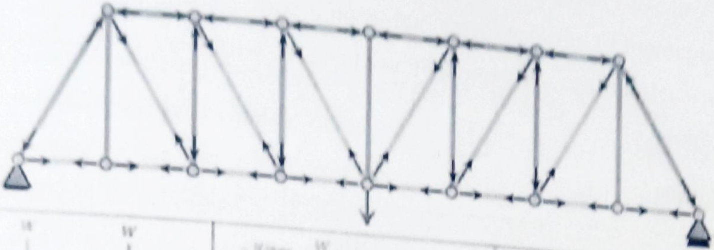
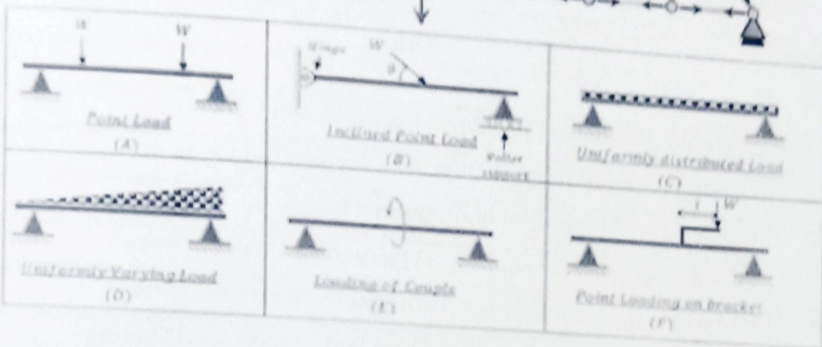
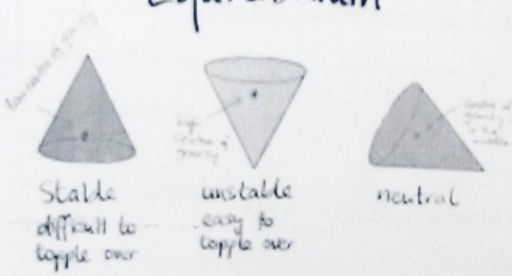
Module 2: Equilibrium of System of Coplanar Forces & Trusses

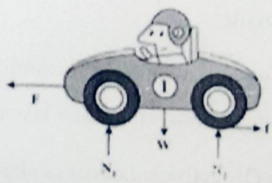
Infographics





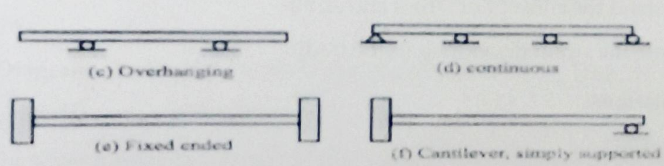
Equilibrium





Beam Types

- Types of beams- depending on how they are supported.



Lecture 10

2.1 Equilibrium

2.1.1 Motivation:

The balanced bodies are said to be in equilibrium. So, it becomes necessary to understand arrangement of various forces acting on a body causing balanced condition. All structures (trusses), buildings, dams, machines etc. in the world satisfy conditions of equilibrium while designing.

2.1.2 Syllabus:

Lecture No.	Title	Duration (Hrs.)	Self-(Hrs.)
10	Laws and theorems, Free Body Diagram, Lami's Theorem	1	2
11	Conditions of Equilibrium of general & concurrent system	1	2
12	Problems on equilibrium of Connected bodies – I	1	2
13	Reactions of different support, Types of loads, types of beams	1	2
14	Problems on reactions of beam with pulley & external mass	1	2
15	Problems on Simply supported beam with complicated loading	1	2
16	Trusses, Types of Trusses, Condition of Perfect.	1	2
17	Problems cn method of joints	1	2

2.1.3 Weightage: 19 to 23 Marks (Approximately)

2.1.4 Pre-Requisite:

- Knowledge of Law of transmissibility of force is needed.
- Knowledge of Newton's third law is needed.

2.1.5 Learning Objectives: Learners shall be able to

1) Explain free body diagram
2) Apply Lami's theorem to different equilibrium problems
3) Apply conditions of equilibrium to different equilibrium problems
4) Find reactions of simply supported beam.
5) Understand the concept of stress and strain
6) Calculate the strain developed in the body

2.1.6 Key notations:

➢ **Equilibrium:**

$\sum F_X$ = Summation of all horizontal components of forces

$\sum F_Y$ = Summation of all vertical components of forces

$\sum M$ = Summation of Moments of all forces taken about a point

2.1.7 Theoretical Background:

A very basic concept when dealing with forces is the idea of equilibrium or balance. In general, an object can be acted on by several forces at the same time. A force is a vector quantity which means that it has both a magnitude (size) and a direction associated with it. If the size and direction of the forces acting on an object are exactly balanced, then there is no net force acting on the object and the object is said to be in equilibrium. Because there is no net force acting on an object in equilibrium, then from Newton's first law of motion, an object at rest will stay at rest, and an object in motion will stay in motion.

The ends of a truss are pinned, so that they don't carry moments. The only reactions at the ends of a truss member are forces. External forces on trusses act only on the end points. Truss problems are solved by the method of sections, where an imaginary cut is made through the member(s) of interest, and global equilibrium of forces and moments are used to determine the forces in the members, or by the method of joints, in which a single joint is isolated and analyzed and the resulting forces (not necessarily with a numerical value) are transferred to adjacent joints, where the process is repeated. The resulting set of equations can then be solved by linear algebra, or substitution.

2.1.8 Formulae:

Equilibrium:

- Lami's theorem: $\dfrac{P}{\sin\alpha}=\dfrac{Q}{\sin\beta}=\dfrac{R}{\sin\gamma}$
- For Concurrent force system: Conditions of Equilibrium are $\sum F_X=0\ \sum F_Y=0$
- For General force system: Conditions of Equilibrium are $\sum F_X=0;\sum F_Y=0$ & $\sum M=0$

2.1.9 Key definitions:

1) **Equilibrium:** If the resultant of the force system happens to be zero, the system is said to be in a state of equilibrium.
2) **Equilibrant:** A single force which when acting with all other forces keeps the body at rest or in equilibrium.
3) **Free Body Diagram:** A diagram formed by isolating the body from its surrounding and then showing all the forces acting on it.
4) **Reaction:** Whenever a body is supported, the support offers resistance, known as reaction.

2.1.10 Laws and theorems:

- **Law of equilibrium of two forces:** Two forces can be in equilibrium only if they are equal in magnitude, opposite in direction and collinear in action.
- **Lami's Theorem:** If three concurrent forces are in equilibrium then their magnitudes are proportional to the angle between the other two forces. If a body is in equilibrium under the action of three non-collinear coplanar and concurrent forces, then each force is proportional to the sine of the angle between the other two forces.

2.1.11 Theory:

- **Free Body Diagram:** It is diagram of body under consideration. The diagram shows magnitude, direction and point of applications of all external, active and reactive forces acting on the body. In the diagram unknown forces, corresponding directions are labeled. By including the necessary dimension in FBD, moments of the forces can be easily analyzed. Thus, it is very important and very first step in analysis of problems on equilibrium.

Rules to be followed while drawing the free body diagram:

1) Identify the object for which FBD need to draw. (If multiple objects then there will be multiple FBD).
2) Draw only object and forces acting on it.
3) Draw a shape that is roughly the same shape as the original object. You do not need to include details which are not important to the problem. For example, a standing person might be drawn by a narrow vertical box.
4) All FBD's must include the coordinate system that applies that diagram. You can use different coordinate systems for different FBD's within the same problem
5) Forces should be drawn as arrows coming from the side of the object where the force originates. The arrow should point in the direction of the force
6) All forces of known magnitude should be labeled with that magnitude (e.g. "50N" or "8.4lb"). All unknown forces should be labeled with the symbol you will use when writing your equations of motion for the object (e.g. F_{12} might be used for the force of object 1 on object 2. F_G might be used to identify the force of gravity on an object of unknown mass. F_N can represent the normal for an object resting on a table.)
7) Weight of a particle or rigid body always acts vertically downward through its center of gravity irrespective of its position (Horizontal or Inclined).
8) Axial forces in any member may be assumed as tensile, if result comes negative then force will have to considered compressive. (The direction of reactions generated due to support can be assumed)

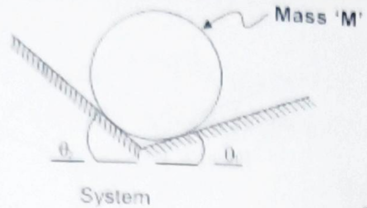


Fig 2.1

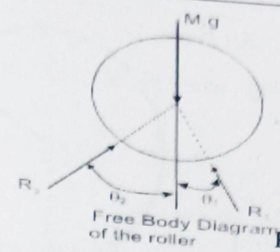


Fig 2.2

Solved Problems on F.B.D.

1) Draw Free body diagram of a lamp weighing 150 N is supported by two cables AC and BC. **(Ans: Fig 2.4)**

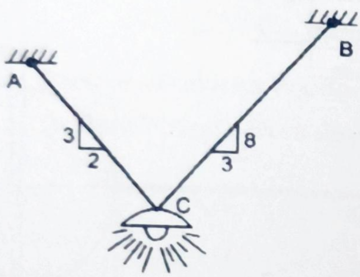


Fig 2.3

Solution:

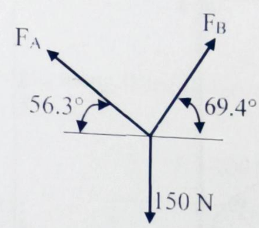


Fig 2.4

2) Draw Free body diagram for given fig 2.5, showing a 10 kg lamp supported by two cables AB and AC. Find the tension in each cable. **(Ans: Fig 2.6)**

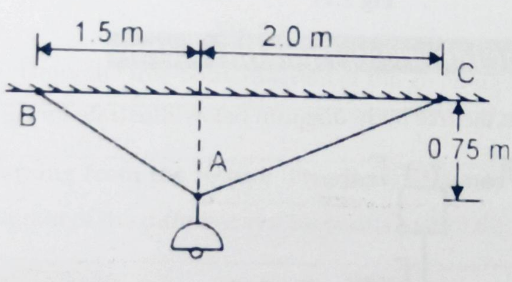


Fig 2.5

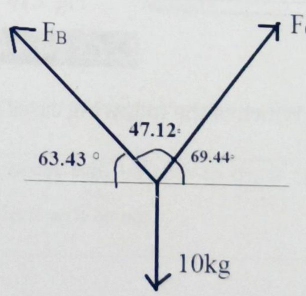


Fig 2.6

3) A roller of weight W = 1000 N rest on a smooth incline plane. It is kept from rolling down the plane by a string AC. Draw free body diagram for given fig 2.7.

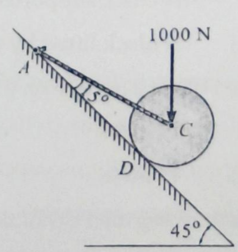


Fig 2.7

4) The weight of roller is 1500 N. 'P' is the minimum force required to start the roller over the block. Draw the FBD for given fig 2.8. **(Ans: Fig 2.9)**

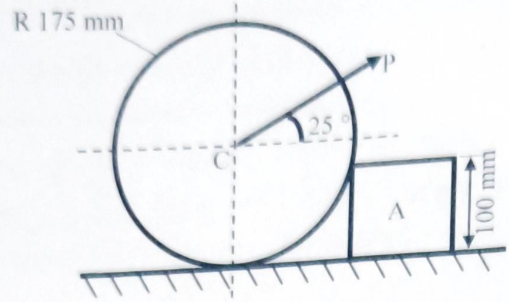


Fig 2.8

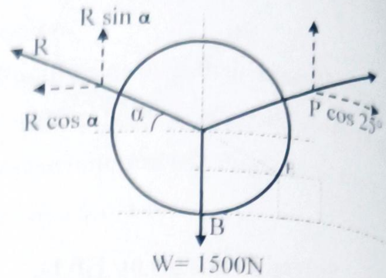


Fig 2.9

5) Draw FBD of cylinder and frame as shown in fig 2.10. **(Ans: Fig 2.11 & 2.12)**

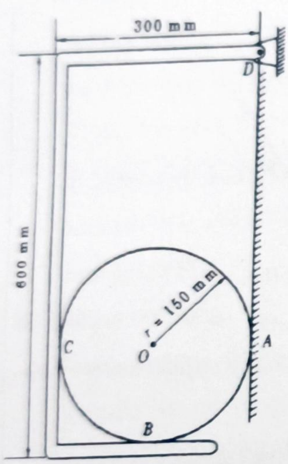


Fig 2.10

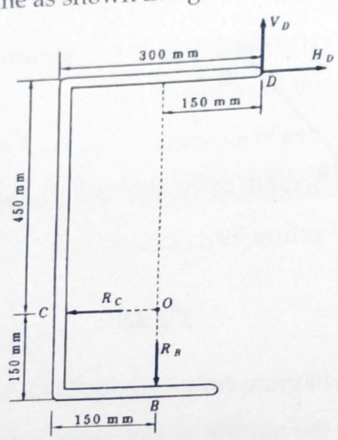


Fig 2.11

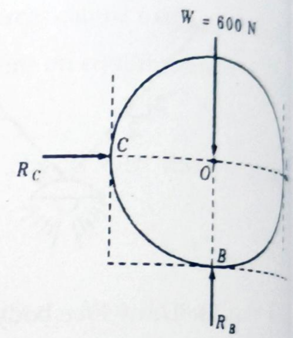


Fig 2.12

Let's check the takeaway from this lecture

1. Which of the following cases does the free body diagram below illustrate for fig 2.13?

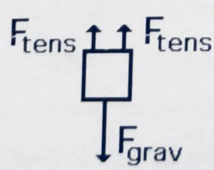


Fig 2.13

i) A block supported by two strings ii) A block supported by two springs

iii) A block lifted by two forces iv) A block falling off a table

2. The FBD of a Sphere on a table will have

i) 2 Forces ii) 3 Forces

iii) 4 Forces iv) 1 Force

3. For drawing the FBD of the door, the door will have to be

i) Cut in 2 sections ii) Supported by the wall

iii) Isolated from the hinges iv) Inverted upside down

Exercise:

1) Draw a Free Body Diagram for a book is at rest on a tabletop shown in fig 2.15.
2) Draw a Free Body Diagram for given fig 2.14

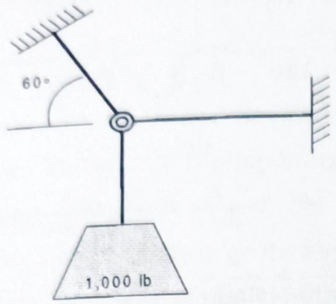


Fig 2.14

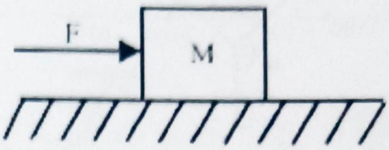


Fig 2.15

Questions/Problems Practice for the day:

3) Draw FBD of a block suspended to the ceiling using three strings shown in fig 2.16.

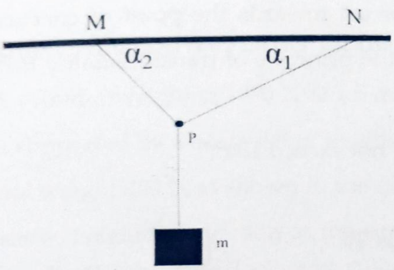


Fig 2.16

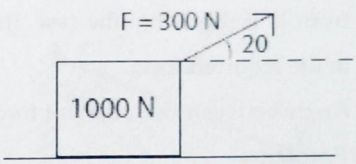


Fig 2.17

4) Draw the free-body diagram of the given fig 2.17

Learning from the lecture 'Free Body Diagram': Learner will be able to draw free body diagram of the different system so analysis of the system will be easy.

2.2 Lami's Theorem

Learning Objective: Student will be able to apply Lami's theorem to different equilibrium problems

Lami's Theorem: *It states, if a body is in equilibrium under the action of 3 coplanar & concurrent forces, then each force is proportional to the 'sine' of the angle between other two forces.*

Proof: Let P, Q and R be the 3 forces acting at point 'O' in equilibrium.

Let R_1 be the resultant of two forces P and Q. Now point 'O' is subjected to only two forces R and R_1. As per equilibrium under two forces R and R_1 must be equal, opposite and collinear.

Now, by sine rule

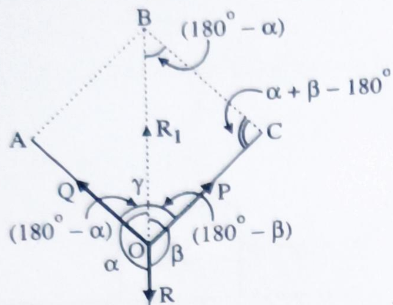


Fig 2.18

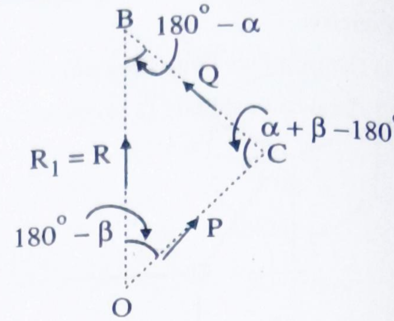


Fig 2.19

$$\frac{P}{\sin(180°-\alpha)}=\frac{Q}{\sin(180°-\beta°)}=\frac{R}{\sin(\alpha+\beta-180°)}$$

$$\frac{P}{\sin(\alpha)}=\frac{Q}{\sin(\beta)}=\frac{R}{\sin(360°-\gamma-180°)}$$

$$\frac{P}{\sin(\alpha)}=\frac{Q}{\sin(\beta)}=\frac{R}{\sin(\gamma)}$$

Limitations: -

1) It is applicable only when three forces acting at a point are in equilibrium.
2) The three concurrent forces should either act towards the point of concurrence or away from it. If this is not the case, then using the principle of transmissibility they can be made in the required form.
3) Angle between two adjacent forces should not exceed 180°

Solved Problems:

1) A circular roller of weight 1000 N and radius 20 cm hangs by a tie rod AB =40 cm and rests against a smooth vertical wall at C as shown in Fig. 2.20. Determine the tension in the tie rod and reaction RC at point C.

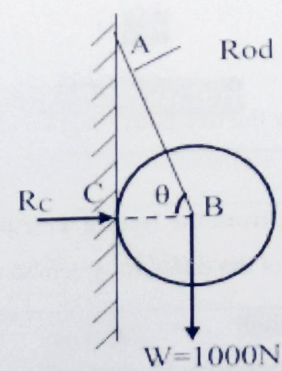


Fig 2.20

Solution: Draw the F.B.D. of the roller in equilibrium.

From geometry of Right-angled triangle ABC

Cos θ = BC/AB = 20/40, θ =60°

Let, T = Tension in rod

R_C = Reaction at point C

Using Lami's theorem we get,

$$\frac{T}{\sin 60°}=\frac{R_c}{\sin 150°}=\frac{1000}{\sin 120°}$$

Therefore T =1154.7 N, R_C = 577.35 N

Let's check the takeaway from this lecture

1. The Lami's theorem is applicable only for
   - i) Coplanar – Non-concurrent Forces
   - ii) Coplanar – Concurrent Forces
   - iii) Non-Coplanar - Non-concurrent Forces
   - iv) Non-Coplanar – Concurrent Forces
2. The Total angle between all forces to apply Lami's theorem, should not exceed

i) 120° ii) 180°

iii) 360° iv) 720°

3. Angle between 2 forces should not cross

i) 120° ii) 180°

iii) 360° iv) 720°

Exercise

1) A system of connected flexible cables shown in fig 2.21 is supporting two vertical forces 200 N and 250 N at points B and D. Determine the forces in various segments of the cable. **(Ans: T_{DE} 224.14 N, T_{BD} = T_{BC} = 183.01 N 336.6 N, T_{AB}=326.79 N)**

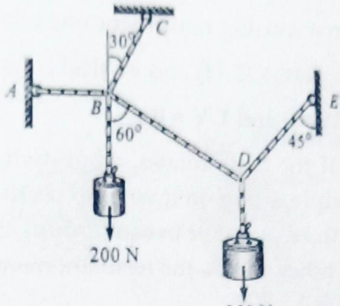


Fig 2.21

Questions/Problems for Practice for the day:

2) A cylinder weighing 1000 N & 1.5 m dia. is supported by a beam AB of length 6 m and weight 400 N as shown in the figure below. Neglecting friction at the surface of contact of the cylinder, determine (i) Wall reaction at D, (ii) Tension in the cable BC and (iii) Hinged reaction at support A. **(Ans: R_D=1000N, T_{BC}= 588.08 N, H_A= 490.7 N, V_A= 1105.96 N)**

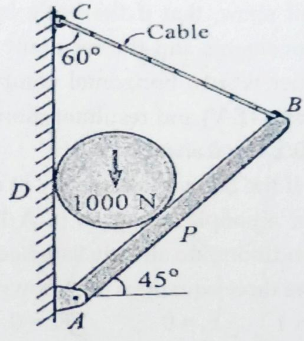


Fig 2.22

3) An electric light weighs 15N hangs from point C, ssas shown in fig 2.23. AC is inclined at 60° to horizontal & BC at 45° to vertical. Using Lami's theorem, find the forces in the strings AC & BC. **(Ans: 10.98N, 13.44N)**

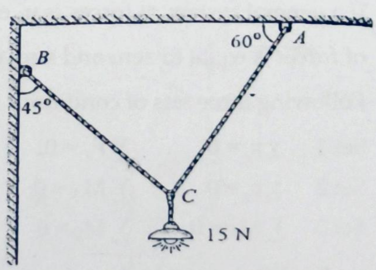


Fig 2.23

Learning from the lecture 'Lami's theorem': Student will be able to apply Lami's theorem to the different system so calculation of different forces can be done.

2.3 Conditions of Equilibrium of general & concurrent system of forces

Learning Objective-Student will be able to apply conditions of equilibrium to sphere system.

- **Conditions of equilibrium of concurrent system of forces.**

1. If the body moves in any direction, it means that there is a resultant force acting on it. A little consideration will show, that if the body is to be at rest or in equilibrium, the resultant force causing movement must be zero. Or in other words, the horizontal component of all the forces (Σ H) and vertical component of all the forces (ΣV) must be zero. Mathematically, **$\Sigma H = 0$** and **$\Sigma V = 0$**

2. If the body rotates about itself, without moving, it means that there is a single resultant couple acting on it with no resultant force. A little consideration will show, that if the body is to be at rest or in equilibrium, the moment of the couple causing rotation must be zero. Or in other words, the resultant moment of all the forces (Σ M) must be zero. Mathematically, **$\Sigma M = 0$**

3. If the body moves in any direction and at the same time it rotates about itself, if means that there is a resultant force and also a resultant couple acting on it. A little consideration will show, that if the body is to be at rest or in equilibrium, the resultant force causing movements and the resultant moment of the couple causing rotation must be zero. Or in other words, horizontal component of all the forces (Σ H), vertical component of all the forces (Σ V) and resultant moment of all the forces (Σ M) must be zero. Mathematically, **$\Sigma H = 0$ $\Sigma V = 0$** and **$\Sigma M = 0$**

4. If the body is completely at rest, it necessarily means that there is neither a resultant force nor a couple acting on it. A little consideration will show that, in this case the following conditions are already satisfied: **$\Sigma H = 0$ $\Sigma V = 0$** and **$\Sigma M = 0$**

The three equations are known as the conditions of equilibrium.

Set 1	$\sum F_x = 0$	$\sum F_y = 0$
Set 2	$\sum F_x = 0$	$\sum M_A = 0$, where A is not on X – axis.
Set 3	$\sum M_A = 0$	$\sum M_B = 0$, where A and B are non collinear with concurrence.

- **Conditions of equilibrium for general system of forces:**

If a general system of forces is in equilibrium, then sum of forces along any axis in the plane of forces is equal to zero and sum of moments about any point in the plane is equal to zero. Following three sets of conditions can be used.

Set 1	$\Sigma F_x = 0$	$\Sigma F_y = 0$;	$\sum M_A = 0$	
Set 2	$\Sigma F_x = 0$	$\Sigma M_A = 0$;	$\sum M_B = 0$	[line AB not perpendicular to x axis]
Set 3	$\Sigma M_A = 0$	$\Sigma M_B = 0$	$\sum M_C = 0$	[where A, B, C are not on one line]

- **Equilibrium of body subjected to two forces (Fig 2.24 & 2.25):** If the body is subjected to forces acting only at two points, then it is called a Two – Force body. A body will be in equilibrium

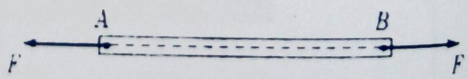


Fig 2.24

only when two forces are equal in magnitude, opposite in direction and have the same line of action.

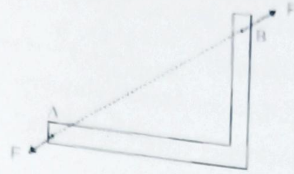


Fig 2.25

- **Equilibrium of body subjected to three force (Fig 2.26):** When a body is in equilibrium and acted upon by three forces then these three forces will be either concurrent or parallel forces

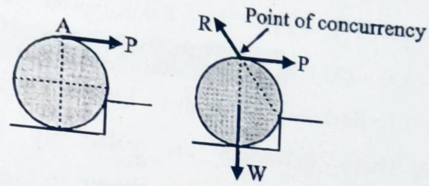


Fig 2.26

Solved Problems:

2) **Two cylinders P & Q in a channel as shown in fig 2.27. The cylinder 'P' has a diameter of 100mm & weight 200N and 'Q' has 180 mm diameter & weight 500N. Determine the reaction at all the contact surfaces.**

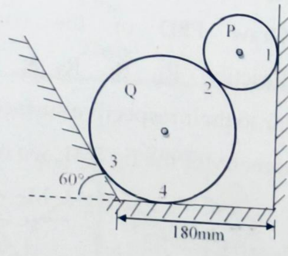


Fig 2.27

Solutions: Fig 2.28 shows FBD of combined cylinders P and Q. Reactions R_1, R_2, R_3 and R_4 are perpendicular to their respective surfaces.

From geometry of figure, AD is the angle bisector of angle HAG.

angle DAG = angle DAH = 30°

From triangle DAG, $\tan 30^\circ = DG/AG = X_1/90$

$X_1 = 90 \tan 30^\circ = 51.96$ mm

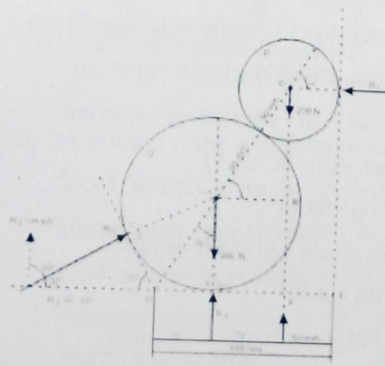


Fig 2.28

Also, length DE = 180 = $X_1 + X_2 + 50 = 51.96 + X_2 + 50$; $X_2 = 78.04$ mm

Applying COE to combined cylinders

(+ ve Anticlockwise) $\Sigma M_A = 0$; $R_1 \times BC - 200 \times AB = 0$

$R_1 \times 116.23 - 200 \times 78.04 = 0$; $R_1 = 134.3$ N $(\leftarrow)$

$\Sigma F_x = 0\ (\rightarrow)$; $-R_1 + R_2 \sin 60° = 0$;

$-134.3 + 0.866\ R_3 = 0$; $\therefore R_3 = 155$ N at $\theta = 30°$

$\Sigma F_y = 0\ (\uparrow)$; $-500 - 200 + R_4 + R_3 \cos 60° = 0$

$-500 - 200 + R_4 + 155 \cos 60° = 0$; $R_4 = 622.5$ N

$(\uparrow)$ To find reaction R_2, draw **FBD** of cylinder P

3) Three cylinders are piled up in a rectangular channel as shown in Fig 2.30. Determine the reaction at point 6 between cylinder A and the vertical wall of the channel. (Cylinder A : R_A = 4 cm, M_A = 15 Kg, Cylinder B : R_B = 6 cm, M_B = 40 Kg, Cylinder C : R_C = 5 cm, M_C = 20Kg)

Solution: Draw FBD of the combined cylinders. Reaction R_1, R_3, R_5 & R_6 are perpendicular to their respective surfaces.

From the geometry of the fig 2.31, we have R_A = 4 cm & R_C = 5 cm; M_A = 15 Kg & M_C = 20 Kg ; R_B = 6 cm, M_B = 40 Kg

Length AB = $R_A + R_B$ = 10 cm, BC = $R_C + R_B$ = 11 cm

Length BP = (18 − C1 − B3) = (18 − 5 − 6) = 7 cm

Length AQ = (18 − A6 − B3) = (18 − 4 − 6) = 8 cm

In ΔBCP; $\cos\theta_1 = BP/BC = 7/11$

$\theta_1 = \cos^{-1}(7/11) = 50.48°$

and in ΔABQ; $\cos\theta_2 = AQ/AB = 8/10$

$\theta_2 = \cos^{-1}(8/10) = 36.87°$

Now Draw FBD of individual cylinder

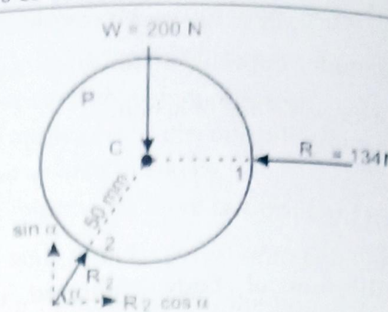


Fig 2.29

separately as shown in fig 2.29

Applying COE to cylinder P;

$\Sigma F_y = 0\ (\uparrow)$; $R_2 \sin \alpha - W = 0$;

$R_2 \sin 56.12° - 200 = 0$; $R_2 = 240.8$ N

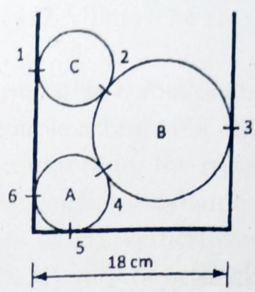


Fig 2.30

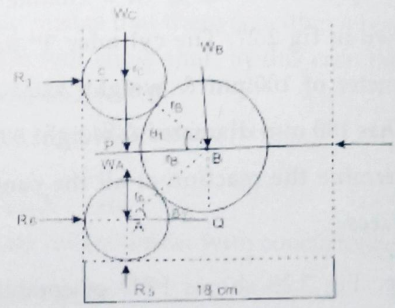


Fig 2.31

FBD of cylinder C; (Fig 2.32)

Applying Lami's theorem, $[R_1 / \sin(90+50.48)] = [R_2 / \sin 90] = [196.2 / \sin(180-50.48)]$;

$R_1 = 161.84$N, $R_2 = 254.34$N

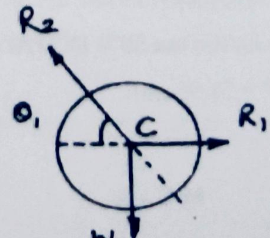


Fig 2.32

FBD of cylinder B; (Fig 2.33)

Apply the Condition of equilibrium

$\sum F_x = 0\ (+\rightarrow)\ -R_3 + R_2\cos\theta_1 + R_4\cos\theta_2 = 0$

$\sum F_y = 0\ (+\uparrow)\ R_4\sin\theta_2 - R_2\sin\theta_1 - (40\times9.81) = 0$

$R_4 = 981.22$ N, $R_3 = 946.99$ N

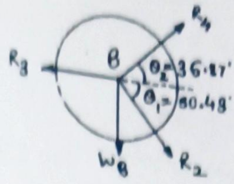


Fig 2.33

FBD of cylinder A; (Fig 2.34)

$\sum F_x = 0\ (+\rightarrow)\ R_6 - R_4\cos\theta_2 = 0$

$\sum F_y = 0\ (+\uparrow)\ R_5 - R_4\sin\theta_2 - (15\times9.81) = 0$

$R_6 = 785$ N; R5=737.74 N

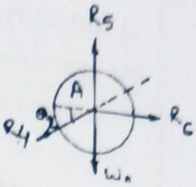


Fig 2.33

Let's check the takeaway from this lecture

1. There are ____ conditions of equilibrium.
   i) One
   ii) Two
   iii) Three
   iv) Four
2. There are ____ conditions of equilibrium for Concurrent force system
   i) One
   ii) Two
   iii) Three
   iv) Four
3. There are ____ conditions of equilibrium for Parallel force system
   i) One
   ii) One
   iii) Three
   iv) Three

Exercise:

1) Two spheres A & B are resting in a smooth channel as given in fig 2.34. Draw FBD of A & B, find reactions at contact points. R_A=250mm, R_B=200mm respectively. **(Ans: 138.07N, 243.03N, 159.43N, 620.28N)**

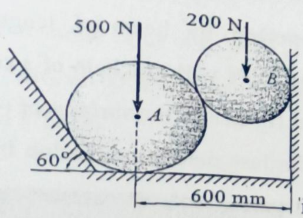


Fig 2.34

2) Sphere A '1000N' rests on two spheres each of weight 900N for fig 2.25. The spheres B & C are connected by an inextensible string of length L=600mm. Radius of spheres = 200mm. determine reactions at all contact points & force in string. **(Ans: 1467.13N, 756.07N)**

3) A smooth roller with 50mm radius and 200N weight is to be pulled over a 10mm high step as shown in fig 2.26. Find minimum possible value of P and corresponding value of 'θ'. **(Ans: θ= 36.87°; P_{min} =120N).**

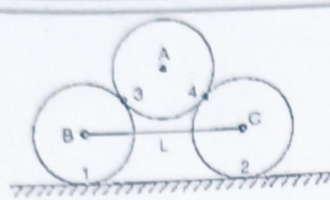


Fig 2.25

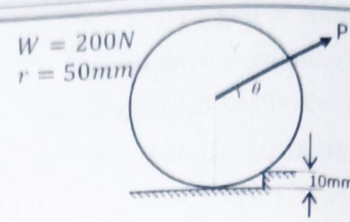


Fig 2.26

Questions/Problems for practice for the day:

4) Two smooth cylinders are supported between stationary surfaces as shown in fig 2.27. Find reaction at all points of contact. **(Ans.: R_E = 188N; R_D = 565.8N; 30°; R_C =166.2N; R_f = 349.2N,57.42°)**

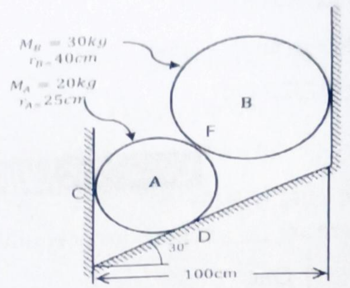


Fig 2.27

Learning from the lecture 'Lami's theorem': Student will be able to apply conditions of equilibrium to different system so reactions coming on bodies can be calculated

2.4 Problems on equilibrium of Connected bodies.

Learning Objective: Students will be able to apply conditions of equilibrium Connected bodies

Solved Problems

1) **A cylinder of diameter 1m and weighing 1000N and another block weighing 500N are supported by beam of length 7m weighing 250N with the help of a cord as shown in fig 2.28. If the surfaces of contact are frictionless determine tension in cord and reaction at points of contact.**

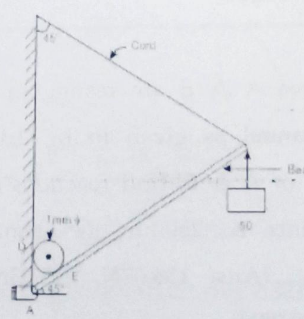


Fig 2.28

Solution: Draw F.B.D of cylinder & beam AB.

Using Lami's Theorem for cylinder,

R_D/ sin 135 = R_E/ sin 90 = 1000/ sin135

By calculation, R_D=1000 N; R_E=1414.2 N

From the geometry of fig. ∠DAE = 45°

since AO is the angle bisector of ∠DAE, hence

∠OAE = 45°/2 = 22.5°

From right angle triangle OAE,

tan22.5° = OE/AE = 0.5/AE; AE = 1.21m

Length BL=7 cos45°=4.95m; Length AG = 3.5 cos45°=2.5m; Length AL=7 sin45°= 4.95m

Applying COE to beam AB,

$\sum M_A = 0$ (Anti clockwise +ve)

$(T \times 7) - (R_E \times AE) - (250 \times AG) - (500 \times BL) = 0$

$7T - (1414.2 \times 1.21) - (250 \times 2.5) - (500 \times 4.95) = 0$

7T = 4800.2 ; T = 685.75N

Fig 2.29

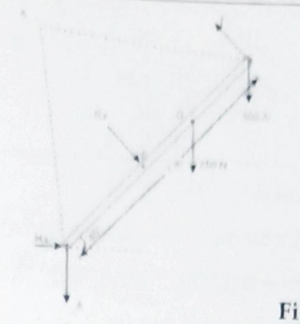


Fig 2.30

$\sum F_x = 0(\rightarrow)$ +ve ; $-T\cos45° + R_E\cos45° + H_A = 0$

$H_A = -515.09$ N; $H_A = 515.09$ N $(\leftarrow)$

$\sum F_y = 0(\uparrow)$ +ve; $T\sin45° - 500 - 250 - R_E\sin45° + V_A = 0$; $V_A = 1265.09$N $(\uparrow)$

$R_A = \sqrt{(H_A)^2 + (V_A)^2} = \sqrt{(515.09)^2 + (1265.09)^2}$

$= 1365.93$ N

$\tan\theta = (V_A / H_A)$; $\theta = \tan^{-1}(1265.09/515.09)$

$\theta = 67.85°$

2) **A weightless bar is placed in a horizontal position on the smooth inclines as shown in fig 2.31. Find 'x' at which 200N force should be placed from point B to keep the bar horizontal.**

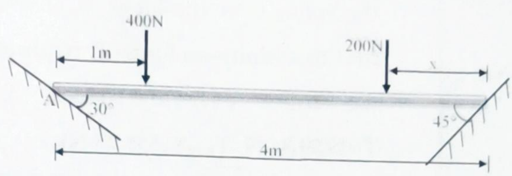


Fig 2.31

Solution: Draw F.B.D. of following fig. 2.32

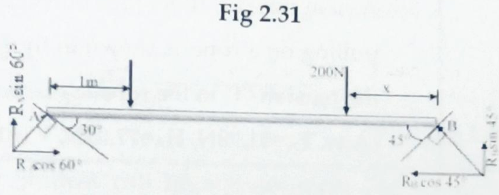


Fig 2.32

Applying COE, Moment about point B

$\sum M_B = 0$ (Anticlockwise) +ve;

$-(R_A\sin60° \times 4) + (400 \times 3) + (200 \times d) = 0$ — (I)

$\sum F_x = 0(\rightarrow)$ +ve; $R_A\cos60° - R_B\cos45° = 0$

$R_A(0.5) - R_B(0.71) = 0$ — (II)

$\sum F_y = 0(\uparrow)$ +ve; $R_A\sin60° - 400 - 200 + R_B\sin45° = 0$

$R_A(0.866) + R_B(0.71) = 600$ — (III)

Solving equation (II) & (III) simultaneously,

we get, $R_A = 439.2$N, $R_B = 309.3$N

Substitute value of R_A & R_B in equation (I)

$-(R_A\sin60° \times 4) + (400 \times 3) + (200 \times d) = 0$,

$d = 1.61$m.

Let's check the takeaway from this lecture

1. If a body is in equilibrium, we may conclude that

    i) The moment of all the forces about any point is zero
    ii) The resultant of all the forces acting on it is zero
    iii) No force is acting on the body
    iv) Both (i) & (ii)

2. There are ____ conditions of equilibrium for Concurrent force system

    i) One
    ii) Two
    iii) Three
    iv) Four

3. There are ____ conditions of equilibrium for Parallel force system

i) One   ii) One

iii) Three   iv) Three

Exercise:

1) Determine the forces in cables AB and BC needed to hold the 50 kg ball B in equlibrium for fig 2.33. Given F=300N, d = 1m. **(Ans: F=426.7N(T) ; F_{BC} =155.7N (T))**

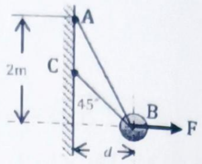


Fig 2.33

2) The roller shown in fig. 2.34 is of weight 1500N. What force 'T' is necessary to start the roller over the block A, if θ=25°, Also find the minimum force 'T' required to start the roller over the block A. **(Ans.: T=1759.34 N, T_{min}=1355.23 N)**

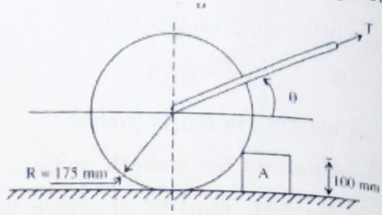


Fig 2.34

3) A man raises a 10 Kg joist of length 4 m by pulling on a rope as shown in fig 2.35. Find the tension 'T' in the rope & reaction at 'A'. **(Ans: T_A=81.98N, H_A=77.03N, V_A=126.03N)**

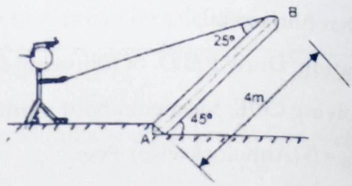


Fig 2.35

4) A crane is pivoted at end B and is supported by a smooth guide at A as shown in fig 2.36. Determine the reaction produced at A and B by a vertical load W=5kN applied at C. **(Ans.: R_A=6.667kN(←),R_B=8.334kN, 36.87°)**

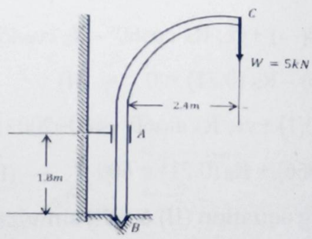


Fig 2.36

Practice Problem:

5) A 5m rigid homogeneous rod hinged to a wall at its end A. It is supported by an inextensible wire at B as shown in the fig 2.37. Weight of the rod is 20kg. The rod is supporting a smooth cylinder of diameter 2m and mass 30kg. Determine reaction at 'A' & Tension in wire 'BC'. **(Ans.: T_{BC} = 382.3N; R_{AX} = 88N(→), R_{AY} = 495.5N(↑), R_A = 503.3N, 79.83°)**

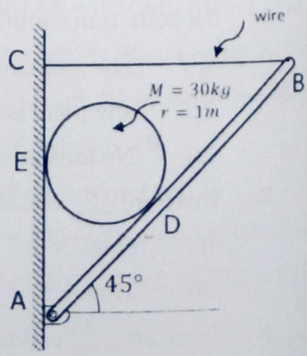


Fig 2.37

6) A bar is hinged at 'A' and rests on cylinder at 'C'. The center of cylinder is connected to the bar by a horizontal wire OE as shown in fig 2.35. Determine i) Reaction at A, ii) Tension in wire, iii) Reaction at C & D, Neglect weight of bar & assume smooth surfaces. **(Ans: V_A = 79.3N(↓), H_A = 0, T = 551.7N, R_C = 800N, R_D = 779.34N)**

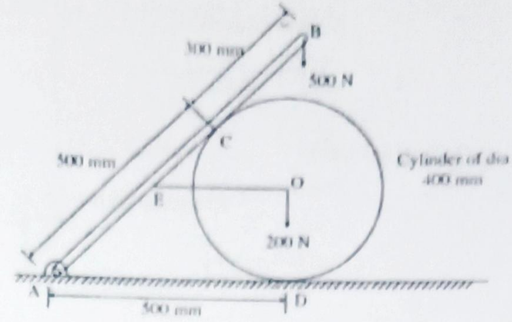


Fig 2.38

7) Two smooth rollers are connected by a bent rod ABC as shown in fig 2.36. Find reactions at D, E & F. Point 'A' & 'C' are frictionless pin connections. **(Ans: R_D = 4359N(→); R_E = 3202N(↑); R_F=1165N)**

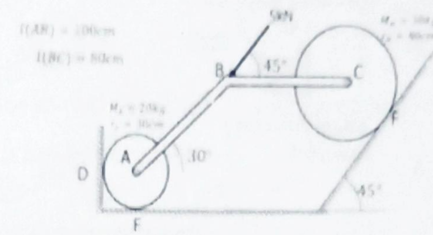


Fig 2.39

8) A cylinder of weight 300N is held in equilibrium as shown in fig 2.40. Determine the tension in the string AD and reaction at C and B. The length of AE =750 mm. **(Ans: 692.3N, 271.55N, 992N)**

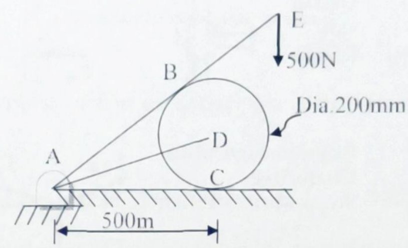


Fig 2.40

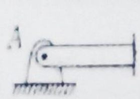
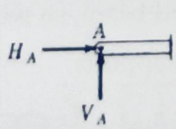
Learning from the lecture 'Lami's theorem': Student will be able to apply conditions of equilibrium to connected bodies so reactions coming on bodies can be calculated

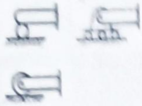
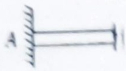
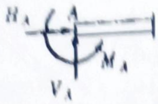
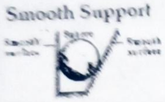
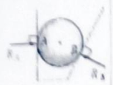
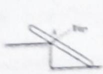
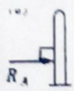
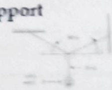
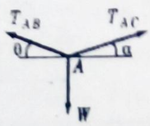
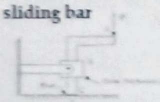
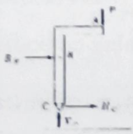
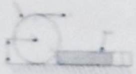
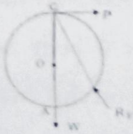
2.5 Reactions of different support, Types of loads, types of beams

Learning Objective: Students will be able to draw free body diagram of several types of beams.

Types of supports and reactions: Whenever a body is supported, the support offers resistance, known as Reaction. E.g You are sitting on chair while reading book, your weight is being supported by the chair which offers a force of resistance upwards.

There are distinct types of support shown in tabular form: -

Sr. No.	Types of Support	Reaction & FBD	Number of Unknowns
1	Hinge support 		Hinge support allows only rotation and does not allow movement either horizontally or vertically. Hence, Hinge support offers two unknown reactions. In which one acts in horizontal direction other reaction acts in vertical direction at the point of contact.

2	Roller support			Roller support offers only one unknown reaction. This acts perpendicular to the surface at the point of contact.
3	Fixed support			Fixed support does not allow any type of motion, neither translation nor rotational. Hence, Fixed support offers three unknown reaction. In which one acts in horizontal direction other reaction acts in vertical direction and third is moment at the point of contact.
4	Smooth Support			A smooth surface offers a similar reaction as that of roller support. i.e. a force reaction perpendicular to the surface at the point of contact.
5	Edge(Fulcrum) Support			It offers reactive force, only one unknown reaction normal to the surface of the body at the point of contact.
6	Collar/ slot/ Guide			One Unknown: It offers a force reaction normal to the surface of the body.
7	Rope/String/Cable/ Chain/Belt Support			In FBD of Rope/string/cable/chain/ belt support the force is always taken as tension and away from the body.
8	FBD of bar pivoted at one end and has a sliding bar			Here Pivoted point is act like a rough surface or hinge support which will have two unknown reaction
9	Force 'P' applied to turn the roller over rectangular block			In this type of support contact between roller and the surface is taken as dummy contact i.e., in FBD force at point A will not be consider at that instance.

Types of beams:

(i) Simply supported beam: A beam that has hinged connection at one end and roller connection in other end is called simply supported beam, so when transverse loads are applied they resists by generating reactive forces on their supports.

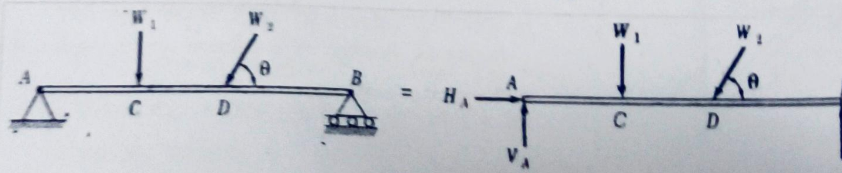


Fig 2.41 (a) & (b)

(ii) Over hanging beam: A beam which extends beyond the supports is known as overhanging

2.42 (a) & (b)

Single overhanging beam Double overhanging beam

(iii) **Cantilever beam:** A beam fixed at one end & free at other end is called cantilever beam. At fixed support, there are three reactions- (a) horizontal reaction (b) vertical reaction & (c) Moment. as shown in fig 2.43

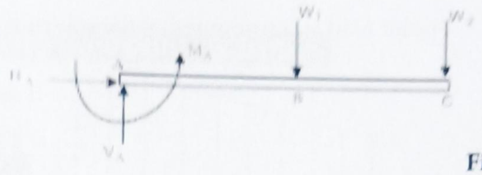


Fig 2.43

(iv) Compound beam: When two or more beams are joined together by means of internal hinge or internal roller, the combinations called compound beam. As shown

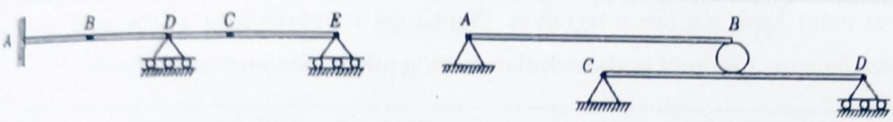


Fig 2.44

- In the analysis of compound beam, always moment of forces acting on individual beam or combination of beams about internal hinge is zero.
- We can disconnect the beams at internal hinge or internal rollers and draw FBD of each beam separately by assuming internal reaction components at hinge as equal and opposite.
- Conditions of equilibrium to each beam can be separately applied after disconnection.

Types of load on the beam:

There are mainly four types of load on the beam.

(i) **Point load or concentrated load:**

A load acting at a single point on the beam is called point load.

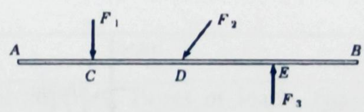


Fig 2.45

(ii) **Uniformly distributed load or rectangular load (UDL)**: A load which is spread over the beam (partly or fully) uniformly is called UDL. UDL can be converted into point load by taking product of intensity of UDL and span of UDL. This point load will be represented at the center of UDL.

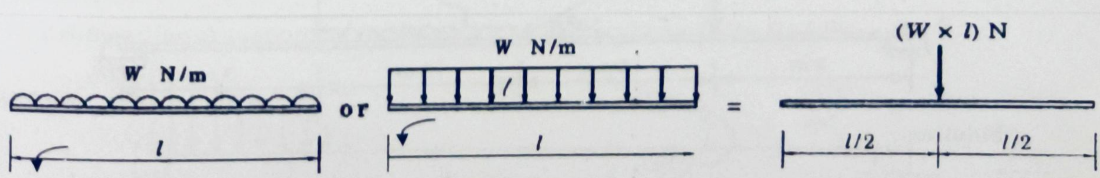


Fig 2.46

(iii) Uniformly varying load or triangular load (UVL): A load whose intensity is linearly varying between two points is known as Uniformly Varying Load.

(iv) Trapezoidal load (UVL): Combination of UDL and UVL: A load whose intensity is linearly varying between two points is known as Uniformly Varying Load. Uniformly varying load can also be converted into point load by calculating the area of the loading and point load is represented at the centroid of the area.

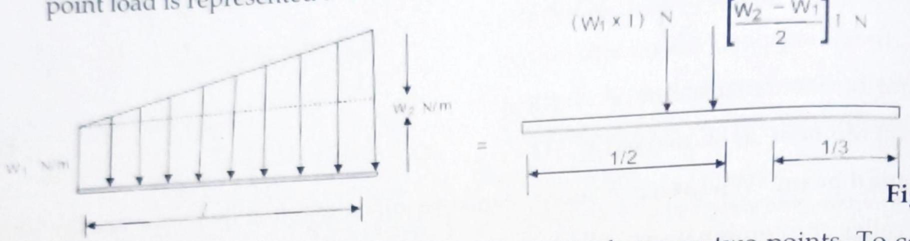


Fig 2.47 (1) & (2)

(v) Varying load (VL): A load whose intensity varies between two points. To convert varying load to point load, we use integration. Point load is represented at the centroid of the loading diagram. Centroid is also calculated using integration method.

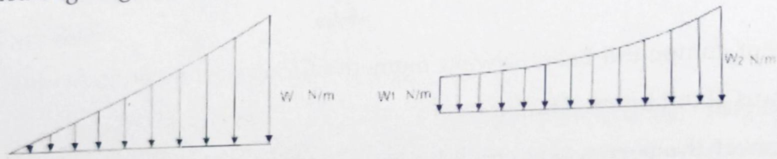


Fig 2.48 (1) & (2)

Solved Problem:

3) **Draw Free body diagram for a given fig 2.49.**

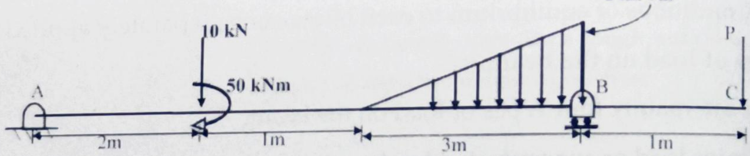


Fig 2.49

Solution:

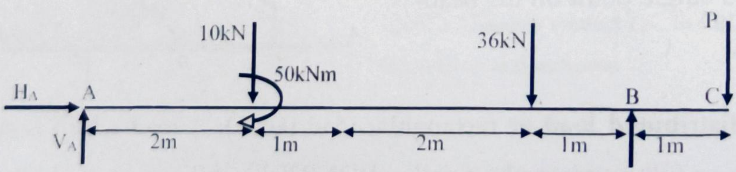


Fig 2.50

4) **A beam AB is loaded as shown in fig 2.51. Draw Free body diagram.**

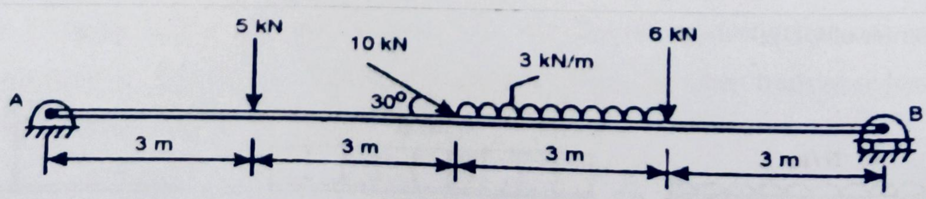


Fig 2.51

Solution:

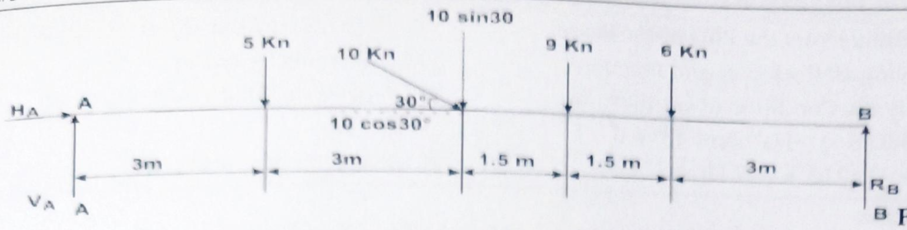


Fig 2.52

Let's check the takeaway from this lecture

1) Number of reactions in a double roller support

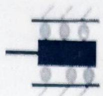


Fig 2.53

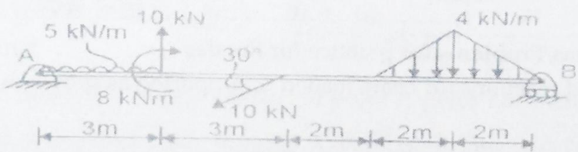
a) 3 b) 2 c) 1 d) 0

2) Reaction of a roller support is always ________

a) parallel to rolling direction b) perpendicular to rolling direction c) depends on the direction of loading d) none of above

Exercise:

1) Draw FBD for a given beam shown in fig 2.54.



ig 2.54

Problem for Practice of the day

2) Draw FBD for a given beam shown in fig 2.55.

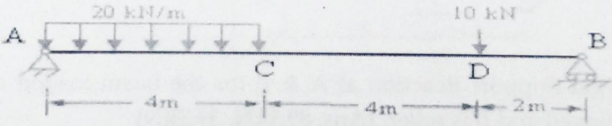


Fig 2.55

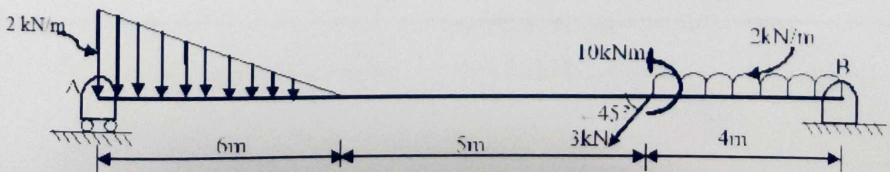
Learning from the lecture 'Reactions of different support, Types of loads, types of beams': Student will be able to draw free body diagram of several types of beams & able to convert different loads into point load.

Problems on Simply supported beam

Learning outcome: Student will be able to find reactions of simply supported beam.

Solved problems:

5) **Find the reaction at supports of the Beam AB loaded as shown in the fig 2.56.**



'ig 2.56

Solution: Draw the FBD of the Beam showing all the forces and reactions.
Apply the Condition of equilibrium

$\sum F_x = 0$ (+→) $-H_A - 3\cos 45° = 0$

$H_A = -2.12$ N $= 2.12$ N(→)

$\sum F_y = 0$ (+↑) $R_D - 3\sin 45° - 6 - 8 + V_A = 0$ – (i)

$\sum M_A = 0$ (+anticlockwise)

$8\times2 + 10 + 3\sin45°\times4 + 6\times11 - R_D\,15 = 0$ --(ii)

On solving

$R_D = 6.69$ N(↑), $V_A = 9.422$ N(↑) ....Ans

Exercise:

1) Find analytically the support reaction at B and load P for the beam shown in fig 2.57 if the reaction at support A is zero. **(Ans: 26kN, 72kN)**

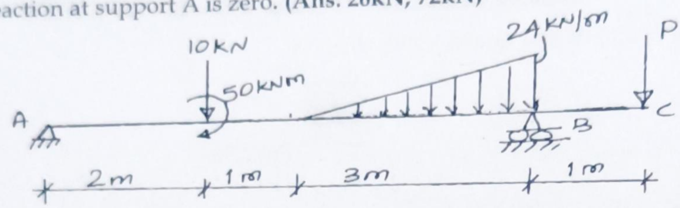


Fig 2.57

2) Find the Reactions at supports of the beam AB loaded as shown in the fig 2.58 below. **(Ans: R_B= 136.03 kN, H_A=0, V_A=103.97 kN)**

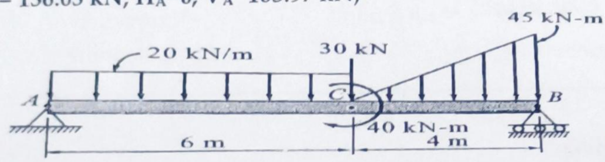


Fig 2.58

Questions/Problems for practice for the day

3) Find reactions for beam loaded & supported as shown in fig 2.59. **(Ans: 50kN, 60kN)**

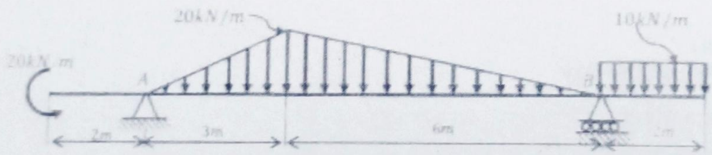


Fig 2.59

4) Find Support Reaction at A & B for the beam loaded as shown in the fig 2.60. A is hinged and B is roller. **(Ans: 89.5kN, 39.5kN)**

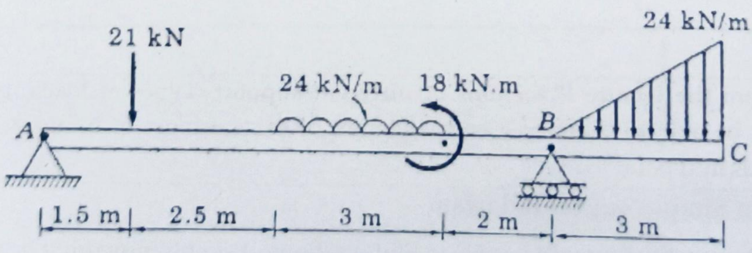
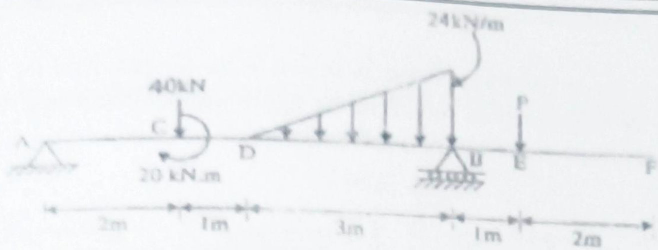


Fig 2.60

5) Find the support reaction at B and the load P, for the beam shown in fig 2.61. if the reaction at support A is zero. **(Ans: 56kN, 102kN)**



ig 2.61

Learning from the lecture 'Simply supported beam: Student will be able to apply conditions of equilibrium to Simply supported beam & can calculate reactions of simply supported beam

2.6 Problems on reactions of beam with pulley & external mass (Complicated loading)

Learning objective: Students will be able to find the reactions of beam containing complicated loads

<u>Solved problems</u>

6) **Determine minimum weight of the block required to keep beam in horizontal equilibrium as shown in fig 2.62. Assume rough pulley with coefficient of friction as 0.2.**

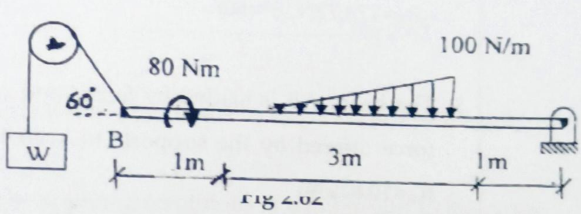


Fig 2.62

Solution: Draw the FBD of beam AB. Hinge support at point A gives horizontal and vertical reaction. UVL is converted into point load. Resolve T at point B into two components. as shown in the fig 2.63.

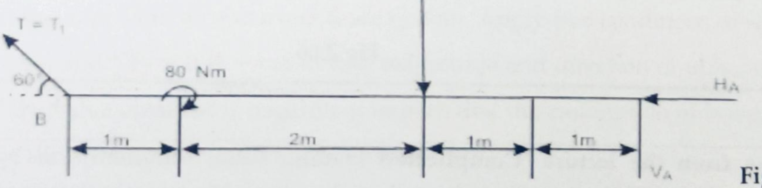


Fig 2.63

Apply the condition of equilibrium, we have $\sum M_A = 0$ (+anticlockwise)

$150\times2-80+ T \sin 60°\times5 =0$

Hence T = 50.8 N

Now pulley has coefficient of friction so tension on both side of the pulley will be Different

$T= T_1$ Tension on the tight side of pulley

T_2 =Tension on the slack side of pulley

$W = T_2$ = Minimum Weight of the block

Ratio of tension

$T_1/ T_2 = e^{\mu\theta}$

μ = Coefficient of friction =0.2

θ = Angle of lap (measured in radians)

So, $50.8/W = e^{(0.2\times50\Pi/180)}$

W= 30.1 N…… Ans

Exercise:

1) A beam is loaded and supported as shown in the fig 2.64. Find the maximum mass M that can be suspended as shown in the figure without disturbing its equilibrium. The small pulley is frictionless. **[Ans.:M=26.82kg]**

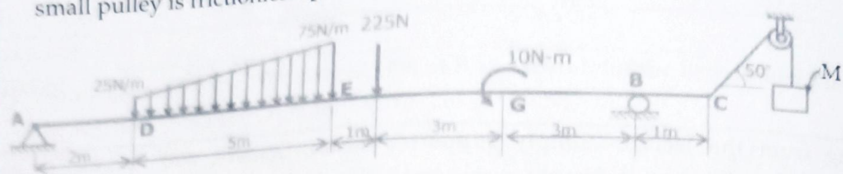


Fig 2.64

2) Find support reaction of the beam as shown in fig 2.65. (**Ans: - H_A=116.86 N (→), V_A=82.77 N (↑), R_B=173.72N, θ°=60)**

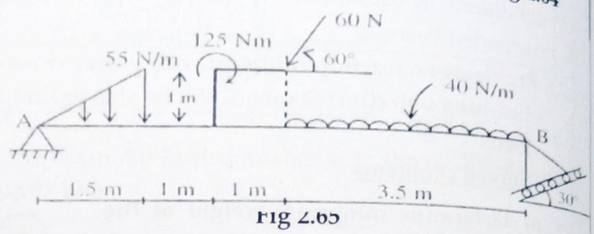


Fig 2.65

3) The beam AB is loaded by forces and couples as shown in fig 2.66. Find the reaction force offered by the supports to keep the system in equilibrium. **(Ans: R_A=19.74kN, R_B=10.82kN)**

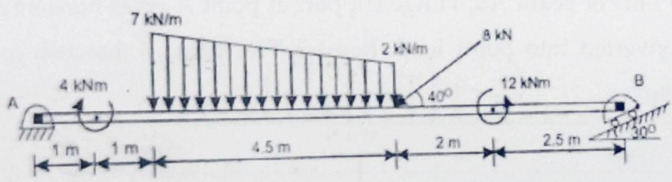


Fig 2.66

Learning from the lecture 'Complicated loading beam': Student will be able to apply conditions of equilibrium to Complicated loading beam'& can calculate reactions of simply supported beam

Lecture 17

Trusses, Types of Trusses, Condition of Perfect

Learning Objective: Students will able to identify Perfect truss

2.1.12 Trusses:

➢ **Assumptions made in truss analysis:**

1) All the members of the truss lie in one plane.
2) The loads acting on the truss lie in the plane of the truss.
3) The members of the truss are joined at the ends by internal hinges known as pins.
4) Loads act only at the joints and not directly on the members.

5) Each member is a two force body thereby resulting in axial forces which are either tensile or compressive
6) The self-weight of the members being small as compared to the loads applied is neglected.
7) The truss is statically determinate i.e. forces can be determined using equilibrium conditions.

➢ **Types of trusses:**

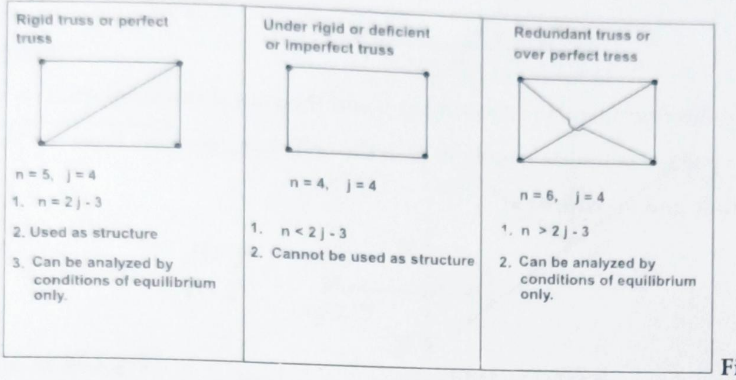


Fig 2.67

➢ **Working rules for Method of Joints:**

Principle: If the entire truss is in equilibrium, then each joint of the truss is also in equilibrium.

1) Draw Free body diagram of the entire truss.
2) Find the reactions at the supports of the truss by applying conditions of equilibrium to the entire truss.
3) Isolate a joint from the truss which has not more than two members with unknown force.
4) Assume that the members carry tensile force and hence show direction of the arrowheads (representing force direction) away from the joint.
5) The forces at the joint form a concurrent force system. Apply two conditions of equilibrium viz. $\Sigma F_X = 0$ (→ +ve) and $\sum F_Y = 0$ (↑ + ve) to find magnitude and direction of unknown force in the members. If the value obtained is negative it implies that the assumption of being a tensile force was incorrect and the member carries compressive force.
6) Now isolate another joint having not more than two members with unknown force. Follow steps 3 to 5 and solve each joint to find forces in all the members of the truss.
7) Tabulate the results indicating the member, magnitude of force and nature of the force.

➢ **Special Cases in truss analysis (ZERO Force Members) :**

1. If a joint has only two members meeting at it and the joint is not supported or loaded then the members meeting at that joint has zero force along them.

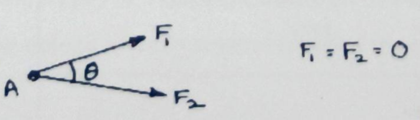


Fig 2.68

2. If a joint has three members meeting at it and the joint is not supported or loaded, also out of three members two members form a pair of collinear members then the third member meeting at that joint has zero force along it and the collinear members have forces equal in magnitude and nature.

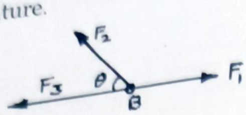


Fig 2.69

3. If a joint has four members meeting at it and the joint is not supported or loaded, also there are two pairs of collinear members then the collinear members have forces equal in magnitude and in nature.

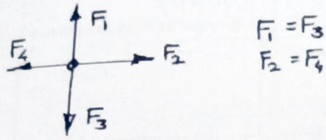


Fig 2.70

2.1.13 *Solved Problems:*

1. *Identify members carrying zero forces in the plane truss as shown below:*

Solution:

a)

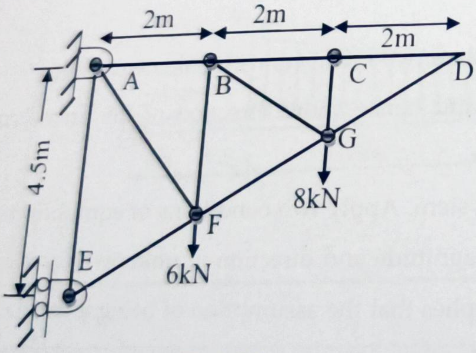


Fig 2.71

b)

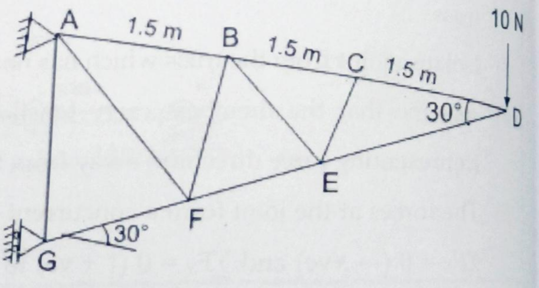


Fig 2.72

a) FBD of entire truss:

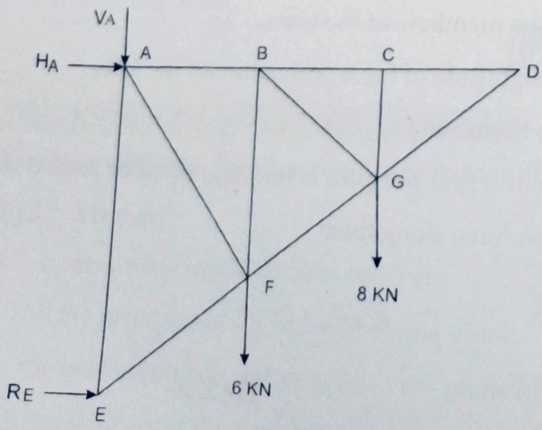


Fig 2.73

b) FBD of entire truss:

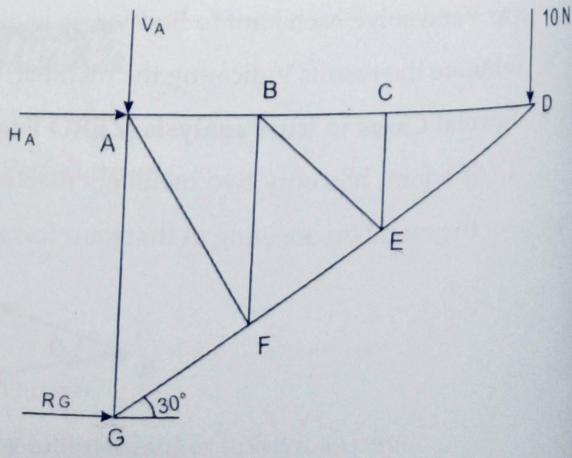


Fig 2.74

[i] FBD of joint D:

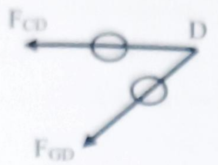


Fig 2.75

[i] FBD of joint C

Fig 2.76

[ii] FBD of joint C:

Fig 2.77

[ii] FBD of joint E:

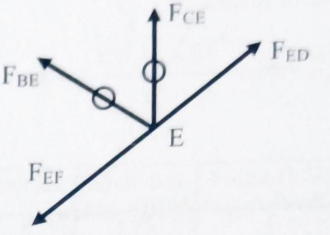


Fig 2.78

[iii] FBD of joint B:

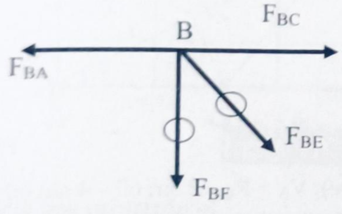


Fig 2.79

[iv] FBD of joint F:

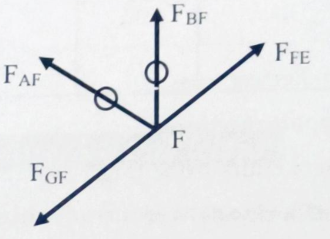


Fig 2.80

<u>Exercise:</u>

1) **Find the reaction of & zero force member in given truss.**

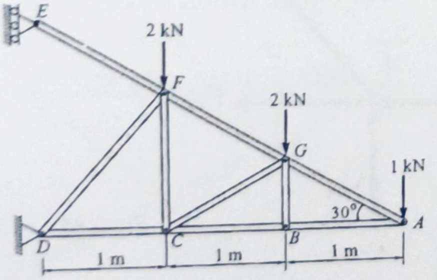


Fig 2.81

<u>Practice:</u>

Find the reaction of & zero force member in given truss. (Ans: $F_{CB} = F_{CD} = F_{AE} = F_{DE} = 0$; 4kN; -4kN)

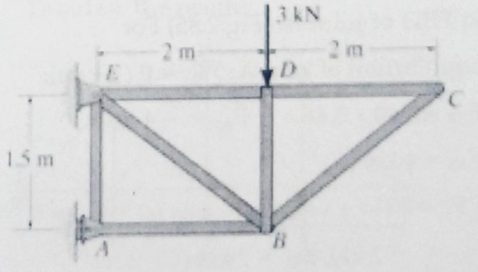


Fig 2.82

Learning from the lecture 'Trusses, Types of Trusses, Condition of Perfect': Student will able to apply conditions of Perfect truss & able to identify several types of trusses

Lecture 18

Problems on method of joints

Learning objective: Student will able to find the forces in different member of truss.

2.1.14 Solved Problems:

12) Determine forces in all the members of the plane truss shown in the sketch fig 2.83 by method of joints.

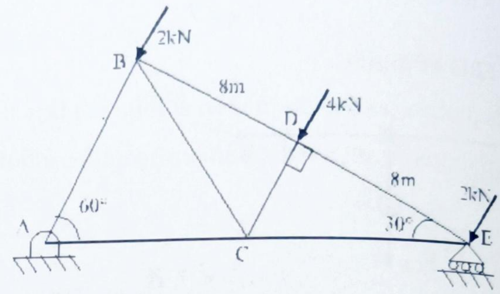


Fig 2.83

Solution: (1) Checking stability of truss: –

No. of members, m = 7; No. of joints, j = 5

No. of reactions, r = 3; m = 2 j – r

i.e. 7 = (2 x 5) – 3

7 = 7............so Perfect truss

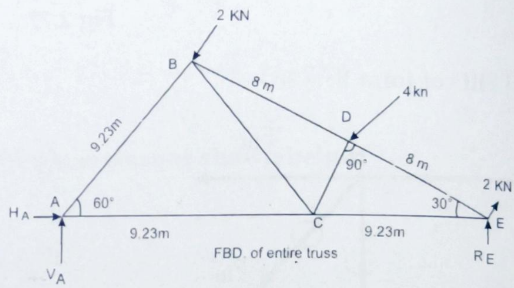


Fig 2.84

(2) Finding support reactions:

For equilibrium of entire truss, (Fig 2.84)

a) $\sum M_A = 0$ (Anticlockwise) +ve

so (– 4 x 8) – (2 x 16) + (RE x 18.47) = 0

$R_E = 3.46$kN (↑)

b) $\sum F_Y = 0$ (↑ + ve); $V_A + R_E - 2 \sin 60 - 4 \sin 60 - 2 \sin 60 = 0$; $V_A = 3.46$ kN (↑)

c) $\sum F_X = 0$ (→ + ve); $H_A - 2 \cos 60 - 4 \cos 60 - 2 \cos 60 = 0$; $H_A = 4$ kN (→)

(3) Method of joints: –

a) FBD of joint A: (Fig 2.85) For equilibrium of joint A; $\sum F_Y = 0$ (↑ + ve);

$F_{AB} \sin 60 + 3.46 = 0$; $F_{AB} = -4$ kN;

$F_{AB} = 4$ kN (C)

$\sum F_X = 0$ (→ + ve); $4 + F_{AB}\cos 60 + F_{AC} = 0$

$F_{AC} = -2$ kN; $F_{AC} = 2$ kN (C)

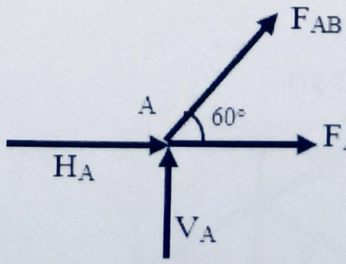


Fig 2.85

b) FBD of joint E: (Fig 2.86)

$\sum F_Y = 0$ (↑ + ve); $F_{DE} \sin 30 - 2 \sin 60 + 3.46 = 0$; $F_{DE} = -3.46$ kN; $F_{DE} = 3.46$ kN (C)

$\sum F_X = 0$ (→ + ve); $-F_{CE} - F_{DE}\cos 30 - 2\cos 60 = 0$; $F_{CE} = 2$ kN (T)

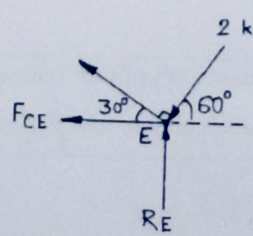


Fig 2.86

c) FBD of joint D: (Fig 2.87) By observation, joint D comprises two pairs of collinear members. So, $F_{DE} = F_{BD}$ and $F_{DC} = 4kN$ (C)

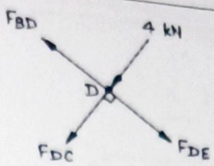


Fig 2.87

$F_{BD} = 3.46$ kN (C); $F_{DC} = 4$ kN (C)

d) FBD of joint C: (Fig 2.88)

$\sum F_X = 0$ ($\rightarrow$ + ve);

$F_{CE} - F_{AC} - F_{BC}\cos 60 + F_{DC}\cos 60 = 0$

$2 - (-2) - F_{BC}\cos 60 + (-4\cos 60) = 0$

$F_{BC} = 4$ kN (T)

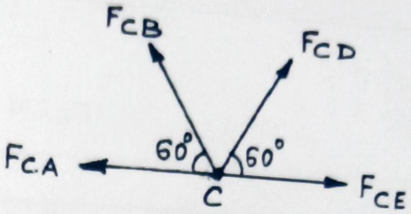


Fig 2.88

Sr.No.	Member	Force (kN)	Nature	Sr.No.	Member	Force (kN)	Nature
1	AB	4	C	5	CD	4	C
2	AC	2	C	6	CE	2	T
3	BC	4	T	7	DE	3.46	C
4	BD	3.46	C				

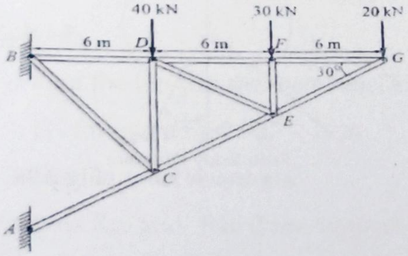
Let's check the take away from this lecture

1. We use principle of ………………… for determining the forces in the members of a Truss.

a) Statics b) Dynamics c) Moment d) None of these

Exercise:

1) Find the forces in the members DF, DE, CE and EF by method of joints only for the pin jointed frame shown in fig 2.89.



ig 2.89

2) Find the forces in the members of the pin jointed truss loaded as shown in fig 2.90. Tabulate the results.

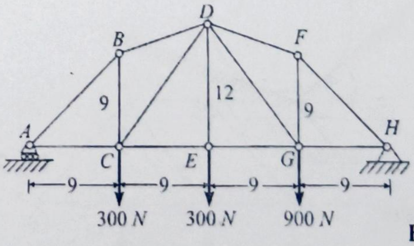


Fig 2.90

Practice:

3) Determine the forces in the members BC and AD of the truss as shown in the fig. 2.91 by using method of joints. (Ans: BC = 70.7 kN (C), AD = 95.6 kN (T))

4) A vertical load of 30 kN is applied to joint A of the truss for given fig 2.92. Determine the member forces and the support reactions.

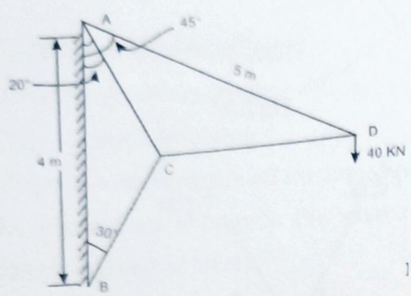


Fig 2.91

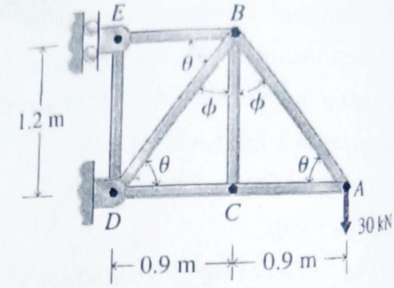


Fig 2.92

Learning from the lecture 'Problems on method of joints': Student will able to Find the forces in different member of the truss & will able to analyze different members

2.1.15 Short answer questions:

Equilibrium

1. **Define Equilibrium:**

Ans: If the resultant of the force system happens to be zero, the system is said to be in a state of equilibrium.

2. **Define Free body diagram:**

Ans: It is diagram of body under consideration. The diagram shows magnitudes directions and point of applications of all external, active and reactive forces acting on the body. In the diagram unknown forces, corresponding directions are labeled. Thus, it is very important and very first step in analysis of problems on equilibrium.

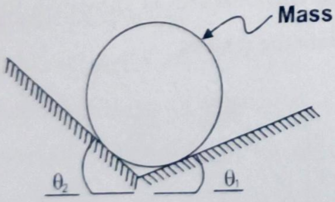


Fig 2.93

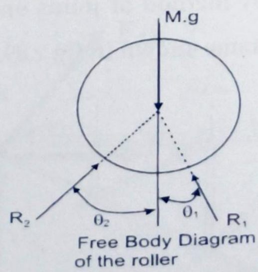


Fig 2.94

Trusses

1) **What is principle for method of joints?**

Ans: If the entire truss is in equilibrium, then each joint of the truss is also in equilibrium.

2) **What is statically determinate truss?**

Ans: A truss which can be analyzed by applying conditions of equilibrium is called statically determinate.

3) **What is perfect truss?**

Ans: A truss which satisfies condition $m = 2j - r$ is known as perfect truss.

Where m = number of members; J = number of joints; r = number of reactions

4. **State some applications of trusses:**

Ans:

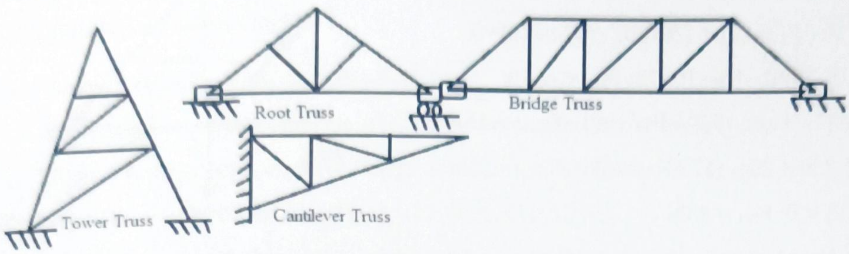


Fig 2.95

➢ **Differentiate between Perfect, Imperfect and over perfect trusses**

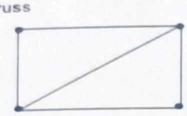
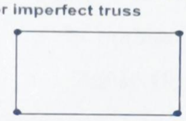
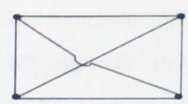
Rigid truss or perfect truss	Under rigid or deficient or imperfect truss	Redundant truss or over perfect tress
		
n = 5, j = 4	n = 4, j = 4	n = 6, j = 4
1. n = 2 j - 3	1. n < 2 j - 3	1. n > 2 j - 3
2. Used as structure	2. Cannot be used as structure	2. Can be analyzed by conditions of equilibrium only.
3. Can be analyzed by conditions of equilibrium only.		

Fig 2.96

Working rule for method of section:

The method involves dividing the truss into sections by cutting through the selected members and analyzing the section as a rigid body. This is an alternative to the method of joints for finding the internal axial forces in truss members. It works by cutting through the whole truss at a single section and using global equilibrium to solve for the unknown axial forces in the members that cross the cut section.

Solved:

1) Find the force in the members DG, DF and EG for given fig 2.97

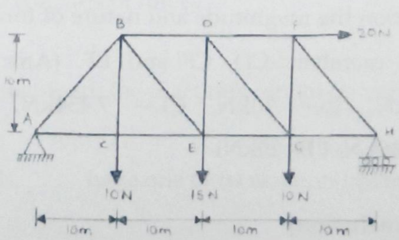


Fig 2.97

Finding support reactions:

Assume R_{AV} and R_{AH} be the support reactions at point A and R_H be the support reaction at point H; For equilibrium of entire truss,

$\sum M_A = 0$ (Anticlockwise) +ve;

$(10\times10)+(15\times20)+(10\times30) + (20\times10) - (R_H\times40)=0$

$\mathbf{R_H=22.5N}$; $\sum F_X = 0$ (→ + ve)

$R_{AH}+20=0$; $\mathbf{R_{AH} = 20N}$

$\sum F_Y = 0$ (↑ + ve);

$R_{AV} + R_H -10 - 15 -10=0$

$R_{AV} + R_H -35 = 0$;

$R_{AV} + 22.5-35 = 0$; $\mathbf{R_{AV} = 12.5N}$

$\sum F_y = 0$ ($\uparrow$ + ve); $F_{DG}\sin45 - 10 + R_H = 0$

$F_{DG}\sin45 - 10 + 22.5 = 0$; $F_{DG}\sin45 + 12.5 = 0$

$F_{DG}\sin45 = -12.5$; $F_{DG}\sin45 = 17.67N$

$\sum M_G = 0$; $(-F_{DF}\times10) + (20\times10) - (22.5\times10) = 0$

$(-F_{DF}\times10) - 25 = 0$; $-F_{DF}\times10 = 25$; $\mathbf{F_{DF} = -2.5N}$

$\sum F_x = 0$ ($\rightarrow$ +ve); $-F_{EG} - F_{DF} + 20 - F_{DG}\cos45 = 0$

$-F_{EG} - (-2.5) + 20 - (17.67\cos45) = 0$

$-F_{EG} + 35 = 0$; $\mathbf{F_{EG} = 35N}$

FBD of RHS part:

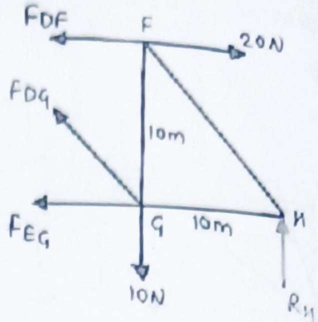


Fig 2.98

Practice Problems:

1) A truss is shown in the fig 2.99. Find the magnitude and nature of forces in the members BC, AC and AE by the method of section. **(Ans: BC=36.373kN, AC= 41.569kN, AE=18kN)**

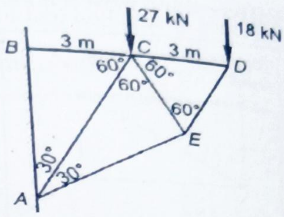


Fig 2.99

2) The cantilever truss is shown in fig 2.100. Find the magnitude and nature of forces in the members 3-4, 4-7, and 6-7 by the method of section. **(Ans: 3-4= 20kN, 4-7=10kN, 6-7=8.66kN)**

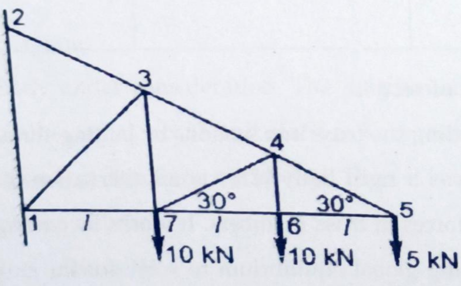


Fig 2.100

3) A plane truss having a point load of 30kN at E as shown in fig 2.101. Find by method of section the magnitude and nature of forces in the members CD, CF and EF. **(Ans: R_A= 20kN, R_B= 10kN, CD= 7.454kN, CF= 18.86kN, EF= 20kN)**

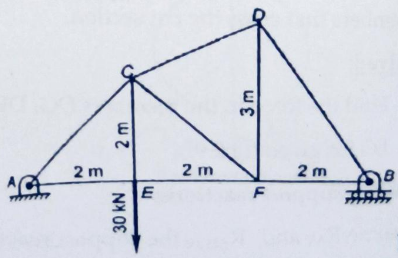


Fig 2.101

2.1 Conclusion:

Learning Outcomes: Learners should be able to

Know, Comprehend

1) Understand the conditions of equilibrium and its applications based on type of system

2) Understand Lami's Theorem and its application

Apply, Analyse

3) Identify the types of supports and its reactions

4) Identify the types of beams and supports and find the reactions of support

Synthesize

5) Calculate stress and strain

6) Calculate poison's ratio

2.2 Add to Knowledge (Content Beyond Syllabus)

a) Analysis of Truss can be done with software named as ANSYS.

b) https://www.youtube.com/watch?v=sKZAipnrV9E

c) https://www.youtube.com/watch?v=J0cCA1PAINU

d) https://www.youtube.com/watch?v=Hn_iozUo9m4

2.3 Set of Multiple-Choice Questions:

1. According to Lami's Theorem, the three forces

(a) Must be equal.
(b) Must be at 120° to each other.
(c) Must be both of above.
(d) May not be any of the two.

2. The Lami's Theorem is applicable only for

(a) Coplanar forces
(b) Concurrent forces
(c) Coplanar and concurrent forces
(d) Any type of forces

3. If a body is in equilibrium. We may conclude that

(a) No force is acting on the body
(b) The resultant of all forces acting on it is zero.
(c) The moments of the forces about any point is zero.
(d) Both (b) and (c)

4. If the sum of all the forces acting on a body is zero, then the body may be in equilibrium provided the forces are

(a) Concurrent
(b) Parallel
(c) Like parallel
(d) Unlike parallel

5. A body is said to be in equilibrium, if it has no linear motion.

(a) True
(b) False

6. Lami's Theorem cannot be applied in case of concurrent forces

(a) Agree (b) Disagree

7. In a simply supported beam carrying triangular load, the reactions cannot be vertical

(*a*) True
(*b*) False

8. An overhanging beam with downward loads............have one of its reaction upward and the other downward.

(*a*) can
(*b*) can not

9. The reaction at the roller supported end of a beam is always

(*a*) vertical
(*b*) horizontal
(*c*) none of the above

10. If the reaction of a beam, at one of its supports is the resultant of horizontal and vertical forces, then it is a

(*a*) simply supported end (*b*) roller supported end (*c*) hinged end

11. A couple acting at the mid-point of a simply supported beam has some horizontal and vertical components.

(*a*) Agree (*b*) Disagree.

1.(d) 2. (a) 3. (d) 4. (a) 5. (b) 6. (b) 7. (b) 8. (a) 9. (c) 10. (c) 11. (b)

2.4 Short Answer Questions:

1) Define Equilibrium:
2) Define Free body diagram:
3) State and Prove Varignon's Theorem?
4) State and prove Lami's Theorem?
5) Define Couple. Why the couple moment is said to be a free vector?
6) Distinguish between couple and moment?
7) What is meant by force-couple system? Give one example.
8) Can a coplanar non concurrent system with zero resultant force necessarily be in equilibrium?
9) When is moment of force zero about a point and maximum about a point?
10) How would you find out the equilibrium of non-coplanar forces?
11) When is moment of force zero about a line? Give one example.
12) Explain free body diagram with one example?
13) State the necessary and sufficient conditions for equilibrium of rigid bodies in two dimensions?
14) Write the equation of equilibrium of a rigid body?
15) Write the conditions equilibrium of a system of parallel force acting in a plane?
16) What are the reactions at Hinged, Roller and Fixed support of a plane beam that are possible?
17) How many scalar equations can be obtained for equilibrium of rigid body in three dimensions?
18) State difference between stress and strain.

2.5 Long Answer Questions:

1) Three cylinders are piled up in a rectangular channel as shown in Figure 2.102. Determine the reaction at point 6 between cylinder A and the vertical wall of the channel. (Cylinder A: radius= 4 cm, m =15 Kg, Cylinder B: Radius =6 cm, m= 40 Kg, Cylinder C: Radius = 5 cm, m = 20Kg)
(Ans: 735.75N, 981N, 784.8N)

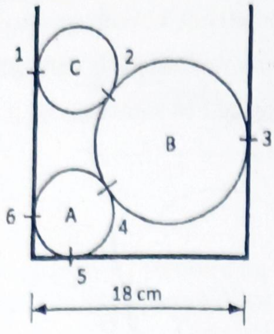


Fig 2.102

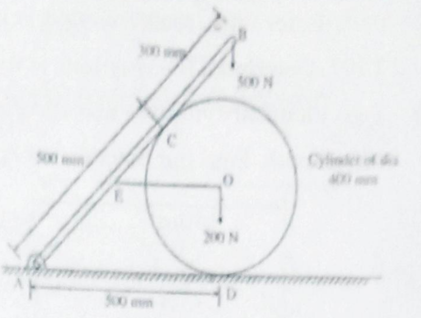


Fig 2.103

2) A bar is hinged at A and rests on cylinder at C. AC=500mm, CB= 300mm, diameter of cylinder is 400mm and its weight is 200N. The center of cylinder is connected to the bar by a horizontal wire OE as shown in figure 2.103. A weight of 500N is suspended at B. Determine i) Reaction of Hinge A, ii) Tension in the wire, iii) Reaction at C and D. Neglect the weight of the bar and assume all surfaces smooth. **(Ans: 552.13N,79.33N, 800N)**

3) Three identical smooth rollers, each of mass M and radius r are stacked on two inclined surfaces as shown in the fig 2.104 Each surface is inclined at an angle α with the horizontal. Determine the smallest angle α to prevent the stack from collapsing. **(Ans: α =10.89°)**

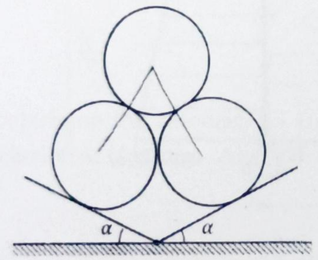


Fig 2.104

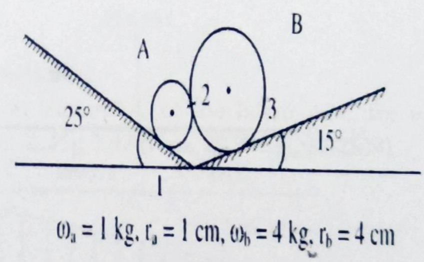


Fig 2.105

4) Determine the reactions at points of contact 1, 2 and 3, in figure 2.105. Assume smooth surfaces. **(Ans: - R_1 =19.73 N; R_2= 11.6 N; R_3=32.22N)**

5) A cylinder of weight 500N is kept on two inclined planes as shown in the fig 2.106. Determine the reactions at the contact points A and B. **(Ans: 253.85N, 388.93N)**

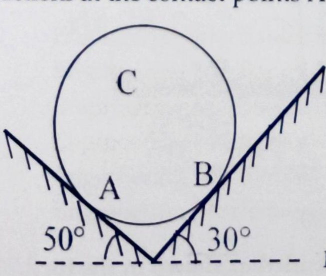


Fig 2.106

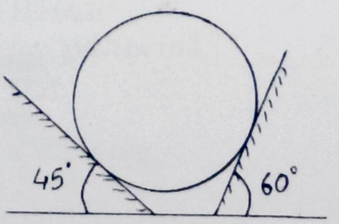


Fig 2.107

6) A cylinder with 1500N weight is resting in an unsymmetrical smooth groove as shown in fig 2.107. Determine the reactions at the points of contacts. **(Ans: 1344.86N, 1098.07N)**

7) Two identical cylinders dia. 100 mm weight 200N are placed as shown in fig 2.108. All contacts are smooth. Find out reactions at A, B and C.

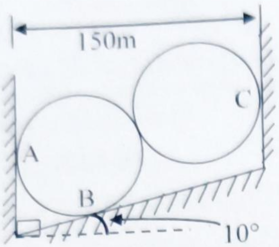


Fig 2.108

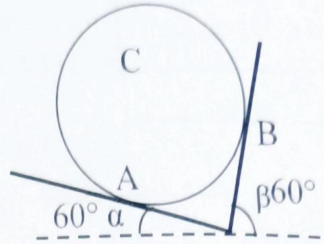


Fig 2.109

8) A smooth circular cylinder of weight W and radius R rests in a V shape groove whose sides are inclined at angles α and to the horizontal as shown in fig 2.109. Find the reactions R_A and R_B at the points of contact.

9) Find reactions at A and B for a bent beam ABC loaded is as shown in fig 2.92. **(Ans: H_A=18kN, V_A =13.15kN, R_B = 4.85kN.)**

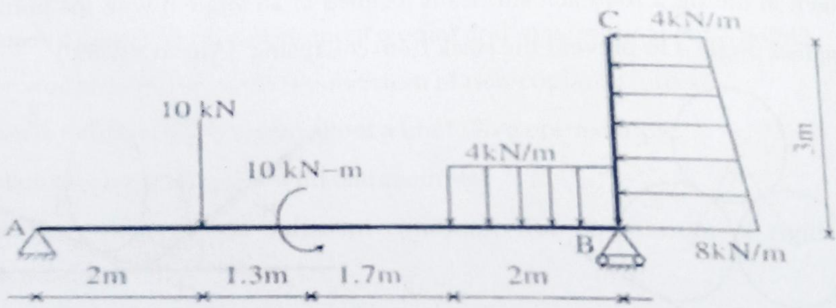


Fig 2.92

10) Find the support reactions at A and B for the beam shown in fig 2.93 **(Ans: 157.03kN, 108kN,314.07kN)**.

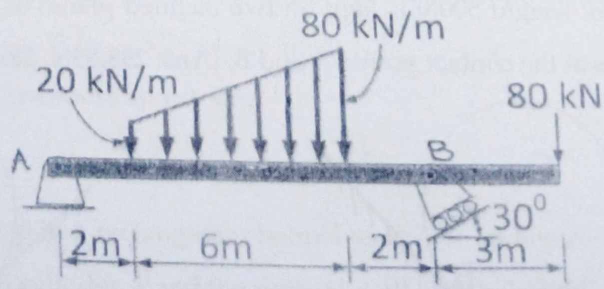


Fig 2.93

11) Find the reactions at supports B and F for the beam loaded as shown in the fig 2.94 below. **(Ans: 360kN, 449.45kN, 293.22kN)**

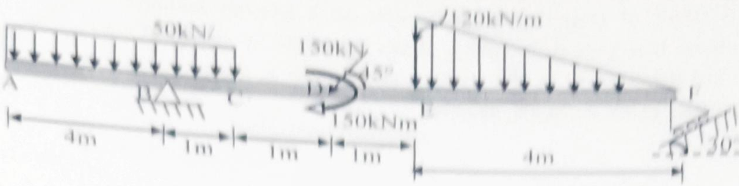


Fig 2.94

12) Find the support reactions for the beam loaded and supported as shown in fig. 2.95 **(Ans: 192.43kN, 140.43kN, 140.56kN)**

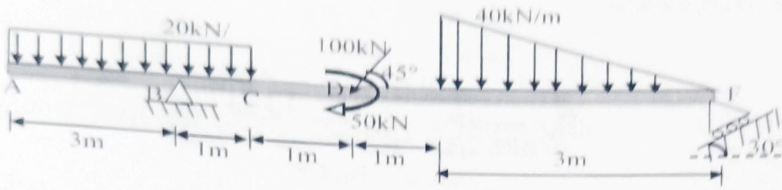


Fig 2.95

13) Determine the reactions at A, B and C for the frame shown in fig 2.96. **(Ans:**

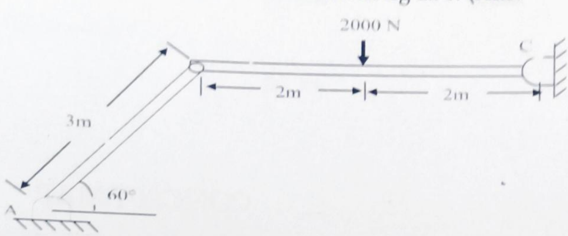


Fig 2.96

14) Determine the intensity of distributed load W at the end C of the beam ABC for which the reaction at C is zero. Also calculate the reaction at B. Fig 2.97 **(Ans: 22.53kN, 3.52kN)**

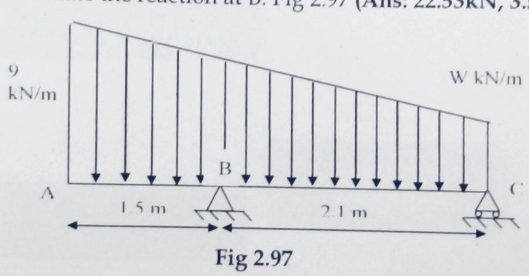


Fig 2.97

2.6 References:

1) Engineering Mechanics by Tayal, Umesh Publication
2) Engineering Mechanics by Beer & Johnson, Tata McGraw Hill
3) Engineering Mechanics by F.L. Singer by Harper
4) Engineering Mechanics – Statics, R. C. Hibbler
5) Engineering Mechanics – Statics, J. L. Merium, I. G. Kraig
6) Engineering Mechanics – P. J. Shah, R. Bade

Self-Assessment

1) A roller of weight W=1000N rests on a smooth inclined plane. It is kept from rolling down the plane by string AC. Find the tension in the string and reaction at the point of contact D. **(Ans: 73.2N, 89.65N)**

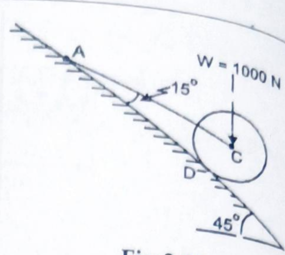


Fig 2.98

2) Find the reactions for the beam loaded and supported as shown in fig 2.99 **(Ans: 18kN, 13.15kN, 4.85kN)**

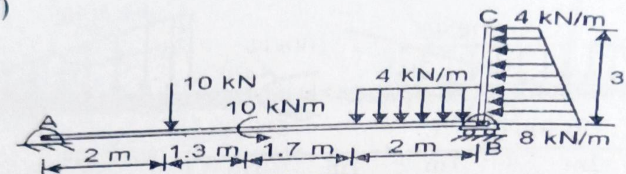


Fig 2.99

Q.3. A cylinder B, W_B =1000N, Dia. 40cm rests against a smooth wall. Find out reaction at C and T_{AB}. **(Ans:**

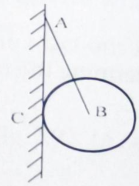
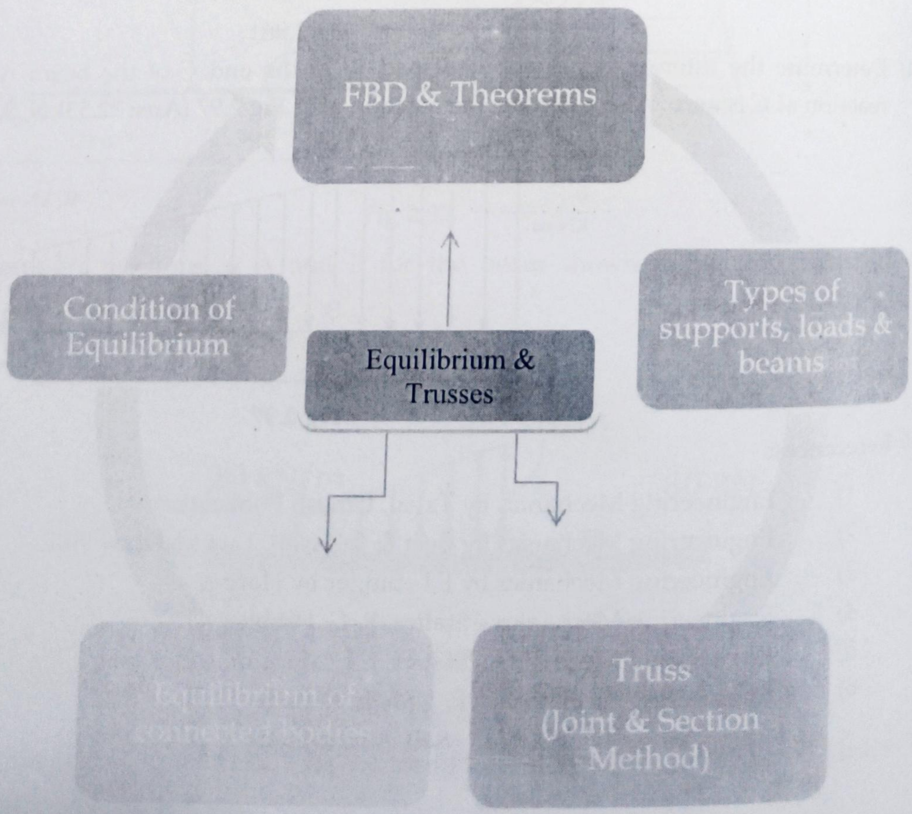


Fig 2.99a

CONCEPT MAP



Self Evaluation

Name of Student:

Class & Div.:

Course Code:

Roll No.:

1. Can you define stress and strain ?

   (a) Yes (b) No

2. Are you able to state difference between stress and strain?

   (a) Yes (b) No

3. Are you able to define the term equilibrant, free body diagram?

   (a) Yes (b) No

4. Are you able to calculate the strain developed in the body?

   (a) Yes (b) No

5. Do you understand this module?

   (a) Yes (b) No