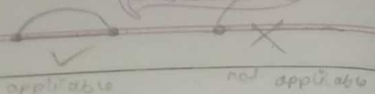
# MODULE - 5

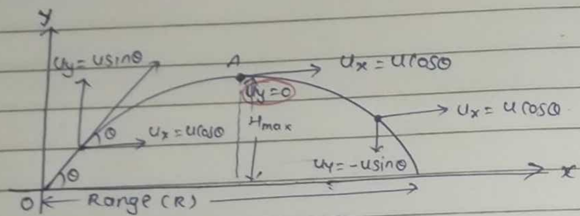
very imp

Note :- R, T, H this formulae only applicable when initial point and final point at same level




most imp # Projectile motion :

- when a particle is projected in space, its motion is a combination of horizontal and vertical motion



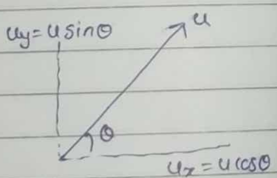

# Derivation of time of flight, horizontal range and Maximum height attained by a projectile on horizontal plane :

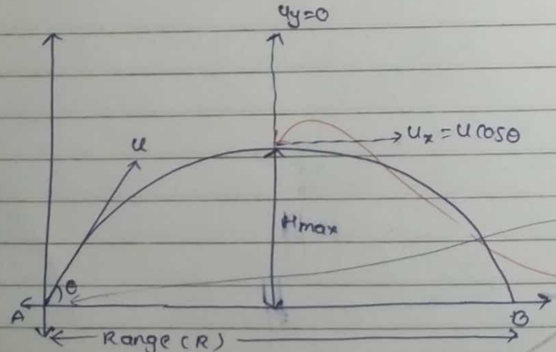
→ consider a particle projected from point 'A' and lands at point B

let u = initial velocity of projection

θ = angle of projection

t = total time of flight




(3) Maximum Height (H):

↳ vertical distance b/w point of projection and Highest point on path of projectile

- where vertical component of velocity ZERO

since air resistance is to be neglected

x motion is uniform motion and Y motion is motion under gravity

(1) Time of flight (T):

↳ Total time taken to complete the projectile path.

consider y-motion from A → B (MUG)

$S_y = u_y t - (\frac{1}{2}) g t^2$ --- $\{ S = ut + \frac{1}{2} a t^2 \}$ ↳ $a_y = -g$

displacement ↳ shortest distance b/w initial to final.

vertical motion → displacement zero ho raha hai

$0 = (u\sin\theta) t - \frac{1}{2} g t^2$

$u\sin\theta \cancel{t} = \frac{1}{2} g t^{\cancel{2}}$ $\qquad \frac{2u\sin\theta}{g} = t$

① $t = \frac{2u\sin\theta}{g}$ ...(1)

(2) Range (R): Horizontal distance b/w point of projection A and point of landing B

↳ consider x-motion from A → B (UM)

S = velocity × time

$= u_x \times t$

So, $R = u\cos\theta \times \frac{2u\sin\theta}{g}$

② $R = \frac{u^2 \sin 2\theta}{g}$

For the range to be maximum

↳ angle of projection (θ) should be 45°

$$\boxed{R_{max} = u^2/g}$$

- consider Y-motion from A → B (MUG) motion under gravity

$v_y^2 = u_y^2 - 2 g S_y$

$0 = u_y^2 - 2gh$

$u^2 \sin^2\theta = 2gh$

③ $h = \frac{u^2 \sin^2\theta}{2g}$

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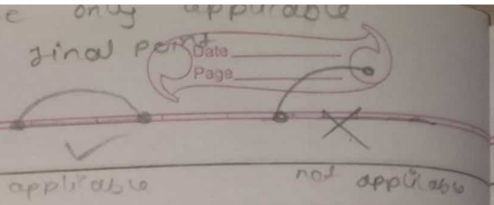
Note :- R, T, H this formulae only applicable when Initial point and final point at same level

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# Projectile motion :

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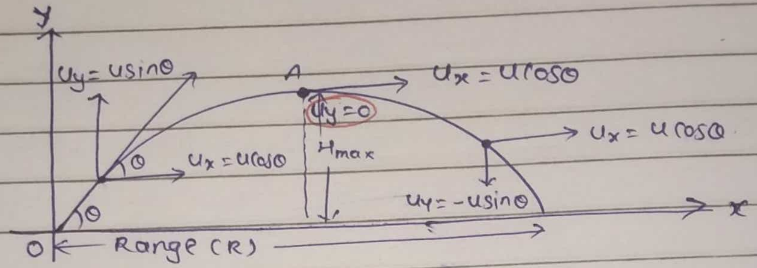


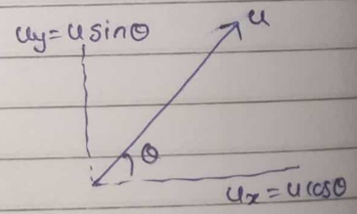

# Derivation of time of flight, horizontal range and Maximum height attained by a projectile on horizontal plane :

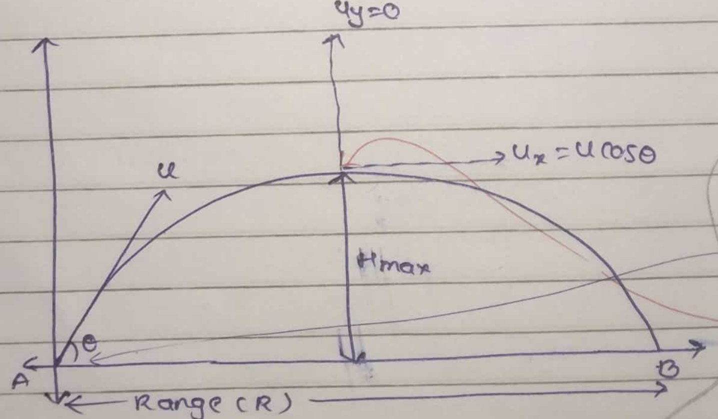
→ consider a particle projected from point 'A' and lands at point B

let $u$ = initial velocity of projection

$\theta$ = angle of projection

$t$ = total time of flight




(3) Maximum Height (H):

- vertical distance b/w Point of projection and Highest point on path of projectile
- where vertical component of velocity Zero

Date
Page

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x motion is uniform motion
and
Y motion is motion under gravity

(1) Time of flight (T):

↳ Total time taken to complete the projectile path.

consider y-motion from $A \rightarrow B$ (MUG)

$S_y = U_y t - (1/2) g t^2$ --- $\{ S = ut + \frac{1}{2} at^2 \}$

displacement ↳ shortest distance b/w initial to final.

$a_y = -g$

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$0 = (u\sin\theta)t - \frac{1}{2} g t^2$

$u\sin\theta \cancel{t} = \frac{1}{2} g t^{\cancel{2}}$   $\frac{2u\sin\theta}{g} = t$

1 $t = \frac{2u\sin\theta}{g}$ ...(1)

(2) Range (R): Horizontal distance b/w point of projection A and point of landing B

↳ consider x-motion from $A \rightarrow B$ (UM)

S = velocity × time

$= u_x \times t$

so,

$R = u\cos\theta \times \frac{2u\sin\theta}{g}$

2 $R = \frac{u^2 \sin 2\theta}{g}$

(i): For the range to be maximum

b/w ↳ angle of projection ($\theta$) should be 45°

$\boxed{R_{max} = u^2/g}$

• consider y-motion from $A \rightarrow B$ (MUG) motion under gravity

$V_y^2 = U_y^2 - 2g S_y$

$0 = U_y^2 - 2gh$

$u^2 \sin^2\theta = 2gh$

3 $h = \frac{u^2 \sin^2\theta}{2g}$

Equation of path → used only when (4 mo se 3) value are known which is $y, x, \theta, u$ — any 3 value are known

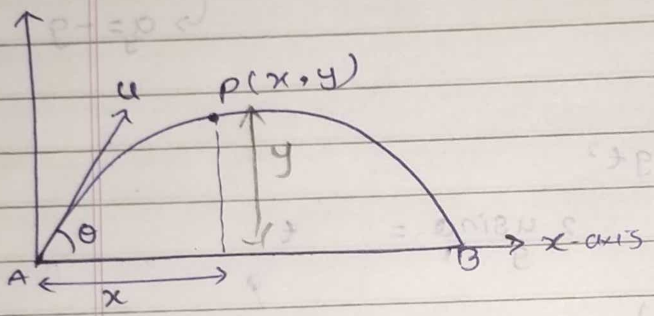
Note :

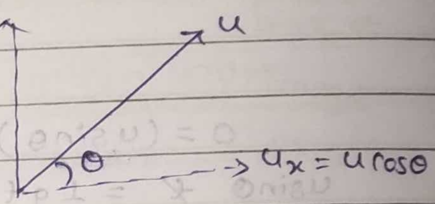
# Derivation of the equation for the path of projectile : (Equation of trajectory):

let the particle be projected from A with initial velocity 'u' , angle of projection '$\theta$'

let $P(x, y)$ be some point on the path of projectile at time $t_1$

$u_y = u \sin\theta$




let after time $t_1$ , particle be reached at point $P(x, y)$

consider x-motion from A – B (UM)

$s = \text{velocity} \times t_1$

$x = u_x \times t_1$

$x = u\cos\theta \times t_1$

$$\boxed{t_1 = \frac{x}{u\cos\theta}}$$

consider y-motion from (A to B) (MUG)

$S_y = U_y t - \frac{1}{2} g t^2$

$y = u\sin\theta \, t_1 - \frac{1}{2} g t_1^2$

puting $t_1$ in above eqution

$y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$

$y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}$ or $y = x\tan\theta - \frac{g x^2 (1+\tan^2\theta)}{2u^2}$

(MUG)

$\frac{gx^2(1+\tan^2\theta)}{2u^2}$

Imp

$\theta$ angle is alway take $x$-axis

$t_H = t_V$

Note : • Time for $\boxed{\text{Horizontal motion}}$ and $\boxed{\text{Vertical motion}}$ is alway same

• Acceleration for Horizontal motion $= 0$ and

for Vertical motion $= -9.81$

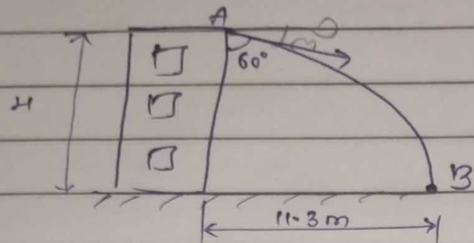
most most imp

hint → projectile formula R, T, H only applicable when initial and final point at same level

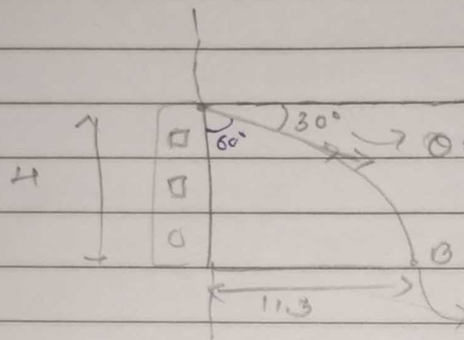
e.g Numerical

(1) A ball thrown with speed of 12 m/s at an angle of 60° with building strikes the ground 11.3 horizontally from the foot of the building as shown. Determine the height of the building.

⟹




1st we take initial as origin




→ $\theta = -30^\circ$

2nd $\theta$ is alway with x-axis

because below x-axis

→ (x, y) → in 4th quadrant

(11.3, -H)

$x = 11.3$ $y = -H$ $\theta = -30^\circ$ $u = 12\,m/s$ → which is given

applying equation of path

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$-H = 11.3\,(-0.577) - \frac{9.8 \times (11.3)^2}{2 \times 144 \times 0.75}$$

$$= -6.52 - \frac{9.8 \times 11.3 \times 11.3}{216}$$

$$= -6.52 - \frac{1251.36}{216}$$

$$= -6.52 - 5.793$$

$$\cancel{-}H = \cancel{-}12.313$$

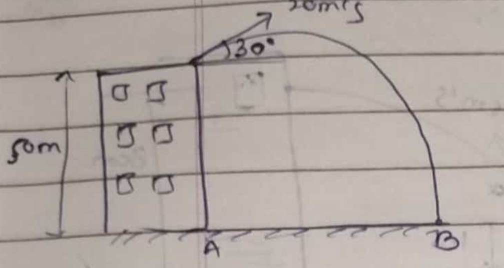
$$\boxed{H = 12.313\,m}$$

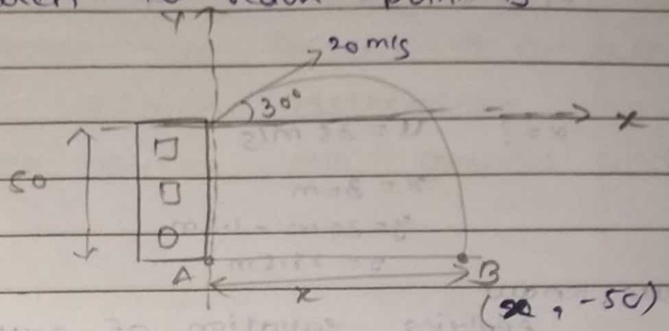
(2) A particle is projected from the top of tower of height 50m with velocity of 20m/s at an angle 30° to the horizontal determine :

(1) Horizontal distance AB

(2) The velocity with which it strikes the ground at B

(3) Total time taken to reach point B




$x = x \qquad y = -50 \qquad \theta = 30° \qquad u = 20 m/s$

Applying equation of path

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$-50 = x\tan 30° - \frac{9.8x^2}{2\times(20)^2\times(\cos 30)^2}$$

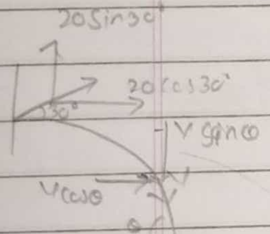
$$-50 = (0.57)x - \frac{9.8\,x^2}{600}$$

$$-50 = (0.57)x - (0.0163)x^2$$

$$(0.016)x^2 - (0.57)x - 50 = 0$$

calculated by calculator

(1) $\boxed{x_1 = 75.56}$ $\qquad x_2 \neq -40.59$




$AB = x = 75.56$

| | H.M | V.M |
|---|---|---|
| u | $u\cos\theta$ $= 20\cos 30$ | $u\sin 30°$ $= 20\sin 30°$ |
| v | $v\cos\theta$ | $-v\sin\theta$ |
| s | $x = 75.56$ | $y = -50$ |
| a | 0 | $-9.8$ |
| t | t | t |

Horizontal motion

(i)

$v = u + at$

$v\cos\theta = 20\cos 30° + 0$

$\boxed{v\cos\theta = 20\cos 30°}$ ...(1)

$s = ut + \frac{1}{2}at^2$

$75.56 = 20\cos 30\, t + \frac{1}{2}\times 0\times t^2$

$\frac{1}{t} = \frac{20\cos 30°}{75.56}$ $\qquad \frac{1}{t} = \frac{17.32}{75.56}$

$t = \frac{75.56}{17.32}$

(3) $\boxed{t = 4.36}$

(ii) Vertical motion

$v = ut + at$

$-v\sin\theta = 20\sin 30° - 9.8\times 4.36$

$\boxed{-v\sin\theta = 20\sin 30° - 42.728}$ ...(2)

equation (1) and (2)

$$\frac{-v\sin\theta}{v\cos\theta} = \frac{20\sin 30° - 42.72}{20\cos 30°}$$

$$\tan\theta = \frac{10 - 42.72}{17.32}$$

$$\tan\theta = \frac{32.72}{17.32}$$

$$\theta = \tan^{-1}\left(\frac{32.72}{17.32}\right)$$

$\boxed{\theta = 62.105°}$

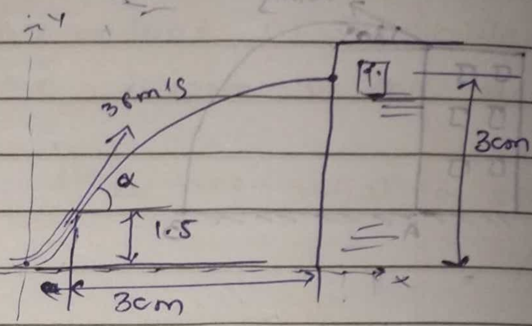
putting in eq (1)

$$v = \frac{20\cos 30°}{\cos(62.105)}$$

$$v = \frac{17.32}{0.46}$$

(2) $\boxed{v = 37.68 \text{ m/s}}$

(3) A fire nozzle located at A discharged water initial velocity $v = 36\ m/s$. knowing that the stream of water strikes the building at a height $h = 30\ m$ above the ground determine the angle α made by the nozzle with the horizontal




⇒ $\alpha = ?$ $\quad u = 36\ m/s$

$x = 30m$

~~$y = 30m - 1.5m$~~

$y = 28.5m$

we know
Applying equation of path

x, y, θ, u

$$y = x\tan\alpha - \frac{gx^2}{2u^2}(1+\tan^2\theta)$$

$$28.5 = 30\tan\alpha - \frac{\overset{4.9}{9.8}\times 900}{2\times 36\times 36}(1+\tan^2\alpha)$$

$$= 30\tan\alpha - (3.4)(1+\tan^2\alpha)$$

$$28.5 = 30\tan\alpha - 3.4 - 3.4\tan^2\alpha$$

$$28.5 + 3.4 = 30\tan\alpha - 3.4\tan^2\alpha$$

$$31.9 = 30\tan\alpha - 3.4\tan^2\alpha$$

$$3.4\tan^2\alpha - 30\tan\alpha + 31.9 = 0$$

{ calculated by calculator }

~~tanα = 1.596  tanα = 0.655~~

$\tan\alpha = 7.58$ $\qquad \tan\alpha = 1.23$

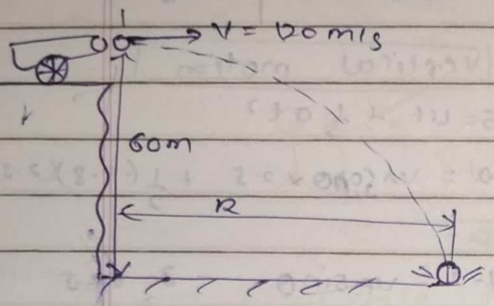
$\alpha = \tan^{-1}(7.58)$ $\qquad \alpha = \tan^{-1}(1.23)$

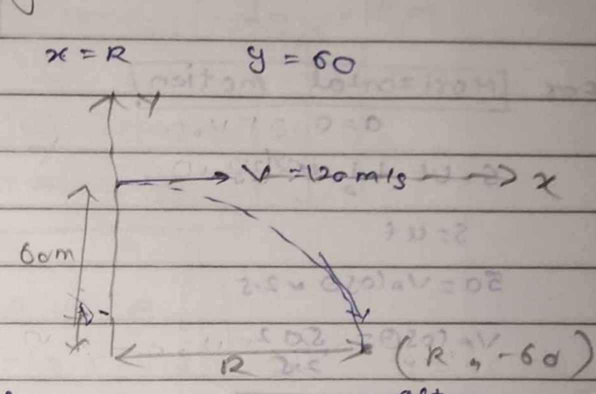
$\boxed{\alpha = 82.48^\circ}$ $\qquad \boxed{\alpha = 50.88^\circ}$

~~Now, checking the possible angle of projection for maximum height~~

(4) A cannon ball is fired from point A with a horizontal muzzle velocity 120 m/s as shown in figure. If the cannon is located at an elevation of 60m above the ground. determines the time for cannon ball to strike the ground and the Range R'

$\theta = 0$




$x = R \quad y = -60 \quad \theta = 0°$ → because velocity act along the x-axis

$u = 120$ m/s

Applying the equation of path

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$-60 = x\tan 0° - \frac{9.8\,(R)^2}{2\times(120)^2(\cos 0°)^2}$$

$$-60 = 0 - \frac{9.8R^2}{2\times(120)^2}$$

$$+60 = +\frac{4.9R^2}{(120)^2}$$

$$R^2 = \frac{120}{4.9}$$

$$= \frac{14400\times 60}{4.9}$$

$$= ~~2938077~~$$

$$R^2 = 176326.5306$$

$$R = \sqrt{176326.5306}$$

$$\boxed{R = 419.915}$$

Using kinematical equation
Horizontal motion → $a = 0$

$$S = ut + \frac{1}{2}at^2$$

$$\boxed{S = ut}$$

$$R = 120t + \frac{(0)}{2}t^2$$

$$419.915 = 120t - 0\,t^2$$

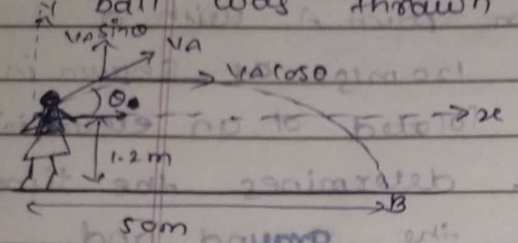
$$~~4.9t^2 - 120t + 419.9 = 0~~$$

$$419.9 = 120t$$

$$t = \frac{419.9}{120}$$

$$\boxed{t = 3.499 \text{ sec}}$$

(5) It is observed that the time for the Ball to strike the ground at B is 2.5 seconds, after throwing from A, Determine the speed $V_A$ and $\theta$ angle at which the ball was thrown




| | Horizontal motion | vertical motion |
|---|---|---|
| s | 50 | -1.2 |
| v | 0 | 0 |
| a | 0 | -9.8 |
| u | $V_A \cos\theta$ | $V_A \sin\theta$ |
| t | 2.5 | 2.5 |

For Horizontal motion

$a = 0$

$S = ut + \frac{1}{2}at^2$

$S = ut$

$50 = V_A \cos\theta \times 2.5$

$V_A \cos\theta = \frac{50}{2.5}$

$\boxed{V_A \cos\theta = 20}$ ---(1)

For vertical motion

$S = ut + \frac{1}{2}at^2$

$-1.2 = V_A \sin\theta \times 2.5 + \frac{1}{2}(-9.8)(2.5)^2$

$-1.2 = V_A \sin\theta - 30.625$

$30.62 - 1.2 = V_A \sin\theta \; 2.5$

$\boxed{V_A \sin\theta = \frac{29.42}{2.5}}$ ---(2)

$\boxed{V_A \sin\theta = 11.176}$

eq(1) and (2)

$\frac{V_A \sin\theta}{V_A \cos\theta} = \frac{11.176}{20}$

$\tan\theta = 0.5884$

$\theta = \tan^{-1}(0.58)$

$\boxed{\theta = 30.11°}$

0.0588

$\theta$ put in eq(1)

$V_A = \frac{20}{\cos(30.11°)}$

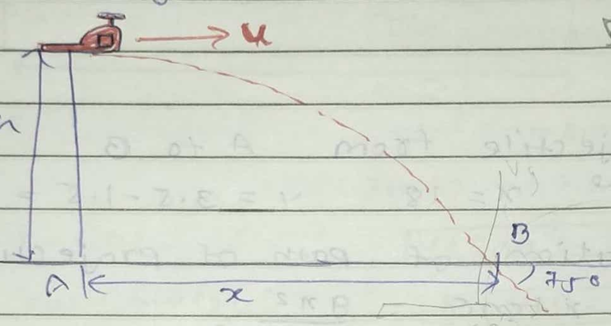
$\boxed{V_A = 23.1196 \text{ m/s}}$

Imp Note *** :
- H.M → uniform motion ⟶ $a=0$
- V.M → motion under gravity ⟶ $9.8$

(1) A Box released from a Helicopter moving Horizontally with constant velocity 'u' from a certain Height 'H' from the ground takes 5 seconds to reach the ground Hitting at an angle of 75° as shown in figure.

Determine
(i) the horizontal distance 'x'
(ii) the Height 'h'
(iii) the velocity 'u'

H.M
$S = ut + \frac{1}{2}at^2$
$S = \frac{1}{2}at^2$
$S = ut$
$S = 5 \times u$




Sol:- The motion of Box from point A to B

(a) $\boxed{\text{Horizontal motion}}$ :

$S = ut + \frac{1}{2}at^2$

$S = ut$ --- {projectile is having uniform motion}
($a=0$)

$x = u \times 5$

$\boxed{x = 5u}$ --- (1)

(b) $\boxed{\text{Vertical motion}}$ :

$S = ut + \frac{1}{2}at^2$

$S = ut + \frac{1}{2}9.8t^2$ {projectile is having motion under gravity}
($a = 9.8$)

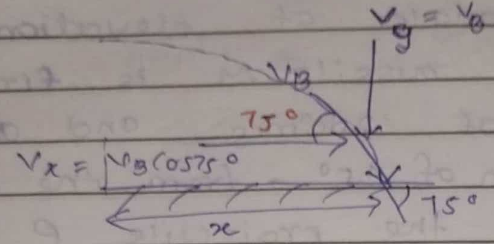
$h = (0 \times t) + \frac{1}{2}9.8t^2$

$h = 4.9 \times 25$

$\boxed{h = 122.62m}$

Your choice which one we take either (+) or (−)

(c) At point B or At traget B :




$$\frac{v_y}{v_x} = \frac{\cancel{V_B} \sin 75°}{\cancel{V_B} \cos 75°}$$

$$\frac{V_y}{V_x} = \tan 75°$$

$$\boxed{\frac{V_y}{u} = 3.732}$$

(d) The vertical velocity can be taken of ... { Initially Box at rest }

$$v = 0 + at$$

$$V_y = 0 + (9.8) \times 5$$

$$u(3.732) = 49$$

$$\boxed{u = 13.1296}$$

Putting the value of $u$ in equation (1)

$$x = 5u$$

$$= 5 \times 13.1296$$

$$\boxed{x = 65.64 \text{ m}}$$

Imp

(2) A projectile P is fired at a muzzle velocity of 200 m/s at an angle of elevation of 60°. After sometime a missile M is fired at muzzle velocity of 2000 m/s and at an angle of elevation of 45°, from the same point, to destroy the projectile P.

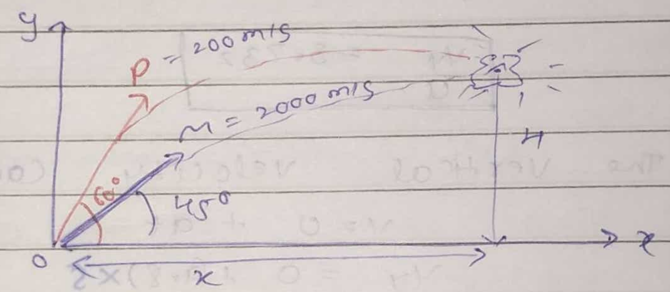
Find

(i) Height

(ii) Horizontal distance

(iii) Time with respect to P firing at which ~~firing at~~ the destruction take place.

⇒




Sol :-

(a) projectile motion of P muzzle

we know that

Applying equation of path

$$y = x \tan 60° - \frac{g x^2}{2\times(200)^2 (\cos 60°)^2}$$

$$= (1.73)\, x - \frac{\cancel{9.8}\ 4.9\ x^2}{\cancel{2} \times \cancel{40000} \times \frac{1}{4}\ 40000}$$

$$\boxed{y = 1.7x - 49 \times 10^{-3} x^2}$$

$\cancel{1.7x - y = 49 \times 10^{-3} x^2 \ \text{—(1)}}$

$$\boxed{y = 1.7x - 4.9\times10^{-4} x^2} \quad \text{—(1)}$$

$\cancel{1.7x - y = 49 \times 10^{-4} x^2}$

(b) projectile motion ~~of~~ muzzle

Applying equation of path

$$y = x\tan 45^\circ - \frac{9.8 x^2 (1+\tan^2 45^\circ)}{2\times(2000)^2}$$

$$y = x - \frac{9.8\times x^2\times \cancel{2}}{\cancel{2}(2000)^2}$$

$$\boxed{y = x - 2.45\times10^{-6}x^2} \quad \cdots(2)$$

~~$x - y = 2.45\times10^{-6}x^2 \quad \cdots(2)$~~

~~add eq(1) and eq(2)~~

~~$x - y = 2.45\times10^{-6}x^2$~~

~~$-1.7x + y = 0.049\times10^{-6}x^2$~~

~~$-0.7x =$~~

$$\boxed{y = x - 0.0245\times10^{-4}x^2} \quad \cdots(2)$$

put y in equation (1)

$$1.7x - 4.9\times10^{-4}x^2 = x - 0.0245\times10^{-4}x^2$$

$$1.7x - x = 4.9\times10^{-4}x^2 - 0.0245\times10^{-4}x^2$$

$$(0.7)\cancel{x} = 4.8755\times10^{-4}x^{\cancel{2}}$$

$$\frac{0.7\times10^4}{4.8755} = x$$

$$\boxed{x = 1497.28\ m}$$