Note ①: If A is a symmetric matrix with real entries

⑪ The Diagonal Entries of skew symmetric matrix $\forall = 0$ $[A = -A^T]$

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$a_{ij} = -a_{ji} \quad \forall ij$

$a_{ii} = -a_{ii}$

$a_{ii} = 0$

Matrix → Square

- Symmetric - $A = A^T$
- Skew Symmetric - $A = -A^T$
- Upper Triangular - $a_{ij} = 0, \forall i > j$
- Lower Triangular - $a_{ij} = 0 \; \forall j > i$
- Identity - $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- Scalar - $\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$
- Diagonal - $\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$
- Transpose - interchange in the Row & Column
- Conjugate - $\bar{A}$
- Hermitian - $A = (\bar{A})^T = \overline{(A^T)} = A^\theta$
- Skew Hermitian = $A = -A^\theta$

Note 3:
- Diagonal entries of hermitian matrix is always Real.

$A = A^\theta$

$a_{ij} = \overline{a_{ji}} \quad \forall ij$

$a_{ii} = \overline{a_{ii}}$

$x + iy = x - iy$

$2iy = 0$

$\boxed{y = 0}$

$\boxed{a_{ii} = x}$

Note 4: Diagonal Entries of skew hermitian matrix are either 0 or purely imaginary.

$A = -A^\theta$

$a_{ij} = -(\overline{a_{ji}})^T$

$a_{ii} = -\overline{a_{ii}}$

$x + iy = -(x - iy)$

$2x = 0, \; \boxed{x = 0}$

$\rightarrow \boxed{a_{ii} = iy}$

~~Note If A~~

Orthogonal Matrix:-

- A non-zero square matrix is said to be orthogonal if $\boxed{A \cdot A^T = I}$

note* - The Determinant of Orthogonal matrix = ±1

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Q1 Ex 3.1 3.1 - Ex

$A.A^T = I$

$|A.A^T| = |I|$

$|A| |A^T| = 1$

$|A| |A| = 1$

$|A|^2 = 1$

$|A| = \pm 1$

Unitary Matrix $(A.(\bar{A})^T = I)$ $\boxed{A^{-1} = A^\theta}$

Show that any Non-zero square matrix can be represented as sum of symmetric matrix & skew symmetric matrix.

Proof:

$$A = \frac{A}{2} + \frac{A}{2} + \frac{A^T}{2} - \frac{A^T}{2}$$

$$= \frac{A + A^T}{2} + \frac{A - A^T}{2}$$

$$= P + Q$$

$A = P + Q$

$P^T = P$

$$P^T = \left(\frac{A + A^T}{2}\right)^T$$

$$= \frac{A^T + (A^T)^T}{2}$$

$$= \frac{A^T}{2} + \frac{A}{2} = P$$

$$\frac{A^T + A}{2} = P.$$

Now,

$$Q = \left(\frac{A - A^T}{2}\right)^T$$

$$= \frac{A^T}{2} - \frac{(A^T)^T}{2}$$

$$= \frac{A^T}{2} - \frac{A}{2} = -Q$$

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Ex 25 — Q. Express matrix A as the sum of Symmetric & Skew Symmetric

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Now, $A + A^T$

$$\begin{bmatrix} 2 & 6 & 0 \\ 6 & 10 & 14 \\ 10 & 14 & 18 \end{bmatrix}$$

$$= 2 \begin{bmatrix} 1 & 3 & 5 \\ 3 & 5 & 7 \\ 5 & 7 & 9 \end{bmatrix} = \left[\frac{A+A^T}{2} = \left[1 \ 3 \ 5 \right. \right. \quad \text{— (I)}$$

Also, $A - A^T = \begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -2 \\ 4 & 2 & 0 \end{bmatrix}$

$$= 2 \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\frac{A - A^T}{2} = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} \quad \text{— (II)}$$

(I) + (II)

Q Prove that Any Square matrix can be represented as the sum of Hermitian & Skew Hermitian Matrix

Proof

$$A = \frac{A}{2} + \frac{A}{2} + \frac{A^{\theta}}{2} - \frac{A^{\theta}}{2}$$

$$= \frac{A + A^{\theta}}{2} + \frac{A - A^{\theta}}{2}$$

$$A = \underset{\text{Herm}}{P} + \underset{\text{Skew}}{Q}$$

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Ex. 30

Q $A = \begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3+2i & -1+2i & 0 \end{bmatrix}$

$= \begin{bmatrix} -3i & -1-i & 3+2i \\ 1-i & i & 1-2i \\ -3-2i & -1-2i & 0 \end{bmatrix}$

$A^{\theta} = \begin{bmatrix} -3i & 1-i & -3-2i \\ -1-i & i & -1-2i \\ 3+2i & 1-2i & 0 \end{bmatrix}$

$P = \frac{A + A^{\theta}}{2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ - sin = → Hermitian Matrix

→ $A = P + Q$

$A - A^{\theta} \begin{bmatrix} 6i & -2-2i & 6-4i \\ 2+2i & -2i & 2+4i \\ -6-4i & 2-4i & 0 \end{bmatrix}$

$= 2 \begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3+2i & -1+2i & 0 \end{bmatrix}$

$Q = \frac{A - A^{\theta}}{2} = \begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ 3-2i & -1-2i & 0 \end{bmatrix}$

$\left[\frac{A - A^{\theta}}{2}\right]^{\theta} = \begin{bmatrix} -3i & 1-i & -3-2i \\ 1-i & i & -1-2i \\ 3+2i & -1+2i & 0 \end{bmatrix} = -\left(\frac{A - A^{\theta}}{2}\right)$

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$A^{-1} = \frac{adj A}{|A|}$

$AA^T = I$ —① → $|A| = \pm 1$ — 1

→ Since Orthogonal $|A| = \pm 1$. ⇒ Non singular

→ ∴ It always have Inverse of matrix (orthogonal)

Now, premultiply by A^{-1} in eqn ①.

→ $A.A^T = I$

$A^{-1} A A^T = A^{-1}$ $[A^{-1}.A = I]$

$A^T = A^{-1}$

$\frac{AA^T = I}{|A| = I}$

Q $A = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 9 \end{bmatrix}$ Prove Orthogonal

$A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 1 & 4 & -8 \\ 4 & 7 & 9 \end{bmatrix}$

$A^T = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$

$A.A^T = \frac{1}{81}\begin{bmatrix} 81 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 81 \end{bmatrix}$

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Q Find a, b, c & A^{-1} if $A = \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix}$ is orthogonal.

$$A^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & b & c \end{bmatrix}$$

$$A \cdot A^T = \frac{1}{3}\begin{bmatrix} 1+4+a^2 & 2+2-ab & 2-4+ac \\ 2+2+ab & \cancel{4}+1+b^2 & 4-2+bc \\ 2-4+ac & 4-2+bc & 4+4+c^2 \end{bmatrix} = I$$

$\cancel{2+2+ab=1}$

$1+4+a^2 = 1$

$\cancel{a^2 = -4}$

$\cancel{a = \pm 2}$

$\cancel{a^2 = 9+3}$

$a^2 = 4$

$a = \pm 2$

$4+1+b^2 = 1$

$b^2 = -4$

$b = \pm 2$

$\frac{1}{9}[8+c^2] = 1$

$8+c^2 = 9$

$\boxed{c = \pm 1}$

$\frac{4 + ab}{9} = 0.$

$ab = -4$

if $a=2$, $b=-2$

$-2+ac=0$

$ac=-2$

$c=1$

$(a,b,c) = (2,-2,1)$

$\cancel{\frac{4+ab}{9} = 0}$

$\cancel{ab = -4}$

if $a=-2$, $b=2$

If $a=-2, b=2, c=-1$

$(a,b,c) = (-2,2,-1)$.

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$\Rightarrow A_1 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ $A^{-1} = A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

$A_2 = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$ $A^{-1} = A^T = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & 2 & -1 \end{bmatrix}$,

EX 32 — Q1 wrong

Q $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ Show that A is Unitary

$A^\theta = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

$A \cdot A^\theta = I$

$\Rightarrow \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

$= \frac{1}{3} \begin{bmatrix} 1+1+1 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

Q 2 $A = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ Show the $(I-A)(I+A)^{-1}$ unit

$A^\theta = \begin{bmatrix} 0 & -1-2i \\ 1-2i & 0 \end{bmatrix}$

~~$A \cdot A^\theta = I$~~

$X = I - A$, $Y = I + A$

$Z = Y^{-1}$

$W = X \cdot Z$

$W \cdot W^\theta = I$

$X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1+2i \\ 1-2i & \end{bmatrix}$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

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Rank of Matrix

- Rank of Any Matrix are r if it has followings properties. There is atleast one minor (non-zero) of order r every minor of order greater than r is non zero & the rank of matrix is represented ~~by matrix~~ as row (ρ) of A.

Q $\begin{vmatrix} 2 & 4 & 5 \\ 6 & 7 & 8 \\ 9 & 10 & 11 \end{vmatrix}$

$\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

One 2 order
9 order matrix

$(-1)^2$

a_{11} a_{12} a_{13}

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

8 $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

9 $[a]$

1 $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Minor example

$A \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$

I^{st} Minor - $\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 0$

II^{nd} min - $\begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$

III^{rd} Minor - $\begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = 0$

$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\boxed{\text{Rank } 1}$ $\rho(A) = 1$

$\rho(B) = 1$

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Echelon form

→ A Matrix A is set to be echelon form if .

1. Every zero row of the matrix A occurs below a nonzero row
2. No of zeroes before a non-zero element in a row is less than the number of such zeroes in the next row
3. The ~~length~~ Rank of matrix in echelon form is equal to non-zero rows of the matrix

1) Transfor Using Row Operations

make $a_{11} = 1$

a_{21} A a_{31} k o a_{11} ko

Appr subt ... that

Procedure to convert / transform matrix in echelon form

① We first make $a_{11} = 1$. – By (i) interchanging rows of
(ii) Dividing $\frac{a_{11}}{a_{11}} = 1$.
(iii)
(iii) By Subtracting ~~rows~~

(ii) With the help of a_{11} we make 0 the element below a_{11} (a_{21}, a_{31}).

Using row operation

(iii) Again we make $a_{22} = 1$ by Step ①. & making the element zero below a_{22} Using row operation

(iv) we Repeat same process either we get Last Row as 0 ~~Row~~ OR the Last Diagonal Element (a_{33}) as non zero.

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$R_i \leftrightarrow R_j$

$R_i - R_j$

$R_i \rightarrow \alpha R_j$

$R_i \rightarrow R_i \rightarrow \alpha R_j$

Q. $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}_{4\times4}$

Soln:-

$R_2 \rightarrow R_2 + 2R_1$

$R_3 \rightarrow R_3 - R_1$

$\sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -2+2\times2 & -3+2\times3 & -1+2\times(-1) \\ 0 & 0-2 & 1-3 & \\ 0 & 1 & 1 & \end{bmatrix}$

$R_4 \longleftrightarrow R_2$

$\sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -2 & -2 & 2 \\ 0 & 3 & 3 & -3 \end{bmatrix}$

$R_3 \rightarrow R_3 + 2R_2$

$R_4 \rightarrow R_4 - 3R_2$

$\sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\rho(2)$.

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Q $A = \begin{bmatrix} 2 & 1 & 1 & 3 \\ 1 & 0 & 1 & 2 \\ 3 & 1 & 2 & 5 \end{bmatrix}$ 3×4

A $R_1 \leftrightarrow R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 1 & 2 & 5 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 3 & 1 & 2 & 5 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank $\boxed{2}$ $\rho(2)$

Q. $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & 7 \end{bmatrix}$

$R_2 \leftrightarrow R_1$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & 7 \end{bmatrix} \quad R_4 - 6R_1$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

~~$R_2 \to R_2 - R_3$~~ ; $R_2 \to \frac{R_2}{6} - R_3$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_3 - 4R_2$, $R_4 - 9R_2$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 66 & 44 \end{bmatrix}$$

$R_4 \to R_4 - 2R_3$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \rho(3)$$

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Rank of Matrix by Reduction to Normal form

DM — PAQ

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II Reduction of Matrix in PAQ Normal Form

(I) This is the another method to find the Rank of the matrix where $[A]_{m \times n} = I_m A I_n$.

(II) First we apply the Row Transformation on the Left side matrix of equation I_m. Then we only apply the Row matrix on the matrix I_m on RHS.

Then By continous Row Transformation I_m is converted to P. The LHS Matrix A is converted into echelon form.

(III) Now, we apply the column transformation of LHS matrix A. to convert it into A Normal form & the same operation (i.e. Column Transformation). Will be applied on I_n to convert it into Q.

$$[A]_{m \times n} = I_m A I_n$$

$$\begin{array}{ccc} I_m & A & I_n \\ \Big\downarrow{\scriptstyle \text{Row Tran}} & & \Big\downarrow{\scriptstyle \text{Column Transfn}} \\ P & A & Q \end{array}$$

– Rank of $I_3 \to P$ & Q Remains same after transf

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Q $A = \begin{bmatrix} 2 & -2 & 3 \\ 3 & -1 & 2 \\ 1 & 2 & -1 \end{bmatrix}_{3\times3}$

$A = I_3 A I_3$

$\begin{bmatrix} 2 & -2 & 3 \\ 3 & -1 & 2 \\ 1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A I_3$

$R_1 \leftrightarrow R_3$

$\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A I_3$

$R_2 - 3R_1$ $R_3 - 2R_1$

$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & -6 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix}$

$R_3 - R_2$

$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -3 \\ 1 & -1 & 1 \end{bmatrix} A I_3$

$R_3 \leftrightarrow R_2$

$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & -7 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 3 \end{bmatrix} A I_3$

$R_3 + 7R_2$

$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 12 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 6 & 10 \end{bmatrix} A I_3$

$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 1 \\ \frac{7}{5} & \frac{-6}{5} & \frac{4}{5} \end{bmatrix}$

$\Downarrow$

P

$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = PA \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$C_2 - 2C_1$ $C_3 + C_1$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = PA \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$I_3 = PAQ$

$I_3 = PAQ$

$I_3 = (PAQ)^{-1}$

$I_3 = Q^{-1} A^{-1} P^{-1}$

$Q = A^{-1} P^{-1}$

Note:

If A is a square matrix $A_{n\times n}$ of order n and $|A| \neq 0$, then A is non-singular & the Rank of the matrix is equal to the order of the matrix.

Since $|A| \neq 0 \rightarrow A^{-1}$ exists.

P & Q is always non singular having Rank as P^{-1} & Q^{-1} exists.

$\rightarrow \boxed{A^{-1} = QP}$

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Ex. 35 Q 1. 1st part

3×4 matrix — find its Rank

(1) $A = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 1 & 4 & -5 \\ -1 & -5 & -5 & 7 \end{bmatrix}$

Soln:-

Note*

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System Of Equations

Homogeneous eqn

$2x+3y=0$
$3x+6y=0$

$3x=4y$

~~a & t~~

$\boxed{C=0}$

→ C is zero

↓ → Consistent

(∀ solution exists)

(0,0) - Always $ax+by=0$.

Unique

$[A][X]=0$
$\rho(A)$ = no of variable

Note: if A is square matrix of order n & Determinant of A is not equal to zero. → ~~Rank of~~

$\rho(A)$ = order of A = no of variable

In this case system of equation has unique solutions ie zero solution

$(x,y)=(0,0)$

$$\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$|A|\neq 0$

$\rho(A)=2$

$x_1+2x_2=0$, $3x_1+4x_2=0$

∞ (many) - when

$r(A)\neq$ no of variable/unknowns

→ Infinite many solutions

$2x+3y=0$
$4x+6y=0$

- $|A|=0$.

Non-Homogeneous eqn

$2x+3y=1$
$3x+9y=0$

$2x+4y+3=0$.
$2x+4y=-3$

$\boxed{C\neq 0}$

→ C is non-zero.

Non- Homogeneous eq

$AX = B$; $B \neq 0$. [equation mein atleast one non-zero

$AX = B$

Consistent

(Augmented Matrix) $[A|B]_{m \times (n+1)}$

$\boxed{\rho(A) = \rho[A|B]}$

$\boxed{\text{unique}}$

$\rho(A) = \text{No of variable}$

$\rho(A) = \text{no of columns}$

Unique means only Soln is zero Soln

0 is the trivial Solution

$\boxed{\infty \text{ many solutions}}$

$\rho(A) < \text{No of variable}$

Inconsistent

$\rho(A) \neq \rho(A|B)$

↳ no solutions

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Q $x + y + z = 6$

$x + 2y + 3z = 14$

$2x + 4y + 7z = 30.$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

A X = B

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 2 & 4 & 7 & 30 \end{array}\right]$$

$R_2 - R_1$, $R_3 - 2R_1$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 2 & 5 & 18 \end{array}\right]$$

$R_3 - 2R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\rho(A) = 3$

$\rho(A|B) = 3$

$\rho(A) = \rho(A|B) = 3 =$ no of variable

$\rightarrow$ unique Solution

$x + y + z = 6$

$1 + 2z = 8$

$z = 2$

$x = 0, \quad y = 4, \quad z = 2$

Q2 $x+y+z=5$ — check consistency
$x+2y+3z=10$
$x+2y+3z=8$

$$\begin{bmatrix}1&1&1\\1&2&3\\1&2&3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}5\\10\\8\end{bmatrix}$$
$$A \quad X = B$$

$$[A|B] = \left[\begin{array}{ccc|c}1&1&1&5\\1&2&3&10\\1&2&3&8\end{array}\right]$$

R_2-R_1, R_3-R_1

$$= \left[\begin{array}{ccc|c}1&1&1&5\\0&1&2&5\\0&1&2&3\end{array}\right]$$

R_3-R_2

$$\sim \begin{bmatrix}1&1&1&5\\0&1&2&5\\0&0&0&-2\end{bmatrix}$$

$\rho(A)=2$, $\rho(A|B)=3$

$\rho(A) \neq \rho(A|B)$.
inconsistent.

Q3 $x-2y+z-w=2$
$x+2y+4w=1$
$4x-z+3w=-1$.

$$\begin{bmatrix}1&-2&1&-1\\1&2&0&4\\4&0&-1&3\end{bmatrix}\begin{bmatrix}x\\y\\z\\w\end{bmatrix}=\begin{bmatrix}2\\1\\-1\end{bmatrix}$$
$$A \quad X = B$$

$$\left[\begin{array}{cccc|c}1&-2&1&-1&2\\1&2&0&4&1\\4&0&-1&3&-1\end{array}\right]$$

R_2-R_1, R_3-4R_1

$$\left[\begin{array}{cccc|c}1&-2&1&-1&2\\0&4&-1&5&-1\\0&+8&-5&7&-9\end{array}\right]$$

R_3-2R_2

$$\left[\begin{array}{cccc|c}1&-2&1&-1&2\\0&4&-1&5&-1\\0&0&-3&-3&-7\end{array}\right]$$

$\rho(A)=3$, $\rho(A|B)=3$
→ Consistent
$\rho(A|B)=\rho(A) <$ no of variable
→ Infinitely many solution

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$x - 2y + z - w = 2$
$4y - z + 5w = -1$
$-3z - 3w = -7$

$\boxed{w = t, \quad t \in R}$

$z = \frac{7 - 3t}{3}$

$z = \frac{7 - 3t}{3} = \frac{7}{3} - t$

$4y - z + 5w = -1$

$y = \frac{1}{4}\left[-1 + z - 5w\right]$

$y = \frac{1}{4}\left[-1 + \frac{7}{3} - t - 5t\right]$

$y = \frac{1}{4}\left(-1 + \frac{7}{3} - t - 6t\right)$

$y = \frac{1}{4}\left[\frac{4}{3} - 6t\right]$

$= \frac{1}{3} - \frac{3}{2}t$

$x = 2 + 2y - z + w$
$= 2 + \frac{2}{3} - 3t - \frac{7}{3} + t + t$

$x = \frac{-1}{3} - t$

Soln
$(x, y, w, z) = \left(\frac{1}{3} - t, \frac{1}{3} - \frac{3}{2}t, \frac{7}{3} - t, t\right)$

Check putting t=0 in any equation

3-

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For what value of λ & μ the following system of equation

$2x + 3y + 5z = 9$
$7x + 3y - 2z = 8$
$2x + 3y + \lambda z = \mu$

— such that (i) unique soln
(ii) infinite many soln
(iii) no soln

$$(A|B) \quad \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_1 \to \frac{1}{2} R_1$

$$\left[\begin{array}{ccc|c} 1 & 1.5 & 2.5 & 4.5 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_2 - 7R_1, R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 1.5 & 2.5 & 4.5 \\ 0 & -7.5 & -19.5 & -23.5 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

(i) Unique Soln
$\partial(A) = \partial(A|B) = 3$
$\lambda - 5 \neq 0$
$\lambda \neq 5, \forall \mu$

(ii) ∞
$\partial(A) = \partial(A|B) < 3$
$\lambda - 5 = 0 \qquad \mu - 19 = 0$
$\boxed{\lambda = 5} \qquad \boxed{\mu = 19}$

(iii) $\partial(A) \neq \partial(A|B)$
$\lambda - 5 = 0, \quad \mu - 9 \neq 0$
$\lambda = 5, \quad \mu \neq 9$

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Q For what value of

$$x + y + z = 6$$
$$x + 2y + 5z = 10$$
$$2x + 3y + \lambda z = \mu$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 2 & 3 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$R_2 - R_1$, $R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 4 \\ 0 & 1 & \lambda-2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ \mu-12 \end{bmatrix}$$

$[A|B]$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 4 & 4 \\ 0 & 1 & \lambda-2 & \mu-12 \end{array}\right]$$

$R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 4 & 4 \\ 0 & 0 & \lambda-6 & \mu-16 \end{array}\right]$$

0

(I) Unique Soln

$\partial(A) = \partial(A|B) =$ no of variable (3)

$\lambda - 6 \neq 0$ ~~$\mu \neq 16$~~

$\lambda \neq 6$, $\forall \mu$

(II) Infinite ~~[illegible]~~

$\rho(A) = \partial(A|B) < (3)$ (no of var,

$\lambda - 6 = 0$, $\mu - 16 = 0$

$\boxed{\lambda = 6}$ $\boxed{\mu = 16}$

LI LD → Column form men banate hai

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No soln

$\rho(A) = \rho(A|B)$ — $\lambda - 6 = 0$, & $\mu - 16 \neq 0$.

$\lambda = 6$, $\mu \neq 16$.

Linearly Independent & Linearly Dependent

① Linearly Independent. (LI)

$x_1, x_2 \ldots x_n$ are n vectors are said to be Linearly independent. If there are n scalar $k_1, k_2 \ldots k_n$ such that $k_1 x_1 + k_2 x_2 \ldots + k_n x_n = 0$.

(i) iff $k_1 = k_2 = \ldots = k_n = 0$. the vector is Linear Independent

$\forall k_i = 0$.

(ii) Linearly Dependent. (LD)

n vector $x_1, x_2 \ldots x_n$ are said to be LD. if for scalar $k_1, k_2 \ldots k_n$ such that $k_1 x_1 + k_2 x_2 \ldots + k_n x_n = 0$.

① Iff $\exists$ One $k_i \neq 0$.

$k_1 x_1 = -(k_2 x_2 + \ldots k_n x_n)$

$x_1 = \frac{-1}{k_1}(k_2 x_2 + \ldots k_n x_n)$

(For checking LI or LD of vector we write the vector inside the matrix in column)

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Q Check where vector is LI or LD.

$[1,1,1,3]$, $[1,2,3,4]$, $[2,3,4,7]$
x_1 x_2 x_3

a, b, c are scalars

$ax_1 + bx_2 + cx_3 \rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \\ 3 & 4 & 7 \end{bmatrix}$

$R_2 - R_1$, $R_3 - R_1$, $R_4 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_3 - 2R_2$, $R_4 - R_2$

$$\sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 < 3$

$\rightarrow$ LD.

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$a + b + 2c = 0$

$b + c = 0$

$c = t$

$b = -t$

$a = -2c - b = -2t + t = t.$

$(-t, -t, t)$.

$23 - \frac{12}{11}^2$

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$[1, 2, -1, 0] = x_1$
$[1, 3, 1, 2] = x_2$
$[4, 2, 1, 0] = x_3$
$[6, 1, 0, 1] = x_4$

a, b, c, d are scala

$ax_1 + bx_2 + cx_3 + dx_4 = 0$

$$\begin{bmatrix} 1 & 1 & 4 & 6 \\ 2 & 3 & 2 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

$R_2 - 2R_1$; $R_3 + R_1$, R

$$\begin{bmatrix} 1 & 1 & 4 & 6 \\ 0 & 1 & -6 & -11 \\ 0 & 2 & 5 & 6 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

$R_3 - 2R_2$, $R_4 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 4 & 6 \\ 0 & 1 & -6 & -11 \\ 0 & 0 & 17 & 28 \\ 0 & 0 & 12 & 23 \end{bmatrix}$$

$R_4 - \frac{12}{11} R_3$

$$\begin{bmatrix} 1 & 1 & 4 & 6 \\ 0 & 1 & -6 & -11 \\ 0 & 0 & 17 & 28 \\ 0 & 0 & 1 & \frac{250}{17} \end{bmatrix}$$

$\rho(A) = 4$ = no of variabl/scalar

$\Longrightarrow$ LI

$a + b + 4c + 6d = 0$

$b - 6c - 11d = 0$

$17c + 28d = 0$

$c + \frac{55}{17} d = 0.$

$d = 0$

$c = 0.$

$b = 0.$

$a = 0.$

$23 - \frac{12}{17} \times 28$

H.W

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