrent forces
   - iii) Non-Coplanar Non-concurrent forces
   - iv) Non-coplanar Concurrent
4. **The process of finding out the resultant force, of a few given forces, is called**
   - i) Composition
   - ii) Resolution
   - iii) Equilibrant
   - iv) None of these
5. **Non-concurrent Non-parallel, coplanar forces are called as**
   - i) General force system
   - ii) Space forces
   - iii) None of above
   - iv) Both i& ii
6. **..........................is an extension of Triangle law of forces for more than two forces**
   - i) Parallelogram law
   - ii) Coulomb's law
   - iii) Polygon law
   - iv) Sine Rule
7. **Which of the following statement is correct?**
   - i) A force is an agent which produces or tends to produce motion
   - ii) A force is an agent which stops or tends to stop motion
   - iii) A force may balance a given number of forces acting on a body
   - iv) Both a & b
8. **To determine the effects of force acting on a body, we must know**
   - i) Its magnitude and direction of the line along which it acts
   - ii) Its nature (where push or pull)
   - iii) Point through which it acts on the body
   - iv) All of the above

9. **If number of forces acting simultaneously on a particle, then the resultant of these forces will have the same effect as produced by the all the forces, this is known as**
   - i) Principle of physical independences of forces
   - ii) Principle of transmissibility of forces
   - iii) Principle of resolution of forces
   - iv) None of the above
10. **The moment of a force about any point is geometrically equal to .... area of the triangle whose base is the line representing the force and vertex is the point about which the moment is taken**
    - i) Half
    - ii) Same
    - iii) Twice
    - iv) None of these
11. **In a clockwise moment, we use wall clock to know time for which the moment is applied**
    - i) Right
    - ii) Wrong
    - iii) Can't Say
    - iv) Partially Correct
12. **According to the Lami's theorem, the three forces**
    - i) Must be equal
    - ii) Must be at 120° to each other
    - iii) Must be both of above
    - iv) May not be any of the two
13. **The Lami's theorem is applicable only for**
    - i) Coplanar forces
    - ii) Concurrent forces
    - iii) Coplanar and concurrent forces
    - iv) Any type of forces
14. **If a body is in equilibrium, we may conclude that**
    - i) The moment of all the forces about any point is zero.
    - ii) The resultant of all the forces acting on it is zero
    - iii) No force is acting on the body
    - iv) Both i & ii

Exercise:

1. What do you mean by resolution of a force into a force and a couple? Convert the given force into a force couple at point B.

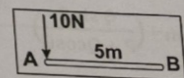


Fig 1.32

2. How many types of forces can exist?

Questions/Problems for Practice for the day:

1. List out all the Force Systems.
2. Where will the resultant of a Concurrent Force System pass from?

Learning from the lecture 'Definition & Theory of Force': Learner will be able to know the definitions of forces and different other concepts & procedures related to analysis of forces.

Lecture: 6

1.1.1 Determine Unknown Force

Learning Objectives: Learners will be able to find the unknown forces when resultant is given

1.1.2 <u>Solved Problems</u>

8) **R=800N is the resultant of 4 concurrent forces. Find the fourth force F_4.**

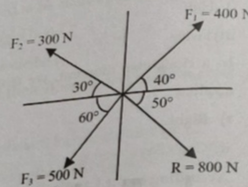


Fig 1.33

Solution: This is a concurrent system of four forces.

R=800N at θ=50°; Rx= R cos50°= 800 cos50°;

Ry= R sin50°= 800 sin50°

Σ Fx= Rx(→ +ve); 400 cos45°–300 cos30°-500 cos60°+ (F_{4x}) = 800 cos50°

F_{4x} = 741.2N (→)

ΣFy= Ry(↑ +ve)

400 sin45° – 300 sin30° -500 sin60° + (F_{4y}) = 800 cos50°

F_{4y} = -612.6N = 612.6N (↓)

$$F_4 = \sqrt{(F_{4x})^2 + (F_{4y})^2} = 961.6N$$

$$\theta_4 = \tan^{-1}\left(\frac{F_{4y}}{F_{4x}}\right) = \tan^{-1}\left(\frac{612.6}{741.2}\right) = 39.6°$$

9) **The sum of two concurrent forces P and Q is 270 N and their resultant is of 180 N. If resultant is perpendicular to P. Find P & Q.**

Solution: Let θ be the angle between two forces P and Q.

Here, P + Q = 270N, R = 180N and α=90°

Using $\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$

$\therefore 90° = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right) = \infty$

∴ P + Q cosθ = 0

∴ Q cosθ = - P

Also, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

∴ $(180)^2 = P^2 + Q^2 + 2P(-P)$

∴ $(180)^2 = Q^2 - P^2$

∴ $(180)^2 = (Q - P)(Q + P)$

∴ $(180)^2 = (Q - P)(270)$

∴ (Q – P) = 120

As (Q + P) = 270 & (Q – P) = 120,

Solving, we get, **P = 75N & Q = 195N**

10) For two forces P and Q acting at a point, maximum resultant is 2000N and minimum magnitude of resultant is 800N. Find values of P and Q.

Solution:

We know that, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

For maximum value of R, θ = 0.

$\therefore R_{max} = \sqrt{P^2 + Q^2 + 2PQ}$

$\therefore R_{max} = \sqrt{(P + Q)^2} = (P + Q)$

∴ (P + Q) = 2000N.................. (1)

For minimum value of R, θ = 180°

$\therefore R_{max} = \sqrt{P^2 + Q^2 - 2PQ}$

$\therefore R_{max} = \sqrt{(P - Q)^2} = (P - Q)$

∴ (P + Q) = 800N............... (2)

Solving we get, **P = 1400N & Q = 600N**

1. **If the resultant of a force system is vertical, then**
   i) $\Sigma F_y = R_y$ ii) $\Sigma F_x = R_x$
   iii) Both iv) None
2. **If the resultant of a force system is horizontal, then**
   i) $\Sigma F_y = R_y$ ii) $\Sigma F_x = R_x$
   iii) Both iv) None

Exercise:

1. A force R = 25N has components F_a, F_b and F_c as shown in figure 1.34. If F_c=20N, find F_a and F_b. **[Ans: F_a=33.9N, F_b= 35.09N]**

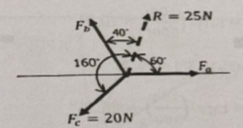


Fig 1.34

2. Three forces act on the bracket. Determine the magnitude and the direction of the force F_1 so that the resultant forces are directed along the line-x and has magnitude of 800N for given fig 1.35. **[Ans: F_1= 193.8N, θ = 24.63°]**

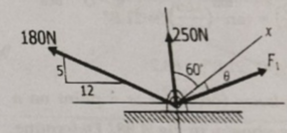


Fig 1.35

Learning from the lecture 'Determine unknown force': Learner will able to find the unknown forces when magnitude & nature of resultant is given.

Lecture: 7

1.1.3 *Resultant of Concurrent force systems*

Learning Objectives: Learners will be able to find the resultant of Concurrent force systems

Solved Problems

11) The striker of carom board lying on the board is being pulled by four players as shown in given fig 1.36. The players are sitting exactly at the center of the four sides. Find the resultant forces in magnitude & direction.

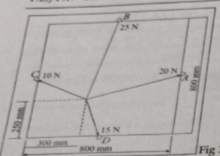


Fig 1.36

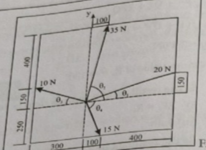


Fig 1.37

Solution:

$\theta_1 = \tan^{-1}\left(\frac{AG}{OG}\right) = \tan^{-1}\left(\frac{150}{500}\right) = 16.7°$

$\alpha = \tan^{-1}\left(\frac{EB}{OE}\right) = \tan^{-1}\left(\frac{100}{550}\right) = 10.3°$

$\theta_2 = (90° - \alpha) = (90° - 10.3°) = 79.7°$

$\theta_3 = \tan^{-1}\left(\frac{CH}{OH}\right) = \tan^{-1}\left(\frac{150}{300}\right) = 26.56°$

$\beta = \tan^{-1}\left(\frac{FD}{OF}\right) = \tan^{-1}\left(\frac{100}{250}\right) = 21.8°$

$\theta_4 = (90° - \beta) = (90° - 21.8°) = 68.2°$

$\Sigma F_x (\rightarrow +ve) = 25\cos\theta_2 + 20\cos\theta_1 - 10\cos\theta_3 + 15\cos\theta_4 = 20.25N (\rightarrow)$

$\Sigma F_y (\uparrow +ve) = 25\sin\theta_2 + 20\sin\theta_1 + 10\sin\theta_3 - 15\sin\theta_4 = 20.89N (\uparrow)$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(20.25)^2 + (20.89)^2} = 29.1N$

$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right) = \tan^{-1}\left(\frac{20.89}{20.25}\right) = 45.89°$

12) **Three coplanar forces act at a point on a bracket as shown in fig 1.38. Determine the value of the angle ά such that the resultant of the three forces will be vertical. Also find the magnitude of the resultant.**

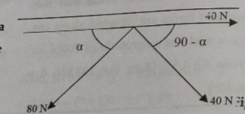


Fig 1.38

Solution:

Resultant of three forces will be vertical.

$\Sigma F_y (\uparrow +ve) = R_y$

$\therefore -80\sin\alpha - 40\sin(90-\alpha) = R$ ......(I)

$\Sigma F_x (\rightarrow +ve) = 0$

$\therefore -80\cos\alpha + 40\cos(90-\alpha) + 40 = 0$

$\therefore -2\cos\alpha + \cos(90-\alpha) + 1 = 0$

$\therefore -2\cos\alpha + \sin\alpha + 1 = 0$ ($\because \cos(90-\alpha) = \sin\alpha$)

Or, $2\cos\alpha = 1 + \sin\alpha$

$\therefore 2\left(\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}\right) = \left(\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}\right) + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$\therefore 2\left(\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\right)\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right) = \left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right)^2$

$\therefore 2\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right) = \left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right)$

Divide by $\cos\frac{\alpha}{2}$

$\therefore 2\left(1 - \tan\frac{\alpha}{2}\right) = \left(\tan\frac{\alpha}{2} + 1\right)$

$\therefore 3\tan\frac{\alpha}{2} = 1$

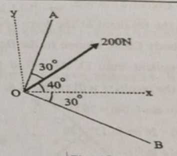
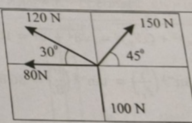
$\therefore \alpha = 2\tan^{-1}\left(\frac{1}{3}\right) = 36.86°$ ......(II)

From (I) & (II),

$-80\sin 36.86° - 40\sin(90° - 36.86°) = R$

$\therefore R = -80N$

1. **The Lami's theorem is applicable only for .... forces**
   i) One
   ii) Two
   iii) Three
   iv) Four
2. **The Lami's theorem is applicable only for ....**
   i) Coplanar forces
   ii) Concurrent forces
   iii) Coplanar and concurrent forces
   iv) Any type of forces

Exercise:	
1) Resolve 200N force into components along A & B directions. Refer fig. 1.39 **[Ans: 190.84N, 101.54N]**	 Fig1.39
2) Find the resultant system of four concurrent forces as shown in the figure analytically. **[Ans: R=100.7 N, α = 143.6°]**	 Fig 1.40

Learning from the lecture 'Resultant of Concurrent force systems': Learner will able to find the resultant of Concurrent force systems where all forces meet at a common point.

Lecture: 8

1.1.4 Resultant of Parallel & General force systems

Learning Objectives: Learners will be able to find the resultant of Parallel & General force systems

Solved Problems

13) Determine the magnitude and position of the resultant with respect to point A of the parallel forces shown in fig 1.41

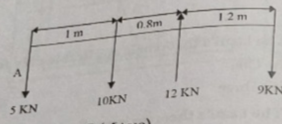


Fig 1.41

Solution: $\Sigma Fy\ (\uparrow +ve) = R$

$\therefore -5 - 10 + 12 - 9 = R$

$\therefore R = 12N\ (\downarrow)$

Using Varignon's theorem

$\Sigma M_A{}^F = M_A{}^R\ (\circlearrowleft +ve)$

$\therefore -(10 \times 1) + (12 \times 1.8) - (9 \times 3) = -R \times d$

$\therefore -15.4 = -12x\ d, \quad \therefore d = 1.3m$

14) Find the resultant of the force system acting on a body OABC shown in the fig 1.42. Find the resultant from O. Also find the points where the resultant will cut the X & Y axis.

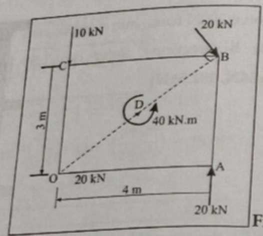


Fig 1.42

Solution: $\alpha = \tan^{-1}(3/4) = 36.86°$

$\Sigma Fx\ (\rightarrow +ve) = 20 \cos 53.13° - 20$

$\therefore \Sigma Fx = -8\ KN = 8kN\ (\leftarrow)$

$\Sigma Fy\ (\uparrow +ve) = -10 - 20 \sin 53.13 + 20 = -6KN = 6KN(\downarrow)$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{8^2 + 6^2} = 10kN$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{6}{8}\right) = 36.86°$$

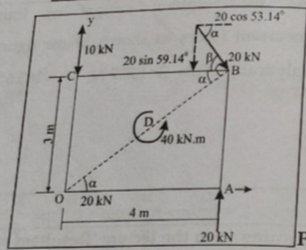


Fig 1.43

15) A system of forces acting on a bell crank as shown in fig 1.44. Determine the magnitude, direction and point of application of the resultant with respect to O.

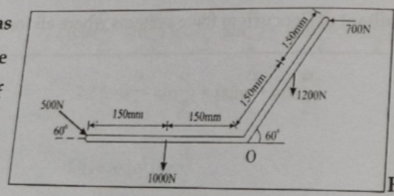


Fig 1.44

Solution:

$\Sigma F_x\ (\rightarrow +ve) = 500 \cos 60° - 700$

$\therefore \Sigma F_x = -450N = 450N\ (\leftarrow)$

$\Sigma F_y\ (\uparrow +ve) = -500 \sin 60° - 1000 - 1200$

$\therefore \Sigma F_y = -2633N = 2633N\ (\downarrow)$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\therefore R = \sqrt{450^2 + 2633^2} = 2671.2N$

$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right) = \tan^{-1}\left(\frac{2633}{450}\right) = 80.3°$

16) Determine completely the resultant of the four coplanar forces shown in the fig 1.45. Locate the line of action of the resultant with respect to 'A'

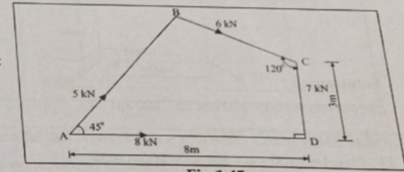


Fig 1.45

Solution:

$\Sigma F_x\ (\rightarrow +ve) = 5 \cos 45° + 8 + 6 \cos 30° = 16.73KN\ (\rightarrow)$

$\Sigma F_y\ (\uparrow +ve) = 5 \sin 45° - 7 - 6 \sin 30° = -6.46KN = 6.46KN\ (\downarrow)$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{16.73^2 + 6.46^2}$

$\therefore R = 17.93KN$

$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right) = \tan^{-1}\left(\frac{6.46}{16.73}\right) = 21.1°$

Using Varignon's theorem,

$\Sigma M_A{}^F = M_A{}^R\ (\circlearrowleft) +ve$

$\therefore (7 \times 8) + (6 \times 6 \sin 30°) - (4.15 \times 6 \cos 30°) = 17.93 \times d$

d = 5.33m ($\because$ d is +ve, moment will be $\circlearrowleft$)

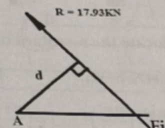


Fig 1.46

17) Determine the resultant of general coplanar force system shown in fig 1.47.

Solution: $\Sigma Fx(\rightarrow +ve) = 7 \cos 42° + 13 + 10 \sin 30° = 23.2KN\ (\rightarrow)$; $\Sigma Fy\ (\uparrow +ve) = -18 - 6 - 10\cos 30 + 7\sin 42 = -27.98KN = 27.98KN\ (\downarrow)$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \therefore R = \sqrt{23.2^2 + 27.98^2}$

$= 36.3KN$

$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right) = \tan^{-1}\left(\frac{27.98}{23.2}\right) = 50.33°$

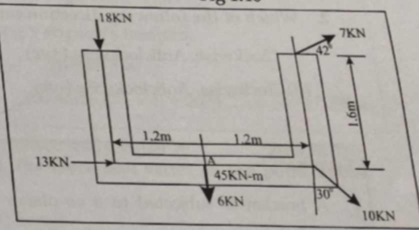


Fig 1.47

Moment about point A,

Using Varignon's theorem,

$\Sigma M_A{}^F = M_A{}^R\ (\circlearrowleft) +ve$

$\therefore (18 \times 1.2) - (1.6 \times 7 \cos 42°) + (1.2 \times 7 \sin 42°) = 36.3 \times d; \qquad d = 1.474m$

18) Determine the resultant of four forces tangential to the circle of radius 4 cm as shown. What will be the location of the resultant with respect to the center of the circle?

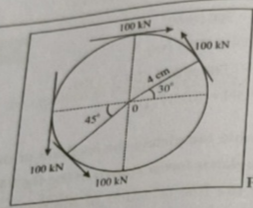


Fig 1.48

Solution:

$\Sigma F_x (\rightarrow +ve) = 100 + 100 \cos 45 - 100 \sin 30$

$\therefore \Sigma F_x (\rightarrow +ve) = 120.7$ kN $(\rightarrow)$

$\Sigma F_y (\uparrow +ve) = -100 - 100 \sin 45 + 100 \cos 30$

$\therefore \Sigma F_y (\uparrow +ve) = -84.1$ kN $= 84.1$ kN $(\downarrow)$

Using $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\therefore R = \sqrt{120.72^2 + 84.1^2} = 147.1$KN

$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right) = \tan^{-1}\left(\frac{84.1}{120.7}\right) = 34.86°$

Location of resultant force,

Using Varignon's theorem

$\Sigma M_o^F = M_o^R$ (↺) +ve

$\therefore -100 \times 4 + 100 \times 4 + 100 \times 4 + 100 \times 4 = -147.1 \times d$

d = - 5.44 m or d = 5.44 m (left of 'O')

1. *How to locate the resultant of a given force system?*
   - i) Lami's Theorem
   - ii) Varignon's Theorem
   - iii) Sine Rule
   - iv) Ohm's Law
2. *Which of the following direction convention of moment of a force is correct?*
   - i) Clockwise, Anticlockwise (+ve)
   - ii) Clockwise (-ve), Anticlockwise (+ve)
   - iii) Clockwise, Anticlockwise (-ve)
   - iv) None of the above

Exercise:

1. A bracket is subjected to a co-planer force system as shown in figure Determine the magnitude and line of action of the resultant. **[Ans. R = 252.18 N, θ = 37.52°, x = 350 m]**

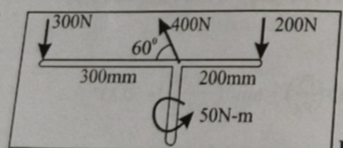


Fig 1.49

Questions/Problems for Practice for the day:

1. Find the resultant of coplanar force system given below and locate the same on AB with consideration of applied moment of 4800N-mm **[Ans: R= 510N, θ = 66.95°, passing through point A]**

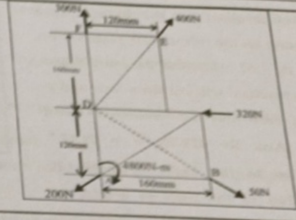


Fig 1.50

Learning from the lecture 'Resultant of Parallel & General force systems': Learner will able to find the resultant of Parallel & General force systems and position of the resultant

Lecture: 9

1.1.5 *Shifting of a Force*

Learning Objectives: Learners will be able to know shifting of a force from one point to another.

Solved Problems

19) Resolve the system of forces shown in fig. 1.51 into a force & couple at point A.

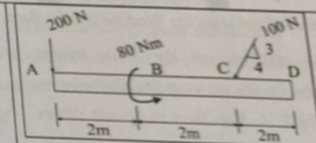


Fig 1.51

Solution: $\theta = \tan^{-1}(3/4) = 36.87°$

$\Sigma Fx (\rightarrow +ve) = -100 \cos 36.87°$

$\therefore \Sigma Fx = -80N = 80N (\leftarrow)$

$\Sigma F_y (\uparrow +ve) = -200 - 100 \sin 36.87°$

$\therefore \Sigma F_y = -260N = 260N (\downarrow)$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\therefore R = \sqrt{80^2 + 260^2} = 272.03$KN

$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right) = \tan^{-1}\left(\frac{260}{80}\right) = 72.89°$

Using Varignon's theorem,

$\Sigma M_A^F = M_A^R$ (↺) +ve

$80 - (100 \sin 36.87° \times 4) = 2671.2 \times d$; $d = 0.59$m

1. **The moment of a force about any point is geometrically equal to ………. area of the triangle whose base is the line representing the force and vertex is the point about which the moment is taken**
   - i) Half
   - ii) Same
   - iii) Twice
   - iv) Zero
2. **Which of the following statement is correct?**
   - i) A force is an agent which produces or tends to produce motion
   - ii) A force is an agent which stops or tends to stop motion

Exercise:

1. The resultant of the force system acting on the rectangular plate shown in fig 1.52. Also find the point where the resultant will cut the x- axis and y- axis. Also shift the resultant to point 'A' **[Ans: R= 471.7N, θ = 21.12 °, 3.217m to the right of point B, x=13.24m to the right of origin, y= 5.11m above the origin]**

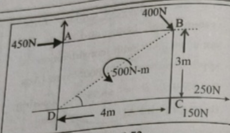


Fig 1.52

2. Resolve the force 'F' equal to 900N acting at B, as shown in fig 1.53 into
   (i) Parallel components at A & 0,
   (ii) A couple and force at O.

[Ans. F_A = 2700 N (↓) F_o = 1800 N (↓),F =900 N (↓) M=2700 N-m]

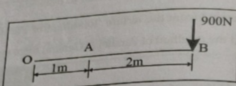


Fig 1.53

Questions/Problems for Practice for the day:

1. The resultant of the force system acting on the rectangular plate shown in fig 1.54. Also find the point where the resultant will cut the x- axis & y- axis. Also shift the resultant to point 'B' **[Ans: R=201.4N, θ = 59.87 °, x= 0.3866m to the left of origin, Y= 0.666m above the origin]**

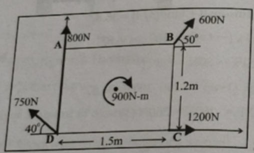


Fig 1.54

Learning from the lecture 'Shifting of a force': Learner will be able to know shifting of a force from one point to another

1.3 Conclusion:

Learning Outcomes: Learners should be able to

- **Know, Comprehend**

1. Define different types of forces and fundamental parameters such as tensile, compressive, point of application, etc.
2. Identify and locate the forces in a figure with respect to different axis.

- **Apply, Analyze**

3. Locate different types of forces in a figure by considering position of resultant and the moment of forces.
4. Find the equilibrant of a system, which can bring a system into equilibrium.

- **Synthesize**

5. Analyze fundamental parameters for different forces and centroid for plane lamina.

1.4 Add to Knowledge (Content Beyond Syllabus)

- Centroids indicates centre of mass of a uniform solid. Stick a pivot at a centroid and the object will be in perfect balance.
- Lots of construction applications & engineering applications to design things so that minimal stress and energy is used to stabilize a component.
- In stress and deflection analysis of a beam, the location of centroid is very important.
- Recent Research: https://calcresource.com/centroid-how-to-find.html

1.5 Set of Multiple-Choice Questions:

1) Density is best given by ______
   a) Product of volume and density
   b) Ratio of mass to Volume
   c) Addition of mass and density
   d) Subtraction of mass and density
2) If solving the question in 3D calculations is difficult, then use the 2D system and then equate the ratio of the product of the centroid of the section to its mass to the total mass of the body to the centroid.
   a) True
   b) False
3) One of the uses of the centroid is as in the simplification of the loading system the net force acts at the __________ of the loading body.
   a) Centroid
   b) The centre axis
   c) The corner
   d) The base
4) The use of centroid comes in picture as if the non-Uniform loading is of the type of parabola then what will be the best suited answer among the following?
   a) The net load will not be formed as all the forces will be cancelled
   b) The net force will act the centre of the parabola
   c) The net force will act on the base of the loading horizontally
   d) The net force will act at the centroid of the parabola
5) The x axis coordinate and the y axis coordinate of the centroid are having different types of calculations to calculate them.
   a) True
   b) False
6) The centre of __________ is the ratio of the product of centroid and volume to the total volume.
   a) Centroid axis
   b) Density
   c) Mass
   d) Volume

7) If the force vector F acting along the centroid is having its x-axis component being equal to Z N, y-axis component be X N and z-axis component be Y N then vector F is best represented by?

a) Xi + Yj + Zk b) Yi + Xj + Zk c) Zi + Yj + Xk d) Zi + Xj + Yk

8) Centroid of a body does depends upon the small weights of tiny particles. Which statement is right for force acting by the small particles of the body having it's vector form as = Ai + Bj + Ck?

a) In rectangular components representation of any vector we have vector F = Ai + Bj + Ck

b) In rectangular components representation of any vector we have vector F = Ax + By + Cz

c) In rectangular components representation of any vector we have vector F = Fx + Fy + Fz

d) In rectangular components representation of any vector we have vector F = Fi + Fj + Fk

9) Centroid determination involves the calculations of various forces. In that forces are having various properties. That is force is developed by a support that does not allow the ________ of its attached member.

a) Translation b) Rotation c) Addition d) Subtraction

10) Determine the y coordinate of centroid of the area in the shape of circle as shown.

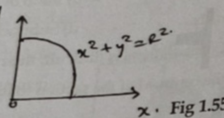


Fig 1.55

a) 4R/π b) 2/π c) 2R/3 d) 2R/5

11) What is not the condition for the equilibrium for the calculations used for the determination of the centroid in three dimensional system of axis?

a) ∑Fx=0 b) ∑Fy=0 c) ∑Fz=0 d) ∑F≠0

12) Determine the x coordinate of centroid of the area in the shape of circle as shown.

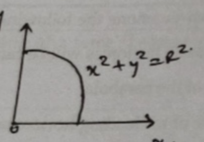


Fig 1.56

a) 4R/π b) 2/π c) 2R/3 d) 2R/5

1) B	2) B	3) A	4) D	5) B	6) D	7) D	8) C	9) A	10) A	11) D	12) A

1.6 Short Answer Questions:

1) Find the difference between centroid, centre of gravity & centre of mass.

2) Determine centroid of an equilateral triangle having base & height on x-axis & y-axis

respectively.

3) Locate the centroid of a semicircle having center O at origin and diameter on y-axis.

4) Locate the centroid of an area as shown in fig 1.57.

5) Locate the centroid of a L section having base of 10cm on x-axis and 8cm height on y-axis.

6) Locate the centroid of an area as shown in fig 1.58. having radius R

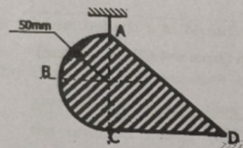


Fig 1.57

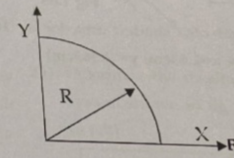


Fig 1.58

7) Forces act on the plate ABCD as shown in the fig 1.59. The distance AB is 4m. Given that the plate is in equilibrium find: Force F, Angle α [Ans: F= 13N, α= 67.4°]

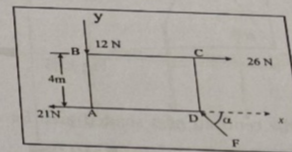


Fig 1.59

8) State and prove Varignon's Theorem

9) Define Lami's Theorem and explain it with neat diagram

10) Determine the magnitude and direction of the resultant of the forces shown in the given fig 1.60. **[Ans.: 178 N, 355°]**

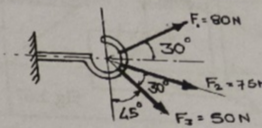


Fig 1.60

11) Determine the equilibrant of the co-planer concurrent forces shown in fig 1.61. **[Ans: R= E = 97.95N, θ =26.11°]**

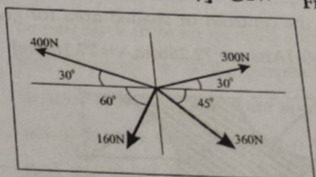


Fig 1.61

12) Determine the resultant of the following parallel forces. **[Ans: R= 80N]**

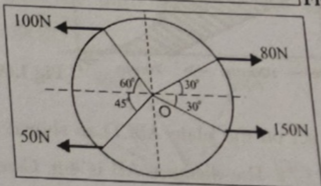


Fig 1.62

1.7 Long Answer Questions:

13) Locate the centroid of a plane lamina for

14) Locate the centroid of a shaded area fo

given fig 1.63(**Ans: x= 1.532 m, y= 1.721 m**)

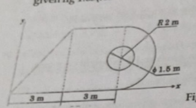


Fig 1.63

given fig 1.62 (**Ans: x=0.349a, y= 0.637a**)

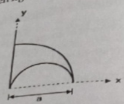


Fig 1.64

15) Locate the centroid of a shaded area for a given fig 1.65 (**Ans: x=2.63cm, y= 2.365cm**)

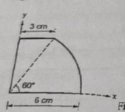


Fig 1.65

16) Locate the centroid of a shaded area for a given fig 1.66 (**Ans: x=4.625cm, y= 1cm**)

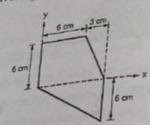


Fig 1.66

17) Locate the centroid of a shaded area for a given fig 1.67 (**Ans: x=5a/12 y= 5a/12**)

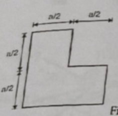


Fig 1.67

18) Find the centroid of the shaded area as shown in fig 1.68. [**Ans.: x = 17.673cm, y = 11.835cm**]

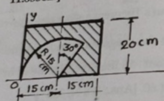


Fig 1.68

19) Find the centroid of shaded area for given fig 1.69. [**Ans: x= 72.26mm, y= 77.16mm**]

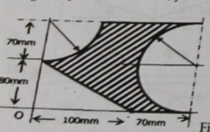


Fig 1.69

20) Determine the CG of the shaded area for given fig. 1.70. [**Ans: x= 125cm, y= 31.81cm**]

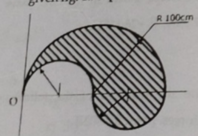


Fig 1.70

21) Forces act on the plate ABCD as shown in the fig. 1.71. The distance AB is 4m. Given that the plate is in equilibrium find - Force F, Angle α, Distance AD [**Ans: F= 13N, α=**

22) Two concurrent forces P & Q acts at O such that their resultant acts along x-axis for given fig 1.72. Determine the magnitude of Q and hence resultant. [**Ans: Q=300N, R=**

67.4°, AD= 8.66m]

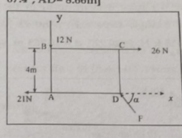


Fig 1.71

595.69N]

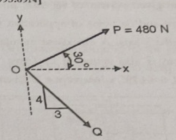


Fig 1.72

23) Determine the resultant of the forces as given in the fig. 1.73. Find the angle which the resultant makes with positive x-axis. [**Ans: R= 100.919 N, θ=30.45°**]

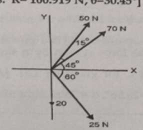


Fig 1.73

24) Determine the resultant of the vertical force system shown in fig 1.74. [**Ans: M=4500 N-m (↺)**]

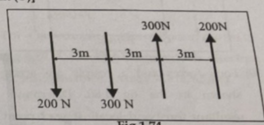


Fig 1.74

25) Determine the resultant of the non-concurrent, non-parallel system of forces as shown in fig 1.75. [**Ans: R= 175N, θ = 55°, 1.317m to the right of point O.**]

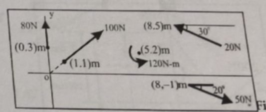


Fig 1.75

26) For the system shown, determine (a) The required value of if resultant of three forces is to be vertical, (b) The corresponding magnitude of resultant [**Ans: 21.8°, 229.29N (↓)**]

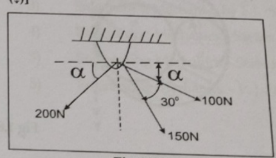


Fig 1.76

27) To pull hook from an inclined surface, total pull required is 800N perpendicular to inclined surface. 3 forces are applied out of which two of them are shown in fig 1.77, find the third force.[**Ans: R= 426N, θ = 58.83°**]

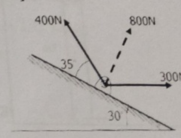


Fig 1.77

28) For the given system of fig 1.78, find the resultant and its point of application w.r.t. 'O' on the x-axis. A clockwise moment of 5000N-cm is also acting at O. **[Ans: R= 341.42 N, θ=64.79°, x-intercept = 17.23 cm]**

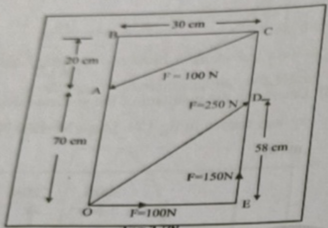


Fig 1.78

29) Replace the force system given in the fig 1.79 by a single force w.r.t point C. **[Ans: R= 78.115 k N, θ=20.22°, d = 10.763 m right of C]**

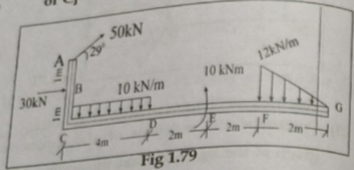


Fig 1.79

30) Four forces and a couple are acting as shown in the fig 1.80. Determine the resultant force and locate it w.r.t point A. **[Ans: R= 305.28 N, θ=31.6°, d = 248.95m]**

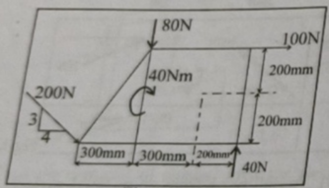


Fig 1.80

31) Replace the force system by a single force w.r.t point C for given fig 1.81. **[Ans: R= 22.36 N, θ=26.56°, d = 35 m right of C]**

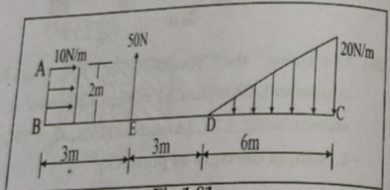


Fig 1.81

32) Replace loading on frame fig 1.82, by a force & moment at point A. **[Ans: R= 921.9N, d= 3.47m]**

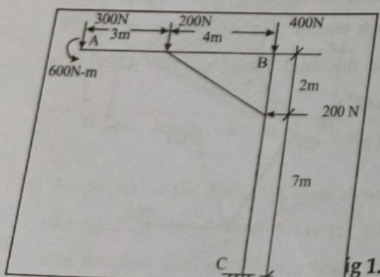


Fig 1.82

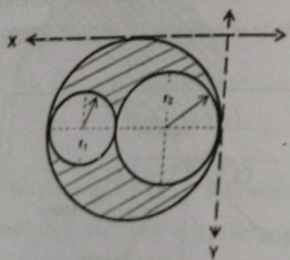


Fig 1.83

33) Determine the centroid of the shaded area shown in fig 1.83 with respect to given reference axes (Given: r_1 = 13cm, r_2 = 23cm) **(Ans: -40.99cm, -36cm)**

34) Determine the centroid for the given shaded area for fig 1.84 **(Ans: 107.19mm,107.19mm)**

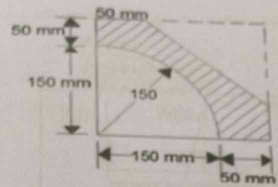


Fig 1.84

35) Determine the centroid for the given shaded area for fig 1.85 **(Ans: 7.50, 5.08)**

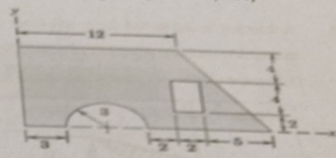


Fig 1.85

36) Determine the centroid for the given shaded area for fig 1.86 **(Ans: -10mm, 87.12mm)**

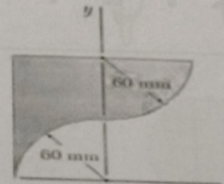


Fig 1.86

37) For a non-concurrent force system shown in fig 1.87. Determine magnitude & direction of resultant force. **(Ans: 6.71kN, 63.49°)**

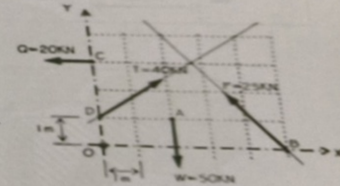


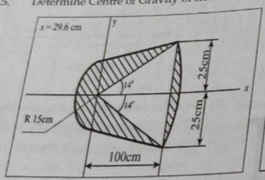
Fig 1.87

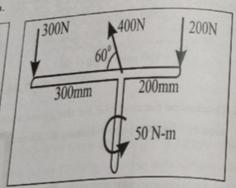
1.8 References:

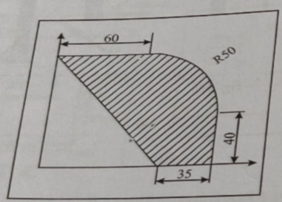
1) Engineering Mechanics byTayal, Umesh Publication
2) Engineering Mechanics by Beer & Johnson, Tata McGraw Hill
3) Engineering Mechanics by F.L. Singer by Harper
4) Engineering Mechanics – Statics, R. C. Hibbler
3) Engineering Mechanics – Statics, J. L. Merium, I. G. Kraig
5) Engineering Mechanics – P. J. Shah, R. Bade

Self-Assessment

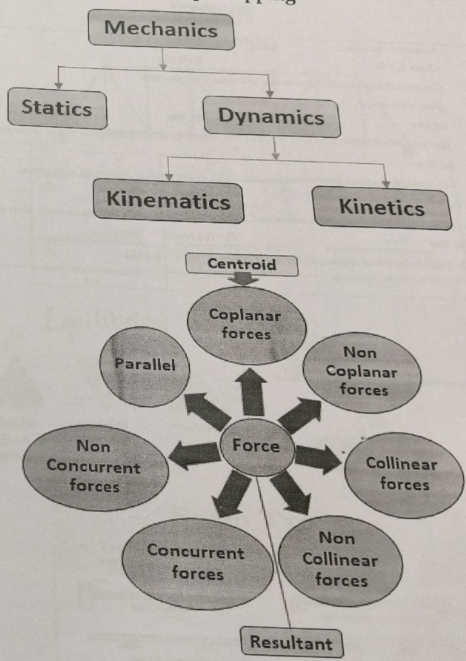
1. What is the difference between moment & couple? (Level 1)
2. Why is force called a sliding vector? (Level 2)
3. Can you guess where will the centroid of this figure will lie? (Level 3)
4. A bracket is subjected to a co-planer force system as shown in fig. Determine the magnitude and line of action of the resultant. (Level 4)
5. Determine Centre of Gravity of shaded area. (Level 5)







Concept Mapping



Self-Evaluation

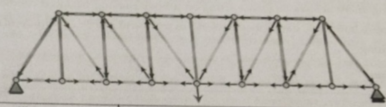
Name of Student: Course Code:

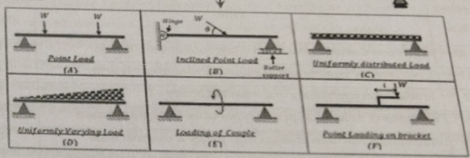
Class & Div.: Roll No.:

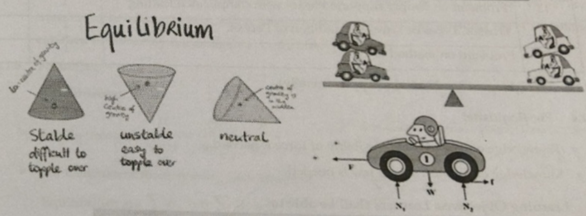
1. Can you define parallelogram law of forces, Couple, Varignon's theorem?
   (a) Yes (b) No
2. Are you able to state different types of forces & formulae of different figure in centroid?
   (a) Yes (b) No
3. Are you able to state the properties of couple, derivation of Varignon's theorem?
   (a) Yes (b) No
4. Are you able to solve numerical based on Centroid of plane lamina and resolution of forces?
   (a) Yes (b) No
5. Do you understand this module? (a) Yes (b) No

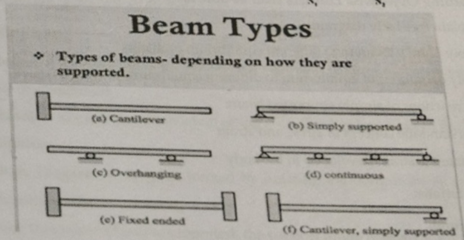
Module 2: Equilibrium of System of Coplanar Forces & Trusses

Infographics









Lecture 10

2.1 Equilibrium

2.1.1 Motivation:

The balanced bodies are said to be in equilibrium. So, it becomes necessary to understand arrangement of various forces acting on a body causing balanced condition. All structures (trusses), buildings, dams, machines etc. in the world satisfy conditions of equilibrium while designing.

2.1.2 Syllabus:

Lecture No.	Title	Duration (Hrs.)	Self-Study (Hrs.)
10	Laws and theorems, Free Body Diagram, Lami's Theorem	1	2
11	Conditions of Equilibrium of general & concurrent system	1	2
12	Problems on equilibrium of Connected bodies – I	1	2
13	Reactions of different support, Types of loads, types of beams	1	2
14	Problems on reactions of beam with pulley & external mass	1	2
15	Problems on Simply supported beam with complicated loading	1	2
16	Trusses, Types of Trusses, Condition of Perfect.	1	2
17	Problems on method of joints	1	2

2.1.3 Weightage: 19 to 23 Marks (Approximately)

2.1.4 Pre-Requisite:

- Knowledge of Law of transmissibility of force is needed.
- Knowledge of Newton's third law is needed.

2.1.5 Learning Objectives: Learners shall be able to

1) Explain free body diagram
2) Apply Lami's theorem to different equilibrium problems
3) Apply conditions of equilibrium to different equilibrium problems
4) Find reactions of simply supported beam.
5) Understand the concept of stress and strain
6) Calculate the strain developed in the body

2.1.6 Key notations:

➢ **Equilibrium:**

$\sum F_x$ = Summation of all horizontal components of forces

$\sum F_y$ = Summation of all vertical components of forces

$\sum M$ = Summation of Moments of all forces taken about a point

2.1.7 Theoretical Background:

A very basic concept when dealing with forces is the idea of equilibrium or balance. In general, an object can be acted on by several forces at the same time. A force is a vector quantity which means that it has both a magnitude (size) and a direction associated with it. If the size and direction of the forces acting on an object are exactly balanced, then there is no net force acting on the object and the object is said to be in equilibrium. Because there is no net force acting on an object in equilibrium, then from Newton's first law of motion, an object at rest will stay at rest, and an object in motion will stay in motion.

The ends of a truss are pinned, so that they don't carry moments. The only reactions at the ends of a truss member are forces. External forces on trusses act only on the end points. Truss problems are solved by the method of sections, where an imaginary cut is made through the member(s) of interest, and global equilibrium of forces and moments are used to determine the forces in the members, or by the method of joints, in which a single joint is isolated and analyzed and the resulting forces (not necessarily with a numerical value) are transferred to adjacent joints, where the process is repeated. The resulting set of equations can then be solved by linear algebra, or substitution.

2.1.8 Formulae:

Equilibrium:

➢ Lami's theorem: $\dfrac{P}{\sin\alpha} = \dfrac{Q}{\sin\beta} = \dfrac{R}{\sin\gamma}$

➢ For Concurrent force system: Conditions of Equilibrium are $\sum F_X = 0 \; \sum F_Y = 0$

➢ For General force system: Conditions of Equilibrium are $\sum F_x = 0; \sum F_y = 0$ & $\sum M = 0$

2.1.9 Key definitions:

1) **Equilibrium:** If the resultant of the force system happens to be zero, the system is said to be in a state of equilibrium.
2) **Equilibrant:** A single force which when acting with all other forces keeps the body at rest or in equilibrium.
3) **Free Body Diagram:** A diagram formed by isolating the body from its surrounding and then showing all the forces acting on it.
4) **Reaction:** Whenever a body is supported, the support offers resistance, known as reaction.

2.1.10 Laws and theorems:

- **Law of equilibrium of two forces:** Two forces can be in equilibrium only if they are equal in magnitude, opposite in direction and collinear in action.
- **Lami's Theorem:** If three concurrent forces are in equilibrium then their magnitudes are proportional to the angle between the other two forces. If a body is in equilibrium under the action of three non-collinear coplanar and concurrent forces, then each force is proportional to the sine of the angle between the other two forces.

2.1.11 Theory:

- **Free Body Diagram:** It is diagram of body under consideration. The diagram shows magnitude, direction and point of applications of all external, active and reactive forces acting on the body. In the diagram unknown forces, corresponding directions are labeled. By including the necessary dimension in FBD, moments of the forces can be easily analyzed. Thus, it is very important and very first step in analysis of problems on equilibrium.

Rules to be followed while drawing the free body diagram:

1) Identify the object for which FBD need to draw. (If multiple objects then there will be multiple FBD).
2) Draw only object and forces acting on it.
3) Draw a shape that is roughly the same shape as the original object. You do not need to include details which are not important to the problem. For example, a standing person might be drawn by a narrow vertical box.
4) All FBD's must include the coordinate system that applies that diagram. You can use different coordinate systems for different FBD's within the same problem
5) Forces should be drawn as arrows coming from the side of the object where the force originates. The arrow should point in the direction of the force
6) All forces of known magnitude should be labeled with that magnitude (e.g. "50N" or "8.4lb"). All unknown forces should be labeled with the symbol you will use when writing your equations of motion for the object (e.g. F_{12} might be used for the force of object 1 on object 2. F_G might be used to identify the force of gravity on an object of unknown mass. F_N can represent the normal for an object resting on a table.)
7) Weight of a particle or rigid body always acts vertically downward through its center of gravity irrespective of its position (Horizontal or Inclined).
8) Axial forces in any member may be assumed as tensile, if result comes negative then force will have to considered compressive. (The direction of reactions generated due to support can be assumed)

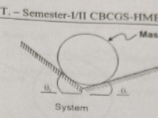


Fig 2.1

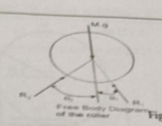


Fig 2.2

Solved Problems on F.B.D.

1) Draw Free body diagram of a lamp weighing 150 N is supported by two cables AC and BC. (**Ans: Fig 2.4**)

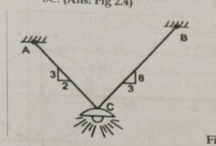


Fig 2.3

Solution:

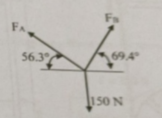


Fig 2.4

2) Draw Free body diagram for given fig 2.5, showing a 10 kg lamp supported by two cables AB and AC. Find the tension in each cable. (**Ans: Fig 2.6**)

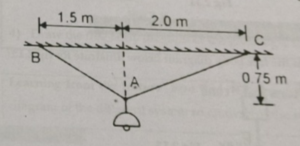


Fig 2.5

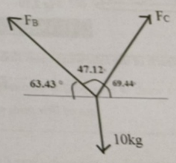


Fig 2.6

3) A roller of weight W = 1000 N rest on a smooth incline plane. It is kept from rolling down the plane by a string AC. Draw free body diagram for given fig 2.7.

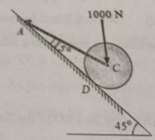


Fig 2.7

4) The weight of roller is 1500 N. 'P' is the minimum force required to start the roller over the block. Draw the FBD for given fig 2.8. (**Ans: Fig 2.9**)

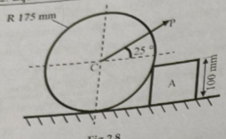


Fig 2.8

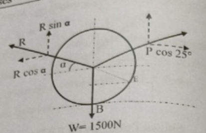


Fig 2.9

5) Draw FBD of cylinder and frame as shown in fig 2.10. (Ans: Fig 2.11 & 2.12)

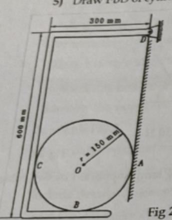


Fig 2.10

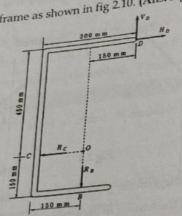


Fig 2.11

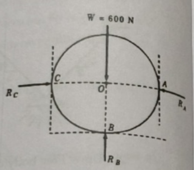


Fig 2.12

Let's check the takeaway from this lecture

1. Which of the following cases does the free body diagram below illustrate for fig 2.13?

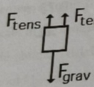


Fig 2.13

i) A block supported by two strings
ii) A block supported by two springs
iii) A block lifted by two forces
iv) A block falling off a table

2. The FBD of a Sphere on a table will have

i) 2 Forces
ii) 3 Forces
iii) 4 Forces
iv) 1 Force

3. For drawing the FBD of the door, the door will have to be

i) Cut in 2 sections
ii) Supported by the wall
iii) Isolated from the hinges
iv) Inverted upside down

Exercise:

1) Draw a Free Body Diagram for a book is at rest on a tabletop shown in fig 2.15.
2) Draw a Free Body Diagram for given fig 2.14

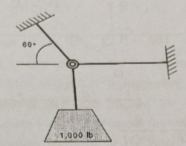


Fig 2.14

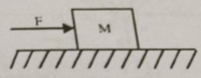


Fig 2.15

Questions/Problems Practice for the day:

3) Draw FBD of a block suspended to the ceiling using three strings shown in fig 2.16.

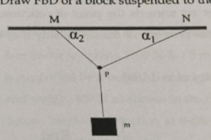


Fig 2.16

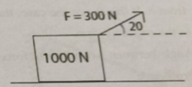


Fig 2.17

4) Draw the free-body diagram of the given fig 2.17

Learning from the lecture 'Free Body Diagram': Learner will be able to draw free body diagram of the different system so analysis of the system will be easy.

2.2 Lami's Theorem

Learning Objective: Student will be able to apply Lami's theorem to different equilibrium problems

Lami's Theorem: *It states, if a body is in equilibrium under the action of 3 coplanar & concurrent forces, then each force is proportional to the 'sine' of the angle between other two forces.*

Proof: Let P, Q and R be the 3 forces acting at point 'O' in equilibrium.

Let R_1 be the resultant of two forces P and Q. Now point 'O' is subjected to only two forces R and R_1. As per equilibrium under two forces R and R_1 must be equal, opposite and collinear.

Now, by sine rule

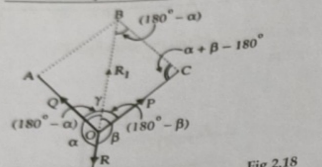


Fig 2.18

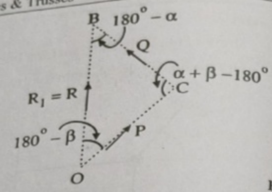


Fig 2.19

$$\frac{P}{\sin(180^\circ-\alpha)} = \frac{Q}{\sin(180^\circ-\beta^\circ)} = \frac{R}{\sin(\alpha+\beta-180^\circ)} \qquad \frac{P}{\sin(\alpha)} = \frac{Q}{\sin(\beta)} = \frac{R}{\sin(360^\circ-\gamma-180^\circ)}$$

$$\frac{P}{\sin(\alpha)} = \frac{Q}{\sin(\beta)} = \frac{R}{\sin(\gamma)}$$

Limitations: -

1) *It is applicable only when three forces acting at a point are in equilibrium.*
2) *The three concurrent forces should either act towards the point of concurrence or away from it. If this is not the case, then using the principle of transmissibility they can be made in the required form.*
3) *Angle between two adjacent forces should not exceed 180°*

Solved Problems:

1) *A circular roller of weight 1000 N and radius 20 cm hangs by a tie rod AB =40 cm and rests against a smooth vertical wall at C as shown in Fig. 2.20. Determine the tension in the tie rod and reaction RC at point C.*

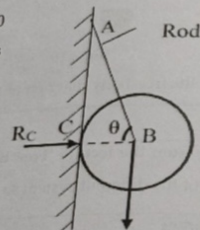


Fig 2.20

Solution: *Draw the F.B.D. of the roller in equilibrium.*

From geometry of Right-angled triangle ABC

$\cos\theta = BC/AB = 20/40$, $\theta = 60°$

Let, T = Tension in rod

R_C = Reaction at point C

Using Lami's theorem we get,

$$\frac{T}{\sin 60^\circ} = \frac{R_c}{\sin 150^\circ} = \frac{1000}{\sin 120^\circ}$$

Therefore T =1154.7 N, R_C = 577.35 N

Let's check the takeaway from this lecture

1. *The Lami's theorem is applicable only for*
   - i) Coplanar – Non-concurrent Forces
   - ii) Coplanar – Concurrent Forces
   - iii) Non-Coplanar – Non-concurrent Forces
   - iv) Non-Coplanar – Concurrent Forces
2. The Total angle between all forces to apply Lami's theorem, should not exceed

- i) 120°
- ii) 180°
- iii) 360°
- iv) 720°

3. Angle between 2 forces should not cross
   - i) 120°
   - ii) 180°
   - iii) 360°
   - iv) 720°

Exercise

1) A system of connected flexible cables shown in fig 2.21 is supporting two vertical forces 200 N and 250 N at points B and D. Determine the forces in various segments of the cable. (**Ans: T_{DE} 224.14 N, T_{BD} = T_{BC} = 183.01 N 336.6 N, T_{AB}=326.79 N)**

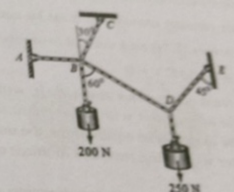


Fig 2.21

Questions/Problems for Practice for the day:

2) A cylinder weighing 1000 N & 1.5 m dia. is supported by a beam AB of length 6 m and weight 400 N as shown in the figure below. Neglecting friction at the surface of contact of the cylinder, determine (i) Wall reaction at D, (ii) Tension in the cable BC and (iii) Hinged reaction at support A. (**Ans: R_D=1000N, T_{BC}= 588.08 N, H_A= 490.7 N, V_A= 1105.96 N)**

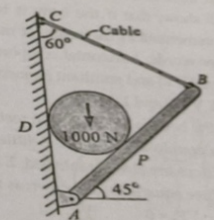


Fig 2.22

3) An electric light weighs 15N hangs from point C, ssas shown in fig 2.23. AC is inclined at 60° to horizontal & BC at 45° to vertical. Using Lami's theorem, find the forces in the strings AC & BC. (**Ans: 10.98N, 13.44N)**

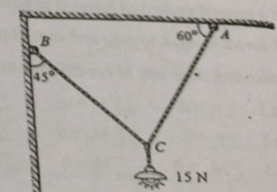


Fig 2.23

Learning from the lecture 'Lami's theorem': Student will be able to apply Lami's theorem to the different system so calculation of different forces can be done.

2.3 Conditions of Equilibrium of general & concurrent system of forces

Learning Objective-Student will be able to apply conditions of equilibrium to sphere system.

- **Conditions of equilibrium of concurrent system of forces.**

1. If the body moves in any direction, it means that there is a resultant force acting on it. A little consideration will show, that if the body is to be at rest or in equilibrium, the resultant force causing movement must be zero. Or in other words, the horizontal component of all the forces (ΣH) and vertical component of all the forces (ΣV) must be zero. Mathematically, $\Sigma H = 0$ and $\Sigma V = 0$

2. If the body rotates about itself, without moving, it means that there is a single resultant couple acting on it with no resultant force. A little consideration will show, that if the body is to be at rest or in equilibrium, the moment of the couple causing rotation must be zero. Or in other words, the resultant moment of all the forces (ΣM) must be zero. Mathematically, $\Sigma M = 0$

3. If the body moves in any direction and at the same time it rotates about itself, if means that there is a resultant force and also a resultant couple acting on it. A little consideration will show, that if the body is to be at rest or in equilibrium, the resultant force causing movements and the resultant moment of the couple causing rotation must be zero. Or in other words, horizontal component of all the forces (ΣH), vertical component of all the forces (ΣV) and resultant moment of all the forces (ΣM) must be zero. Mathematically, $\Sigma H = 0$ $\Sigma V = 0$ and $\Sigma M = 0$

4. If the body is completely at rest, it necessarily means that there is neither a resultant force nor a couple acting on it. A little consideration will show that, in this case the following conditions are already satisfied: $\Sigma H = 0$ $\Sigma V = 0$ and $\Sigma M = 0$

The three equations are known as the conditions of equilibrium.

Set 1 $\sum F_x = 0$ $\sum F_y = 0$

Set 2 $\sum F_x = 0$ $\sum M_A = 0$, where A is not on X - axis.

Set 3 $\sum M_A = 0$ $\sum M_B = 0$, where A and B are non collinear with concurrence.

- ***Conditions of equilibrium for general system of forces:***

If a general system of forces is in equilibrium, then sum of forces along any axis in the plane of forces is equal to zero and sum of moments about any point in the plane is equal to zero.

Following three sets of conditions can be used.

Set 1 $\Sigma F_x = 0$ $\sum F_y = 0;$ $\sum M_A = 0$

Set 2 $\Sigma F_x = 0$ $\sum M_A = 0;$ $\sum M_B = 0$ [line AB not perpendicular to x axis]

Set 3 $\sum M_A = 0$ $\sum M_B = 0$ $\sum M_C = 0$ [where A, B, C are not on one line]

- **Equilibrium of body subjected to two forces (Fig 2.24 & 2.25):** If the body is subjected to forces acting only at two points, then it is called a Two – Force body. A body will be in equilibrium

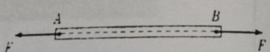


Fig 2.24

only when two forces are equal in magnitude, opposite in direction and have the same line of action.

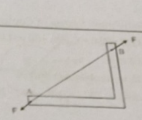


Fig 2.25

- **Equilibrium of body subjected to three force (Fig 2.26):** When a body is in equilibrium and acted upon by three forces then these three forces will be either concurrent or parallel forces

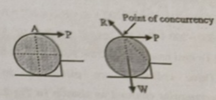


Fig 2.26

Solved Problems:

2) **Two cylinders P & Q in a channel as shown in fig 2.27. The cylinder 'P' has a diameter of 100mm & weight 200N and 'Q' has 180 mm diameter & weight 500N. Determine the reaction at all the contact surfaces.**

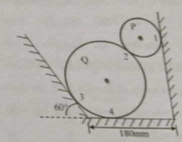


Fig 2.27

Solutions: Fig 2.28 shows FBD of combined cylinders P and Q. Reactions R_1, R_2, R_3 and R_4 are perpendicular to their respective surfaces.

From geometry of figure, AD is the angle bisector of angle HAG.

angle DAG = angle DAH = 30°

From triangle DAG, tan 30° = DG/AG = X_1/90

X_1 = 90 tan 30° = 51.96 mm

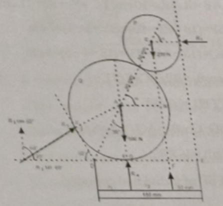


Fig 2.28

Also, length DE = 180 = $X_1 + X_2 + 50 = 51.96 + X_2 + 50$; $X_2 = 78.04$ mm

Applying COE to combined cylinders

(+ ve Anticlockwise) $\Sigma M_A = 0$; $R_1 \times BC - 200 \times AB = 0$

$R_1 \times 116.23 - 200 \times 78.04 = 0$; $R_1 = 134.3$ N (←)

$\Sigma F_x = 0$ (→); $-R_1 + R_3 \sin 60° = 0$;

$-134.3 + 0.866\, R_3 = 0$; ∴$R_3 = 155$ N at $\theta = 30°$

$\Sigma F_y = 0$ (↑); $-500 - 200 + R_4 + R_3 \cos 60° = 0$

$-500 - 200 + R_4 + 155 \cos 60° = 0$; $R_4 = 622.5$ N

(↑) To find reaction R_2, draw FBD of cylinder P separately as shown in fig 2.29

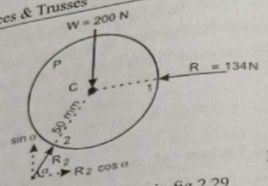


Fig 2.29

Applying COE to cylinder P;

$\Sigma F_y = 0$ (↑); $R_2 \sin \alpha - W = 0$;

$R_2 \sin 56.12° - 200 = 0$; $R_2 = 240.8$ N

3) Three cylinders are piled up in a rectangular channel as shown in Fig 2.30. Determine the reaction at point 6 between cylinder A and the vertical wall of the channel. (Cylinder A : R_A = 4 cm, M_A =15 Kg, Cylinder B : R_B =6 cm, M_B= 40 Kg, Cylinder C : R_C = 5 cm ,M_C = 20Kg)

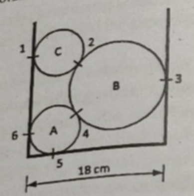


Fig 2.30

Solution: Draw FBD of the combined cylinders. Reaction R_1, R_3, R_5 & R_6 are perpendicular to their respective surfaces.

From the geometry of the fig 2.31, we have R_A = 4 cm & R_C =5 cm; M_A =15 Kg & M_C =20 Kg ;

R_B =6 cm, M_B=40 Kg

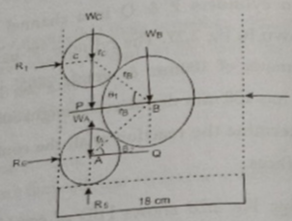


Fig 2.31

Length AB =R_A+R_B=10 cm, BC =R_C+R_B= 11 cm

Length BP=(18 −C1 −B3)=(18 −5 −6)=7 cm

Length AQ=(18 −A6 −B3)=(18 −4 −6)=8 cm

In ΔBCP; $\cos\theta_1$ =BP/BC=7/11

$\theta_1 = \cos^{-1}(7/11) = 50.48°$

and in ΔABQ; $\cos\theta_2$ =AQ/AB =8/10

$\theta_2 = \cos^{-1}(8/10) = 36.87°$

Now Draw FBD of individual cylinder

FBD of cylinder C; (Fig 2.32)

Applying Lami's theorem, [R_1 /sin (90+50.48)] = [R_2/sin90] = [196.2/ sin (180-50.48)];

R_1 = 161.84N, R_2 = 254.34N

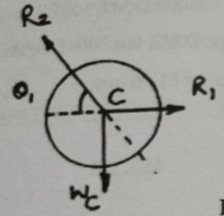


Fig 2.32

FBD of cylinder B; (Fig 2.33)

Apply the Condition of equilibrium

$\sum F_x = 0$ (+→) $-R_3 + R_2 \cos\theta_1 + R_4 \cos\theta_2 = 0$

$\sum F_y = 0$ (+↑) $R_4 \sin\theta_2 - R_2 \sin\theta_1 - (40\times 9.81) = 0$

R_4 = 981.22 N, R_3 = 946.99 N

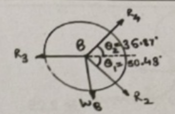


Fig 2.33

FBD of cylinder A; (Fig 2.34)

$\sum F_x = 0$ (+→) $R_6 - R_4 \cos\theta_2 = 0$

$\sum F_y = 0$ (+↑) $R_5 - R_4 \sin\theta_2 - (15\times 9.81) = 0$

R_6 =785 N; R5=737.74 N

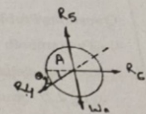


Fig 2.33

Let's check the takeaway from this lecture

1. There are ____ conditions of equilibrium.
   i) One ii) Two
   iii) Three iv) Four
2. There are ____ conditions of equilibrium for Concurrent force system
   i) One ii) Two
   iii) Three iv) Four
3. There are ____ conditions of equilibrium for Parallel force system
   i) One ii) One
   iii) Three iv) Three

Exercise:

1) Two spheres A & B are resting in a smooth channel as given in fig 2.34. Draw FBD of A & B, find reactions at contact points. R_A=250mm, R_B=200mm respectively. **(Ans: 138.07N, 243.03N, 159.43N, 620.28N)**

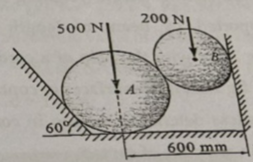


Fig 2.34

2) Sphere A '1000N' rests on two spheres each of weight 900N for fig 2.25. The spheres B & C are connected by an inextensible string of length L=600mm. Radius of spheres = 200mm. determine reactions at all contact points & force in string. **(Ans: 1467.13N, 756.07N)**

3) A smooth roller with 50mm radius and 200N weight is to be pulled over a 10mm high step as shown in fig 2.26. Find minimum possible value of P and corresponding value of 'θ'. **(Ans: θ= 36.87°; P_{min} =120N).**

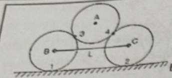


Fig 2.25

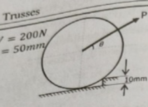


Fig 2.26

Questions/Problems for practice for the day:

4) Two smooth cylinders are supported between stationary surfaces as shown in fig 2.27. Find reaction at all points of contact. (**Ans.:** $R_E = 188N$; $R_D = 565.8N$; $30°$; $R_C = 166.2N$; $R_f = 349.2N, 57.42°$)

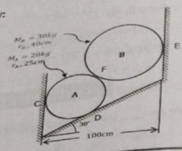


Fig 2.27

Learning from the lecture 'Lami's theorem': Student will be able to apply conditions of equilibrium to different system so reactions coming on bodies can be calculated

2.4 Problems on equilibrium of Connected bodies.

Learning Objective: Students will be able to apply conditions of equilibrium Connected bodies

Solved Problems

1) **A cylinder of diameter 1m and weighing 1000N and another block weighing 500N are supported by beam of length 7m weighing 250N with the help of a cord as shown in fig 2.28. If the surfaces of contact are frictionless determine tension in cord and reaction at points of contact.**

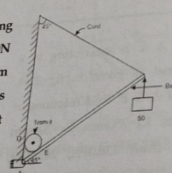


Fig 2.28

Solution: Draw F.B.D of cylinder & beam AB.

Using Lami's Theorem for cylinder,

$R_D / \sin 135 = R_E / \sin 90 = 1000 / \sin 135$

By calculation, $R_D = 1000$ N; $R_E = 1414.2$ N

From the geometry of fig. $\angle DAE = 45°$

since AO is the angle bisector of $\angle DAE$, hence

$\angle OAE = 45°/2 = 22.5°$

From right angle triangle OAE,

$\tan 22.5° = OE/AE = 0.5/AE$; $AE = 1.21$m

Length $BL = 7 \cos 45° = 4.95$m; Length $AG = 3.5 \cos 45° = 2.5$m; Length $AL = 7 \sin 45° = 4.95$m

Applying COE to beam AB,

$\sum M_A = 0$ (Anti clockwise +ve)

$(T \times 7) - (R_E \times AE) - (250 \times AG) - (500 \times BL) = 0$

$7T - (1414.2 \times 1.21) - (250 \times 2.5) - (500 \times 4.95) = 0$

$7T = 4800.2$; $T = 685.75$N

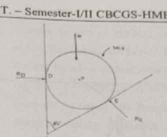


Fig 2.29

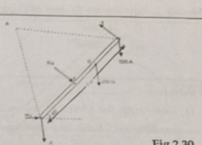


Fig 2.30

$\sum F_x = 0(\rightarrow)$ +ve ; $-T\cos 45° + R_E \cos 45° + H_A = 0$

$H_A = -515.09$ N; $H_A = 515.09$ N $(\leftarrow)$

$\sum F_y = 0(\uparrow)$ +ve; $T \sin 45° - 500 - 250 - R_E \sin 45° + V_A = 0$; $V_A = 1265.09$N $(\uparrow)$

$R_A = \sqrt{(H_A)^2 + (V_A)^2} = \sqrt{(515.09)^2 + (1265.09)^2}$
$= 1365.93$ N

$\tan\theta = (V_A / H_A)$; $\theta = \tan^{-1}(1265.09/515.09)$

$\theta = 67.85°$

2) **A weightless bar is placed in a horizontal position on the smooth inclines as shown in fig 2.31. Find 'x' at which 200N force should be placed from point B to keep the bar horizontal.**

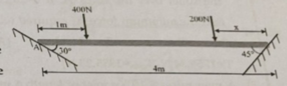


Fig 2.31

Solution: Draw F.B.D. of following fig. 2.32

Applying COE, Moment about point B

$\sum M_B = 0$ (Anticlockwise) +ve;

$-(R_A \sin 60° \times 4) + (400 \times 3) + (200 \times d) = 0$ – (I)

$\sum F_x = 0(\rightarrow)$ +ve; $R_A \cos 60° - R_B \cos 45° = 0$

$R_A(0.5) - R_B(0.71) = 0$ – (II)

$\sum F_y = 0(\uparrow)$ +ve; $R_A \sin 60° - 400 - 200 + R_B \sin 45° = 0$

$R_A(0.866) + R_B(0.71) = 600$ – (III)

Solving equation (II) & (III) simultaneously,

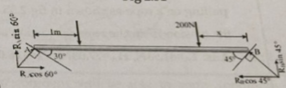


Fig 2.32

we get, $R_A = 439.2$N, $R_B = 309.3$N

Substitute value of R_A & R_B in equation (I)

$-(R_A \sin 60° \times 4) + (400 \times 3) + (200 \times d) = 0$,

$d = 1.61$m.

Let's check the takeaway from this lecture

1. If a body is in equilibrium, we may conclude that
   i) The moment of all the forces about any point is zero
   ii) The resultant of all the forces acting on it is zero
   iii) No force is acting on the body
   iv) Both (i) & (ii)
2. There are ____ conditions of equilibrium for Concurrent force system
   i) One
   ii) Two
   iii) Three
   iv) Four
3. There are ____ conditions of equilibrium for Parallel force system

i) One ii) One

iii) Three iv) Three

Exercise:

1) Determine the forces in cables AB and BC needed to hold the 50 kg ball B in equlibrium for fig 2.33. Given F=300N, d = 1m. **(Ans: F=426.7N(T) ; F_{BC} =155.7N (T))**

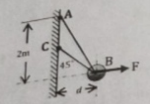


Fig 2.33

2) The roller shown in fig. 2.34 is of weight 1500N. What force 'T' is necessary to start the roller over the block A, if θ=25°, Also find the minimum force 'T' required to start the roller over the block A. **(Ans.: T=1759.34 N, T_{min}=1355.23 N)**

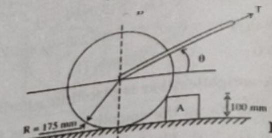


Fig 2.34

3) A man raises a 10 Kg joist of length 4 m by pulling on a rope as shown in fig 2.35. Find the tension 'T' in the rope & reaction at 'A'. **(Ans: T_A=81.98N, H_A=77.03N, V_A=126.03N)**

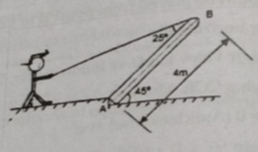


Fig 2.35

4) A crane is pivoted at end B and is supported by a smooth guide at A as shown in fig 2.36. Determine the reaction produced at A and B by a vertical load W=5kN applied at C. **(Ans.: R_A=6.667kN(←),R_B=8.334kN, 36.87°)**

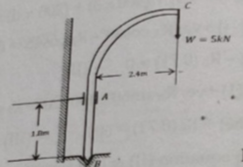


Fig 2.36

Practice Problem:

5) A 5m rigid homogeneous rod hinged to a wall at its end A. It is supported by an inextensible wire at B as shown in the fig 2.37. Weight of the rod is 20kg. The rod is supporting a smooth cylinder of diameter 2m and mass 30kg. Determine reaction at 'A' & Tension in wire 'BC'. **(Ans.: T_{BC} = 382.3N; R_{AX} = 88N(→), R_{AY} = 495.5N(↑), R_A = 503.3N, 79.83°)**

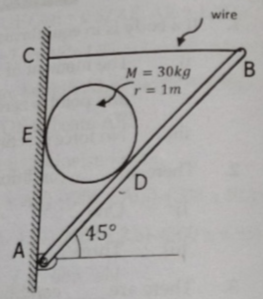


Fig 2.37

6) A bar is hinged at 'A' and rests on cylinder at 'C'. The center of cylinder is connected to the bar by a horizontal wire OE as shown in fig 2.35. Determine i) Reaction at A, ii) Tension in wire, iii) Reaction at C & D, Neglect weight of bar & assume smooth surfaces. **(Ans: V_A = 79.3N(↓), H_A = 0, T = 551.7N, R_C = 800N, R_D = 779.34N)**

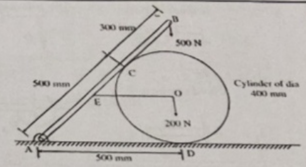


Fig 2.38

7) Two smooth rollers are connected by a bent rod ABC as shown in fig 2.36. Find reactions at D, E & F. Point 'A' & 'C' are frictionless pin connections. **(Ans: R_D = 4359N(→); R_E = 3202N(↑); R_F=1165N)**

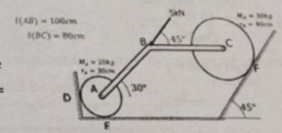


Fig 2.39

8) A cylinder of weight 300N is held in equilibrium as shown in fig 2.40. Determine the tension in the string AD and reaction at C and B. The length of AE =750 mm. **(Ans: 692.3N, 271.55N, 992N)**

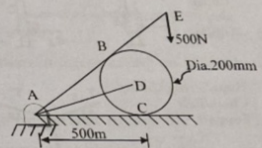


Fig 2.40

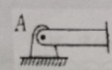
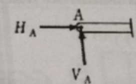
Learning from the lecture 'Lami's theorem': Student will be able to apply conditions of equilibrium to connected bodies so reactions coming on bodies can be calcuiated

2.5 Reactions of different support, Types of loads, types of beams

Learning Objective: Students will be able to draw free body diagram of several types of beams.

Types of supports and reactions: Whenever a body is supported, the support offers resistance, known as Reaction. E.g You are sitting on chair while reading book, your weight is being supported by the chair which offers a force of resistance upwards.

There are distinct types of support shown in tabular form: -

Sr. No.	Types of Support	Reaction & FBD	Number of Unknowns
1	Hinge support 		Hinge support allows only rotation and does not allow movement either horizontally or vertically. Hence, Hinge support offers two unknown reactions. In which one acts in horizontal direction other reaction acts in vertical direction at the point of contact.

2	Roller support		Roller support offers only one unknown reaction. This acts perpendicular to the surface at the point of contact.
3	Fixed support	H_A, M_A, V_A	Fixed support does not allow any type of motion, neither translation nor rotational. Hence, Fixed support offers three unknown reaction. In which one acts in horizontal direction other reaction acts in vertical direction and third is moment at the point of contact.
4	Smooth Support		A smooth surface offers a similar reaction as that of roller support. i.e. a force reaction perpendicular to the surface at the point of contact.
5	Edge(Fulcrum) Support		It offers reactive force, only one unknown reaction normal to the surface of the body at the point of contact.
6	Collar/ slot/ Guide	R_A	One Unknown: It offers a force reaction normal to the surface of the body.
7	Rope/String/Cable/ Chain/Belt Support	T_{AB}, T_{AC}, W	In FBD of Rope/string/cable/chain/ belt support the force is always taken as tension and away from the body.
8	FBD of bar pivoted at one end and has a sliding bar		Here Pivoted point is act like a rough surface or hinge support which will have two unknown reaction
9	Force 'P' applied to turn the roller over rectangular block		In this type of support contact between roller and the surface is taken as dummy contact i.e., in FBD force at point A will not be consider at that instance.

Types of beams:

(i) Simply supported beam: A beam that has hinged connection at one end and roller connection in other end is called simply supported beam, so when transverse loads are applied they resists by generating reactive forces on their supports.

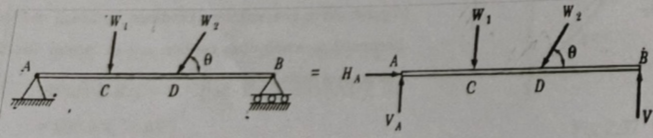
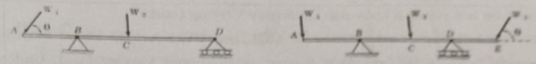


Fig 2.41 (a) & (b)

(ii) Over hanging beam: A beam which extends beyond the supports is known as overhanging



Single overhanging beam Double overhanging beam

2.42 (a) & (b)

(iii) Cantilever beam: A beam fixed at one end & free at other end is called cantilever beam. At fixed support, there are three reactions- (a) horizontal reaction (b) vertical reaction & (c) Moment. as shown in fig 2.43

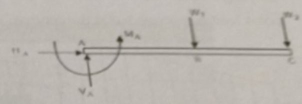


Fig 2.43

(iv) Compound beam: When two or more beams are joined together by means of internal hinge or internal roller, the combinations called compound beam. As shown

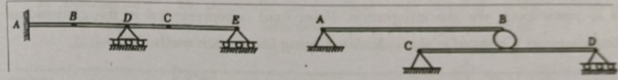


Fig 2.44

- In the analysis of compound beam, always moment of forces acting on individual beam or combination of beams about internal hinge is zero.
- We can disconnect the beams at internal hinge or internal rollers and draw FBD of each beam separately by assuming internal reaction components at hinge as equal and opposite.
- Conditions of equilibrium to each beam can be separately applied after disconnection.

Types of load on the beam:

There are mainly four types of load on the beam.

(i) Point load or concentrated load:

A load acting at a single point on the beam is called point load.

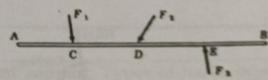


Fig 2.45

(ii) Uniformly distributed load or rectangular load (UDL): A load which is spread over the beam (partly or fully) uniformly is called UDL. UDL can be converted into point load by taking product of intensity of UDL and span of UDL. This point load will be represented at the center of UDL.

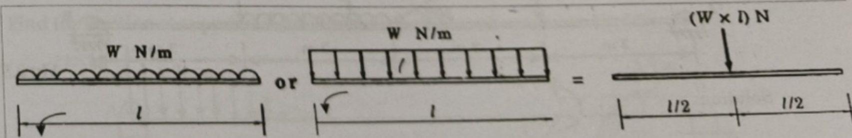
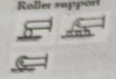
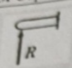
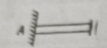
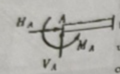
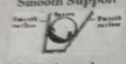
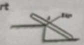
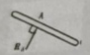
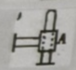
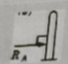
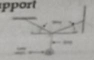
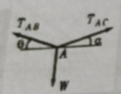
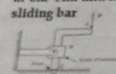
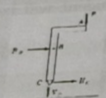
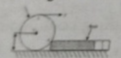
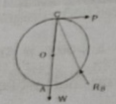


Fig 2.46

No.	Support	FBD	Description
	Roller support 		Roller support offers only one unknown reaction. This acts perpendicular to the surface at the point of contact.
2 / 3	Fixed support 		Fixed support does not allow any type of motion, neither translation nor rotational. Hence, Fixed support offers three unknown reaction. In which one acts in horizontal direction other reaction acts in vertical direction and third is moment at the point of contact.
4	Smooth Support 		A smooth surface offers a similar reaction as that of roller support. i.e. a force reaction perpendicular to the surface at the point of contact.
5	Edge(Fulcrum) Support 		It offers reactive force, only one unknown reaction normal to the surface of the body at the point of contact.
6	Collar/ slot/ Guide 		One Unknown: It offers a force reaction normal to the surface of the body.
7	Rope/String/Cable/ Chain/Belt Support 		In FBD of Rope/string/cable/chain/ belt support the force is always taken as tension and away from the body.
8	FBD of bar pivoted at one end and has a sliding bar 		Here Pivoted point is act like a rough surface or hinge support which will have two unknown reaction
9	Force 'P' applied to turn the roller over rectangular block 		In this type of support contact between roller and the surface is taken as dummy contact i.e., in FBD force at point A will not be consider at that instance.

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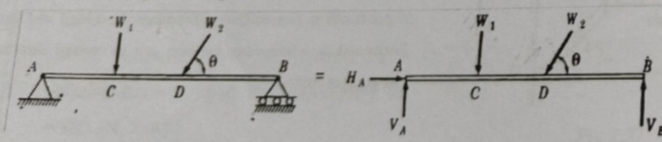

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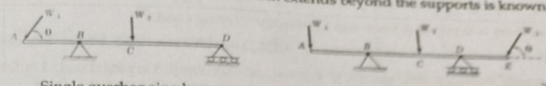

Single overhanging beam Double overhanging beam

2.42 (a) & (b)

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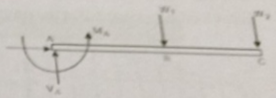

Fig 2.43

(iv) Compound beam: When two or more beams are joined together by means of internal hinge or internal roller, the combinations called compound beam. As shown

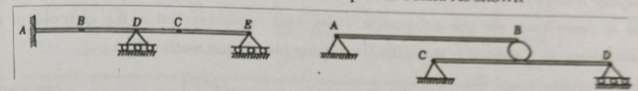

Fig 2.44

- In the analysis of compound beam, always moment of forces acting on individual beam or combination of beams about internal hinge is zero.
- We can disconnect the beams at internal hinge or internal rollers and draw FBD of each beam separately by assuming internal reaction components at hinge as equal and opposite.
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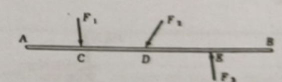


Fig 2.45

(ii) Uniformly distributed load or rectangular load (UDL): A load which is spread over the beam (partly or fully) uniformly is called UDL. UDL can be converted into point load by taking product of intensity of UDL and span of UDL. This point load will be represented at the center of UDL.

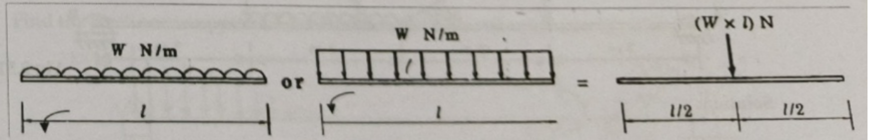


Fig 2.46

(iii) Uniformly varying load or triangular load (UVL): A load whose intensity is linearly varying between two points is known as Uniformly Varying Load.

(iv) Trapezoidal load (UVL): Combination of UDL and UVL: A load whose intensity is linearly varying between two points is known as Uniformly Varying Load. Uniformly varying load can also be converted into point load by calculating the area of the loading and point load is represented at the centroid of the area.

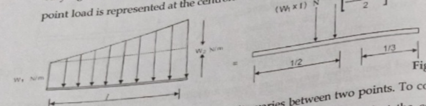


Fig 2.47 (1) & (2)

(v) Varying load (VL): A load whose intensity varies between two points. To convert varying load to point load, we use integration. Point load is represented at the centroid of the loading diagram. Centroid is also calculated using integration method.

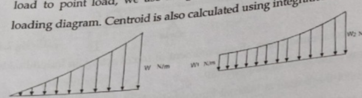


Fig 2.48 (1) & (2)

__Solved Problem:__

3) Draw Free body diagram for a given fig 2.49.

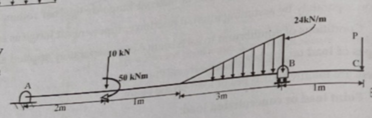


Fig 2.49

Solution:

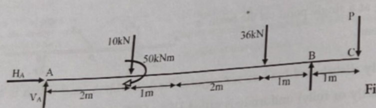


Fig 2.50

4) **A beam AB is loaded as shown in fig 2.51. Draw Free body diagram.**

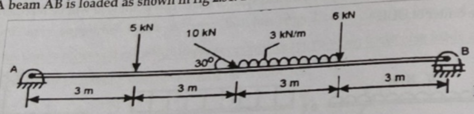


Fig 2.51

Solution:

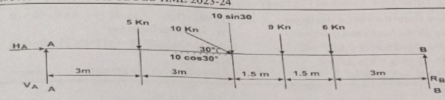


Fig 2.52

Let's check the takeaway from this lecture

1) Number of reactions in a double roller support

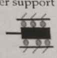


Fig 2.53

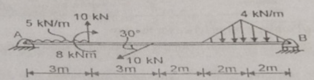
a) 3 b) 2 c) 1 d) 0

2) Reaction of a roller support is always ________

a) parallel to rolling direction b) perpendicular to rolling direction c) depends on the direction of loading d) none of above

Exercise:

1) Draw FBD for a given beam shown in fig 2.54.



ig 2.54

Problem for Practice of the day

2) Draw FBD for a given beam shown in fig 2.55.

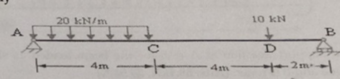


Fig 2.55

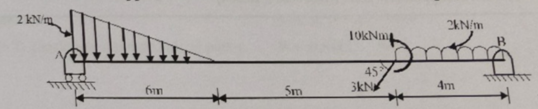
Learning from the lecture 'Reactions of different support, Types of loads, types of beams': Student will be able to draw free body diagram of several types of beams & able to convert different loads into point load.

Problems on Simply supported beam

Learning outcome: Student will be able to find reactions of simply supported beam.

__**Solved problems:**__

5) **Find the reaction at supports of the Beam AB loaded as shown in the fig 2.56.**



ig 2.56

Solution: Draw the FBD of the Beam showing all the forces and reactions.

Apply the Condition of equilibrium

$\sum F_x = 0\ (+\rightarrow)\ -H_A - 3\cos 45° = 0$

$H_A = -2.12\ N = 2.12\ N(\rightarrow)$

$\sum F_y = 0\ (+\uparrow)\ R_D - 3\sin 45° - 6 - 8 + V_A = 0$ — (i)

$\sum M_A = 0$ (+anticlockwise)

$8\times2 + 10 + 3\sin45°\times4 + 6\times11 - R_D\,15 = 0$ — (ii)

On solving

$R_D = 6.69\ N(\uparrow)$, $V_A = 9.422\ N(\uparrow)$ ………Ans

Exercise:

1) Find analytically the support reaction at B and load P for the beam shown in fig 2.57 if the reaction at support A is zero. **(Ans: 26kN, 72kN)**

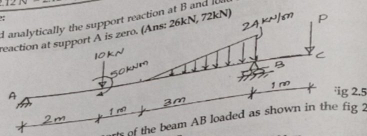


Fig 2.57

2) Find the Reactions at supports of the beam AB loaded as shown in the fig 2.58 below. **(Ans: R_B= 136.03 kN, H_A=0, V_A=103.97 kN)**

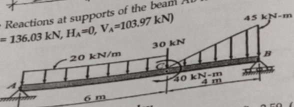


Fig 2.58

Questions/Problems for practice for the day

3) Find reactions for beam loaded & supported as shown in fig 2.59. **(Ans: 50kN, 60kN)**

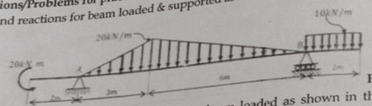


Fig 2.59

4) Find Support Reaction at A & B for the beam loaded as shown in the fig 2.60. A is hinged and B is roller. **(Ans: 89.5kN, 39.5kN)**

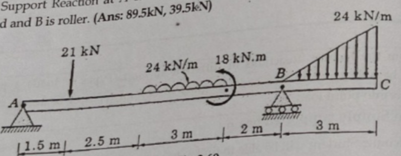
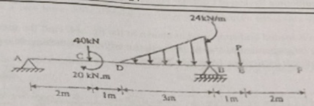


Fig 2.60

5) Find the support reaction at B and the load P, for the beam shown in fig 2.61. if the reaction at support A is zero. **(Ans: 56kN, 102kN)**



ig 2.61

Learning from the lecture 'Simply supported beam: Student will be able to apply conditions of equilibrium to Simply supported beam & can calculate reactions of simply supported beam

2.6 **Problems on reactions of beam with pulley & external mass (Complicated loading)**

Learning objective: Students will be able to find the reactions of beam containing complicated loads

Solved problems

6) **Determine minimum weight of the block required to keep beam in horizontal equilibrium as shown in fig 2.62. Assume rough pulley with coefficient of friction as 0.2.**

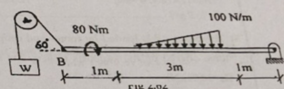


Fig 2.62

Solution: Draw the FBD of beam AB. Hinge support at point A gives horizontal and vertical reaction. UVL is converted into point load. Resolve T at point B into two components. as shown in the fig 2.63.

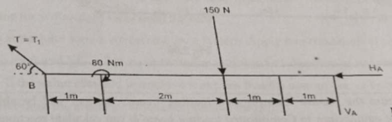


Fig 2.63

Apply the condition of equilibrium, we have $\sum M_A = 0$ (+anticlockwise)

$150\times2 - 80 + T\sin 60°\times5 = 0$

Hence T = 50.8 N

Now pulley has coefficient of friction so tension on both side of the pulley will be Different

$T = T_1$ Tension on the tight side of pulley

T_2 =Tension on the slack side of pulley

$W = T_2$ = Minimum Weight of the block

Ratio of tension

$T_1/T_2 = e^{\mu\theta}$

μ = Coefficient of friction =0.2

θ = Angle of lap (measured in radians)

So, $50.8/W = e^{(0.2\times50\Pi/180)}$

W= 30.1 N…… Ans

Exercise:

1) A beam is loaded and supported as shown in the fig 2.64. Find the maximum mass M that can be suspended as shown in the figure without disturbing its equilibrium. The small pulley is frictionless. **[Ans.:M=26.82kg]**

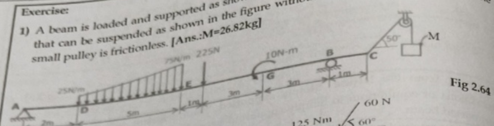


Fig 2.64

2) Find support reaction of the beam as shown in fig 2.65. **(Ans: - H_A=116.86 N (→), V_A=82.77 N (↑), R_B=173.72N, $\theta°$=60)**

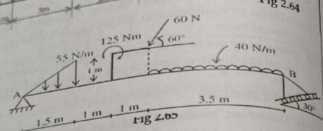


Fig 2.65

3) The beam AB is loaded by forces and couples as shown in fig 2.66. Find the reaction force offered by the supports to keep the system in equilibrium. (Ans: **R_A=19.74kN, R_B=10.82kN)**

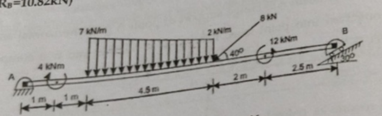


Fig 2.66

Learning from the lecture 'Complicated loading beam': Student will be able to apply conditions of equilibrium to Complicated loading beam'& can calculate reactions of simply supported beam

Lecture 17

Trusses, Types of Trusses, Condition of Perfect

Learning Objective: Students will able to identify Perfect truss

2.1.12 Trusses:

➤ **Assumptions made in truss analysis:**

1) All the members of the truss lie in one plane.
2) The loads acting on the truss lie in the plane of the truss.
3) The members of the truss are joined at the ends by internal hinges known as pins.
4) Loads act only at the joints and not directly on the members.

5) Each member is a two force body thereby resulting in axial forces which are either tensile or compressive
6) The self-weight of the members being small as compared to the loads applied is neglected.
7) The truss is statically determinate i.e. forces can be determined using equilibrium conditions.

➤ **Types of trusses:**

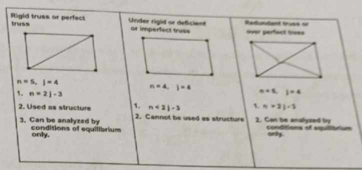


Fig 2.67

➤ **Working rules for Method of Joints:**

Principle: If the entire truss is in equilibrium, then each joint of the truss is also in equilibrium.

1) Draw Free body diagram of the entire truss.
2) Find the reactions at the supports of the truss by applying conditions of equilibrium to the entire truss.
3) Isolate a joint from the truss which has not more than two members with unknown force.
4) Assume that the members carry tensile force and hence show direction of the arrowheads (representing force direction) away from the joint.
5) The forces at the joint form a concurrent force system. Apply two conditions of equilibrium viz. $\Sigma F_X = 0$ (→ +ve) and $\sum F_Y = 0$ (↑ + ve) to find magnitude and direction of unknown force in the members. If the value obtained is negative it implies that the assumption of being a tensile force was incorrect and the member carries compressive force.
6) Now isolate another joint having not more than two members with unknown force. Follow steps 3 to 5 and solve each joint to find forces in all the members of the truss.
7) Tabulate the results indicating the member, magnitude of force and nature of the force.

➤ **Special Cases in truss analysis (ZERO Force Members) :**

1. If a joint has only two members meeting at it and the joint is not supported or loaded then the members meeting at that joint has zero force along them.

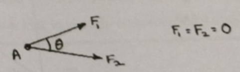
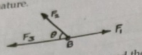


Fig 2.68

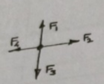
2. If a joint has three members meeting at it and the joint is not supported or loaded, also out of three members two members form a pair of collinear members then the third member meeting at that joint has zero force along it and the collinear members have forces equal in magnitude and nature.



$F_2 = 0 \quad F_1 = F_3$

Fig 2.69

3. If a joint has four members meeting at it and the joint is not supported or loaded, also there are two pairs of collinear members then the collinear members have forces equal in magnitude and in nature.



$F_1 = F_3$
$F_2 = F_4$

Fig 2.70

2.1.13 Solved Problems:

1. Identify members carrying zero forces in the plane truss as shown below:

Solution:

a)

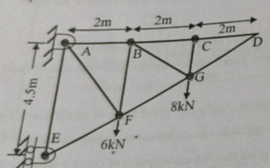


Fig 2.71

b)

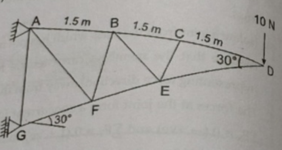


Fig 2.72

a) FBD of entire truss:

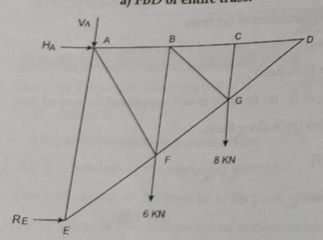


Fig 2.73

b) FBD of entire truss:

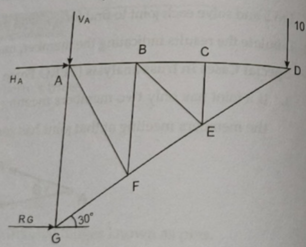


Fig 2.74

[i] FBD of joint D:

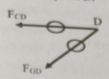


Fig 2.75

[i] FBD of joint C

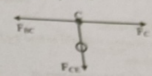


Fig 2.76

[ii] FBD of joint C:

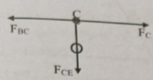


Fig 2.77

[ii] FBD of joint E:

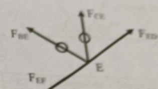


Fig 2.78

[iii] FBD of joint B:

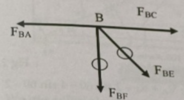


Fig 2.79

[iv] FBD of joint F:

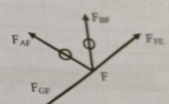


Fig 2.80

Exercise:

1) **Find the reaction of & zero force member in given truss.**

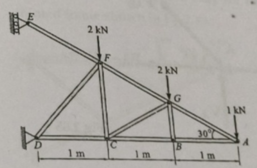


Fig 2.81

Practice:

Find the reaction of & zero force member in given truss. (Ans: $F_{CB} = F_{CD} = F_{AE} = F_{DE} = 0$; 4kN; -4kN)

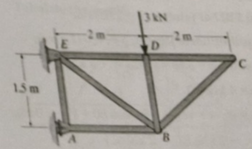


Fig 2.82

Learning from the lecture 'Trusses, Types of Trusses, Condition of Perfect': Student will able to apply conditions of Perfect truss & able to identify several types of trusses

Lecture 18

Problems on method of joints

Learning objective: Student will able to find the forces in different member of truss.

2.1.14 Solved Problems:

12) Determine forces in all the members of the plane truss shown in the sketch fig 2.83 by method of joints.

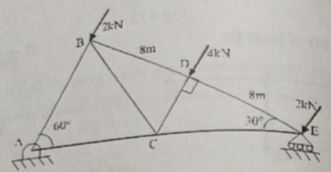


Fig 2.83

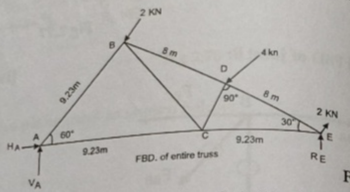


Fig 2.84

Solution: (1) Checking stability of truss: –

No. of members, m = 7; No. of joints, j = 5

No. of reactions, r = 3; m = 2 j – r

i.e. 7 = (2 x 5) – 3

7 = 7............so Perfect truss

(2) Finding support reactions:

For equilibrium of entire truss, (Fig 2.84)

a) $\sum M_A = 0$ (Anticlockwise) +ve

so (– 4 x 8) – (2 x 16) + (RE x 18.47) = 0

$R_E = 3.46$kN (↑)

b) $\sum F_Y = 0$ (↑ + ve); $V_A + R_E - 2\sin 60 - 4\sin 60 - 2\sin 60 = 0$;

$V_A = 3.46$ kN (↑)

c) $\sum F_X = 0$ (→ + ve); $H_A - 2\cos 60 - 4\cos 60 - 2\cos 60 = 0$;

$H_A = 4$ kN (→)

(3) Method of joints: –

a) FBD of joint A: (Fig 2.85) For equilibrium of joint A; $\sum F_Y = 0$ (↑ + ve);

$F_{AB}\sin 60 + 3.46 = 0$; $F_{AB} = -4$ kN;

$F_{AB} = 4$ kN (C)

$\sum F_X = 0$ (→ + ve); $4 + F_{AB}\cos 60 + F_{AC} = 0$

$F_{AC} = -2$ kN; $F_{AC} = 2$ kN (C)

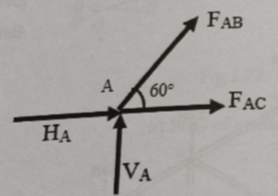


Fig 2.85

b) FBD of joint E: (Fig 2.86)

$\sum F_Y = 0$ (↑ + ve); $F_{DE}\sin 30 - 2\sin 60 + 3.46 = 0$; $F_{DE} = -3.46$ kN; $F_{DE} = 3.46$ kN (C)

$\sum F_X = 0$ (→ + ve); $-F_{CE} - F_{DE}\cos 30 - 2\cos 60 = 0$; $F_{CE} = 2$ kN (T)

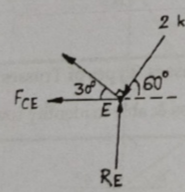


Fig 2.86

c) FBD of joint D: (Fig 2.87) By observation, joint D comprises two pairs of collinear members. So, $F_{DE} = F_{BD}$ and $F_{DC} = 4$kN (C)

$F_{BD} = 3.46$ kN (C); $F_{DC} = 4$ kN (C)

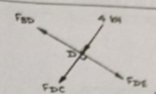


Fig 2.87

d) FBD of joint C: (Fig 2.88)

$\sum F_X = 0$ (→ + ve);

$F_{CE} - F_{AC} - F_{BC}\cos 60 + F_{DC}\cos 60 = 0$

$2 - (-2) - F_{BC}\cos 60 + (-4\cos 60) = 0$

$F_{BC} = 4$ kN (T)

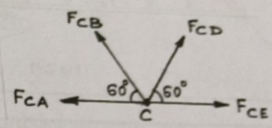


Fig 2.88

Sr.No.	Member	Force (kN)	Nature	Sr.No.	Member	Force (kN)	Nature
1	AB	4	C	5	CD	4	C
2	AC	2	C	6	CE	2	T
3	BC	4	T	7	DE	3.46	C
4	BD	3.46	C				

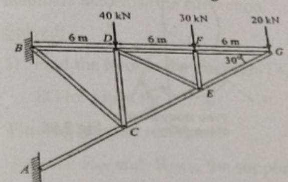
Let's check the take away from this lecture

1. We use principle of .... for determining the forces in the members of a Truss.

a) Statics b) Dynamics c) Moment d) None of these

Exercise:

1) Find the forces in the members DF, DE, CE and EF by method of joints only for the pin jointed frame shown in fig 2.89.



ig 2.89

2) Find the forces in the members of the pin jointed truss loaded as shown in fig 2.90 Tabulate the results.

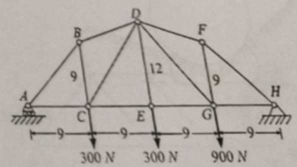


Fig 2.90

Practice:

3) Determine the forces in the members BC and AD of the truss as shown in the fig. 2.91 by using method of joints. (Ans: BC = 70.7 kN (C), AD = 95.6 kN (T))

4) A vertical load of 30 kN is applied to joint A of the truss for given fig 2.92. Determine the member forces and the support reactions.

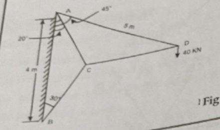


Fig 2.91

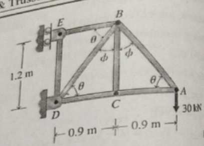


Fig 2.92

Learning from the lecture 'Problems on method of joints': Student will able to Find the forces in different member of the truss & will able to analyze different members

2.1.15 Short answer questions:

Equilibrium

1. **Define Equilibrium:**

Ans: If the resultant of the force system happens to be zero, the system is said to be in a state of equilibrium.

2. **Define Free body diagram:**

Ans: It is diagram of body under consideration. The diagram shows magnitudes directions and point of applications of all external, active and reactive forces acting on the body. In the diagram unknown forces, corresponding directions are labeled. Thus, it is very important and very first step in analysis of problems on equilibrium.

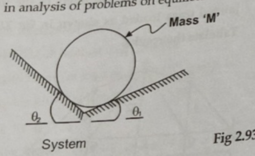


Fig 2.93

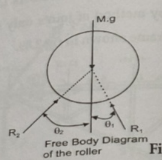


Fig 2.94

Trusses

1) ***What is principle for method of joints?***

Ans: If the entire truss is in equilibrium, then each joint of the truss is also in equilibrium.

2) ***What is statically determinate truss?***

Ans: A truss which can be analyzed by applying conditions of equilibrium is called statically determinate.

3) ***What is perfect truss?***

Ans: A truss which satisfies condition $m = 2j - r$ is known as perfect truss.

Where m = number of members; j = number of joints; r = number of reactions

4. **State some applications of trusses:**

Ans:

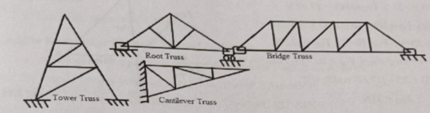


Fig 2.95

➢ **Differentiate between Perfect, Imperfect and over perfect trusses**

Rigid truss or perfect truss	Under rigid or deficient or imperfect truss	Redundant truss or over perfect tress
n = 5, j = 4	n = 4, j = 4	n = 6, j = 4
1. n = 2j - 3	1. n < 2j - 3	1. n > 2j - 3
2. Used as structure	2. Cannot be used as structure	2. Can be analyzed by conditions of equilibrium only.
3. Can be analyzed by conditions of equilibrium only.		

Fig 2.96

Working rule for method of section:

The method involves dividing the truss into sections by cutting through the selected members and analyzing the section as a rigid body. This is an alternative to the method of joints for finding the internal axial forces in truss members. It works by cutting through the whole truss at a single section and using global equilibrium to solve for the unknown axial forces in the members that cross the cut section.

Solved:

1) Find the force in the members DG, DF and EG for given fig 2.97

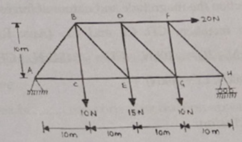


Fig 2.97

Finding support reactions:

Assume R_{AV} and R_{AH} be the support reactions at point A and R_H be the support reaction at point H; For equilibrium of entire truss,

$\sum M_A = 0$ (Anticlockwise) +ve;

$(10\times10)+(15\times20)+(10\times30) + (20\times10) - (R_H\times40)=0$

$R_H=22.5N$; $\sum F_X = 0$ ($\rightarrow$ + ve)

$R_{AH}+20=0$; $\mathbf{R_{AH} = 20N}$

$\sum F_Y = 0$ ($\uparrow$ + ve);

$R_{AV} + R_H - 10 - 15 - 10 = 0$

$R_{AV} + R_H - 35 = 0$;

$R_{AV} + 22.5 - 35 = 0$; $\mathbf{R_{AV} = 12.5N}$

$\sum F_Y = 0\ (\uparrow +ve);\ F_{DG}\sin 45 - 10 + R_H = 0$

$F_{DG}\sin 45 - 10 + 22.5 = 0;\ F_{DG}\sin 45 + 12.5 = 0$

$F_{DG}\sin 45 = -12.5;\ F_{DG}\sin 45 = 17.67N$

$\sum M_G = 0;\ (-F_{DF}\times 10) + (20\times 10) - (22.5\times 10) = 0$

$(-F_{DF}\times 10) - 25 = 0;\ -F_{DF}\times 10 = 25;\ F_{DF} = -2.5N$

$\sum F_X = 0\ (\rightarrow +ve);\ -F_{EG} - F_{DF} + 20 - F_{DG}\cos 45 = 0$

$-F_{EG} - (-2.5) + 20 - (17.67\cos 45) = 0$

$-F_{EG} + 35 = 0;\ F_{EG} = 35N$

FBD of RHS part:

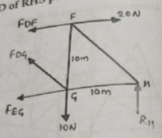


Fig 2.98

Practice Problems:

1) A truss is shown in the fig 2.99. Find the magnitude and nature of forces in the members BC, AC and AE by the method of section. **(Ans: BC=36.373kN, AC= 41.569kN, AE=18kN)**

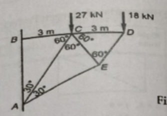


Fig 2.99

2) The cantilever truss is shown in fig 2.100. Find the magnitude and nature of forces in the members 3-4, 4-7, and 6-7 by the method of section. **(Ans: 3-4= 20kN, 4-7=10kN, 6-7=8.66kN)**

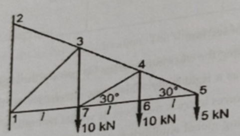


Fig 2.100

3) A plane truss having a point load of 30kN at E as shown in fig 2.101. Find by method of section the magnitude and nature of forces in the members CD, CF and EF. **(Ans: R_A= 20kN, R_B= 10kN, CD= 7.454kN, CF= 18.86kN, EF= 20kN)**

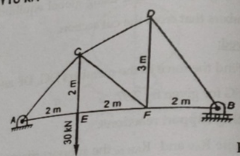


Fig 2.101

2.1 Conclusion:

Learning Outcomes: Learners should be able to

Know, Comprehend

1) Understand the conditions of equilibrium and its applications based on type of system
2) Understand Lami's Theorem and its application

Apply, Analyse

3) Identify the types of supports and its reactions
4) Identify the types of beams and supports and find the reactions of support

Synthesize

5) Calculate stress and strain
6) Calculate poison's ratio

2.2 Add to Knowledge (Content Beyond Syllabus)

a) Analysis of Truss can be done with software named as ANSYS.
b) https://www.youtube.com/watch?v=sKZAipnrV9E
c) https://www.youtube.com/watch?v=J0cCA1PAINU
d) https://www.youtube.com/watch?v=Hn_iozUo9m4

2.3 Set of Multiple-Choice Questions:

1. According to Lami's Theorem, the three forces

(a) Must be equal. (b) Must be at 120° to each other.
(c) Must be both of above. (d) May not be any of the two.

2. The Lami's Theorem is applicable only for

(a) Coplanar forces (b) Concurrent forces
(c) Coplanar and concurrent forces (d) Any type of forces

3. If a body is in equilibrium. We may conclude that

(a) No force is acting on the body (b) The resultant of all forces acting on it is zero.
(c) The moments of the forces about any point is zero. (d) Both (b) and (c)

4. If the sum of all the forces acting on a body is zero, then the body may be in equilibrium provided the forces are

(a) Concurrent (b) Parallel
(c) Like parallel (d) Unlike parallel

5. A body is said to be in equilibrium, if it has no linear motion.

(a) True (b) False

6. Lami's Theorem cannot be applied in case of concurrent forces

(a) Agree (b) Disagree

7. In a simply supported beam carrying triangular load, the reactions cannot be vertical

(*a*) True (*b*) False

8. An overhanging beam with downward loads............have one of its reaction upward and the other downward.

(*a*) can (*b*) can not

9. The reaction at the roller supported end of a beam is always

(*a*) vertical (*b*) horizontal (*c*) none of the above

10. If the reaction of a beam, at one of its supports is the resultant of horizontal and vertical forces, then it is a

(a) simply supported end (b) roller supported end (c) hinged end

11. A couple acting at the mid-point of a simply supported beam has some horizontal and vertical components.

(a) Agree (b) Disagree.

1.(d) 2. (a) 3. (d) 4. (a) 5. (b) 6. (b) 7. (b) 8. (a) 9. (c) 10. (c) 11. (b)

2.4 Short Answer Questions:

1) Define Equilibrium:
2) Define Free body diagram:
3) State and Prove Varignon's Theorem?
4) State and prove Lami's Theorem?
5) Define Couple. Why the couple moment is said to be a free vector?
6) Distinguish between couple and moment?
7) What is meant by force-couple system? Give one example.
8) Can a coplanar non concurrent system with zero resultant force necessarily be in equilibrium?
9) When is moment of force zero about a point and maximum about a point?
10) How would you find out the equilibrium of non-coplanar forces?
11) When is moment of force zero about a line? Give one example.
12) Explain free body diagram with one example?
13) State the necessary and sufficient conditions for equilibrium of rigid bodies in two dimensions?
14) Write the equation of equilibrium of a rigid body?
15) Write the conditions equilibrium of a system of parallel force acting in a plane?
16) What are the reactions at Hinged, Roller and Fixed support of a plane beam that are possible?
17) How many scalar equations can be obtained for equilibrium of rigid body in three dimensions?
18) State difference between stress and strain.

2.5 Long Answer Questions:

1) Three cylinders are piled up in a rectangular channel as shown in Figure 2.102. Determine the reaction at point 6 between cylinder A and the vertical wall of the channel. (Cylinder A: radius= 4 cm, m =15 Kg, Cylinder B: Radius =6 cm, m= 40 Kg, Cylinder C: Radius = 5 cm, m = 20Kg) **(Ans: 735.75N, 981N, 784.8N)**

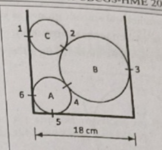


Fig 2.102

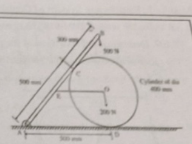


Fig 2.103

2) A bar is hinged at A and rests on cylinder at C. AC=500mm, CB= 300mm, diameter of cylinder is 400mm and its weight is 200N. The center of cylinder is connected to the bar by a horizontal wire OE as shown in figure 2.103. A weight of 500N is suspended at B. Determine i) Reaction of Hinge A, ii) Tension in the wire, iii) Reaction at C and D. Neglect the weight of the bar and assume all surfaces smooth. **(Ans: 552.13N,79.33N, 800N)**

3) Three identical smooth rollers, each of mass M and radius r are stacked on two inclined surfaces as shown in the fig 2.104 Each surface is inclined at an angle α with the horizontal. Determine the smallest angle α to prevent the stack from collapsing. **(Ans: α =10.89°)**

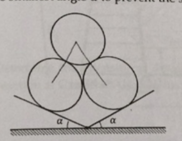
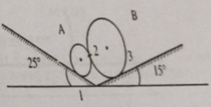


Fig 2.104



$m_a = 1$ kg, $r_a = 1$ cm, $m_b = 4$ kg, $r_b = 4$ cm

Fig 2.105

4) Determine the reactions at points of contact 1, 2 and 3, in figure 2.105. Assume smooth surfaces. **(Ans: - R_1 =19.73 N; R_2= 11.6 N; R_3=32.22N)**

5) A cylinder of weight 500N is kept on two inclined planes as shown in the fig 2.106. Determine the reactions at the contact points A and B. **(Ans: 253.85N, 388.93N)**

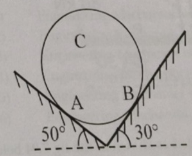


Fig 2.106

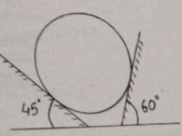


Fig 2.107

6) A cylinder with 1500N weight is resting in an unsymmetrical smooth groove as shown in fig 2.107. Determine the reactions at the points of contacts. **(Ans: 1344.86N, 1098.07N)**

7) Two identical cylinders dia. 100 mm weight 200N are placed as shown in fig 2.108. All contacts are smooth. Find out reactions at A, B and C.

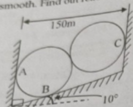


Fig 2.108

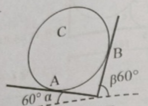


Fig 2.109

8) A smooth circular cylinder of weight W and radius R rests in a V shape groove whose sides are inclined at angles α and to the horizontal as shown in fig 2.109. Find the reactions R_A and R_B at the points of contact.

9) Find reactions at A and B for a bent beam ABC loaded is as shown in fig 2.92. **(Ans: H_A=18kN, V_A =13.15kN, R_B = 4.85kN.)**

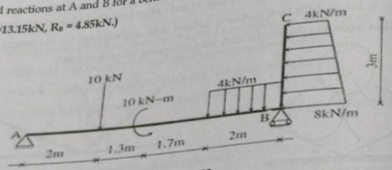


Fig 2.92

10) Find the support reactions at A and B for the beam shown in fig 2.93 **(Ans: 157.03kN, 108kN,314.07kN).**

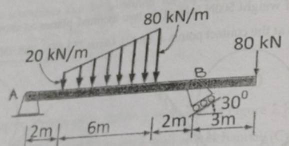


Fig 2.93

11) Find the reactions at supports B and F for the beam loaded as shown in the fig 2.94 below. **(Ans: 360kN, 449.45kN, 293.22kN)**

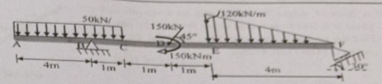


Fig 2.94

12) Find the support reactions for the beam loaded and supported as shown in fig. 2.95 **(Ans: 192.43kN, 140.43kN, 140.56kN)**

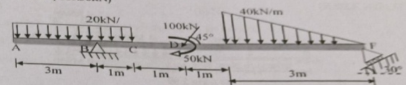


Fig 2.95

13) Determine the reactions at A, B and C for the frame shown in fig 2.96. **(Ans:**

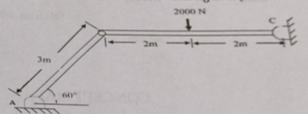


Fig 2.96

14) Determine the intensity of distributed load W at the end C of the beam ABC for which the reaction at C is zero. Also calculate the reaction at B. Fig 2.97 **(Ans: 22.53kN, 3.52kN)**

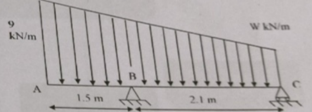


Fig 2.97

2.6 References:

1) Engineering Mechanics by Tayal, Umesh Publication
2) Engineering Mechanics by Beer & Johnson, Tata McGraw Hill
3) Engineering Mechanics by F.L. Singer by Harper
4) Engineering Mechanics – Statics, R. C. Hibbler
5) Engineering Mechanics – Statics, J. L. Merium, L. G. Kraig
6) Engineering Mechanics – P. J. Shah, R. Bade

Self-Assessment

1) A roller of weight W=1000N rests on a smooth inclined plane. It is kept from rolling down the plane by string AC. Find the tension in the string and reaction at the point of contact D. **(Ans: 73.2N, 89.65N)**

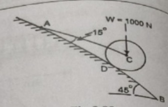


Fig 2.98

2) Find the reactions for the beam loaded and supported as shown in fig 2.99 **(Ans: 18kN, 13.15kN, 4.85kN)**

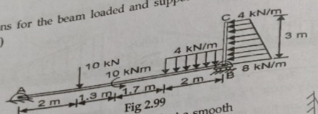


Fig 2.99

Q.3. A cylinder B, W_B =1000N, Dia. 40cm rests against a smooth wall. Find out reaction at C and T_{AB}. **(Ans:**

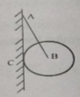
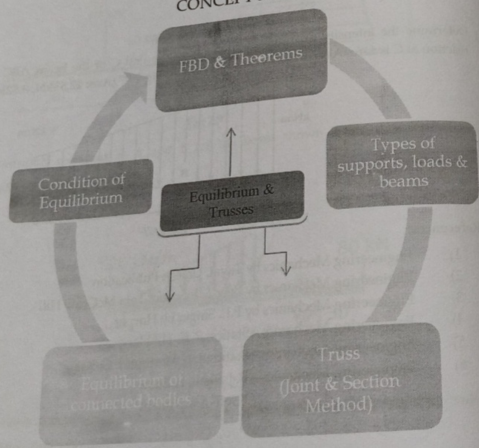


Fig 2.99a

CONCEPT MAP



Self Evaluation

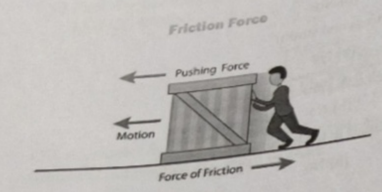
Name of Student: **Course Code:**

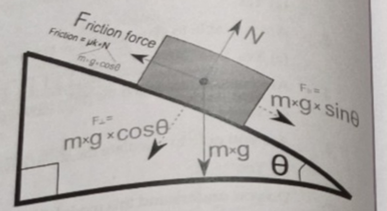
Class & Div.: **Roll No.:**

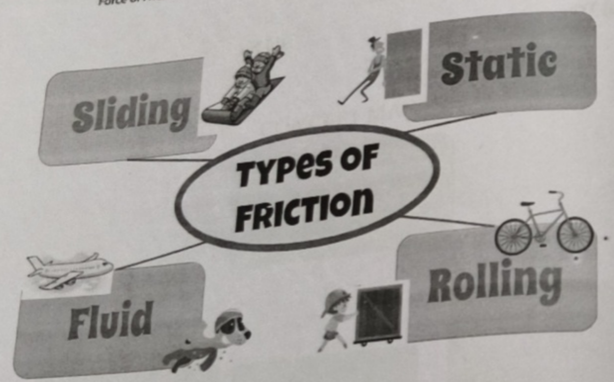
1. Can you define stress and strain ?
   (a) Yes (b) No
2. Are you able to state difference between stress and strain?
   (a) Yes (b) No
3. Are you able to define the term equilibrant, free body diagram?
   (a) Yes (b) No
4. Are you able to calculate the strain developed in the body?
   (a) Yes (b) No
5. Do you understand this module?
   (a) Yes (b) No

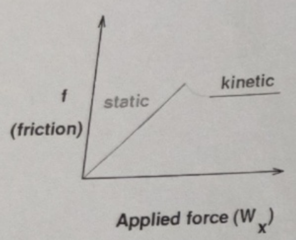
Module 3: Friction

Infographics

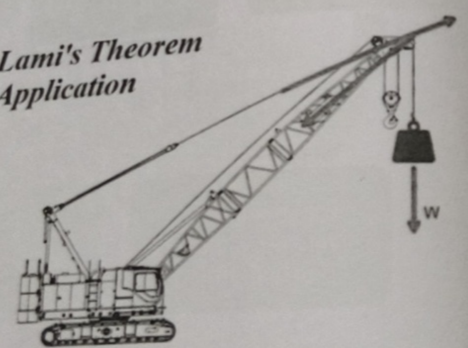








Lami's Theorem Application



Lecture: 19

3.1 *Friction: Introduction, Laws and Definitions*

3.1.1 Motivation:

Perfectly frictionless surface does not exist. The biggest gift to mankind is 'FRICTION'. It is boon and curse for human being. In machines, friction is both a liability and an asset. It causes loss of power and/or wear which is undesirable. On the other hand, friction is essential for various holding and fastening devices as well as for friction drives and brakes. When subject to load, it acts opposite to the direction of motion. In this chapter, we shall consider the application of the principles of friction to engineering problems.

3.1.2 Syllabus:

Lecture No.	Title	Duration (Hrs.)	Self-Study (Hrs.)
19	Introduction, Laws of friction, Angle of Repose, Angle of friction and Cone of Friction	1	2
20	Problems based on single block on horizontal and inclined plane	1	2
21	Problems on multiple blocks (Horizontal/Inclined plane)	1	2
22	Problems on wedge & blocks (Horizontal/Inclined plane)	1	2
23	Problems on Ladder supported by wall and ground	1	2
24	Problems on Tipping/ Slipping of block	1	2

3.1.3 Weightage: 14 to 18 marks (Approximately)

3.1.4 Pre-requisite:

- Knowledge of fundamentals of physics (forces and motion) and mathematical formulation
- Understanding of concepts of equilibrium of forces
- Knowledge of drawing Free Body Diagrams etc.

3.1.5 Learning Objectives: Learners will be able to

1. Learners shall be able to Explain the theory of friction and its characteristics
2. Learners shall be able to Define static and dynamic friction, laws of friction.
3. Learners shall be able to Define the various terms associated with Friction
4. Learners shall be able to Understand application of friction to blocks, wedges & blocks
5. Learners shall be able to Explain applications of friction on ladder and ladder having external weight acting on it.

6. Learners shall be able to Understand the conditions of tipping and sliding

3.1.6 Abbreviations: FBD = Free body Diagram

3.1.7 Notations:

μ = Coefficient of friction,

μ_S = Coefficient of static friction,

μ_K = Coefficient of kinetic friction

$F_{Limiting}$ = Limiting frictional force,

N = Normal reaction,

ϕ = Angle of friction,

α = Angle of repose

F = Frictional force,

W = Weight of block, wedge, ladder etc.,

P = Power transmitted in kW

3.1.8 Formulae:

- Limiting frictional force, $F_{Limiting} = \mu_S N$, where μ_S = coefficient of static friction & N = Normal reaction
- $\tan\phi = \frac{F}{N} = \frac{\mu N}{N} = \mu$, where ϕ = Angle of friction, F = Frictional force & N = Normal reaction
- $\tan\varphi = \tan\alpha = \tan\theta$ so, $\phi = \alpha = \theta$ where ϕ = Angle of friction, α = Angle of repose
- θ = Angle of inclined plane with horizontal

3.1.9 Definitions:

- **Friction:**
  Friction may be defined as a contact resistance exerted by on body upon a second body when the second body moves or tends to move past the first body. From this definition, it should be observed that friction is a retarding force always acting opposite to the motion or the tendency to move.
- **Static Friction:**
  It is the friction experienced between surfaces in contact when there is no sliding between them relative to each other under the action of external forces.
- **Limiting Friction**
  It is the maximum possible static friction. It is frictional force when sliding between the surfaces is about to start under the action of external forces. Limiting friction is directly proportional to normal reaction between the surfaces in contact, corresponding constant of proportionality is known as coefficient of static friction (μ_S)., $F_{Limiting} = \mu_S N$
- **Kinetic Friction**
  It is the friction experienced during the sliding motion between the surfaces in contact after attaining limiting friction under the action of external force. Kinetic friction is directly proportional to normal reaction between the surfaces in contact. $F_{Kinetic} = \mu_K N$

- **Angle of Friction (ϕ)**
  It is the angle made by the resultant reaction (force) of normal reaction and limiting frictional force to the normal reaction. It is denoted by (ϕ).
- **Angle of Repose (α)**
  The term" angle of repose" is used with reference to an inclined surface. It is the maximum angle made by the inclined plane with the horizontal for which a body kept on the incline remains in impending sliding motion without any external force acting on it other than its own weight only.

3.1.10 Theoretical Background:

The following experiment is useful in explaining theory of friction as applied to dry un-lubricated surfaces. Let a block of weight 'W' rest on a rough horizontal surface and assume a horizontal force 'P' to be applied to the block as shown.

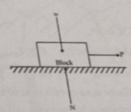


Fig 3.1

a. **No friction: (frictional resistance F is also zero):**
When P is zero, the block is in equilibrium, applying conditions of equilibrium,
$\sum F_Y = 0, (\uparrow +ve)$ $N - W = 0$ So, $N = W$

b. **Equilibrium:** When P is given increasing values that are insufficient to cause motion, the frictional resistance F increases correspondingly to maintain static equilibrium. Forces acting on the body in equilibrium are as shown.

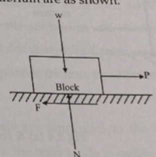


Fig 3.2

$\sum F_X = 0, (\rightarrow +ve)$

$P - F = 0$

$\sum F_Y = 0, (\uparrow +ve)$

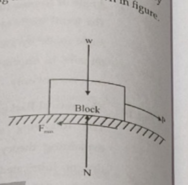
$N - W = 0$

Additionally, $F < F_{Max}$ where F_{Max} is limiting force of friction.

c. **Impending Motion:** Eventually, the block is on the verge of motion and at this instant, F attains its maximum available value F_{Max}. Any further increase in P then causes motion. The stage when F becomes F_{Max} when the body is just about to slide, the motion is called impending sliding motion. F_{Max} is called the limiting or maximum value frictional force. Body is still in static equilibrium. The region up to the point of impending motion is called the range of static friction, and in this stage the value of the friction force is determined by the equations of equilibrium. The FBD of block in impending motion is as shown in figure.

For body in static equilibrium,

$$\sum F_X = 0, (\rightarrow +ve)$$
$$P - F_{Max} = 0$$
$$P = F_{Max}$$
$$\sum F_Y = 0, (\uparrow +ve)$$
$$N - W = 0$$



$N = W$

Additionally, $F = F_{Max}$, where $F_{Max} = \mu_{Max} \times N = \mu_{Limiting} \times N = \mu_S \times N$,

Fig 3.3

Where μ_S is coefficient of static frction

d. **Dynamic Friction:** Any further increase in P then causes motion, but surprisingly, the value of F does not stay at its maximum value but decreases rapidly to a kinetic value which remains constant, as depicted on the graph along with the FBD of the block in figure 3.1.4.

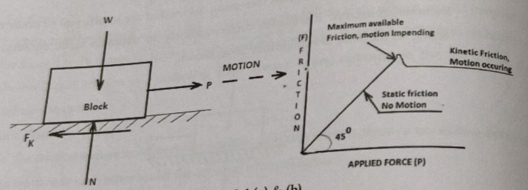


Fig 3.4 (a) & (b)

One way of understanding these results is to examine a magnified view of the contact surfaces. These are shown in figure 3.1.5 together with a FBD of the block. The surfaces are assumed to be composed of irregularities (which can look like hills and valleys) which mesh together. Support is necessarily intermittent and exists at the mating humps. The direction of each of the reactions on the block R_1, R_2, R_3 etc. depends not only on the geometric profile of the irregularities but also on the extent of local deformation at each contact point. The total normal force N is the sum of n-components of the R's and the total

frictional force F is the sum of tangential components of the R's. When the surfaces are in relative motion, the contacts are more nearly along the tops of the humps, and the tangential components of R's are smaller than when the surfaces are at rest relative to one another. The frictional resistance F is developed by the effort of P to break this meshing or interlocking of irregularities.

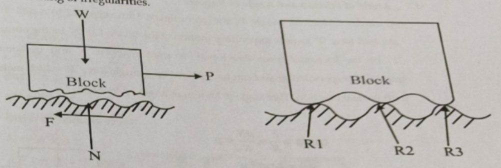


Fig 3.5 (a) & (b)

It is apparent that frictional resistance depends on the amount of wedging action between hills and valleys of the contact surfaces. The measure of this wedging action depends on the normal pressure N between the surfaces. As a result, the maximum frictional resistance that may exist is proportional to the normal pressure and is expressed as $F_{Max} \alpha N$. This relation may be reduced to an equation by adding a constant of proportionality, say μ, which depends on the roughness of the contact surfaces. This constant is called the Coefficient of friction, and above relation may be written as $F_{Max} = \mu \times N$, where F_{Max} is the maximum available frictional force developed when motion is impending. In an actual situation, the equations of equilibrium will determine the value of F to maintain the equilibrium. Of course, if F as determined from equilibrium conditions is less than or just equal to the maximum available friction, equilibrium will exist.

e. **Laws of Friction:**

1) The frictional force is always tangential to the contact surface and acts in a direction opposite to that in which the body tends to move.
2) The magnitude of frictional force is self-adjusting to the applied force till the limiting frictional force is reached and at the limiting frictional force the body will have impending motion.
3) Limiting frictional force F_{max} is directly proportional to normal reactions (i.e., $F_{max} = \mu_S N$)
4) For a body in motion, kinetic frictional force F_K developed is less than that of limiting frictional force F_{max} and the relation $F_K = \mu_K N$ is applicable.

5) Frictional force depends upon the roughness of the surface and the material in contact.
6) Frictional force is independent of the area of contact between the two surfaces.
7) Frictional force is independent of the speed of the body.
8) Coefficient of static friction μ_S is always greater than the coefficient of kinetic friction μ_K

f. **Angle of Friction and Angle of Repose:**
Consider a block of weight W shown in figure 3.1.6 subjected to a pull 'P'. The Value of applied force 'P' causes impending motion of the block. Let 'N' be the normal reaction and 'F' be the frictional force developed. Normal reaction, N and frictional force, F are mutually perpendicular and can be replaced by a single force 'R' which makes an angle 'ϕ' with normal reaction. This angle is known as angle of friction.

$$\tan\phi = \frac{F}{N} = \frac{\mu N}{N} = \mu$$
$$\phi = \tan^{-1}\mu$$
$$R = \sqrt{N^2 + F^2} = \sqrt{N^2 + (\mu N)^2}$$
$$R = N\sqrt{1 + \mu^2}$$

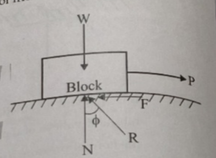


Fig 3.6

Consider a block of weight 'W' resting on an inclined plane as shown in figure 3.1.7. Let the inclined plane makes an angle of θ with the horizontal. When θ is small, block is will be in equilibrium on inclined plane. The component of weight W sin θ down the plane is balanced by frictional force F. As θ kept on increasing, W sin θ will also increase but being balanced by increase in frictional force F. when θ reaches a certain value say $\theta = \alpha$, block is at the verge of sliding and frictional force reaches the limiting condition. This value $\theta = \alpha$ is known as angle of repose. Now, consider the equilibrium of the block.

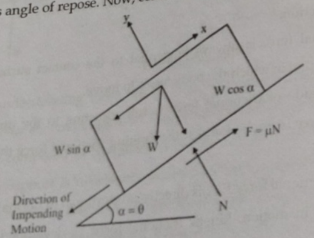


Fig 3.7

Applying conditions of equilibrium,

$$\sum F_Y = 0, (\uparrow +ve)$$
$$N - W\cos\alpha = 0$$
$$N = W\cos\alpha \,.........(1)$$
$$\sum F_X = 0, (\rightarrow +ve)$$
$$F - W\sin\alpha = 0$$
$$F = W\sin\alpha \,.........(2)$$

But $F = \mu \times N$

$$\mu N = W\sin\alpha$$

But from equation (1)

$$N = W\cos\alpha$$
$$\mu W\cos\alpha = W\sin\alpha$$
$$\mu = \tan\alpha$$

But $\theta = \alpha$ and $\mu = \tan\varphi$

$$\tan\varphi = \tan\alpha = \tan\theta$$
$$\varphi = \alpha = \theta$$

Thus, mathematically we can say: **Angle of Friction = Angle of Repose** (only at the time of impending motion)

When, $\theta < \varphi$ Body is in equilibrium,

$\therefore \theta = \alpha = \varphi$, Body is in impending motion,

$\therefore \theta > \varphi$, Body is in motion

g. **Cone of Friction:** When a body is having impending motion in the direction of applied force P, the maximum frictional resistance will cause angle of friction 'ϕ' with the normal reaction to be maximum. If force P is applied in some other direction, for impending motion again the resultant reaction makes limiting angle of friction with the normal reaction and the direction of frictional resistance will be tangential to the surfaces in contact opposite to, this other direction of applied force, P. Thus, when the direction of applied force P is gradually changed through 360⁰, the resultant R generates an inverted right circular cone whose semi-vertex angle equal to ϕ. This inverted cone with semi vertex angle ϕ is known as *Cone of friction.*

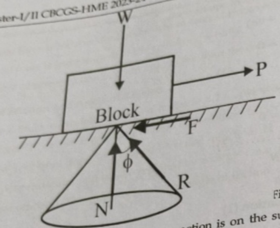


Fig 3.8

Significance of friction: If the resultant reaction is on the surface of this inverted right circular cone whose semi vertex angle is limiting angle of frictionϕ, the motion of the body is impending. If the resultant is within this cone, the body is stationary.

Let's check the takeaway from this lecture

1. The value of Coefficient of Friction is always
   i) Less than 1 ii) Between 0 & 1
   iii)Less than 0 iv)More than 1
2. Limiting friction is always applied in the direction
   i) Against force ii) Against angle
   iii)Against motion iv)Against moment

Exercise:

1. Define the term friction and how does it come into play?
2. What is cone of friction and state its significance.
3. Define the term angle of repose and angle of friction.

Questions/Problems for Practice for the day

4. State and explain laws of friction

Learning from the lecture: Student will be able to understand the various terms and definitions related to Friction.

Lecture: 20

3.2 *Problems based on single blocks on horizontal and inclined plane*

Learning Objective: Learners will be able to understand the concept of solving the problems based on single blocks.

1. If a horizontal force of 1200N is applied horizontally on a block weighing 1000N then what will be direction of motion of the block. Take μ =0.3.

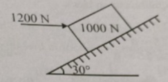


Fig 3.9a

Solution: Draw F.B.D of block

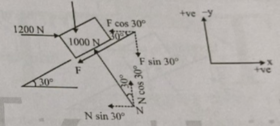


Fig: 3.9b

Applying conditions of equilibrium on block

$$\sum F_X = 0, (\rightarrow +ve)$$

$$1200 - F\cos 30 - N\sin 30 = 0$$

$$1200 - \mu N(0.866) - N(0.5) = 0$$

$$1200 = (0.3)N(0.866) + N(0.5)$$

$$N = 1579.36N$$

Frictional Force,$F = \mu \times N = 0.3 \times 1579.36 = 473.7N$,

Since applied force$P = 1200N > F = 473.7N$, hence block will move in upward direction.

2. A support block is acted upon by two forces as shown in the figure. Knowing that the coefficients of friction between the block and the incline are μ_S= 0.35 and μ_k= 0.25, determine the force P required, i) to start the block moving up the incline. ii) To keep it moving up. iii) To prevent it from sliding down. Where μ_S= Coefficient of static friction, μ_k= Coefficient of kinetic friction

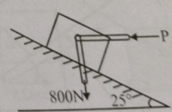


Fig: 3.10

Solution: i) To start the block moving up the incline.

FBD of block moving in upward direction.

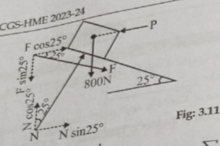


Fig: 3.11

Applying condition of equilibrium on block

$$\sum F_X = 0, (\rightarrow +ve)$$

$$-P + N\sin 25 + F\cos 25 = 0$$

$$-P + N(0.422) + (0.35N)\cos 25 = 0$$

$$P = 0.740N\ldots\ldots(1)$$

$$\sum F_Y = 0, (\uparrow +ve)$$

$$-800 + N\cos 25 - F\sin 25 = 0$$

$$-800 + N(0.906) - (0.35N)\sin 25 = 0$$

$$N = 1054.01N\ldots\ldots(2)$$

Substitute value N in equation (1)

$P = 0.74N = 0.74(1054.01) = 779.96N$

(ii) To keep it moving up.

Draw F.B.D of block moving up

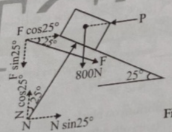


Fig: 3.12

Applying condition of equilibrium on block

$$\sum F_X = 0, (\rightarrow +ve)$$

$$-P + N\sin 25 + F\cos 25 = 0$$

$$-P + N(0.423) + (0.25N)\cos 25 = 0$$

$$P = 0.648N\ldots\ldots(1)$$

$$\sum F_Y = 0, (\uparrow +ve)$$

$$-800 + N\cos 25 - F\sin 25 = 0$$

$$-800 + N(0.906) - (0.25N)\sin 25 = 0$$

$$N = 999.5N\ldots\ldots(2)$$

Substitute value N in equation (I), $P = 0.648N P = 0.648(999.5) = 648.67N$

(iii) To prevent it from sliding down

Draw F.B.D of block sliding down

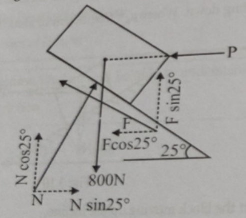


Fig 3.13

Applying condition of equilibrium on block

$$\sum F_X = 0, (\rightarrow +ve)$$

$$-P + N\sin 25 - F\cos 25 = 0$$

$$-P + N(0.422) - (0.35N)\cos 25 = 0$$

$$P = 0.105N\ldots\ldots(1)$$

$$\sum F_Y = 0, (\uparrow +ve)$$

$$-800 + N\cos 25 + F\sin 25 = 0$$

$$-800 + N(0.906) + (0.35N)\sin 25 = 0$$

$$N = 759.7N\ldots\ldots(2)$$

Substitute value N in equation (I), $P = 0.105N; P = 0.105(759.7) = 79.76N$

3. Two identical blocks A and B are pin connected by rod as shown in figure. If sliding impends when $\theta = 75°$, determine coefficient of friction assuming it to be same at both floor and wall.

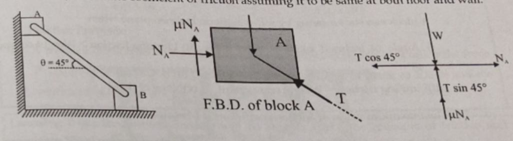


Fig 3.14

Solution: Let T be the thrust exerted by the rod connecting block A and B. Body A will have a tendency to move downward and body B will have tendency to move to the right.

Draw F.B.D. of body A and body B and apply conditions of equilibrium.

From FBD of block A

$\Sigma F_X = 0; -T\cos 45° + N_A = 0 \; N_A = \frac{T}{\sqrt{2}}\ldots\ldots\ldots(i)$

$$\sum F_Y = 0; -W + T\sin 45° + \mu N_A = 0$$

Put value of N_A from equation (i) to get

$$\frac{\mu T}{\sqrt{2}} - W + \frac{T}{\sqrt{2}} = 0 W = \frac{T}{\sqrt{2}}(1+\mu)\ldots\ldots(ii)$$

From FBD of block B

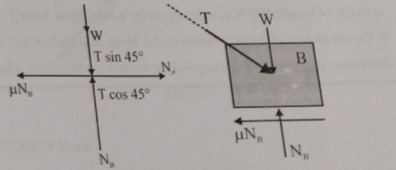


Fig 3.15

$\Sigma F_X = 0;\ T\cos 45° - \mu N_B = 0\frac{T}{\sqrt{2}} - \mu N_B = 0 \ldots\ldots (iii)$

Put value of N_B from equation (iii) to get

$\frac{T}{\sqrt{2}} - \mu\left(W + \frac{T}{\sqrt{2}}\right) = 0\frac{T}{\sqrt{2}}(1-\mu) - \mu W = 0\frac{T}{\sqrt{2}}\left(\frac{1-\mu}{\mu}\right) = W \ldots\ldots (iv)$

Equation (ii) & (iv) we get

$\frac{T}{\sqrt{2}}(1+\mu) = \frac{T}{\sqrt{2}}\left(\frac{1-\mu}{\mu}\right)\mu(1+\mu) = 1 - \mu or \mu^2 + 2\mu - 1 = 0 or \mu = \frac{-2\pm\sqrt{4+4}}{2} = 0.4142$

Coefficient of friction, $\mu = 0.4142$

Let's check the takeaway from this lecture

1. A block can start moving by self-weight on inclined plane when
   i) Angle of inclined plane is more than 45° ii) Limiting friction > Weight component
   iii)Limiting friction < Weight component iv)None of these
2. The maximum value of friction coefficient is 1 because
   i) Limiting and applied force is equal ii) Angle of friction is 45° maximum
   iii)Angle of repose is 45° maximum iv)All of the above

Exercise:

1. A body resting on a horizontal plane required a pull of 200 N inclined at 40° to the plane, to initiate the motion. It was also found that a push of 250 N inclined at 40° to the plane, just moved the body, as shown in Fig. Determine the weight of the body and coefficient of friction. **(Ans. W=1285.59N & μ=0.132)**

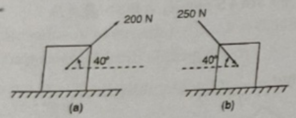


Fig 3.16

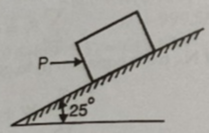


Fig 3.17

2. A block of weight 800 N is acted upon by a horizontal force P as shown in Fig. 3.17 If the coefficient of friction between the block and incline are μ_S=0.35 and μ_K=0.25, determine the value of P for impending motion up the plane. **(Ans. P=780.41N)**

3. Determine the minimum value and the direction of a force P required to cause motion of a 100 kg block to impend upon 30° plane. The coefficient of friction is 0.20. (Ans:

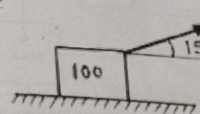


Fig 3.18

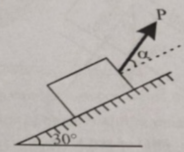


Fig 3.19

Practice Problem:

4. A wooden block rests on a horizontal plane as shown in figure. Determine the force 'P' required to (a) pull it (b) push it. Assume the weight of block as 100 N and the coefficient of friction μ = 0.4. **(Ans: 37.4N, 46.38N)**

Learning from the lecture: Student will be able to apply the concepts of friction and its formulae for block problems.

Lecture: 21

3.3 *Problems on multiple blocks separately connected with string (Horizontal/Inclined plane)*

Learning Objective: Learners will be able to understand the concept of solving the problems based on multiple blocks connected separately.

1. Find the least force P that will just start the system of blocks moving to the right. Takeμ = 0.3. Assume smooth pulley.

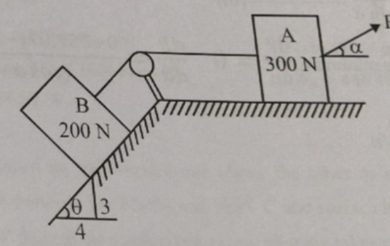
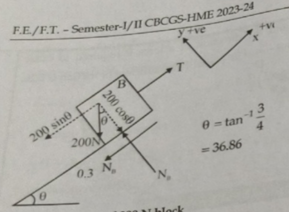


Fig 3.20

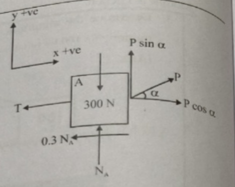
Solution:

FBD OF 200 N block



$\theta = \tan^{-1}\frac{3}{4}$

$= 36.86$



FBD of 300 N block

Applying Conditions of equilibrium to 200 N block

$\Sigma F_X = 0\ (\rightarrow +ve)$

$T - (200 \sin 36.86) - 0.3\ N_B = 0$

$T - 0.3\ N_B = 119.07$ ———(I)

$\Sigma F_Y = 0\ (\uparrow +ve)$

$N_B - (200 \cos 36.86) = 0$

$N_B = 160.02\ N$

Putting this value in equation (I)

$T - (0.3 \times 160.02) = 119.07$

$\therefore T = 167.07\ N$

Applying Conditions of equilibrium to 300 N block

$\Sigma F_X = 0\ (\rightarrow +ve)$

$P \cos\alpha - T - 0.3\ N_A = 0$

$P \cos\alpha - 167.07 - 0.3\ N_A = 0$

$\therefore P \cos\alpha = 167.07 + 0.3\ NA$ ———(II)

$\Sigma F_Y = 0\ (\uparrow +ve)$

$P \sin\alpha + N_A - 300 = 0$

$N_A = 300 - P \sin\alpha$

Putting this value in equation (II)

$\therefore P \cos\alpha = 167.07 + 0.3\,(300 - P \sin\alpha)$

$P\,(\cos\alpha + 0.3 \sin\alpha) = 257.07$

$$P = \frac{257.07}{\cos\alpha + 0.3 \sin\alpha} \text{ ———(III)}$$

$P = f(\alpha)$, for P to be minimum, $\frac{dP}{d\alpha} = 0$, $\frac{dP}{d\alpha} = \frac{0 - 257.07(-\sin\alpha + 0.3\cos\alpha)}{(\cos\alpha + 0.3\sin\alpha)^2} = 0$

$-\sin\alpha + 0.3\cos\alpha = 0$

$\sin\alpha = 0.3\cos\alpha$

$\tan\alpha = 0.3$

$\alpha = \tan^{-1}0.3 = 16.7^\circ$

Putting value of α in equation (III)

$$P_{Least} = \frac{257.07}{\cos 16.7 + 0.3 \sin 16.7} = 247.12\ N$$

2. Two blocks W1& W2 resting on two inclined planes are connected by a horizontal bar AB as shown in fig. If W1 equals 1000 N, determine the maximum value of W2 for which the

equilibrium can exist. The angle of friction is 20o for all rubbing surfaces.

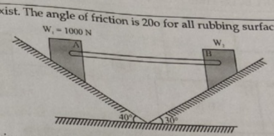
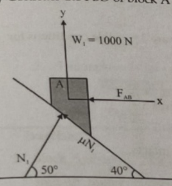


Fig: 3.21

Solution: For maximum weight of the block B in limiting equilibrium condition, the tendency of block B will be to impend upwards. So, impending motion of block A will be downward.

(i) Consider the FBD of block A



F.B.D. of Block A

$$\sum F_Y = 0;$$

$$N_1 \sin 50^\circ + \mu N_1 \sin 40^\circ - 1000 = 0$$

$$N_1 = \frac{1000}{\sin 50^\circ + \tan 20^\circ \sin 40^\circ}$$

$$N_1 = 1000N$$

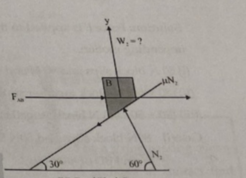
$$\sum F_X = 0;$$

$$N_1 \cos 50^\circ - \mu N_1 \cos 40^\circ - F_{AB} = 0$$

$$F_{AB} = 1000 \cos 50^\circ - \tan 20^\circ \times 1000 \cos 40^\circ$$

$$F_{AB} = 363.97N$$

(ii) Consider the FBD of block B



F.B.D. of Block B

$$\sum F_X = 0;$$

$$F_{AB} - N_2 \cos 60^\circ - \mu N_2 \cos 30^\circ = 0$$

$$N_2 = \frac{363.97}{(\cos 60^\circ + \tan 20^\circ \cos 30^\circ)}$$

$$N_2 = 446.48N$$

$$\sum F_Y = 0;$$

$$N_2 \sin 60^\circ - \mu N_2 \sin 30^\circ - W_2 = 0$$

$$W_2 = 446.48 \sin 60^\circ - \tan 20^\circ \times 446.48 \sin 30^\circ$$

$$W_2 = 305.41N$$

3. Three blocks are placed on 'the surface one above the other as shown in figure. The static coefficient of friction between the blocks and block C and surface is also shown. Determine the maximum value of P that can be applied before any slipping takes place.

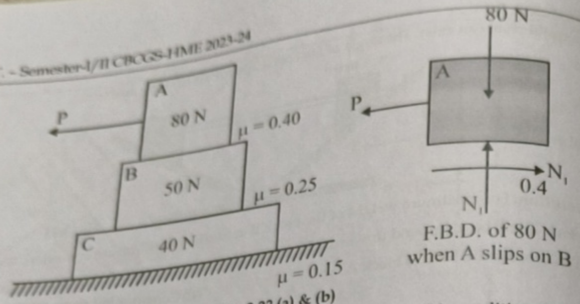


Fig 3.22 (a) & (b)

Solution: Force P is applied to the top block A. Here there are 3 possible conditions for impending motion.

(i) 80 N block slips and 50 N and 40 N blocks remain intact.

(ii) (80 + 50) N block together slips, and 40 N block remain intact.

(iii) (80 + 50+ 40) N blocks together slip on surface.

Case(i) 80N block slips and 50N & 40 N block may remain intact

From FBD of 80N block

$\Sigma F_Y = 0;\ N_1 - 80 = 0\ N_1 = 80N$

$\sum F_X = 0; 0.4N_1 - P = 0 (0.4 \times 80) - P = 0 P = 32N........(A)$

Case(ii) (80 + 50) N as a unit slips and 40 N block may remain intact

From FBD of (80 + 50) N block

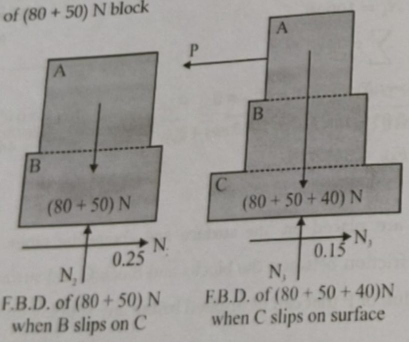


Fig 3.23

$\Sigma F_Y = 0;\ N_2 - (80 + 50) = 0\ N_2 = 130N$

$\sum F_X = 0; 0.25N_2 - P = 0 (0.25 \times 130) - P = 0 P = 32.5N........(B)$

Case(iii) (80 + 50 + 40) N as a unit slips on the surface

From FBD of (80 + 50 + 40) N together

$\Sigma F_Y = 0;\ N_3 - (80+50+40) = 0;\ N_3 = 170N\ \Sigma F_X = 0; 0.15N_3 - P = 0P = 25.5N........(C)$

From (A), (B) & (C) maximum value of P that can be applied is the minimum value obtained from these equations

Therefore, at P=25.5N all the three blocks slip on the ground surface as a single unit

Let's check the takeaway from this lecture

1. For blocks kept one above the other, the system will be disturbed completely only if the frictional resistance between block is mor than that of the support
   i) True ii) False
   iii) Maybe
2. The pulleys in the problems are considered to be
   i) Weightless ii) Frictionless
   iii) Rigid iv) All of the above

Exercise:

1. Block A of mass 12 kg and block B of mass 6 kg are connected by a string passing over a smooth pulley shown in Fig. (below left). If μ=0.12 at all surfaces of contact find smallest value of force P to maintain equilibrium. **(Ans. 163.5 N)**

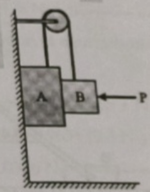


Fig 3.24

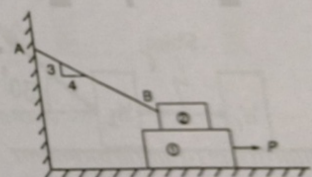


Fig 3.25

2. A block of weight W_1 = 1290N rests on a horizontal surface and supports another block of weight W_2 = 570N on top of it as shown in Fig. (above right) Block of weight W_2 is attached to a vertical wall by an inclined string AB. Find the Force 'P' applied to the lower block, that will be necessary to cause the slipping to impend.
   Coefficient of friction between blocks 1 and 2 = 0.25
   Coefficient of friction between block 1 and horizontal surface = 0.40 **(Ans. P=823N)**
3. Two inclined planes AC and BC inclined at 60°and 30° to the horizontal meet at a ridge C. A mass of 100 kg rests on the inclined plane BC and is tied to a rope which passes

over a smooth pulley at the ridge, the other end of the rope being connected to a block of W kg mass resting on the plane AC shown in Fig. (below left).. Determine the least and greatest value of W for the equilibrium of the whole system. **(Ans. 243.88N, 973.05N)**

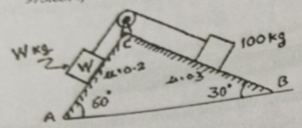


Fig 3.26

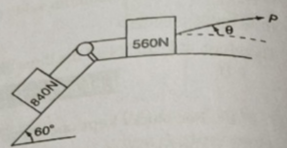


Fig 3.27

4. What is the least value of 'P' required to cause the motion impend, the system shown in Fig. (above right) Assume coefficient of friction on all contact surfaces as 0.2. **(Ans. P=905.52N, θ=11.31°)**

Practice Problem:

5. In the figure below, the two blocks (W_1 = 30N and W_2= 50N) are placed on rough horizontal plane. Coefficient of friction between the block of weight W_1 and plane is 0.3 that between block of weight W_2 and plane is 0.2. Find the minimum value of the force 'P' to just move the system. Also find the tension in the string. **(Ans: 19.67N, 9N)**

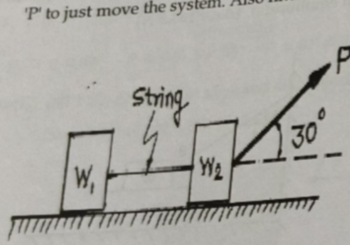


Fig 3.28

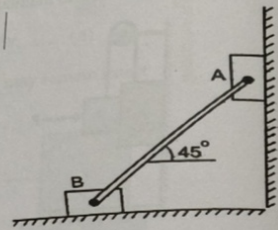


Fig 3.29

6. Two blocks A and B which are identical weighing W each are supported by a rod inclined at 45° to the horizontal as shown in Fig. (above right). If the blocks are in limiting equilibrium, find the coefficient of friction assuming it to be the same at both the floor and the wall. **(Ans. μ=0.414)**

Learning from the lecture: Student will be able to understand and solve complicated problems based on blocks and connected links

Lecture: 22

3.4 Problems on wedge & blocks (Horizontal/Inclined plane)

Learning Objective: Learners will be able to understand the concept of solving the problems based on wedge and blocks.

1. If coefficient of friction at all sliding surfaces is 0.3, find whether the 4 KN force is enough to move 5 KN load of inclined surfaces.

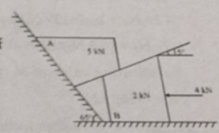


Fig 3.30

Sol: F.B.D of block A and B

Applying condition of equilibrium to block A,

$\sum F_X = 0$ (→+ve)

$- N_2 \sin 15 - F_2\cos 15 + F_1\cos 65 + N_1\sin 65 = 0$

$- 0.258\ N_2 - (0.3\ N_2)\ 0.965 + (0.3\ N_1)\ 0.422 + 0.906\ N_1 = 0$

$- 0.548\ N_2 + 1.0326\ N_1 = 0$ ---------------(I)

$\sum F_Y = 0$ (↑+ve)

$- 5 + N_2\ \cos 15\ -\ F_2\sin 15\ +\ N_1\cos\ 65 - F_1\sin 65 = 0$

$-5 + 0.965\ N_2 - (0.3\ N_2)0.258 + 0.422\ N_1 - (0.3\ N_1)\ 0.906 = 0$

$0.8876\ N_2 + 0.1502\ N_1 = 5$ ---------------(II)

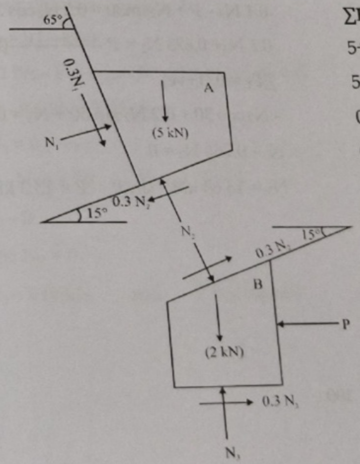
Applying condition of equilibrium to block A,

$\sum F_X = 0$ (→+ve)

$- N_2 \sin 15 - F_2\cos 15 + F_1\cos 65 + N_1\sin 65 = 0$

$- 0.258\ N_2 - (0.3\ N_2)\ 0.965 + (0.3\ N_1)\ 0.422 + 0.906\ N_1 = 0$

$- 0.548\ N_2 + 1.0326\ N_1 = 0$---------------(I)



$\sum F_Y = 0$ (↑+ve)

$5 + N_2 \cos 15 - F_2\sin 15 + N_1\cos 65 - F_1\sin 65 = 0$

$5 + 0.965\ N_2 - (0.3\ N_2)0.258 + 0.422\ N_1 - (0.3\ N_1)\ 0.906 = 0$

$0.8876\ N_2 + 0.1502\ N_1 = 5$ ---------------(II)

Solving equations (I) and (II) simultaneously

N_1= 2.74 kN and N_2 = 5.16 kN

Applying condition of equilibrium to wedge B,

$\sum F_X = 0$ (→+ve)

$F_3 - P + F_2\cos 15 + N_2\sin 15 = 0$

$(0.3\ N_3) - P + (0.3N_2)\ 0.965 + 0.258\ N_2 = 2.86$

$0.3\ N_3 + 0.5475\ N_2 = P$

Substituting value of N_2 in above equation

so $P = 0.3\ N_3 + 2.8251$ ---------------(III)

$\sum F_Y = 0\ (\uparrow +ve)$

$-2 + N_3 + F_2 \sin 15 - N_2 \cos 15 = 0$

$-2 - N_3 + (0.3\ N_2)0.258 - 0.965\ N_2 = 0$

Substituting value of N_2 in above equation

$N_3 = -6.58$ kN

$N_3 = 6.58$ kN(-ve sign indicates assumed direction is wrong hence direction is to be reverse)

Substituting value of N_3 in equation (III)

$P = 0.3N_3 + 2.8251$

$P = 4.8$kN

As P (4.8 kN) is more than the given applied force of 4 kN, it is not sufficient to move the block of 5 kN.

2. A wedge A and block B are subjected to known weight Q = 10 kN and force P as shown in figure. Determine the range of force P for which there is no motion.

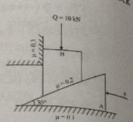


Fig: 3.31

Solution:

Case I: Block B has impending motion in upward direction

FBD of blocks A and B

For equilibrium of block B,

$\sum F_X = 0\ (\rightarrow +ve)$

$N_1 - 0.2\ N_2 \cos 30 - N_2 \sin 30 = 0$

$N_1 - 0.67\ N_2 = 0$ ---------------(I)

$\sum F_Y = 0\ (\uparrow +ve)$

$-0.3\ N_1 - 10 - 0.2\ N_2 \sin 30 + N_2 \cos 30 = 0$

$-0.3\ N_1 + 0.77\ N_2 = 10$ ---------------(II)

$N_1 = 11.77$ kN and $N_2 = 17.57$ kN

For equilibrium of block A,

$\sum F_X = 0\ (\rightarrow +ve)$

$0.1\ N_3 - P + N_2 \sin 30 - 0.2\ N_2 \cos 30 = 0$

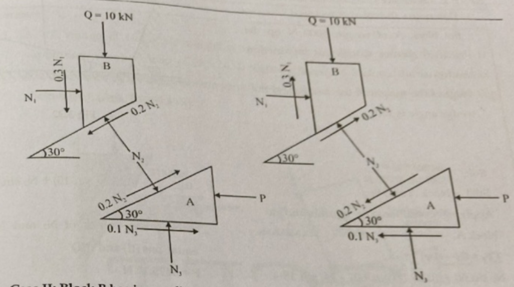
$0.1\ N_3 + 0.673\ N_2 = P$ ---------------(III)

$\sum F_Y = 0\ (\uparrow +ve)$

$-N_2 \cos 30 + 0.2\ N_2 \sin 30 + N_3 = 0$

$N_3 - 0.766\ N_2 = 0$

$N_3 = 13.63$ kN and $P = 13.3$ kN



Case II: Block B has impending motion in downward direction

FBD of blocks A and B

For equilibrium of block B,

$\sum F_X = 0\ (\rightarrow +ve)$

$N_1 + 0.2\ N_2 \cos 30 - N_2 \sin 30 = 0$

$N_1 - 0.326\ N_2 = 0$ ---------------(III)

$\sum F_Y = 0\ (\uparrow +ve)$

$0.3\ N_1 - 10 + 0.2\ N_2 \sin 30 + N_2 \cos 30 = 0$

$0.3\ N_1 + 0.966\ N_2 = 10$

$N_1 = 3.064$kN and $N_2 = 9.40$ kN

For equilibrium of block A,

$\sum F_X = 0\ (\rightarrow +ve)$

$-0.1\ N_3 - P + N_2 \sin 30 - 0.2\ N_2 \cos 30 = 0$

$P = -0.1\ N_3 + 0.326\ N_2$

$\sum F_Y = 0\ (\uparrow +ve)$

$N_3 - N_2 \cos 30 - 0.2\ N_2 \sin 30 = 0$

$N_3 - 0$

$.966\ N_2 = 0$

$N_3 = 9.08$kN and $P = 2.156$ kN

3. Determine the force P required to move the block A of weight 5000 N up the inclined plane. Coefficient of friction between all contact surfaces is 0.25. Neglect the weight of the wedge and the wedge angle is 150.

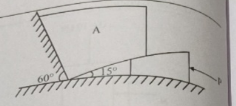


Fig 3.32

Sol:

FBD of block A and B:

Applying conditions of equilibrium to block A,

$\sum F_X = 0\ (\rightarrow +ve)$

$N_1 \sin 60 + (0.25\, N_1 \cos 60) - N_2 \sin 15 - (0.25\, N_2 \cos 15) = 0$

$0.991\, N_1 - 0.5\, N_2 = 0$ ---------------(I)

$\sum F_Y = 0\ (\uparrow +ve)$

$N_1 \cos 60 - (0.25\, N_1 \sin 60) + N_2 \cos 15 - (0.25\, N_2 \sin 15) - 5000 = 0$

$0.283\, N_1 + 0.9012\, N_2 = 5000$ ---------------(II)

Solving equations (I) and (II), we get

$N_1 = 2416.4$ N and $N_2 = 4789.34$ N

Applying conditions of equilibrium to block B,

$\sum F_Y = 0\ (\uparrow +ve)$

$N_3 + (0.25\, N_2 \sin 15) - N_2 \cos 15 = 0$

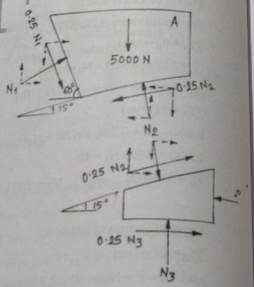
$N_3 = 4316.25$ N ---------------(III)

$\sum F_X = 0\ (\rightarrow +ve)$

$0.25\, N_3 + (0.25\, N_2 \sin 15) + N_2 \sin 15 - P = 0$

Substituting values of N_2 and N_3 from equations (II) and (III)

P = 3475.17 N



Let's check the takeaway from this lecture

1. Wedges have used to
   i) Lift blocks by horizontal effort
   ii) Move the block horizontally with vertical effort
   iii) None of these
2. Wedges are considered to be weightless
   i) True
   ii) False

   iii) Can't Say
   iv) Not always true

Exercise:

1. A 15° wedge of negligible weight is to be driven to tighten a body B which is supporting a vertical load of 1000 N. If the co-efficient of friction for all surfaces of contact is to be 0.25. Find minimum force P required to drive the wedge in the given Fig (below left). **(Ans. P=233.89N)**

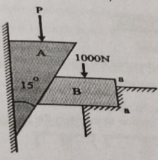


Fig 3.33

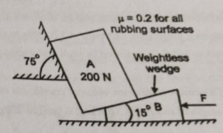
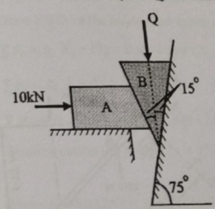


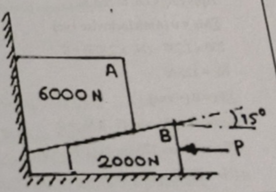
Fig 3.34

2. Find the force F to have motion of Block A impending up the plane. Take coefficient of friction for all the surfaces in contact as 0.2. Consider the wedge B as weightless (above right). **(Ans. F=134.6N)**
3. The wedge B is used to raise the weight of 5 kN resting on a block A. what horizontal force P, is required to do this, if the coefficient of friction for all the surfaces in contact is 0.2? **(Ans:3.84 kN)**

Practice Problems:

4. The Fig (below left) shows a wedge B held between the block A and the surface C. A horizontal push of 10kN is acting on the block A. Find the vertical force Q on the wedge B so as to just move it downward. Assume co-efficient of friction as 1/3 for all the surfaces of contact. **(Ans. Q=16.77kN)**





3. Determine the force P required to move the block A of weight 5000 N up the inclined plane. Coefficient of friction between all contact surfaces is 0.25. Neglect the weight of the wedge and the wedge angle is 150.

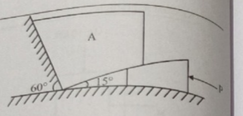


Fig 3.32

Sol:

FBD of block A and B:

Applying conditions of equilibrium to block A,

$\sum F_X = 0$ (→+ve)

$N_1 \sin 60 + (0.25\ N_1 \cos 60) - N_2 \sin 15 - (0.25\ N_2 \cos 15) = 0$

$0.991\ N_1 - 0.5\ N_2 = 0$ ————(I)

$\sum F_Y = 0$ (↑+ve)

$N_1 \cos 60 - (0.25\ N_1 \sin 60) + N_2 \cos 15 - (0.25\ N_2 \sin 15) - 5000 = 0$

$0.283\ N_1 + 0.9012\ N_2 = 5000$ ————(II)

Solving equations (I) and (II), we get

$N_1 = 2416.4$ N and $N_2 = 4789.34$ N

Applying conditions of equilibrium to block B,

$\sum F_Y = 0$ (↑+ve)

$N_3 + (0.25\ N_2 \sin 15) - N_2 \cos 15 = 0$

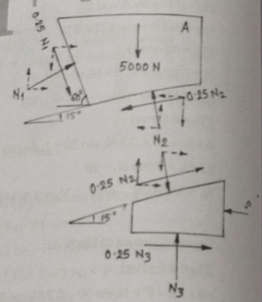
$N_3 = 4316.25$ N ————(III)

$\sum F_X = 0$ (→+ve)

$0.25\ N_3 + (0.25\ N_2 \sin 15) + N_2 \sin 15 - P = 0$

Substituting values of N_2 and N_3 from equations (II) and (III)

P = 3475.17 N



Let's check the takeaway from this lecture

1. Wedges have used to
   - i) Lift blocks by horizontal effort
   - ii) Move the block horizontally with vertical effort
   - iii) None of these
2. Wedges are considered to be weightless
   - i) True
   - ii) False

   - iii) Can't Say
   - iv) Not always true

Exercise:

1. A 15° wedge of negligible weight is to be driven to tighten a body B which is supporting a vertical load of 1000 N. If the co-efficient of friction for all surfaces of contact is to be 0.25. Find minimum force P required to drive the wedge in the given Fig (below left). **(Ans. P=233.89N)**

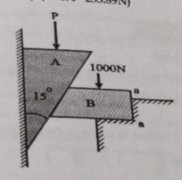


Fig 3.33

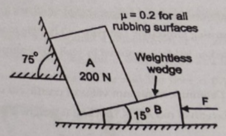
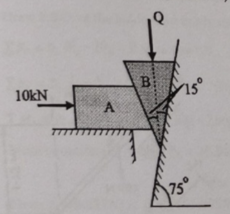


Fig 3.34

2. Find the force F to have motion of Block A impending up the plane. Take coefficient of friction for all the surfaces in contact as 0.2. Consider the wedge B as weightless (above right). **(Ans. F=134.6N)**
3. The wedge B is used to raise the weight of 5 kN resting on a block A. what horizontal force P, is required to do this, if the coefficient of friction for all the surfaces in contact is 0.2? **(Ans:3.84 kN)**

Practice Problems:

4. The Fig (below left) shows a wedge B held between the block A and the surface C. A horizontal push of 10kN is acting on the block A. Find the vertical force Q on the wedge B so as to just move it downward. Assume co-efficient of friction as 1/3 for all the surfaces of contact. **(Ans. Q=16.77kN)**



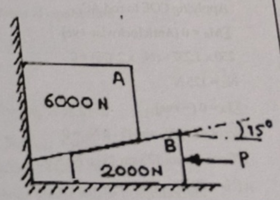


Fig 3.36

Fig 3.35

5. Find force P required to lift 6000 N block A. Take μ at surfaces in contact as 0.3. (Ans: 7354.47N)

Learning from the lecture: Student will be able to solve problems based on wedges.

Lecture: 23

3.5 *Problems on Ladder supported by wall and ground*

Learning Objective: Learners will be able to understand the concept of solving the problems based on ladder supported by wall and ground surface.

1. Determine minimum value of coefficient of friction to maintain the position shown in figure. Length of rod AB is 3.5m and it weighs 25kg.

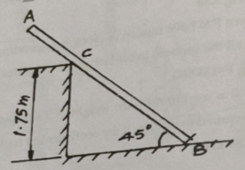
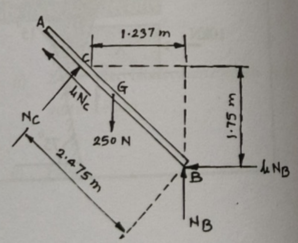


Fig 3.37

Solution: The rod AB is supported by a rough surface at B and a rough edge at C. The ladder loses its equilibrium position by slipping to the right.

FBD of the ladder



Applying COE to rod AC

$\sum M_B = 0$ **(Anticlockwise +ve)**

$250 \times 1.237 - (N_C \times 2.475) = 0$

$N_C = 125$ N

$\sum F_X = 0$ **(→+ve)**

$N_C \cos 45 - \mu N_C \sin 45 - \mu N_B = 0$

$125 \cos 45 - \mu \times 125 \sin 45 - \mu N_B = 0$

$\mu (N_B + 88.39) = 88.39$ --------------(I)

$\sum F_Y = 0$ **(↑+ve)**

$N_C \sin 45 + \mu N_C \cos 45 + N_B - 250 = 0$

$125 \sin 45 + \mu \times 125 \cos 45 + N_B - 250 = 0$

$N_B = 161.61 - 88.39\,\mu$ --------------(II)

Substituting above value of N_B in equation (I)

$\mu\,[(161.61 - 88.39\mu) + 88.39] = 88.39$

$88.39\,\mu^2 - 250\,\mu + 88.39 = 0$

Solving above quadratic equation, $\mu = 0.414$ or $\mu = 2.414$

Since μ cannot be more than 1, the feasible value of $\mu = 0.414$

2. A non-homogeneous ladder shown in figure rests against a smooth wall at 'A' and a rough horizontal floor at 'B'. The mass of the ladder is 30 kg and is concentrated at 2 m from the bottom. The coefficient of static friction between the ladder and the floor is 0.35. Will the ladder stand in position?

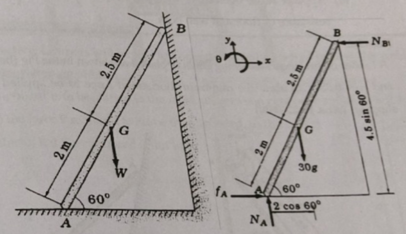


Fig 3.39

Sol: In this problem, first calculate frictional force F_A, By applying conditions of equilibrium. Then, if $F_A \leq \mu N_A$ ladder will be in equilibrium, otherwise it is under motion.

Draw F.B.D. of the ladder and apply conditions of equilibrium.

$\sum F_Y = 0;\ N_A - 30g = 0\ N_A = 294.3N$.......(i)

$\sum F_X = 0;\ f_A - N_B = 0\ f_A = N_B$.......(ii)

$\sum M_A = 0;\ N_B \times 4.5 \sin 60^\circ - 30g \times 2 \cos 60^\circ = 0\ N_B = 75.517N$.......(iii)

From equation (ii), we get $f_A = 75.517N$, But $f_{Max} = \mu N_A$

$\therefore f_{Max} = 0.35 \times 294.3 = 103.005N$

Here, $f_A(75.517N) < f_{Max}(103.005N)$. Hence ladder will be in equilibrium.

Let's check the takeaway from this lecture

1. If the ladder supported on smooth vertical wall and rough horizontal surface is applied with forces at the lower end it will move upwards only if
   i) Applied force overcomes frictional resistance of floor
   ii) Applied force overcomes frictional resistance of wall
   iii) Applied force overcomes frictional resistance of floor and wall both
   iv) More information required
2. If the ladder supported on smooth vertical wall and rough horizontal surface is applied with forces at the lower end it will move downwards only if
   i) Applied force overcomes frictional resistance of floor
   ii) Applied force overcomes frictional resistance of wall
   iii) Applied force overcomes frictional resistance of floor and wall both
   iv) More information required

Exercise:

1. A ladder of 4m length weighing 200N is placed as shown in the Fig (below left). μ_B=0.25 and μ_A= 0.35. Calculate the minimum horizontal force to be applied at A to prevent slipping. **(Ans. P=22.08N)**

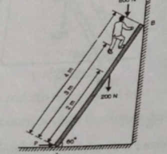


Fig 3.40

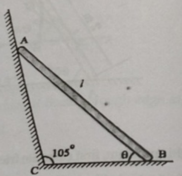


Fig 3.41

2. The uniform ladder of length 'l' and mass "m" is placed against the surfaces as shown in Fig (above right). If the co-efficient of friction μ_S=0.25 at A & B, find the maximum angle θ at which the ladder can be just before slipping begins. **(θ=59.86°)**

Practice Problem:

3. A uniform ladder 3.6 m long weighs 180 N. it is held in position as shown in the figure. The coefficient of friction between the wall and the ladder is 0.25 and that between the floor and ladder is 0.35. The ladder in addition to its weight supports a man of 80 kg at its top end. Calculate the horizontal force P to be applied to the ladder at the floor level

to prevent slipping. if the force P is not applied what should be the minimum inclination of the ladder with horizontal so that there is no slipping? **(Ans: 224.12 N, 70.09°)**

4. A uniform ladder of length 5m rests against a rough vertical wall with its lower end on a rough horizontal floor, the ladder being inclined at 60° to the horizontal. The coefficient of friction between all the contact surfaces is 0.25. A man of weight 600 N ascends up the ladder. What is maximum length up along to ladder the man will be able to ascend before the ladder commences to slip. The weight of the ladder is 100 N. **(Ans: 2.304 m)**

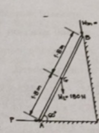


Fig 3.42

Learning from the lecture: Student will be able to solve problems based on ladder.

Lecture: 24

3.6 *Problems on Tipping/ Slipping of block*

Learning Objective: Learners will be able to understand the concept of solving the problems based on tipping/slipping of block.

1. A 120 kg cupboard is to be shifted to the right. μS between cupboard and floor is 0.3. Determine: i) the force P required to move the cupboard, ii) the largest allowable value of h if the cupboard is not to tip over.

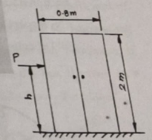


Fig 3.43

Solution:

i) The force P required moving the cupboard

Applying conditions of equilibrium to the cupboard,

$\sum F_Y = 0$ (↑+ve)

$N - 1177.2 = 0$

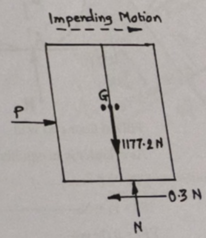
$N = 1177.2$ N

$\sum F_X = 0$ (→+ve)

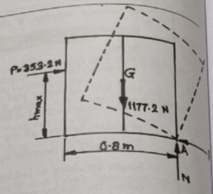
$P - 0.3\,N = 0$

$P - 0.3 \times 1177.2 = 0$

$P = 353.2$ N



ii) The largest allowable value of h if the cupboard is not to tip over.



At maximum height condition, the cupboard is on the verge of tipping. The normal reaction N shifts to the corner A. Applying conditions of equilibrium to the cupboard,

$\sum M_A = 0$ (Anticlockwise +ve)

$-353.2 \times h_{max} + 1177.2 \times 0.4 = 0$

$h_{max} = 1.33$ m

1. For the system given in figure (i) if applied force P is 180 N will the cylinder rotate? Take weight of the cylinder W = 900 N and coefficient of friction as 0.25.

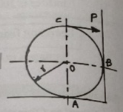
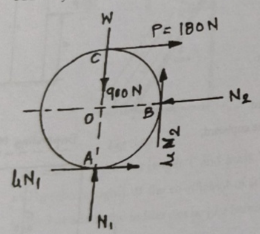
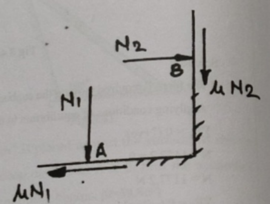


Fig 3.44

Solution: Now because of force P whether the cylinder will rotate or not is not known. Hence frictional forces F_1 and F_2 may or may not have reached its maximum limiting values μN_1 and μN_2 respectively. Therefore, to begin with frictional forces F_1 and F_2 and not as μN_1 and μN_2.

FBD of cylinder





FBD of floor and wall

As cylinder is in equilibrium,

$\sum F_X = 0$ (→+ve)

$180 + F_1 = N_2$ ---------------- (I)

$\sum F_Y = 0$ (↑+ve)

$900 = N_1 + F_2$ ---------------- (II)

$\sum M_C = 0$ (Anticlockwise +ve)

$(-180 \times 2r) + (N_2 \times r) + (F_2 \times r) = 0$ ------ ---------- (III)

Now let us assume that F_2 has reached its limiting value μN_2 i.e. 0.25 N_2 and find the values of N_2, N_1 and F_1 solving above equations.

From equation (III), $180 \times 2r = (N_2 \times r) + (0.25 N_2 \times r)$

$N_2 = \frac{360 r}{1.25 r} = 288$ N

From equation (II), $900 = N_1 + F_2 = N_1 + (0.25 \times 288)$

$N_1 = 828$ N

From equation (I), $180 + F_1 = 288$

$F_1 = 108$ N

Now from the Free body diagram, it is seen that the forces F_1, F_2 and P create moments about O.

Moments of F_1 and F_2 about O is anticlockwise $= (F_1 \times r) + (F_2 \times r)$

$= (108 \times r) + (0.25 \times 288 \times r)$

$= 180 r$

Moment of P about O is clockwise $= P \times r = 180 r$

Both clockwise and anticlockwise moments are equal and hence the cylinder will not rotate. Now maximum possible limiting value of $F_1 = \mu N_1 = 0.25 \times 828 = 207$ N

But here it has reached only 108 N; there is a scope for F_1 to increase from 108 N to 207 N before the cylinder start rotating.

2. A wooden box of 200 'kg mass is placed on an inclined as shown in figure. The coefficient of friction between the wooden box and the incline is 0.35. Find (i) The value of P for impending sliding motion up the plane. (ii) For what height above the plane this force may be applied if tipping of the box is not to occur. (iii) If the force P is removed will the box tip over in a counter clockwise sense? (iv) If the force P is removed, will the box slide down the plane?

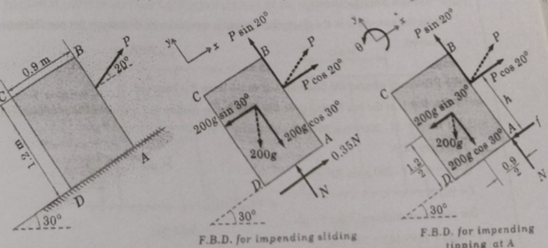


Fig 3.45

Solution:

(i) Impending sliding motion up the plane

In this case $f = 0.35N$ will act downwards. Draw F.B.D and apply conditions of equilibrium

$\sum Fy = 0;\quad N + P\sin 20° - 200\,g\cos 30° = 0$

$N = 1699.142 - 0.342\,P$ ....(I)

$\sum Fx = 0;\quad -0.35\,N + P\cos 20° - 200\,g\sin 30° = 0$

Put value of N from equation (I)

$P\cos 20° - 200\,g\sin 30° - 0.35\,(1699.142 - 0.342\,P) = 0$

$P = 1486.93$ N... Ans

(ii) Height of force P for which tipping will not occur

Let 'h' be the height at which P is applied so that tipping motion is impended. Note that frictional force and normal reaction act at point A. Now

$\sum M_A = 0;\quad (200g\sin 30° \times 0.6) - (P\cos 20° \times h) + (200g\cos 30° \times 0.45) = 0$

But $P = 1486.93$ N (already calculated)

$h = (1353.214 / 1486.93\cos 20°) = 0.968\text{m} < 1.2$ m

Tipping will not occur when P is applied at a height of 0.968 m or less than 0.968 m

(iii) To check whether tipping is possible is anticlockwise direction or not.

If the box has to tip in counterclockwise direction counterclockwise moment must be positive.

Now taking moment about D point.

$\sum M_D = 0;$

$(200g\sin 30° \times 0.6) - (200g\cos 30° \times 0.45)$

$= -176.014$ Nm

$= 176.014$ Nm (clockwise)

Since the moment is in the clockwise, tipping in anticlockwise direction is not possible when P is removed.

(iv) To check whether the box will slide or not when P is removed.

Draw F.B.D of the box having downward impending motion and apply condition of equilibrium

$\sum Fy = 0;\quad N - 200\,g\cos 30° = 0$

$N = 1699.142N$

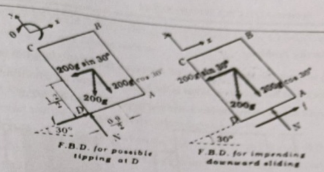
$\sum Fx = 0;\quad f - 200\,g\sin 30° = 0$

$f = 981$ N

But maximum frictional force $f = \mu N$

$f = 0.35 \times 1699.142 = 594.7$ N

Since $f > f_{max}$ the box will slide down the plane in the absence of external force



Let's check the takeaway from this lecture

1. Tipping generally occurs because
   i) Moment of Weight is more than that of moment of applied force
   ii) Moment of Weight is less than that of moment of applied force
   iii) It is natural phenomena
   iv) None of these
2. For solving Tipping related problems, we may have to use _____ conditions of equilibrium
   i) One
   ii) Two
   iii) Three
   iv) Four

Exercise:

1. Two homogeneous blocks are freely resting with their weights and coefficients of friction at surfaces of contact, given is as shown in the figure. Find the value of P which will destroy the equilibrium of the system. (Ans: 13.2 N)

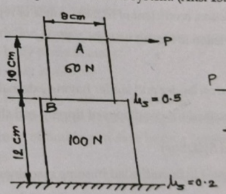


Fig 3.46

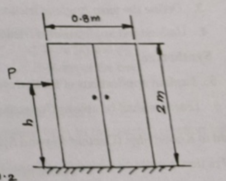


Fig 3.47

2. A 120 kg cupboard is to be shifted to the right μ_S between cupboard and floor is 0.3. Determine: i) the force P required for moving the cupboard, ii) the largest allowable

value of h if the cupboard is not to tip over. (Ans: 353.2 N, 1.33 m)

Practice Problem:

3. A rectangular block of mass 'm' rests on a floor. The coefficient of friction between the block and the floor is μ. What is the highest position for a horizontal force P that would permit to just move the block without tipping? **(Ans: d = b/2 μ)**

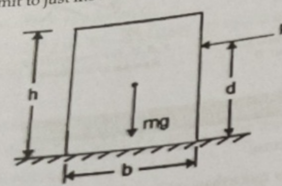


Fig 3.48

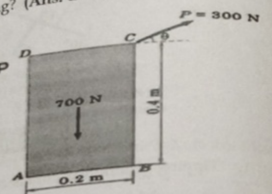


Fig 3.49

4. In the figure 3.49 shown determine the range of values of θ for which the block will slide without tipping. **(Ans: θ=19°)**

Learning from the lecture: Student will be able to learn and apply the concept of Tipping

3.7 Conclusion:

Learning Outcomes:

Learners should be able to

Know, Comprehend

1. Explain the theory of friction and its characteristics
2. Define static and dynamic friction, laws of friction.

Apply, Analyse

3. Define the term Angle of friction, coefficient of friction, Angle of repose and cone of friction
4. Understand application of friction to blocks, wedges & blocks

Synthesize

5. Explain applications of friction on ladder and ladder having external weight acting on it.
6. Learners shall be able to Understand the conditions of tipping and sliding

3.8 Add to Knowledge (Content Beyond Syllabus)

Friction is quite undesirable and needs to be mitigated in some machines & processes such as

a) power screws

b) bearings and gears

c) flow of fluids in pipes

Its presence would cause loss of power, wearing out of parts and huge economic losses.

However the working of many devices such as

a) friction brakes and clutches

b) belt and rope drives

c) holding and fastening devices

depends on friction and there the presence is advantageous

Belts and Rope Drive: These are used when the distance between the axes of the two shafts to be connected which are considered to be non-rigid and can undergo strain while in motion.

Types of belt: Flat Belt, V Belt, Round Belt

Chain drive: It is used when distance between the shaft centres is short and no slip is required.

Gears: It is used for transmitting motion and power when the distance between the driving and driven shafts is relatively small and when constant velocity ratio is desired. Types of gears: Spur Gears, Helical Gears, Bevel Gears, Spiral Gears, Worm Gears, Rack & Pinion, Internal & External Gear.

Screw Jack: It is a simple machine used for lifting heavy loads, through short distances with the help of small effort applied at its handle. E.g: Raising a vehicle to change the wheel or tyre.

Research Work: https://www.codecogs.com/library/engineering/theory_of_machines/belt-and-rope-drives-brakes.php

3.9 Set of Multiple-Choice Questions:

1) The magnitude of the force of friction between two bodies, one lying above the other, depends upon the roughness of the

a) Upper body
b) Lower body
c) Both the bodies
d) The body having more roughness

2) The force of friction always acts in a direction opposite to that

a) In which the body tends to move
b) In which the body is moving
c) Both (a) and (b)
d) None of the two

3) Which of the following statement is correct?

a) The force of friction does not depend upon the area of contact
b) The magnitude of limiting friction bears a constant ratio to the normal reaction between the two surfaces
c) The static friction is slightly less than the limiting friction.
d) All (a), (b) and (c)

4) The magnitude of the force of friction between two bodies, one lying above the other, depends upon the roughness of the……

a) Upper body
b) Lower body
c) Both the bodies
d) The body having more roughness

5) **The force of friction always acts in a direction opposite to that……**
a) In which the body tends to move
b) In which the body is moving
c) Both (*a*) and (*b*)
d) None of the two

6) **Which of the following statement is correct?**
a) The force of friction does not depend upon the area of contact
b) The magnitude of limiting friction bears a constant ratio to the normal reaction between the two surfaces
c) The static friction is slightly less than the limiting friction.
d) All (a), (b) and (c)

7) **The force of friction between two bodies in contact……**
a) Depends upon the area of their contact
b) Depends upon the relative velocity between them
c) Is always normal to the surface of their contact
d) All of the above

8) **The maximum value of friction force which comes into play when a body tends to move on a surface is called**
a) Sliding friction
b) Limiting friction
c) Dynamic friction

9) **The coefficient of friction depends on……….**
a) Area of contact
b) shape of the body
c) nature of contact surfaces

10) **The force required to move a body up an inclined plane will be least when the angle of inclination is**
a) equal to friction angle
b) greater than friction angle
c) less than friction angle

11) **We can walk or run as…….**
a) friction on a foot is equal to forward thrust
b) friction is greater than forward thrust
c) friction is less than forward thrust

12) **The angle of friction is …….**
a) the angle between the normal and the resultant of normal reaction and limiting friction force
b) the ratio of friction force to normal reaction
c) the angle between the horizontal and the resultant of normal reaction and friction force

13) **The angle of inclination of an inclined plane when a body is about to slide down is called**
a) angle of friction
b) angle of repose
c) angle of kinetic friction

14) **When a ladder is resting on a smooth ground and leaning against a rough vertical wall, then the force of friction acts**
a) towards the wall at its upper end
b) downwards at its upper end
c) upwards at its upper end

15) **If the angle of inclination of a plane is less than the friction angle, then we require a force**
a) to move the body upwards only
b) to move the body downwards only
c) to move the body upwards and downwards only

16) **A cube rests on a rough horizontal surface. If the cube is gradually tilted by tilting the inclined plane, then sliding will occur without toppling of the cube if coefficient of friction is ________.**
a) greater than 1
b) equal to 1
c) less than 1

17) **Friction on the wheel of a cycle acts ________.**
a) forwards
b) backwards
c) upwards

18) **The cycle stops when wheels are stopped rolling with the help of the brake as________.**
a) rolling of the wheels decreases
b) power to the wheels stops
c) sliding friction is much higher than rolling friction which acts against motion

19) **The cause of friction between the two surfaces is ……….**
a) Roughness
b) Material
c) Both a & b
d) None of these

20) **Dynamic friction as compared to static friction is…….**
a) Less
b) Equal
c) Greater

1.(c) 2.(c) 3.(d) 4.(c) 5.(c) 6.(d) 7.(d) 8.(b) 9.(c) 10.(c) 11.(b) 12.(a) 13.(b) 14.(b) 15.(b) 16.(a) 17.(a) 18.(c) 19.(c) 20.(c)

3.10 Short Answer Questions:

1. What do understand by limiting friction?
2. What is the difference between angle of friction and angle of repose?
3. Why is the value of static friction always higher than that of kinetic friction?
4. On what basis is the coefficient of friction between two surfaces in contact decided?
5. With a neat-labelled diagram explain all the terms related to friction.
6. What is cone of friction? Explain its significance.
7. What do you understand by Tipping?
8. Explain the mechanism of Rolling.
9. Explain Laws of Friction.
10. What are the various applications of friction? Explain its advantages and disadvantages.

3.11 Long Answer Questions:

1. Two blocks A and B weighing 800 N and 1000 N respectively rest on two inclined planes each inclined at 30° to the horizontal. They are connected by a rope passing through a smooth pulley as shown. Ropes carrying loads of W1 and 5000 N (W2) and passing over pulleys at the tops of the planes are also connected to the two blocks as shown in figure. Coefficient of friction may be taken as 0.1 and 0.2 for blocks A and B respectively. Determine the least and greatest value of W1 for the equilibrium of the whole system. (Ans: 4657.52N, 5142.48N)

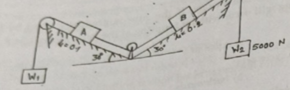


Fig 3.50

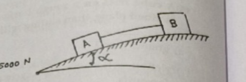


Fig 3.51

2. A cord connects two bodies A and B of weights 450 N and 900 N. The two bodies are placed on an inclined plane and cord is parallel to inclined plane. The coefficient of friction for body A is 0.16 and that for B is 0.42. Determine the inclination of the plane to the horizontal and tension in the cord when the motion is about to take place down the plane. The body A is below the body B on the inclined plane. (Ans: 18.434°, 73.98N)
3. Calculate the magnitude of the horizontal force P acting on the wedges B and C to raise a load of 200 kN resting on A. Assume μ between the wedges and the ground as 0.25 and between the wedges and A as 0.2. Also assume symmetry of loading and neglect the weight of A, B and C. Wedges are resting on horizontal surface and their slope is 1:10. (Ans: 55.665N)

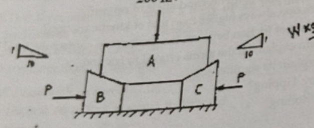


Fig 3.52

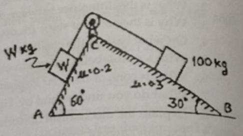


Fig 3.53

4. Two inclined planes AC and BC inclined at 60° and 30° to the horizontal meet at a ridge C. A mass of 100 kg rests on the inclined plane BC and is tied to a rope which passes over a smooth pulley at the ridge, the other end of the rope being connected to a block of W kg mass resting on the plane AC. Determine the least and greatest value of W for the equilibrium of the whole system. (Ans: 243.88N, 973.05N)
5. Two blocks W1& W2 resting on two inclined planes are connected by a horizontal bar AB as shown in fig. If W1 equals 1000 N, determine the maximum value of W2 for which the equilibrium can exist. The angle of friction is 20o for all rubbing surfaces.

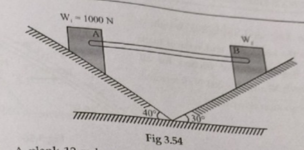


Fig 3.54

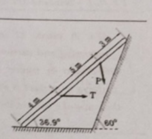


Fig 3.54

6. A plank 12 m long and negligible weight is supported and carries the horizontal and vertical loads as shown in figure. If coefficient of friction for all surfaces of contact is μ = 0.30 and T = 300 N determine the value of P to start the motion downward.
7. Calculate the range of 'P' for which equilibrium is maintained. Weight of 'AB' = 100N (Ans: 8.29N, 80.58N)

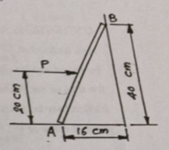


Fig 3.55

3.12 References:

1) Engineering Mechanics by Tayal, Umesh Publication
2) Engineering Mechanics by Beer & Johnson, Tata McGraw Hill
3) Engineering Mechanics by F.L. Singer by Harper
4) Engineering Mechanics - Statics, R. C. Hibbler
5) Engineering Mechanics - Statics, J. L. Merium, I. G. Kraig
6) Engineering Mechanics - P. J. Shah, R. Bade

Self-Assessment

Q.1. What is cone of friction? Explain its significance.

Q.2. What do you understand by Tipping?

Q.3. Explain the mechanism of Rolling.

Q.4. A plank 12 m long and negligible weight is supported and carries the horizontal and vertical loads as shown in figure. If coefficient of friction for all surfaces of contact is $\mu = 0.30$ and $T = 300$ N determine the value of P to start the motion downward.

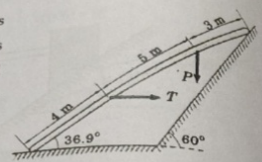


Fig 3.56

Q.5. Two blocks A and B weighing 800 N and 1000 N respectively rest on two inclined planes each inclined at 30° to the horizontal. They are connected by a rope passing through a smooth pulley as shown. Ropes carrying loads of W1 and 5000 N (W2) and passing over pulleys at the tops of the planes are also connected to the two blocks as shown in figure. Coefficient of friction may be taken as 0.1 and 0.2 for blocks A and B respectively. Determine the least and greatest value of W1 for the equilibrium of the whole system. (Ans: 4657.52N, 5142.48N)

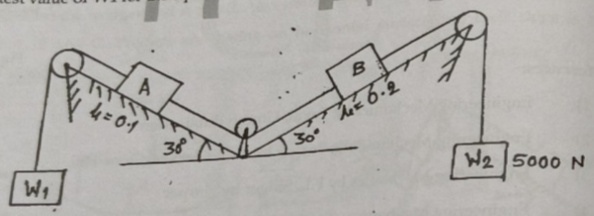
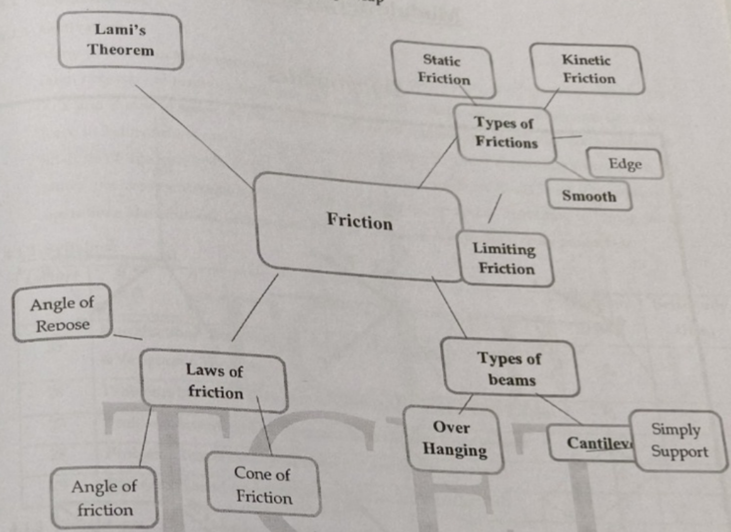


Fig 3.57

Concept Map



Self-Evaluation

Name of Student: **Course Code:**

Class & Div.: **Roll No.:**

1. Can you define Kinetic and Static Coefficient of Friction?

   (a) Yes **(b) No**

2. Are you able to state the difference between angle of repose and angle of friction?

   (a) Yes **(b) No**

3. Are you able to solve problems based on blocks and wedges?

   (a) Yes **(b) No**

4. Are you able to solve problems based on ladders and tipping?

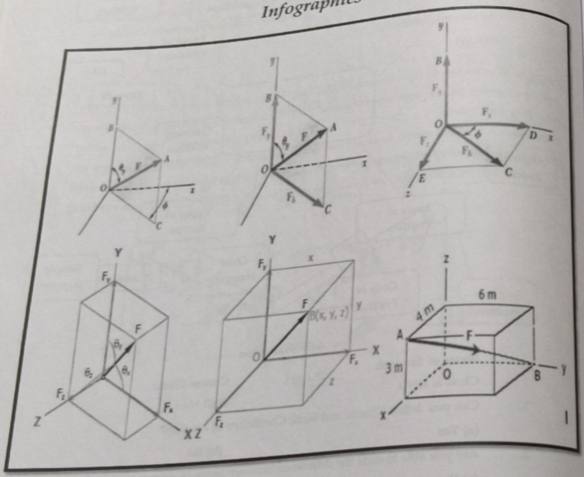
   (a) Yes **(b) No**

5. Do you understand this module?

   (a) Yes **(b) No**

Module 4: Forces in Space

Infographics



Lecture: 25

4.1 *Forces in Space*

4.1.1 Motivation:

Many problems in Mechanics require analysis in three dimensions, and for such problems it is often necessary to resolve a force into its three mutually perpendicular components i.e., along X, Y and Z axes in space. A force system in three dimensions is called Space Force system. A force in 3-dimensions can be expressed as vectors using X, Y and Z coordinate system. In the analysis of this force system, the vector analysis is very much involved. Thus, it is required to study the representations of forces and their effects as vector quantities by using vector operations like addition, unit vectors, direction cosines, dot product, cross product etc.

4.1.2 Syllabus:

Lecture No.	Title	Duration (Hrs.)	Self-Study (Hrs.)
25	Introduction, Resultant of forces, Condition of equilibrium &Varignon's Theorem	2	3
26	Problems based on Resultant of General & Parallel force system	1	3
27	Problems based on Equilibrium of Concurrent force system	1	1
28	Problems based on Equilibrium of General force system	1	2
29	Problems based on Equilibrium of Parallel force system	1	1

4.1.3 Weightage: 15 to 16 Marks (Approximately)

4.1.4 Pre-requisite:
Basic knowledge of vector, Basic understanding of forces and its characteristics, types, finding resultants of various systems of forces, effects of forces like moments and couples, Equilibrium of forces arranged in different force systems are required to be known.

4.1.5 Learning Objectives:

1) Learners will be able to represent force in vector form, angles made by its components with rectangular axes
2) Learners will be able to define the moment of force about a point and about a line
3) Learners will be able to enlist vector component of a force
4) Learners will be able to evaluate resultant of concurrent, parallel and general force system
5) Learners will be able to identify, problems based on equilibrium of concurrent &parallel force system.
6) Learners will be able to solve problems based on equilibrium of general force system.

4.1.6 Abbreviations: NA

4.1.7 Notations:

$\bar{r}$ = Position vector $\bar{i}$ = unit vector along x-axis $\bar{j}$ = unit vector along y-axis $\bar{k}$ = unit vector along z-axis

4.1.8 Formulae:

$$\bar{F} = F\bar{e}$$

$$\bar{F} = F_X\bar{i} + F_Y\bar{j} + F_Z\bar{k}$$

$$\cos^2\theta_X + \cos^2\theta_Y + \cos^2\theta_Z = 1$$

$$\overline{M_O} = \bar{r} \times \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ r_X & r_Y & r_Z \\ F_X & F_Y & F_Z \end{vmatrix}$$

4.1.9 Definitions: NA

4.1.10 Theoretical Background:

A. Rectangular components of a force in space:

The space force acting along diagonal OA $\bar{F} = F_X\bar{i} + F_Y\bar{j} + F_Z\bar{k}$ can be resolved into three components.

$F_X = F\cos\theta_X$, $F_Y = F\cos\theta_Y$,

$F_Z = F\cos\theta_Z$ ....(1)

Where $\theta_X, \theta_Y, \theta_Z$ are known as direction of the force along x, y & z direction respectively.

F= magnitude of force $\bar{F}$

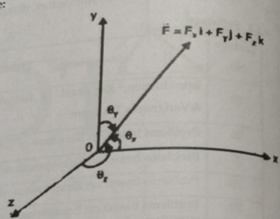


Fig: 4.1

$F = \sqrt{F_X^2 + F_Y^2 + F_Z^2}$ (Magnitude of vector force)

If i, j and k are the unit vectors along x, y and z axes respectively, the force vector can be expressed as

$\bar{F} = F_X\bar{i} + F_Y\bar{j} + F_Z\bar{k}$ ....(2)

From equations (1), we can write direction cosines of force F as,

$$l = \cos\theta_X = \frac{F_X}{F}, m = \cos\theta_Y = \frac{F_Y}{F}, n = \cos\theta_Z = \frac{F_Z}{F}$$

$$\bar{F} = (F\cos\theta x)i + (F\cos\theta y)j + (F\cos\theta z)k$$

$$\therefore l^2 + m^2 + n^2 = \cos^2\theta_X + \cos^2\theta_Y + \cos^2\theta_Z = \frac{F_X^2}{F^2} + \frac{F_Y^2}{F^2} + \frac{F_Z^2}{F^2} = \frac{F_X^2 + F_Y^2 + F_Z^2}{F^2}$$

$$\therefore l^2 + m^2 + n^2 = 1$$

1) If $\overline{F}$ = (- 238 i + 157 j +312 k) kN, determine the magnitude and directions of the force.

Solution: Magnitude of the force,

$$F = \sqrt{F_X{}^2 + F_Y{}^2 + F_Z{}^2}$$

$$\therefore F = \sqrt{(-238)^2 + (157)^2 + (312)^2}$$

$$\therefore F = 422.6547 \text{ kN}$$

Direction of the force,

$F_X = F\cos\theta_X \therefore -238 = 422.6547\cos\theta_X \quad \therefore \theta_X = 124.27°$

$F_Y = F\cos\theta_Y \therefore 157 = 422.6547\cos\theta_Y \quad \therefore \theta_Y = 68.19°$

$F_Z = F\cos\theta_Z \therefore 312 = 422.6547\cos\theta_Z \quad \therefore \theta_Z = 42.42°$

2) A force of 1000N forms angles of 60°, 45°, and 120° with x, y and z axes respectively. Write equation in the vector form.

Solution: Given: $|F| = 1000N$, $\theta_x = 60°$, $\theta_y = 45°$ and $\theta_z = 120°$

Vector representation of the force:

$\bar{F} = (F\cos\theta x)i + (F\cos\theta y)j + (F\cos\theta z)k$

= (1000 cos 60°) i + (1000 cos 45°)j + (1000 cos 120°) k

= [500 i +707.1j - 500 k] N ... Ans.

3) A force acts at origin in a direction defined by angles θy = 65° and θz= 40°. knowing that x component of force is -750N find (I) the other components, (ii) magnitude of force and (iii) the value of θx

Solution: Given: $\theta_y = 65°$, $\theta_z = 40°$ and Fx = -750 N. Using the equation,

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$; $\cos^2\theta_x + \cos^2 65° + \cos^2 40° = 1$; $\cos^2\theta_x = 0.2346$; $\cos\theta_x = \pm 0.4843$

$\theta_x = 61.03°$ or 180 - 61.03 = 118.97° $\quad \theta_x = 118.97°$ since Fx = -750 N has negative magnitude

We know that $F_X = |F|\cos\theta_X \quad -750 = |F|\cos 118.97° \quad \therefore |F| = 1548.46$ N

Also, $F_y = |F|\cos\theta_y$ and $F_z = |F|\cos\theta_z$

$F_y = 1548.46\cos 65°$ $\quad F_z = 1548.46\cos 40°$

$F_y = 654.41$ N $\quad F_z = 1186.19$ N .... Ans.

B. Unit Vector ($\bar{e}$): A vector whose magnitude is equal to 1 and which is directed along the original force is called unit vector.

$\therefore \quad \bar{F} = F\bar{e} \qquad \bar{e} = \frac{\text{Force vector}}{\text{magnitude of the force vector}} = \frac{\bar{F}}{|F|}$

$\bar{F} = F_X\cos\theta_X\bar{i} + F_Y\cos\theta_Y\bar{j} + F_Z\cos\theta_Z\bar{k} \qquad \bar{F} = F(\cos\theta_X\bar{i} + \cos\theta_Y\bar{j} + \cos\theta_Z\bar{k})$

Where $\bar{e}$= Unit vector in the direction of force, $\bar{F}$

C. Unit Vector when force is specified by two points: Consider a force 'F' passing through two points A (X_1, Y_1, Z_1) and B (X_2, Y_2, Z_2) the force vector can be expressed as -

$$\therefore \bar{F} = F\bar{e} \therefore \bar{F} = F\left[\frac{(X_2 - X_1)\bar{i} + (Y_2 - Y_1)\bar{j} + (Z_2 - Z_1)\bar{k}}{\sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2}}\right]$$

4) A 150 KN force acts at P (8, 12, 0) and passes through Q (2, 0, 4). Put the force in vector form.

Solution: We know that $\bar{F} = F(\bar{e})$ where F = 150 kN and $\bar{e}$ =Unit vector in the direction of force F.

Let $(X_1, Y_1, Z_1) \equiv (8, 12, 0)$ and $(X_2, Y_2, Z_2) \equiv (2, 0, 4)$

$$\overline{F} = F\left[\frac{(X_2 - X_1)\overline{i} + (Y_2 - Y_1)\overline{j} + (Z_2 - Z_1)\overline{k}}{\sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2}}\right]$$

$$\therefore \overline{F} = 150\left[\frac{(2-8)\overline{i} + (0-12)\overline{j} + (4-0)\overline{k}}{\sqrt{(2-8)^2 + (0-12)^2 + (4-0)^2}}\right]$$

$$\therefore \overline{F} = (-64.2857\, i - 128.5714\, j + 42.85714\, k)\ kN$$

D. **Unit vector of line($\bar{\lambda}$):** It is the ratio of vector force along the line to magnitude of force.

$$\bar{\lambda} = \frac{\textit{Vector force along the line}}{\textit{magnitude of the force}}$$

5) **$|P|$ = 200 N. Coordinates of points are A = (0,15,0) cm, E= (30,0,0). Determine unit vector along line EA and express the force P in vector form.**

Solution: Force in vector form with unit vector $\bar{\lambda}_{EA}$

$$\overline{P} = |P|\overline{\lambda_{EA}} = 200\left(\frac{(0-30)i + (15-0)j + (0-0)k}{\sqrt{(-30)^2 + 15^2}}\right) = -178.9i + 89.45j$$

E. **Components of force when orientations of planes are given:**

Consider a force 'F' at origin O. the direction of force F with Y axis is 'θ_Y'.

$\therefore$Y – Component of force $F_Y = \cos\theta_Y$ and component along direction 'OC' is $F_{OC} = F \sin\theta_Y$

We can resolve 'F_{OC}' along two rectangular components i.e. X and Z axes.

$F_X = F_{OC}\cos\alpha$ or $F_X = F\sin\theta_Y \cos\alpha$ $\quad F_Z = F_{OC}\sin\alpha$ or $\quad F_Z = F\sin\theta_Y \sin\alpha$

F. **Resultant of concurrent forces in space:**

When a system of concurrent forces in space is given the resultant will be obtained by summing their rectangular components. To determine resultant, resolve each fssssorce into rectangular components and express as

$$R_X\overline{i} + R_Y\overline{j} + R_Z\overline{k} = \sum(F_X i + F_Y j + F_Z k) = \left(\sum F_X\right)\overline{i} + \left(\sum F_Y\right)\overline{j} + \left(\sum F_Z\right)\overline{k}$$

$$\therefore R_X = \sum F_X,\ R_Y = \sum F_Y,\ R_Z = \sum F_Z,\ R = \sqrt{R_X^2 + R_Y^2 + R_Z^2},\ \cos\theta_X = \frac{R_X}{R},\ \cos\theta_Y = \frac{R_Y}{R},\ \cos\theta_Z = \frac{R_Z}{R}$$

6) **A force P_1 = 10 N in magnitude acts along direction AB whose coordinates of points A and B are (3,2,-1) m and (8, 5, 3) m respectively. Another force P_2 = 5 N in magnitude acts along BC where C has coordinates (-2, 11, -5) m. Determine a) the resultant of P_1 and P_2 in its vector form b) the moment of the resultant about a point D (1, 1, 1) m .**

Solution: P_1 = 10 N, P_2 = 5 N

Coordinates, A = (3, 2, -1), B = (8, 5, 3), C = (-2, 11, -5), D = (1, 1, 1) & K = (5, 8, 3)

Putting forces in vector form, $\overline{P_1} = P_1\ \overline{e_{AB}}$ .... (Multiplication)

$$= 10\left[\frac{(8-3)\overline{i} + (5-2)\overline{j} + [3-(-1)]\overline{k}}{\sqrt{(8-3)^2 + (5-2)^2 + [3-(-1)]^2}}\right] = 10\left[\frac{5\,\overline{i} + 3\,\overline{j} + 4\,\overline{k}}{7.071}\right]$$

$$= 10\ (0.7071\, i + 0.4242\, j + 0.5656\, k) = 7.071\, i + 4.242\, j + 5.656\ kN$$

$\overline{P_2} = P_2\ \overline{e_{BC}}$........(Multiplication)

$$= 5\left[\frac{(-2-8)\overline{i} + (11-5)\overline{j} + (-5-3)\overline{k}}{\sqrt{(-2-8)^2 + (11-5)^2 + (-5-3)^2}}\right] = 5\left[\frac{-10\,\overline{i} + 6\,\overline{j} - 8\overline{k}}{14.142}\right]$$

$$= 5\ (-0.7071\, i + 0.4242\, j - 0.5656\, k)$$

$$\overline{P_2} = -3.5355\, i + 2.121\, j - 2.828\, k\ N$$

Therefore, Resultant force, $\overline{R} = \overline{P_1} + \overline{P_2}$

$$\overline{R} = (7.071\, i + 4.242\, j + 5.656\, k) + (-3.5355\, i + 2.121\, j - 2.828\, k)$$

$$= 3.5355\, i + 6.363\, j + 2.828\ k\ N$$

a) Forces are concurrent at point B, so resultant also passes through this point

Moment of resultant about point D (1, 1, 1) = $\overline{M_D} = \overline{r_{DB}} \times \overline{R}$ ..... (Cross Product)

$\overline{r_{DB}}$= Position vector extending from point D to point B = (8 - 1) i + (5 - 1) j + (3 - 1) k

$\therefore \overline{r_{DB}}$ = (7 i + 4 j + 2 k) m. So, $\overline{M_D} = \begin{vmatrix} i & j & k \\ 7 & 4 & 2 \\ 3.5355 & 6.363 & 2.828 \end{vmatrix}$

= (- 1.414 i - 12.725 j + 30.399 k) Nm

G. **Conditions of equilibrium for concurrent forces in space:**

If a particle is in equilibrium the components of resultant must be equal to zero.

$R_X = 0, R_Y = 0, R_Z = 0\ \sum F_X = 0, \sum F_Y = 0, \sum F_Z = 0$

H. **Moment of a force about a point:**

Moment of a force about a point 'O' is defined as the vector product of $\overline{r}$ and $\overline{F}$., $\overline{M_O} = \overline{r} \times \overline{F}$

Where $\overline{r}$= Position vector of point of application 'O', $\overline{F}$= Force Vector,

$$\overline{M_O} = \overline{r} \times \overline{F} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ r_X & r_Y & r_Z \\ F_X & F_Y & F_Z \end{vmatrix}$$

I. **Moment of a force about origin:**

Consider a force $\overline{F} = F_X\overline{i} + F_Y\overline{j} + F_Z\overline{k}$ passing through points $P(X_1, Y_1, Z_1)$ and $Q(X_2, Y_2, Z_2)$.

Moment about origin, $\overline{M_O} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ X_1 & Y_1 & Z_1 \\ F_X & F_Y & F_Z \end{vmatrix}$ or $\overline{M_O} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ X_2 & Y_2 & Z_2 \\ F_X & F_Y & F_Z \end{vmatrix}$

J. **Moment of a force about any other point:**

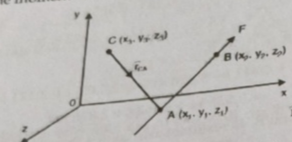
Consider a force $\overline{F} = F_X\overline{i} + F_Y\overline{j} + F_Z\overline{k}$ passing through points $P(X_1, Y_1, Z_1)$ and $Q(X_2, Y_2, Z_2)$.

The moment of this force about a point $C(X_3, Y_3, Z_3)$

$$\overline{M_C} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ (X_1 - X_3) & (Y_1 - Y_3) & (Z_1 - Z_3) \\ F_X & F_Y & F_Z \end{vmatrix} \text{ or } \overline{M_C} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ (X_2 - X_3) & (Y_2 - Y_3) & (Z_2 - Z_3) \\ F_X & F_Y & F_Z \end{vmatrix}$$

Steps to find moment of a force about a point:

Let force F is passing through points $A(X_1, Y_1, Z_1)$ and $B(X_2, Y_2, Z_2)$ on its line of action. Let $C(X_3, Y_3, Z_3)$ be the moment centre.



ig: 4.2

Step 1: Put force in vector form i.e. $\overline{F} = F_X\overline{i} + F_Y\overline{j} + F_Z\overline{k} \therefore \overline{F} = F\overline{e}$

Step 2: Find position vector extending from moment centre to any other point on the force. i.e.$\overline{r} = r_X\overline{i} + r_Y\overline{j} + r_Z\overline{k}$

Step 3: Perform cross product of the position vector and the force vector to get moment vector.

$$\overline{M} = \overline{r} \times \overline{F} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ r_X & r_Y & r_Z \\ F_X & F_Y & F_Z \end{vmatrix}$$

7) A force $\overline{F}$ = (3 i – 4 j + 12 k) N acts at a point P (1, -2, 3) m. Find a) moment of the force about origin b) moment of the force about point Q (2, 1, 2) m.

Solution: a) Moment of the force about origin 'O':

$$\overline{M_O} = \overline{r_{PO}} \times \overline{F} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ r_X & r_Y & r_Z \\ F_X & F_Y & F_Z \end{vmatrix} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 1 & -2 & 3 \\ 3 & -4 & 12 \end{vmatrix}$$

$\therefore \overline{M_O} = [(12 \times -2) - (3 \times -4)]\, i - [(1 \times 12) - (3 \times 3)]\, j + [(1 \times -4) - (3 \times -2)]\, k$

$\therefore \overline{M_O} = -12\,i - 3\,j + 2\,k$ Nm

b) Moment of the force about point Q (2, 1, 2) m:

$$\overline{M_Q} = \overline{r_{PQ}} \times \overline{F} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ r_X & r_Y & r_Z \\ F_X & F_Y & F_Z \end{vmatrix} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ (1-2) & (-2-1) & (3-2) \\ 3 & -4 & 12 \end{vmatrix} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ -1 & -3 & 1 \\ 3 & -4 & 12 \end{vmatrix}$$

$\therefore \overline{M_Q} = [(12 \times -3) - (1 \times -4)]\, i - [(-1 \times 12) - (3 \times 1)]\, j + [(-1 \times -4) - (3 \times -3)]\, k$

$\therefore \overline{M_Q} = -32\,i + 15\,j + 13\,k$ Nm

K. Varignon's Theorem:

Moment of resultant of a force system about a point is equal to the sum of moments of various forces about the same point. $\overline{M_O} = (\overline{r} \times \overline{F_1}) + (\overline{r} \times \overline{F_2})$ $\quad \overline{M_O} = \overline{r} \times (\overline{F_1} + \overline{F_2})$

$\overline{M_O} = \overline{r} \times \overline{F} \therefore \overline{M_R} = \sum M = 0$ (About same point)

L. Conditions of equilibrium for Non-concurrent and parallel forces in space:

$R_x = 0 \quad R_y = 0 \quad R_z = 0, \sum M = 0 \quad \sum F_X = 0 \quad \sum F_Y = 0 \quad \sum F_Z = 0 \quad \sum M = 0$

<u>Problems based on force & Moment of force</u>

8) A force acts at the origin in a direction defined by the angles $\theta_x = 56°$ & $\theta_y = 35°$. Knowing that the Z component of the force is 345 N. Determine a) the other components b) the magnitude of the force c) the value of θ_z.

Solution: We know that $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\cos^2 56 + \cos^2 35 + \cos^2\theta_z = 1$

$\therefore \cos^2\theta_z = 0.01629 \qquad \therefore \cos\theta_z = \pm\, 0.1276 \qquad \therefore \theta_z = 82.66°$ or $\theta_z = 97.33°$

Since F_Z = 345 N, force component is directed towards Positive Z-direction. So $\theta_z = 82.66°$

$F_Z = F\cos\theta_Z \qquad \therefore 345 = F\cos 82.66 \qquad \therefore F = 2700.43$ N

$F_Y = F\cos\theta_Y \qquad \therefore F_Y = 2700.43\cos 35 \qquad \therefore F_Y = 2212.06$ N

$F_X = F\cos\theta_X \qquad \therefore F_X = 2700.43\cos 56 \qquad \therefore F_X = 1510.06$ N

1) Represent the force F = 150 N shown in figure as a Cartesian vector.

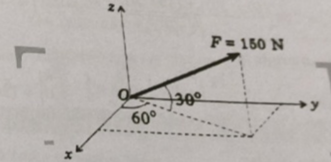


Fig: 4.3

Solution: Referring the diagram, Force F_{OB} = 150 N is resolved into two components as

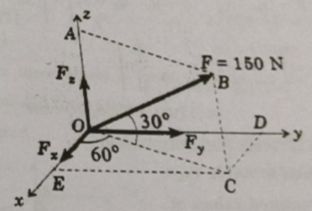


Fig: 4.4

$F_{OC} = F_{OB}$ COS 30° = 150 cos 30° = 129.9 N& $F_{OA} = F_Z = F_{OB}$ sin 30° = 150 sin 30° = 75 N

Now, force F_{OC} = 129.9 N is resolved into two components as

$F_{OE} = F_X = F_{OC}$ cos 60° = 129.9 cos 60° = 64.95 N&$F_{OD} = F_Y = F_{OC}$ sin 60° = 129.9 sin 60° = 112.5 N

Hence, three components of forces are: F_X = 64.95 N, F_Y = 112.5 N & F_Z = 75 N

$F = F_X i + F_Y j + F_Z k$ = 64.95 i + 112.5 j + 75 k

2) A parallelepiped as shown in figure 3.1.8 is acted upon by a force 200 N. Determine the moment of this force about (I) origin 0 (II) point H

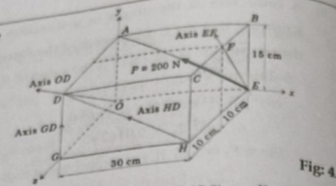


Fig: 4.5

Given: |P| = 200 N. Coordinates of different points are O= (0,0,0), A = (0,15,0) cm, E= (30,0,0) cm, D = (0,15,20) cm, H= (30,0,20) cm, F = (30, 15, 10) cm, G = (0, 0, 20) cm.

Solution: First we express the force in vector form as

$$\overline{P} = |P|\overline{\lambda_{EA}} = 200\left(\frac{(0-30)i+(15-0)j+(0-0)k}{\sqrt{(-30)^2+15^2}}\right) = -178.9i + 89.45j$$

(i) Moment of force $\overline{P}$ about origin 'O'

$$\overline{M_O^P} = \overline{r_{E/O}} \times \overline{P} = \begin{vmatrix} i & j & k \\ x_E & y_E & z_E \\ P_X & P_Y & P_Z \end{vmatrix} = \begin{vmatrix} i & j & k \\ 30 & 0 & 0 \\ -178.9 & 89.45 & 0 \end{vmatrix} = (2683.5k)N - cm$$

(i) Moment of force $\overline{P}$ about point 'H'

$$\overline{M_H^P} = \overline{r_{E/H}} \times \overline{P} = \begin{vmatrix} i & j & k \\ x_E - x_H & y_E - y_H & z_E - z_H \\ P_X & P_Y & P_Z \end{vmatrix} = \begin{vmatrix} i & j & k \\ (30-30) & (0-0) & (0-20) \\ -178.9 & 89.45 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & -20 \\ -178.9 & 89.45 & 0 \end{vmatrix}$$

$$= (1789i + 3578j)N - cm$$

Problem on resultant of concurrent force system

3) Knowing that the tension in AC is T_{AC}= 20 kN, determine the required values of tension T_{AB} and T_{AD} so that the resultant of the three forces applied at point A is vertical. Find their resultant.

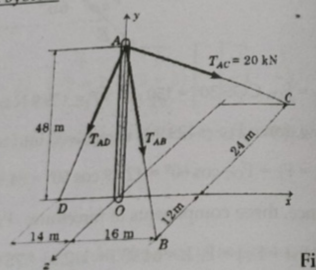


Fig: 4.6

Given: T_{AC} = 20 kN, R_x = 0 and R_z= 0. Coordinates of points: O = (0, 0, 0), A = (0, 48, 0) m, B = (16, 0, 12) m, C = (16,0, -24) m and D = (-14,0,0) m.

From the data given in the problem, we have

$$\overline{R} = \overline{T_{AB}} + \overline{T_{AC}} + \overline{T_{AD}} = |T_{AB}|\lambda_{AB} + |T_{AC}|\lambda_{AC} + |T_{AD}|\lambda_{AD}$$

$$= T_{AB}\left[\frac{(16-0)i+(0-48)j+(-24-0)k}{\sqrt{16^2+(-48)^2+(-24)^2}}\right] + 20\left[\frac{(16-0)i+(0-48)j+(-24-0)k}{\sqrt{16^2+(-48)^2+(-24)^2}}\right]$$

$$+T_{AD}\left[\frac{(-14-0)i+(0-48)j+(0-0)k}{\sqrt{(-14)^2+(-48)^2+(0)^2}}\right]$$

$$\overline{R} = (0.31T_{AB} + 5.71 - 0.28T_{AD})i + (0.92T_{AB} - 17.14 - 0.96T_{AD})j + (0.23T_{AB} - 8.57)k$$

Equating i, j and k components , we get

$R_X = 0.31T_{AB} + 5.71 - 0.28T_{AD}$……(i) $R_Y = -0.92T_{AB} - 17.14 - 0.96T_{AD}$……(ii) $R_Z = 0.23T_{AB} - 8.57$……(iii)

It is given that $R_X = 0 \& R_Z = 0$

$$\therefore R_Z = 0 = 0.23T_{AB} - 8.57 \therefore T_{AB} = 37.26kN$$

Also $R_X = 0 = 0.31T_{AB} + 5.71 - 0.28T_{AD}$

$$0 = (0.31 \times 37.26) + 5.71 - 0.28T_{AD} \therefore T_{AD} = 61.65kN$$

Putting values of T_{AB} & T_{AD} in equation (ii), we get

$$R_Y = (-0.92 \times 37.26) - 17.14 - (0.96 \times 61.65), R = -110.60kN$$

Let's check the takeaway from this lecture

1. **To find force in vector form, ________ is performed between magnitude of force and unit vector of the line of action of the force.**
   i) Cross product ii) Multiplication
   iii)Dot product iv)Addition
2. **Moment of a force about a point is obtained by performing ________ of position vector and force vector.**
   i) Cross product ii) Multiplication
   iii)Dot product iv)Addition

Exercise:

1) Define the terms unit vector & position vector.
2) What is meant by concurrent forces in space?
3) State the conditions of equilibrium for a particle in space.
4) A force of 1200N acts along PQ, P(4,5,-2) & Q(-3,1,6)m. Calculate its moment about a point A(3,2,0)m. **(Ans: 105.65 [16i +6j +17k] Nm)**

5) Knowing that the tension in AC is T_{AC}= 20 KN, determine the required values of tension T_{AB} and T_{AD} so that the resultant of the three forces applied at point A is vertical. Find their resultant. **(Ans: T_{AD} = 61.22KN, T_{AB} = 37.14KN, R = -110.32j KN)**

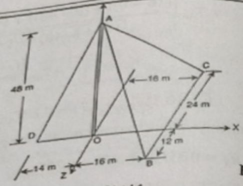


Fig: 4.7

6) The lines of action of three forces concurrent at origin 'O' pass respectively through A, B, C having coordinates of G (0,0,0). The magnitudes of the forces are, F_{AB} = 40 N, F_{AC} = 30 N, F_{AD} = 40 N. Find the magnitude and direction of the resultant. **(Ans: 4i - 87.77j + 17.88 k & 88.21 N)**

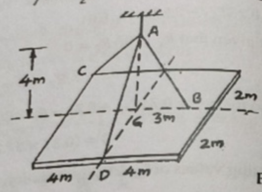


Fig: 4.8

Questions/Problems for Practice for the day

1) Two vectors A & B are defined by the relations A = 3i+4j+6k, B=4i+5j-9k. Determine the sum, difference and dot product of these vectors and find angle between them. **(Ans: 7i+9j-3k, -i-j+15k, 109.2°)**

2) A force acts at the origin in a direction defined by the angles θ_y = 65° and θ_z = 40°. Knowing that the x-component of the force is -750kN, determine (i) the other components (ii) magnitude of the force and (iii) the value of θ_x. **(Ans: 654.45 kN, 1186.26kN, 1548.55kN, 118.97°)**

3) A force P_1 = 10N in magnitude acts along direction AB whose coordinates of points A & B are (3,2,-1) and (8,5,3). Another force P_2 = 5N in magnitude acts along BC where C has coordinate (-2,11,-5). Determine: a) the resultant of P_1 and P_2 in its vector form. b) moment of the resultant about a point D whose coordinates are (1,1,1). **(Ans: R = 3.53i + 6.36j +2.828kN, M = -1.408i - 12.736j + 30.4kNm, Component = 2N)**

4) The cable exerts forces F_{AB} = 100N and F_{AC} = 120N on the ring at A as shown in fig 4.9. Determine the magnitude of the resultant force acting at A. **(Ans: R = 217N)**

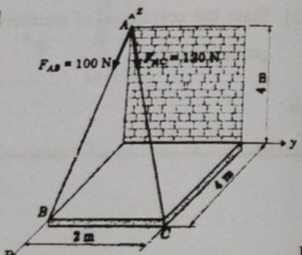


Fig: 4.9

Learning from lecture: Learners will be able to define forces acting in spaces and solve the problems based on moment resultant of forces.

Lecture 26

4.2 ***Problems based on Resultant of General & Parallel force system***

Learning Objective: Learners will be able to understand the concept of solving the problems based on resultant of general & parallel force system.

4) A rectangular parallelepiped surface subjected to four forces in the direction and magnitude as indicated. Reduce them to a resultant force at origin and moment about origin.

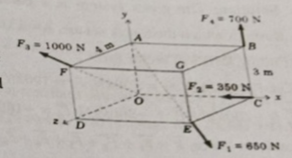


Fig: 4.11

Solution: Given: $|F_1|$ = 650 N, $|F_2|$ = 350 N, $|F_3|$ = 1000 N, $|F_4|$ = 700 N.

Coordinates of points : O= (0, 0, 0), A = (0, 3, 0), B = (5, 3, 0), C = (5, 0, 0), D = (0, 0, 4), E = (5, 0, 4),

F = (0, 3, 4). The resultant force is given by

$$\overline{R} = |F_1|\overline{\lambda_{AE}} + |F_2|\overline{\lambda_{CO}} + |F_3|\overline{\lambda_{DF}} + |F_4|\overline{\lambda_{CB}}$$

$$= 650\left[\frac{(5-0)i + (0-3)j + (4-0)k}{\sqrt{5^2 + (-3)^2 + 4^2}}\right] + 350\left[\frac{(5-0)i + (0-0)j + (0-0)k}{\sqrt{5^2}}\right]$$

$$+1000\left[\frac{(0-0)i + (3-0)j + (4-0)k}{\sqrt{(3)^2 + 4^2}}\right] + 700\left[\frac{(5-5)i + (0-3)j + (0-0)k}{\sqrt{(-3)^2}}\right]$$

$$= (459.62i - 275.77j + 367.69k) - 350j + (600j + 800k) + 700j$$

$$\therefore \overline{R} = (109.62i + 1024.23j + 1167.7k)$$

Moment of resultant about origin O is given by, $\overline{M_O^R} = \overline{M_O^{F_1}} + \overline{M_O^{F_2}} + \overline{M_O^{F_3}} + \overline{M_O^{F_4}}$

$\overline{M_O^{F_2}} = \overline{M_O^{F_3}}$ = 0as these forces pass through the origin O

$$\overline{M_O^R} = \overline{M_O^{F_1}} + \overline{M_O^{F_4}} = \overline{r_{A/O}} \times \overline{F_1} + \overline{r_{C/O}} \times \overline{F_4} = \begin{vmatrix} i & j & k \\ x_A & y_A & z_A \\ F_{1x} & F_{2x} & F_{3x} \end{vmatrix} + \begin{vmatrix} i & j & k \\ x_C & y_C & z_C \\ F_{4x} & F_{4x} & F_{4x} \end{vmatrix}$$

$$= \begin{vmatrix} i & j & k \\ 0 & 3 & 0 \\ 459.62 & -275.77 & 367.69 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 5 & 0 & 0 \\ 0 & 700 & 0 \end{vmatrix} = 1103.07i - 1378.86k + 3500k$$

$$\overline{M_O^R} = 1103.07i + 2121.14k$$

5) **A plate foundation is subjected to five vertical forces as shown. Replace these five forces by means of a single vertical force and find the point of replacement.**

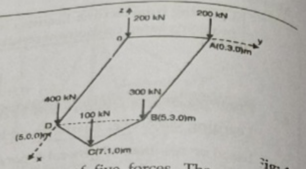


Fig: 4.12

Solution: The given system is a parallel force system of five forces. The coordinates through which the forces act are, A=(0,3,0), B = (5,3,0), C= (7,1,0), D=(5,0,0), O=(0,0,0).

Putting forces in vector form

Let, $F_1 = 200kN, F_2 = 300kN, F_3 = 100kN, F_4 = 400kN, F_5 = 200kN$

$\overline{F_1} = -200k\text{kN}, \overline{F_2} = -300k\text{kN}, \overline{F_3} = -100k\text{kN}, \overline{F_4} = -400k\text{kN}, \overline{F_5} = -200k\text{kN}$

Resultant force, $\overline{R} = \overline{F_1} + \overline{F_2} + \overline{F_3} + \overline{F_4} + \overline{F_5} = [(-200k) + (-300k) + (-100k) + (-400k) + (-200k)]$

$$\overline{R} = (-1200k) \text{ kN}$$

Point of application of Resultant: Let the resultant act at a point P (x, y, 0) m in the plane of a plate. To use Varignon's theorem, we need to find moments of all the forces and also of the resultant about point 'O' $\overline{M_O^{F_1}} = \overline{r_{OA}} \times \overline{F_1}$ ……where $\overline{r_{OA}} = (3\bar{j})$ m

$\overline{M_O^{F_1}} = 3\bar{j} \times (-200)\bar{k} = (-600i)$ kN-m

$\overline{M_O^{F_2}} = \overline{r_{OB}} \times \overline{F_2}$…………where $\overline{r_{OB}} = (5\,\bar{i} + 3\,\bar{j})$ m

$= (5\,\bar{i} + 3\,\bar{j}) \times (-300)\,\bar{k} = (-900\,i + 1500\,j)$ kN m

$\overline{M_O^{F_3}} = \overline{r_{OC}} \times \overline{F_3}$……..where $\overline{r_{OC}} = (7\bar{i} + \bar{j})m$

$= (7\,\bar{i} + \bar{j}) \times (-100)\,\bar{k} = (-100\,i + 700\,j)$ kN m

$\overline{M_O^{F_4}} = \overline{r_{OD}} \times \overline{F_4}$……..where $\overline{r_{OD}} = (5\bar{i})$ m

$= (5\bar{i}) \times (-400)\,\bar{k} = (2000j)$ kN m

$\overline{M_O^{R}} = \overline{r_{OP}} \times \overline{R}$……where $\overline{r_{OP}} = (x\bar{i} + y\bar{j})$ m

$= (x\bar{i} + y\bar{j}) \times (-1200)\,\bar{k} = (-1200y)i + (1200x)j$ kN m

Using Varignon's Theorem, $\sum \overline{M_O^{F}} = \sum \overline{M_O^{R}}$ $\quad \therefore\ \overline{M_O^{F_1}} + \overline{M_O^{F_2}} + \overline{M_O^{F_3}} + \overline{M_O^{F_4}} + \overline{M_O^{F_5}} = \overline{M_O^{R}}$

$\therefore\ (-600\,i) + (-900\,i + 1500\,j) + (-100\,i + 700\,j) + (2000\,j) + 0 = (-1200\,y)\,i + (1200\,x)\,j$

$\therefore\ -1600\,i + 4200\,j = (-1200\,y)\,i + (1200\,x)\,j$

Equating the coefficient, -1600 = - 1200y, y = 1.33m, Similarly 4200 = 1200x, x=3.5m

So resultant, R= - 1200k acts at point P = (3.5, 1.33, 0) m

Let's check the takeaway from this lecture

1. **Magnitude of moment of a force about a line is obtained by performing _________ of moment vector and unit vector of the line**
   i) Cross product ii) Multiplication
   iii)Dot product iv)Addition

2. **Magnitude of the force component in finding vector component is obtained by performing** ________ of the force vector and unit vector of the line.
   i) Cross product ii) Multiplication
   iii)Dot product iv)Addition

Ans – 1. Dot product , 2. Dot product

Exercise:

1) Determine the resultant force and the resultant couple of the force system shown in Fig. 4.13 when F_1 = 100 N, F_2 = 20 N, F_3 = 40 N and F_4 = 40 N. **(Ans: R = -76.64i + 11.1j +80k, M_O^R = - 72i – 220j -150.7k Nm)**

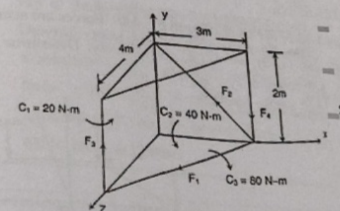


Fig: 4.13

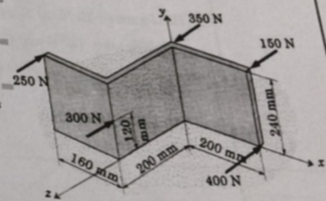


Fig: 4.14

2) Determine the x and y coordinates of a point through which the resultant of the parallel forces passes. Also determine the resultant for a given Fig. 4.14. **(Ans: $\bar{K}$ = - 450k acts at (22.22, -53.33, 0))**

Questions/Problems for Practice for the day

3) Determine the resultant of the non-concurrent, non-parallel system of forces as shown in fig. 4.15 **(Ans : 7.24 i + 54.48 j – 62 k)**

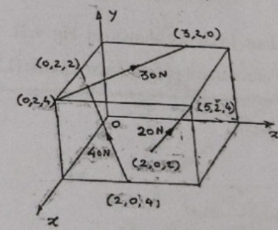


Fig: 4.15

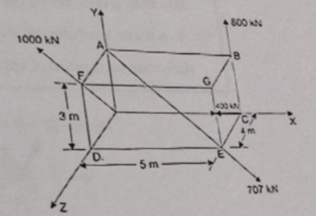


Fig: 4.16

4) Fig.4.16 shows a rectangular parallelepiped subjected to four forces in direction. Reduce

them to a resultant force at the origin & a moment. (Ans: 1631kN, (1200i + 2500k) m)

5) Find the resultant force system at A for the forces acting on the bent cantilever beam. BC is parallel to z-axis. (Ans: (500i – 1270j)N, (240i – 2860k)Nm

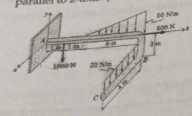


Fig: 4.17

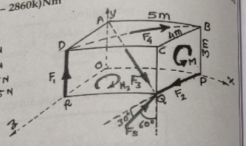


Fig: 4.18

6) For fig. 4.18, replace the system of forces by a force and a couple at 'O'. Take M_1 = 60 Nm and M_2 = 100 Nm. (Ans: 123.67 Nm)

7) The forces of 20 N, 10 N and 30 N are as shown in fig. 4.19. Forces are acting in the x-z plane at coordinates (x, z) are (2, 3), (4, 2) and (7, 4) respectively. Determine the resultant and locate it. (Ans : 40 N upward)

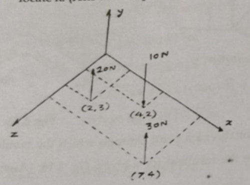


Fig: 4.19

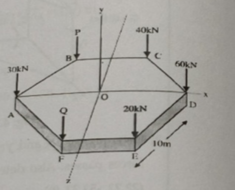


Fig: 4.20

8) A concrete mat in the shape of regular hexagon of side 10 m, supports four loads as shown in figure 4.20. Determine loads P and Q if resultant of six loads is to pass through centre of the mat. (Ans: 50kN, 70kN)

9) A square foundation supports four loads as shown in Fig 4.21. Determine magnitude, direction and point of application of resultant of four forces. (Ans: (1.75,0,1.5) m)

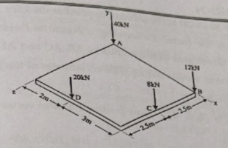


Fig: 4.21

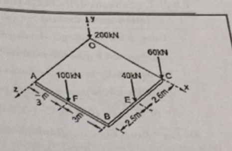


Fig: 4.22

10) A square foundation mat supports the four columns shown in fig. 4.22. Determine the magnitude and point of application of the resultant of four loads. (Ans: 400kN, (1.5, 1.75, 0) m)

11) Three parallel bolting forces act on the rim of the circular cover plate as shown. Determine the magnitude and direction of a resultant force equivalent to the given force system and locate its point of application P, on the cover plate. (Ans: 650N, (0.24,-0.12,0)m)

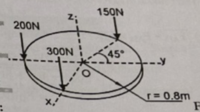


Fig: 4.23

Learning from lecture: Learners will be able to solve the problems based on resultant of parallel force and general force system.

Lecture 27

4.3 *Problems based on Equilibrium of Concurrent force system*

Learning Objective: Learners will be able to understand the concept and solve problems based on concurrent force system and its equilibrium condition.

6) Figure shows a space truss. Find forces in members AB, AC and AD of the truss loaded at joint A by a force F = (40i – 30j – 20k) kN.

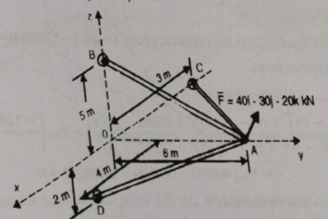


Fig: 4.24

Solution: Since the load F = (40 i - 30 j - 20 k)kN is being applied at the joint A of the space truss, axial forces get developed in the members forming a concurrent system at A. Let F_{AB}, F_{AC} and F_{AD} be the axial forces developed in the members AB, AC and AD respectively. Let us initially assume the forces to be of tensile nature. The coordinates of the joints through which the forces pass are shown on the figure.

A= (0, 6, 0), B= (0, 0, 5), C= (-3, 0, 0), D= (4, 0, -2) …Now putting forces in vector form,

a) $\overline{F_{AB}} = F_{AB} \times \overline{e_{AB}}$………(Multiplication)

$$= F_{AB}\left[\frac{(0-0)\overline{i}+(0-6)\overline{j}+(5-0)\overline{k}}{\sqrt{(0-0)^2+(0-6)^2+(5-0)^2}}\right] = F_{AB}\left[\frac{-6\overline{j}+5\overline{k}}{7.8102}\right] = F_{AB}\,(-0.7682\,j + 0.6401\,k)\ kN$$

b) $\overline{F_{AC}} = F_{AC} \times \overline{e_{AC}}$………(Multiplication)

$$= F_{AC}\left[\frac{(-3-0)\overline{i}+(0-6)\overline{j}+(0-0)\overline{k}}{\sqrt{(-3-0)^2+(0-6)^2+(0-0)^2}}\right] = F_{AC}\left[\frac{-3\overline{i}-6\overline{j}}{6.7082}\right]$$

$$= F_{AC}(-0.4472\,i - 0.8944\,j)\ kN$$

c) $\overline{F_{AD}} = F_{AD} \times \overline{e_{AD}}$ ………… (Multiplication)

$$= F_{AD}\left[\frac{(4-0)\overline{i}+(0-6)\overline{j}+(-2-0)\overline{k}}{\sqrt{(4-0)^2+(0-6)^2+(-2-0)^2}}\right] = F_{AD}\left[\frac{4\overline{i}-6\overline{j}-2\overline{k}}{7.4833}\right]$$

$$= F_{AD}(0.5345i - 0.8017j - 0.2672k)\ kN$$

d) $\overline{F} = (40i - 30j - 20k)$ kN

Applying Conditions of equilibrium, $\sum F_X = 0;\ -0.4472F_{AC} + 0.5345F_{AD} + 40 = 0$

$\sum F_Y = 0;\ -0.7682F_{AB} - 0.8944F_{AC} - 0.8017F_{AD} - 30 = 0,\quad \sum F_Z = 0;\ 0.6401F_{AB} - 0.2672F_{AD} - 20 = 0$

Solve above equations, we get F_{AB}= - 5.7189 kN = 5.7189 kN (Compressive),

F_{AC}= - 16.3580 kN =16.3580 kN (Compressive) F_{AD}= 61.15 kN (Tensile)

7) **A square steel plate 2400 mm x 2400 mm has a mass of 1800 kg with mass center at G. Calculate the tension in each of the three cables with which the plate is lifted while remaining horizontal. Length DG = 2400 mm.**

Solution: At support D, tensions in the cables and reaction R_D form a concurrent system in equilibrium. Drawing FBD of the joint D and finding the coordinates of the points through which these forces pass.

A= (-1200, 0, 1200), B= (1200, 0, 1200), C= (0, 0, -1200), D= (0, 2400, 0)

Putting forces in vector form,

1) $\overline{F_{DA}} = F_{DA}\ \overline{e_{DA}}$ ………… (Multiplication)

$$= F_{DA}\left[\frac{(-1200-0)\overline{i}+(0-2400)\overline{j}+(1200-0)\overline{k}}{\sqrt{(-1200-0)^2+(0-2400)^2+(1200-0)^2}}\right] = F_{DA}\left[\frac{-1200\overline{i}-2400\overline{j}+1200\overline{k}}{2939.387}\right]$$

$$= F_{DA}(-0.4082i - 0.8164j + 0.4082k)\ kN$$

2) $\overline{F_{DB}} = F_{DB}\ \overline{e_{DB}}$ ………… (Multiplication)

$$= F_{DB}\left[\frac{(1200-0)\overline{i}+(0-2400)\overline{j}+(1200-0)\overline{k}}{\sqrt{(1200-0)^2+(0-2400)^2+(1200-0)^2}}\right] = F_{DB}\left[\frac{1200\overline{i}-2400\overline{j}+1200\overline{k}}{6.7082}\right]$$

$$= F_{DB}(0.4082i - 0.8164j + 0.4082k)\ kN$$

3) $\overline{F_{DC}} = F_{DC}\ \overline{e_{DC}}$ ………… (Multiplication)

$$= F_{DC}\left[\frac{(0-0)\overline{i}+(0-2400)\overline{j}+(-1200-0)\overline{k}}{\sqrt{(0-0)^2+(0-2400)^2+(-1200-0)^2}}\right] = F_{DC}\left[\frac{-2400\overline{j}-1200\overline{k}}{2683.281}\right]$$

$$= F_{DC}(-0.8944j - 0.4472k)\ kN$$

4) $\overline{R_D} = (1800 \times 9.81)$ N = (17658 j) N, Since reaction R_D would be equal to the weight of the steel plate in magnitude and direction and opposite in sense. Applying Conditions of equilibrium $\sum F_X = 0;\ -0.4082F_{DA} + 0.4082F_{DB} = 0;\ \sum F_Y = 0;\ -0.8164F_{DA} - 0.8164F_{DB} - 0.8944F_{DC} + 17658 = 0$

Solving the above equation, we get F_{DA}= - 5407.27N = 5407.27N (Compressive) F_{DB}= - 5407.27N = 5407.27N (Compressive) F_{DC}= - 9871.422N = 9871.422N (Compressive)

Let's check the takeaway from this lecture

1. **The force system on a tripod carrying a camera is an example of …… Forces**
   i) Coplanar Concurrent
   ii) Non-Coplanar Concurrent
   iii) Coplanar Non-Concurrent
   iv) Non-Coplanar Non-Concurrent
2. **The zero vector is also called as ………. vector**
   i) Null
   ii) Position
   iii) Both a & b
   iv) None of these

Exercise:

1) Three cables are connected at point A for a given fig 4.25. Find the value of force P for which tension in cable AD is 300N. (**Ans: P = 937.91 N, T_{AB} = 427.36 N, T_{AC} = 340.03 N**)

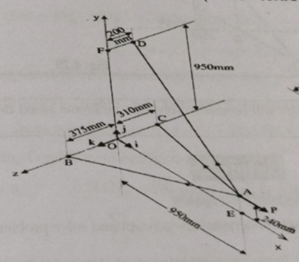


Fig: 4.25

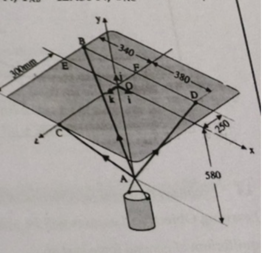


Fig: 4.26

2) Determine the tensions in cables AB, AC and AD, if weight of cylinder is 160 kg show

fig. 4.26. (Ans: T_{AB} = 1316.43 N, T_{AC} = 1892.95 N, T_{AD} = 1211.63N)

Questions/Problems for Practice for the day

3) Determine safe value of load 'W' that can be supported by the tripod shown in Fig. 4.27 without exceeding compressive force of 3 kN in any member. (Ans: 6.52 kN)

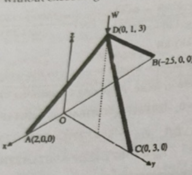


Fig: 4.27

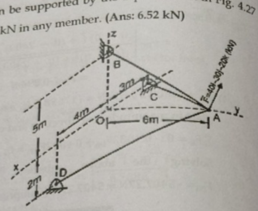


Fig: 4.28

4) Find the forces in the members AB, AC and AD of the truss loaded at joint A by a force F = (40i-30j-20k) kN for a given fig 4.28 **(Ans: 5.7189 kN (C), 16.3580 kN (C), 61.15 kN (T))**

5) A vertical tower DC shown in fig 4.29 is subjected to a horizontal force P = 50kN at its top and is anchored by two similar wires BC and AC. Calculate (I) tension in BC and AC (II)thrust in the tower (III) Hinge reactions at D. Co-ordinates of the points are as below: O (0,0,0), b (0,0,-4), d (3,0,0), A (0,0,4) and C (3,20,0) **(Ans: F_{AC} = F_{BC} = 171.75 kN (C), F_{CD} = 333.4 kN (C))**

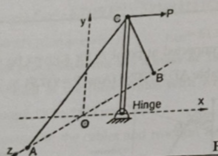


Fig: 4.29

Learning from lecture: Learners will be able to solve the problems based on equilibrium of concurrent force system.

Lecture 28

4.4 *Problems based on Equilibrium of General force system*

Learning Objective: Learners will be able to understand the concept and solve problems based on equilibrium of general force system.

8) A vertical mast AB is supported at A by ball and socket joint and by cables BC and DE as shown in fig 4.30. A force F = (500 i + 400 j – 300 k) N is applied at B. Find the reaction at A.

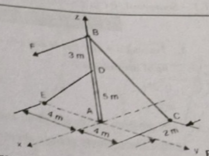


Fig: 4.30

Solution: The given system is a general force system of four forces. Let the forces in the cables are T_{DE} and T_{BC}. Let R_A be the reaction at joint 'A' giving reactions A_X, A_Y and A_Z.

A = (0,0,0), B = (0,0,8), C = (-2,4,0), D = (0,0,5) & E = (0,-4,0)

Putting forces in the vector form, 1) $\overline{T_{DE}} = T_{DE}\overline{e_{DE}}$ .... (Multiplication)

$$= T_{DE}\left[\frac{(0-0)\bar{i} + (-4-0)\bar{j} + (0-5)\bar{k}}{\sqrt{(0-0)^2 + (-4-0)^2 + (0-5)^2}}\right] = T_{DE}\left[\frac{-4\bar{j} - 5\bar{k}}{6.4031}\right] = T_{DE}(-0.6246\,j - 0.7808\,k)\ N$$

2) $\overline{T_{BC}} = T_{BC}\overline{e_{BC}}$ .... (Multiplication)

$$= T_{BC}\left[\frac{(-2-0)\bar{i} + (4-0)\bar{j} + (0-8)\bar{k}}{\sqrt{(-2-0)^2 + (4-0)^2 + (0-8)^2}}\right] = T_{BC}\left[\frac{-2\bar{i} + 4\bar{j} - 8\bar{k}}{9.1651}\right]$$

$$= T_{BC}(-0.2182i + 0.4364j - 0.8728k)\ N$$

3) $\overline{F} = (500i + 400j - 300k)$ N, 4) $\overline{R_A} = (A_X i + A_Y j + A_Z k)$ N

Taking moments of all forces about joint 'A': $\overline{M}_A^{T_{DE}} = \overline{r_{AD}} \times \overline{T_{DE}}$ ......where $\overline{r_{AD}} = 5\bar{k}$ m

$$= (5k) \times T_{DE}(-0.6246j - 0.7808k) = T_{DE}(3.123i)\ Nm$$

$\overline{M}_A^{T_{BC}} = \overline{r_{AB}} \times \overline{T_{BC}}$ ....where $\overline{r_{AB}} = 8\bar{k}$ m

$$= (8k) \times T_{BC}(-0.2182\,i + 0.4364\,j - 0.8728\,k) = T_{BC}(-3.4912i - 1.7456j)\ Nm$$

$\overline{M}_A^{F} = \overline{r_{AB}} \times \overline{F}$ ....where $\overline{r_{AB}} = 8\bar{k}$ m

$$= (8k) \times (500i + 400j - 300k) = (-3200i + 4000j)\ Nm,$$

$\overline{M}_A^{R_A} = 0$ ....since R_A passes through A

Applying Conditions of equilibrium for moments

$\sum M_X = 0;\ 3.123T_{DE} - 3.4912T_{BC} - 3200 = 0,\ \sum M_Y = 0;\ -1.7456T_{BC} + 4000 = 0$

Solving above two equations, we get T_{BC} = 2291.475N, T_{DE} = 3586.294N

Applying Conditions of equilibrium for forces

$\sum F_X = 0;\quad -0.2812T_{BC} + 500 + A_X = 0\quad \sum F_Y = 0; -0.6246T_{DE} + 0.4364T_{BC} + 400 + A_Y = 0$

$$\sum F_Z = 0; -0.7808T_{DE} - 0.8728T_{BC} - 300 + A_Z = 0$$

Solving above equations by substituting values of T_{BC}&T_{DE} we get,

$A_X = 0$, $A_Y = 840N$, $A_Z = 5100.17N$, So $\overline{R_A}$ = (840j + 5100.17k) N

Let's check the takeaway from this lecture

1. Forces (3i + 4j + 5k) N and (-7i -2k) N are acting at a point. The resultant force will be of magnitude
   i) 3N ii) 4N
   iii)5N iv)$5\sqrt{2}$ N
2. The position vector of a point about O is 2i + 3j. A force -9i+10j passes through the point. The moment of force about O is
   i) -7k ii) 47k
   iii)12k iv)None of these

Exercise

1) A homogeneous plate of mass 75kg and subjected to a force and couple moment as shown in the fig 4.31. It is supported in horizontal plane by means of a roller at A, a ball socket joint at B and a cord at C, determine the components of the reaction at the supports. **(Ans: 667.875N, -216.67N, 584.54N)**

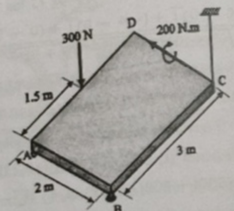


Fig: 4.31

Questions/Problems for Practice for the day

2) The uniform bar of length 7m and 100kg in mass is supported by a ball and socket joint at A in the horizontal floor. The ball end B rests against the vertical walls. Find reaction components at A & B as shown in the fig. 4.32 **(Ans: 981N, 327N, 1425.36N)**

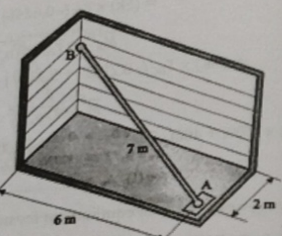


Fig: 4.32

Learning from lecture: Learners will be able to solve the problems based on equilibrium of general force system.

Lecture 29

4.5 *Problems based on Equilibrium of Parallel force system*

Learning Objective: Learners will be able to understand the concept and solve problems based one equilibrium of parallel force system.

9) **A rectangular table 1 m x 2 m is mounted on three equal supports to 1, 2 and 3. The table weighs 2kN which acts at the C.G of the table. If two vertical loads 2 kN and 6 kN are applied on the surface of the table at D and E as shown in fig 4.33, find the reactions at the supports.**

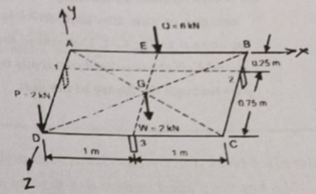


Fig: 4.33

Solution: The given system is a parallel force system of five forces. Let R_1, R_2 and R_3 be the reactions at supports 1, 2 and 3 respectively.FBD of the plate is as shown in the figure 4.31.

1= (0, 0, 0.25), 2= (2, 0, 0.25), 3= (1, 0, 1), G= (1, 0, 0.5), D= (0, 0, 1) & E= (1, 0, 0), B= (2, 0, 0)

Putting forces in vector form,

1) $\overline{F_Q} = -6j$ kN 4) $\overline{R_1} = R_1 j$ kN
2) $\overline{F_P} = -2j$ kN 5) $\overline{R_2} = R_2 j$ kN
3) $\overline{W} = -2j$ kN 6) $\overline{R_3} = R_3 j$ kN

Taking moments of all forces about any point say 'E',

$\overline{M_B^{F_Q}} = \overline{r_{BE}} \times \overline{F_Q}$ ....where $\overline{r_{BE}} = -\overline{i}$ m

$= (-i) \times (-6j) = (6k)$ Nm

$\overline{M_B^{F_P}} = \overline{r_{BD}} \times \overline{F_P}$ ....where $\overline{r_{BD}} = -2\overline{i} + \overline{k}$ m

$= (-2\overline{i} + \overline{k}) \times (-2j) = (2i + 4k)$ Nm

$\overline{M_B^{W}} = \overline{r_{BG}} \times \overline{W}$ ....where $\overline{r_{BG}} = -\overline{i} + 0.5\overline{k}$ m

$= (-\overline{i} + 0.5\overline{k}) \times (-2j) = (i + 2k)$ Nm

$\overline{M_B^{R_1}} = \overline{r_{B1}} \times \overline{R_1}$.......where $\overline{r_{B1}} = -2\overline{i} + 0.25\overline{k}$ m

$= (-2i + 0.25k) \times (R_1 j) = (-0.25R_1 i - 2R_1 k)$ Nm

$\overline{M_B^{R_2}} = \overline{r_{B2}} \times \overline{R_2}$.......where $\overline{r_{B2}} = 0.25\overline{k}$ m

$= (0.25k) \times (R_2 j) = (-0.25R_2 i)$ Nm

$\overline{M_B^{R_3}} = \overline{r_{B3}} \times \overline{R_3}$ ....where $\overline{r_{B3}} = -\overline{i} + \overline{k}$ m

$= (-i + k) \times (R_3 j) = (-R_3 i - R_3 k)$ Nm

Applying Conditions of equilibrium

$\sum M_X = 0;\ (2 + 1 - 0.25R_1 - 0.25R_2 - R_3) = 0$ $\sum M_Z = 0;\ (6 + 4 + 2 - 2R_1 - R_3) = 0$

$\sum F_Y = 0;\ (-6 - 2 - 2 + R_1 + R_2 + R_3) = 0$

Solving above equations, we get $R_1 = 5.67N, R_2 = 3.67N, R_3 = 0.67N$

10) A concrete mat in the shape of regular hexagon of side 10m supports 6 loads as shown in fig 4.34. Determine loads F_2 and F_4 if the resultant of 6 loads is to pass through the centre of the mat.

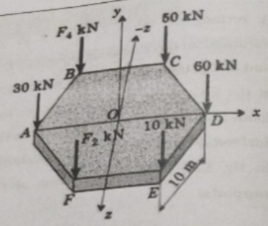


Fig: 4.34

Solution: Given: F_1 = -10j, F_2 = -F_2j, F_3 = -30j, F_4 = -F_4j, F_5 = -50 j, F_6 = -60 j. Co-ordinates of different points are O= (0, 0, 0), A = (-10, 0, 0) m, B = (-5, 0, -8.66) m, C= (5, 0, -8.66) m, D = (10, 0, 0) m, E = (5, 0, 8.66) m, F= (-5,0, 8.66) m. Since resultant force passes through point 0 = (0, 0, 0), moment of resultant about O must be zero.

Using Varignon's theorem, we have

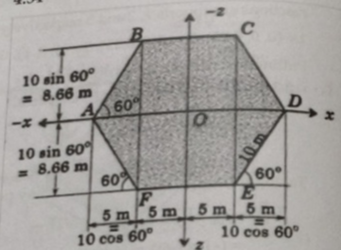


Fig: 4.35

$$\sum \overline{M_O^F} = 0 = \overline{M_O^{F_1}} + \overline{M_O^{F_2}} + \overline{M_O^{F_3}} + \overline{M_O^{F_4}} + \overline{M_O^{F_5}} + \overline{M_O^{F_6}}$$

$$0 = \overline{r_{E/O}} \times \overline{F_1} + \overline{r_{F/O}} \times \overline{F_2} + \overline{r_{A/O}} \times \overline{F_3} + \overline{r_{B/O}} \times \overline{F_4} + \overline{r_{C/O}} \times \overline{F_5} + \overline{r_{D/O}} \times \overline{F_6}$$

$$0 = \begin{vmatrix} i & j & k \\ 5 & 0 & 8.66 \\ 0 & -10 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ -5 & 0 & 8.66 \\ 0 & -F_2 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ -10 & 0 & 0 \\ 0 & -30 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ -5 & 0 & -8.66 \\ 0 & -F_4 & 0 \end{vmatrix}$$
$$+ \begin{vmatrix} i & j & k \\ 5 & 0 & -8.66 \\ 0 & -50 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 10 & 0 & 0 \\ 0 & -60 & 0 \end{vmatrix}$$

$$0 = 86.6i - 50k + 8.66F_2 i + 5F_2 k - 300k - 8.66F_4 i + 5F_4 k - 43.3i - 250k - 600k$$

$$0 = (86.6 + 8.66F_2 - 8.66F_4 - 43.3)i + (-50 + 5F_2 - 300k + 5F_4 - 250 - 600)k$$

Equating i & k components to zero separately,

$8.66F_2 - 8.66F_4 = 43.3\ldots\ldots(i)$ &$5F_2 + 5F_4 = 900\ldots..(ii)$

From equation (i) & (ii), $F_2 = 92.75kN(\downarrow) F_4 = 87.5kN(\downarrow)$

Let's check the takeaway from this lecture

1. **Two vectors will be ……… to each other if their cross product is zero**
   i) Parallel ii) Perpendicular
   iii)Inclined iv)None of the Above
2. **The unit vector is obtained by dividing the vector by its ……**
   i) Direction ii) Zero
   iii)Magnitude iv)None of the Above

Exercise

1. A rectangular table 1 m x 2 m is mounted on three equal supports to 1, 2 and 3. The table weighs 2kN which acts at the CG of the table. If two vertical loads 2 kN and 6 kN are applied on the surface of the table at D and E as shown in fig 4.36, find the reactions at the supports. **(Ans: R_1 = 5.67k, R_2 = 3.67k, R_3 = 0.667k)**

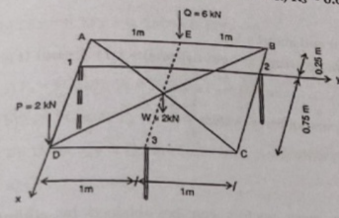


Fig: 4.36

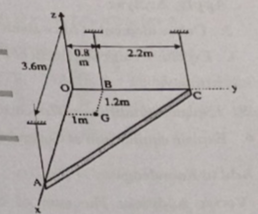


Fig: 4.37

2. A plate of weight 60kN is supported by three wires in horizontal plane as shown in fig 4.37. Find tensions in each cable. **(Ans: 20kN, 27.26kN, 12.73kN)**

Questions/Problems for Practice for the day

3. A square plate of size 0.2m x 0.2m having weight 250 N is supported by three vertical wires as shown in fig 4.38. Determine tension in each wire. AD = 0.1m, DC = 0.16m, EB = 0.16m **(Ans: T_A = 93.75N, T_B = 78.125N, T_C = 78.125N)**

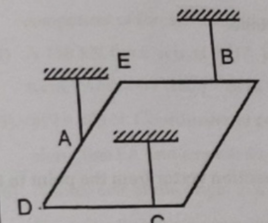


Fig: 4.38

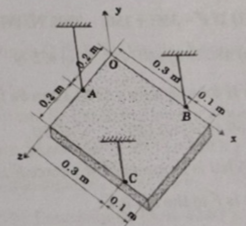
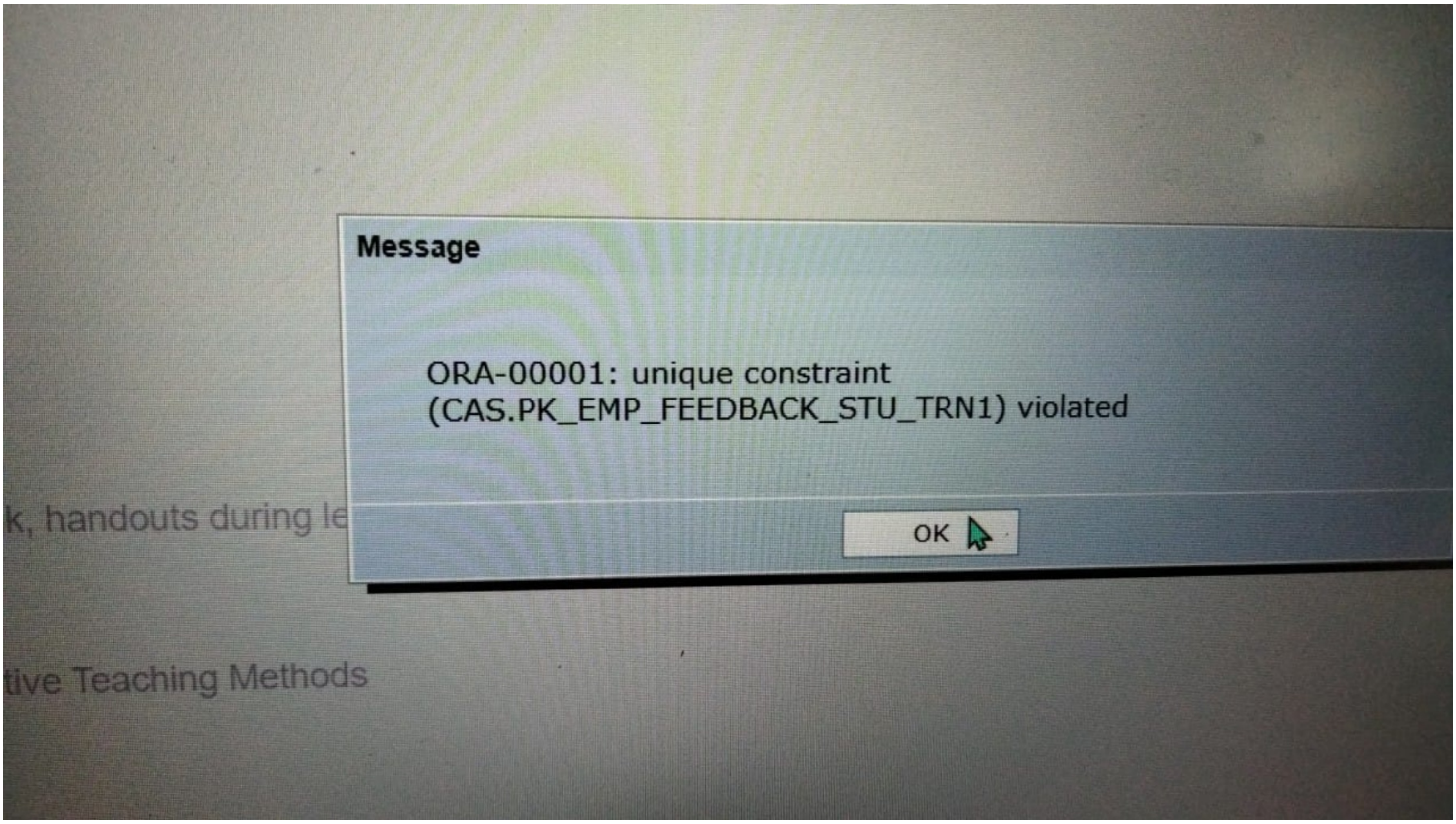


Fig: 4.39

4. A square plate of size 0.4m x 0.4m having a weight of 500N is supported by three vertical wires as shown in fig 4.39. Determine tension in each wire. **(Ans: 166.67N, 166.66N,**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166.66N)

Learning from lecture: Learners will be able to solve the problems based on equilibrium of parallel force system.

4.6 **Conclusion:**

Learning Outcomes: Learners should be able to

Know, Comprehend

1. Define forces in space in all the three directions (i.e., X, Y, Z directions),
2. Explain the position of different forces in space in vector notation providing magnitude and direction.

Apply, Analyse

3. Define Moment of force about a point and about a line.
4. Explain vector component of a force

Synthesize

5. Explain equilibrium of Concurrent & parallel force system
6. Explain equilibrium of General force system

4.7 **Add to Knowledge:**

- **Vector Addition:** The sum of two or more vectors can be obtained by adding the components i.e. A+B = B+A (Commutative Law)
- **Cross Product:** Also known as vector product i.e. $A \times B = AB \sin \theta$& if $\theta = 0$ or п then $A \times A = 0$, $A \times B = 0$, $(A \times B)$ is not equal to $(B \times A)$.
- **Dot Product:** Also known as scalar product i.e. A.B = AB cos θ. If θ = 0, A.B = AB & if θ=90 then A.B = 0, A.B = B.A(Commutative Law)

Research Work: https://www.mak.com/products/simulate/vr-forces

4.8 **Set of Multiple-Choice Questions:**

1) If $\bar{F}$ = 345i + 150j – 290k N. What will be its magnitude?

a) 480 N b) 460 N c) 475 N d) 500 N

2) If $\theta x = 66^0$&$\theta y = 140^0$. Then θz is

a) 69.140 b) 60.140 c)65.70 d)66.50

3) What is moment $\bar{M}$ of a force $\bar{F}$ about a font where position vector from the point to the fore is $\bar{F}$ to the for is $\bar{r}$

a) $\bar{r} \times \bar{f}$ b) $\bar{f} \times \bar{r}$ c) $\bar{r}.\bar{f}$ d) $\bar{f}.\bar{r}$

4) Unit vector has magnitude ________.

a) 1 b) 2 c) 3 d) 4

5) Resultant $\bar{R}$ of four concurrent forces $\bar{F}1, \bar{F}2, \bar{F}3,$ &$\bar{F}4$ is ________

a) $\bar{F_1} + \bar{F_2} + \bar{F_3} + \bar{F_4}$ b) $\bar{F_1} \times \bar{F_2} \times \bar{F_3} \times \bar{F_4}$ c) $\sqrt{(F_1^2 + F_2^2 + F_3^2 + F_4^2)}$ d) $\bar{F_1}.\bar{F_2}.\bar{F_3}.\bar{F_4}$

6) A force $\bar{F}$ passing form points P (5, 6, 7) & Q (8, 8, 9) directing from Q to P is given by

a) $\bar{F} = 3i + 2j + 2$ b) $\bar{F} = 3i - 2j - 2$ c) $\bar{F} = -3i + 2j + 2$ d) $\bar{F} = -3i - 2j - 2k$

7) A force $\bar{F}$ passing through origin in -ve y- direction & having magnitude of 20 KN is given by ________

a) $\bar{F} = 20$ b) $\bar{F} = 20$ c) $\bar{F} = 0$ d) $\bar{F} = -20j$

8) Conditions of equilibrium for comment space forces are

a) $\Sigma Fx = 0, \Sigma Fy = 0, \Sigma Fz = 0$

b) $\Sigma Fx = 0, \Sigma Fy = 0, \Sigma Mx = 0, \Sigma My = 0$

c) $Mx = 0, \Sigma Fy = 0, \Sigma Fz = 0, \Sigma M = 0$

d) $\Sigma Fx = 0, \Sigma Fy = 0$ & $M = 0$

9) If force $\bar{F} = F_x i + F_y j + F_z k$ having θx & θz as force direction with +ve axes then

a) $F_x = F\cos\theta x, F_y = F\sin\theta y, F_Z = F\cos\theta z$

b) $F_x = F\cos\theta x, F_y = F\cos\theta y, F_z = F\cos\theta z$

c) $F_x = FSM\theta x, F_y = F\sin\theta y, F_z = F\sin\theta z$

d) $F_x = F\cos\theta x, F_y = F\cos\theta y, F_z = F\sin\theta z$

10) If Qx, Qy & Qz are force directions with five axes then

a) $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

b) $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 2$

c) $\cos\theta_x + \cos\theta_y + \cos\theta_z = 1$

d) $\cos\theta x + \cos\theta_y + \cos\theta_z = 2$

4.9 **Short Answer Questions:**

1) A force of 1000N forms angles of 60°, 45°, and 120° with x, y and z axes respectively. Write equation in the vector form. **(Ans: 500i + 707.1j – 500k N)**
2) Å force acts at origin in a direction defined by angles $\theta_y = 65°$ and $\theta_z = 40°$. knowing that x component of force is -750N find the other components **(Ans: 118.97°, -654.41N, 1186.19N, 1548.46N)**
3) A force acts at origin in a direction defined by angles θy = 35° and θz= 40°. knowing that x component of force is -870N find the value of θx **(Ans: 120°)**
4) A 150 kN force acts at P (17, 12, 0) and passes through Q (9, 0, 4). Put the force in vector form. **(Ans: -80 i -120 j + 40 k)**
5) |P| = 200 N. Coordinates of points are A = (0,15,20) cm, E= (30,10,0). Determine unit vector along line EA and express the force P in vector form. **(Ans: -165 i + 27.5 j + 110 k)**
6) A force acts at the origin in a direction defined by the angles $\theta_X = 56^0$ & $\theta_Y = 35^0$ Knowing that the Z component of the force is 545 N. Determine θ_Z **(Ans: 82.66°)**
7) Two vectors A & B are defined by the relations A = 6i+4j+6k, B=4i+7j-9k. Determine the sum, difference. **(Ans: Sum = 10i + 11j – 3k & Difference = 2i -3k + 15k)**

8) Two vectors A & B are defined by the relations A = 3i+8j+6k, B=5i+5j-9k. Determine dot product of these vectors **(Ans: 1)**

9) Two vectors A & B are defined by the relations A = 3i+4j+9k, B=4i+5j-3k. find angle between them. **(Ans: 89.21°)**

10) A rectangular parallelepiped carries four forces as shown in fig 4.40. Reduce the force system to a resultant force applied at the origin & a moment about origin. OA = 5m OB = 2m OC = 4m. **(Ans: R = 1.27i + 10.37j +13.176k, M = 8.432i + 29.46k)**

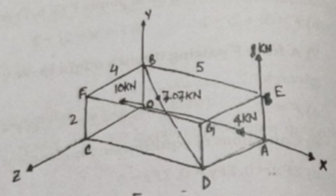


Fig: 4.40

11) Force F = 80i + 50j – 60k passes through a point A (6,2,6). Compute its moment about a point B (8,1,4). **(Ans: - 160i + 40j - 180k)**

12) Force F = (3i-4j+12k) N acts at point A (1,-2,3). Find (i) Moment of force about origin (ii)Moment of force about point B (2,1,2)m. **(Ans: - 12i - 3j + 2k , 32i - 15j - 13k)**

13) A force of 10kN acts at a point P (2,3,5) and has its line of action passing through Q (10,-3,4)m. Calculate moment of this force about a point S (1,-10,3)m. **(Ans: 0.995i - 16.92j - 109.45k)**

14) A force of 1200N acts along PQ, P (4, 5, -2) & Q (-3, 1, 6) m. Calculate its moment about a point A (3,2,0) m **(Ans:1690.4i +633.96j +1796.14k)**

15) Discuss the resultant of concurrent forces in space.

16) A T shaped rod is suspended using three cables as shown in fig 4.41. Neglecting the weight of the rods, find the tension in each cable. **(Ans: 26.67N, 40N, 33.34N)**

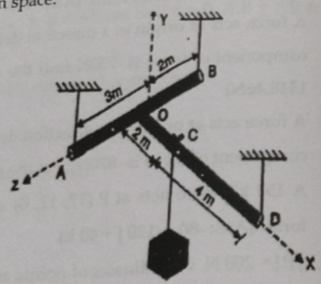


Fig: 4.41

17) The resultant of three concurrent space forces at $\overline{R} = (-788j)$ N as shown in fig 4.42. Find the magnitude of forces F_1, F_2& F_3 respectively. **(Ans: 154N, 320N, 400N)**

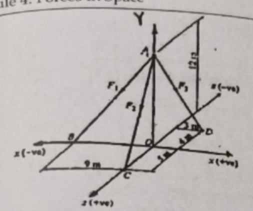
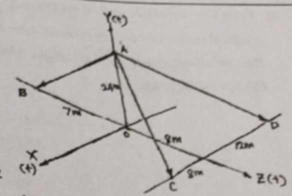


Fig: 4.42



ig: 4.43

18) Coordinate distance are in 'm' units for the space frame in fig 4.43. there are 3 members. AB, AC & AD. There is a force W= 10kN acting at A in a vertically upward direction. Determine the tension in AB, AC & AD. **(Ans: 5.53kN, 3.10kN, 2.16kN)**

19) A force of 140kN passes through point C (-6,2,2) and goes to point B (6.6, 8). Calculate moment of force about origin. **(Ans: 769.4 kN-m)**

20) A force acts at origin in a direction defined by angles θ_y = 65° and θ_z= 45°. knowing that x component of force is -660N find magnitude of force **(Ans:124.523°)**

4.10 Long Answer Questions:

1) Knowing that the resultant of three parallel forces is 500N acting in the positive y-direction and passing through the centre of the rectangular plate as shown in fig 4.44, determine the forces F₁, F₂, F₃ **(Ans: 200j , 100 j & 100k)**

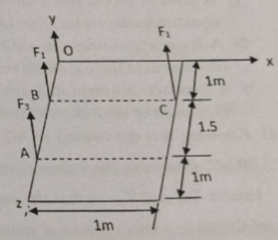


Fig: 4.44

2) A pole is held in place by three cables. If the force of each cable acting on the pole is as shown in fig 4.45, determine the resultant. **(Ans: 500 i + 197.3 j + 156k)**

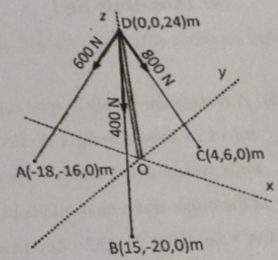


Fig: 4.45

3) F_1=500N, F_2=800N and F_3=600N act on a vertical mast AB as shown in fig 4.46. Find the resultant force at the origin (Ans : 155.62i + 148. 29k)

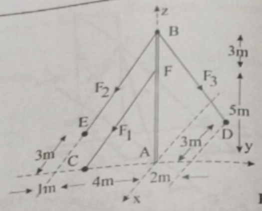


Fig: 4.46

4.11 References:

1) Engineering Mechanics by Tayal, Umesh Publication
2) Engineering Mechanics by Beer & Johnson, Tata McGraw Hill
3) Engineering Mechanics by F.L. Singer by Harper
4) Engineering Mechanics – Statics, R. C. Hibbler
5) Engineering Mechanics – Statics, J. L. Merium, I. G. Kraig
6) Engineering Mechanics – P. J. Shah, R. Bade

Self-Assessment

1) A force of 1000N forms angles of 65°, 45°, and 130° with x, y and z axes respectively. Write equation in the vector form. **(Ans: -562.45 i+ 525.32 j-367.29 k)**
2) A force acts at origin in a direction defined by angles θ_y = 75° and θ_z= 50°. knowing that x component of force is -550N find the other components **(Ans: 52°)**
3) A force acts at origin in a direction defined by angles θy = 35° and θz= 40°. knowing that x component of force is -870N find the value of θx. **(Ans: 120.5°)**
4) Knowing that the tension in AC is T_{AC}= 20 kN, determine the required values of tension T_{AB} and T_{AD} so that the resultant of the three forces applied at point A is vertical. Find their resultant for given fig 4.47. **(Ans: -110.05 KN)**

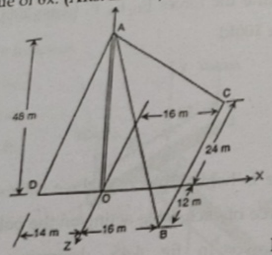


Fig: 4.47

5) A rectangular parallelepiped carries four forces as shown in fig 4.48. Reduce the force system to a resultant force applied at the origin and a moment about origin. OA = 5m OB = 2m OC = 4m. **(Ans: R = 1.27i + 10.37j +13.176k, M = 8.432i + 29.46k)**

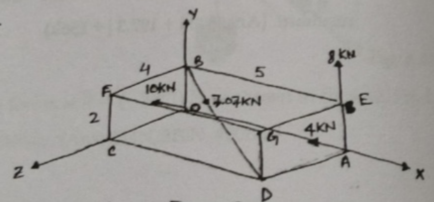
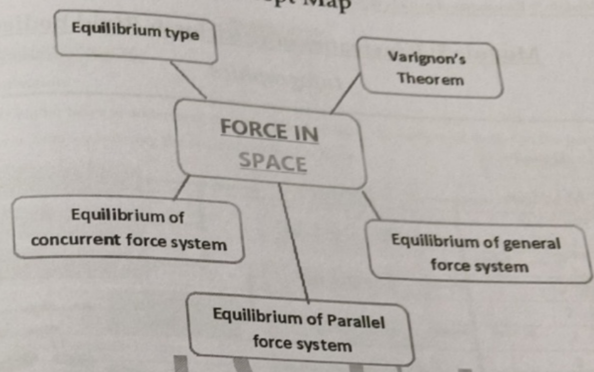


Fig: 4.48

Concept Map



Self-Evaluation

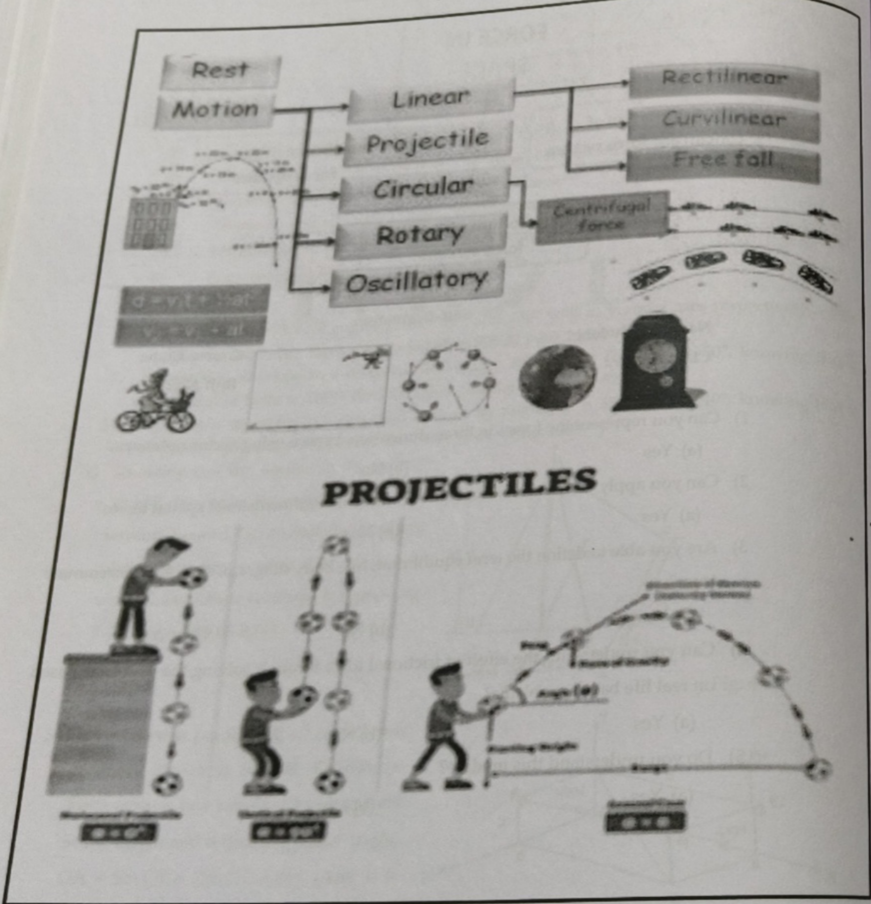
Name of Student: **Course Code:**

Class & Div.: **Roll No.:**

1) Can you represent the forces in three-dimensional space using vector notations?
   (a) Yes (b) No
2) Can you apply the conditions of equilibrium to three dimensional spatial forces?
   (a) Yes (b) No
3) Are you able to define the term equilibrant, free body diagram, statically determinate truss?
   (a) Yes (b) No
4) Can you understand the effect of frictional force through solving the problems based on real life based problems?
   (a) Yes (b) No
5) Do you understand this module?
   (a) Yes (b) No

Module 5: Kinematics of Particles & Rigid Bodies

Infographics



Lecture 30

5.1 *Kinematics of Particles*

5.1.1 Motivation:

This chapter helps to understand the concepts of motion. The analysis of motion of the particle is done without considering the cause of motion i.e., force.

5.1.2 Syllabus:

Lecture	Content	Duration of Lecture (Hrs.)	Self-Study (Hrs.)
30	Rectilinear Motion	1	2
31	Motion Under Gravity	1	2
32	Curvilinear Motion	1	2
33	Projectile Motion	1	2
34	Projectile Motion	1	2
35	Introduction: (Types of Motion & ICR)	1	2
36	ICR with two links system	1	2
37	ICR of rollers	1	2

5.1.3 Weightage: 15 -20 Marks

5.1.4 Learning Objectives: Learners shall be able to

1) State Newton's Laws of Motion to solve problems on motions with uniform velocity-acceleration as well as variable acceleration using
2) Derive the equations for velocities and accelerations of the body moving along the curved path
3) Illustrate the graphical solution to find the displacement, velocity, acceleration and time of the particle' motion through motion curves
4) Determine the Range, Maximum Height, Time of Flight and velocities of the parabolic motion of the particle i.e. projectile motion.
5) Apply the concept of dependent motion in motion under Gravity
6) Derive the equations of distance, speed and acceleration for the given equation of time

5.1.5 Key Notations:

➤ **Rectilinear Motion**
1) u = initial velocity
2) v = final velocity
3) a = acceleration

4) t = time
5) s = displacement

- **Projectile Motion**
  1) U = initial velocity
  2) R = horizontal range
  3) H_{MAX} = Maximum Height
  4) T = time of flight
  5) α = Angle of Projection

- **Curvilinear motion**
  1) a_n = normal acceleration
  2) a_t = tangential acceleration
  3) a = total acceleration
  4) ρ = radius of curvature

5.1.6 Theoretical Background:

Kinematics is the area of mechanics concerned with the study of motion of particles and rigid bodies without consideration of what has caused the motion. When we take into consideration the factors causing the motion, the area of study is called dynamics. A particle has no size but has mass. This is a completely hypothetical concept. However, many times object can be approximated as particle. For example, a car is moving. Compared to distance, it travels, its size is very small, and it can be treated as particle. Moreover, a rigid body can be considered as a combination of small particles. Thus, the concepts learned for a particle can clarify the kinematics of rigid body. If the particle is moving along straight path, then it is called as rectilinear motion.

E.g., A train moving on single track, a stone released from top of tower etc.

- **Time**:
  - Measure of durations of events and the intervals between them.
  - Units: seconds (sec)
- **Distance:**
  - It's a scalar quantity.
  - Path covered by a body during a motion.
  - Unit: meter (m), kilometer (km), millimeter (mm) etc.
- **Displacement**
  - It's a vector quantity.
  - Shortest distance covered by the body.
  - Unit: SI - meter (m), kilometer (km), millimeter (mm) etc. CGS – centimeter (cm)
- **Speed:**
  - It's a scalar quantity. i.e. magnitude of velocity vector.

  - Distance travelled per unit of time.
  - Units: SI - meter per second (m/s), kilometer per second (km/s or kmps), kilometer per hour (kmph), CGS - centimeter per second (cm/s)
  - In everyday usage, kilometers per hour or miles per hour are the common units of speed. At sea, knots or nautical miles per hour are a common speed.
- **Velocity**:
  - It's a vector quantity
  - Rate of change of displacement
  - Units: SI - meter per second (m/s), kilometer per second (km/s or kmps), kilometer per hour (kmph), CGS - centimeter per second (cm/s)
  - In everyday usage, kilometers per hour or miles per hour are the common units of speed. At sea, knots or nautical miles per hour are a common speed.
- **Acceleration:**
  - It's a vector quantity
  - Rate of change of velocity
  - Units: SI - meters per second square (m/s2), CGS - centimeter per second square (cm/s2)

5.1.7 Formulae:

- **Rectilinear Motion:**

Equations of motion:

1) For Uniform Acceleration Motion:
   - $v = u + at$
   - $s = ut + {}^1/_2 at^2$
   - $v^2 = u^2 + 2as$
2) For Uniform Motion:

   $S = u\,t$

3) Displacement in n[th] second:

$$S^{nth} = u + \frac{a}{2}(2n-1)$$

- **Motion under gravity:**

As 'g' is always downward, the kinematic equations can be formulated as

- $v = u - gt$
- $s = ut - {}^1/_2 gt^2$
- $v^2 = u^2 - 2gs$

where's' is the vertical distance between point of projection and second point which may or may not be a point of landing.

➢ **Sign conventions:**

1) All upward quantities as positive
2) All downward quantities as negative
3) If particle motion is upward, consider 's' as positive
4) If motion of the particle is downward, consider 's' as negative

➢ **Curvilinear Motion**

For particles moving along the curve with uniform tangential acceleration

- $v = u + a_t t$
- $s = ut + {}^1\!/_2\, a_t t^2$
- $v^2 = u^2 + 2a_t s$
- $v = \frac{ds}{dt}$

Tangential Acceleration,

$$a_t = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$$

Normal Acceleration, $a_n = \frac{v^2}{\rho}$

Total Acceleration: $a = \sqrt{a_t^2 + a_n^2}$

➢ **Radius of Curvature(ρ):**

When a particle is moving along a curve y = f(x), radius of curvature at a point is given by

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} \qquad \rho = \frac{\left[v_x^2+v_y^2\right]^{3/2}}{v_x a_y - v_y a_x}$$

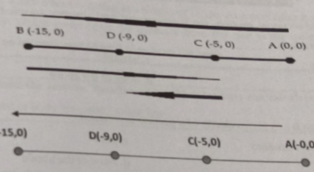
5.1.8 Key Definitions:

1) **Dynamics:** It is the branch of mechanics which deals with the particles and bodies which are in motion.
2) **Kinematics**: It is the branch of dynamics which deals with the motion of particles and bodies without reference to the forces which cause the motion as-well as mass of particle or body. It is the study of geometry of motion.
3) **Particle**: In a particular problem if the dimensions of the body under consideration are not relevant in solving that problem, then we consider the body as a particle. i.e. its entire mass is assumed to be concentrated at a point.
4) **Rectilinear motion**: The motion of the particle in the straight line. E.g. car traveling along a straight road, a stone thrown straight up in the air, etc.
5) **Motion under Gravity:** Motion under gravitational field: it may be controlled (e.g. Rising helicopter, balloon, etc.) or motion under the action of gravity (e.g. Freely falling body)
6) **Curvilinear Motion**: It is the motion in which the particle travels along a curved path. E.g. car traveling along a curved road, a stone thrown at an angle with the horizontal, etc.

7) **Projectile motion:** In projectile motion, the particle moves in space along vertical and horizontal directions simultaneously. For projectile motion, the particle is projected in space at a certain angle with horizontal.
8) **Absolute motion:** The motion of the particle with respect to the fixed frame of reference is called absolute motion of a particle.
9) **Relative motion:** The motion of a particle relative to a set of axes which are moving is called as relative motion.
10) **Dependent motion:** When motion of one particle depends upon the motion of other particle or several particles, the motion is called as dependent motion

Solved Problems:

1) **The path of travel of the particle is represented by following diagram. Determine displacement and distance traveled**



5.1

Path of travel: 1) A to B 2) B to C 3) C to D

Solution: starting point A (0,0)

Displacement: final point – initial point

Point of reversal-> B, C

Point	Displacement (-> +ve) W.R.T origin 'A'	Distance
A	0	0
B	-15	15
C	-5	25
D	-9	29

As the direction changes v=0 at B, C

2) **A car covers 100 m in 6 seconds and it takes another 5 seconds to cover next 120 m. Find the initial velocity of the car and uniform acceleration of the car.**

Given: In t=6 sec, s= 100 m

s=100+120=220 m

In t= (6+5) = 11 sec

Let, u= Initial velocity of car

a= uniform acceleration of car

Now using equation of motion, $s = ut + \frac{1}{2}at^2$

$= u \times 6 + \frac{1}{2} \times a \times 6^2$

$\therefore 100 = 6 \times u + 18 \times a$....(i)

$220 = u \times 11 + \frac{1}{2} \times a \times 11^2$

$\therefore 220 = 11 \times u + 60.5 \times a$ .... (ii)

Solving equation (i) & (ii), we get

Initial velocity, u=12.6666 m/s,

Uniform acceleration, a=1.333 m/s^2

Let's check the takeaway from this lecture

1) During unidirectional motion, the displacement and distance traveled by a particle with uniform acceleration is ______

a. different

b. same

c. variable

d. none of the above

2) When acceleration is ______, velocity of a particle is constant.

a. constant but non-zero

b. maximum

c. zero

d. none of the above

Exercise:

1) A motorist is travelling at 90 kmph, when he observes a traffic light 250 m ahead of him turns red. The traffic light is timed to stay red for 12 sec. if the motorist wishes to pass the light without stopping, just as it turns green. Determine i) The required uniform deceleration of the motor. ii) the speed of the motor as it passes the traffic light. **[Ans: a= -0.6944 m/s^2, v= 60kmph]**

2) The position of a particle which moves along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$ where x is expressed in m and t in seconds. Determine (a) the time at which the velocity will be zero, (b) the position and distance traveled by the particle at that time, (c) the acceleration of the particle at the time, (d) the distance traveled by the particle from t =4 see to t =6 sec. **[Ans.: t = 5 sec x_s= -60m, 100m, 18m/s^2, 18m]**

3) Two electric trains A and B leave the same station on parallel lines. The train A starts

from rest with a uniform acceleration of 0.2 m/s2 and attains a speed of 45 kmph, which is maintained constant afterwards. The train B leaves 1 minute after with a uniform acceleration of 0.4 m/s2 to attain a maximum speed of 72 kmph, which is maintained constant afterwards. When will the train B overtake the train A? **[Ans.: t= 114.6 s]**

Practice Problems for the Day:

1) A vehicle moves at a uniform velocity of 54 km/h for the first 75 seconds. Then it accelerates uniformly at 2 m/s^2 and attains a maximum velocity of 180 km/h. It now moves further with this uniform velocity for the next 4 minutes and then moves with uniform retardation and comes to rest in 25 seconds. Find the total time of travel and the total distance covered.**[Ans: t = 357.5 secs, S =14.32 km]**

2) An NCC parade is going at a uniform speed of 6kmph through a place under a berry tree on which a bird is sitting at a height of 12.1m. At a particular instant the bird drops a berry. Which cadet (give the distance from the tree at the instant) will receive the berry on his uniform? **(Ans: 2.62m from the tree)**

Learning from the lecture 'Rectilinear Motion': Learner can calculate velocity acceleration, displacement of body travelling along line

Lecture 31

5.2 *Motion Under Gravity*

Learning Objective: Learners will be able to calculate different Motion parameters for the system under gravity. They will be able to calculate motion parameters of bodies having relative motion in vertical motion.

Theory:

- **Motion under gravity:**

The vertical motion of a particle under the influence of constant gravitational acceleration 'g' is known as motion under gravity.

Referring to the sign conventions following examples will illustrate the details:

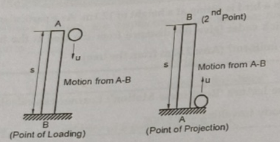


Figure 5.2

Solved Problems:

1) ***A stone is thrown vertically upwards and returns to the ground in 6 sec. How high does it go?***

Sol: Let the stone projected with an initial velocity $V = 0$ travel h meters to the peak. If it takes 6 sec to return back, implies that 3 sec were spent in going up and 3 sec in coming down.

Motion of stone (Ground to peak)

M.U.G. ↑ +ve	$v = u + a \times t$
$u = v_0$ m/s	$0 = v_0 - 9.81 \times 3$
$v = 0$	$V_0 = 29.43$ m/s
$s = h$ meters	$V^2 = u^2 + 2as$
$g = -9.81$ m/s^2	$0 = (29.43)^2 + 2 \times -9.81 \times h$
$t = 3$ sec	$h = 44.145$ m

2) ***From the top of a tower 100m high, a stone was dropped down at the same time, another stone was thrown up from the foot of the tower with a velocity of 30m/s. when and where the 2 stones***

will cross each other? Find the velocity of each stone at the time of crossing

Sol:

Stone 1

$U_1 = 0$

x_1 downwards $a_1 = -9.81 m/s^2$

Using $x = ut + ½ at^2$

$-x_1 = 0 - 9.81/2\, t^2$

Now we know that mod $x_x + x_2 = 100$

$-(100 - x_2) = -9.81/2\, t^2$

hence $t = 3.33s$

$X_{1\, t=3.33} = 9.81 * 3.33^2 = 54.3m$

$X_{1\, t=3.33} = 100 - 54.3 = 45.7m$

$V_1 = u + at = 0 - 9.81 * 3.33 = 32.7 m/s$ downwards

$V_2 = u + at = 30 - 9.81 * 3.33 = 2.7 m/s$ downwards

Stone 2

$U_2 = 30 m/s$ upwards x_2 upwards

$a_2 = -\ 9.81 m/s^2$

$x_2 = 30t - 9.81/2\, t^2$

implies $x_1 = 100 - x_2$

$100 - x_2 = 9.81/2\, t^2$

$100\, [\, 30t - ½ * 9.81\, t^2] = 9.81/2\, t^2$

3) **A stone has fallen 12m after being dropped from the top of tower. Another stone is dropped from a point 20m below the top of the tower. If both stones reach the ground together, find the height of the tower.**

Soln. The stone 1 is dropped from top of tower, when it reaches 12m below the top (at point B) its velocity is given by

$V_B^2 = U_A^2 - 2gs = 0 - 2 \times 9.8 \times (-12)$

$V_B = 15.34 m/s$ downwards

Let t be the time taken by the stone 2 to reach ground (C to D)

Therefore, time taken by stone 1 from B to D must be t sec.

For stone 1 (B to D)

Use $s = ut - ½ gt^2$

$-(H - 12) = -15.34t - ½\, 9.8\, t^2$

$H = 15.34t + 4.9\, t^2 + 12$..........(i)

For stone 2 (C to D)

Use $s = ut - ½ gt^2$

$-(H - 20) = 0 - ½\, 9.8\, t^2$

$H = 20 + 4.9t^2$..................(ii)

Equating I and ii we get

$20 + 4.9t^2 = 15.34t + 4.9t^2 + 12$

$t = 0.52s$

Substituting in ii

$H = 20 + 4.9\,(0.52)^2$

$H = 21.33m$

4) **In a flood relief area, a helicopter going up with a constant velocity, first batch of food packet is released which takes 4s to reach the ground. No sooner than this batch reaches the ground, second batch of food packet is released, which takes 5s to reach the ground. From what height the first batch of packet was released and what is the velocity with which the helicopter is going up?**

Soln. Let u be the constant velocity of the helicopter

For first food packet $s_1 = ut - ½gt^2$

$-s_1 = (u \times 4) - ½ \times 9.8 \times 4^2$

$s_1 = 78.48 - 4u$……. (i)

for second food packet

$-s_2 = (u \times 5) - ½ \times 9.8 \times 5^2$

$S_2 = 122.62 - 5u$……(ii) for helicopter ,

$(h - s)$ = velocity x time (as velocity is constant)

$s_2 - s_1 = u \times 4$…(iii)

substituting the values of s and s_2 from i and ii

$122.62 - 5u - (78.48 - 4u) = 4u$,

$5u = 44.14$ $u = 8.83$ m/s By equation (i)

$s_1 = 43.16$ m

Thus, first batch of packets is released from 43.16m.

Let's check the takeaway from this lecture

1) Ball is thrown up and attains a maximum height of 100 m. Its initial speed was

i. 9.8 m/s iii. 19.6 m/s

ii. 44.2 m/s iv. 98 m/s

2) A ball dropped from a wall of height h travels 50 m in last two seconds before landing. What is the height of the wall from which the ball was dropped?

i. 120.15 m iii. 183.48 m

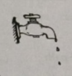
ii. 127.37 m iv. Insufficient data

<u>Exercise :</u>

1) A particle falling under gravity falls 30 m in a certain second. Find the time required to cover the next 30 m. **[Ans: 0.775 secs]**

2) A ball is dropped from a height of 5m onto a sandy floor and penetrates the sand up to 10cm before coming to rest. Find the retardation of the ball in sand assuming it to be uniform. **[Ans: 490m/s²]**

3) Water drips from a faucet at the rate of 5 drops per second as shown in figure. Determine the vertical separation between two consecutive drops after the lower drop has attained a velocity of 3 m/s.**[Ans.: 0.4 m]**



4) The depth of a well up to water surface in it is 'H' meters. A stone is dropped into the well from the ground. After 3 seconds, the sound of the splash is heard at the ground. If the velocity of sound is 330 m/s, find the value of 'H'. **[Ans. H = 40. 59 m]**

5) From the top of a tower, 100m high, a stone was dropped down at the same time, another stone was thrown up from the foot of the tower with a velocity of 30 m/s. When & where the two stone will cross each other? Find the velocity of each stone at the time of crossing.**[Ans. t = 3.33 s, h_1 = 54.5m, h_2 = 45.5 m, 32.7 m/s (↓), 2.7 m/s (↓)]**

<u>Practice Problems for the Day:</u>

1) A particle falls from rest and in the last second of its motion it passes 70 m. Find the height from which it fell and the time of Its fall.**[Ans: 286.1 m]**

2) A ball is dropped from a height. If it takes 0.200s to cross the last 6m before hitting the ground, find the height from which it was dropped. Take $g = 10m/s^2$. **[Ans: 48m]**

3) A balloon starts moving upwards from the ground with const. acceleration of 1.6 m/s^2. 4 seconds later, a stone is thrown upwards from same point:(i) What velocity should be imparted to the stone so that it just touches the ascending balloon?(ii) At what height, will the stone touch balloon?**[Ans.: (i) V_s = 23.57 m/sec. (ii) h = 24.02 m.]**

4) A ball is thrown vertically upward from the 12m levels in an elevator shaft, with an initial velocity of 18m/s. At the same instant, an open platform elevator passes the 5m level, moving

upward with a constant velocity of 2 m/s. Determine (a) when and where the ball will hit the elevator, (b) the relative velocity of the ball with respect to the elevator when the ball hits the elevator. [Ans. t = 3.6 sec, elevation from ground = 12.3 m, $V_{B/E}$ = 19.81 m/s]

Learning from the lecture 'Motion under Gravity': Learner can calculate velocity acceleration, displacement of body travelling along line in vertical direction.

Lecture 32

5.3 *Curvilinear Motion*

Learning Objective: Learner will be able to calculate different motion parameter when the body is in curvilinear motion.

Theory

A motorcycle travels up a hill for which the path can be approximated by a function y = f(x). If the motorcycle starts from rest and increases its speed at a constant rate, how can we determine its velocity and acceleration at the top of the hill? How would you analyze the motorcycle's "flight" at the top of the hill?

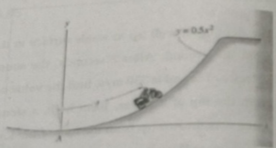


Figure 5.3

Coordinate systems in curvilinear motion:

1) Rectangular coordinates system
2) Normal and tangential coordinate system (path variables)

Rectangular coordinates In this system, for plane curve we use x and y coordinates and for space curve we use x, y and z coordinates.

Position Vector: Location of the particle P situated at point x, y and z is defined by position vector, $\bar{r}$.

$\bar{r} = x\,i + y\,j + z\,k$ (for space curve)

$\bar{r} = x\,i + y\,j$ (for plane curve)

The magnitude of position vector is $|r| = r = \sqrt{x^2 + y^2 + z^2}$ and its direction is

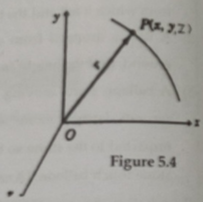


Figure 5.4

$$\cos\alpha = \frac{x}{r},\ \cos\beta = \frac{y}{r}\cos\gamma = \frac{z}{r}$$

➢ **Velocity:** The velocity of the particle can be expressed as,

$$v = \frac{dr}{dt} = \frac{d}{dt}xi + yj$$
$$= \frac{dx}{dt}i + \frac{dy}{dt}j$$
$$= v_x i + v_y j$$

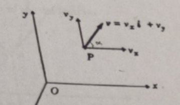


Figure 5.5

The magnitude & direction of velocity is given by,|v|=v = $\sqrt{v_x^2 + v_y^2}$& $\tan\alpha = \frac{v_y}{v_x}$

Acceleration: Acceleration of the particle can be expressed as

$$a = \frac{dv}{dt} = \frac{d}{dt}(v_x i + v_y j) = a_x i + a_y j$$

The magnitude& direction of acceleration 'a' is given by, |a| =

$$a = \sqrt{a_x^2 + a_y^2}\ \&\ \tan\beta = \frac{a_y}{a_x}$$

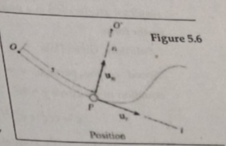


Figure 5.6

When a particle moves along a curved path, it is sometimes convenient to describe its motion using coordinates other than Cartesian. When the path of motion is known, normal (n) and tangential (t) coordinates are often used. In the n-t coordinate system, the origin is located on the particle (the origin moves with the particle). The t-axis is tangent to the path (curve) at the instant considered, positive in the direction of the particle's motion. The n-axis is perpendicular to the t-axis with the positive direction towards the center of curvature of the curve.

Special cases of motion:

There are some special cases of motion to consider

1) The particle moves along a straight line. When ρ= infinity $a_n = v^2/\rho$ $a = a_t$

   The tangential component represents the time rate of change in the magnitude of the velocity

2) The particle moves along a curve at constant speed. When $a_t = 0$ $a = a_n = v^2/\rho$

   The normal component represents the time rate of change in the direction of the velocity.

3) The tangential component of acceleration is constant, In this case, we can apply equation of motion with acceleration as tangential acceleration.

4) The particle moves along a path expressed as $y = f(x)$. The radius of curvature, ρ, at any point on the path can be calculated from

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Solved Problems:

1) A particle moves with a constant speed of 3 m/s along the path shown in figure. What is the resultant acceleration at a position on the path where x = 0.5 m? Also represent the acceleration in vector form.

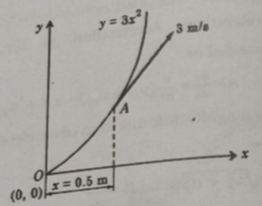


Figure 5.7

Solution: Given Data

Speed of the particle, v = 3 m/s (constant)

equation of the curve $y = 3x^2$

$$a = (a_n^2 + a_t^2)^{1/2}$$

body is moving with constant speed, hence

$a_t = 0$

normal acceleration at A, $a_n = \frac{v^2}{\rho} = \frac{9}{\rho}$

$$y = 3x^2$$

$\frac{dy}{dx} = 6x$ At point A, $\left(\frac{dy}{dx}\right)_{x=0.5} = 6 \times 0.5 = 3$

$$\frac{d^2y}{dx^2} = 6(constant)$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(3)^2\right]^{3/2}}{6}$$

$$\rho_{at\ x=0.5} = 5.27\ m$$

$a = a_n = 9/5.27 = 1.708\ m/s^2$

To represent acceleration in vector form

As shown in figure 5.8, tangential acceleration is tangent to the curve at point A and normal acceleration is towards the centre of curvature and is perpendicular to the direction of tangential acceleration. From the figure

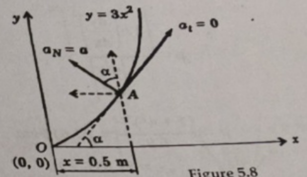


Figure 5.8

$\tan\alpha = \left(\frac{dy}{dx}\right)_{x=0.5} = 3$ $\alpha = 71.565°$

$$a = a_x i + a_y j$$

$$a = (-a_n \sin\alpha)i + (a_n \cos\alpha)j$$

$$a = (-1.708 \sin 71.565°)i + (1.708 \cos 71.565°)j$$

$$a = -1.62i + 0.54j$$

2) A Jet plane travels along the parabolic path as shown in fig. When it is at point A, it has a speed of 200 m/s which is increasing at the rate of 0.8 m/s2. Determine the magnitude of the acceleration of the plane when it is at A.

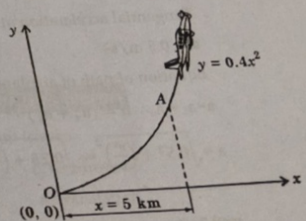


Figure 5.9

Sol: Given data:Equation of parabolic path, $y = 0.4x^2$

Speed of the Jet plane at A, v = 200 m/s

Tangential acceleration at A, a_t = 0.8 m/s²

$$a = (a_n^2 + a_t^2)^{1/2} = \sqrt{0.8^2 + a_n^2}$$

$$= \sqrt{0.64 + \left(\frac{v^2}{\rho}\right)^2} = \sqrt{0.64 + \left(\frac{200^2}{\rho}\right)^2} \quad ...(i)$$

$$y = 0.4x^2$$

$$\therefore \frac{dy}{dx} = 0.8x \therefore \left(\frac{dy}{dx}\right)_{x=5\ km} = 0.8 \times 5 = 4$$

$$\left(\frac{dy}{dx}\right)^2 = 0.8\ (constant)$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$\therefore \rho = \frac{(1+4^2)^{3/2}}{0.8} = 87.616\ km$$

$$= 87.616 \times 10^3$$

$$a = \sqrt{0 + \frac{200^2}{(87.616 \times 10^3)^2}} = 0.921\ m/s^2$$

3) **An airplane travels along a curved path. At point P it has a speed of 360 kmph and it is increasing at the rate of 0.5 m/s", Determine at P (i) the magnitude of total acceleration. (ii) the angle made by the acceleration vector with positive x-axis.**

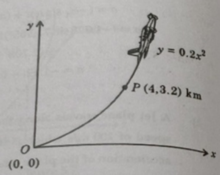


Figure 5.10

Sol: Speed of airplane at point P,

$v = 360\ kmph = 100\ m/s$

Tangential acceleration at point P,

$a_t = 0.5\ m/s^2$

Equation of path of airplane, $y = 0.2\,x^2$

$a = a_t + a_n \therefore a = (a_n^2 + a_t^2)^{1/2}$

$$a = \sqrt{0.5^2 + \left(\frac{v^2}{\rho}\right)^2} == \sqrt{0.25 + \left(\frac{100^2}{\rho}\right)^4}$$

$y = 0.2\,x^2$

$$\therefore \frac{dy}{dx} = 0.4x \therefore \left(\frac{dy}{dx}\right)_{x=4} = 0.4 \times 4 = 1.6 = \tan\alpha$$

$$\therefore \alpha = 58°$$

$$\left(\frac{dy}{dx}\right)^2 = 0.4\ (constant)$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$\therefore \rho = \frac{(1+1.6^2)^{3/2}}{0.4} = 16.793\ km = 16793\ m$$

$$a = \sqrt{0.25 + \frac{100^4}{16793^2}} = 0.778\ m/s^2$$

Normal acceleration,

$a_n = v^2/\rho = 1000^2/16793 = 0.596\ m/s^2$

$\tan\beta = (a_n / a_t) = 0.596/ 0.5$

$\beta = 50°$

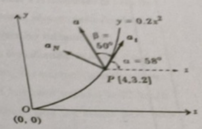


Figure 5.11

Vector diagram at point P is as shown in fig.

Magnitude of total acceleration

$a = 0.778\ m/s^2$

Angle made by acceleration vector

$\theta = \alpha + \beta = 58° + 50° = 108°$ with positive x-axis

Let's check the takeaway from this lecture

1) When motion is ______, the normal component of acceleration is zero.
   a. curvilinear
   b. rotational
   c. rectilinear
   d. translation

2) The radius of curvature of trajectory for a profile is minimum, if ______
   a. velocity is minimum
   b. acceleration is maximum
   c. both a. and b.
   d. none of the above

Exercise

1) The position of a particle is given by $r = 2t^2 i + (4/t^2)\ j$ m where t is in seconds. Determine when t =1 sec (i) the magnitude of normal and tangential components of acceleration of

the particle and (ii) the radius of curvature of the path. **[Ans.: (i) a_t = 19.68 m/s², a_n = 14.32 m/s²]**

2) A particle at the position (4,6,3) at start, is accelerated at $a = 4ti - 10t^2 j$ m/s². Determine the acceleration, velocity and the displacement after 2 seconds. **[Ans.: a = 40.792 m/s² V = 27.84 m/sec. S = 14.67 m.]**

3) A skier travels with a constant speed at 6 m/s along the parabolic path $y = x^2/20$. Determine his velocity and acceleration at the instant he arrives at 'A'. Neglect the size of the skier. **[Ans. V_A = 6 m/s, 45°, a_A = 1.27 m/s², 135°]**

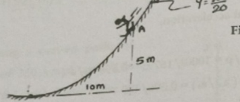


Figure 5.12

4) The jet plane travels along the vertical parabolic path. When it is at point A it has a speed of 200 m/s, which is increasing at the rate of 0.8 m/s², determine the magnitude of the plane's acceleration when it is at point A. **[Ans. 0.521 m/s²]**

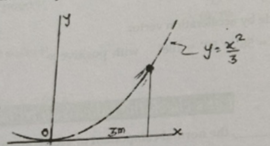


Figure 5.13

5) A car travels along a depression in a road, the equation of depression being $x^2 = 200y$. The speed of the car is constant and being equal to 72 km/h. Find the acceleration when the car is at the deepest point in the depression. What is the radius of curvature at the depression at the point? **[Ans: a = 4 m/s², ρ = 100 m]**

__Practice Problems for the Day:__

1) A particle moves in the x - y plane with velocity components $V_x = 8t - 2$ and $V_y = 2$. If it passes through the point (x,y) = (14,4) at t = 2 seconds determine, the equation of the path traced by the particle. Find also the resultant acceleration at t = 2 seconds. **[Ans: $x = y^2 - y + 2$, 8m/s²]**

2) A rocket follows the path such that its acceleration is given by $a = (4\,i + t\,j)$ m/s². At r = 0 it

starts from rest. At t = 10 seconds, determine, (i) speed of the rocket. (ii) radius of curvature of its path, (iii) Magnitude of normal and tangential components of acceleration. **[Ans.: (i) V_x = 40 m/s, V_y = 50 m/s, (ii) R = 1312.6 m, (iii) a_N = 3.123 m/s² & a_T = 10.307 m/s²].**

3) A particle moves along a hyperbolic path $(x^2/16) - y^2 = 28$. If the x - component of velocity is V_x = 4 m/s and remains constant, determine the magnitudes of particle's velocity and acceleration when it is at point (32, 6) m **[Ans.: V = 4.216 m/sec. a = 0.13m/s²]**

4) A motorist is traveling on a curved section of a highway of radius 750 m at the speed of 108 km/h. The motorist suddenly applies the break causing the automobile to slow down at a constant rate. Knowing that after 8 sec the speed has been reduced to 72 km/h. determine the acceleration of the automobile immediately after the breaks have been applied. **[Ans: 1.732 m/s²]**

5) A car starts from rest at t = 0 along a circular track of radius 200m. The rate of increase in the speed of the car is uniform. At the end of 60 seconds the speed of the car is 24 km/hr. Find the tangential and normal components of acceleration at t = 30 sec. **[Ans. a_t = 0.11 m/s², a_n = 0.054 m/s²]**

6) A point moves along the path $y = (1/3)x^2$ with a constant speed of 8 m/s. what are the x and y components of the velocity when x = 3m? What is the acceleration at the point when x = 3m. **[Ans: x = 3.578 m/s, y = 7.155 m/s]**

7) The truck travels in a circular path having a radius of 50 m at a speed of 4 m/s. For a short distance from s = 0, its speed is increased by a = (0.05 s) m/s², where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved s = 10m. **[Ans.: a = 0.653 m/s²]**

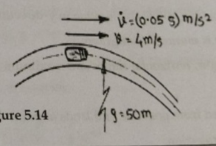


Figure 5.14

Learning from the lecture 'Motion under gravity': Learner can calculate velocity, acceleration, displacement of bodies travelling along curve path.

Lecture 33

5.4 Problems on Projectile Motion

Learning Objective: Learner's will be able to apply equations of Projectile motion &will be able to find different motion parameter

Theory:

➤ **Projectile Motion:**

When a particle is projected in space, its motion is a combination of horizontal and vertical motion (rectangular co-ordinates). The motion of such a particle is called as projectile motion. There are two rectangular components of acceleration.

$a_x = 0$ and $a_y = -g$

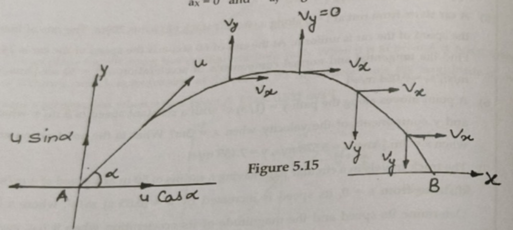


Figure 5.15

If we observe motion in x-direction, we see that acceleration in x-direction, $a_x = 0$ so the motion in x-direction is uniform motion. While acceleration in y-direction is gravitational acceleration 'g' so the motion in y-direction is motion under gravity.

➤ ***Derivation of time of flight, horizontal range and maximum height attained by a projectile on horizontal plane:***

Consider a particle projected from point 'A' and lands at point 'B' both on HP.

Let, u = Initial velocity of projection

α = Angle of projection

t = Total time of flight

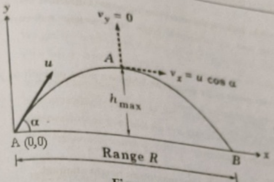


Figure 5.16

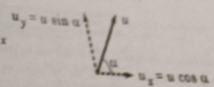


Figure 5.17

Since air resistance is to be neglected, x motion is uniform motion and y motion is motion under gravity.

1) **Time of flight (T) :**
   - Total time taken to complete the projectile path
   - Consider y-motion from A→B (MUG)

     $S_y = U_y t - (1/2)\, gt^2$

     $0 = (u \sin \alpha)\, t - (1/2) gt^2$

     $t = (2u \sin \alpha)/g$ .... Expression for the time of flight of the projectile

2) **Range (R):**
   - Horizontal distance between point of projection and point of landing.
   - Consider x-motion from A→B (UM)

     S = velocity x time = $v_x \cdot t$ So, $R = u \cos \cdot t$

     Substituting value of $t = (2u \sin \alpha)/g$ in above equation

     $R = (u \cos \alpha)(2u \sin \alpha)/g = u^2 (2 \sin \alpha \cos \alpha)/g$

     $R = (u^2 \sin 2\alpha)/g$ .... Expression for horizontal range
   - For the range to be maximum,

     $(d/d\alpha) \sin 2\alpha = 0$

     $\cos 2\alpha = 0$ $2\alpha = 90^\circ$ $\alpha = 45^\circ$

     For maximum range, angle of projection (α) should be 45°

     $R_{max} = u^2/g$ ....Expression for maximum range on horizontal range

For maximum range, $\alpha = 45^0$ only when there is no restriction in vertical direction. For a situation involving such restriction always first find the allowable angle of projection and corresponding maximum possible range.

3) **Maximum Height (h):**

- Vertical distance between point of projection and highest point on path of projectile where vertical component of velocity is zero
- Consider y-motion from A→C (MUG)

$V_y^2 = U_y^2 - 2gS_y$

$0 = (u \sin \alpha)^2 - 2gh$

$h = (u^2 \sin^2\alpha)/2g$ .... Expression for maximum height attained

The above sets of formulae are applicable only when point of projection and point of landing are at the same level on HP. If the two points at different levels use x-motion and y-motion for analysis.

➢ **Derivation of the equation for the path of the projectile: (Equation of trajectory):**

Let the particle be projected from A(0,0) with initial velocity 'u' and angle of projection 'α'.

Let P (x, y) be some point on the path of projectile at time t_1

Consider horizontal and vertical motion of projectile

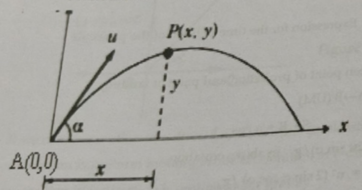


Figure 5.18

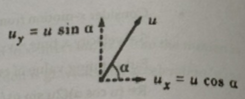


Figure 5.19

Let, after a time 't_1' particle be reached at point P(x, y)

Consider x-motion from A→P (UM)

$\therefore \; x = u (\cos \alpha) t_1$

$\therefore \; t_1 = x / u \cos \alpha$ .... (I)

Consider y-motion from A to P (MUG)

$\therefore S_y = U_y t - (1/2) g t^2$

$\therefore y = u (\sin \alpha)\, t_1 - (1/2)\, g t_1^2$ .... (II)

We know that the path equation is independent of time, we substitute value of time $t_1 = x / u \sin \alpha$ in eqn. (II)

We get, $y = u (\sin \alpha) (x / u \cos \alpha) - (1/2) g (x^2 / u^2 \cos^2\alpha)$

$y = x \tan \alpha - (g x^2 / 2u^2 \cos^2\alpha)$...... Expression of trajectory

Solved Problems:

1) **An aeroplane flying at an altitude of 2000 m accidently loses a rivet. Determine thelocation of the rivet on the ground for the following cases.**

(i)aeroplane is moving with a horizontal velocity of 400 km/hr. (ii) aeroplane is moving with a velocity of 400 km/hr inclined at an angle of 30° in the upward direction. iii)aeroplane is moving with a velocity of 400 km/hr inclined at an angle of 30° in the downward direction.

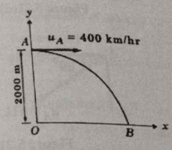


Figure 5.20(a)

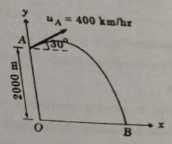


Figure 5.20(b)

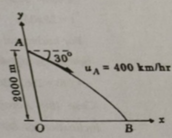


Figure 5.20(c)

Soln: Given y = -2000 m; $u_A = 400 \times 5/18 = 111.11$ m/s

Case (i) Aeroplane is moving horizontally. Rivet will start off horizontally with the same velocity of aeroplane.

Initial horizontal velocity of rivet, $u_{AX} = 111.11$ m/s

Initial vert.cal velocity of rivet, $u_{AY} = 0$

For vertical motion:

$\therefore y = u_{ay} t + (1/2) g t^2$

$\therefore -200 = 0 + (1/2)(-9.81) \times t^2$

$\therefore t = 20.1928$ sec.

For horizontal motion: $x = u_{Ax} t$

$x = 111.11 \times 20.1928$

$x = OB = 2243.6368$m

Alternate: We can use equation of path of projectile to solve this problem, For this, u_A = 111.11 m/s;

$\alpha = 0$, $y = -2000$ m. Now, $y = x \tan\alpha + \frac{\frac{1}{2}gx^2}{u^2\cos^2\alpha}$

$$-2000 = x\tan 0^\circ + \frac{\frac{1}{2}(-9.81)x^2}{(111.11)^2\cos^2 0^\circ}$$

$$x = OB = 2243.6368\text{m}$$

Case (ii) Aeroplane is moving at an upward inclination of 30° with horizontal. Falling rivet will have horizontal and vertical component of velocity at pointA.$u_{AX} = 111.11 \cos 30^\circ = 96.224$ m/s

$u_{AY} = 111.11 \sin 30^\circ$ 55.555 m/s

For vertical motion:$y = u_{Ay}t + \frac{1}{2}gt^2$

$-2000 = 55.555t + \frac{1}{2}(-9.81)t^2$

$4.905t^2 - 55.555t - 2000 = 0$

$t = 26.635$ s

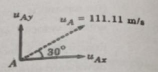


Figure 5.21(d)

For horizontal motion:$x = u_{Ax} .t$

$x = 96.224 \times 26.635$

$x = OB = 2562.926$m

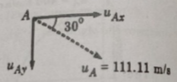


Figure 5.21(e)

Case (iii) Aeroplane is moving downward at an inclination of 30° with horizontal. Falling rivet will have horizontal and vertical component of velocity at point A.

$U_{AX} = 111.11 \cos 30^\circ = 96.224$ m/s

$U_{AY} = -111.11 \sin 30^\circ = -55.555$ m/s

For vertical motion:

$y = u_{Ay}t + \frac{1}{2}gt^2$

$-2000 = -55.555t + \frac{1}{2}(-9.81)t^2$

$4.905t^2 + 55.555t - 2000 = 0$

$t = 15.3087$ s

2) A ball is thrown upward from a high cliff with a velocity of 100 m/s at an angle of elevation of 30 degrees with horizontal. The ball strikes the inclined ground at right angles. If inclination of ground is 30 degrees as shown, determine (i) velocity with which the ball strikes the ground, (ii) the time at which the ball strikes the ground, (iii) coordinates (x, y) of point of strike.

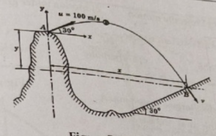


Figure 5.22(a)

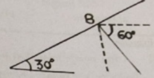


Figure 5.22(b)

Let v be the velocity of striking ball at B, since ball strikes the incline at right angle the components of velocity at B are $v_x = v\cos 60^\circ = 0.5$ v and $v_y = -v\sin 60^\circ = -0.866$ v

At point of projection A

X-component of velocity $u_x = u\cos 30^\circ = 100\cos 30^\circ = 86.6025$ m/s

Y-component of velocity $u_y = u\sin 30^\circ = 100\sin 30^\circ = 50$ m/s

For projectile motion, x-component of velocity remains constant at point A and B.

$\therefore\ u_x = v_x$

$\therefore\ 86.6025 = 0.5$ v

$\therefore\ v = 173.025$ m/s….this is velocity when ball strikes the ground…

$\therefore$ y component of velocity at B, $v_y = -0.866v = -0.866 \times 173.025$

$v_y = -150$m/s

Using equations of motion in the vertical direction

$\therefore\ v_y = u_y + gt$ here $v_y = -150$ m/s, $u_y = 50$ m/s,

$\therefore\ -150 = 50 - 9.81\ t$

$\therefore\ t = 20.387$ s = time taken to hit the incline………. Ans

For motion in horizontal and vertical direction, we use

$x = u_x \times t$ and $y = u_y t + (1/2)gt^2$

$x = 85.5025 \times 20.387$ $\quad y = 50 \times 20.387 + (1/2) \times (-9.81) \times (20.387)^2$

$x = 1765.565$ m $\quad y = -1019.314$ m

Coordinates of (x,y) is as (1765.565, -1019.314)

Let's check the takeaway from this lecture

1) When a particle is projected in space, its motion is combination of horizontal and vertical motion. The motion of such a particle is called as……….

a) Curvilinear motion b) Projectile motion c) Motion under gravity.

2) The path of a projectile is ……………in nature

a) Parabolic b) Cubic c) Hyperbolic d) Helix

Exercise:

1) A fire nozzle located at A discharges water initial velocity v =36 m/s. Knowing that the stream of water strikes the building at a height h = 30 meters above the ground determine the angle αmade by the nozzle with the horizontal.**[Ans.: α= 82.42° or 50.885°.]**

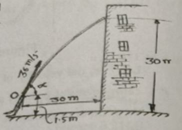


Figure 5.23

2) Find the least initial velocity which a projectile may have, so that it may clear a wall 3.6 m high and 4.8 m distant (from the point of projection) and strike the horizontal plane through the foots of the wall at a distance 3.6 m beyond the wall. The point of projection is at the same level as the foot of the wall.**(Ans= u = 9.77 m/s)**

3) An antiaircraft gun fires a shell as a plane passes directly over the position of the gun, at an altitude of 6000 m the muzzle velocity of the shell is 1500 m/s. Knowing that the plane is flying horizontally at 450 km/h. determine: (a) the required firing angle if the shell is to hit the plane, (b) the velocity and acceleration of the shell relative to the plane at the time of impact. **[Ans: θ= 85.22°, $V_{M/P}$ = 1455.54 m/s (↑), $a_{M/P}$ = 9.81 m/s² (↓)]**

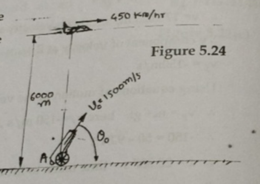


Figure 5.24

Practice Problems for the Day:

1) A stone is thrown in a vertical plane with an initial velocity or 30 m/sec at an angle of 60° above the horizontal. Compute the radius of curvature of its path at the position when it is 15 m horizontally away from its initial position.**[Ans.: R = 70.193 m]**

2) A cannon ball is fired from point A with a horizontal muzzle velocity of 120 m/s, as shown in figure. If the cannon is located at an elevation of 60 m above the ground, determine the time for cannon ball to strike the ground and the range 'R'.**[Ans=3.5 sec R=**

420 m]

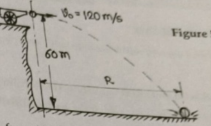
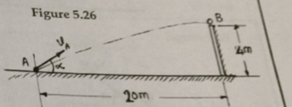


Figure 5.25

3) When a ball is kicked from 'A' as shown in fig. it just clears the top of the wall at 'B' as it reaches the maximum height. Knowing that the distance from 'A' to the wall is 20 m and the wall is 4m high, determine the initial speed at which the ball was kicked. Neglect the size of the ball.**[Ans. 23.9 m/s]**

Figure 5.26



4) A projectile P is fired at a muzzle velocity of 200 m/s at an angle of elevation of 60°. After sometime, a missile M is fired at muzzle velocity of 2000 m/s and at an angle of elevation of 45°, form the same point, to destroy the projectile P. Find:(I) Height, (ii) Horizontal distance and(iii) Time with respect to P firing at which the destruction takes place.**[Ans.: (i) h = 1494.4m; (ii) x =1499.9m; (iii) Time lag = 15 sec.]**

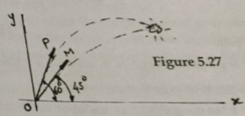


Figure 5.27

Learning from the lecture 'Projectile Motion':Learner can calculate velocity, acceleration, displacement of bodies travelling along projectile.

Lecture 34

5.5 *Problems on Projectile Motion*

Learning Objective: Learners will be able to solve complex problems based on projectile motion.

Solved Problems:

1) **A player throws a ball with an initial velocity u 16m/s from point A located 1.5m above the floor, if h = 3.5m, determine the angle α for which the ball will strike the wall at point B**

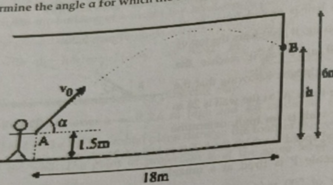


Figure 5.28

Soln:

For projectile from A to B we have $x = 18m$ $\quad y = 3.5 - 1.5 = 2m$ $\quad u = 16m$

By equation of path of projectile

$y = x \tan\alpha - g x^2/(2 * u^2 \cos^2\alpha)$

$2 = 18 \tan\alpha - 9.81 * 18^2 * \sec^2\alpha / (2 * 16^2)$

$2 = 18 \tan\alpha - 6.21 (1 + \tan^2\alpha)$

Solving the above quadratic equation

$\tan\alpha = 2.33$ $\qquad \tan\alpha = 0.567$

$\alpha = 66.77$ degrees $\qquad \alpha = 29.55$ degrees

Now checking the possible angle of projection for maximum height to not exceed $(6 - 1.5) = 4.5m$

$H_{max} = u2 \sin^2\alpha / (2 * 9.81)$

$4.5 = 162 \sin^2\alpha / (2 * 9.81)$

$\alpha = 35.97$degrees

Thus for given arrangement the angle of projection should not exceed 35.97 degrees

So the permissible angle to hit the point B is 29.55 degrees.

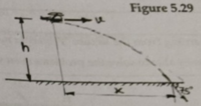
Let's check the takeaway from this lecture

1) Equation of path in projectile motion is given by ....
   a) $y = x^2 \tan\alpha - g x^2/(2 * u^2 \cos^2\alpha)$
   b) $y = x \tan\alpha - g x^2/(2 * u^2 \cos^2\alpha)$
   c) $y = x \tan\alpha - g x^2/(u^2 \cos^2\alpha)$
2) What remains constant in projectile motion?
   a) Horizontal Velocity b) Gravitational Pull c) Both d) None

Exercise:

1) A box released from a helicopter moving horizontally with constant velocity 'u' from a certain height 'h' from the ground takes 5 seconds to reach the ground hitting at an angle of 75°as shown in figure. Determine (i) the horizontal distance 'x' (ii) the height 'h' and (iii) the velocity 'u'.**[Ans.: (i) x = 65.72 m (ii) h =122.63 m (iii) u =13.143 m/s →]**

Figure 5.29



2) A projectile is aimed at an object on the horizontal plane through the point of projection and falls 15m short when the angle of projection is 16°, while it overshoots the object by 20m when the angle of projection is 42°. Determine the angle of projection to hit the object correctly. Assume no air resistance on the projectile.**[Ans. 23.5°]**
3) It is observed that a skier leaves the platform at A and then hits the ramp at B as shown in the figure 4.30 in 5 seconds. Calculate the initial speed 'u' and the launch angle. **(Ans: 19.836m/s, 36.23°)**

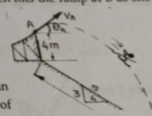


Figure 5.30

Practice Problems for the Day:

1) Two guns are pointed at each other, one upward at an angle of 30°, and the other at the same angle of depression the muzzles being 30 m apart. If the guns are shot with velocities of 350 m/s upwards and 300 m/s downwards respectively, find when and where they will meet?**(Ans=(x,y)=(13.94 m, 8.04 m)**
2) A projectile is aimed at a mark on the horizontal plane through the point of projection. It falls 12m short when the angle of projection is 15°; while it overshoots the mark by 24m when the same angle is 45°. Find the angle of projection to hit the mark. Assume no air resistance.**(Ans: 20.9°)**
3) A shot is fired with a velocity of 40 meters per second from a point 20 meters in front of a vertical wall 10 meters high. Find the angle of projection to the horizontal to enable the shot just to clear the top of the wall. **[Ans: α = 36.13°]**
4) A gun fires a projectile with velocity 300m/s. Find the angle of inclination so that it

strikes a target at horizontal distance of 4000 m from gun and 200 m above it.[76.6°]

5) A mortar fires a projectile across a level field so that the range 'r' is maximum and to equal to 1,000 meters. Find the time of flight.[Ans.: 14.14 sec]

6) A missile thrown at 30° to horizontal falls 10 m short of target, and goes 20 m beyond the target when thrown at 40° to horizontal. Determine correct angle of projection if velocity remains the same in all the cases.[Ans: 32.49°]

7) A ball is thrown by a player with an initial velocity of 15 m/s from a point 1.5 m above ground. If the ceiling is 6 m high, determine the highest point on the wall at which the ball strikes the wall, 18 m away.[Ans: h = 4.2 m]

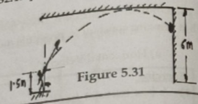


Figure 5.31

Learning from the lecture 'Projectile motion': Learners can identify the given data precisely, thereby able to solve the problems even when two projectiles are given.

Lecture: 35

5.6 Methods to find ICR

Learning Objectives: Learners will be able to know different methods to find ICR.

Method 1: When body slides on two surfaces,

$$\omega = v_A / IA = v_B / IB$$

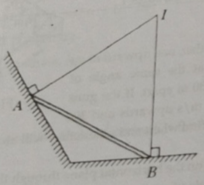


Figure 5.32(a)

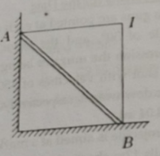


Figure 5.32(b)

Method 2: When one part of the body slides & another part rotates about a hinge point.

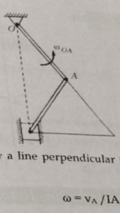


Figure 5.33(a)

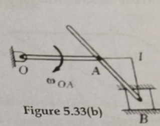


Figure 5.33(b)

To locate ICR draw a line perpendicular to the sliding surface from point B and extend rotating link about point O

$$\omega = v_A / IA = v_B / IB \text{ and } v_A = (OA) \times \omega_{OA}$$

Method 3: When two links of the system rotates about separate hinge points

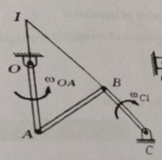


Figure 5.34(a)

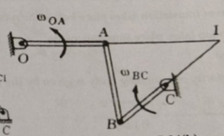


Figure 5.34(b)

Locate ICR by extending the two links.

$$\omega = v_A / IA = v_B / IB \text{ and } v_A = (OA) \times \omega_{OA} \text{ and } v_B = (CB) \times \omega_{CB}$$

Method 4: When body rolls on fixed surface.

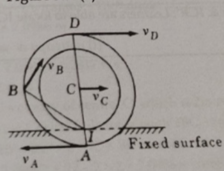


Figure 5.35(a)

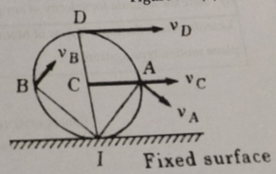


Figure 5.35(b)

The point which is in contact with fixed surface becomes Instantaneous Centre of Rotation.

$$\omega = v_A / IA = v_B / IB = v_C / IC = v_D / ID$$

Method 5: When body lies between two moving surfaces.

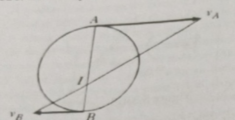


Figure 5.36(a)

$\omega = v_A / IA = v_B / IB$

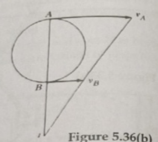


Figure 5.36(b)

Let's check the takeaway from this lecture

1) *In a combined motion of rotation and translation*
   a) *The motion of rotation takes place before the motion of translation*
   b) *The motion of translation takes place before the motion of rotation*
   c) *Both the motion takes place simultaneously*
   d) *All the above*
2) *The linear velocity of a rotating body is given by the relation*
   a) $v = r\omega$ b) $v = r/\omega$
   c) $v = \omega/r$ d)) ω^2/r

Exercise:

1) List the methods to locate ICR
2) How to locate the ICR when body lies between two moving surfaces.

Questions/problems for practice for the day:

1) State the formula for velocity of any point a rigid body performing general plane motion.

Learning from the topic 'Types of Motion & ICR': Learners are able to locate ICR of general plane motion type systems.

Lecture 36

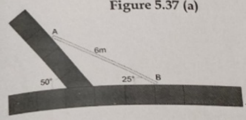
5.7 *ICR with two links system*

Learning Objectives: Learners will be able to locate ICR with two links

Solved Problems:

1) **The rod is in contact with two smooth stationary surfaces. At the instant shown in figure its end B has velocity2 m/s rightward. Find velocity of end A and angular velocity of the rod. Also find velocity of a point on the rod, which is two meters from end B, at the same instant.**

Figure 5.37 (a)



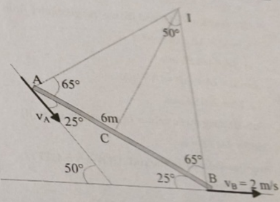


Figure 5.37(b)

Solution:

We have,

$\omega = vA / IA = vB / IB$ ------ (I)

And $vB = 2$ m/s;

So, $\omega = 2 / IB$

Applying Sine Rule for ΔIAB

$6 / \sin 50 = IA / \sin 65 = IB / \sin 65$

$IA = IB = 7.0986$ m

Put these in equation (I)

$\Omega = 2 / 7.0986 = 0.2817$ r/s

$VA = 2$ m/s

To find velocity of centre 'C' which is 2m away from point 'B'

Applying Cosine Rule for triangle IBC, We have;

$IC^2 = IB^2 + BC^2 - 2 \times IC \times BC \times \cos 65$

$IC^2 = 7.09862 + 22 - 2 \times 2 \times 7.0986 \times \cos 65$

$IC = 6.5108$ m

Now, $vC = IC \times \omega = 6.5108 \times 0.2817 = 1.8344$ m/s

2) Locate the instantaneous centre of rotation of link AB. Find also the angular velocity of link OA. Take velocity of slider at B = 2500 mm/s. The link and slider mechanism is as shown in the figure.

Solution:

Rod OA rotates about point 'O' with angular velocity ωOA. Velocity of point A is perpendicular to OA. Slider at B moves along the incline with velocity vB. Drawing lines perpendicular to vA and vB, Intersect at ICR 'I'

$vB = 2500$ mm/s $= IB \times \omega$ ---------(I)

$vA = OA \times \omega OA = IA \times \omega$

$vA = 200 \times \omega OA = IA \times \omega$ ---------(II)

To find length IA and IB, we use geometry from ΔBAI

$\cos 30 = AB / IB = 400 / IB$

i.e. IB = 461.88 mm

$\tan 30 = IA / AB = IA / 400$

i.e. IA = 230.94 mm

Substituting these values in equations (I) & (II)

$2500 = 461.88 \times \omega$

Angular Velocity of the rod AB is $\omega = 5.4127$ r/s

From eq. (II),

$200\ \omega OA = IA \times \omega = 230.94 \times 5.4127$

Angular velocity of the link OA = 6.25 r/s

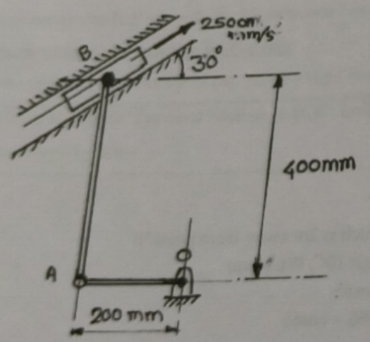


Figure 5.38(a)

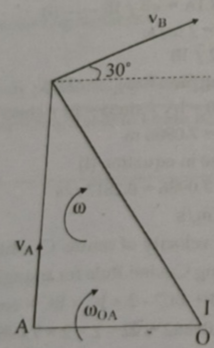


Figure 5.38(b)

3) In the engine system shown, the crank AB has a constant clockwise angular velocity of 2000 rpm. For the crank position indicated, determine the angular velocity of the connecting rod BD and the velocity of the piston P.

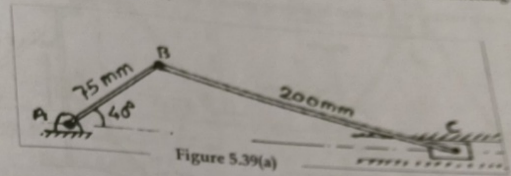


Figure 5.39(a)

Solution:

Now AB has a constant clockwise angular velocity of 200 r.p.m. Then,

$\omega AB = 2 \pi N / 60 = 2 \pi \times 2000 / 60$

$\omega AB = 209.4395$ r/s

The angular velocity of the connecting rod BC

For angle ACB by Sine Rule,

$200 / \sin 40 = 75 / \sin\theta$

$\sin\theta = 75 \times \sin 40 / 200$

$\theta = 13.94°$

Now

$vB = \omega AB \times AB$

$vB = 15.7073$ r/s

Now by sine rule in ΔIBC

$IB / \sin 76.0518 = 200 / \sin 50 = IC / \sin 53.94$

IB = 0.253 m

IC = 211.1135 mm = 0.211 m

So,

$vB = \omega BC \times IB$

$15.707 = \omega BC \times 0.253$

$\omega BC = 62.083$ r/s

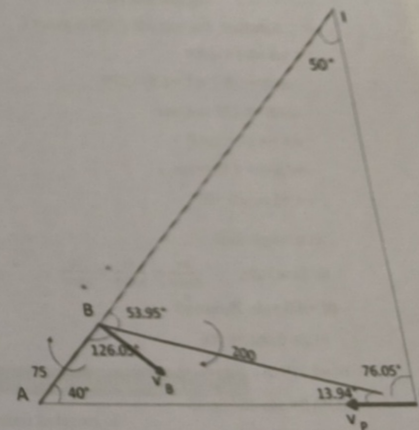


Figure 5.39(b)

4) Figure shows a collar B which moves upwards with a constant velocity of 1.5 m/s. At the instant when θ = 50°, determine (i) the angular velocity of rod AB which is pinned at B and freely resting at A against 25° sloping ground. (ii) the velocity of end A of the rod.

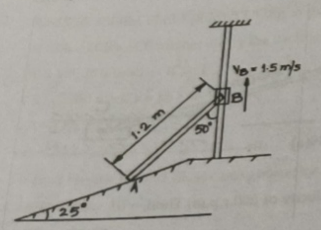


Figure 5.40(a)

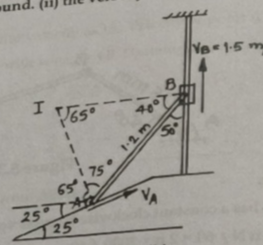


Figure 5.40(b)

Solution: For rod AB :(ICR is point I)

vB = B I × ωAB

ωAB = vB / B I = 1.5/1.279

ωAB = 1.172 rad/sec

vA = A I1 × ωAB

= 0.850 × 1.172 ≈ 1m/s

vA = 1m/s, θ = 25°

In triangle ABI

By Sine rule, $\frac{AB}{Sin65} = \frac{BI}{Sin75} = \frac{AI}{Sin40}$

BI = AB × sin 75/sin 65

= 1.2× 0.966/0.906

BI = 1.279 m

AI = AB× sin 40/sin 65

= 1.2×0.642/0.906

AI = 0.850 m

4) In a mechanism shown in figure, piston C is constrained to move in a vertical slot. A and B moves on horizontal surface. Rods CA and CB are connected with smooth hinges. If vA=0.45 m/s to the right. find velocity of C and B. Also find angular velocity of two rods.

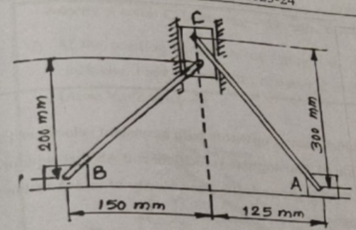


Figure 5.41(a)

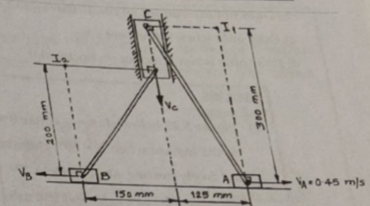


Figure 5.41(b)

Solution:

a) For rod AC: (ICR is point I1)

vA = A I1 × ωAC

ωAC = vA / AI1 = 0.45/0.3

ωAC = 1.5 rad/sec

vC = CI1 × ωAC

= 0.125 × 1.5

vC = 0.1875 m/sec (↓)

b) For rod BC: (ICR is point I2)

vC = CI2 × ωBC

ωBC = vC / CI2 = 0.1875/0.150

ωBC = 1.25 rad/sec

vB = BI2 × ωBC

= 0.2 × 1.25

vC = 0.25 m/sec (←)

Let's check the takeaway from this lecture

1) The angular velocity of rotating body is expressed in terms of

a) Revolution per minute

b) Radian per second

c) Any one of two

d) None of the two

2) The relationship between linear velocity and angular velocity of a cycle

a) Exist under all condition

b) Does not exist under all condition

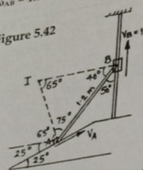
c) Exist only when it does not slip

d) Exist only when it moves on horizontal

Exercise:

1) Figure 5.42 below shows a collar B which moves upwards with a constant velocity of 1.5 m/s At the instant when $\theta = 50°$. Determine (i) The angular velocity of rod AB that is pinned at B and freely resting at A against 25° sloping ground, (ii) The velocity of end A of the rod. [Ans.: $\omega_{AB} = 1.173$ r/s, $V_A = 0.998$ m/s, 25°]

Figure 5.42



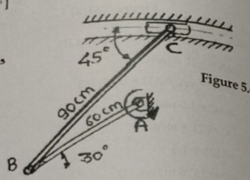


Figure 5.43

2) If the angular velocity of link AB is AB = 3 rad/s. Determine the velocity of the block C and the angular velocity of the connecting link CB at the instant shown in the above figure 5.43.[Ans.: 2.45 rad/s, 67 cm/s (←)]

Questions/problems for practice for the day:

1) The link shown in figure 5.44 is guided by two block at A and B which move in the fixed slots if the velocity of A is 2 m/s downward, determine velocity of B at the instant $\theta = 45°$. [Ans.: ω = 14.1 rad/s, VB=2 m/s (→)]

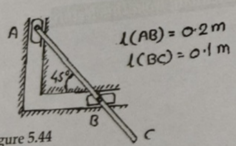


Figure 5.44

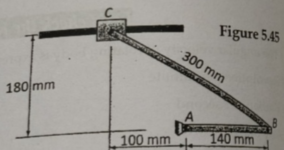


Figure 5.45

2) In figure 5.45 collar 'C' slides on a horizontal rod. In the position shown rod AB is horizontal and has angular velocity of 0.6 rad/sec clockwise. Determine angular velocity of BC and velocity of collar C. [Ans.: $\omega_{BC} = 12$ rad/s, V_C=1.2 m/s (→)]

3) At the position shown in the figure 5.46, the crank AB has angular velocity of 3 rad/sec clockwise. Find the velocity of slider C and the point D at the instant shown. AB= 100mm. [Ans.: V_C=2 m/s (→)]

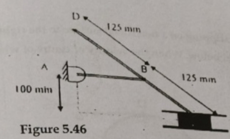


Figure 5.46

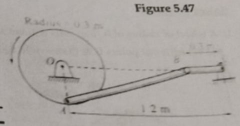


Figure 5.47

4) In the device shown in figure 5.47, find the velocity of point B and angular velocity of both the rods. The wheel is rotating at 2rad/sec anticlockwise. [Ans.: ωAB = 2 r/s ↺, VB=1.8 m/s↑ , ωBC = 6 r/s↺]

5) At the instant shown in figure 5.48, velocity of rolling wheel is 0.25 m/s towards right. For rod AB, AC = CB = 0.5 m. Rod CD is 1.3m long. Find the velocity of slider D [Ans.: V_D=1 m/s]

Soln.:

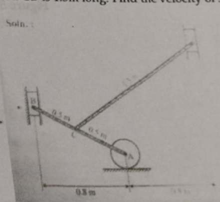


Figure 5.48

Learning from the lecture 'ICR with two links system': Learners will be able to locate ICR of systems with two links and analyze such problems

Lecture: 37

5.8 ICR of rollers

Learning Objectives: Learners will be able to locate ICR of rollers.

Solved Problems:

1) **A wheel of radius of 0.75 m rolls without slipping on a horizontal surface to the right. Determine the velocities of points C & Q shown in fig. below. When the velocity of centre of wheel is 10 m/s towards right.**

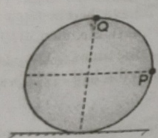


Figure 5.49(a)

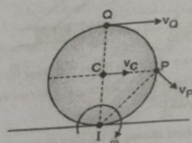


Figure 5.49(b)

Solution:

Point I is the ICR

$V_C = IC \times \omega$

$10 = 0.75 \times \omega$

$\omega = 13.33$ r/s

$VP = IP \times \omega$

$= 0.75\sqrt{2} \times 13.33$

$= 14.14$ m/s (45°)

$V_Q = IQ \times \omega$

$= 1.5 \times 13.33 = 20$ m/s

2) **There is a uniform cylinder to which a rod AB is pinned at A and the other end of the rod B is moving along a vertical wall as shown in figure. If the end B of the rod is moving upward along the wall at the speed of 3.3 m/s, find the angular velocity of the cylinder. Assume cylinder is rolling without slipping.**

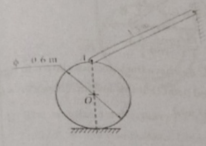


Figure 5.50(a)

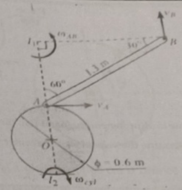


Figure 5.50(b)

(i)Rod AB (Performs GPM)

At the given instant shown I1 is the ICR

$V_B = I1B \times \omega_{AB}$

$\therefore \omega_{AB} = 3.3/(1.3\cos 30)$

$\omega_{AB} = 2.931$ r/s ↺

$v_A = I1A \times \omega AB = 1.3 \sin 30 \times 2.931$

$v_A = 1.9$ m/s

(ii)Cylinder (Performs GPM)

At the given instant shown I2 is the ICR

$v_A = I2A \times \omega cyl$

$\therefore \omega cyl = 1.9/0.6$

$\therefore \omega cyl = 3.167$ r/s ↻

Exercise:

1) In the figure 5.51, the disc rolls without slipping on the horizontal plane with an angular velocity of 10 rev/min clockwise. The bar AB is attached as shown. Line OA is horizontal. Point B moves along the horizontal plane. Determine the velocity of point B for the phase. **[Ans: $V_B = 1.099$ m/s (→)]**

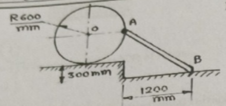


Figure 5.51

2) Due to slipping points A and Bon the rim of the disk have the velocities shown in figure 5.52. Determine the velocities of the center point C and point D at this instant. **[Ans.: V_C = 0.76 m/s, V_D = 2.87 m/s]**

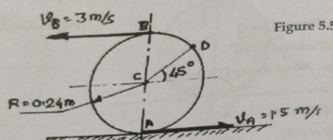


Figure 5.52

Questions/problems for practice for the day:

1) The disk of radius r is confined to roll without slipping at A and B. If the plates have the velocities shown in figure 5.53, determine the angular velocity of the disk. **[Ans.: 1.5 v/r]**

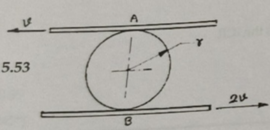


Figure 5.53

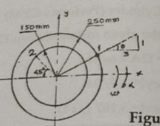


Figure 5.54

2) A circular lamina rotates in XY plane about an axis perpendicular to the XY plane and passing through a centre point 0 as shown in figure 5.54. Angular velocity of the lamina is 2 radian/sec clockwise and it has an angular acceleration 1 radian/sec². Find the velocity and acceleration of the points 1 and 2 for the position shown. **[Ans :300 mm/s,618.46 mm/sec²]**

3) At the instant shown in figure 5.55 the disk is rotating at ω = 4 rad/s. If the end of the cord wrapped around the disk is fixed at D. determine the velocities of point A, B and C.**[Ans: V_A= 0, V_B = 16 m/s (↓), V_C = 11.31 m/s (45°)]**

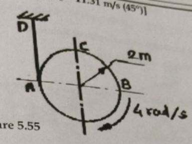


Figure 5.55

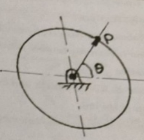


Figure 5.56

4) The angular acceleration of a wheel having a diameter of 60 cm is defined by a = (3 + 0.1θ) rad/s², where θ in radians. Determine the magnitude of the velocity and acceleration of a point p on its rim when 8 =2 rad, how much time is needed to reach the angular position? Originally, θ = 0 & ω = 0 when t = 0. **[Ans.: 1.07 m/s, 3.9m/s² and 1.15sec.]**

5) A bar BC slides at C in a collar by 4 m/s velocity as shown in figure 5.57. The other end B is pinned on a roller. Find the angular velocity of bar BC and the roller. **[Ans.: ω_{BC} = 0.386 r/s ↺, ω_{roller}= 1.891 r/s ↺]**

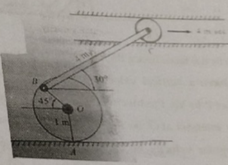


Figure 5.57

Learning from the lecture 'ICR of rollers': Learners will be able to locate ICR of systems containing rollers and analyze such problems

5.9 Conclusion:

Learning Outcome: Students shall be able to

Know, Comprehend

1. Know the difference between different types of motions.

2. Know the concept of GPM

Apply, Analyze

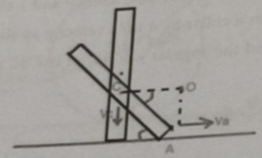
3. Elaborate the basic concepts of the Instantaneous Centre of Rotation
4. Find relationship between linear velocity and angular velocity

Synthesize

5. Locate the ICR graphically and find the find linear velocity and angular velocity of different points on the link
6. Analyze & solve systems containing two links.

5.10 Add to Knowledge (Content beyond Syllabus)

Let's take a rod that is kept on a frictionless floor a slight impulse is given such that it starts rotating while falling down. We have to find the velocity of center of mass.



Since there is no frictional force hence there is no force in the horizontal direction, so the centre of mass will only have a vertical velocity say Vc. Also lower most point will only have horizontal velocity. Let it be Va. First we will find the ICOR drawing lines perpendicular to the velocity vectors. They intersect at O. So the whole body will seem to be in pure rotation from that point. Let the angular velocity be Va. Now the center of mass has come down by 0.5 (1 − sinθ).

By energy conservation,0.5mg (1 −sinθ) = 0.5IICORω2 — —- (1)

By parallel axis theorem, IICOR = ml2/12 = ml2cos2θ/4 — —- (2)

Using (1) and (2), we can find the value of ω.

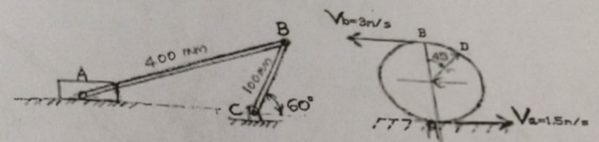
Once it is known we can find,

Vc = ω × 0.5cosθ

Research Work:https://ed.fnal.gov/arise/guides/phys03-kinematics.pdf

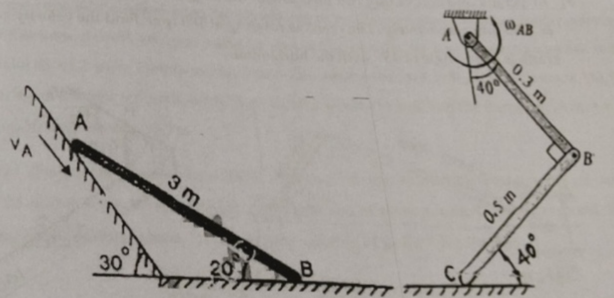
5.11 Short Answer Questions:

1) The crank BC of a slider crank mechanism is rotating at constant speed of 30rpm clockwise. Determine the velocity of the piston A at the given instant. AB= 400mm, BC=100mm.



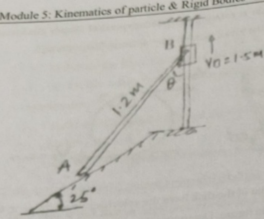
2) Due to slipping, points A and B on the rim of the disk have the velocities Va =1.5m/s to the right and V_b=3m/s to the left as shown in figure. Determine the velocities of the centre point C and point D on the rim at this instant. Take radius of disk 0.24m.

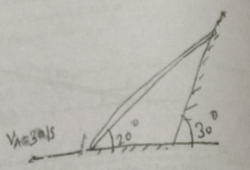
3) Rod AB of length 3m is kept on smooth planes as shown in fig. The velocity of end A is 5m/s along the inclined plane. Locate the ICR and find the velocity of end B. (Dec 2016)



4) A rod AB has an angular velocity of 2 rad/sec, counter clockwise as shown. End C of rod BC is free to move on a horizontal surface. Determine (i) Angular velocity of BC and (ii) Velocity of C. **[Ans:** $\omega_{BC} = 1 rad/s (anticlockwise)$**]**

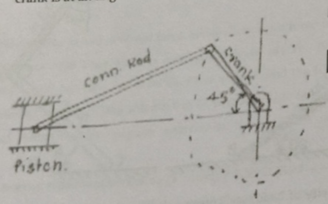
5) Fig. shows a collar B which moves upward with constant velocity of 1.5 m/s. At the instant when θ=500 determine (i) the angular velocity of rod pinned at B and freely resting at A against 25⁰ sloping ground and (ii) the velocity of end A of the rod. **[Ans: V_A = 0.996 m/s ω_{AB} = 1.17r/s]**

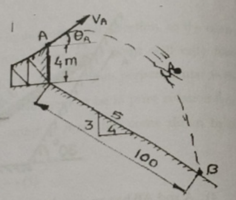




6) A bar AB 2 m long slides down the plane as shown. The end A slides on the horizontal floor with a velocity of 3 m/s. Determine the angular velocity of Rod AB and the velocity of end B for the position shown. **[Ans: V_B = 2.867 m/s ω_{AB} = 0.763 r/s↺]**

7) In a crank and connecting rod mechanism the length of crank and connecting rod are 300 mm & 1200 mm respectively. The crank is rotating at 180 rpm. Find the velocity of piston, when the crank is at an angle of 45° with the horizontal.

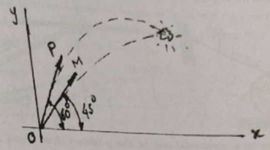




8) It is observed that a skier leaves the platform at A and then hits the ramp at B as shown in the figure in 5 seconds. Calculate the initial speed 'u' and the launch angle. **(Ans: 19.836m/s, 36.23°)**

9) A projectile P is fired at a muzzle velocity of 200 m/s at an angle of elevation of 60°. After sometime, a missile M is fired at muzzle velocity of 2000 m/s and at an angle of elevation of 45°, form the same point, to destroy the projectile P. Find:(I) Height, (ii) Horizontal distance and (iii)

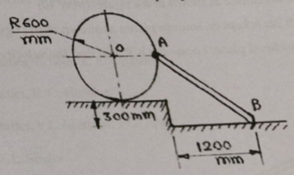
Time with respect to P firing at which the destruction takes place.**[Ans.: (i) h = 1494.4m; (ii) x =1499.9m; (iii) Time lag = 15 sec.]**



10) A balloon starts moving upwards from the ground with const. acceleration of 1.6 m/s². 4 seconds later, a stone is thrown upwards from same point:(i) What velocity should be imparted to the stone so that it just touches the ascending balloon?(ii) At what height, will the stone touch balloon?**[Ans.: (i) V_s = 23.57 m/sec. (ii) h = 24.02 m.]**

11) A ball is thrown vertically upward from the 12m levels in an elevator shaft, with an initial velocity of 18m/s. At the same instant, an open platform elevator passes the 5m level, moving upward with a constant velocity of 2 m/s. Determine (a) when and where the ball will hit the elevator, (b) the relative velocity of the ball with respect to the elevator when the ball hits the elevator. **[Ans. t = 3.6 sec, elevation from ground = 12.3 m**

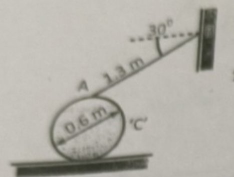
12) In the figure shown, the disc rolls without slipping on the horizontal plane with an angular velocity of 10 rev/min clockwise. The bar AB is attached as shown. Line OA is horizontal. Point B moves along the horizontal plane. Determine the velocity of point B for the phase.

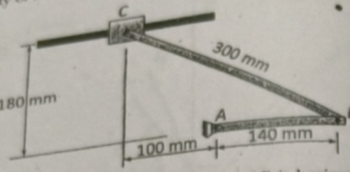


12.2 Long Answer Questions:

1) 'C' is a uniform cylinder to which a rod AB is pinned at A and the other end of the rod B is moving

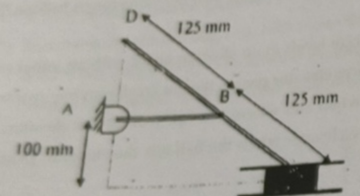
along a vertical wall as given in the fig. If the end B of the rod is moving upward along the wall at a speed of 3.3m/s find the angular velocity of the cylinder assuming that it is rolling without slipping.

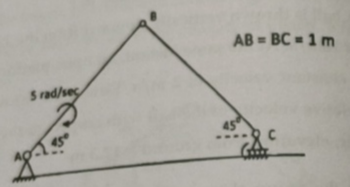




2) In figure collar 'C' slides on a horizontal rod. In the position shown rod AB is horizontal and has angular velocity of 0.6 rad/sec clockwise. Determine angular velocity of BC and velocity of collar C.

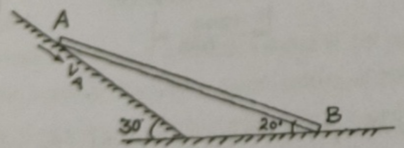
3) At the position shown in the figure, the crank AB has angular velocity of 3 rad/sec clockwise. Find the velocity of slider C and the point D at the instant shown. AB= 100mm.

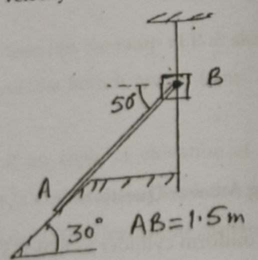




4) In the mechanism shown find the velocity of point C and angular velocity of link BC if angular velocity of link AB is 5rad/s. solve the problem when the link AB and link BC make an angle of 45 degrees with the horizontal as shown in the figure. (May'10)

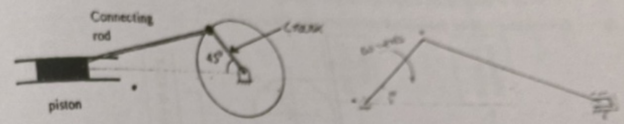
5) Rod AB of length 3m is kept on smooth planes as shown in the figure. The velocity of the end A is 5 m/sec along the inclined plane. Locate the ICR and find the velocity of the end B





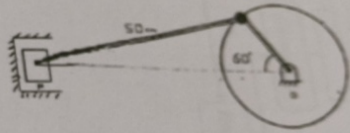
6) Collar B moves up with a constant velocity VB = 2m/s. Rod AB is pinned at B. Find out angular velocity of AB and velocity of A. [Jun 13]

7) In a crank and connecting rod mechanism the length of crank and the connecting rod are 300mm and 1200mm respectively. The crank is rotating clockwise at 180rpm. Find the velocity of piston when the crank is at an angle of 45° with the horizontal.



8) Fig. shows the crank and connecting rod mechanism. The crank AB rotates with angular velocity of 2 rad/sec in clockwise direction. Determine the angular velocity of connecting rod BC and the velocity of piston C using ICR method. AB = 0.3 m , CD = 0.8 m [Dec 14]

9) For crank of concentric mechanism shown in fig. determine the ICR of connecting rod at position shown. The crank OQ rotates clockwise at 310 rpm. Crank length =10 cm, connecting rod length = 50. Also find the velocity of P & angular velocity of rod at that instant. [May 15] [Ans: vp = 1.9142 m/s, ωpq= 5.63]



10) Define the terms with neat sketches: Direct impact, oblique impact & line of impact.

10.2 References:

1) Mechanics for Engineers – Statics, F. Beer, E. R. Johnston

2) Engineering Mechanics – Statics, R. C. Hibbler

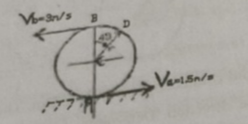
3) Engineering Mechanics – Statics, J. L. Merium, I. G. Kraig

4) Engineering Mechanics – F. L. Singer

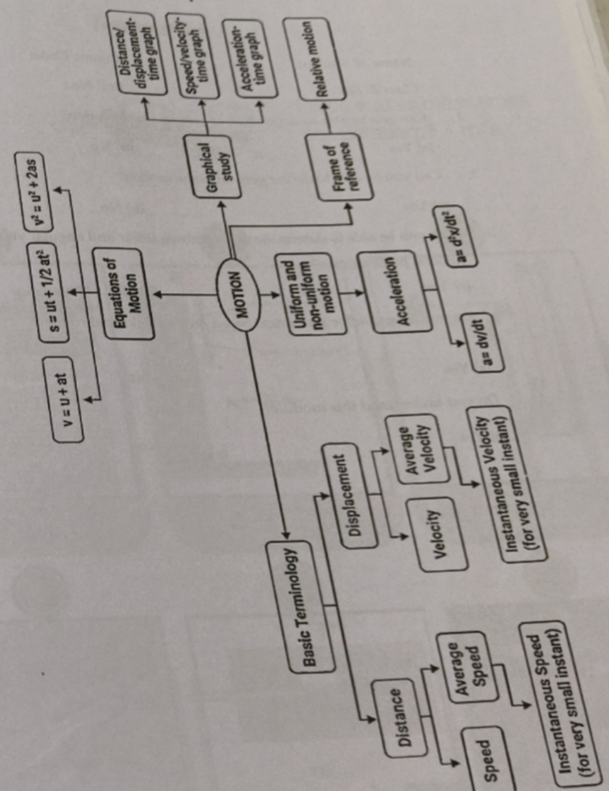
5) Engineering Mechanics – P. J. Shah, R. Bade

Self-Assessment

1) What are the different types of motion?(Level 1)
2) Define GPM?(Level 2)
3) What is the point of zero velocity called as?(Level 3)
4) A roller is rolling on a straight road. Where will the ICR lie?(Level 4)
5) Locate the ICR of the roller shown in fig.?(Level 5)



Concept Map



Self-Evaluation

Name of Student: Course Code:

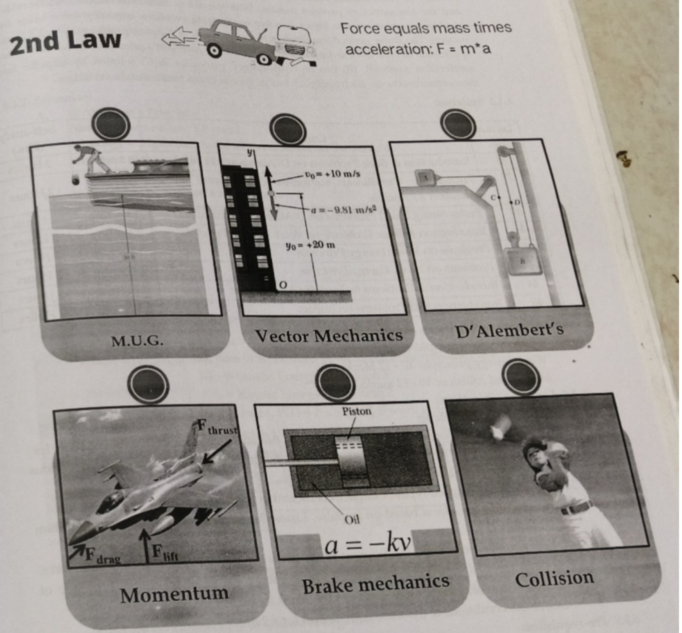
Roll No.:

Class & Div.:

1. Can you be able to define ICR of general plane motion?
   (a) Yes (b) No
2. Can you be able to define general plane motion?
   (a) Yes (b) No
3. Can you be able to determine the unknown linear and angular velocities for a given links in general plane motion?
   (a) Yes (b) No
4. Are you able to solve numerical based on two links and rollers?
   (a) Yes (b) No
5. Do you understand this module?
   (a) Yes (b) No

Module: 6 Kinetics of Particles

Infographics



Lecture: 38

6.1 Kinetics of Particles

6.1.1 Motivation:

According to Newton's second law, a particle will accelerate when it is subjected to unbalanced forces. Kinetics is the study between unbalanced forces and resulting changes in motion. This module will combine our knowledge of properties of forces, developed in statics and the kinematics of particles motion. With the aid of Newton's second law, there is a combination of these two topics to solve engineering problems involving force, mass and motion. The three general approaches to the solution of kinetics problems are: (A) Direct application of Newton's second law in the form of D'Alembert's principle (called force-mass acceleration method), (B) use of Work-Energy principles, &(C) solution by impulse and momentum methods. Each approach has its special characteristics and advantages.

6.1.2 Syllabus:

Lecture No.	Content	Duration (Hrs.)	Self-study (Hrs.)
38	Introduction & Basic Problems on D' Alembert's Principle	1 Lecture	2 Hours
39	Friction & Curvilinear motion-based Problems on D' Alembert's Principle	1 Lecture	2 Hours
40	Pulley String based Problems on D'Alembert's Principle	1 Lecture	2 Hours
41	Introduction & Basic Problems on Work Energy Principle	1 Lecture	2 Hours
42	Problems on Work Energy Principle	1 Lecture	2 Hours
43	Problems on Work Energy Principle	1 Lecture	2 Hours
44	Introduction & Problems on Impulse-Momentum	1 Lecture	2 Hours
45	Introduction and Problems on Impact	1 Lecture	2 Hours

6.1.3 Weightage: 30 Marks (Approximately)

D'Alembert's principle: 8 - 12 Marks

Work Energy principle: 10 - 12 Marks

Impact and collision: 10 - 12 marks

6.1.4 Learning Objectives:

Learners shall be able to:

1. State the concept of Kinetics of Particles and its difference from Kinematics of Particles
2. Use Newton's second law of motion and hence D'Alembert's Principle in various problems based on kinetics of particle
3. Understand Impact, types of impact, coefficient of restitution
4. Classify the problems based on Impulse, Linear Momentum and Impulse-Momentum Theorem
5. Differentiate work done by different types of forces like external force, gravity force, friction force, spring force etc., under Work-Energy Principle by using Principle of Conservation of Energy
6. Derive the conditions of dynamic equilibrium for solving problems on the same

6.1.5 Pre-requisites:

Newton's Second Law: The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and

inversely proportional to the mass of the object.

D'Alembert's Principle: The reaction due to the inertia of an accelerated body is equal and opposite to the force causing the acceleration and results in a condition of dynamic equilibrium

Work-Energy Principle: The principle of work–energy (also known as the work and kinetic energy principle) states that the work done by all forces acting on a particle (the work of the resultant force) equals the change in the kinetic energy of the particle.

6.1.6 Abbreviations& Notations:

W.E.: Work-Energy

K.E.: Kinetic Energy

ma = Inertia force

U = Work done

e = Coefficient of restitution

6.1.7 Formulae:

A. D'Alembert's Principle:

i. **Newton's second law: $\sum F = m\,a$**

where $\sum F$ = Algebraic Sum of all forces acting on the particle

m = mass of the particle in kg

a = acceleration in m/s²

ii. **Application of Newton's second law:**

For Rectangular Co-ordinate system:

$\sum F_X = m\,a_X$

$\sum F_Y = m\,a_Y$

$\sum F_Z = m\,a_Z$

For Curvilinear Motion:

$\sum F_t = m\,a_t = m\dfrac{dv}{dt}$

$\sum F_n = m\,a_n = m\dfrac{v^2}{\rho}$

B. Work-Energy Principle:

i. **Kinetic energy, $KE = \frac{1}{2}\,m\,v^2$**

ii. **Work-Energy Principle, $U_{1\text{-}2} = KE_2 - KE_1$**

Where $U_{1\text{-}2}$ = Algebraic sum of work done by all forces from position 1 to 2

KE_1 = Kinetic Energy at position 1

KE_2 = Kinetic Energy at position 2

iii. **Work done by External force, WD = F x S (+ve) if work is done in the direction of force**

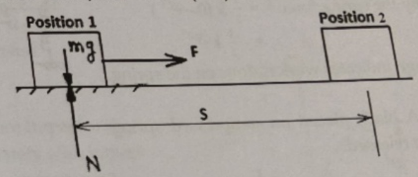


Fig. 6.1 Work done by External force.

iv. **Work done by Frictional force, WD = $-F_r$ x S** Where F_r = Frictional force = $\mu_K N$

WD by frictional force is always negative.

Always use coefficient of kinetic friction.

If a block is moving on horizontal plane,

$N = m\,g$; and
If a block is moving on an inclined plane, $N = mg\cos\theta$

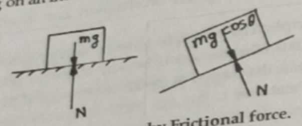


Fig. 6.2 Work done by Frictional force.

v. **Work done by Gravitational Force, WD = m g h**
Work done is (+ve) if particle is moving in the direction of gravity (↓).
Work done is (-ve) if particle is moving in opposite direction of gravity (↑).
Gravity force does no work if body moves horizontally.
$WD\ (A \rightarrow A_2) = -\,m\,g\,h_2$
$WD\ (A \rightarrow A_1) = m\,g\,h_1$
WD by Gravity $= -\,m\,g\,h\ (1 \rightarrow 2)$
$= -\,m\,g\,(s.\sin\theta)$

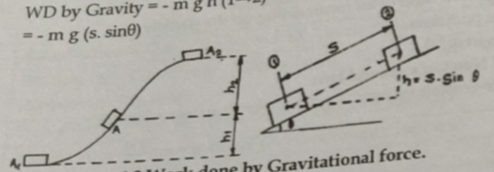


Fig. 6.3 Work done by Gravitational force.

vi. **Work done by Spring force: WD =** $\frac{1}{2}k\,(X_1^2 - X_2^2)$

a) ***When spring is not connected with the particle during motion***
Where k = Spring constant (N/m) = Stiffness of the spring
X_1 = Deflection in spring in position 1 of the particle
X_2 = Deflection in spring in position 2 of the particle

E.g. 1:A block when dropped on to an Unstretched spring:
WD by the spring force $= = \frac{1}{2}k\,(0 - \delta^2)$
$= -\frac{1}{2}k\delta^2$

(-ve) sign indicates work is done on the spring.

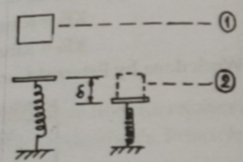


Fig. 6.4

E.g. 2: A block placed on compressed spring and then released.

WD by the spring force $= = \frac{1}{2}\ k\,(\delta^2 - 0)$
$= \frac{1}{2}\ k\ \delta^2$

(+ve) sign indicates work is done by the spring.

$WD = \frac{1}{2}k\,[(L_1 - L_0)^2 - (L_2 - L_0)^2]$

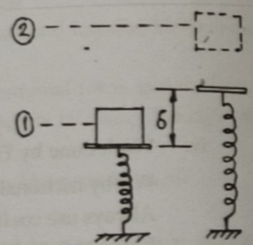


Fig. 6.5 Work done by Spring force (Free)

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b) ***When spring is connected to the particle during motion***
Where k = Spring constant (N/m)
= Stiffness of the spring
L_0 = Unstretched or unreformed length of spring
L_1 = Length of spring in position 1 of particle
L_2 = Length of spring in position 2 of particle
Deflection in spring in position 1, $X_1 = L_1 - L_0$
Deflection in spring in position 2, $X_2 = L_2 - L_0$

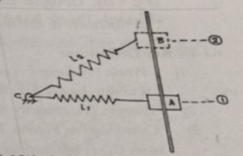

Fig. 6.6 Work done by Spring force (Connected)

C. **Impulse-Momentum theorem and Impact:**
i. **Linear Momentum = mass x velocity = m V**
ii. **Linear Impulse, Impulse$_{1\text{-}2}$ =** $\int_{t1}^{t2} F\,dt$

where F is a large force acting over a short interval of time t1 to t2

iii. **Impulse Momentum Theorem:** $\int_{t1}^{t2} F\,dt = m \int_{v1}^{v2} dv$

∴ Impulse $= m\,(V_2 - V_1) = m\,V_2 - m\,V_1$ = Final momentum – initial momentum
Conservation of momentum:
Final momentum = Initial momentum

$$\sum mV_{\,2} = \sum mV_{\,1}$$

For two particles if velocities before impact are u_1 and u_2 and velocities after impact are $v_1$and v_2, then the total momentum is conserved as no external impulsive forces are acting.
$\therefore\ m_1v_1 + m_2\,v_2 = m_1u_1 + m_2u_2$ (→ +ve and ← -ve)

iv. **Coefficient of Restitution, (e):**

$$e = \frac{\text{Impulse during restoration period}}{\text{Impulse during deformation period}}$$

$$e = \frac{\text{Relative Velocity of Separation}}{\text{Relative Velocity of Approach}}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

where,
$v_1$and $v_2$are the velocities of particles 1 and 2 immediately after impact
$u_1$and $u_2$are the velocities of particles 1 and 2 just before impact

Impact with infinite mass:
$e = \sqrt{\frac{h_2}{h_1}}$
h_1 = Height just before Impact and
h_2 = Height immediately after impact
For perfectly elastic impact, e = 1
For Semi Elastic Impact, 0 < e < 1
For plastic Impact, e = 0
(Eg: Quartz Fiber & Phosphor Bronze - Perfectly Elastic Body)
(Eg: Rubber Ball &Plastic bottles - Semi - Elastic Body)

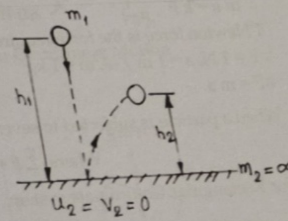

Fig. 6.7 Coefficient of Restitution

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(Eg: Clay, Dough& Cow Dung (Gobar)
–Perfectly Plastic Body)

6.1.8 Key Definition:

1) **Work:** If a force F is acting on a body and the body displaces by a distance S, then work is said to be done by the force on the body.
2) **Energy:** The capacity to do the work is Energy.
3) **Conservative Forces:** The force acting on the particle is said to be conservative if its work $U_{1\text{-}2}$ is independent of the path followed by the particle as it moves from position 1 to 2.
4) **Power:** The rate of change of doing work is power.
5) **Linear Momentum:** Total quantity of motion possessed by a particle is linear momentum.
6) **Impulsive Force:** Large force acting on a particle during a short interval of time is known as impulsive force.
7) **Impact:** It is the collision between two bodies which continue for a short interval of time and during which both bodies exert a large amount of force on each other.
8) **Coefficient of Restitution:** It is the ratio of relative velocity of separation to the relative velocity of approach (in opposite direction).

6.1.9 Theory:

➢ **Newton's 2nd Law:** The rate of change of linear momentum is directly proportional to impressed force and takes place in the direction of the force.

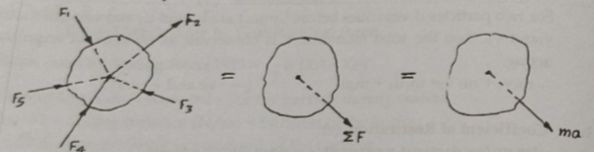


Fig. 5.8 Newton's Second Law

$\frac{d}{dt}(m \times V) \propto F$

$\therefore m \frac{dv}{dt} = k\,F$……k = Constant of proportionality

$\therefore m\,a = k\,F$

1 Newton force is the force required to produce unit acceleration on unit mass.

$\therefore F = 1\,N, a = 1\,m/s^2, m = 1\,kg \qquad \therefore k = 1$

$\therefore F = m\,a$

When a particle is subjected to several forces, $\sum F = m\,a$ ------------------- (I)

Where $\sum F$ = Algebraic sum of all forces acting on the particle

For Rectangular Co-ordinate system: $\sum F_X = m\,a_X \quad \sum F_Y = m\,a_Y \quad \sum F_Z = m\,a_Z$

For Curvilinear Motion: $\sum F_t = m\,a_t = m\left(\frac{dv}{dt}\right) \qquad \sum F_n = m\,a_n = m\left(\frac{v^2}{\rho}\right)$

➢ ***D'Alembert's Principle:***

Referring to equation (I), we have $\sum F = m\,a$

Transposing the RHS of the equation (I), we have $\sum F - m\,a = 0$

The above equation is a dynamic equilibrium equation put forth by D'Alembert's.

The ma vector is treated as an inertia force and when added with a negative sign to all other forces, results in equilibrium state of particle.

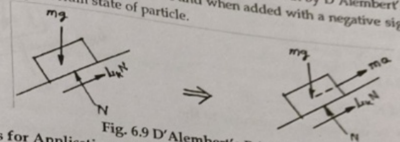


Fig. 6.9 D'Alembert's Principle

➢ **Working rules for Applications of D'Alembert's principle: For Rectilinear Motion**

1. Draw free body diagram of each particle separately showing all the forces acting on it. Also locate x and y axis.
2. Decide or assume direction of acceleration.
3. Apply D'Alembert's force in opposite direction of acceleration, to create dynamic equilibrium.
4. Apply conditions of equilibrium, $\sum F_X = 0$ and $\sum F_Y = 0$.

➢ **Working rules for Applications of D'Alembert's principle (For Curvilinear Motion):**

1. Draw free body diagram of each particle separately showing all the forces acting on it. Also locate n and t axis.
2. Decide directions of a_T and a_N.
3. Apply D'Alembert's forces (m a_T) and (m a_N) in opposite directions of accelerations a_T and a_N respectively to create dynamic equilibrium.
4. Apply conditions of equilibrium in normal and tangential direction $\sum F_n = 0$ and $\sum F_t = 0$.

If particle starts from rest, $a_T = 0$ at $t = 0$.

If the particle moves with constant speed $\frac{dv}{dt} = 0$, hence $a_T = 0$.

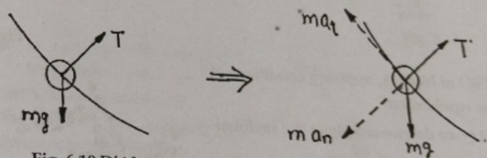


Fig. 6.10 D'Alembert's Principle for Curvilinear Motion

6.1.10 Solved Problems

1. **A 500 N crate kept on the top of a 15°-sloping surface is pushed down the plane with an initial velocity of 20 m/s. If $\mu_S = 0.5$ and $\mu_K = 0.4$, determine the distance travelled by the block and the time it will take as it comes to rest.**

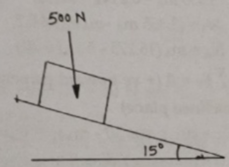


Fig. 6.11

Sol **By D'Alembert's principle, for dynamic equilibrium,**

$\sum F_X = 0$ **(+ ve downward along the inclined plane)**

$500 \sin 15 - \mu_K N - ma = 0$

$129.41 - (0.4\ N) - (\frac{500}{9.81})\ a = 0$

$50.96\ a + 0.4\ N = 129.41$ ---------------(I)

$\sum F_Y = 0$ **(+ ve upward perpendicular to the inclined plane)**

$N - 500 \cos 15 = 0$

$N = 482.96\ N$ ---------------(II)

Substituting value of N in equation (I)

$a = -1.25\ m/s^2$ (-ve sign indicates block travels down the slope)

This is a case of rectilinear motion- Uniform acceleration,

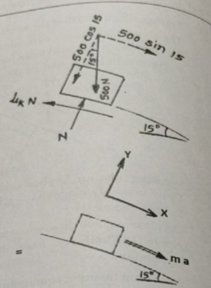
$U = 20\ m/s,\ V = 0,\ s = ?\ a = -1.25\ m/s^2,\ t = ?$

Using Newton's laws of motion,

$V^2 = u^2 + 2\ a\ s$

$0 = (20)^2 + (2 \times -1.25 \times s)$

$S = 160\ m$



Using $V = u + a\ t$

$0 = 20 + (-1.25\ t)$

$t = 16$ sec.

2. **Two blocks A and B are held on an inclined plane 5 m apart as shown in figure. For A, $\mu_A = 0.2$ and for B, $\mu_B = 0.1$. If the blocks begin to slide down the plane simultaneously calculate the time and distance travelled by each block before collision...**

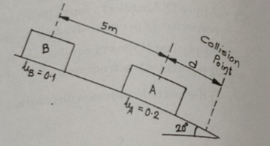


Fig.6.12

Sol From FBD of block A, applying conditions of dynamic equilibrium

$\sum F_X = 0$ **(+ ve downward along the inclined plane)**

$m_A\ g \sin 20 - \mu_K N_A - m_a a_{ax} = 0$

$(m_A \times 9.81 \times 0.342) - (0.2\ N_A) = m_A a_{ax}$

$3.355\ m_A - 0.2\ N_A = m_A\ a_{ax}$

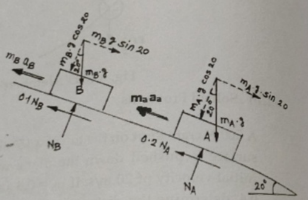
$N_A = (3.355\ m_A - m_A\ a_{ax}) / 0.2$

$N_A = m_A (16.775 - 5\ a_{ax})$ ----(I)

$\sum F_Y = 0$ **(+ ve upward perpendicular to the inclined plane)**

$N_A - m_A\ g \cos 20 - m_A a_Y = 0$ (but $a_{YA} = 0$ as acceleration in Y direction is zero)

$N_A = m_A \times 9.81 \times 0.939$



$N_A = 9.218\ m_A$ ---------------(II)

Substituting value of N_A in eq (I)

$9.211\ m_A = m_A (16.775 - 5\ a_{ax})$

$a_{ax} = 1.512\ m/s^2$

Similarly, for block B,

$\sum F_X = 0$ **(+ ve downward along the inclined plane) gives**

$N_B = (3.355\ m_B - m_B a_{BX}) / 0.1$

$N_B = m_B (33.55 - 10\ a_{BX})$ -------(III)

$\sum F_Y = 0$ **(+ ve upward perpendicular to the inclined plane) gives**

$N_B = 9.218\ m_B$---------------(IV)

be (d + 5) meter in time 't'.

Using kinematic equations,

Substituting value of N_B in equation (IV)

$a_{BX} = 2.43\ m/s^2$

Here $a_B > a_A$ so blocks will collide.

Let 'd' be the distance travelled by block A just before collision in time 't'.

So, distance travelled by block B will

For block A:

$S_A = u_A t + ½\ a_A t^2$

$d = 0 + (½ \times 1.512 \times t^2)$

so, $d = 0.755\ t^2$ ---------------(V)

For block B:

$S_B = u_B t + ½\ a_B t^2$

$(d + 5) = 0 + (½ \times 2.43 \times t^2)$ --------------- (VI)

Substituting value of 'd' from equation (V) to equation (VI), we get

$t = 3.3$ sec.and $d = 8.22$ m

So, distance travelled by block A before collision is 8.22 m and of block B is (8.22 + 5) = 13.22 m.

3. **Two wires AC and BC are tied at C to a sphere of mass 5 kg which revolves at a constant speed 'V' in the horizontal circle of radius 1.5 m as shown in the figure. Determine maximum and minimum value of V, if both wires are to remain tight and tension in either of the wires is not to exceed 70 N.**

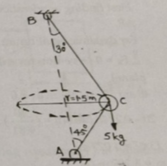


Fig. 6.13

Sol As 5 kg mass is rotating with constant velocity, its tangential and normal acceleration is zero. So $a_t = 0$ and $a_N = 0$.

Referring to the FBD of the mass, apply conditions of dynamic equilibrium

$\sum F_N = 0$ **(→+ ve)**

$T_{BC} \cos 60 + T_{AC} \cos 45 - 5\ a_N = 0$

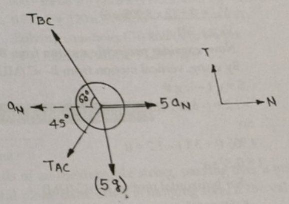
But $a_N = \frac{v^2}{\rho}$

$\therefore 0.5\ T_{BC} + 0.707\ T_{AC} - 5\,(\frac{v^2}{1.5}) = 0$

$0.5\ T_{BC} + 0.707\ T_{AC} = 3.333\ V^2$ -----(I)

$\sum F_T = 0$ **(↑+ ve)**

$T_{BC} \sin 60 - T_{AC} \sin 45 - (5 \times 9.81) = 0$



Substituting $T_{BC} = 70$ N and $T_{AC} = 16.36$ N in equation (I), we get

$(0.5 \times 70) + (0.707 \times 16.36) = 3.333\ V^2$

$\therefore$ Velocity, $V = 3.738\ m/\sum$

$0.866\ T_{BC} - 0.707\ T_{AC} = 49.05$ ——(II)

Let $T_{AC} = 70$ N. Substituting this value in equation (II)

$\therefore T_{BC} = 113.78\ N > 70\ N$,

hence this is not valid.

Let $T_{BC} = 70$ N. Substituting this value in equation (II)

$\therefore T_{AC} = 16.36\ N < 70\ N$,

this is a valid value.

This is a maximum velocity.

For Minimum velocity

Let $T_{AC} = 0$, substituting in equation (II)

$\therefore 0.866\ T_{BC} = 49.05$

$T_{BC} = 56.64\ N < 70\ N$,

this is a valid value.

Putting $T_{BC} = 56.64$ N and $T_{AC} = 0$ in equation (I) to get minimum velocity

$(0.5 \times 56.64) + (0.707 \times 0) = 3.333\ V^2$

$\therefore$ Minimum Velocity, $V = 2.915\ m/s$

4. **A suitcase of weight of weight 40 N slides from rest 6 m down a ramp. If $\mu_K = 0.2$, determine the point where it strikes the ground at C. How much time does it take to move from A to C?**

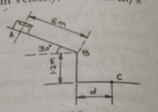
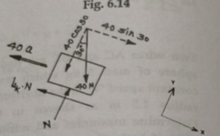


Fig. 6.14



Sol Consider motion from A to B (Linear motion)

$u = 0, s = 6\ m, V_B = ?$

First finding acceleration of suitcase on plane AB.

For dynamic equilibrium,

$\sum F_X = 0$ **(+ ve downward along the inclined plane)**

$40 \sin 30 - \mu_K N - m\ a_X = 0$

$20 - 0.2\ (40 \cos 30) = (\frac{40}{9.81})\ a_X$

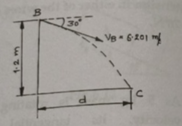
$13.07 = 4.077\ a_X$

$a_X = 3.205\ m/s^2$

Velocity at B, using $V^2 = u^2 + 2\ a\ s$

$V_B^2 = 0 + (2 \times 3.205 \times 6)$

$V_B = 6.201\ m/s$



Now, consider projectile motion from B→C.

By using, vertical motion from B→C (MUG)

$S = u\ t - ½\ g\ t^2$

$- 1.2 = (- 6.201 \sin 30) - (½ \times 9.81 \times t^2)$

So,

$4.905\ t^2 + 3.1\ t - 1.2 = 0$

$t = 0.27$ sec

For horizontal motion B→C (UM)

$s = u \cos \theta . t$

$d = 6.201 \cos (+30) \times t$

$= 6.201 \cos (+30) \times 0.27 = 1.45$ m

Time taken from A → B is,

$V_B = u + a\ t$

$6.201 = 0 + 3.205\ t$

$t_{AB} = 1.934$ sec

so, total time,

$t = t_{AB} + t_{BC} = 1.934 + 0.27 = 2.204$ sec

5. **A horizontal force P = 600 N is exerted on block A of mass 120 kg as shown in the figure. The coefficient of friction between block A and horizontal plane is 0.25. Block B has a mass of 30 kg and the coefficient of friction between it and the plane is 0.4. The wire between the two blocks makes an angle 30^0 with the horizontal. Calculate the tension in the wire and acceleration of blocks.**

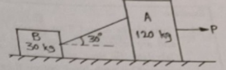
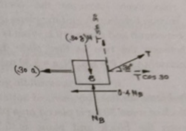


Fig. 6.15

Sol Draw FBD of block A and block B in dynamic equilibrium and applying conditions of equilibrium,



From FBD of block B:

$\sum F_X = 0$ **(→+ ve)**

$T \cos 30 - 0.4\ N_B - 30\ a = 0$

$0.866\ T - 0.4\ N_B = 30\ a$ ——————(I)

$\sum F_Y = 0$ **(↑+ ve)**

$N_B + T \sin 30 - (30\ g) = 0$ (No acceleration in Y direction)

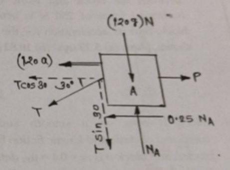
$N_B = (30 \times 9.81) - 0.5\ T$

$N_B = 294.3 - 0.5\ T$ —————— (II)

Substituting equation (II) in equation (I), we get

$0.866\ T - [0.4\ (294.3 - 0.5\ T)] = 30\ a$

$1.066\ T - 30\ a = 117.72$ ————(III)



From FBD of block A:

$\sum F_X = 0$ **(→+ ve)**

$P - T \cos 30 - 0.25\ N_A - 120\ a = 0$

$600 - 0.866\ T - 0.25\ N_A = 120a$ ----(IV)

$\sum F_Y = 0$ **(↑+ ve)**

$N_A - 120\ g - T \sin 30 = 0$ (No acceleration in Y direction)

$N_A = 1172.2 - 0.5\ T$ ——————(V)

Substituting equation (V) in equation (IV), we get

$600 - 0.866\ T - [0.25 \times (1172.2 - 0.5\ T)] = 120\ a$

$0.741\ T + 120\ a = 306.95$ ——————(VI)

Solving equations (III) and (VI), we get

$T = 155.16$ N and $a = 1.589\ m/s^2$

Let's check the takeaway from this lecture

1. **D' Alembert's principle is expressed by……………**

   a) F = ma b) F - ma = 0 c) F x ma = 0

2. **If two masses are connected to the two ends of an inextensible string, passing over a pulley. One the mass is lying on a rough horizontal plane &the other is hanging free. If the value of coefficient of friction is increased, it will increase its…………………**

   a) Acceleration b) Tension c) Both

3. **If a body slides down an incline, the acceleration of the body is given by……….**

   a) g b) g c) Both

Exercise

Q1. Two blocks A and B are placed 15 m apart and are released simultaneously from rest. Calculate the time taken and the distance travelled by each block before they collide. **[Ans: 4.74 sec, 16.64 m, 31.64 m)**

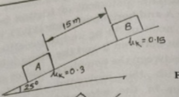


Fig.6.16

Q2. Masses A and B are 7.5 kg and 27.5 kg respectively. Assume the coefficient of friction between A and the plane is 0.25 and between B and plane is 0.1. What is the force between the two as they slide down the incline? **(Ans: P = 6.62 N, a = 5.39 m/s²)**

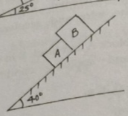


Fig.6.17

Practice Problems for the Day

Q3. A block of mass 30 kg is placed on a plane, μ_K between the block and plane is 0.3. if a horizontal force of 250 N is acting on the block, find its acceleration for the two cases shown. **[Ans: (a) 5.39 m/s² (b) 10.82 m/s²]**

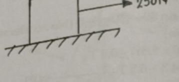


Fig. 6.18

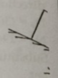


Fig. 6.19

Q4. Block B rests on a smooth surface. If coefficient of static and kinetic fiction between block A and block B is $\mu_S = 0.4 = \mu_K$, determine the acceleration of each block, if (a) P = 30 N, (b) P = 250 N. **[Ans: (a) $a_A = a_B = 0.84$ m/s² (b) $a_A = 20.6$ m/s², $a_B = 1.57$ m/s²]**

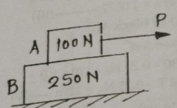


Fig.6.20

Q5. The 10-kg block is pulled upwards from rest position by a constant cable tension. Determine the velocity of the block when it reaches the elevation of the pulley. Take coefficient of kinetic friction between block and bar $\mu_K = 0.16$. **(Ans: v = 3.47 m/s)**

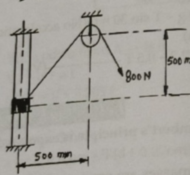


Fig. 6.21

Learning from this Lecture:

1. Learners will be able to understand the cause of motion and their effects.
2. Learners will be able to understand the basic difference between Newton's second law of motion and D'Alembert's Principle

Lecture 39

6.2 *Connected bodies-based Problems on D'Alembert's Principle*

Learning Objective: In this Lecture, Learners will be able to understand the concept of D'Alembert's Principle and apply it in the connected body problems

Theory:

➢ **Working rules for Applications of D'Alembert's principle: For Connected Bodies (When two or more particles are connected by strings)**

1. Find relation of accelerations among the particles by using kinematic constraints (by direct string law or by concept of dependent motion)
2. Assume directions of accelerations if frictional force is absent.
3. Decide carefully the direction of accelerations if frictional forces are involved.
4. Draw free body diagram of each particle showing all forces acting on it.
5. Apply D'Alembert's force in opposite of acceleration to create dynamic equilibrium.
6. Apply conditions of equilibrium $\sum F_X = 0$ and $\sum F_Y = 0$.

Solved Problems

1. **Determine the weight W required to be attached to 150 N block to bring the system in the figure shown to stop in 5 seconds if at any stage 500 N is moving down at 3 m/s. Assume pulley to be massless and frictionless.**

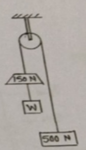


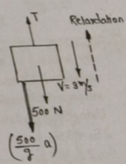
Fig. 6.22

Sol When weight W is added, 500 N undergoes retardation 'a' and comes to stop after 5 seconds. Using kinematic equation for motion of 500 N block

$V = u + a\,t$

$0 = 3 + (a \times 5)$

$a = -0.6\ \text{m/s}^2$ or $a = 0.6\ \text{m/s}^2$ (retardation)



Applying conditions of equilibrium to 500 N block weights (150 + W)

$\sum F_Y = 0$ (↑+ ve)

$$500 - T - \left(\frac{500}{g}\right)a = 0$$

$$T = 500 - \left(\frac{500}{g}\right)(-0.6)$$

$T = 530.58$ N

Applying conditions of equilibrium to

weights (150 + W)

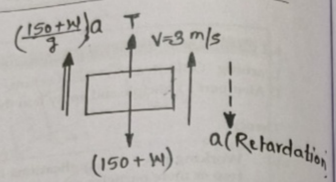
$\sum F_y = 0$ (↑+ ve)

$T - (150 + W) - (\frac{150 + W}{g})\,a = 0$

$530.58 - 150 - W = (\frac{150 + W}{g})(-0.6)$

$530.58 - 150 + (\frac{150 + W}{9.81})(0.6) - 150 - W = 0$

So, W = 415.15 N



2. **A system shown in figure is initially at rest. Neglecting friction, determine velocity of block A after it has moved 2.7 m when pulled by a force of 90 N.**

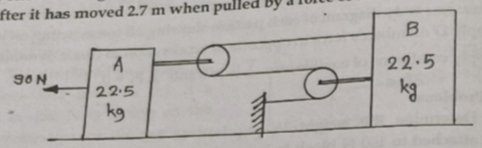
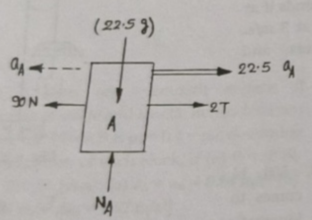
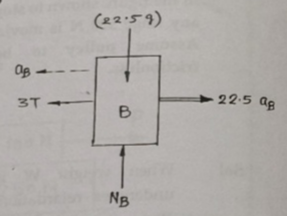


Fig. 6.23

Sol





By looking at the two blocks, block A is subjected to tension 2T and block B is subjected to tension 3T.

Using String's law, $N_A S_A = N_B S_B$

$2 S_A = 3 S_B$

So, $2 a_A = 3 a_B$ --------------(I)

For dynamic equilibrium of block, A and B,

For Block A:

$\sum F_x = 0$ (→+ ve)

$-90 + 2T + 22.5 a_A = 0$

$2T + 22.5 a_A = 90$ ----------------(II)

For Block B:

$\sum F_x = m a_x$ (→+ ve)

$-3T + 22.5 a_B = 0$

from equation (I),

$a_B = \frac{2}{3} a_A$

$-3T + (22.5 \times \frac{2}{3} a_A) = 0$

$3T - 15 a_A = 0$ ----------------(III)

Solving equations (II) and (III) we get

Tension, T = 13.846 N

Acceleration, $a_A = 2.769\ m/s^2$ (←)

Acceleration, $a_B = 2.769\ m/s^2$(←)

Velocity of block A after 2.7 m displacement:

using kinematic equation of motion,

$V^2 = u^2 + 2as$

$V_A^2 = 0 + (2 \times 2.769 \times 2.7)$

$V_A = 3.867$ m/s

Let's check the takeaway from this lecture

1. **In a D'Alembert's principle problems on imaginary force is applied to the body opposite to the direction of acceleration..........**
 a) Frictional force (F_k) b) Inertia (ma) c) Reactive force (NR)
2. **The work done is valid to be zero when a ....**
 a) Some force acts on a body but displacement is zero.
 b) No force acts on the body, but displacement takes place
 c) Either a or b

Exercise

Q1. The coefficient of kinetic friction between 2 kg block A and incline is $\mu_K = 0.15$. Block B has a mass of 5 kg. The system is released from rest. Determine the velocity of block A after it has moved 1 m. **(Ans: V_A = 2.53 m/s)**

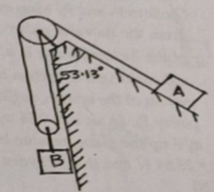


Fig. 6.24

Q2. Determine the acceleration of a 100 N weight shown below after the motion has begun. Also calculate the tensions in the string. Assume the pulleys frictionless. **(Ans: a = 1.38 m/s², T_1 = 4.81 N, T_2 = 42.99 N)**

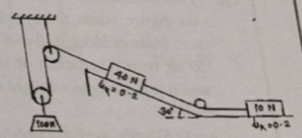


Fig. 6.25

Q3. Masses A (5 kg), B (10 kg) and C (20 kg) are connected as shown in the figure by inextensible chord passing over massless and frictionless pulleys. The coefficient of friction for mass A and B and ground is 0.20. if the system is released from rest, find the acceleration a_A, a_B and a_C, also tension T in the chord. **(Ans: a_A = 7.46 m/s² (→), a_B = 2.75 m/s² (←) and a_C = 5.10 m/s² (↓), T = 47.09 N)**

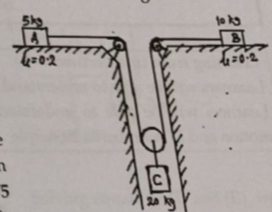


Fig. 6.26

Practice Problems for the Day

Q1. An airplane has a mass of 25000kg and its engine develop a total thrust of 40kN along the runway. The force of air resistance to motion of air plane is given by $D = 2.25v^2$, where v is in m/s and D is newton. Determine the length of runway required if the plane takes off and becomes airborne at a speed of 240km/hr. **(Ans: 1.6km)**

Q2. A vertical lift of weight 10kN moving from rest with constant acceleration acquires an upward velocity of 4m/s over 5m. Determine the tension in the cables supporting the lift (Ans: 11.6 kN)

Q3. The bob of 2 m pendulum describes an arc of circle in a vertical plane. If the tension in the chord is 2.5 times the weight of the bob for the position shown, find the velocity and acceleration of the bob in that position. **(Ans: V = 5.66 m/s, a_T = 4.9 m/s², a_N= 16.02 m/s²)**

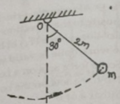


Fig.6.27

Q4. Figure shows two masses m_1 = 1 kg and m_2 = 2 kg connected by rope passing over two smooth pulleys P_1 and P_2. Mass m_3 = 5 kg is supported from the movable pulley P_2. If the inclination of the inclined plane is α, where tan $\alpha = 3/4$ and coefficient of friction is 0.1, determine the motion of the system, neglecting the weight of pulley P_2. **(Ans: a_3 = 0.54 m/s² ↓, a_1 = a_2 = 1.08 m/s² up the plane, tension between m_2 and m_3 = 23.64 N and that between m_1 and m_2 = 7.88 N)**

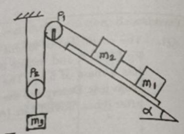


Fig. 6.28

Q5. Find acceleration of block A, B and C shown in the figure when the system is released from rest. Mass of block A, B and C is 5 kg, 10 kg and 50 kg resp. Coefficient of friction for block A and B is 0.3. Neglect weight of pulley and rope friction. (Ans:
a_A=12.518m/s²,a_B=0.9175m/s²,a_C=6.718m/s²)

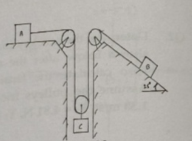


Fig. 6.29

Learning from this Lecture:
Learners will be able to understand the cause of motion and their effects.
Learners will be able to understand the basic difference between Newton's second law of motion and D'Alembert's Principle

Lecture 40

6.3 *String-Pulley based Problems on D'Alembert's Principle*

Learning Objective: In this Lecture, Learners will be able to apply D'Alembert's Principle on problems based on bodies connected by string

Theory

- **String Law:** According to string law, when two particles are connected by a continuous, inextensible string passing over smooth pulleys then the relation between the two strings can

be given as:
$N_{s1} = N_{s2}$, $N_{v1} = N_{v2}$, $N_{a1} = N_{a2}$.
Here, N1 and N2 are string parts which are connected to particles two particles respectively.
- This relation determines different quantities in magnitude.
- Therefore. It has limitations such as:
This law is valid only if single string connects two particles.
This law does not give results regarding direction of motion.

Solved Problems

1. **Determine the acceleration of block B. Also determine its displacement after 1 sec. if system starts from rest. Assume frictionless pulleys.**

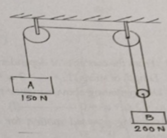


Fig. 6.30

Sol Using String's law, $N_A S_A = N_B S_B$

$S_A = 2 S_B$

So $a_A = 2 a_B$

As shown in the figure, block A is undergoing tension T and block B is subjected to tension 2T. To decide motion of the blocks, assume block B to be stationary. Then

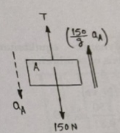


Fig. 6.31

$\sum F_Y = 0$ (↑+ ve)

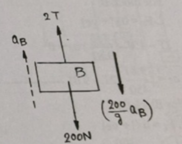
2T – 200 = 0 so T = 100 N

For block A, downward force = 150 N

Upward force = 100 N

Net downward force = 150 – 100 = 50 N.

Hence block A moves down and block B moves up.



For dynamic equilibrium,

FBD of 150 N block:

$\sum F_Y = 0$ (↑+ ve)

$$T - 150 + \left(\frac{150}{9.81}\right) a_A = 0$$

$$T + \left(\frac{150}{9.81}\right) a_A = 150 \text{ -----------------(I)}$$

From FBD of 150 N block:

$\sum F_Y = 0$ (↑+ ve)

$$2T - 200 - \left(\frac{200}{9.81}\right) a_B = 0$$

$$\text{But } a_B = \frac{a_A}{2}, \text{ So, } 2T - \left(\frac{200}{9.81}\right)\frac{a_A}{2} = 200$$

Solving equations (I) and (II), we get

a_A= 2.452 m/s² (↓)

so a_B = 1.226 m/s² (↑) (+ve answers indicate assumed directions are correct)

Now using kinematic equation of motion for block B,

$y_B = u_B t + ½ a_B t^2$

$y_B = 0 + ½ \times 1.226 \times 1^2 = 0.614$ m

$y_B = 0.614$ m

$T - (\frac{50}{9.81})\, a_A = 100$ ————(II)

2. **Three weights A, B and C are connected as shown in the figure. Determine the acceleration of each weight and tension in the string.**

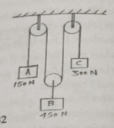
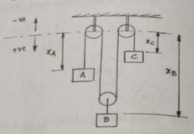
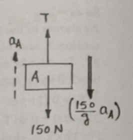


Fig. 6.32



Sol From the concepts of dependent motion,
Length of string, $L_1 = x_A + 2\, x_B + x_C$
Differentiating above equation twice, we get
$a_A + 2\, a_B + a_C = 0$ ————(I)
Assuming upward motion for all the three blocks,



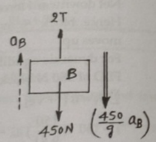
For dynamic equilibrium, applying conditions of equilibrium
For Block A:

$\sum F_Y = 0$ (↑+ ve)

$T - 150 - (\frac{150}{9.81}) a_A = 0$

$a_A = (\frac{9.81}{150})\, T - 9.81$ ————(II)

For Block B:



$\sum F_Y = 0$ (↑+ ve)

$2T - 450 - (\frac{450}{9.81})\, a_B = 0$

$a_B = (\frac{9.81}{450})\, 2T - 9.81$ ————(III)

For Block C:

$\sum F_Y = 0$ (↑+ ve)

$T - 300 - (\frac{300}{9.81})\, a_C = 0$

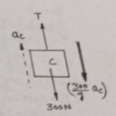
$a_C = (\frac{9.81}{300})\, T - 9.81$ ————(IV)

substituting equations (II), (III) and (IV) in equation (I), we get

$[(\frac{9.81}{150})\, T - 9.81] + 2\, [(\frac{9.81}{450})\, 2T - 9.81] + [(\frac{9.81}{300})\, T - 9.81] = 0$

Solving above equation, we get T = 211.76 N

$a_A = 4.039\ m/s^2$ (↑), $a_B = -0.577\ m/s^2 = 0.577\ m/s^2$ (↓)
& $a_C = -2.885\ m/s^2 = 2.885\ m/s^2$ (↓)



3. **In the system shown in figure the pulleys are to be considered massless and frictionless. Determine the acceleration of each mass and tension in the fixed cord.**

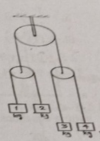
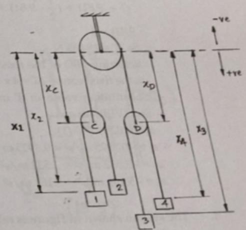


Fig. 6.33

Sol Using concept of dependant motion,
Let string 1 - connecting block 1 and 2,
string 2 - connecting pulleys C and D and
string 3 - connecting block 3 and 4.



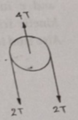
Length of string 1, $L_1 = (x_1 - x_C) + (x_2 - x_C) = x_1 + x_2 - 2\, x_C$
differentiating above equation twice $a_1 + a_2 - 2\, a_C = 0$

so, $a_C = \frac{a_1 + a_2}{2}$ ————(I)

Length of string 2, $L_2 = x_C + x_D$
differentiating above equation twice $a_C + a_D = 0$, so $a_D = -\, a_C$ ————(II)
Length of string 3, $L_3 = (x_3 - x_D) + (x_4 - x_D) = x_3 + x_4 - 2\, x_D$
differentiating above equation twice $a_3 + a_4 - 2\, a_D = 0$ ————(III)
so, from eq (II), $a_3 + a_4 - 2\,(-\, a_C) = 0$

from eq (I) $a_3 + a_4 + 2[\frac{a_1 + a_2}{2}] = 0$

so $a_1 + a_2 + a_3 + a_4 = 0$ ————(IV)



For dynamic equilibrium,
Block A:

$\sum F_Y = 0$ (↑+ ve)

$T - 9.81 - a_1 = 0$

$a_1 = T - 9.81$ ————(V)

Block B:

$\sum F_Y = 0$ (↑+ ve)

$T - (2 \times 9.81) - 2\,a_2 = 0$

$a_2 = \frac{T}{2} - 9.81$ ——————(VI)

Block C:

$\sum F_Y = 0$ (↑+ ve)

$T - (3 \times 9.81) - 3\,a_3 = 0$

$a_3 = \frac{T}{3} - 9.81$ ——————(VII)

Block D:

$\sum F_Y = 0$ (↑+ ve)

$T - (4 \times 9.81) - 4\,a_4 = 0$

$a_4 = \frac{T}{4} - 9.81$ ——————(VIII)

putting equations (V) to (VIII) in equation (IV)

$(T - 9.81) + (\frac{T}{2} - 9.81) + (\frac{T}{3} - 9.81) + (\frac{T}{4} - 9.81) = 0$

So, $T = 18.835$ N

Referring FBD of the fixed pulley, tension in the fixed cord $= 4T = 4 \times 18.835 = 75.34$ N

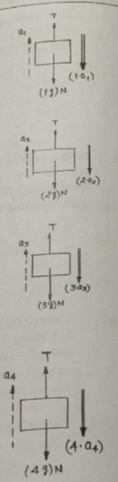
Substituting value of T in equations (V) and (VIII), we get,

$a_1 = 9.025\ m/s^2$ (↑)

$a_2 = -0.3925\ m/s^2 = 0.3925\ m/s^2$ (↓)

$a_3 = -3.532\ m/s^2 = 3.532\ m/s^2$ (↓)

$a_4 = -5.101\ m/s^2 = 5.101\ m/s^2$ (↓)



4. The system shown in figure is released from rest. What is the height lost by bodies A, B and c in 2 seconds? Take coefficient of kinetic friction at rubbing surfaces as 0.4. Also find tension in the wires.

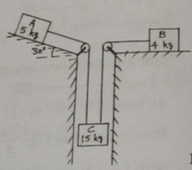


Fig. 6.34

Sol Let T_A be the tension in the string connecting 5 kg and 4 kg block and T_B be the tension in the string connecting 4 kg and 15 kg block. Let 'a' be the acceleration of the system.

From FBD of 5 kg block A:

$\sum F_Y = 0$ (↑+ ve)

$N_A - (5 \times 9.81 \times \cos 30) = 0$

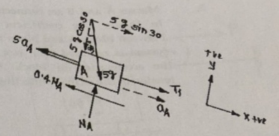
$\therefore N_A = 42.48$ N

$\sum F_X = 0$ (→+ ve)

$T_1 + (5 \times 9.81 \times \sin 30) - 0.4\,N_A - 5\,a = 0$

$T_1 + 24.525 - (0.4 \times 42.28) - 5\,a = 0$

$T_1 = 5\,a - 7.533$ ——————(I)



From FBD of 4 kg block B:

$\sum F_Y = 0$ (↑+ ve)

$N_B - (4 \times 9.81) = 0$

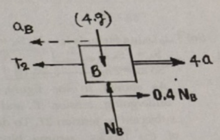
$\therefore N_B = 39.24$ N

$\sum F_X = 0$ (→+ ve)

$T_2 - 0.4\,N_B - 4\,a = 0$

$T_2 - (0.4 \times 39.24) - 4\,a = 0$

$T_2 = 15.696 + 4\,a$ ——————(II)

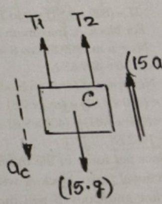


From FBD of 15 kg block C:

$\sum F_Y = 0$ (↑+ ve)

$-T_1 - T_2 - 15\,a + (15 \times 9.81) = 0$

$T_1 + T_2 = 147.15 - 15\,a$ ——————(III)



Substituting equations (I) and (II) in equation (III)

$(5\,a - 7.533) + (15.696 + 4\,a) = 147.15 - 15\,a$

$a = 5.72\ m/s^2$

from equation (I), $T_A = 21.422$ N

from equation (II), $T_B = 38.86$ N

For height lost by block A and C in 2 seconds,

For block C, $H_C = u\,t + \frac{1}{2}\,a\,t^2 = 0 + (\frac{1}{2} \times 5.791 \times 2^2)$

$H_C = 11.58$ m

Block A will move by the same amount but along the inclined plane. Hence height lost by block A will be, $H_A = 11.58 \sin 30 = 5.8$ m

Block B will not lose the height as it will move along the horizontal plane.

5. **Masses A and B are connected as shown in figure. The coefficient of friction between block A and plane is 0.2. If the system is released from rest, determine the acceleration of block A and the tension in the string connecting block B.**

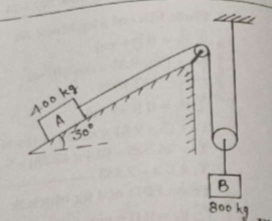


Fig. 6.35

Sol Using String's law, $N_A S_A = N_B S_B$

$S_A = 2 S_B$

So $a_A = 2 a_B$ ---------------(I)

As shown in the figure, block A is undergoing tension T and block B is subjected to tension 2T. To decide motion of the blocks, assume block B to be stationary. Then

$\sum F_Y = 0$ (↑+ ve)

$2T - (800 \times 9.81) = 0$ so $T = 3924$ N

For block A, maximum frictional force = μ $N_A = \mu\, m_A \times 9.81 \cos\theta = 0.2 \times 400 \times 9.81 \times \cos 30 = 679.65$ N

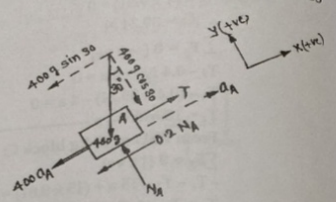
Net upward force on block

$= (400 \times 9.81) - (m_A \times 9.81 \times \sin\theta)$

$= (400 \times 9.81) - (400 \times 9.81 \times \sin 30)$

$= 1962$ N

Since net force in the upward direction > frictional force, block A will move up the incline and body B will move down, the incline.

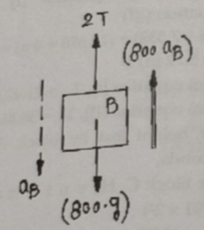


From FBD of 400 kg block in dynamic equilibrium,

$\sum F_X = 0$ (→+ ve upward along the inclined plane)

$T - (400 \times 9.81 \times \sin 30) - 679.65 - 400\, a_A = 0$

$T - 400\, a_A = 2641.65$ ---------(II)



From FBD of 800 kg block B in dynamic equilibrium

$\sum F_Y = 0$ (↑+ ve)

$(800 \times 9.81) - 2T - (800\, a_B) = 0$

$T + 400\, a_B = 400 \times 9.81$

From equation (I), $a_B = \dfrac{a_A}{2}$

$\therefore T + 200\, a_A = 3924$ --------------------(III)

Solving equations (II) and (III), we get

Tension in the string, T = 3496.55 N

Tension in the string connecting Block B,

2T = 6993.10 N

Acceleration of block A, $a_A = 2.137$ m/s²

Acceleration of block B, $a_B = 1.069$ m/s²

6. **If the coefficient of static and kinetic friction between 20 kg block A and 100 kg block B are both essentially the same value of 0.5 and there is no friction between wheels of cart B and surface, determine the acceleration of each block when (i) P = 40 N (ii) P = 60 N. Assume pulley to be massless and frictionless.**

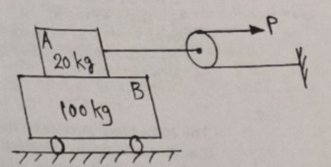
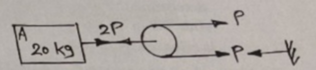


Fig.6.36

Sol By looking at the above figure, block A is subjected to a force 2P.



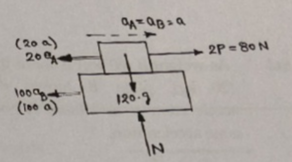
Maximum frictional force between A and B,

$F_{MAX} = \mu N = \mu\, m_A\, g = 0.5 \times 20 \times 9.81 = 98.1$ N

Frictional force between ground surface and block B = 0.

(i) When P = 40 N

Block A is subjected to force 2P = 80 N < F_{MAX}, hence block A and block B will move together as a single unit with same acceleration a.



Referring to the FBD of two blocks

$\sum F_X = 0$ (→+ ve)

$80 - 100\,a - 20\,a = 0$

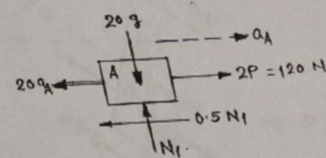
$a = 0.667$ m/s² (→)

(ii) When P = 60 N

Block A is subjected to force

2P = 120 N > F_{MAX}

Both the blocks will have different accelerations.



Referring to FBD of block A and block B,

Applying conditions of equilibrium,

For Block A:

$\sum F_Y = 0$ (↑+ ve)

$N_1 - (20 \times 9.81) = 0$

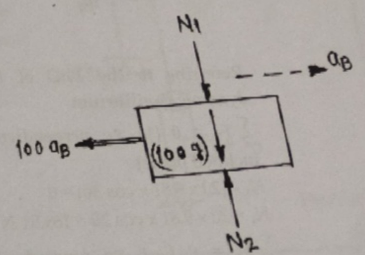
$\therefore N_1 = 196.2$ N

$\sum F_X = 0$ (→+ ve)

$120 - 0.5\, N_1 - 20\, a_A = 0$

$120 - (0.5 \times 196.2) - 20\, a_A = 0$

$a_A = 1.905$ m/s² (→)



For Block B:

$\sum F_X = 0$ (→+ ve)

$\sum F_Y = 0$ (↑+ ve)

$N_2 - (100 \times 9.81) - N_1 = 0$

But $N_1 = 20 \times 9.81$ from equation (I)

$\therefore N_2 - (100 \times 9.81) - (20 \times 9.81) = 0$

$\therefore N_2 = 1177.2$ N

$0.5 N_1 - 100 a_B = 0$

$(0.5 \times 20 \times 9.81) - 100 a_B = 0$

$\therefore a_B = 0.981$ m/s² (→)

7. **The 20 kg block A rests on the 60-kg plate B as shown in figure. Neglecting the mass of the rope and pulley and using the coefficients of friction indicated, determine the time needed for block A to slide 0.5 m on the plate when the system is released from rest.**

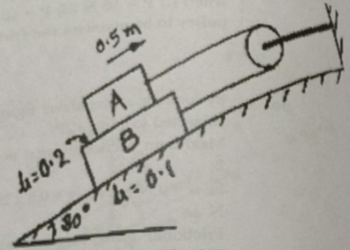
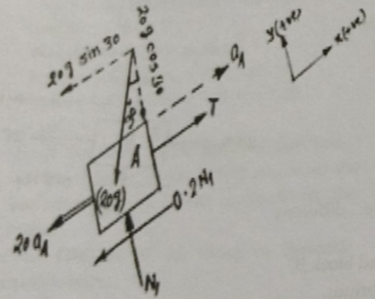
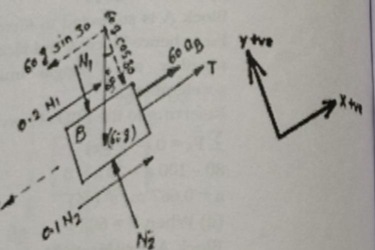


Fig. 6.37

Sol As weight B (60 kg) is more than block A (20 kg), plate B moves down with acceleration 'a' and block A goes up with same acceleration.





Referring to the FBD of block A in dynamic equilibrium

$\sum F_Y = 0$ **(↑+ ve perpendicular to the inclined plane)**

$N_1 - (20 \times 9.81 \times \cos 30) = 0$

$N_1 = 20 \times 9.81 \times \cos 30 = 169.91$ N

$\sum F_X = 0$ **(→+ ve upward along the inclined plane)**

$T - (20 \times 9.81 \times \sin 30) - 0.2 N_1 - 20 a = 0$

Substituting equation (I) in above equation,

$N_2 - (20 \times 9.81 \times \cos 30) - (60 \times 9.81 \times \cos 30) = 0$

$N_2 = (80 \times 9.81 \times \cos 30)$

$N_2 = 679.65$ N ------------------(III)

$\sum F_X = 0$ **(→+ ve upward along the inclined plane)**

$(60 \times 9.81 \times \sin 30) - T - 0.2 N_1 - 0.1 N_2 - 60 a = 0$

$294.3 - T - 0.2 N_1 - 0.1 N_2 - 60 a = 0$

Using equations (I), (II) and (III)

$294.3 - (132.04 + 20 a) - (0.2 \times 169.91) - (0.1 \times 679.65) - 60 a = 0$

$a = 0.753$ m/s²

so, $a_A = 0.753$ m/s² up the plane and $a_B = 0.753$

$T = (10 \times 9.81) + (0.2 \times 20 \times 9.81 \times \cos 30) + 20 a$

$T = (13.46 \times 9.81) + 20 a$

$T = 132.04 + 20 a$ ------------------(II)

Referring to the FBD of plate B in dynamic equilibrium

$\sum F_Y = 0$ **(↑+ ve perpendicular to the inclined plane)**

$N_2 - N_1 - (60 \times 9.81 \times \cos 30) = 0$

m/s² down the plane.

$\therefore$ relative acceleration of A w.r.t. B, $a_{A/B} = a_A - a_B = 0.753 - (-0.753) = 1.506$ m/s²

Using kinematic equation of motion,

$S_{A/B} = u_{A/B} \times t + (½ \times a \times t^2)$

$\therefore 0.5 = 0 + (½ \times 1.506 \times t^2)$

$\therefore t = 0.815$ sec.

Let's check the takeaway from this lecture

1. String Law can be applied to ______ no of bodies connected
   a) 2   b) not more than 3   c) more than 3
2. **Which other method can be used to find relation in between two connected bodies**
   a) Method of joints b) Method of Sections   c) Concept of dependent motion

Exercise

Q1. Determine the tension developed in chords attached to each block and the accelerations of the blocks when the system shown is released from rest. Neglect the mass of the pulleys and chords.**(Ans: T=19.344N,a_A=1.244m/s²,a_B= 4.976m/s²↓)**

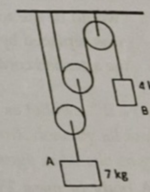


Fig.6.38

Q2. Determine the tension developed in the two chords and the acceleration of each block. Neglect the mass of pulleys and chords. **(Ans: a_A = 0.76 m/s² ↓, a_B = 1.51 m/s² ↑, T_{Lower} = 45.3 N, T_{Upper} = 90.6 N)**

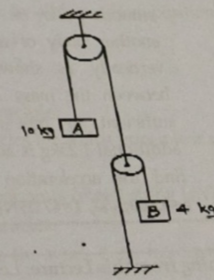


Fig.6.39

Practice Problems for the Day

Q6. The three weights A, B and C of weights 3 kg, 2 kg and 7 kg are connected as shown in figure. Determine the accelerations of A, B and C. Also find the tension on the string. **(Ans:T=27.935N, a_A=0.5m/s²↓, a_B=4.15m/s²↑, a_C= 1.83m/s²↓)**

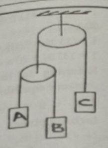


Fig.6.40

Q7. The system of pulleys, masses and connecting inextensible cables as shown pulleys are massless and frictionless. If the system is released from rest, Find the acceleration of each of the three masses and the tension in cable. **(Ans: T=27.70N, a_A=4.04m/s²↑, a_B=2.885m/s²↓, a_C=0.557m/s² ↓)**

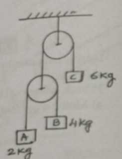


Fig.6.41

Q8. A 25 N block B rests on a smooth surface as shown in fig. Determine its acceleration when the 15 N block A is released from rest. What would be the acceleration of B if the block of A was replaced by a 15N vertical force acting on the attached cord? **(Ans: 2.56m/s², 2.94m/s²)**

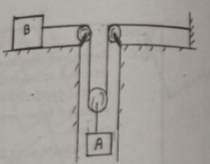


Fig.6.42

Q9. A body of mass 25kg resting on a horizontal table is connected by string is passing over a smooth pulley at the edge of the table to another body of mass 3.75kg and hanging vertically as shown. Initially the friction between the mass A and the table is just sufficient to prevent the motion. If an additional 1.25kg is added to the 3.75kg mass, find the acceleration of the masses. **(Ans: a=0.409m/s², T=47.05N) (Dec 2013)**

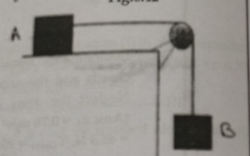


Fig.6.43

Learning from this Lecture: Learners will be able to string law to get the relation of motion in between two connected bodies.

Lecture 41

6.4 *Introduction & Basic Problems on Work Energy Principle*

Theory

➤ **Work done by a force:**

- Work is said to be done by a force when it displaces the body.
- Work is a scalar quantity
- SI Unit: **joule** (J); MKS Unit: **kgm²/s²**; CGS Unit: **gm-cm²/s²** or **erg** (**Conversion:** 1 joule = 10^7 erg)
- Dimensional Formula: **[ML^2T^{-2}]**
- Work done may be positive, negative or zero
- Conservative work done is independent of path.

In evaluating work done, four types of forces are usually considered. They are 1) Externally applied force 2) Gravitational force 3) Frictional force 4) Spring Force

➤ **Work –Energy Principle:** The work done by force in displacing a body is equal to change in kinetic energy of the body

By Newton's second law of motion,

$$F = m\,a \therefore F = m\,V\,\frac{dv}{ds}$$

$$F\,ds = m\,V\,dV$$

Integrating both sides,

$$\int_{s1}^{s2} F ds = m \int_{u}^{v} v\,dv \int_{s1}^{s2} F ds = m\,\frac{v^2}{2} - m\,\frac{u^2}{2}$$

The term $\int_{s1}^{s2} F ds$ = Work done from position 1 to 2

$\therefore U_{1\to2}$ = Final KE – Initial KE

$\therefore U_{1\to2}$ = Change in KE

The work-energy principle for a particle can be stated as follows, " If a particle of mass m, subjected to unbalanced force system, the total work done by all forces during the displacement (from position 1→2) is equal to the change in kinetic enegy during that displacement.

$\therefore$ **$U_{1\to2} = KE_2 - KE_1$**

Where $U_{1\to2}$ = Algebraic sum of work done by all forces from position 1 to 2

KE_1 = Kinetic energy of system at position 1

KE_2 = Kinetic energy of system at position 2

➤ **Working rules for application of work-energy principle:**

1. Decide two positions of particle or system of particles for which principle is required to be used.
2. In case of connected particles find kinematic constraints (direct string law) to find out relation among the velocities and positions of particles.
3. Without drawing free body diagram, calculate the work done from position 1 to 2, denoted by ($U_{1\to2}$)
4. Calculate kinetic energy at both positions.
5. Apply work-energy principle between two positions.
6. When analysis involves two or more unknown quantities, along with work-energy

principle, use D' Alembert's principle to support the analysis.

➤ **Key Points:**

i. Work done by normal reaction (N) is always zero because it has no component in the direction of displacement.

ii. For horizontal displacement of particle, no work is done by gravity force.

iii. Work done by tension in the string is always zero as it is internal force.

iv. The reaction at frictionless pin (hinge) when body rotates about pin does no work.

v. Forces applied to fixed points where ds = 0 and the forces acting perpendicular to displacement do not work.

Solved Problems

1. **A 20-kg crate is released from rest on the top of incline at A. It travels on the incline and finally comes to rest on the horizontal surface at C. Find the distance 'x' it travels on the horizontal surface and the maximum velocity it attains during the motion. Take $\mu_K = 0.3$.**

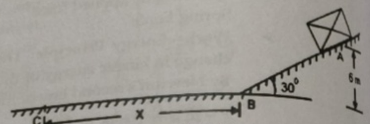


Fig.6.44

Sol The crate acquires maximum velocity at the lower most point B on the incline.

For motion A→B

Work done calculations:

a) Work done by Gravity force = m g h = 20 x 9.81 x 6 = 1177.2 J

b) Work done by frictional force = - μ_K N S

Here, for the inclined surface, normal reaction N = W cos 30 = 20 x 9.81 cos 30 = 169.9 N and distance travelled by block, S = $\frac{6}{\sin 30}$ = 12 m

So, work done = - 0.3 x 169.9 x 12 = - 611.64 J

Total work done, $U_{A\to B}$ = 1177.2 + (611.64) = 1177.2 – 611.64 = 565.56 J

Kinetic energy calculations:

$KE_A = \frac{1}{2} m V_A^2 = 0$

(At rest in position A)

$KE_B = \frac{1}{2} m V_B^2 = \frac{1}{2} \times 20 \times V_B^2$

Applying Work-energy principle:

$U_{A\to B} = KE_B - KE_A$

$\therefore 565.56 = (\frac{1}{2} \times 20 \times V_B^2) - 0$

$\therefore V_B = 7.52$ m/s i.e. $V_{MAX} = 7.52$ m/s

For motion B→C

Work done calculations:

Work done by frictional force = - μ_K N S

For the horizontal surface, N = w = 20 x 9.81 = 196.2 N

Work done, $U_{B\to C}$ = - 0.3 x 196.2 x S

Kinetic energy calculations:

$KE_B = \frac{1}{2} m V_B^2 = \frac{1}{2} \times 20 \times (7.52)^2 = 565.56$ J

$KE_C = \frac{1}{2} m V_C^2 = \frac{1}{2} \times 20 \times 0 = 0$ (Block comes to rest at C)

Applying Work-energy principle:

$U_{B\to C} = KE_C - KE_B$

$\therefore$ - 0.3 x 196.2 x S = 0 – 565.56

$\therefore$ S = 9.608 m i.e. x = 9.608

2. **A block of mass 50 kg is placed on a rough horizontal floor as shown in figure. A force P is applied on to the block such that block attains a velocity of 10 m/s covering 10 m. Determine the value of P. Take coefficient of friction between block and surface as 0.25.**

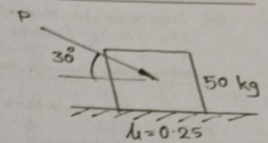
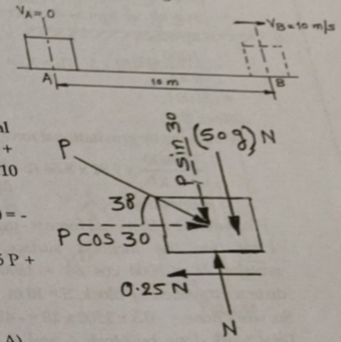


Fig.6.45

Sol Block moves from position (A) to (B) covering 10 m distance.



For motion A→B

Work done by external force = F x S = (P cos 30) x 10 = 8.66 P

Work done by frictional force = - μ_K N S

Here, for the horizontal surface, normal reaction N = (Psin 30) + (50 x 9.81) = 0.5 P + 490.5 and distance travelled by block, S = 10 m.

So work done = - 0.25 x (0.5 P + 490.5) x 10 = - 1.25 P – 1226.25 = - (1.25 P + 1226.25)

Total work dine, $U_{A\to B}$ = 8.66 P + [- (1.25 P + 1226.25)] = 7.41 P – 1226.25

Kinetic energy calculations:

$KE_A = \frac{1}{2} m V_A^2 = 0$ (At rest in position A)

$KE_B = \frac{1}{2} m V_B^2 = \frac{1}{2} \times 50 \times 10^2 = 2500$ J

Applying Work-energy principle:

$U_{A\to B} = KE_B - KE_A$

$\therefore$7.41 P – 1226.25 = 2500 – 0

$\therefore$P = 502.86 N

3. **Block B of weight 300 N having a speed of 2 m/s in position (1) travels 10 m along and down the slope. Block A of weight 1000 N is connected to it by an inextensible string. Find the velocities of the blocks in the new position. Take μ_S = 0.35 and μ_K = 0.3 at the inclined surface.**

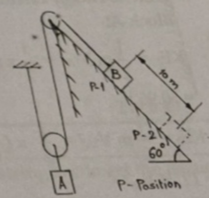


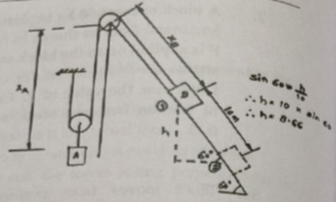
Fig.6.46

Sol From String's law, $N_A X_A = N_B X_B$

$2X_A = 1X_B \therefore X_B = 2X_A$ so for $X_B = 10$ m, $X_A = 5$ m

Differentiating above equation we get, $V_B = 2V_A$

Considering motion of block A and B from position (1) to (2)



Work done calculations:

Block A: Work done by gravitational force = - m g h (- ve sign as displacement of block A is upwards)

$= -\frac{1000}{9.81} \times 9.81 \times 5 \quad (h = X_A = 5\text{ m})$

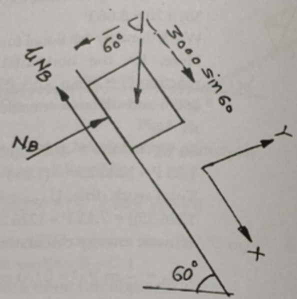
$= -5000$ J

Block B:

Work done by gravitational force = m g h

$= \frac{3000}{9.81} \times 9.81 \times 8.66 \; (h = 8.66\text{ m})$

$= 25980.76$ J



Work done by frictional force = $-\mu_K N S$

Here, for the inclined surface, normal reaction N = 3000 cos 60 = 1500 N and distance travelled by block, S = 10 m

So, work done = - 0.3 x 1500 x 10 = - 4500 J

Total work done by block A and B during motion of block B from position (1) to (2)

= - 5000 + (25980.76 + (- 4500))

= 16480.76 J

Kinetic energy calculations:

Block A:

$KE_1 = \frac{1}{2} m V_{1A}^2 = \frac{1}{2} \times \left(\frac{1000}{9.81}\right) \times (1)^2$

$= 50.96$ J

$KE_2 = \frac{1}{2} m V_{2A}^2 = \frac{1}{2} \times \left(\frac{1000}{9.81}\right) \times \left(\frac{V_{2B}}{2}\right)^2$

$= 12.74\, V_{2B}^2$

Block B:

$KE_1 = \frac{1}{2} m V_{1B}^2 = \frac{1}{2}\left(\frac{3000}{9.81}\right)(2)^2$

$= 611.62$ J

$KE_2 = \frac{1}{2} m V_{2B}^2 = \frac{1}{2}\left(\frac{3000}{9.81}\right) V_{2B}^2$

$= 152.90\, V_{2B}^2$

Applying Work-energy principle:

$U_{1\to 2} = KE_2 - KE_1$

$16480.76 = (12.74\, V_{2B}^2 + 152.90\, V_{2B}^2) - (50.96 + 611.62)$

$\therefore V_{2B} = 10.17$ m/s

Also, $V_{2A} = 0.5 \times V_{2B} = 0.5 \times 10.17 = 5.086$ m/s

Let's check the takeaway from this lecture

1. The kinetic energy of a body of mass (m) & velocity (V) is equal to

a) m V b) m V / 2 c) m² V / 2 d) m V² / 2

2. ………..can be defined as the capacity to do work.

a) Power b) Momentum c) Energy

Exercise

Q1. In the figure, the block P of weight 50 N is pulled so that the extension in the spring is 10 cm. the stiffness of the spring is 4 N/cm and the coefficient of friction between the block and the plane OX is μ = 0.3. Find (i) the velocity of the block as the spring returns to its un-deformed state (ii) the maximum compression in the spring [Ans: **(i) 0.447 m/s, (ii) 2.5 cm]**

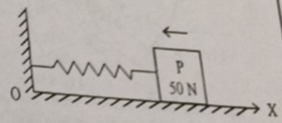


Fig.6.47

Q2. Find the velocity of block A and B when block A has travelled 1.2 m long along inclined plane. Mass of A is 10 kg and that of B is 50 kg. Coefficient of friction between block A and inclined plane is 0.25. Pulleys are massless and frictionless. Use work energy principle. **(Ans: 4.175m/s, 2.087m/s)**

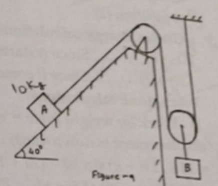


Fig.6.48

Practice Problems for the Day

Q1. The system shown in the figure is released from rest. Determine the velocity of each block when block B has descended 1.5 m. The mass of the blocks are m_A = 12 kg, m_B = 6 kg and coefficient of kinetic friction between block A and the horizontal surface is 0.20. **[Ans: V_A = 1.981 m/s (→), V_B = 3.962 m/s (↓)] (Dec 2013)**

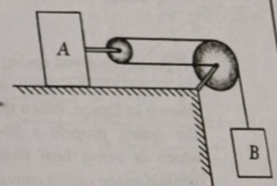


Fig.6.49

Learning from this Lecture: Learners will be able to use string law to get the relation of motion in between two connected bodies.

Lecture 42

6.5 *Problems on Work Energy Principle-I*

Solved Problems

1. **A 30 N block is released from rest. It slides down a rough incline having μ = 0.25. Determine maximum compression of the spring.**

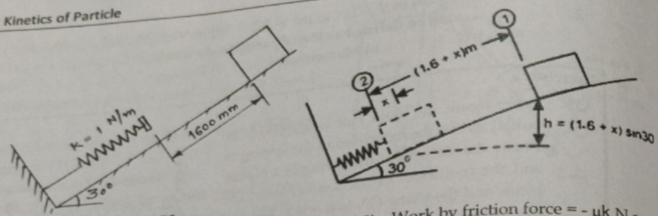


Fig.6.50

So
I
The block travels from rest at position (1), slides down the incline and after 1.6 m of travel down the incline hits the spring, compresses it by x meters and comes to a halt at position (2).

Kinetic Energy calculations:

$KE_1 = 0$ ………. Since it starts from rest

$KE_2 = 0$ ………. Since it comes to rest

Work done calculations:

1) Work by weight force = + m g h (+ve since displacement is downward) ……[mg = 30 N]

= 30 x (1.6 + x) sin 30 = (24 + 15 x) J

0.25 (30 cos 30) (1.6 + x) … [N = mg cos 30 = 30 cos 30]

= (- 10.39 – 6.495 x) J ……… [s = 1.6 + x]

2) Work by friction force = - μk N s = -

3) Work by spring force = $\frac{1}{2}$ K $(x_1^2 - x_2^2)$

= $\frac{1}{2}$ x 1000 x (0 – x^2) ………. [x_1 = 0, x_2 = x]

= (- 500 x^2)

Total work done = (24 + 15 x) + (- 10.39 – 6.495 x) + (- 500 x^2)

Applying Work-Energy principle:

U1→2 = KE2 – KE1

∴(24 + 15 x) + (– 10.39 – 6.495 x) + (– 500 x2) = 0, ∴- 500 x2 + 8.505 x + 13.61 = 0, ∴ x = 0.1737 m

Hence the maximum compression of the spring is x = 0.1737 m

2. **A pre-compressed spring compressed by 0.2 m is held by a latch mechanism OA as shown in figure. When the latch is released the spring propels a 30-kg machine part which is being heat treated at A up the inclined plane onto a conveyer belt at B. The coefficient of friction between machine part and incline is 0.1. The desired speed of machine part when it reaches the top of the incline is 5 m/s. Determine the spring constant 'K' in kN/m that engineer must use. Angle of inclination of plane is 30⁰ with horizontal.**

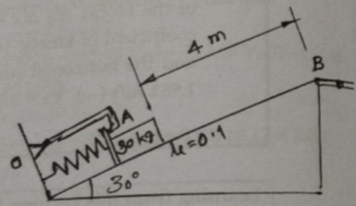


Fig.6.51

So
1
After releasing the spring, the block will be propelled up the inclined plane by 4m. spring will attain its original position. Referring to position (1) and (2) of the block as shown in the figure.

Work done calculations:

Work done by gravity force = - m g h = - 30 x 9.81 x (4 sin 30) = - 30 x 9.81 x 2 = - 588.6 J

Work done by frictional force = - μ_K N S = - 0.1 x (30 x 9.81 x cos 30) x 4 = - 101.95 J

Work done by spring force = $\frac{1}{2}$ K $(x_1^2 - x_2^2)$ =

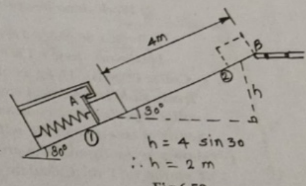
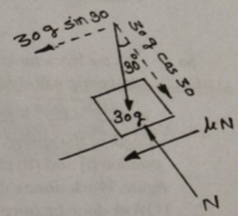


Fig.6.52

$\frac{1}{2}$ x K x (0.2² – 0) = (0.02 K) J

Total work done = (- 588.6) + (- 101.95) + (0.02 K) = - 690.55 + (0.02 K)



Kinetic energy calculations:

$KE_1 = 0$

$KE_2 = \frac{1}{2}$ m V² = $\frac{1}{2}$ x 30 x 5² = 375 J

Applying work energy principle:

$U_{1\to2} = KE_2 - KE_1$

∴ - 690.55 + (0.02 K) = 375

∴K = 53277.45 N/m = 53.27 kN/m

3. **A 10-kg slider 'A' moves with negligible friction along the horizontal guide. The attached spring has a stiffness of 60 N/m and is stretched 0.6 m in position 'A' where the slider is released from rest. The 250 N force is constant and the pulley offers negligible resistance to the motion of the cord. Calculate the velocity of the slider as it passes point C.**

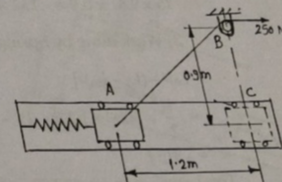
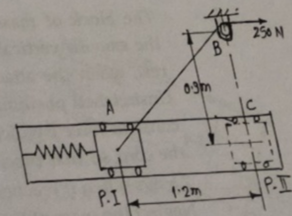


Fig.6.53

So
1
As the block moves from A to C, there is extension of the spring from x_1 = 0.6 m to x_2 = 1.8 m. Referring to position (I) and (II) shown in the figure,



Work done calculations:

1) Work done by force 250 N = 250 x [Length AB (L_1) – Length BC (L_2)]

= 250 x ($\sqrt{1.2^2 + 0.9^2}$ – 0.9) = 250 x (1.5 – 0.9) = 150 J

Kinetic energy calculations:

$KE_1 = 0$

$KE_2 = \frac{1}{2}$ m V² = $\frac{1}{2}$ x 10 x V² = 5 V²

2) Work done by spring force = $\frac{1}{2}$ K (x_1^2 - x_2^2) = ½ x 60 (0.6^2 – 1.8^2) = - 86.4 J

Total work done = 150 + (- 86.4) = 63.6 J

Applying work energy principle:

$U_{1\to2}$ = KE_2 – KE_1

63.6 = 5 V^2 – 0, ∴ V = 3.566 m/s

4. **Collar of mass 15 kg is at rest at A. it can freely slide in a vertical smooth rod AB. The collar is pulled by a constant force F = 800 N acting at an angle of 30º with the vertical as shown in the figure. The unstretched length of spring is 1 m. Calculate the velocity of collar when it reaches B. Take spring constant K = 3000 N/m and AC is horizontal.**

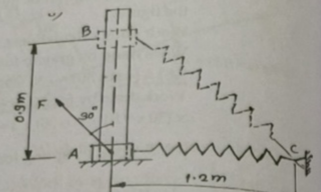
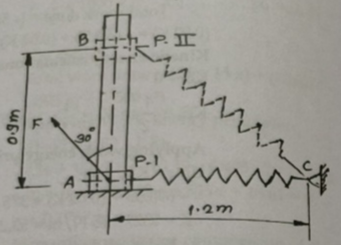


Fig.6.54

So
1

As the block moves from A to B, length of the spring will change from L_1 = 1.2 m to L_2 = $\sqrt{1.2^2+0.9^2}$ = 1.5 m. Also, unstretched length of the spring, L_o = 1 m. Referring to position (I) and (II) shown in the following figure, **Work done calculations:**



1) Work done by force F = 800 N = F cos 30 x S = (800 x cos 30) x 0.9 = 623.53 J

2) Work done by gravity force = - m g h = - 15 x 9.81 x 0.9 = - 132. 43 J

3) Work done by Spring force = $\frac{1}{2}$ k [(L_1 - L_o)^2 – (L_2 – L_o)^2]

= $\frac{1}{2}$ x 3000 x [(1.2 – 1)^2 – (1.5 – 1)^2]

= - 315 J

Total work done = 623.53 + (-132.43) + (-315) =176.1 J

Kinetic energy calculations:

KE_1 = 0

KE_2 = $\frac{1}{2}$ m V^2 = $\frac{1}{2}$ x 15 x V^2 = 7. 5 V^2

Applying work energy principle

$U_{1\to2}$ = KE_2 – KE_1

176.1 = 7.5 V^2 – 0

∴ V = 4.851 m/s

5. **The block of mass 0.5 kg moves within the smooth vertical slot. If it starts from rest, when the attached spring is in the Unstretched position at A. Determine the constant force F which must be applied to the cord so that block attains a speed of 2.5 m/s when it reaches B, i.e. S_B = 0.15 m. Neglect the mass of the cord and pulley.**

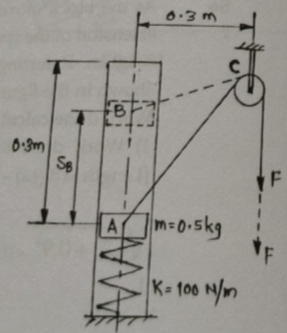
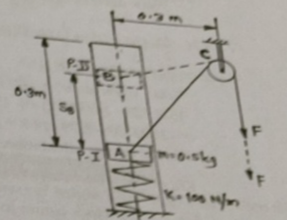


Fig.6.55

So
1

As the block moves from A to B, it goes up by 0.15 m and spring is stretched by 0.15 m. Referring to position (I) and (II).



Work done calculations:

Work done by force F = F x S = F x [Length AC (L_1) – Length BC (L_2)]

= F x [$\sqrt{0.3^2+0.3^2}$ - $\sqrt{0.3^2+(0.3-0.15)^2}$] = 0.0889 F

Work done by gravity force = - m g h = - 0.5 x 9.81 x 0.15 = - 0.735 J

Work done by Spring force = $\frac{1}{2}$ k (x_1^2 – x_2^2) = ½ x 100 x (0 – 0.15^2) = - 1.125 J

Total work done = 0.0889 F + (-0.735) + (-1.125) = 0.0889 F – 1.86

Kinetic energy calculations:

KE_1 = 0

KE_2 = $\frac{1}{2}$ m V^2 = $\frac{1}{2}$ x 0.5 x 2.5^2 = 1.5625 J

Applying work energy principle

$U_{1\to2}$ = KE_2 - KE_1

0.0889 F – 1.86 = (1.5625 – 0)

∴F = 38.498 N

6. **A spring is used to stop 100 kg package which is moving down a 300 incline. The spring has a constant K = 30 kN/m and is held by cables so that it is initially compressed 90 mm. If the velocity of the package is 5 m/s when it is 9 m from spring, determine the maximum additional deformation of spring in bringing the package to rest. Assume coefficient of friction between block and incline as 0.2.**

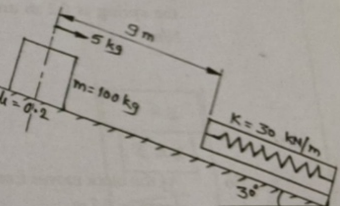
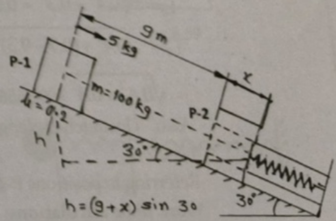


Fig.6.56

So
1

Let 'X' be the additional deformation of the spring beyond 0.09 m till the block comes to rest and velocity of the block becomes zero.

Referring to the position (1) and (2)



Work done calculations:

Work done by gravity force = m g h = 100 x 9.81 x (9 + X) sin 30 = 4414.5 + 490.5 X

Work done by frictional force = - μ_K N S = - μ_K x m g cos 30 x ∑

= - 0.2 x 100 x 9.81 x cos 30 x (9 + X) = - 1529.228 – 169.914 X

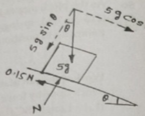
Work done by spring force = $\frac{1}{2}K(x_1^2 - x_2^2)$ =

$\frac{1}{2} \times (30 \times 10^3) \times [0.09^2 - (0.09 + X)^2]$

$= -15000\,X^2 - 2700\,X$

Total work done = $(4414.5 + 490.5\,X) + (-1529.228 - 169.914\,X) + (-15000\,X^2 - 2700\,X)$

$= -15000\,X^2 - 2379.414\,X + 2885.272$



Kinetic energy calculations:

$KE_1 = \frac{1}{2} m V_1^2 = ½ \times 100 \times 5^2 = 1250$ J

$KE_2 = 0$ (As block comes to rest, velocity V_2 will be zero)

Applying work energy principle

$U_{1\to2} = KE_2 - KE_1$

$-15000\,X^2 - 2379.414\,X + 2885.272 = (0 - 1250)$

$15000\,X^2 + 2379.414\,X - 4135.272 = 0$

Solving above quadratic equation, we get

$X = 0.45169$ m = 451.69 mm

7. **A collar of mass 10 kg moves on a vertical guide as shown in figure. Neglecting friction between the guide and the collar, find the velocity of the collar after it has fallen 0.7 m starting from rest from the position shown. The unstretched length of the spring is 0.2 m and its stiffness is 200 N/m.**

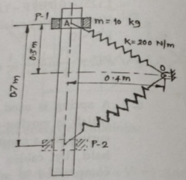


Fig.6.57

So
l

As the block moves from P-1 to P-2, it goes down by 0.7 m.

Length of the spring will change from

$L_1 = \sqrt{0.4^2 + 0.3^2} = 0.5$ m to

$L_2 = \sqrt{0.4^2 + (0.7 - 0.3)^2}$

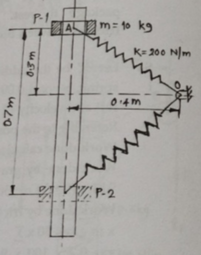
$= \sqrt{0.4^2 + 0.4^2} = 0.5656$ m.

Also, Unstretched length of the spring, L_0=0.2 m.

Referring to positions P-1 and P-2

Work done calculations:

Work done by gravity force = m g h = 10 x 9.81 x 0.7 = 68.67 J



Work done by Spring force = $\frac{1}{2} k\,[(L_1 - L_0)^2 - (L_2 - L_0)^2]$

$= \frac{1}{2} \times 200 \times [(0.5 - 0.2)^2 - (0.5656 - 0.2)^2]$

$= -4.366$ J

Total work done = 68.67 + (4.366)

= 64.304 J

Kinetic energy calculations:

$KE_1 = 0$

$KE_2 = \frac{1}{2} m V^2 = \frac{1}{2} \times 10 \times V^2 = 5\,V^2$

Applying work energy principle

$U_{1\to2} = KE_2 - KE_1$

$64.304 = 5\,V^2$

$\therefore V = 3.586$ m/s

Let's check the takeaway from this lecture

1. **A 3kg body is dropped from the top of a tower of height 135m. If g=10m/s², then the K.E of body after 3 seconds will be**
   a)950J b)10J c)1150J d)1350J
2. ____________ **is responsible for work done by spring**
   a) Length of spring b) Stiffness of spring c) No. of turns of spring d) None

Exercise

Q1. A wagon weighing 490 kN starts from rest runs 30 m down on inclined surface and strikes a post. If the rolling resistance of the track 5 N / kN, find the velocity of wagon when it strikes the post. If the impact is to be cushioned by means of bumper spring having K = 14.7 kN/mm, determine the compression of the bumper spring. **(Ans: x = 100 mm)**

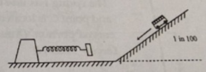


Fig.6.58

Q2. A 3 kg block rests on 2 kg block that is attached to a spring of constant K = 40 N/m. The upper block is suddenly removed. Determine maximum height and maximum velocity reached by 2 kg block. **(Ans: H_{MAX} = 1.47 m, V_{MAX} = 3.29 m/s)**

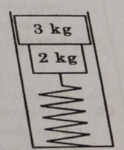


Fig.6.59

Q3. A spring of stiffness k is placed horizontally and a ball of mass m strikes the spring with a velocity v. Find the maximum compression of the spring. Take m= 5kg, k = 500 N/m, V = 3m/s**(Ans: 0.3m)(Dec' 12)**

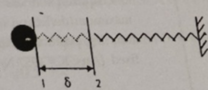


Fig.6.60

Practice Problems for the Day

Q1. A 10-kg slider A moves with negligible friction up the inclined guide. The attached spring has stiffness of 60 N/m and is stretched 0.6 m in position A where the slider is released from rest. The 250 N is constant and the pulley offers negligible resistance to the motion of the cord. Determine the velocity V_C of the slider as it moves from A to C. **(Ans: V = 0.974 m/s)**

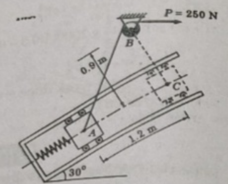


Fig.6.61

Q2. A 25 N collar is released from rest at A and travels along the smooth guide. Determine the speed when its centre reaches point C and normal force it exerts on the rod at that point. The spring has un-stretched length of 300 mm and point C is located just before the end of the curved portion. **(Ans: V_C = 3.796 m/s, Normal force = 94.55 N)**

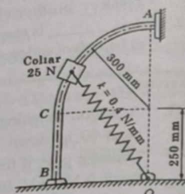


Fig.6.62

Q3. A collar has a mass of 5 kg can slide without friction on a pipe. If it is released from rest at the position B shown in the figure where the spring is un-stretched. What speed will the collar have after moving 50 mm to position C? Take spring constant as 2000 N/m. **(Ans: V_C = 0.39 m/s)**

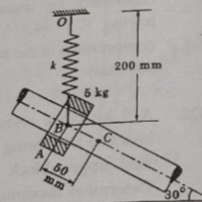


Fig.6.63

Q4. The 15-kg collar is released from rest in the position shown and slides with negligible friction up the fixed rod inclined at 30⁰ from the horizontal under the action of a constant force P = 200 N applied to the cable. Calculate the required stiffness 'K' of the spring so that maximum deflection of spring is equal to 180 mm. the position of the small pulley at B is fixed. **(Ans: K = 1957 N/m)**

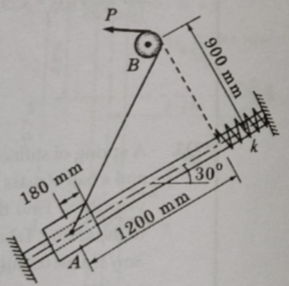


Fig.6.64

Learning from this Lecture: Learners will be able to apply concepts of springs in work energy

principle to find the various aspects of the given objects

Lecture 43

6.6 Problems on Work Energy Principle-II

Solved Problems

1. **The slider of mass 1 kg attached to a spring of stiffness 400 N/m and Unstretched length 0.5 m is released from A as shown in figure. Determine the velocity of the slider as it passes through B and C. Also compute the distance beyond C where the slider will come to the rest.**

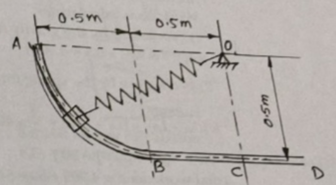
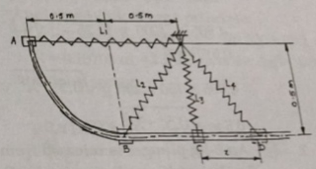


Fig.6.65



Sol Referring to the positions A, B, C and D as shown in the figures,

Unstretched length of the spring, L_0 = 0.5 m

For Motion A → B:

Length of the spring at position A of the block, $L_1 = (0.5 + 0.5) = 1$ m

Length of the spring at position B of the block, $L_2 = \sqrt{0.5^2 + 0.5^2} = 0.707$ m

Work done calculations:

1) Work done by the gravity force = m g h = 1 x 9.81 x 0.5 = 4.905 J

2) Work done by the spring force = $\frac{1}{2}$ k $[(L_1 - L_0)^2 - (L_2 - L_0)^2]$

$= \frac{1}{2} \times 400 \times [(1 - 0.5)^2 - (0.707 - 0.5)^2]$

= 41.4302 J

Total work done = 4.905 + 41.4302 = 46.335 J

Kinetic energy calculations:

$KE_1 = 0$

$KE_2 = \frac{1}{2} m V_2^2 = \frac{1}{2} \times 1 \times V_2^2 = 0.5 V_2^2$

Applying Work energy principle:

$U_{1\to2} = KE_2 - KE_1$

$46.335 = 0.5 V_2^2 - 0$

$\therefore V_2 = 9.62$ m/s (Velocity of slider at

Kinetic energy calculations:

$KE_1 = 0$

$KE_2 = \frac{1}{2} m V_2^2 = \frac{1}{2} \times 1 \times V_2^2 = 0.5 V_2^2$

Applying Work energy principle:

$U_{1\to2} = KE_2 - KE_1$

$54.905 = 0.5 V_2^2 - 0$

$\therefore V_2 = 10.48$ m/s (Velocity of slider at position C = 10.48 m/s)

For Motion A → D:

Let 'X' be the distance moved by slider beyond C and the slider comes to rest at D.

Length of the spring at position A of the block, $L_1 = (0.5 + 0.5) = 1$ m

Length of the spring at position D of the block, $L_4 = \sqrt{0.5^2 + X^2}$

Work done calculations:

1) Work done by the gravity force = m g h = 1 x 9.81 x 0.5 = 4.905 J

position $B = 9.62\ m/s$)

For Motion $A \rightarrow C$:

Length of the spring at position A of the block, $L_1 = (0.5 + 0.5) = 1$ m

Length of the spring at position C of the block, $L_3 = 0.5$ m

Work done calculations:

1) Work done by the gravity force = m g h = $1 \times 9.81 \times 0.5 = 4.905$ J

2) Work done by the spring force = $\frac{1}{2}$ k $[(L_1 - L_0)^2 - (L_3 - L_0)^2] = \frac{1}{2} \times 400 \times [(1 - 0.5)^2 - (0.5 - 0.5)^2] = 50$ J

Total work done = 4.905 + 50 = 54.905 J

Applying Work energy principle:

$U_{1\rightarrow2} = KE_2 - KE_1$

$4.905 + (200 \times [0.25 - (\sqrt{0.5^2+X^2} - 0.5)^2]) = 0$

$4.905 + (50 - 200\ [(\sqrt{0.5^2+X^2})^2 - 2 \times 0.5 \times \sqrt{0.5^2+X^2} + 0.5^2] = 0$

2) Work done by the spring force $= \frac{1}{2}$ k $[(L_1 - L_0)^2 - (L_4 - L_0)^2] = \frac{1}{2} \times 400 \times [(1 - 0.5)^2 - (\sqrt{0.5^2+X^2} - 0.5)^2]$

$= 200 \times [0.25 - (\sqrt{0.5^2+X^2} - 0.5)^2]$ J

Total work done = $4.905 + [200 \times [0.25 - (\sqrt{0.5^2+X^2} - 0.5)^2]]$ J

$= 4.905 + [50 - 200 (\sqrt{0.5^2+X^2} - 0.5)^2]$

Kinetic energy calculations:

$KE_1 = 0$, $KE_2 = \frac{1}{2} m V_2^2 = 0$

($V_2 = 0$ as slider comes to rest at position D)

$4.905 + (50 - 200\ [0.25 + X^2 - \sqrt{0.5^2+X^2} + 0.25]) = 0$

$4.905 + 50 - 50 - 200 X^2 + 200\sqrt{0.5^2+X^2} - 50 = 0$

Solving above equation we get

$X = 0.893$ m beyond C

2. **An 8-kg plunger is released from rest in the position shown in the figure and is stopped by two nested springs. The constant of the outer spring is $K_1 = 3$ kN/m and that of inner spring $K_2 = 10$ kN/m. Determine the maximum deflection of the outer spring.**

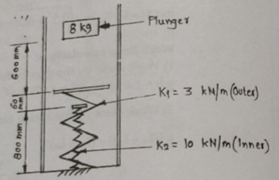


Fig.6.66

Sol Let 'X' be the deflection of the outer spring and hence (X - 0.06) m be the deflection in the inner spring.

Referring to the positions (1) and (2) of the plunger

Work done calculations:

Work done by gravity force = m g h = $8 \times 9.81 \times (0.6 + X) = 78.48\ (0.6 + X) = 47.088 + 78.48\ X$

2) Work done by the spring = $\frac{1}{2} K (x_1^2 - x_2^2)$

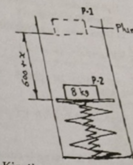
$= [\frac{1}{2} \times 3000 \times (0 - X^2)] + [\frac{1}{2} \times 10000 \times (0 - (X-0.06)^2)]$

$= -1500 X^2 - 5000 (X^2 - 0.12 X + 0.0036)$

$= -1500 X^2 - 5000 X^2 + 600 X - 18$

$= -6500 X^2 + 600 X - 18$

Total work done = $(47.088 + 78.48 X) + (-6500 X^2 + 600 X - 18) = -6500 X^2 + 678.48 X + 29.088$



Kinetic energy calculation:

$KE_1 = 0$ (As the plunger is at rest in position (1) so $V_1 = 0$)

$KE_2 = 0$ (As the plunger comes to rest in position (2) so $V_2 = 0$)

Applying Work energy principle:

$U_{1\rightarrow2} = KE_2 - KE_1$; $-6500 X^2 + 678.5 X + 29.1 = 0$

$\therefore X = 0.13703$ m = 137.03 m is the deflection in the outer spring

3. **A cylinder has a mass 20 kg and is released from rest when h = 0. Determine the speed when h = 3 m. Each spring has an unstretched length of 2 m.**

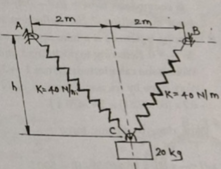


Fig.6.67

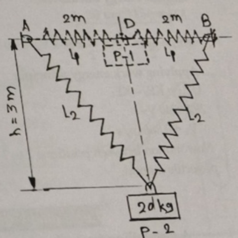
Sol Unstretched length of each spring, L_0 = 2 m

Length of each spring at position (1), $L_1 = 2$ m

Length of each spring at position (2),

$L_2 = AC = \sqrt{AD^2 + DC^2} = \sqrt{2^2 + 3^2} = 3.605$ m

Referring to the positions (1) and (2) shown in above figure,



Work done calculations:

1) Work done by gravity force = m g h
$= 20 \times 9.81 \times 3 = 588.6$ J
2) Work done by spring force
$= \frac{1}{2} k [(L_1 - L_0)^2 - (L_2 - L_0)^2]$
$= \frac{1}{2} \times 40 \times 2 [(2 - 2)^2 - (3.605 - 2)^2]$
$= - 103.041$ J
Total work done $= 588.6 + (- 103.041) =$ 485.559 J

Kinetic energy calculations:
$KE_1 = 0$
$KE_2 = \frac{1}{2} m V_2^2 = \frac{1}{2} \times 20 \times V_2^2 = 10 V_2^2$
Applying Work energy principle:
$U_{1\to2} = KE_2 - KE_1$
$485.559 = 10 V_2^2$
$V_2 = 6.968$ m/s

4. **A block of mass m = 80 kg is compressed against a spring as shown in figure. How far from point B (distance x) will the block strike on the plane at point A. Take free length of spring as 0.9 m and spring stiffness as $K = 40 \times 10^2$ N/m.**

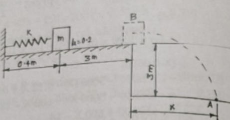
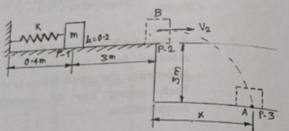


Fig.6.68



Sol Initially at position 1 of the block, spring is compressed to $x_1 = (0.9 - 0.4) = 0.5$ m and at position 2 the deflection in the spring is zero as it attains its free length, so $x_2 = 0$. Referring to positions 1 and 2,
Work done calculations: (from 1 → 2)
Work done by frictional force $= -\mu_K N S =$
$- 0.2 \times (80 \times 9.81) \times 3 = -470.4$ J
Work done by Spring force $= \frac{1}{2} k (x_1^2 - x_2^2) = \frac{1}{2} \times (40 \times 10^2)(0.5^2 - 0) = 500$ J
Total work done $= (- 470.4 + 500) = 29.6$ J
Kinetic energy calculations:
$KE_1 = 0$
$KE_2 = \frac{1}{2} m V_2^2 = \frac{1}{2} \times 80 \times V_2^2 = 40 V_2^2$
Applying work energy principle
$U_{1\to2} = KE_2 - KE_1$
$29.6 = 40 V_2^2$
$\therefore V_2 = 0.866$ m/s
Now motion between positions 2 to 3 is a projectile motion.

Considering vertical motion of the block from position 2 to 3 (MUG):
$S_Y = u t - \frac{1}{2} g t^2$
$\therefore -3 = V_2 t - (\frac{1}{2} \times 9.81 \times t^2)$
$\therefore -3 = (0.866 \times t) - (\frac{1}{2} \times 9.81 \times t^2)$
$\therefore t = 0.78$ sec
Now considering horizontal motion of the block from position 2 to 3 (UM):
$S_X = u t = V_2 t$
$\therefore X = 0.866 \times 0.78$
$\therefore X = 0.675$ m

Let's check the takeaway from this lecture

1. **A man falling from a height h starts rotating midway of his fall. The vertical velocity**

with which the man touches the ground will be …….. $\sqrt{(2gh)}$.
a) equal to b) less than c) greater than d) unpredictable

2. **In WE method, FBD of each particle is not required.**
a) True b) False

Exercise

Q1. Figure shows a collar of mass 20 kg which is supported on a smooth rod. The attached springs are un deformed when d = 0.5 m. Determine the speed of the collar after the applied force of 1000 N causes it to displace so that d = 0.3 m. the collar is at rest when d = 0.5 m.**(Ans: V = 4.6 m/s)**

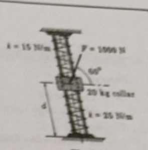


Fig.6.69

Q2. The system shown is in equilibrium when φ = 0. Knowing that initially φ = 90° and block C is given a slight nudge when the system is in that position, determine the velocity of block as it passes through the equilibrium position φ = 0. Neglect weight of the rod.**(Ans: V = 3.756 m/s)**

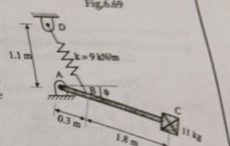


Fig.6.70

Practice Problems for the Day

Q1. A collar having a mass of 2 kg is attached to a spring and is sliding along a smooth circular rod. The spring constant is 5 N/cm. If the collar is released from rest at position B, find the velocity of the collar when it reaches at point C. Assume that circular rod lies in vertical plane. **(Ans: V = 3.5 m/s)**

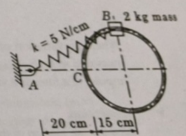


Fig.6.71

Q2. Find the velocity of block A and B when block A has travelled 1.2 m long along inclined plane. Mass of A is 10 kg and that of B is 50 kg. Coefficient of friction between block A and inclined plane is 0.25. Pulleys are massless and frictionless. Use work energy principle. **(Ans: 4.175m/s, 2.087m/s)**

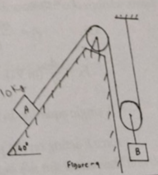


Figure-9

Fig.6.72

Learning from this Lecture: Learners will be able to apply the concepts of springs and work energy principle & kinematics of particles

Lecture 44

6.7 Introduction & Basic Problems on Impulse Momentum

Theory:

➢ **Impulse (J):**

When a large force acts over a short period of time, that force is called as Impulsive force.

The impulse of a force F acting over a time interval for t_1 to t_2 is defined by the integral,

$$J = \int_{t_1}^{t_2} F\, dt$$

The impulse of a force, therefore can be visualized as the area under the force vs. time diagram as shown in figure.

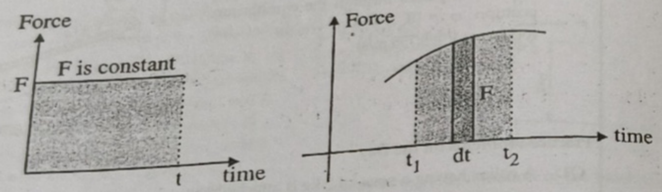


Fig.6.73 Impulse

When the variation of the force w.r.t. the time is unknown, the impulse can also be measured as $J = F_{average} \times \Delta t$

Impulse of a force is a vector quantity and has the unit of Newton second (Ns).

➢ **Momentum:**

Consider the motion of the particle of mass m acted upon by a force F.

The equation of motion of the particle in the x and y direction are,

$F_X = m\, a_X$ and $F_Y = m\, a_Y$

Or $F_X = \dfrac{m\, dv_x}{dt}$ and $F_Y = \dfrac{m\, dv_y}{dt}$

$F_X = \dfrac{d}{dt}(m\, V_X)$ and $F_Y = \dfrac{d}{dt}(m\, V_Y)$

A single equation in the vector from can be written as, $F = \dfrac{d}{dt}(m\, V)$which states that the force F acting on the particle is equal to the rate of change of momentum of the particle. The vector (m V) is called momentum or the linear momentum. It has the same direction as the velocity of the particle. The unit of momentum is

$m\, V = (kg)\left(\frac{m}{s}\right) = (kg)\left(\frac{m}{s^2}\right) s = N\, s$

➢ **Principle of Impulse (J) and Momentum(p):**

We know that by Newton's second law of motion,

$F = m \times a \therefore F = m \times \dfrac{dv}{dt}$

$\therefore F.\, dt = m\, .\, dt$

Integrating both sides, $\int_{t_1}^{t_2} F\, dt = m \int_u^v v\, dv$

Where $\int_{t_1}^{t_2} F\, dt$ = represents impulse

$\therefore$ Impulse (J) = m (v – u)

Impulse (J) = m v – m u

Impulse (J) = Final momentum – Initial momentum

Impulse (J) = change in momentum (Δp)

So, impulse-momentum principle states, "when an unbalanced force system is acting on a particle for a short interval of time, the impulse produced by all the impulsive forces is equal to change in momentum of the particle."

➢ **Working rules for Application of Impulse-momentum principle:**

1. Draw a free body diagram of particle showing all the impulsive forces acting during time interval t_1 to t_2.
2. Calculate impulse of each force.
3. To calculate impulse of each force use following concepts:
   1. When force is constant for finite interval of time
      $Impulse_{1\to 2} = F\,(t_2 - t_1)$
   2. When force is a function of time (F = F(t)]
      $Impulse_{1\to 2} = \int_{t_1}^{t_2} F\, dt$
   3. When force is very large and time interval is very small
      $Impulse_{1\to 2} = F \times \Delta t$
4. Calculate initial momentum (m u) and final momentum (m v)
5. Use Impulse-momentum principle in the direction of impulsive forces.

➢ **Conservation of Momentum:**

If the sum of impulses of the forces is zero then Impulse-Momentum equation becomes

Final momentum = Initial momentum

$\Sigma m\, v = \Sigma m\, u$

So, the total momentum of the particle is conserved.

The momentum is conserved when

1. Resultant of forces is zero
2. Time interval Δt is very small
3. All the external forces are non-impulsive.

The total momentum is conserved only in one direction but not in another direction.

Solved Problems

1. **Two men lined up at one end of a boat initially at rest and run in succession with velocity of 3m/sec relative to boat and dive off at the fore end. Neglecting resistance of water to horizontal motion of the boat, find its velocity after the second man dives. Each man weighs 750N and the boat weighs 4500N.**

Sol We know that,

Change in velocity, $\Delta V = mV/(m+M)$

Change in velocity of the boat, when first man dives off, is given by

$\Delta V_1 = (750x\ 3)/\ (750 + (750+4500)) = 0.3+75m/sec$

Similarly, change in velocity of the boat, when second man dives off, is given by,

$\Delta V_2 = (750x\ 3)/\ (750+4500) = 0.4286m/sec$

Velocity after second man dives = Initial velocity + $\Delta V_1 + \Delta V_2$

$= 0+0.375+0.4286$

$=0.8036m/sec$

2. **A block of wood weighing 10 kg is suspended as shown in figure. A bullet weighing 30 gms is fired with a velocity of 1200 m/s into the block. Calculate (i) the velocity of the block and bullet after the bullet becomes fully embedded in the block and (ii) the angle by which the string will swing from the vertical.**

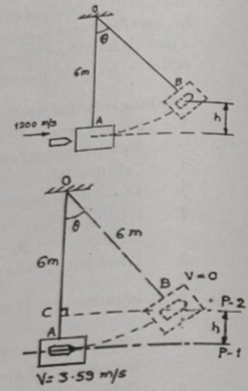


Fig.6.74

Sol Velocity of the block and bullet:

Mass of the wooden block, $m_1 = 10$ kg

Mass of the bullet, $m_2 = 0.03$ kg

Initial velocity of the wooden block, $u_1 = 0$

Initial velocity of the bullet, $u_2 = 1200$ m/s

Let V be the final velocity of block and bullet,

Applying Principle of conservation of momentum,

$m_1 u_1 + m_2 u_2 = (m_1 + m_2) V$

$(10 \times 0) + (0.03 \times 1200) = (10 + 0.03) V$

$\therefore$ Velocity of block and bullet after impact, V $= 3.59$ m/s

Angle of the string with vertical:

Let θ be the angular displacement of block and bullet before it comes to rest. Let 'h' be the rise in the vertical height after impact.

Applying conservation of energy principle,

$(KE + PE)_1 = (KE + PE)_2$

$\frac{1}{2} \times (10 + 0.03) \times (3.59)^2 + 0$

$= 0 + (10 + 0.03) \times 9.81 \times h$

$\therefore h = 0.657$ m

From ΔOBC,

$\cos\theta = \frac{OC}{OB} = \frac{6-0.657}{6} = 0.8905$

$\therefore \theta = 27.06^0$

3. **Two boxes are placed on an incline. $\mu_s = 0.35$ and $\mu_K = 0.22$ between the incline and box A and $\mu_s = 0.2$ and $\mu_K = 0.15$ between the incline and box B. the boxes are in contact when released, determine 1) velocity of each box after 2.5 sec,2) the force exerted by A on B.**

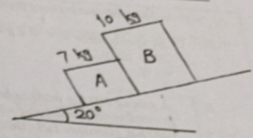
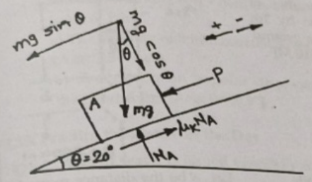
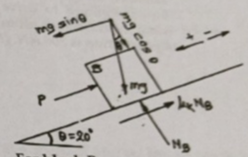


Fig.6.75

Sol





For block A:

by impulse momentum theorem,

$I_{1\to2} = M_2 - M_1$

Σ F. t = m (v – u)

$(P + mg \sin\theta - \mu_K N_A)\ t = m\ (v - u)$

$[P + (7 \times 9.81 \times \sin 20) - (0.22 \times 7 \times 9.81 \times \cos 20)]\ t = 7\ (v - 0)$

$(P + 9.29) \times 2.5 = 7\ v$

$\therefore P + 9.29 = 2.8\ v$ ---------------------(I)

For block B:

$I_{1\to2} = M_2 - M_1$

$(mg \sin\theta - P - \mu_K N_B)\ t = m\ (v - u)$

$(mg \sin\theta - P - \mu_K\ m\ g \cos\theta)\ t = m\ (v - u)$

$[(10 \times 9.81 \times \sin 20) - P - (0.15 \times 10 \times 9.81 \times \cos 20)]\ 2.5 = 10\ (v - 0)$

$19.72 - P = 4\ v$ ——————(II)

Solving equations (I) and (II), we get

$v = 4.266$ m/s and $P = 2.656$ N

$\therefore v_A = v_B = 4.266$ m/s

Force existing between the blocks P = 2.656 N

4. **A 20-gm bullet is fired horizontally into 300gm block which rests on the smooth surface. After bullet becomes embedded into the block, the block moves to the right 0.3m before momentarily coming to rest, determine the speed of the bullet. The spring has stiffness of k =200 N/m and is originally unstretched.**

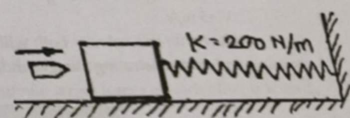


Fig.6.76

Sol For plastic impact between bullet and block

Bullet Block

$m_1 = 0.02$ Kg $\qquad m_2 = 0.3$ Kg

$u_1 = u_2 = 0$

'v' velocity will be same after the impact.

By Law of Conservation of Momentum for plastic impact

$m_1.u_1 + m_2.u_2 = m_1.V_1 + m_2.V_2$

$0.02\ u_1 = 0.32\ v$ ----------(I)

Energy calculation

KE1 $= \frac{1}{2}\ (m_1 + m_2)\ v^2$ (just after impact)

$= 0.16 \times v^2$

KE2 = 0

$U1\to2 = KE_2 - KE_1$

$-9 = 0 - 0.16 \times v^2$

$v = 7.5$ m/s

Solving eq (I), $u_1 = 120$ m/s Speed of bullet as it strikes the block

Now by work-energy principle, Work-done by spring force = $\frac{1}{2} \times k \times (x_1^2 - x_2^2) = \frac{1}{2} \times 200 \times (0 - 0.3^2)$
= - 9J.

5. **A pile of 400 Kg mass is being driven in to ground with the help of a hammer of mass 1000 Kg. Hammer falls through a height of 2.5 m. Assuming plastic impact between hammer and Pile, find the number of blows required to drive the pile by 1m when the resistance offered by the ground to penetration is 300 KN. [May 11][10 M]**

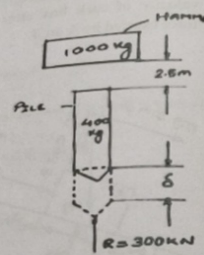


Fig.6.77

Sol Consider impact between hammer and pile,
Hammer Pile
m_h=1000kg, m_p = 400 kg

$u_m = \sqrt{2gh}$

Before impact velocity of pile u_p =0

$= \sqrt{2 \times 9.81 \times 2.5} = 7$ m/s

After impact both pile & hammer move together (plastic impact) so,
By law of conservation of momentum,

$m_h u_h + m_p u_p = m_h v_h + m_p v_p$

$m_h u_h + m_p u_p = (m_h + m_p) v$

$7 \times 1000 \times g + 0 = 1400 \times g \times v$

V =5 m/s

Now with this velocity both will move down and start penetrating into ground.

Let δ be the distance moved by pile in one blow Using work –energy principle just after impact

W.D. Calculation.

W.D. by gravity = m g h = 1400 × 9.81 × δ

W.D. by soil resistance = - 300 × 10^3 × δ

$K.E_1 = \frac{1}{2} \times 1400 \times 5^2$

$K.E_2 = 0$

By work energy principle, U = KE_1-KE_1

$1400 \times 9.81 \times \delta - 300 \times 10^3 \times \delta = 0 - \frac{1}{2} \times 1400 \times 5^2$, δ =0.0611m

Number of blows for '1m' = $\frac{1}{0.061}$ = 16.35

≅ 17 blows

Let's check the takeaway from this lecture

1. **A bullet fired into a target loses half its velocity after penetrating 25cm. how much further will it penetrate before coming to rest.**
   a) √25cm b)25cm c)8.3cm d)75cm
2. **A large force acting on a particle during a short interval of time is known as**
   a) Impact b) Impulsive force c) Torque
3. **The total momentum conserved only in one direction but not in another direction**
   a) True b) False

Exercise

Q1. A 900-kg car travelling at 48km/hr. couples to a 680-kg car travelling at 24km/hr. in the same direction. What is their common speed after coupling? What is the loss in K.E? **(Ans: $V_{coupled}$ = 10.464m/sec, 8.61KJ)**

Q2. Block A is released from rest in the position shown and slide without friction until it strikes the ball B of a simple pendulum. Knowing the coefficient of restitution between A and B as 0.9. Radius of the curve is 0.6m. Find.
a) The velocity of B immediately after impact **(Ans: 2.507m/s)**
b) The maximum angular displacement of the pendulum. **(Ans: θ=49.9°)**

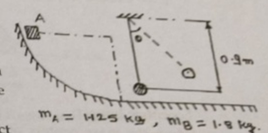


Fig.6.78

L58: Practice Problems for the Day

Q1. A jet of water is issued vertically upwards at a speed of 6m/sec from a nozzle having area of opening of 4mm². If a ball of mass 10gms is perfectly balanced on this jet at a height h above the nozzle, find this height 'h'. **(Ans: 0.983m)**

Q2. A jet of water 5cm in diameter moving with 24 m/s velocity horizontally, strikes a flat vertical plate. If after striking water flows parallel to the plate, find the force exerted on the plate by the jet of water. **(Ans: 1.131kN)**

Learning from this Lecture: Learners will be able to concepts of Impulse-Momentum

Lecture 45 (Part – A)

6.8 *Introduction & Problems on Impact*

Theory:

- **Impact:**
  A collision of two bodies which takes place during a very small interval of time and during which the colliding bodies exert relatively larger forces on each other is known as impact.

- **Line of Impact:**
  When two bodies collide, the line joining the common normal of the colliding bodies is known as line of impact.

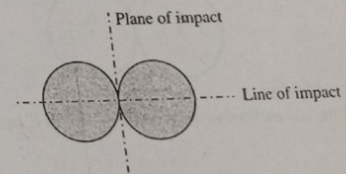


Fig.6.79 Line of Impact

➢ **Central Impact:**
When the mass centers of the colliding bodies lie on the line of impact, the impact is said to be Central Impact.

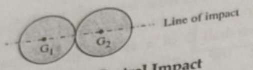


Fig.6.80 Central Impact

➢ **Eccentric Impact:**
When the mass centers of the colliding bodies do not lie on the line of impact, the impact is said to be an Eccentric Impact.

➢ **Direct Central Impact:**
If the velocities of the two particles are along the line of impact, the impact is called as direct central impact.

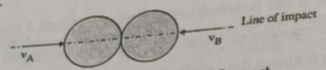


Fig.6.81 Direct Central Impact

➢ **Oblique Central Impact:**
If both particles move along a line other than the line of impact i.e. the velocities of one or both the bodies are not directed along the line of impact.

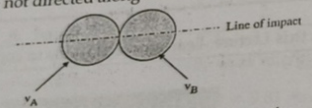


Fig.6.82 Oblique Central Impact

➢ **Deformation and time of deformation:**
During the process of impact both the bodies deform. Deformation is the change in shape and size. The process of deformation continues for a short interval of time known as time of deformation. This is the time from first contact to maximum deformation.

➢ **Restoration and period of restoration:**
Immediately after deformation both bodies tend to regain their original shape and size. This process is called restoration. At the end of restoration two bodies either regain their original shapes fully or partially or they remain permanently deformed.

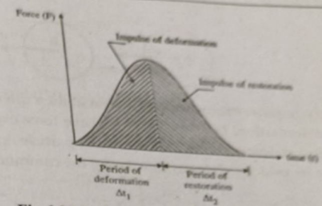


Fig.6.83 Deformation and Restoration Period

➢ **Coefficient of Restitution (e): (Direct Central Impact)**
This can be explained with the help of understanding phenomenon of direct central impact.
Consider two particles A and B with velocities V_A and V_B. If V_A is greater than V_B, the impact will soon take place.

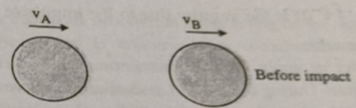


Fig.6.84 Restitution Direct Central Impact (a)

The period of impact is made of period of deformation and period of restoration.
During the period of deformation, the two particles exert large impulsive force on each other. The deformation of both the particles continue till maximum deformation. At this stage both the particles are said to have momentarily united and move with common velocity V.

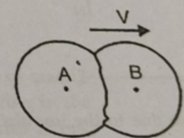


Fig.6.85

Now the period of restitution begins. The two particles restore their shape during this period. Sometimes permanent deformations are also set in the particles. During this period, the impulsive force exerted by the two particles is lesser than during the period of deformation. At the end of the period of restoration, the two particles separate from each other.

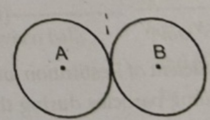


Fig.6.86

The two particles A and B will now have new velocities V_A' and V_B' respectively.

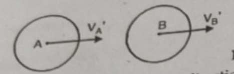


Fig.6.87

Let us apply Impulse Momentum equation in the x direction to the particle A during the period of deformation. Let P be the impulsive force exerted by particle B on A. At the start of period of deformation, the velocity of particle A is v_A and at the end of period of deformation its velocity changes to v, being the common velocity of A and B.

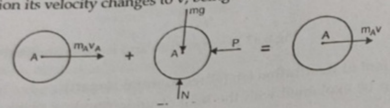


Fig.6.88

$m\,V_1 + \text{Impulse}_{1\text{-}2} = m\,V_2$

$m_A V_A - \int P\,dt = m_A V$ ----------------(I)

here $\int P\,dt$ is the impulse due to the impulsive force P acting during the period of deformation.

Now let us apply impulse momentum equation to particle A during the period of restitution. During this period, a smaller impulsive force say R be exerted by particle B on A. At the start of period of restitution, the velocity of A is V and changes to V_A' at the end of the period of restitution.

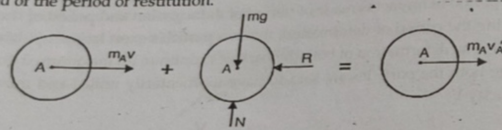


Fig.6.89

$m\,V_1 + \text{Impulse}_{1\text{-}2} = m\,V_2$

$m_A V - \int R\,dt = m_A V_A'$ ----------------(II)

here $\int R\,dt$ is the impulse due to the impulsive force R acting during the period of restitution.

From equations (I) and (II), we have

$$\frac{\int R\,dt}{\int P\,dt} = \frac{v - v_A'}{v_A - v}$$

$$\text{If } e = \frac{\int R\,dt}{\int P\,dt} \text{ then, } e = \frac{v - v_A'}{v_A - v} \quad \text{----------------(III)}$$

e is referred to as the coefficient of Restitution and is defined as the ratio of the impulse exerted between the colliding particles during the period of restitution to the impulse exerted during the period of deformation.

Similarly, if the impulse momentum equation is applied to the particle B,

$$\text{we get } e = \frac{v_B' - v}{v - v_B} \quad \text{----------------(IV)}$$

from basic algebra if $A = \frac{B}{C} = \frac{D}{E}$, then $A = \frac{B+D}{C+E}$

from equations (III) and (IV) we therefore have,

$$e = \frac{v - v_A' + v_B' - v}{v_A - v + v - v_B}$$

$$e = \frac{v_B' - v_A'}{v_A - v_B} \quad \text{----------------(V)}$$

above equation is referred to as Coefficient of restitution.

The value of coefficient of restitution lies between 0 and 1. It mainly depends on the nature of the bodies of collision.

➢ **Procedure to solve problems on Direct Central Impact:**

Given velocities before impact i.e. V_A and V_B and coefficient of restitution e and to find velocities after impact.

1) During impact since there are no external forces acting on the colliding bodies, the momentum of the system is conserved i.e. conservation of momentum is applicable, which is

   Initial momentum = Final momentum

   $\therefore\ m_A V_A' + m_B v_B' = m_A V_A + m_B V_B$ (→ +ve and ← -ve)

2) Use coefficient of restitution equation i.e.

$$e = \frac{V_B' - V_A'}{V_A - V_B}$$

3) Solve two equations to find velocities after impact V_A' and V_B'.

Solved Problems

1. **Three perfectly elastic balls of mass 2 kg, 6 kg and 12 kg are moving in the same direction with velocities 12 m/s, 4 m/s and 2 m/s respectively. If the ball of mass 2 kg strikes with ball of mass 6 kg and then 6 kg strikes with 12 kg. Prove that balls of mass 2 kg and 6 kg will be brought to rest by these impacts.**

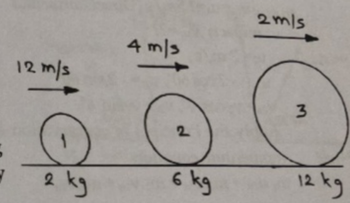


Fig.6.90

Sol First consider impact between balls (1) and (2)

We have, $m_1 = 2$ kg

$m_2 = 6$ kg

$u_1 = 12$ m/s

$u_2 = 4$ m/s

$v_1 = ?$

Now consider impact between balls (2) and (3)

by law of conservation of momentum (→) +ve and (←) - ve

$m_2 u_2' + m_3 u_3 = m_2 v_2' + m_3 v_3$

$(6 \times 8) + (12 \times 2) = 6 v_2' + 12 v_3$

$V_2' + 2 v_3 = 12$ ---------------- (III)

v_2 = ? (v_1 and v_2 are the velocities just after impact)

by law of conservation of momentum (→ +ve and ← - ve)

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$(2 \times 12) + (6 \times 4) = (2 \times v_1) + (6 \times v_2)$

$v_1 + 3 v_2 = 24$ ---------------- (I)

Now for elastic iimpact,

Coefficient of restitution, $e = 1 = \frac{v_2 - v_1}{u_1 - u_2}$

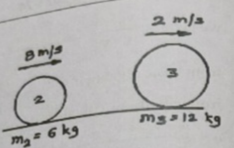
$v_2 - v_1 = u_1 - u_2$

$v_2 - v_1 = 12 - 4$

$v_2 - v_1 = 8$ ---------------- (II)

Solving equations (I) and (II) we get, $v_1 = 0$ and $v_2 = 8$ m/s →

after 1st impact ball (1) is brought to rest and ball (2) will be moving towards right with new velocity and strikes ball (3).



Coefficient of restitution, $e = 1 = \frac{v_3 - v_2'}{u_2 - u_3}$

$v_3 - v_2' = 8 - 2$

$v_3 - v_2' = 6$ ---------------- (IV)

Solving equations (III) and (IV)

We get $v_2' = 0$ and $v_3 = 6$ m/s →

After 2nd impact ball (2) is brought to rest and ball (III) is moving with new velocity of 6 m/s towards right.

So, the balls (1) and (2) are brought to rest after impacts.

2. **Two billiard balls of equal mass collide with velocities u_1=1.5 m/s & u_2=2m/s. Find the velocity of balls after impact and percentage loss in kinematic energy. Coefficient of restitution is 0.9.**

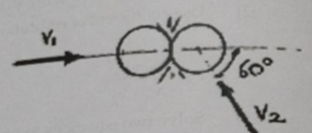


Fig.6.91

Sol Given

$u_{1n}=u_1$=1.5m/s (Direct horizontal impact) ,v_{1n} =??

u_2= 2 m/s ,

u_{2n}= - 2 cos 60°, u_{2t}= - 2 sin 60°

v_{2n}= v_2 cos θ°, v_{2t}= v_2 sin θ°

Apply the Principal of conservation of momentum equation

$m_1.u_{1n} + m_2.u_{2n} = m_1.v_{1n} + m_2.v_{2n}$

$m_1 = m_2 = m$,

$1.5 + (-2\cos 60°) = v_{1n} + v_2 \cos\theta$

$v_{1n} + v_2 \cos\theta = 1$……………………(i)

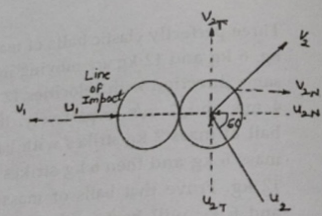
$e = (v_{2n} - v_{1n}) / (u_{1n} - u_{2n})$

$0.9 = (v_2 \cos\theta - v_{1n})/(1.5 - (-2\cos 60°))$

$v_2 \cos\theta - v_{1n} = 2.25$…………(ii)

On solving eq (i) & (ii), we get

v_{1n} = -0.625 m/s



$$v_2 = \sqrt{(v_{2t})^2 + (v2_n)^2}$$

$$= \sqrt{1.732^2 + 1.625^2} = 2.37 \text{ m/s}$$

$$\theta = \tan^{-1}\frac{v_{2t}}{v_{2n}} = \tan^{-1}\frac{1.732}{1.625} = 46.82°$$ Now

Loss in K.E. = Initial K.E. - Final K.E.

v_{2n}= v_2 cos θ= 1.625 m/s

Now we know that tangential component of Velocity in impact will remain same.

So,

$u_{1t} = v_{1t} = 0$

$u_{2t} = v_{2t}$ = - 2 sin 60 = 1.732 m/s

$$\beta_1 \sqrt{(v_{Bn})^2 + (v_{Bt})^2}\,\beta_2 \sqrt{\frac{|v_{Bt}|}{|v_{Bn}|}}^2 + \sqrt{(6.41 \times}$$

$$v_1 = \sqrt{(v_{1t})^2 + (v_{1n})^2} = \sqrt{0 + (-0.625)^2}$$

=0.625m/s

$= [m_1 u_1^2 + m_2 u_2^2] - [m_1 v_1^2 + m_2 v_2^2]$

$= [m \times 1.5^2 + m \times 2^2] - [m \times 0.625^2 + m \times 2.37^2]$

Percentage loss in K.E.=

$$\frac{\text{Initial KE - Final KE}}{\text{Initial KE}} \times 100 = $$

$$\frac{\frac{1}{2}[m \times 1.5^2 + m \times 2^2] - \frac{1}{2}[m \times 0.625^2 + m \times 2.37^2]}{\frac{1}{2}[m \times 1.5^2 + m \times 2^2]}$$

= 3.87 %

3. **Two identical balls of 120 gm collide when they are moving with velocities as shown in figure. Determine the velocities of ball A and B completely after the impact. Take e = 0.8**

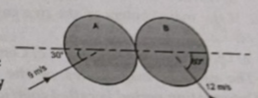


Fig.6.92

Sol In this problem line of impact is along Y-axis, normal to the plane of impact is along X-axis.

As given;

e = 0.8

u_{An} = 9 cos30 = 7.79 m/s

u_{At} = 9 sin30 = 4.5 m/s

u_{Bn} = - 12 cos60 = - 6 m/s

u_{Bt} = 12 sin60 = 10.39 m/s

Let v_{An}, v_{At}, v_{Bn} and v_{Bt} be the component of velocity after collision.

Applying momentum equation and restitution equation along line of impact, X-direction;

Momentum equation

$m_A.u_{An} + m_B.u_{Bn} = m_A.v_{An} + m_B.v_{Bn}$

$0.12 \times 7.79 + 0.12 \times -6 = 0.12 \times v_{An} + 0.12 \times v_{Bn}$

$v_{An} + v_{Bn} = 1.79$ -------(I)

Restitution equation

$e = (v_{Bn} - v_{An}) / (u_{An} - u_{Bn})$

$0.8 = (v_{Bn} - v_{An}) / (7.79 - (-6))$

$v_{Bn} - v_{An} = 11.03$ -------(II)

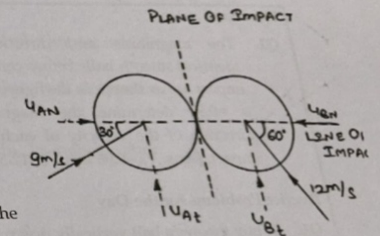
Solving equation (I) & (II)

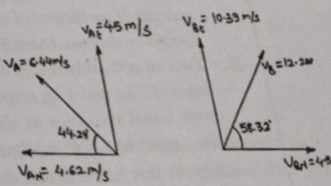
v_{Bn} = 6.4 m/s (→) v_{An} = - 4.62 m/s

= 4.62 m/s (←)

In the normal direction (Y-direction)

i.e. along plane of impact





$$\beta_1 = \tan^{-1}\frac{|v_{At}|}{|v_{An}|} = 44.24°$$

$$\text{Similarly}, v_B = \sqrt{(v_{Bn})^2 + (v_{Bt})^2}$$

$$= \sqrt{(6.41 \times 2) + (10.39 \times 2)}$$

= 12.2082 m/s

And $u_{Ai} = v_{Ai} = 4.5$ m/s (↑)

$u_{Bt} = v_{Bt} = 10.39$ m/s (↑)

Now, $v_A = \sqrt{(v_{An})^2 + (v_{At})^2}$

$= \sqrt{(-4.62)^2 + (4.5)^2}$

$= 6.4494$ m/s

$\beta_2 = \tan^{-1}\frac{|v_{Bt}|}{|v_{Bn}|} = 58.32°$

Velocities of sphere A and B after impact is

Let's check the takeaway from this lecture

1. If the velocities of two particles are along the line of impact, the impact is called as ....impact.
   a) Direct b) Oblique c) Central
2. If the mass centers of the two colliding bodies lie on line of impact, the impact is..................
   a) Direct b) Oblique c) Central
3. The ratio of magnitudes of the impulse during the restoration period and Deformation period is called coefficient of ....
   a) Friction b) Restitution c) Restoration

Exercise

Q1. The magnitude and direction of two identical smooth balls before central oblique impact are as shown in the figure. Assuming e =0.90 determine the magnitude and direction of the velocity of each ball after impact. **[Ans. V_A=6.96 m/s V_B=12.58m/s]**

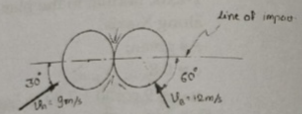


Fig.6.93

Practice Problems for the Day

Q1. A boy throws a ball vertically downwards from a height of 1.5 m. He wants the ball to rebound from floor and just touch the ceiling of room which is at a height of 4 m from ground. If coefficient of restitution e is 0.8, find the initial velocity with which the ball should be thrown. **(Ans: 9.654m/s)**

Q2. Two smooth spheres A and B having a mass of 2 kg and 4 kg respectively collide with initial velocities as shown in fig. If the coefficient of restitution for the spheres is e = 0.8. find the velocities of each sphere after collision. **(Ans: 2.923m/s & 20°, 3.472m/s & 86.053°)**

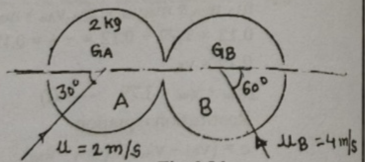


Fig.6.94

Lecture 45 (Part - B)

6.9 Problems on Impact

Theory:

➤ **Impact with infinite mass:**

When a body of small mass collides with a body of very large mass as compared to first, the impact is considered as impact with infinite mass.

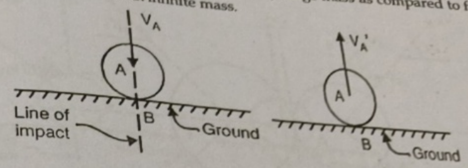
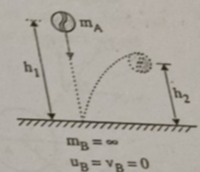


Fig.6.95

For example, when a ball is dropped on a hard floor. There is a loss in kinetic energy due to impact.



Coefficient of restitution, $e = \frac{V_B' - V_A'}{V_A - V_B}$

But velocity of floor before and after impact is zero.

$\therefore V_B = V_B' = 0 \ \therefore \ e = \frac{-V_A'}{V_A}$

(- ve sign indicates that the motion of the ball after impact is in the opposite direction)

But velocity of ball just before impact when it is dropped through height h_1, $V_A = \sqrt{2gh_1}$ (↓)

If after impact ball rises back to height h_2, its velocity after impact is given by, $V_A' = \sqrt{2gh_2}$ (↑)

$\therefore e = \frac{-V_A'}{V_A} = -\left(\sqrt{\frac{2gh_2}{2gh_1}}\right)$ (↓ + ve) (↑ - ve)

$e = \sqrt{\frac{h_2}{h_1}}$ where h_1 = height just before impact

h_2 = height after impact

Relation between 'e' and height of bounce 'h':

Let a ball be dropped from a height 'h' on the ground. Let 'h' be the height of rebound after 'n' bounces. If 'e' is the coefficient of restitution between the ball and the ground, then 'e' is related to 'h' as $e = \sqrt{\frac{h_2}{h_1}}^{1/n}$

➤ **Oblique Central Impact:**

When the velocities of either one or both colliding particles are not directed along the line of impact, the impact is said to be Oblique central impact. In such impact, not only the magnitude of velocities after impact are unknown, but the new direction of travel is also unknown and need to be worked out.

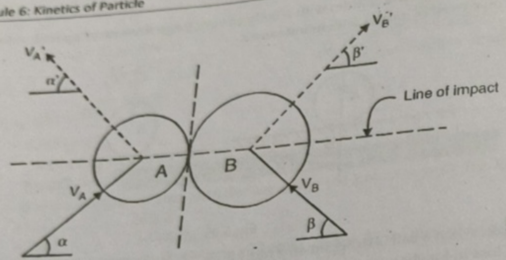


Fig.6.96

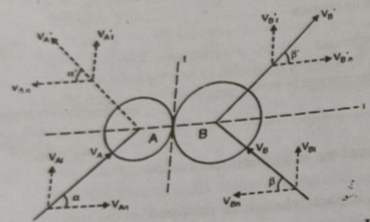


Fig.6.97

In an oblique central impact, the impulsive force acts along the line of impact. Thus, the velocity changes only along the line of impact and no change in velocity takes place in a direction perpendicular to the line of impact.

Given initial velocities of the particle A and B to be V_A at an angle α and V_B at an angle β, coefficient of restitution 'e'.

To find velocities $V_A'$and V_B' and new anglesα' and β'after impact following steps are followed:

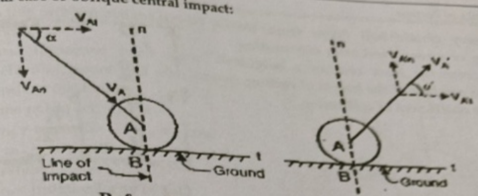
1. Let the line of impact be now called as 'n' direction of impact. Let 't' be the direction perpendicular to the 'n' direction.
2. Resolve the initial velocities V_A and V_B along 'n' and 't' directions so as to get their components V_{An}, V_{Bn}, and V_{At} , V_{Bt} .
3. Work as direct impact problem in the 'n' direction, taking components V_{An}, V_{Bn} as initial velocities. Using conservation of momentum and coefficient of restitution equations, find velocity components $V_{A'n}$, $V_{B'n}$ after impact.
4. Work in the 't' direction. Since velocities do not change in the 't' direction, we have $V_{A't} = V_{At}$ and $V_{B't} = V_{Bt}$
5. The magnitudes of velocities after impact are therefore given as

$$V_A' = \sqrt{(v_{A\,t}')^2 + (v_{A\,n}')^2} \quad \text{and } V_B' = \sqrt{(v_{B\,t}')^2 + (v_{B\,n}')^2}$$

The new direction of velocity is given as

$$\alpha' = \tan^{-1}\frac{V_{A\,t}'}{V_{A\,n}'} \text{ and } \beta' = \tan^{-1}\frac{V_{B\,t}'}{V_{B\,n}'}$$

➤ **Special case of oblique central impact:**



Before Impact **After Impact**

Fig.6.98

When a ball has oblique central impact with a rigid infinite mass such as a rigid wall or ground, apply coefficient of restitution equation in the 'n' direction of impact, we get

$V_{B\,n}' - V_{A\,n}' = e\,(V_{An} - V_{Bn})$

$0 - V_{A\,n}' = e\,(V_{An} - 0)$ if $V_{Bn} = V_{B\,n}' = 0$

So $V_{A\,n}' = -\,e\,V_{An}$

Since velocities do not change in the t direction, we get

$V_{A\,t}' = V_{At}$

a) **If the two bodies are of equal masses:$m_1=m_2=m$ & $V_1 = U_2$ and $V_2 = U_1$.**
Thus if two bodies are of equal masses undergo elastic collision in one dimension, then after the collision, the body will exchange their velocities

b) **If two bodies are of equal masses and second body is at rest:**
$m_1 = m_2$ and initial velocity of second body $U_2 = 0$ & $V_1=0$, $V_2=U_1$
When body A collides against body B of equal mass at rest, the body A comes to rest and body B moves on with the velocity of the body A. In this case transfer of energy is hundred percent. Eg: Billiards Ball"

c) **If the mass of the body is negligible as compared to other:**
$m_1>>m_2$ and $U_2 = 0$; $V_1 = U_1$ and $V_2=2U_1$
When heavy body A collides against a light body B at rest, the body A should keep on moving with same velocity and the body B will move with velocity double that of A.
If $m_2>>m_1$ and $U_2 = 0$ then $V_2 = 0$
When light body A collides against a heavy body B at rest, The body A should start moving with same velocity just in opposite direction while the body B should practically remain at rest.

Solved Problems

1. **A heavy elastic ball drops from the ceiling of a room and after rebounding twice from the floor reaches a height equal to one half of the height of ceiling. Find the coefficient of restitution.**

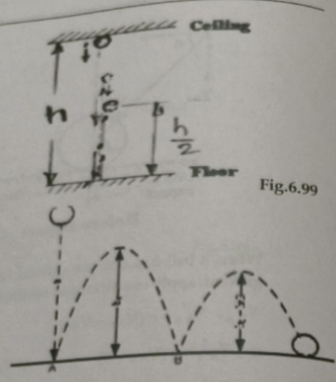


Fig.6.99

Sol At first impact (A), the coefficient of restitution;

$$e_A = \frac{-v_1}{u_1} = \sqrt{\frac{h_1}{h}}$$

i.e. $h_1 = h \sqrt{\frac{v_B - v_A}{u_A - u_B}} \; 1/2 \quad . \quad e^2$ ——————eq(I)

At Second impact (B)

$$e_B = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{h/2}{h_1}}$$

i.e. $e^{2} \times h_1 = \sqrt{\frac{h}{2}}$

$e_A = e_B = e$

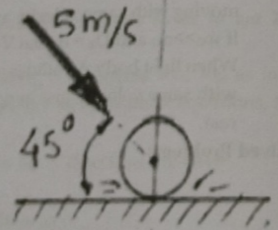
So $h1 = e^{2} \times h$

So from eq (I)

$e^4 . h = \frac{h}{2}$

so $e^4 = 0.2$; $e = 0.841$

2. **A billiard ball moving with a velocity of 5 m/sec strikes a smooth horizontal plane at an angle of 45° with horizontal. If the coefficient of restitution is 0.6, what is the velocity with which the ball rebounds?**



Sol In this problem oblique impact between a body

And horizontal fixed plane. Resolving velocities before and after impact along and normal to the line of impact, we get

$u_{1t} = u_1 \cos 45 \qquad v_{1t} = v_1 \cos\theta$

$u_{1n} = - u_1 \sin 45 \qquad v_{1n} = v_1 \sin\theta$

Where,

u_1= Initial velocity of ball

v_1= Final velocity of ball

u_2= Initial velocity of horizontal plane = 0

v_2= Final velocity of horizontal plane = 0

Now applying restitution equation along

line of impact $e = \frac{-V_{1n}}{u_{1n}}$

i.e. $e = \frac{-v_1 \sin\theta}{-u_1 \sin 45} = 0.6$

$v_1 \sin\theta = 2.1216$ ——————(I)

Now along plane of impact

$u_{1t} = v_{1t}$

$v_1 \cos\theta = 3.536$ ——————(II)

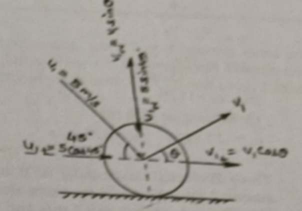


Fig.6.100

Dividing (I) & (II)

$\tan\theta = \frac{2.1216}{3.536}$

$\theta = 30°$

$v_1 \sin 30 = 2.1216$

$v_1 = 4.124$ m/s

Ball rebounds with a velocity v = 4.124 m/s at an angle of 30°

3. **A 2kg sphere A is moving to left with a velocity of 15 m/s when it strikes the vertical face of 4 kg block B which is at rest. The block B is supported on rollers and is attached to a spring of spring of constant k =5000 N/m as shown in figure. If e = coefficient of restitution for the block and the sphere =0.75, determine the maximum compression shortening of the spring due to the impact. Neglect friction.**

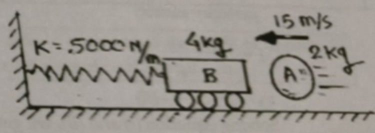


Fig.6.101

Sol Consider the direct impact between sphere and block.

Apply momentum and restitution equation

Momentum equation

$m_A.u_A + m_B.u_B = m_A.v_A + m_B.v_B$

$2.15 \times 15 + 4 \times 0 = 2 \times v_A + 4 \times v_B$

$v_A + 2v_B = 15$ ………….(i)

Restitution equation

Now the spring get compressed velocity of the block B is zero when maximum compression of the spring takes place K.E. is converted into strain energy of spring.

Let x in meters to be maximum compression of the spring by energy principle

$K.E._2 - K.E._1 = \frac{1}{2} k . - \frac{1}{2} k .$

$$e = \frac{v_B - v_A}{u_A - u_B}$$

$$0.75 = \frac{v_B - v_A}{15 - 0}$$

$v_B - v_A = 11.25$..............(ii)

On solving eq (i) & (ii)

$v_A = -2.5\ m/s \quad v_B = 8.75\ m/s$

K.E.$_1 = 0, x_1 = 0$

$\frac{1}{2} m_B = \frac{1}{2} k x^2$

$X = 0.2474$ m maximum compression of the spring

4. Three Perfectly elastic balls A, B and C masses 2 kg ,4 kg and 8 kg move along a line with velocities 4 m/s,1 m/s and 0.75 m/s respectively. If the ball A strikes with ball B which in turn strikes ball C, determine the velocities of the three balls after impact.

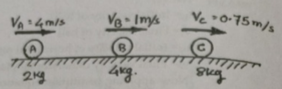


Fig.6.102

Sol Consider impact between A and B. Applying **Principal of Conservation of momentum and coefficient of restitution equation.**

$m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$2 \times 4 + 4 \times 1 = 2v_A + 4\ v_B$

$v_A + 2\ v_B = 6$ ------(i)

$$e = \frac{v_B - v_A}{u_A - u_B}$$ where e= 1 (As balls are perfectly elastic)

$$1 = \frac{v_B - v_A}{u_A - u_B}$$

$v_B - v_A = 3$ ------(ii)

Solving equation (i) and (ii), we get $v_A = 0$ and $v_B = 3\ m/s(\rightarrow)$

After impact ball A comes to rest, ball B moves with velocity $v_B = 3\ m/s$ which will be new initial velocity for the impact between ball B & Ball C.

Now consider impact between B & C.

Principal of Conservation of momentum and coefficient of restitution equation.

$m_B u_B + m_C u_C = m_B\ V_B' + m_C v_C$

$4 \times 3 + 8 \times 0.75 = 4 \times V_B' + 8 \times v_C$

$2\ V_B' + 4\ v_C = 9$ ------(iii)

$$e = \frac{v_C - v_B'}{u_B - u_C}$$ where e= 1 (As balls are perfectly elastic)

$$e = \frac{v_C - v_B'}{3 - 0.75}$$

$v_C - V_B' = 2.25$ ------(iv)

Solving equation (iii) and (iv),we get

$V_B' = 0$, $v_C = 2.25\ m/s$

After impact ball B comes to rest, ball C moves with velocity $v_C = 2.25\ m/s$

After impact velocity of ball A = 0

Velocity of ball B = 0

Velocity of ball C = 2.25m/$\sum(\rightarrow)$

Let's check the takeaway from this lecture

In the plastic impact, coefficient of restitution, e =...............

a) 1 b) $0 < e < 1$ c) 0

In the semi elastic impact, coefficient of restitution, e =...............

a) 1 b) $0 < e < 1$ c) 0

The time taken by the bodies to regain original shape, after compression, is

a) time of compression b) time of restitution c) time of collision

Exercise

Q1. A small steel ball is to be projected horizontally such that it bounces twice on the surface and lands into a cup placed at 8 m as shown. If the coefficient of restitution for each impact is 0.8, determine the velocity of projection 'u' of the ball. (Ans: 3.231 m/s)

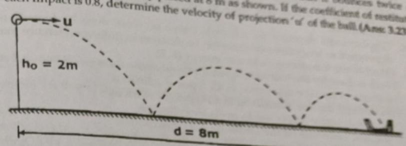


Fig.6.103

Practice Problems for the Day

Q1. Two masses A (8 kg), B (2 kg) moving in the same straight line collide with each other. Before collision and after collision both of them are moving in the same direction. If initial velocities of A and B are U_A=2 m/s, U_B= 1 m/s respectively and if the final velocity of B is V_B= 2 m/s find - (i) Velocity of A after collision =V_A and (ii) Coefficient of restitution 'e' for the two masses. **[Ans. (i) V_A=1.75 m/s (in original direction) (ii) e =0.25]**

Q2. A ball is dropped on to a smooth horizontal floor from a height of 4m. On the second bounce it attains a height of 2.25 m. find the coefficient of restitution between the ball and the floor. **(Ans: 0.889).**

Learning from this Lecture: Learners will be able to understand the types of Impacts & apply the formulae for impact with the use of coefficient of restitution

6.10 Conclusion:

Learning Outcomes: Learner should be able to

Know, Comprehend

1. Define D' Alembert's principle, work energy principle, impulsive force, linear momentum, coefficient of Restitution.
2. Classify the work done by different aspects involved in D' Alembert's Principle

Apply, Analyse

3. Apply D'Alembert's principle, Work energy principle, Impulse momentum theorem in the problems based on Kinetics of Particles
4. Analyze the given data to identify the type of problems based on kinetics of particles

Synthesize

5. Understand the types of Impacts & apply the formulae for impact
6. Apply the concept of coefficient of restitution for collision with infinite mass

6.11 Add to Knowledge:

1. Work is defined for an interval or displacement there is no term like instantaneous work similar to instantaneous velocity.
2. For a particular displacement work is independent of time, work will be same for same displacement whether the time taken is small or large.
3. When several forces acts, work by force for particular displacement is independent of other forces.
4. Displacement depends on reference frame so work done by force is reference frame dependent so work done by force can be different in different reference frame.
5. Effect of work is change in kinetic energy (K.E)
6. Work is done by the source or agent that applies the force
7. Momentum remains conserved in all types of collisions
8. Total energy remains conserved in all types of collisions
9. Only conservative forces works in elastic collisions
10. In inelastic collisions all the forces are not conservative

Research Work: https://mech.subwiki.org/wiki/Coefficient_of_restitution

Atwood's machine

D'Alembert's principle makes the venerable Atwood's machine, trivial to analyze. The state of the machine is determined by the positions of the two masses along the U-shaped coordinate s looping over the frictionless pulley with the string. The work done by gravity on the left-hand mass under the displacement δs is $\delta WL = -mg\delta s$, while the gravity acting on the right-hand mass produces work $\delta WR = +Mg\delta s$. The inertial work on the two masses gives

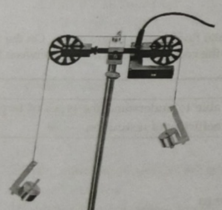


Fig.6.104

$$\delta W_I = -(M+m)(d^2s/dt^2)\delta s,$$

$$\delta W = \left[(M-m)g - (M+m)(d^2s/dt^2)\right]\delta s = 0$$

$$\frac{d^2s}{dt^2} = g\frac{M-m}{M+m}.$$

Note that the force of constraint by the pulley (which is assumed to be free to rotate but held rigidly in place and with negligible moment of inertia) does not enter the problem at all.

Coefficient of Restitution in Sports

A basketball bounces more than a tennis ball, the reason being that when colliding with the ground it suffers fewer energy losses. We can determine the percentage of speed that the ball retains after the collision by use of the coefficient of restitution,

$$e = \frac{V_{after}}{V_{before}}$$

Where, V_{after} and V_{before} are the speeds before and after the collision.

The larger this parameter is, the more elastic the collision, i.e. the fewer the energy losses. Furthermore, if we let a ball drop from a certain height, the height that it will reach after the rebound is higher for balls with a greater coefficient of restitution. More specifically, it can be proved that the coefficient of restitution is approximately given by,

$$e = \sqrt{\frac{h_{after}}{h_{before}}}$$

Where, h_{after} and h_{before} are the heights after and before the collision.

The symbol √ is that of the square root, which is the number than when multiplied

by itself will give the original number. So for example, the square root of 9 is 3. For some sports, the heights at which the balls must rebound to are strictly defined. So in basketball, according to the International Basketball Federation (FIBA), if a ball is dropped from 1.8m it must return to a height between 1.2m and 1.4m. From the above formula we can deduce that the coefficient of restitution will lie between,

$$e = \sqrt{\frac{1.2}{1.8}} = 0.82 \text{ και } e = \sqrt{\frac{1.4}{1.8}} = 0.88$$

In the same way, if a tennis ball is dropped from a height of 100 inches (254cm) on to a concrete floor, it must rebound to a height between 53 inches (134.62cm) and 58 inches (147.32cm). So the limits are,

$$e = \sqrt{\frac{53}{100}} = 0.73 \text{ και } e = \sqrt{\frac{58}{100}} = 0.76$$

A basketball certainly bounces better than a tennis ball.

It is important to note that the coefficient e, depends not only on the type of ball but also on the properties of the ground. This is why the above limits are defined with respect to a concrete floor, as the tennis ball will certainly bounce differently on clay and on grass.

6.12 Multiple Choice Questions:

1) D Alembert's principle is used for
   a) Reducing the problem of kinetics to equivalent statics problem
   b) Determining stresses in the truss
   c) Stability of floating bodies
   d) Designing safe structures
2) When two bodies stick together after collision, the collision is said to be......
   a) partially elastic
   b) elastic
   c) perfectly inelastic
   d) none of the above
3) If momentum is increased by 20%, the K.E increased by......
   a) 44
   b) 55
   c) 66
   d) 77

4) A bullet is shot from a rifle. As a result, the rifle recoils. The K.E of rifle as compared to that of bullet....
   a) is less
   b) is greater
   c) is equal
   d) cannot be concluded
5) Which of the following depend on the choice of the inertial reference frame........
   a) Momentum
   b) Change in momentum
   c) Kinetic Energy
   d) Both a & c
6) A sphere collides with another sphere of identical mass kept at rest. After collision, the two sphere moves. The collision is perfectly inelastic, then the angle between the directions of motion of the two spheres is ......
   a) $0°$
   b) $45°$
   c) different from $90°$
   d) $90°$
7) A 500Kg car moving with a velocity of 36kmph on a straight road uni-directionally, doubles its velocity in 1 min. The average power delivered by the engine for doubling the velocity is
   a) 750 W
   b) 1050 W
   c) 1150 W
   d) 1250 W
8) A bus can be stopped by applying a retarding force 'F' when it is moving with speed 'v' on a level road. The distance covered by it before coming to rest is 's'. If the load of the bus increases by 50% because of passengers, for the same speed and same retarding force, the distance covered by the bus to come to rest shall be ...
   a) 1.5 sec
   b) 2 sec
   c) 1 sec
   d) 2.5 sec
   a) A 1 kg block collides with a horizontal light spring of force constant 2 N/m. The block compresses the spring m from rest position. Assuming that the coefficient of kinetic friction between the block and the horizontal surface is 0.25. What was the speed of the block at the instant of collision?
   b) 6 m/s
   c) 7.2 m/s
   d) 8.6 m/s
   e) 8 m/s
9) A bomb of mass 16kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is 4m/s. The kinetic energy of the other mass is...
   a) 144 J
   b) 288 J
   c) 192 J
   d) 96 J

Answer:

1	a	2	c	3	a	4	a	5	d
6	c	7	d	8	a	9	b	10	b

6.13 Short Answer Questions:

1. State D'Alembert's Principle with two examples.
2. Write short note on Impulse Momentum Equation.
3. Explain coefficient of restitution.
4. Explain work-energy principle.
5. Enlist different types of Impact.
6. A ball drops from the ceiling of a room. After rebounding twice from the floor it reaches

a height equal to one fourth that of ceiling. Find the coefficient of restitution. (Ans: 0.707)

7. Enlist any four formulae for different types of work done.
8. A ball is dropped on to a smooth horizontal floor from a height of 4m. on the second bounce it attains a height of 2.25 m. find the coefficient of restitution between the ball and the floor. (Ans: 0.889).
9. A jet of water 5cm in diameter moving with 24 m/s velocity horizontally, strikes a flat vertical plate. Assuming that after striking water flows parallel to the plate, find the force exerted on the plate by the jet of water. **(Ans: 1.131kN)**
10. A ball falls from a height of 1m hits the ground and rebounds with half its velocity just before impact. Then after rising it falls and hits the ground and again rebounds with half its velocity just before impact, and so on. Find the total distance travelled by the ball till it comes to rest on the ground. **(Ans: 5/3)**
11. A ball of mass m kg hits an inclined smooth surface with a velocity $v_A = 3m/s$. Find out velocity of Rebound. **(Ans: $v = 2.563 m/s$, $\theta = 54.18°$)**

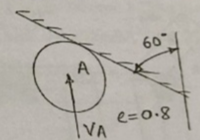


Fig.6.105

12. Block of 10 kg mass is released from position A. the coefficient of friction over length AB = 3 m is $\mu_K = 0.22$ and over length BC = 2 m is $\mu_K = 0.16$. Find the velocity with which the block passes point C. **(Ans: $V_C = 4.73$ m/s)**

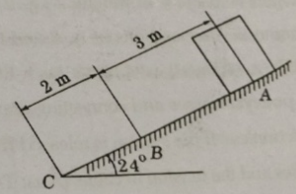


Fig.6.106

13. Two men lined up at one end of a boat initially at rest and run in succession with velocity of 3m/sec relative to boat and dive off at the fore end. Neglecting resistance of water to horizontal motion of the boat, find its velocity after the second man dives. Each man weighs 750N and the boat weighs 4500N.

14. A weightless cable passes over a frictionless pulley as shown in figure. Find the velocity of 50 kg block after it has moved 10 m from rest. Take kinetic coefficient of friction between 50 kg and incline plane as $\mu_s = 0.25$. Neglect inertia of the pulley. **(Ans: V = 9.97 m/s)**

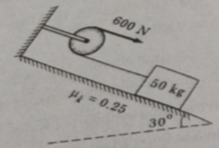


Fig.6.107

6.14 Long Answer Questions:

1. The system shown in figure is released from rest. What is the height lost by the bodies A, B & C in 2sec. Take coefficient of kinetic friction at rubbing surfaces as 0.4. Find also the tension in wires. **(Ans: T_A=21.422N, T_B=38.86N, a= 5.791m/s², h_A=5.791m, h_C= 11.582m).**

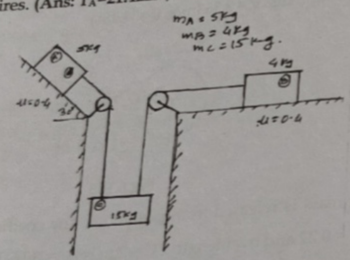


Fig.6.108

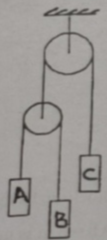


Fig.6.109

2. The three weights A, B and C of weights 3 kg , 2 kg and 7 kg are connected as shown in figure. Determine the accelerations of A, B and C. Also find the tension on the string. **(Ans: T=27.935N, a_A=0.5m/s²↓, a_B=4.15m/s²↑, a_C= 1.83m/s²↓)**

3. The system of pulleys, masses and connecting inextensible cables as shown pulleys are massless and frictionless. If the system is released from rest, Find the acceleration of each of the three masses and the tension in cable. **(Ans: T=27.70N, a_A=4.04m/s² ↑, a_B=2.885m/s² ↓, a_C=0.557m/s² ↓)**

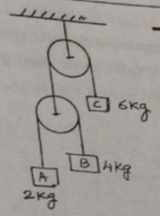


Fig.6.110

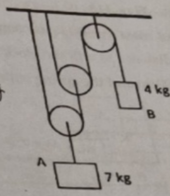


Fig.6.111

4. Determine the tension developed in chords attached to each block and the accelerations of the blocks when the system shown is released from rest. Neglect the mass of the pulleys and chords. **(Ans: T=19.344N, a_A=1.244m/s²↑, a_B= 4.976m/s²↓)**

5. A body of mass 25kg resting on a horizontal table is connected by string is passing over a smooth pulley at the edge of the table to another body of mass 3.75kg and hanging vertically as shown. Initially the friction between the mass A and the table is just sufficient to prevent the motion. If an additional 1.25kg is added to the 3.75kg mass, find the acceleration of the masses. **(Ans: a=0.409m/s², T=47.05N) (Dec 2013)**

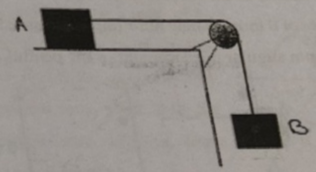
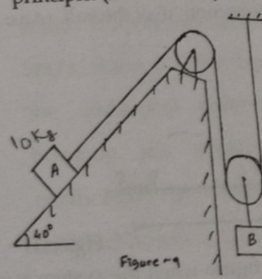


Fig.6.112

6. Find the velocity of block A and B when block A has travelled 1.2 m long along inclined plane. Mass of A is 10 kg and that of B is 50 kg. Coefficient of friction between block A and inclined plane is 0.25. Pulleys are massless and frictionless. Use work energy principle. **(Ans: 4.175m/s, 2.087m/s)**



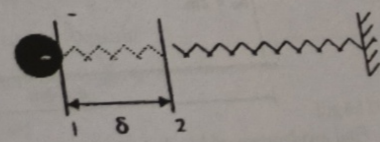


Fig.6.113

Fig.6.114

7. A block of mass m = 80 kg is compressed against a spring as shown in fig. How far from point B (distance x) will the block strike on the plane at point A. Take free length of the spring as 90cm and the spring stiffness as K = 40 N/cm. **(Ans: 0.668m]**

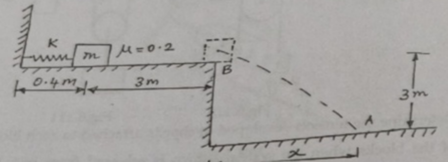


Fig.6.115

8. A spring of stiffness k is placed horizontally and a ball of mass m strikes the spring with a velocity v. Find the maximum compression of the spring. Take m= 5kg, k = 500 N/m, V = 3m/s**(Ans: 0.3m)**

9. Block A is released from rest in the position shown and slide without friction until it strike the ball B of a simple pendulum. Knowing the coefficient of restitution between A and B as 0.9. Radius of the curve is 0.6m. Find.

a) The velocity of B immediately after impact **(Ans: 2.507m/s)**

b) The maximum angular displacement of the pendulum. **(Ans: θ=49.9°]**

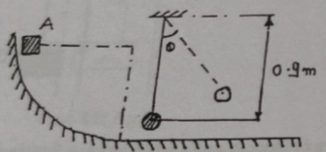


Fig.6.116

10. A small steel ball is to be projected horizontally such that it bounces twice on the surface and lands into a cup placed at a distance of 8 m as shown. If the coefficient of restitution for each impact is 0.8, determine the velocity of projection 'u' of the ball. **(Ans: 3.231m/s)**

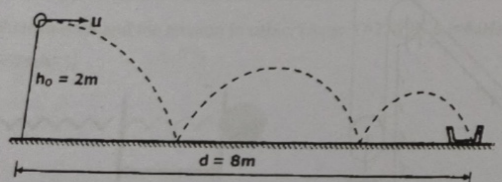
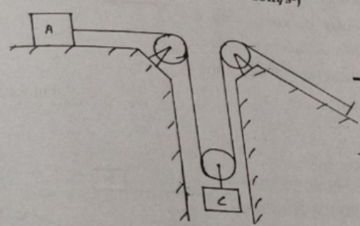


Fig.6.117

11. Find acceleration of block A, B and C shown in the figure when the system is released

from rest. Mass of block A, B and C is 5 kg, 10 kg and 50 kg resp. Coefficient of friction for block A and B is 0.3. Neglect weight of pulley and rope friction. **(Ans: a_A=12.518m/s², a_B=0.9175m/s², a_C=6.718m/s²)**



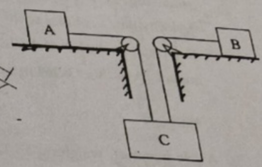


Fig.6.118

12. Masses A (5 kg), B (10 kg), C (20 kg) are connected as shown in the figure by inextensible cord passing over mass less and frictionless pulleys. The coefficient of friction for masses A and B with ground is 0.2. If the system is released from rest, find the acceleration of the blocks and tension in the cords. **(T=47.088N, a_A=7.4556m/s²(→), a_B= 2.7468m/s²(←), a_C=5.1012m/s²(↓)]**

Fig.6.119

13. Two smooth spheres A and B having a mass of 2 kg and 4 kg respectively collide with initial velocities as shown in fig. If the coefficient of restitution for the spheres is e = 0.8. find the velocities of each sphere after collision. **(Ans: 2.923m/s & 20°, 3.472m/s & 86.053°)**

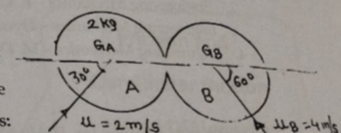


Fig.6.120

14. A spring is used to to stop 100kg package which is moving down a 30° incline. The spring has a constant k=30kN/m is held by cables so that it is initially compressed by 90mm. If the initial velocities of the package is 5m/s, when it is 9m from spring. Determine the additional deformation of spring in bringing the package to rest. Assume coefficient of friction between block and incline as 0.2. **(Ans: 0.4517m)**

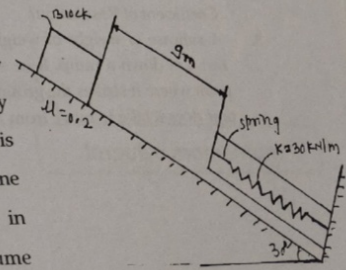


Fig.6.121

15. A smooth spherical ball A of mass 120 grams is moving from left to right with a velocity 2 m/s in a horizontal plane. Another identical ball B traveling in a perpendicular direction with a velocity 6m/s collides with A as shown in figure. Determine velocity of balls A and B after the impact. **[Ans.: V_B = 6.264 m/sec]**

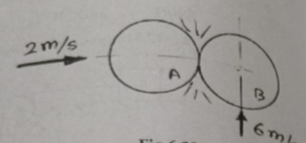


Fig.6.122

16. A pile hammer, weighing 15 kN drops from a height of 600 mm on a pile of 7.5 kN. How deep does a single blow of hammer drive the pile if the resistance of the ground to pile is 140 kN? Assume plastic impact. **[Ans. x =0.051m]**

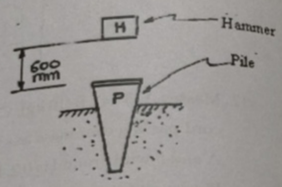


Fig.6.123

6.11 References:

1. Engineering Mechanics – Dynamics, R. C. Hibbler
2. Engineering Mechanics – F. L. Singer
3. Engineering Mechanics – Dynamics, J. L. Merium, I. G. Kraig
4. Engineering Mechanics – M. D. Dayal

Self-Assessment

1. Write the formula for D'Alembert's Principle. State the expression for work energy principle. **(Level 1)**
2. State the condition for equilibrium for General Plane Motion. What do you mean by Coefficient of Restitution? **(Level 2)**
3. A suitcase of weight 40 N slides from rest 6 m down a ramp. If μ_K = 0.2, determine the point where it strikes the ground at C. How much time does it take to move from A to C? **(Level 3)**

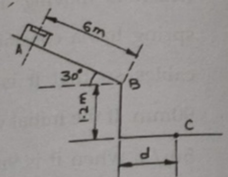


Fig.6.124

4. A collar of mass 10 kg moves on a vertical guide as shown in figure. Neglecting friction between the guide and the collar, find the velocity of the collar after it has fallen 0.7 m starting from rest from the position shown. The unstretched length of the spring is 0.2 m and its stiffness is 200 N/m. **(Level 4)**

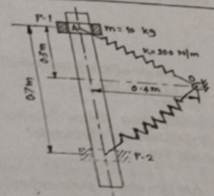


Fig.6.125

5. A billiard ball moving with a velocity of 5 m/sec strikes a smooth horizontal plane at an angle of 45° with horizontal. If the coefficient of restitution is 0.6, what is the velocity with which the ball rebounds? **(Level 5)**

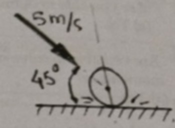
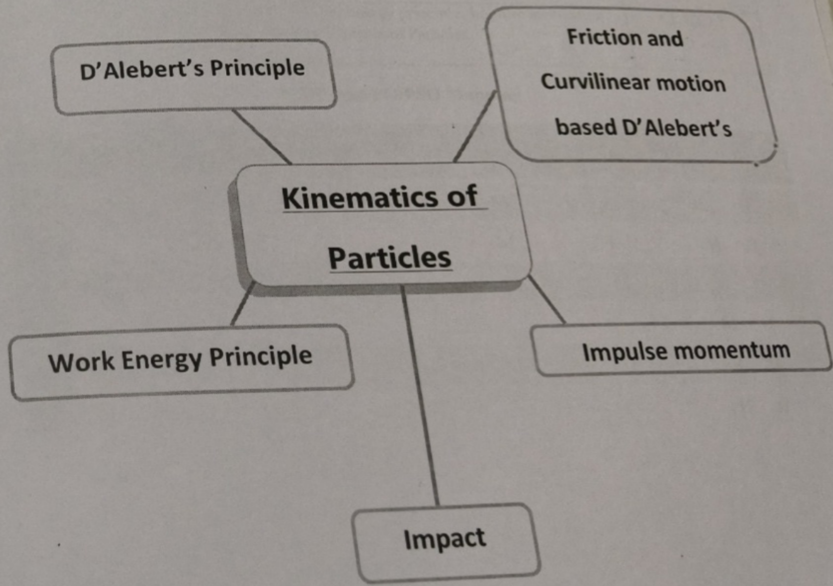


Fig.6.126

Concept Map



Self-Evaluation

Name of Student: Course Code:
Class & Div.: Roll No.:

1. Can you define Newton's second law of motion, Work energy Principle and D'Alembert's Principle?
   (a) Yes **(b) No**
2. Are you able to state different types of work done & Impact?
   (a) Yes **(b) No**
3. Are you able to define the term Impulse, Linear Momentum, Co-efficient of Restitution?
   (a) Yes **(b) No**
4. Are you able to solve numericals based on work energy principle, conservation of momentum?
   (a) Yes **(b) No**
5. Do you understand this module?
   (a) Yes **(b) No**

A. CO Mapping with Revised Bloom Taxonomy Level

Sr. No	Course Outcomes	Cognitive levels of attainment as per Bloom's Taxonomy
CO1	Find resultant/equilibrant of different types of coplanar force system and locate the centroid of plane lamina.	L1, L2
CO2	Construct free body diagram of a coplanar system and calculate the reactions for static equilibrium.	L1, L2, L3
CO3	Analyze problems related to friction for system containing block, wedge, ladder etc.	L1, L2, L3
CO4	Find resultant of different types of non-coplanar force system	L1, L2, L3
CO5	Analyze Projectile motion of the particle and draw motion curves. Locate instantaneous center of rotation and find linear and angular velocity for different links for rigid bodies having plane motion.	L1, L2, L3
CO6	Apply D'Alembert's principle, Work energy principle, Impulse momentum theorem in the problems based on Kinetics of Particles	L1, L2, L3

B. CO and PO/PSO Mapping

PO / CO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO1	H	M							M		L		M	H	
CO2	H	M	M						M		L		M	H	
CO3	H	M		M							M		M	H	
CO4	H												M	H	
CO5	H	L							M		L		H	H	
CO6	H	H		M									H	H	