Page No:
Date:

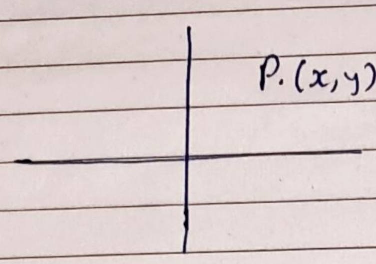
A complex no in cartesian form can be Represented by

$z = x + iy$; $i = \sqrt{-1}$.

$\boxed{|z| = \sqrt{x^2+y^2}}$ Modulus $\Rightarrow$ $\boxed{\bar{z} = x - iy}$ Conjugate

$x = \frac{z+\bar{z}}{2} = x+iy + x-iy$

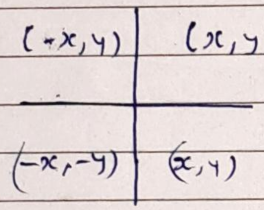
Cartesian



(θ)

The Argument or Amplitude of z is denoted by $\theta = \arg(z)$.

$\theta = \arg(z) = \tan^{-1}\left(\frac{y}{x}\right)$; $-\pi \leq \theta \leq \pi$, $\boxed{-1 \leq \theta \leq 1}$



Point	arg(z)
I (x,y)	$\theta = \tan^{-1}\left\lvert\frac{y}{x}\right\rvert$
II $(-x,y)$	$\pi - \theta = \pi - \tan^{-1}\left\lvert\frac{y}{x}\right)$
III $(-x,-y)$	$-\pi + \theta = -\pi + \tan^{-1}\left(\frac{y}{x}\right)$
IV $(x,-y)$	$-\theta = -\tan^{-1}\left(\frac{y}{x}\right)$

$z = \frac{\sqrt{3}}{2} - \frac{i}{2}$

$|z| = \sqrt{x^2+y^2}$; $x = \frac{\sqrt{3}}{2}$, $y = -\frac{1}{2}$

$= \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$.

Argument is $(+,-)$ ie IV^{th} Quadrant

$-\theta = -\tan^{-1}\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = -\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{-\pi}{6}$

$$z = -\frac{\sqrt{3}}{2} - \frac{1}{2}, \quad \theta = -\pi + \tan^{-1}\left(\frac{y}{x}\right)$$

$$= -\pi + \tan^{-1}\left[\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right]$$

$$= -\pi + \frac{\pi}{6} = -\frac{5\pi}{6}.$$

$$z = -\frac{\sqrt{3}}{2} + \frac{i}{2} \Rightarrow \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}.$$

$$z = \frac{\sqrt{3}}{2} + \frac{1}{2} \Rightarrow \frac{\pi}{6}.$$

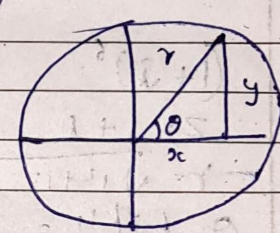
Polar Form:

$$z = r\cos\theta + ir\sin\theta. = r(\cos\theta + i\sin\theta).$$
$$= x + iy$$

$$x = r\cos\theta, \quad y = r\sin\theta.$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right).$$



Conversion Cartesian to Polar view

Exponential Form.

$$z = re^{i\theta}. \quad ; \quad \theta = \tan^{-1}\left(\frac{y}{x}\right) \quad ; \quad r = \sqrt{x^2 + y^2} \; ;$$

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

$$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} \ldots$$ — (expansion of $e^{i\theta}$.)

$$= 1 + i\theta - \frac{\theta^2}{2!} + \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} + \ldots$$

$$= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \ldots\right) + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \ldots\right).$$

$$= \cos\theta + i\sin\theta$$

$i^2 = -1$
$1^2 = 1$

$i = \sqrt{-1}$, $i^2 = -$

$i^x \Rightarrow \frac{x}{4}$ — Divisible by 4 the Remainder 1

$(1+i)^6$

Demouvieers Theorem

De.

for any Rational Number N. one of the value of
$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

$(e^{i\theta})^n = e^{in\theta}$.

Nu. $(1+i)^6$

$z = 1 + i$

$r = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = \tan^{-1}\left|\frac{1}{1}\right| = \frac{\pi}{4}$

$z =$

$(\sqrt{3}-1)^4$

$z = \sqrt{3} - i$

$r = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$

$\theta = -\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

$z = re^{i\theta} = 2 \cdot e^{-i\frac{\pi}{6}}$

$z^4 = \left(2e^{-i\frac{\pi}{6}}\right)^4 = 16 \cdot e$

$z = 2\left[\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right]$

$= 2\left[\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right]$

$z^4 = 2^4\left[\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right]^4$

$\Rightarrow 2^4\left[\cos\frac{4\pi}{6} - i\sin\frac{4\pi}{6}\right]$

$\Rightarrow 2^4\left[\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right]$

$= 2^4\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$= 2^3\left(-1 - i\sqrt{3}\right)$

P.T $(4n)^{th}$ $\boxed{\text{Power } \left[\dfrac{1+7i}{(2-i)^2}\right]^{4n} = (-4)^n}$

P.T $\Rightarrow$

1) $z = \dfrac{1+7i}{(2-i)^2}$ — To

$= \dfrac{1+7i}{4+i^2-4i}$

$= \dfrac{1+7i}{3-4i} \times \dfrac{3+4i}{3+4i}$

$= \dfrac{3+21i+4i+28i^2}{9-16i^2}$

$= \dfrac{-25+25i}{25} = -1+i$

$\tan 0$	$= 0$	$= 0$
$\tan 30$	$= \frac{\pi}{6}$	$= \frac{1}{\sqrt{3}}$
$\tan 45$	$= \frac{\pi}{4}$	$= 1$
$\tan 60$	$= \frac{\pi}{3}$	$= \sqrt{3}$
$\tan 90$	$= \frac{\pi}{2}$	$= \infty$ N.D
$\tan 180$	$= \pi$	$= 0$

$z = -1 + i$

$r = \sqrt{2}$, $\theta = \pi - \tan^{-1}(-1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$z = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$z^{4n} = (\sqrt{2})^{4n}\left[\cos 4n\frac{3\pi}{4} + i\sin 4n\frac{3\pi}{4}\right]$

$= 2^{2n}\left[\cos(3n\pi) + i\sin(3n\pi)\right]$

$= 2^{2n}\left[(-1)^n + i(0)\right]$

$= 4^n(-1)^n$

$= (-4)^n.$

Page No:
Date:

$1 \ 1 \times 1 \quad \underline{i}$

(10) Q $z = \dfrac{1}{2+3i} = \dfrac{1}{2+3i} \times \dfrac{2+3i}{2+3i} = \dfrac{2-3i}{4+9} = \dfrac{2-3i}{13}$

$r = \sqrt{\left(\dfrac{2}{13}\right)^2 + \left(\dfrac{3}{13}\right)^2} = \cancel{\dfrac{4}{\sqrt{13}}} \ \cancel{\dfrac{4}{\sqrt{13}}}$

$\theta = \tan\theta \left(\dfrac{\frac{3}{13}}{-\frac{2}{13}} \right) =$

$\dfrac{1}{1+i} =$

$\dfrac{1}{1+i} \times \dfrac{1-i}{1-i} = \dfrac{1-i}{1+i^2} = \dfrac{1-i}{2}$

$z = -i$ — Polar mein Likho

$= \dfrac{1}{2} - \dfrac{1}{2} i$

$r = \sqrt{0^2 + (-1)^2} = \sqrt{1} = 1$

$\theta = \tan^{-1}\left(\quad \right) = -\dfrac{\pi}{2}$

$\Rightarrow \cos\left(\dfrac{-\pi}{2}\right) + i\left(\sin\left(\dfrac{-\pi}{2}\right)\right)$

$= 0 + i(-1)$

$\underline{= -i}$

21st Oct

(S, M)

If α & β are the Roots of the equation $z^2 \sin^2\theta - z \sin 2\theta + 1 = 0$. then by using D.M Theorem prove

$\alpha^n + \beta^n = 2\cos n\theta \cdot \operatorname{cosec}^n\theta$ where ($n \in I^+$)

$$\boxed{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Soln:-

$z^2 \sin^2\theta - z \sin 2\theta + 1 = 0.$

$a = 1\sin^2\theta$ $b = -1\sin 2\theta$, $c = 1$

$\alpha, \beta = \cancel{+b \pm \sqrt{}}$

$$\frac{+\sin 2\theta \pm \sqrt{(\sin 2\theta)^2 - 4 \cdot \sin^2\theta \cdot 1}}{2 \cdot \sin^2\theta}$$

$$= \frac{2\sin\theta \cdot \cos\theta \pm \sqrt{4\sin^2\theta \cdot \cos^2\theta - 4\sin^2\theta}}{2\sin^2\theta}$$

$$= \frac{2\sin\theta\,(\cos\theta \sqrt{\cos^2\theta - 1})}{2\sin^2\theta}$$

$= \cancel{\cos\theta \pm i \sin\theta}$ — $\sqrt{-1}$

$$= \frac{\cos\theta \pm \sqrt{-\sin^2\theta}}{\cancel{2}\sin\theta}$$ —

$$= \frac{\cos\theta \pm \sqrt{(-1)\sin^2\theta}}{\sin\theta}$$

$$= \frac{\cos\theta + i\sin\theta}{\sin\theta}$$

$\alpha = \dfrac{\cos\theta + i\sin\theta}{\sin\theta}$ $\beta = \dfrac{\cos\theta - i\sin\theta}{\sin\theta}$

$$\alpha^n = \frac{(\cos\theta + i\sin\theta)^n}{\sin^n\theta}$$

$= (\cos n\theta + \cancel{i} i\sin n\theta)\operatorname{cosec}^n\theta$

$=$

$$\beta^n = \frac{(\cos\theta - i\sin\theta)^n}{\sin^n\theta}$$

$= (\cos n\theta - i\sin n\theta)\operatorname{cosec}^n\theta$

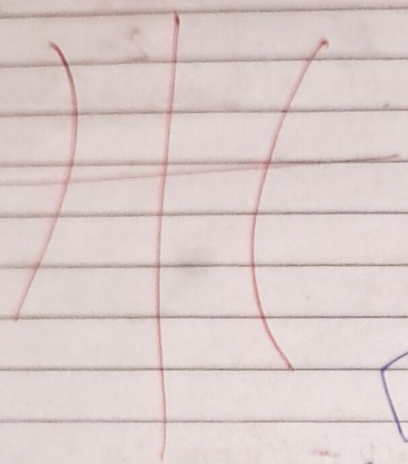
$\alpha^n + \beta^n = 2\cos n\theta \cdot \operatorname{cosec}^n\theta.$

Page No:
Date:

Ex: 24 Hyperbolic functions

θ Abhi tak humne circular function padh $\sin\theta$
— point is in circle

$h\theta$ Ab hyperbolic mein hum by $\sin h\theta$ (sin hyperbolic)



$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

— Basic form

Relation between θ & Euler

$$\boxed{\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}}$$

$$= \frac{\cos\theta + i\sin\theta - (\cos\theta - i\sin\theta)}{2i}$$

$$= \frac{2i\sin\theta}{2i}$$

$$= \sin\theta.$$

$$\boxed{\cos\theta = \frac{e^{i\theta} + \bar{e}^{i\theta}}{2}}$$

$$\boxed{\tan\theta = \frac{e^{i\theta} - e^{i\theta}}{i\left(e^{i\theta} + e^{-i\theta}\right)}}$$

Hyperbolic

$$\boxed{\sinh z = \frac{e^{z} - e^{-z}}{2}} ; \boxed{\cos\theta = \frac{e^{z} + e^{-z}}{2}} ; \boxed{\frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}}$$

Relation btw circular & Hyperbolic function.

$$\sin iz = \frac{e^{i(iz)} - e^{-i(iz)}}{2i} = \frac{e^{-z} + e^{z}}{2i} = -\left(\frac{e^{z} - e^{-z}}{2i}\right)$$

circular

$= (i^2 = -1)$

$\Rightarrow i^2 \frac{(e^{z} - e^{-z})}{2i}$

$= i \sinh z.$

Sin family sign change

Cos family no change

$\sin iz = i \sinh z$

$\cos iz = (\cosh z)$

$\tan iz = i \tanh z.$

(i bahar)

$\sin 2\theta = 2 \sin\theta . \cos\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$1 - 2\sin^2\theta.$

$\sin 2\theta = 2\tan\theta / (1 + \tan^2\theta)$

$\cos 2\theta = 1 - \tan^2\theta / 1 + \tan^2\theta.$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}.$

(Circular)

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\boxed{\sin^2 i\theta + \cos^2 i\theta = 1}$

$i^2 \sinh^2\theta + \cosh^2\theta = 1$

$\boxed{\cos^2 h\theta - \sin^2 h\theta = 1}$

$\boxed{1 + \tan^2\theta = \sec^2\theta} \longrightarrow \boxed{1 - \tanh^2\theta = \operatorname{sech}^2 z}$

$\boxed{1 + \cot^2\theta = \operatorname{cosec}^2\theta} \rightarrow \boxed{1 - \cot^2 h\theta = \operatorname{cosech}^2\theta}$

$\cos 2\theta = \cancel{\cosh^2\theta + \sin^2\theta}$

$= \cancel{\cosh^2\theta - \sinh^2\theta.}$

$\cos 2i\theta = \cos^2 i\theta - \sin^2 i\theta$

$= \cos^2 h\theta + i\sin^2 h\theta.$

$\sin i\theta = 2 \sin hi\theta \cos hi\theta$

$= \cancel{i \sinh 2\theta}.$

$i\sinh 2\theta = 2 i \sinh\theta \cosh\theta$

$\sinh 2\theta = 2\sinh\theta . \cosh\theta$

$\cosh 2\theta =$

$\boxed{\cosh^2\theta + \sinh^2\theta}$

Q. Find the value of $\tanh(\log\sqrt{3})$.

$\tanh(\log\sqrt{3})$

$= \dfrac{e^{\log\sqrt{3}} - e^{-\log\sqrt{3}}}{e^{\log\sqrt{3}} + e^{-\log\sqrt{3}}}$

$\left(x^{2\log b} = z^{2\log x}\right)$

$\boxed{a^{b\log c} = c^{b\log a}}$

$= \dfrac{\sqrt{3} - e^{\log\frac{1}{\sqrt{3}}}}{\sqrt{3} + e^{\log\frac{1}{\sqrt{3}}}}$

$= \dfrac{\sqrt{3} - \frac{1}{\sqrt{3}}}{\sqrt{3} + \frac{1}{\sqrt{3}}} = \dfrac{3-1}{3+1} = \dfrac{2}{4} = \dfrac{1}{2}$

Q. $\{\sinh x - \cosh x = 5\}$; Solve for real values of x.

$\dfrac{e^x - e^{-x}}{2} - \dfrac{e^x + e^{-x}}{2} - 5 = 0$

$e^x - e^{-x} - e^{-x} - e^x - 10 = 0.$

$-2e^{-x} - 10 = 0.$

$e^{-x} = -5$

[$\log(-x)$] Nahi chalega]

This cannot be solved for real value of x

Q. $5\sinh x - \cosh x = 5$

$10e^x - 5e^{-x} - e^{-x} - e^x - 10$

~~$-2e^{-x} + 9e^x - 10 = 0$~~

$4e^x - 6e^{-x} - 10 = 0$

$2e^x - 3e^{-x} - 5 = 0.$

$2e^x - \dfrac{3}{e^x} - 5 = 0$

$2e^x e^x - 3 - 5e^x = 0$

$2e^{2x} - 5e^x - 3 = 0.$

$= \dfrac{5 \pm \sqrt{25 - 4 \cdot 2 \cdot (-3)}}{(2 \cdot 2)}$

$= \dfrac{5 \pm \sqrt{25 + 24}}{4}$

$= \dfrac{5 \pm \sqrt{49}}{4}$

$= \dfrac{5+7}{4}, \dfrac{5-7}{4}$

$= 3, \dfrac{-1}{2}$

$(e^x = 3)$ & $e^x \neq \dfrac{-1}{2}$

$x = \log 3$

This cannot be solved for real values of x

Date:

$\Rightarrow e^x = 3 \Rightarrow x = \log 3.$

$$\tanh x = \frac{3 - \frac{1}{3}}{3 + \frac{1}{3}} = \frac{\frac{9-1}{3}}{\frac{9+1}{3}} = \frac{8}{10} = \frac{4}{5}$$

Q. $\tanh x = \frac{1}{2}$ find value of $\cosh 2x$

$$\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$$ — (Yaha par Relation use karke sab function convert karne hote)

$$\cosh 2\theta = \frac{1 + \tanh^2\theta}{1 - \tanh^2\theta}$$

$$= \frac{1 + \frac{1}{4}}{1 - \frac{1}{4}} = \frac{\frac{5}{4}}{\frac{3}{4}} = \frac{5}{3}$$

Q. $\tanh x = \frac{2}{3}$ find value of x — (Ab Quadratic ban → Iske Quadratic Bana)

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{2}{3}$$

$$3e^x - 3e^{-x} = 2e^x + 2e^{-x}$$

P.T

$$\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1+\sinh^2 x}}} \qquad -\sinh x$$

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow -\sinh^2 x + \cosh^2 x = 1$

$1 + \tan^2\theta = \sec^2\theta \qquad 1 - \tanh^2 x = \operatorname{sech}^2 x$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta \qquad 1 - \coth^2 x = \operatorname{cosech} h$

$$\Rightarrow \cancel{\cfrac{1}{1-\cfrac{1}{\cosh^2 x}}}$$

$$\Rightarrow \cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{\cosh^2 x}}} \Rightarrow \cfrac{1}{1-\cfrac{1}{1-\operatorname{sech}^2 x}} = \cfrac{1}{1-\cfrac{1}{\tanh^2 x}}$$

$$\cancel{\cfrac{1}{1-\cfrac{1}{\cosh^2 x}}}$$

$$= \frac{1}{1-\coth^2 x} = \frac{1}{-\operatorname{cosech}^2 x} = -\sinh^2 x$$

$$\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cosh^2 x}}} \Rightarrow \cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{-\sinh^2 x}}} \Rightarrow \cfrac{1}{1-\cfrac{1}{1+\operatorname{cosech}^2 x}}$$

$$\Rightarrow \cfrac{1}{1-\cfrac{1}{\coth^2 x}} \Rightarrow \frac{1}{1-\tanh^2 x} \Rightarrow \frac{1}{\operatorname{sech}^2 x} = \cosh^2 x$$

Page No:
Date:

$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$-\sinh 2\theta = \cancel{2\tan}$ $\quad \cancel{2\tan} \; \frac{-2\tanh\theta}{1 \cancel{+} \tanh^2\theta}$

$\tan 2\theta = \frac{\cancel{2}\tan\theta}{1-\tan}$

$\tanh 2\theta$

$\boxed{+\tanh 2\theta = \frac{+2\tanh\theta}{1+\tan^2 h\theta}}$

P.T $\left(\frac{1+\tanh x}{1-\tanh x}\right)^3 = \cosh 6x + \sinh 6x$

Soln:.

$\left(\frac{1+\frac{\sinh x}{\cosh x}}{1-\frac{\sinh x}{\cosh x}}\right)^3$

$= \frac{(\cosh x + \sinh x)}{\cosh x - \sinh x}$

$= \left(\frac{\cos ix - i\sin ix}{\cos ix + i\sin ix}\right)^3$

$= \left(\frac{\cos ix - i\sin ix}{\cos ix + i\sin ix} \times \frac{\cos ix - i\sin ix}{\cos ix - i\sin ix}\right)^3$

$= \left(\frac{(\cos ix - i\sin ix)}{(\cos ix)^2 + (\sin ix)^2}\right)^3$

$= \left((\cos ix - i\sin ix)^2\right)^3$

$= \cos i6x - i\sin 6x$

$\left(\frac{1-\tan x}{1+\tan x}\right)$

$1 - \frac{\sin x}{\cos x}$

$\sin ix = i\sinh x$

$\cos ix = \cosh x$

$\Rightarrow \cosh 6x + \sinh 6x$

$\sinh^{-1}x = \log(x + \sqrt{x^2+1})$

$\cosh^{-1}x = \log(x + \sqrt{x^2-1})$

$\tanh^{-1}x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

(I) $(\cosh x + \sinh x)^n$

$(\cos ix + i\sin ix)^n$

$(\cos inx - i\sin inx)$

$\downarrow$

$(\cosh nx + \sinh nx)$

$\downarrow$

$\Rightarrow (\cosh nx \pm \sinh nx)$

(II) $\tan\frac{x}{2} = \tanh\frac{u}{2}$

P.T (I) $\sinh u = \tan x$

$\sinh u = \frac{2\tanh\frac{u}{2}}{1-\tanh^2\frac{u}{2}}$

$\Rightarrow \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$

$= \tan x$

(II) $\cosh u = \sec x$

$\cosh u = \frac{\cancel{2\tan\theta}}{\cancel{1}}$ $\frac{1+\tan^2\theta}{1-\tan^2\theta}$

$= \frac{1-\tanh^2 x}{1+\tanh^2 x}$

$=$

$\cosh u = \sqrt{1+\sinh^2 u}$

$= \sqrt{1+\tan^2 x}$

$= \sqrt{\sec^2 x}$

$\boxed{\cosh u = \sec x}$

(III) $u = \log\left(\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right)$

Page No:

Date:

$u = \log\left(\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right)$

$\Rightarrow \tan\frac{x}{2} = \tanh\frac{u}{2}$

$\Rightarrow \frac{u}{2} = \tanh^{-1}\left(\tan\frac{x}{2}\right)$

$u = 2\tanh^{-1}\left(\tan\frac{x}{2}\right)$

$u = 2\,\frac{1}{2}\log\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$

$u = \log\left(\frac{\tan\frac{\pi}{4} + \tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\cdot\tan\frac{x}{2}}\right)$

$u = \log\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right).$

$\sinh^{-1}x = y$

$\sinh y = x$

$\frac{e^{y} - e^{-y}}{2} = x$

$e^{y} - e^{-y} - 2x = 0$

$e^{y} - \frac{1}{e^{y}} - 2x = 0$

$e^{2y} - 1 - 2xe^{y} = 0$

$a = 1,\ b = -2x,\ c = -1$

$e^{y} = \frac{2x \pm \sqrt{4x^2+4}}{2}$

$e^{y} = x \pm \sqrt{x^2+1}$

$e^{y} = x \pm \sqrt{x^2+1}$

$e^{y} = x - \sqrt{x^2+1}$ — Not valid

(-ve) since $x^2+1 > x$

$e^{y} = x + \sqrt{x^2+1}$

$\boxed{y = \log\left(x + \sqrt{x^2+1}\right)}$

28-10-2024

PAGE No.
DATE / /

D.M Theo

$(\cos\theta + i\sin\theta)^n \rightarrow \cos n\theta + i\sin n\theta, \ n \in \frac{p}{q}$

$x^6 = 1$

$x = (1)^{1/6}$

$z = 1 = x + iy$

$x = 1, \ y = 0$

$\theta = \tan^{-1}\left(\frac{0}{1}\right)$

$\theta = 0$

{Principle Argument}

$\rightarrow (\cos\theta + i\sin\theta)^{1/6}$

General Argument we add $2k\pi$

$= [\cos(0 + 2k\pi) + i\sin(0 + 2k\pi)]^{1/6 = n}$

$k = 0, 1, 2, 3, 4, 5, (6 = 0)$ — $n = (n-1)$

tak jayega k

Application of D.M Theorem

1] For calculating the power of Complex Number

$$\boxed{(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta, \text{ where } n \in I.}$$

ii] For finding the Roots of the Complex Number

$(\cos\theta + i\sin\theta)^{1/n} = [\cos(2k\pi + \theta) + i\sin(2k\pi + \theta)]^{1/n}$

$$\boxed{(\cos\theta + i\sin\theta)^{1/n} = \cos\frac{2k + \theta}{n} + i\sin\frac{2k + \theta}{n} \quad n \in I.}$$

iii]

$(\cos\theta + i\sin\theta)^{p/q} = [(\cos\theta + i\sin\theta)^p]^{1/q}$

$= [\cos p\theta + i\sin p\theta]^{1/q} = \cancel{\cos\theta}\,(\cos(p\theta + 2k\pi) + i\sin(p\theta + 2k\pi))^{1/q}$

$$\boxed{(\cos\theta + i\sin\theta)^{p/q} \Rightarrow \cos\frac{(p\theta + 2k\pi)}{q} + i\sin\frac{(p\theta + 2k\pi)}{q}}$$

Complex Roots Even Range

PAGE No.

DATE / /

Q1 Solve $x^6 - 1 = 0$ using DM Theorem

$x^6 = 1$

$x = (1)^{1/6}$

Let $z = 1$

$x = 1,\ y = 0$

$\theta = \tan^{-1}\left(\frac{0}{1}\right) = 0$

$1 = r(\cos\theta + i\sin\theta)$

$1 = 1(\cos 0 + i\sin 0)$ — ①

From ①

$x = (1)^{1/6} = [\cos(0 + 2k\pi) + i\sin(0 + 2k\pi)]^{1/6}$

$= \cos\frac{2k\pi}{6} + i\sin\frac{2k\pi}{6}$

Now, let find roots.

When $k = 0$

$x_1 = \cos 0 + i\sin 0 = 1$

$\boxed{x_1 = 1}$

$k = 1$

$x_2 = \cos\frac{2\pi}{6} + i\sin\frac{2\pi}{6} = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

$k = 2$

$x_3 = \cos\frac{4\pi}{6} + i\sin\frac{4\pi}{6} = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

$\boxed{x_3 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}}$

$k = 3$

$x_4 = \cos\frac{6\pi}{6} + i\sin\frac{6\pi}{6}$

$= \cos\pi + i\sin\pi$

$= -1 + 0.$

$k = 4.$

$x_5 = \cos\frac{4\pi}{6} + i\sin\frac{4\pi}{6}$

$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

$= \cos\left(\pi + \frac{\pi}{3}\right) + i\sin\left(\pi + \frac{\pi}{3}\right)$

PAGE No.

DATE / /

$= -\frac{1}{2} - \frac{i\sqrt{3}}{2}$

$k=5$

Q2 Find value of $\left((1+i\sqrt{3})^8 + (1-i\sqrt{3})^8\right)$ (Argument change has opposite -ve)

$z = 1+i\sqrt{3}$, $z = 1+i\sqrt{3}$

$x=1, y=\sqrt{3}$

$r = \sqrt{1+3} = 2$

$\theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$1+i\sqrt{3} = r(\cos\theta + i\sin\theta)$

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$\|^{ly}$ $1-i\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$= (1+i\sqrt{3})^8 + (1-i\sqrt{3})^8$

$= \left[2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^8 + \left[2\left(\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\right)\right]^8$

$= 2^8\left(\cos\frac{8\pi}{3} + i\sin\frac{8\pi}{3}\right) + (2^8, \left(\cos\frac{8\pi}{3} + i\sin\frac{8\pi}{3}\right)$

$= 2^8\left[\cos\frac{8\pi}{3} + \cancel{i\sin\frac{8\pi}{3}} + \cos\frac{8\pi}{3} - \cancel{i\sin\frac{8\pi}{3}}\right)$

$= 2^8\left(2\cos\frac{8\pi}{3}\right)$

$= +2^9\cos\left(3\pi - \frac{\pi}{3}\right)$

$-2^9\left(\cos\left(\frac{\pi}{3}\right)\right]$

$= -2^9 \frac{1}{2} = -2^8$

$= -256.$

PAGE No.
DATE / /

Q) find Continued product $(1+i)^{\frac{4}{5}}$ —

(product of roots.

$(1+i)^{4/5} = \left((1+i)^4\right)^{1/5}$

$z = 1+i$, $(x=1, y=1)$

$\boxed{r = \sqrt{2}}$

$\boxed{\theta = \frac{\pi}{4}}$

$= \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^4\right]^{\frac{1}{5}}$

$4^{1/5}\left(\cos\frac{2k\pi+\pi}{5} + i\sin\frac{2k\pi+\pi}{5}\right)$

$= 4^{1/5}\left(e^{i(2k+1)\frac{\pi}{5}}\right)$, $k = 0,1,2,3,4,5$)

when $(k=0)$ $X_1 = 4^{1/5} e^{i\frac{\pi}{5}}$

$4^{1/5} e^{i(2k+1)\frac{\pi}{5}}$

$k=1$

$X_2 = 4^{1/3} e^{i\frac{3\pi}{5}}$

$k=2$

$X_3 = 4^{1/5}$

Continued Product

$(1+i)^{\frac{4}{5}}$

PAGE No.

DATE / /

Q find all the values of $i^{\frac{1}{5}}$.

Sol

$z = i$

$z = \cancel{?}\ i.$

$x = 0, \quad y = 1$

$r = 1$

$\theta = \frac{\pi}{2}$

$$i^{1/5} = \left[r\left[\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right]\right]^{1/5}$$

$$= \left[1\left[\cos\left(2k\pi + \frac{\pi}{2}\right) + i\sin\left(2k\pi + \frac{\pi}{2}\right)\right]\right]^{1/5}$$

$$= \cos(4k+1)\frac{\pi}{10} + i\sin(4k+1)\frac{\pi}{10}$$

$k = 0, 1, 2, 3, 4,$

When $k = 0$

$x_0 = e^{i\frac{\pi}{10}}$

$k = 1 = x_1 = e^{i\frac{5\pi}{10}}$

$k = 2 = x_2 = e^{i\frac{9\pi}{10}}$

$k = 3 = x_3 = e^{i\frac{13\pi}{10}}$

$k = 4, \ x_4 = e^{i\frac{17\pi}{10}}$

If the ~~complex~~ coefficient in a polynomial is real then the complex root are in pair

If the coefficien in a polynomial is complex than the root is not necessarily paired

PAGE No.

DATE / /

$x^6 - 1 = 0$

$x^6 = i$

$x = (i)^{\frac{1}{6}}$

$$x = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{1/6}$$

$$= \left[\cos\left(2k\pi + \frac{\pi}{2}\right) + i\sin\left(2k\pi + \frac{\pi}{2}\right)\right]^{1/6}$$

$$= \cos(4k+1)\frac{\pi}{12} + i\sin(4k+1)\frac{\pi}{12}$$

$k = 0, 1, 2, 3, 4, 5, \cancel{6}$

When $k = 0$

$x_0 = \cos\frac{\pi}{12} + i\sin\frac{\pi}{12}$

$x_1 = \cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}$

$x_2 = \cos\frac{9\pi}{12} + i\sin\frac{9\pi}{12}$

$(2\pi - \theta)$ → To check Pairing

$k = 0 \Rightarrow x_0 = \cos\pi + i\sin\ldots$

5-10M

1 $\frac{x^4}{x^3} = x^1$

PAGE No.

DATE / /

Complex mein roots Single And Pair

Coefficient Real → Pair

Coefficient Complex → Single

Note:-

If the coefficient in a polynomial is real then the complex roots always occur in pair.

Imp #

Q. $x^4 - x^3 - x^2 - x + 1 = 0$. find roots of equation.

$1 - x - x^2 - x^3 + x^4$

Given series is in G.P.

$r = -\frac{1}{x}$, $a =$ $r = -x$, $a = 1$

$$\text{Sum} = \frac{a(1-r^n)}{1-r} = \left(\frac{1(1-(-x)^5}{1-(-x)}\right)$$

$$\boxed{1 - x - x^2 - x^3 + x^4 = \frac{1+x^5}{1+x}}$$

$$1 + x^5 = (1+x)(1 - x - x^2 - x^3 + x^4)$$

$x^5 = -1$

$x = (-1)^{1/5}$

$z = -1$

$z = x = 1, \; y = 0$

$\theta = \tan^{-1}(0) = \pi$

$x = (\cos\pi + i\sin\pi)^{1/5}$ ———— $(\cos\pi + i\sin\pi)$

$x = [\cos(2k\pi + \pi) + i(\sin(2k\pi + \pi)]^{1/5}$

$x = [\cos(2k+1)\frac{\pi}{5} + i\sin(2k+1)\frac{\pi}{5}$

PAGE No.

DATE / /

Q $x^7 + x^4 + i(x^3+1) = 0$

$x^4(x^3+1) + i(x^3+1)$

$(x^4+i) + (x^3+1) = 0$

$x^3 + 1 = 0 \rightarrow x = (-1)^{1/3}$

$x^4 + i = 0 \longrightarrow x = (-i)^{1/4}$

Imp ✩

(10M). # Imp

$(1+x)^n =$

$1 + nx +$

PAGE No.

DATE / /

Using DM Theorem show that all the roots of $(x+1)^7 = (x-1)^7$ are given by $\pm i \cot \frac{k\pi}{7}$, $k = 1, 2, 3$. And also write the conjugate pair of roots.

Expand kar

$$(1+x)^7 = 1 + 7x + \frac{7 \times 6\, x^2}{2!} + \frac{7 \times 6 \times 5 \times x^3}{3!} + 7 \times 6$$

$$-(1-x)^7 =$$

-1

1 Root absurd aayega

Soln

$$(x+1)^7 = (x-1)^7$$

$$\left(\frac{x+1}{x-1}\right)^7 = 1$$

$$\frac{x+1}{x-1} = (\cos 0 + i \sin 0)^{1/7}$$

$$= \left[\cos[0 + 2k\pi] + i \sin[0 + 2k\pi]\right]^{1/7}$$

$$= \cos \frac{2k\pi}{7} + i \sin \frac{2k\pi}{7}$$

$k = 0, 1, 2, 3, 4, 5, 6.$

$$\frac{x+1}{x-1} = \frac{e^{i\frac{2k\pi}{7}}}{1}$$

$$\frac{x+1+x-1}{x+1-x+1} = \frac{e^{i\frac{2k\pi}{7}} + 1}{e^{i\frac{2k\pi}{7}} + 1}$$

$$x = \frac{e^{i\frac{2k\pi}{7}} + 1}{e^{i\frac{2k\pi}{7}} - 1}$$

PAGE No.
DATE / /

Let $\frac{2k\pi}{7} = \theta$.

$x = \frac{e^{i\theta}+1}{e^{i\theta}-1}$

$= \frac{\cos\theta + i\sin\theta + 1}{\cos\theta + i\sin\theta - 1}$

$= \frac{2\cos^2\theta/2 + 2\sin\theta/2\cos\theta/2}{-2\sin^2\frac{\theta}{2} + 2\sin\theta/2\cos\theta/2}$

$= \frac{-\cos\theta/2}{-\sin\theta/2}\left(\frac{\cos\theta/2 + i\sin\theta/2}{-\sin\theta/2 + i\cos\theta/2}\right)$

$= \cot\theta/2\left(\frac{e^{i\theta}}{i^2\sin\theta/2 + i\cos\theta/2}\right)$

$= \cot\theta/2 \; \frac{1}{i}\left[\frac{e^{i\theta}}{i\sin\theta/2 + \cos\theta/2}\right]$

$= \cot\theta/2 \; \frac{1}{i} \cdot \left(\frac{e^{i\theta}}{e^{i\theta}}\right)$

$= \frac{1}{i}\cot\frac{\theta}{2}$

$= \frac{i}{i^2}\cot\frac{\theta}{2} = -i\cot\frac{\theta}{2} = -i\cot\frac{\theta}{2} = -i\cot\frac{2k\pi}{27}$

$= -i\cot\frac{k\pi}{7}$

When $k=0$, $x_0 =$

30 -

PAGE No.

DATE / /

Q Prove that.

(1) $\tanh^{-1}x = \sinh^{-1}\frac{x}{\sqrt{1-x^2}}$ — (Dono Ka formula lagao)

Soln

LHS = $\tanh^{-1}x = \frac{1}{2}\left[\log\frac{(1+x)}{1-x}\right]$

$\sinh^{-1}\frac{x}{\sqrt{1-x^2}}$.

Let $y = \frac{x}{\sqrt{1-x^2}}$

$\sinh^{-1}y = \log\left(y+\sqrt{y^2+1}\right)$

$= \log\left[\frac{x}{\sqrt{1-x^2}}+\sqrt{\left(\frac{x}{\sqrt{1-x^2}}\right)^2+1}\right.$

$= \log\left[\sqrt{\frac{x}{\sqrt{1-x^2}}}+\sqrt{\frac{x^2}{1-x^2}+1}\right)$

$= \log\left[\frac{x}{\sqrt{1-x^2}}+\sqrt{\frac{x^2+1-x^2}{1-x^2}}\right.$

$= \log\left[\frac{x}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-x^2}}\right]$

$= \log\left(\frac{x+1}{\sqrt{1-x^2}}\right) = \cancel{\log}\left(\frac{\sqrt{(1+x)}^2}{\sqrt{1+x\,\cancel{(1-x)}}}\right)$

$= \log\left(\frac{\sqrt{(1+x)}^2}{\sqrt{(1-x)(1+x)}}\right) = \log\left(\sqrt{\frac{(1+x)^2}{(1+x)(1-x)}}\right)$

Q $\operatorname{sech}^{-1}(\sin\theta) = \log \cot\left(\frac{\theta}{2}\right)$

LHS :

Let, $y = \operatorname{sech}^{-1}(\sin\theta)$

$$\operatorname{sech} y = \sin\theta \Rightarrow \frac{1}{\cosh y} = \sin\theta$$

$$\cosh y = \operatorname{cosec}\theta.$$

$$y = \cosh^{-1} \operatorname{cosec}\theta.$$

$$= \log\left[\operatorname{cosec}\theta + \sqrt{\operatorname{cosec}^2\theta - 1}\right]$$

$$= \log\left[\operatorname{cosec}\theta + \cot\theta\right]$$

$$= \log\left[\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right]$$

$$= \log\left[\frac{1+\cos\theta}{\sin\theta}\right]$$

$$= \log\left[\frac{2\cos^2\theta/2}{2\sin\theta/2\,\cos\theta/2}\right]$$

$$y = \log\left[\cot\frac{\theta}{2}\right.$$

$$\Rightarrow \operatorname{sech}^{-1}\sin\theta = \log \cot\theta/2$$

PAGE No.
DATE / /

Q If $\cosh x = \sec\theta$; Then prove that (i) $x = \log(\sec\theta + \tan\theta)$
(ii) $\sinh x = \tan\theta$)
(iii) $\tanh x = \sin\theta$.
(iv) $\tanh \frac{x}{2} = \pm \tan\frac{\theta}{2}$
(v) $\theta = \frac{\pi}{2} - 2\tan^{-1}(e^x)$

Soln:-

(i) $\cosh x = \sec\theta$

$x = \cosh^{-1}(\sec\theta)$

$= \log(\sec\theta + \sqrt{\sec^2\theta - 1})$

$= \log(\sec\theta + \tan\theta)$

$\tan\theta = \frac{2\tan\theta}{1-\tan^2\theta}$.

$-\tanh 2\theta = \frac{2\tanh\theta}{1+\tanh^2\theta}$.

(ii) we know, $\cosh^2 x - \sinh^2 x = 1$

$\sinh^2 x = \cosh^2 x - 1$

$\sinh x = \sqrt{\cosh^2 x - 1}$

$= \sqrt{\sec^2\theta - 1}$

$= \tan\theta$

(iii) $\tanh x = \frac{\sinh x}{\cosh x} = \frac{\tan\theta}{\sec\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} = \boxed{\sin\theta}$

(iv) $\tanh\frac{x}{2}$