MATHS- 2

Module :- I

1^{st} order 1^{st} degree of differential equation (D.E) :-

(1) The general form of 1^{st} order 1^{st} degree D.E is

$$\boxed{\frac{dy}{dx} + f(x, y) = 0}$$

e.g

exact D.E

(1) $\frac{dy}{dx} + 1 = 0$ (2) $\frac{dy}{dx} + \sin x = 0$

exact DE

(3) $x^2 dy + y^2 dx = 0$

Standard form

$x^2 dy = -y^2 dx$ $\frac{dy}{dx} = -y^2/x^2$

convert

General form $\boxed{\frac{dy}{dx} + \frac{y^2}{x^2} = 0}$ → Not exact D.E

- complex question solve by variable separation $\boxed{f(x) dx = g(y) dy}$

- every differential equation not solve by variable separation

1.1

Definition (Exact D.E) :-

M and N are function of x, y

D.E of the form $\boxed{M dx + N dy = 0}$ is said to be exact DE if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

For checking it is exact D.E are not

whose M, N are functions of x and y and $\boxed{M dx + N dy = 0}$ is (Standard form of 1st order 1st degree DE)

1

~~% It's equati~~

90% solution get

∴ It's solution is given by

$$\boxed{\int M dx + \int (\text{terms in } N \text{ free from } x)\, dy = C}$$

OR

10% solution get

$$\boxed{\int (M \text{ term is free from } y)\, dx + \int N dy = C}$$

using when complicate question

e.g ① $N = x^2 y$ — Not get any ~~term~~ then what will be write ⇒ zero

② $N = x^2 - y$ → $N = -y$ → free from x N

③ $N = \frac{x+y}{xy} = \frac{1}{y} + \frac{1}{x}$

free from x $N = \frac{1}{y}$

Note :- general form → $\boxed{\text{Convert into}}$ → standard form

5 marks

(1) $(x^2 - 4xy - 2y^2)\,dx + (y^2 - 4xy - 2x^2)\,dy = 0$

↳ this equation is already in form of standard form

→ Here

$$M = x^2 - 4xy - 2y^2 \qquad N = y^2 - 4xy - 2x^2$$

$$\therefore \frac{\partial M}{\partial y} = -4x - 4y \qquad \frac{\partial N}{\partial x} = -4y - 4x$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}} \quad \text{DE is exact}$$

∴ It's solution is given by

$$\boxed{\int M\,dx + \int (\text{term in } N \text{ free from } x)\,dy = C}$$

$$\int x^2 - 4xy - 2y^2\,dx + \int y^2\,dy = C$$

$$\frac{x^3}{3} - \frac{4y\,x^2}{2} - 2y^2x + \frac{y^3}{3} = C$$

$$\boxed{\frac{x^3}{3} - 2x^2y - 2xy^2 + \frac{y^3}{3} = C}$$

5 marks

(2) Solve $\dfrac{dy}{dx} = \dfrac{\tan y - 2xy - y}{x^2 - x\tan^2 y + \sec^2 y}$

$\downarrow$ convert into standard form

$$(x^2 - x\tan^2 y + \sec^2 y)\,dy = (\tan y - 2xy - y)\,dx$$

$$(x^2 - x\tan^2 y + \sec^2 y)\,dy - (\tan y - 2xy - y)\,dx = 0$$

$N = x^2 - x\tan^2 y + \sec^2 y$ $\qquad$ $M = -(\tan y - 2xy - y)$

$$\frac{\partial N}{\partial x} = 2x - \tan^2 y$$

$$\frac{\partial M}{\partial y} = -(\sec^2 y - 2x - 1) = -\sec^2 y + 2x + 1 = 2x - \tan^2 y$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$ DE is exact

$\therefore$ It's solution is given by

$$\int M\,dx + \int (\text{term in } N \text{ free from } x)\,dy = c$$

$$\int -\tan y + 2xy + y\,dx + \int \sec^2 y\,dy = c$$

$$-x\tan y + 2y\frac{x^2}{2} + xy + \tan y = c$$

$$-x\tan y + (x^2 y + xy) + \tan y = c$$

$$\boxed{(1-x)\tan y + (x+1)xy = c}$$

(3) $(2y^2 - 4x + 5)\,dx = (y - 2y^2 - 4xy)\,dy$

$\Rightarrow$ rearranging the given eq^n as $\boxed{Mdx + Ndy = 0}$

$(2y^2 - 4x + 5)\,dx - (y - 2y^2 - 4xy)\,dy = 0$

Now,
Comparing with $Mdx + Ndy = 0$

Here,

$M = 2y^2 - 4x + 5$ $\quad$ $N = -(y - 2y^2 - 4xy)$

$\frac{\partial M}{\partial y} = 4y - 0 + 0$ $\quad$ $\frac{\partial N}{\partial x} = -(-4y)$

$= 4y$

$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ the given D.E is exact

$\therefore$ The solution of Exact D.E is

$\int M\,dx + \int(\text{term in } N \text{ is free form } x)\,dy = c$

$\int(2y^2 - 4x + 5)\,dx + \int -(y - 2y^2)\,dy = c$

$2xy^2 - \frac{4x^2}{2} + 5x - \frac{y^2}{2} + \frac{2y^3}{3} = c$

$$\boxed{2xy^2 - 2x^2 + 5x + \frac{2y^3}{3} - \frac{y^2}{2} = c}$$

This is the required General solution.

(4) $\left(y+\frac{1}{3}y^3+\frac{1}{2}x^2\right)dx + (x+xy^2)\,dy = 0$

$\Rightarrow$ comparing the given equation with $\boxed{Mdx+Ndy=0}$

Here,

$M = \left(y+\frac{1}{3}y^3+\frac{1}{2}x^2\right)$ $\qquad$ $N = x + xy^2$

$\frac{\partial N}{\partial x} = (1+y^2)$

$\frac{\partial M}{\partial y} = \left(1+\frac{3y^2}{3}\right) = (1+y^2)$

$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ The given D.E is exact

The solution of exact D.E is

$\int M\,dx + \int(\text{term in } N \text{ is free from } x)\,dy = C$

$\int\left(y+\frac{1}{3}y^3+\frac{1}{2}x^2\right)dx + \int 0\,dy = C$

$\boxed{xy + \frac{xy^3}{3} + \frac{1}{6}x^3 = C}$ This is required General solution

(5) Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$

$\Rightarrow$ rerranging the given eqn of $\boxed{Mdx+Ndy=0}$

$\frac{dy}{dx} = -\frac{y\cos x + \sin y + y}{\sin x + x\cos y + x}$

$(\sin x + x\cos y + x)\,dy = -(y\cos x + \sin y + y)\,dx$

$(y\cos x + \sin y + y)\,dx + (\sin x + x\cos y + x)\,dy = 0$

Comparing the given eqn with as $\boxed{Mdx+Ndy=0}$

Here

$M = (y\cos x + \sin y + y)$ $\qquad$ $N = (\sin x + x\cos y + x)$

$\frac{\partial M}{\partial y} = (\cos x + \cos y + 1)$ $\frac{\partial N}{\partial x} = (\cos x + \cos y + 1)$

$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ The given D.E is exact

The solution of exact D.E is

$\int M\,dx + \int(\text{term in } N \text{ is free from } x)\,dy = C$

$\int(y\cos x + \sin y + y)\,dx + \int(0)\,dy = C$

$y\sin x + x\sin y + xy = C$

$\boxed{xy + x\sin y + y\sin x = C}$

This is the required General solution

$\int \log x \, dx = x\log x - x$

(5) $(1+\log xy)dx + \left(1+\frac{x}{y}\right)dy = 0$

$\Rightarrow$ Comparing the given equation as $\boxed{Mdx + Ndy = 0}$

Here. $M = (1+\log xy)$ $N = (1+x/y)$

$\frac{\partial M}{\partial y} = \frac{(x)}{xy} = \frac{1}{y}$ $\frac{\partial N}{\partial x} = \frac{1}{y}$

$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ The given D.E is exact

The solution of exact D.E is

$\int M dx + \int (\text{term in N is free from } x)\, dy = c$

$\int (1+\log xy)\, dx + \int 1\, dy = c$

$x + \frac{xy\log(xy) - xy}{y} + y = c$

$\cancel{x} + x\log(xy) - \cancel{x} + y = c$

$\boxed{x\log(xy) + y = c}$

This the required General Solution

(6) $(e^y+1)\cos x\,dx + e^y \sin x\,dy = 0$

$\Rightarrow$ comparing the given eqn with as $\boxed{Mdx + Ndy = 0}$

Here

$M = (e^y+1)\cos x$ $\quad\quad$ $N = e^y \sin x$

$M = e^y\cos x + \cos x$

$\frac{\partial N}{\partial x} = e^y \cos x$

$\frac{\partial M}{\partial y} = e^y \cos x$

$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ The given D.E is exact

The solution of exact D.E is

$\int M\,dx + \int(\text{term in } N \text{ is free from } x)\,dy = C$

$\int(e^y\cos x + \cos x)\,dx + \int 0\,dy = c$

$(e^y \sin x + \sin x) = C$

$\boxed{(e^y+1)\sin x = c}$

This the required General solution

Note: $f(xy) \neq f(x \cdot y)$ ho bhi sakta hai, nhi bhi ho sakta hai

Imp I. 2

Reducible form of exact DE :-

A No exact DE can be converted into exact D.E by multiplying and integrating factor is called reducible form of exact D.E

$M' = M \times (I.F)$ $N' = N \times (I.F)$

- Integrating factor (I.F) can be determine by following rules :-

(1) If DE of the form → degree should be same. x and y Imp also like $xy + x^2y^2 + x^3y^3$

$$\boxed{y f(xy)\, dx + x\, g(xy)\, dy = 0}$$

then $I.F = \dfrac{1}{(Mx - Ny)}$ provided $(Mx + Ny) = 0$

put $f(x, y) = f(xt, yt)$ → t separately get with function → it is homogeneous

(2) If DE is homogeneous

then $I.F = \dfrac{1}{(Mx + Ny)}$ provided $Mx + Ny = 0$

xy no y term

(3) If $\dfrac{\left(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right)}{N} = g(x)$ i.e function of x - alone

then $If = e^{\int g(x)\, dx}$

no x term

(4) If $\dfrac{\left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right)}{M} = g(y)$ i.e function of y - alone

then $If = e^{\int g(y)\, dy}$

1st check

1	2	3	4
Homogeneous are not	form		

xy degree same check

→ same order

Ex $y(x^0 y^0)\, dx - x(xy + x^2 y^2)\, dy = 0$

$Mdx + Ndy = 0$

e.g

(1) Solve $y(x^2y + e^x)dx - e^x dy = 0$

Here

$M = y(x^2y + e^x)$ $\qquad N = -e^x$

$= x^2y^2 + e^x y$

$\dfrac{\partial M}{\partial y} = 2x^2y + e^x$ $\qquad \dfrac{\partial N}{\partial x} = -e^x$

$\boxed{\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}}$ DE is Non-exact

Consider

$= \dfrac{\left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right)}{M} = \dfrac{\cancel{2x^2y + e^x - e^x}}{\cancel{y(x^2y + e^x)}}$

$= \dfrac{-e^x - 2x^2y - e^x}{y(x^2y + e^x)}$ $\cancel{= \dfrac{2x^2y}{y(x^2y+e^x)}} =$

$= \dfrac{-2e^x - 2x^2y}{y(x^2y + e^x)} = -\dfrac{2}{y}\dfrac{\cancel{(x^2y + e^x)}}{\cancel{(x^2y + e^x)}}$

$\left(\dfrac{\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}}{M}\right) = -\dfrac{2}{y} = g(y)$ i.e function of y - alone

$I.F = e^{\int g(y)dy} = e^{\int -2/y\, dy} = e^{-2\int \frac{1}{y} dy}$

$= e^{-2\log y} = e^{\log_e (y)^{-2}}$

$= y^{-2} = \dfrac{1}{y^2}$

$I.F = \dfrac{1}{y^2}$

On multiplying I.F to given D.E

$M' = M \times I.F = \cancel{y}(x^2y + e^x) \times \dfrac{1}{y^{\cancel{2}}} = \left(\dfrac{x^2y + e^x}{y}\right)$

$N' = N \times I.F = \dfrac{-e^x}{y^2}$

$\dfrac{\partial M'}{\partial y} = \dfrac{y(x^2) - [(x^2y) + e^x)]}{y^2} = \dfrac{\cancel{x^2y} - \cancel{x^2y} - e^x}{y^2}$

$= \dfrac{-e^x}{y^2}$

$\dfrac{\partial N'}{\partial x} = \dfrac{-e^x}{y^2}$

$\boxed{\dfrac{\partial M'}{\partial y} = \dfrac{\partial N'}{\partial x}}$ DE become exact

$\therefore$ It's solution is given by

$\int M' dx + \int (\text{terms in } N' \text{ free from } x)\, dy = c$

$\int \dfrac{x^2y + e^x}{y} dx + \int 0\, dy = c$

$\int \dfrac{x^2\cancel{y}}{\cancel{y}} + \dfrac{e^x}{y} dx + 0 = c$

$\int x^2 dx + \int \dfrac{e^x}{y} dx = c$

$\boxed{\dfrac{x^3}{3} + \dfrac{e^x}{y} + c}$

(2) $(x^2+y^2+1)\,dx - 2xy\,dy = 0$

Here $M = (x^2+y^2+1)$ $\quad N = -2xy$

$\frac{\partial M}{\partial y} = 2y$ $\quad \frac{\partial N}{\partial x} = -2y$

$\therefore \boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$ D.E is not exact

Consider

$\frac{\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)}{N} = \frac{2y+2y}{-2xy} = \frac{2\cancel{y}}{-x\cancel{y}} = \frac{2}{-x}$

$= \frac{2}{-x} = g(x)$ i.e function of x-alone

$I.F = e^{\int g(x)dx}$

$= e^{-2\int \frac{1}{x}dx} = e^{-2\log x}$

$= e^{\log (x)^{-2}}$

$\boxed{I.F = x^{-2} = \frac{1}{x^2}}$

On multiplying I.F to given D.E

$M' = M \times I.F = \frac{(x^2+y^2+1)}{x^2} = \left(1 + y^2/x^2 + 1/x^2\right)$

$N' = N \times I.F = \frac{-2xy}{x^2} = -2y/x$

$\frac{\partial M'}{\partial y} = \frac{2y}{x^2}$ $\quad \frac{\partial N'}{\partial x} = -2\left(-\frac{y}{x^2}\right) = \frac{2y}{x^2}$

$\boxed{\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x} = \frac{2y}{x^2}}$ DE is exact

$\therefore$ It's solution is given by

$\int M'dx + \int (\text{term in } N' \text{ free form } x)\,dy$

$\int \left(1 + y^2/x^2 + 1/x^2\right)dx + \int 0\,dy = C$

$\left(x - \frac{y^2}{x} - \frac{1}{x}\right) = C$

(3) $(x^4+y^4)\,dx - xy^3\,dy = 0$

Comparing with $M\,dx + N\,dy = 0$

$M = x^4 + y^4$ $\qquad N = -xy^3$

$\frac{\partial M}{\partial y} = 4y^3$ $\qquad \frac{\partial N}{\partial x} = -y^3$

$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$ D.E is not exact.

Consider

$\frac{\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)}{N} = \cancel{\frac{x^4+y^4-xy^3}{xy^3}} \cancel{= x^3+}$

$= \frac{4y^3-(-y^3)}{-xy^3} = \frac{5y^3}{-xy^3}$

$= \frac{5}{-x} = g(x)$ i.e function of x alone

$I.F = e^{\int -5/x\,dx} = e^{-5\int \frac{1}{x}dx} = e^{(-5\log x)}$

$= e^{\log_e (x)^{-5}} = x^{-5} = \frac{1}{x^5}$

on multiply I.F to given O.E

$M' = M \times I.F = \frac{x^4+y^4}{x^5} = \frac{1}{x} + \frac{y^4}{x^5}$

$N' = N \times I.F = \frac{-(xy^3)}{x^5} = -\frac{y^3}{x^4}$

x^{-5}

$\frac{\partial M'}{\partial y} = \frac{4y^3}{x^5}$ $\qquad -(y^3(x^{-4}))$ $\qquad \frac{\partial N'}{\partial x} = -y^3 \cdot -4x^{-5} = \frac{4y^3}{x^5}$

$\boxed{\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}}$ OE is exact

$\therefore$ It's solution is given by

$\int M'dx + \int (\text{term in } N' \text{ free from } x)\,dy = C$

$\int \frac{1}{x} + \frac{y^4}{x^5}dx + \int 0\,dy = C$

$\log x + \frac{y^4 x^{-4}}{-4} = C$

$\boxed{\log x - \frac{y^4}{4x^4} = C}$

(3) $y(xy + e^x)dx - e^x dy = 0$

Comparing with $Mdx + Ndy = 0$

$M = y(xy + e^x)$ $\quad N = -e^x$

$= xy^2 + e^x y$

$\frac{\partial M}{\partial y} = 2xy + e^x$ $\quad \frac{\partial N}{\partial x} = -e^x$

$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$ O.E is not exact

consider

$$\frac{\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)}{M} = \frac{-e^x - (2xy + e^x)}{y(xy + e^x)}$$

$$= \frac{-e^x - 2xy - e^x}{y(xy + e^x)} = \frac{-2e^x - 2xy}{y(xy + e^x)}$$

$$= -\frac{2}{y}\frac{(xy + \cancel{e^x})}{\cancel{(xy + e^x)}} = -\frac{2}{y} = g(y)$$

i.e function of y - alone

$$I.F = e^{\int -2/y\, dy} = e^{-2\int \frac{1}{y} dy}$$

$$= e^{-2\log y} = e^{\log_e (y)^{-2}}$$

$$= y^{-2} = \frac{1}{y^2}$$

on multiply I.F of given O.E

$M' = M \times I.F = \frac{y(xy + e^x)}{y^2} = \frac{(xy + e^x)}{y}$

$= (x + \frac{e^x}{y})$

$N' = N \times I.F = \frac{-e^x}{y^2}$

$\frac{\partial M'}{\partial y} = \frac{-e^x}{y^2}$ $\quad \frac{\partial N'}{\partial x} = \frac{-e^x}{y^2}$

$\boxed{\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}}$ O.E is exact

$\therefore$ It's Solution is given by

$\int M'x + \int(\text{term in } N' \text{ free from } x)\, dy = C$

$$\int (x + e^x/y)\, dx + \cancel{\int \frac{-e^x}{y^2} dy} = C$$

$$\frac{x^2}{2} + \frac{e^x}{y} \cancel{= \left(\frac{e^x}{y}\right)} = C$$

$$\frac{x^2}{2} + \frac{e^x}{y} + \cancel{\frac{e^x}{y}} = C$$

$$\boxed{\frac{x^2}{2} + \frac{e^x}{y} = C}$$

$$\frac{x^2}{2} + \frac{e^x}{y}$$

form $y\,f(xy)\,dx + x\,f(xy)\,dy = 0$

(4) $y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0$

Comparing with $M\,dx + N\,dy = 0$

$M = y(xy + 2x^2y^2)$ $\qquad$ $N = x(xy - x^2y^2)$

$= xy^2 + 2x^2y^3$ $\qquad$ $= x^2y - x^3y^2$

$\frac{\partial M}{\partial y} = 2xy + 6x^2y^2$ $\qquad$ $\frac{\partial N}{\partial x} = 2xy - 3x^2y^2$

$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$ D.E is not exact

consider

$(Mx - Ny) = \cancel{x^2y^2} + 2x^3y^3 - \cancel{x^2y^2} + x^3y^3$

$= 3x^3y^3 \neq 0$

$I.F = \frac{1}{3x^3y^3}$

on multiply of given D.E

$M' = M \times I.F$ $\qquad$ $N' = N \times I.F$

$= \frac{xy^2 + 2x^2y^3}{3x^3y^3}$ $\qquad$ $= \frac{x^2y - x^3y^2}{3x^3y^3}$

$= \frac{(y + 2xy^2)}{3x^2y^2}$ $\qquad$ $= \frac{(x - x^2y)}{3x^2y^2}$

$M'\,dx + N'\,dy = 0$

$\left[\frac{y}{3x^2y^2} + \frac{2xy^2}{3x^2y^2}\right]dx + \left[\frac{x}{3x^2y^2} - \frac{x^2y}{3x^2y^2}\right]dy = 0$

$\left[\frac{1}{3x^2y} + \frac{2}{3x}\right]dx + \left[\frac{1}{3xy^2} - \frac{1}{3y}\right]dy = 0$

$\therefore$ It's solution is given by

$\int M\,dx + \int(\text{term in N free from } x)\,dy = C$

$\int\left[\frac{1}{3x^2y} + \frac{2}{3x}\right]dx + \int\left(-\frac{1}{3y}\right)dy = C$

$-\frac{1}{3xy} + \frac{2}{3}\log x - \frac{1}{3}\log y = C$

$\boxed{\frac{1}{3}\left(-\frac{1}{xy} + 2\log x - \log y\right) = C}$

$-\frac{1}{xy} + \log x^2 - \log y = C$

$-\frac{1}{xy} + \log\left(\frac{x^2}{y}\right) = C$

Imp

(5) $(xy\sin xy + \cos xy)\,y\,dx + (xy\sin xy - \cos xy)\,x\,dy = 0$

Comparing with $M\,dx + N\,dy = 0$

$M = (xy\sin xy + \cos xy)y$

$= xy^2\sin xy) + y\cos xy$

$\dfrac{\partial M}{\partial y} = x\left[y^2(\cos xy)x + 2y\sin xy\right] + \left[y(-\sin xy)x + \cos xy\right]$

$= x^2y^2\cos xy + 2xy\sin xy - xy\sin xy + \cos xy$

$\boxed{\dfrac{\partial M}{\partial y} = x^2y^2\cos xy + xy\sin xy + \cos xy}$

$N = (xy\sin xy - \cos xy)x$

$= x^2y\sin xy - x\cos xy$

$= y[x^2\sin xy] - [x\cos xy]$

$\dfrac{\partial N}{\partial x} = y\left[x^2(\cos xy)y + 2(\sin xy)x\right] - \left[x - \sin xy + \cos xy\right]$

$= x^2y^2\cos xy + 2xy\sin xy + x\sin xy - \cos xy$

$\boxed{\dfrac{\partial N}{\partial x} = x^2y^2\cos xy + 3xy\sin xy - \cos xy}$

$\boxed{\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}}$ D.E is not exact

Consider

$(Mx - Ny) = x^2y^2\sin xy + xy\cos xy - x^2y^2\sin xy + xy\cos xy$

$= 2xy\cos xy$

$I.F = \dfrac{1}{2xy\cos xy}$

on multiply the given D.E

$M' = M \times I.F = \dfrac{(xy\sin xy + \cos xy)}{(2x\cos xy)}$

$N' = N \times I.F = \dfrac{(xy\sin xy - \cos xy)}{(2y\cos xy)}$

$M'dx + N'dy = 0$

$\left(\dfrac{xy\sin xy + \cos xy}{2x\cos xy} + \dfrac{xy\sin xy - \cos xy}{2y\cos xy}\right) = 0$

$\therefore$ It's solution of given D.E

$\int M'dx + \int(\text{term in } N' \text{ free from } x)\,dy = C$

$\int\left(\dfrac{xy\sin xy}{x\cos xy} + \dfrac{1}{x}\right)dx + \int -\dfrac{1}{y}\,dy = C$

$(y\tan xy)$

$y\dfrac{-\log(\cos xy)}{y} + \log x - \log y = C$

$\boxed{-\log|\cos(xy)| + \log x - \log y = C}$

OR

$\boxed{\log|\sec(xy)| + \log x - \log y = C}$

$\log(\sec xy) + \log x = \log c + \log y$

$\boxed{x\sec xy = cy}$

(6) $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$

Comparing with $M\,dx + N\,dy = 0$

$M = x^2y - 2xy^2$ $\qquad$ $N = -(x^3 - 3x^2y)$

$\frac{\partial M}{\partial y} = x^2 - 4xy$ $\qquad$ $\frac{\partial N}{\partial x} = -(3x^2 - 6xy)$

$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$ $\quad$ D.E is not exact

Consider

$(Mx + Ny) = \cancel{x^3y} - 2x^2y^2 - \cancel{x^3y} + 3x^2y^2$

$= x^2y^2 \neq 0$

Given D.E is homogeneous D.E

$I.F = \frac{1}{x^2y^2}$

on multiply in given D.E

$M' = M \times I.F = \frac{(x^2y - 2xy^2)}{x^2y^2} = \left(\frac{1}{y} - 2/x\right)$

$N' = N \times I.F = -\frac{(x^3 - 3x^2y)}{x^2y^2} = -\left(\frac{x}{y^2} - \frac{3}{y}\right)$

$\frac{\partial M'}{\partial y} = -\frac{1}{y^2}$ $\qquad$ $\frac{\partial N'}{\partial x} = -\frac{1}{y^2}$

$\boxed{\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}}$ $\quad$ D.E is exact

$\therefore$ It's solution is given by

$\int M\,dx + \int N\,dy = C$ (term N free from x)

$\int\left(\frac{1}{y} - \frac{2}{x}\right)dx + \int \frac{3}{y}\,dy = c$

$\boxed{\frac{x}{y} - 2\log x + 3\log y = c}$

$f(x,y) = f(xt, yt) \Rightarrow t\, f(x,y)$ This is homogeneous

Imp

(7) $(x^2+y^2)\,dx - (x^2+xy)\,dy = 0$

$\Rightarrow$ Comparing the given equation with $M dx + N dy = 0$ we get,

$M = x^2+y^2$ $\qquad N = -(x^2+xy)$

$\frac{\partial M}{\partial y} = 2y$ $\qquad \frac{\partial N}{\partial x} = -(2x+y)$

Here $\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$ The given D.E is not exact

Since the equation is homogeneous

$IF = \frac{1}{Mx+Ny} = \frac{1}{(x^2+y^2)x - (x^2+xy)y} = \frac{1}{x^3+xy^2-x^2y-xy^2}$

$IF = \frac{1}{x^3-x^2y}$ or $\frac{1}{x^2(x-y)}$

multiply the equation by the If

$\frac{x^2+y^2}{x^2(x-y)}\,dx - \frac{(x^2+xy)}{x^2(x-y)}\,dy = 0$ $\qquad -\frac{1}{x} + \frac{1}{y} + \frac{y}{x}$

This is exact

$M' = \frac{x^2+y^2}{x^2(x-y)}$ $\qquad N' = -\frac{(x^2+xy)}{x^2(x-y)} = -\frac{x^2}{x^2(x-y)} + \frac{y}{x(x-y)}$

The solution of this exact D.E is

$\int M'\,dx + \int (\text{term in } N' \text{ free form } x)\,dy = c$

$\int \frac{1}{x-y} + \frac{y^2}{x^2(x-y)}\,dx + \int \frac{1}{y}\,dy = c$

$\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$

$\int \frac{f'(x)}{f(x)} = 9$

$\int \frac{1}{x-y}\,dx + y^2 \int \frac{1}{x^2} \cdot \frac{1}{(x-y)}\,dx \; \cancel{\log y} = c$

$\int \frac{1}{(x-y)}\,dx + (-y^2)\int \left(\frac{-1/x^2}{(x-y)}\right) dx$

$\log(x-y) + (-y^2)\left[\log|(x-y)|\right] \; \cancel{\log y} = c$

$\log(x-y) - y^2 \log(x-y) \; \cancel{\log y} = c$

$\boxed{(1-y^2)\log(x-y) = c}$

This is the required general solution

$\frac{1}{x-y}$

$\frac{1}{x}$

$y(x^{\circ}y^{\circ})\,dx + x(x^{\circ}y^{\circ} - 3x^2y^2)\,dy = 0$

some power

Imp

(8) $y\,dx + x(1-3x^2y^2)\,dy = 0$

Comparing the given equation with

$\boxed{M\,dx + N\,dy = 0}$

$M = y \qquad\qquad N = x(1-3x^2y^2)$

Here $\dfrac{\partial M}{\partial y} = 1 \qquad\qquad \dfrac{\partial N}{\partial x} = 1 - 9x^2y^2$

$\boxed{\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}}$ $\Rightarrow$ The given DE is not exact

But the given equation is type of

$\boxed{f(xy)\,x\,dy + g(xy)\,y\,dx = 0}$

$IF = \dfrac{1}{Mx - Ny} = \dfrac{1}{\cancel{xy} - \cancel{xy} + 3x^3y^3} = \dfrac{1}{3x^3y^3}$

multiply the equation by the IF

$\boxed{\dfrac{y}{3x^3y^3}\,dx + \dfrac{x(1-3x^2y^2)}{3x^3y^3}\,dy = 0}$

$M' = \dfrac{1}{3x^3y^2} \qquad\qquad N' = \dfrac{x - 3x^2y^2}{3x^3y^3}$

$= \dfrac{1}{3x^2y^3} - \dfrac{1}{y}$

$\boxed{\dfrac{\partial M'}{\partial y} = \dfrac{\partial N'}{\partial x}}$ This is exact.

Solution of this exact differential equation.

$\int M'\,dx + \int(\text{term in } N' \text{ free from } x)\,dy = c$

x^{-3}

$\int \dfrac{1}{3x^3y^2}\,dx + \int -\dfrac{1}{y}\,dy = c$

$\dfrac{x^{-2}}{-2}$

$\dfrac{1}{-3\times 2}\,\dfrac{1}{x^2y^2} - \log y = c$

$\boxed{-\dfrac{1}{6x^2y^2} - \log y = c}$ This is required general solution

Date
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(a) $(y-2x^3)\,dx - x(1-xy)\,dy = 0$

$\Rightarrow$ Comparing the given eq^n with $\boxed{Mdx + Ndy = 0}$

$M = y - 2x^3$ $\qquad\qquad$ $N = -x(1-xy)$

$\frac{\partial M}{\partial y} = 1$ $\qquad\qquad$ $\frac{\partial N}{\partial x} = -1 + 2xy$

Here $\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$ The given D.E is not exact

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{1-(-1+2xy)}{-x(1-xy)} = \frac{1+1-2xy}{-x(1-xy)}$$

$$= \frac{2-2xy}{-x(1-xy)} = \frac{2(1-xy)}{-x(1-xy)} = -\frac{2}{x}$$

$$= f(x)$$

$$IF = e^{\int f(x)dx}$$

$$= e^{-\int \frac{2}{x}dx} = e^{-2\int \frac{1}{x}dx} = e^{-2\log x} = e^{\log_e (x^{-2})}$$

$$\boxed{IF = x^{-2} = 1/x^2}$$

on multiply the eq^n by the IF $\qquad -\frac{1}{x} + \frac{xy}{x}$

$$\boxed{\frac{(y-2x^3)}{x^2}\,dx - \frac{x}{x^2}(1-xy)\,dy = 0}$$

$M' = \left(\frac{y}{x^2} - 2x\right)$ $\qquad\qquad$ $N' = -\frac{1}{x} + y$

$\boxed{\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}}$ This is exact

Solution of exact D.E is

$\int M' dx + \int (N' \text{ term in free from } x)\,dy = C$

$\int \left(\frac{y}{x^2} - 2x\right) dx + \int y\,dy = C$ $\qquad \frac{x^{-2+1}}{-1}$

$-\frac{y}{x} - x^2 + \frac{y^2}{2} = C$

$-2y - 2x^3 + xy^2 = 2Cx$

$\boxed{xy^2 - 2y - 2x^3 = 2Cx}$ or $\boxed{xy^2 - 2y - 2x^3 = C_1x}$

This is required general solution.

(10) $x\sin x\,dy + (xy\cos x - y\sin x - 2)\,dx = 0$

$\Rightarrow$ Comparing the equation with as $\boxed{Mdx + Ndy = 0}$

$N = x\sin x$ $\qquad$ $M = xy\cos x - y\sin x - 2$

$\frac{\partial N}{\partial x} = x\cos x + \sin x$ $\qquad$ $\frac{\partial M}{\partial y} = x\cos x - \sin x$

$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$ the given D.E is not exact

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{x\cancel{\cos x} - \sin x - x\cancel{\cos x} - \sin x}{x\sin x}$$

$$= -\frac{2\cancel{\sin x}}{x\cancel{\sin x}} = -\frac{2}{x} = f(x)$$

$$IF = e^{\int f(x)\,dx}$$

$$= e^{\int -2/x\,dx} = e^{-2\int 1/x\,dx} = e^{-2\log x}$$

$$I.F = e^{\log_e (x^{-2})} = x^{-2} = 1/x^2$$

multiply the equation by IF

$$\frac{(xy\cos x - y\sin x - 2)\,dx}{x^2} + \frac{x\sin x\,dy}{x^2} = 0$$

$$\left(\frac{y\cos x}{x} - \frac{y}{x^2}\sin x - \frac{2}{x^2}\right)dx + \frac{\sin x}{x}\,dy = 0$$

$M' = \frac{y}{x}\cos x - \frac{y}{x^2}\sin x - \frac{2}{x^2}$ $\qquad$ $N' = \frac{\sin x}{x}$

$\boxed{\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}}$ This is exact

$\int$(term in M is free from y) dx + $\int N\,dy = 0$

The solution of this exact D.E

$\int$(term in M is free from y) dx + $\int N\,dy = c$

$$\int -\frac{2}{x^2}\,dx + \int \frac{\sin x}{x}\,dy = c$$

$$-2\frac{x^{-2+1}}{-2+1} + \frac{y\sin x}{x} = c$$

$$\frac{2}{x} + \frac{y}{x}\sin x = c$$

$$\boxed{\frac{y}{x}\sin x + \frac{2}{x} = c}$$

This is the required General solutio

***(11) $(2xy^4 e^y + 2xy^3 + y)\,dx + (x^2y^4 e^y - x^2y^2 - 3x)\,dy = 0$

$\Rightarrow$ Comparing the given eq^n with as $\boxed{Mdx + Ndy = 0}$

$M = 2xy^4 e^y + 2xy^3 + y$ $\qquad$ $N = x^2y^4e^y - x^2y^2 - 3x$

$$\frac{\partial M}{\partial y} = 2x[y^4e^y + e^y 4y^3] + 6xy^2 + 1 \qquad \frac{\partial N}{\partial x} = 2xy^4e^y - 2xy^2 - 3$$

$$= 2xy^4e^y + 8xe^y y^3 + 6xy^2 + 1$$

$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$ The Given D.E is not exact

$$\cancel{\frac{-\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x}}{M} = 2xy^4e^y + 8e^y xy^3 + 6y^2x + 1 - 2xy^4e^y + 2xy^2 + 3}$$

$$\cancel{= \frac{8xe^y y^3 + 8xy^2 + 4}{x^2y^4e^y - x^2y^2 - 3x}}$$

$e^{\int \frac{dx}{\sin 2x}}$

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{2xy^4e^y - 2xy^2 - 3 - 2xy^4e^y - 8xe^y y^3 - 6xy^2 - 1}{2xy^4e^y + 2xy^3 + y}$$

$$= \frac{-8xe^y y^3 - 8xy^2 - 4}{2xy^4e^y + 2xy^3 + y}$$

$$= \frac{-4}{y}\left(\frac{2xe^y y^3 + 2xy^2 + 1}{2xe^y y^3 + 2xy^2 + 1}\right) = f(y)$$

$$IF = e^{\int f(y)\,dy} = e^{\int -4/y\,dy} = e^{-4\int \frac{1}{y}dy}$$

$$= e^{-4\log y} = y^{-4} = \frac{1}{y^4}$$

multiply the equation by IF

$$\frac{(2xy^4e^y + 2xy^3 + y)}{y^4}\,dx + \frac{(x^2y^4e^y - x^2y^2 - 3x)}{y^4}\,dy = 0$$

$$\left(2xe^y + \frac{2x}{y} + \frac{1}{y^3}\right)dx + \left(x^2e^y - \frac{x^2}{y^2} - \frac{3x}{y^4}\right)dy = 0$$

$M' = 2xe^y + \frac{2x}{y} + \frac{1}{y^3}$ $\qquad$ $N' = x^2e^y - \frac{x^2}{y^2} - \frac{3x}{y^4}$

$\boxed{\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}}$ This is exact

The solution of this exact D.E is

$$\int M'dx + \int(\text{term in } N' \text{ is free from } x)\,dy = C$$

$$\int\left(2xe^y + \frac{2x}{y} + \frac{1}{y^3}\right)dx + \int 0\,dy = C$$

$$\boxed{x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} = C}$$

This is the required General solution

$$\cancel{= \frac{2xy^4e^y - 8xy^3 - 3 - 2xy^4e^y + 8xe^y y^3 + 6xy^2 + 1}{2xy^4e^y + 8xe^y y^3 + 6x^2y^2 + 1}}$$

$$\cancel{= \frac{-4(2xy^3}{y^4\ /2} \quad ((4\ y/x + 1))}$$

(2)

(12) $(3xy^2 - y^3)\,dx + (xy^2 - 2x^2y)\,dy = 0$

Comparing the eqⁿ with as $\boxed{M\,dx + N\,dy = 0}$

$M = 3xy^2 - y^3$ $\qquad$ $N = xy^2 - 2x^2y$

$\frac{\partial M}{\partial y} = 6xy - 3y^2$ $\qquad$ $\frac{\partial N}{\partial x} = y^2 - 4xy$

$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$ The given D.E is not exact

since the eqⁿ is homogeneous

$IF = \frac{1}{Mx + Ny} = \frac{1}{3x^2y^2 - xy^3 + xy^3 - 2x^2y^2}$

$IF = \frac{1}{x^2y^2}$

multiply the eqⁿ by IF

$\frac{3xy^2 - y^3}{x^2y^2}\,dx + \frac{(xy^2 - 2x^2y)}{x^2y^2}\,dy = 0$

$\left(\frac{3}{x} - \frac{y}{x^2}\right)dx + \left(\frac{1}{x} - \frac{2}{y}\right)dy = 0$

$M' = \frac{3}{x} - \frac{y}{x^2}$ $\qquad$ $N' = \frac{1}{x} - \frac{2}{y}$

$\boxed{\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}}$ This ~~eq~~ is exact

Solution of exact D.E is

$\int M'\,dx + \int (\text{term in } N' \text{ free from } x)\,dy = C_1$

$\int \left(\frac{3}{x} - \frac{y}{x^2}\right)dx + \int -\frac{2}{y}\,dy = C_1$

$3\log x + \frac{y}{x} + (-2)\log y = C_1$

$\log x^3 + \frac{y}{x} - 2\log y = \log c$

$\frac{y}{x} + \log x^3 - \log y^{+2} = \log c$

$y/x + \log\left(\frac{x^3}{y^2}\right) = \log c$

$\frac{y}{x} = \log c - \log(x^3/y^2)$

$= \log c + \log\left(\frac{y^2}{x^3}\right)$

$\frac{y}{x} = \log\left(\frac{cy^2}{x^3}\right)$

$e^{y/x} = \frac{cy^2}{x^3}$

$\boxed{\frac{cy^2}{x^3} = e^{\frac{y}{x}}}$

This is the required general solution

(3)

Definition: $\boxed{\text{Linear D.E of } 1^{st} \text{ order } 1^{st} \text{ degree}}$:

(i) DE of the form $\boxed{\frac{dy}{dx} + Py = Q}$ where P, Q are function of x-alone or constants

is called linear D.E in y

$\Rightarrow$ It's solution is given by

$\boxed{y(I.F) = \int Q(IF)\,dx + C}$ where $IF = e^{\int P dx}$

(ii) D.E of the form $\boxed{\frac{dx}{dy} + Px = Q}$ where P, Q are function of y - alone or constants

is called linear D.E in x

$\Rightarrow$ It's solution is given by

$\boxed{x(IF) = \int Q(IF)\,dy + C}$ where $IF = e^{\int P dy}$

eg (1) $x\cos x\left(\frac{dy}{dx}\right) + y(x\sin x + \cos x) = 1$

$$\frac{dy}{dx} + \left(y\tan x + \frac{1}{x}\right) = \frac{1}{x\cos x}$$

$$\frac{dy}{dx} + \left(y\tan x + \frac{1}{x}\right) = \frac{\sec x}{x}$$

$$\frac{dy}{dx} + \left(\tan x + \frac{1}{x}\right)y = \frac{\sec x}{x}$$

This is of the form $\frac{dy}{dx} + Py = Q$

$P = \tan x + 1/x$ $\qquad Q = \frac{\sec x}{x}$

$IF = e^{\int P dx} = e^{\int \tan x + \frac{1}{x}\, dx}$

$= e^{(\log_e \sec x + \log x)} = e^{\log_e \sec x}\, e^{\log_e x}$

$IF = x\sec x$

The solution of this L.D.E is

$y(IF) = \int Q(IF)\,dx + C$

$y(x\sec x) = \int \frac{\sec x}{\cancel{x}} \times \cancel{x}\sec x + C$

$xy\sec x = \int \sec^2 x + C$

$$\boxed{xy\sec x = \tan x + C}$$

This is the required General solution.

I. 3

(4)

Reducible linear differential equation :

Type I D.E of the form $\boxed{\frac{dy}{dx} + Py = Q\,y^n}$ where P, Q function of x-alone OR Constant

is known as Bernaulli's D.E in y

- To reduce into linear DE

Step 1 : divide throughout by y^n

$$\frac{1}{y^n}\frac{dy}{dx} + \frac{P}{y^{n-1}} = Q \quad \text{----(1)}$$

$\frac{y}{y^n}$

$\frac{1}{y^{n-1}}$

Step 2 : put $\frac{1}{y^{n-1}} = t$

on differentiating w.r.t x

$$-(n-1)\frac{1}{y^n}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{1}{y^n}\frac{dy}{dx} = \frac{-1}{(n-1)}\frac{dt}{dx}$$

$\therefore$ eq^n(1) becomes $\frac{-1}{(n-1)}\frac{dt}{dx} + Pt = Q$

i.e $\boxed{\frac{dt}{dx} + P't = Q'}$ which is linear in $\boxed{t}$

independent variable of x

(4)

Type II D.E of the form $\boxed{\frac{dx}{dy} + Px = Q\,x^n}$ where P, Q func of y-alone or Constant

is known Bernaulli's D.E in x

- To reduce into linear DE

Step 1 : divide throughout by x^n

$$\frac{1}{x^n}\frac{dx}{dy} + \frac{P}{x^{n-1}} = Q \quad \text{----(1)}$$

Step 2 : put $\frac{1}{x^{n-1}} = t$

on differentiating w.r.t y

$$-(n-1)\frac{1}{x^n}\frac{dx}{dy} = \frac{dt}{dy}$$

$$\frac{1}{x^n}\frac{dx}{dy} = -\frac{1}{n-1}\frac{dt}{dy}$$

$\therefore$ eq^n(1) becomes $\frac{-1}{(n-1)}\frac{dt}{dy} + Pt = Q$

i.e $\boxed{\frac{dt}{dy} + P't = Q'}$ which is linear in $\boxed{t}$

independent of y

*** (1) $\frac{dy}{dx} = x^3y^3 - xy$

$\Rightarrow$ Given that $\frac{dy}{dx} + xy = x^3 y^3$. This is Bernaulli's equation

dividing by y^3

$$\boxed{\frac{1}{y^3}\frac{dy}{dx} + \frac{x}{y^2} = x^3} \quad \text{-----(1)}$$

let $\frac{1}{y^2} = t$

$$\frac{-2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{1}{y^3}\frac{dy}{dx} = -\frac{1}{2}\frac{dt}{dx}$$

Substituting above in the equation.

$dy/dx + Py = Q$

$$-\frac{1}{2}\frac{dt}{dx} + xt = x^3 \qquad \frac{dt}{dx} + (-2x)t = -2x^3$$

This is in the form of $\boxed{\frac{dt}{dx} + Pt = Q}$

$P = -2x \qquad Q = -2x^3$

$$IF = e^{\int P dx} = e^{\int -2x dx} = e^{-2\int x dx} = e^{-\frac{2x^2}{2}}$$

$\boxed{IF = e^{-x^2}}$

$\therefore$ Solution of L.D.E is

$$t(IF) = \int Q(IF)\,dx + c$$

$$t\,e^{-x^2} = \int e^{-x^2}(-2x^3)\,dx + c$$

LIATE

$$\boxed{\int uv\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx}$$

$$t\,e^{-x^2} = -\int e^{-x^2}(2x)\,x^2\,dx + c$$

put $x^2 = u$

$2x\,dx = du$

$$t\,e^{-u} = -\int u.e^{-u}\,du + c$$

$$= -\left[u\int e^{-u}du - \int\left(\frac{du}{du}\int e^{-u}du\right)du\right] + c$$

$$= -\left[u\,e^{-u}(-1) - \int (1)(e^{-u})(-1)\,du\right] + c$$

$$= -\left[-u e^{-u} + \int e^{-u}du\right] + c$$

$$t\,e^{-u} = -\left[-u e^{-u} - e^{-u}\right] + c$$

$$t\,e^{-x^2} = u e^{-u} + e^{-u} + c$$

$$t\,e^{-x^2} = x^2 e^{-x^2} + e^{-x^2} + c$$

$$t\,e^{-x^2} = e^{-x^2}(x^2+1) + c$$

$$t = x^2+1 + \frac{c}{e^{-x^2}}$$

$$t = (x^2+1) + ce^{x^2}$$

$$\boxed{\frac{1}{y^2} = x^2 + 1 + ce^{x^2}}$$

This is the required general Solution

*** (2) $y\,dx + x(1-3x^2y^2)\,dy = 0$

$\Rightarrow$ rearranging given equation

$$y\,dx = -x(1-3x^2y^2)\,dy$$

$$y\frac{dx}{dy} = -x(1-3x^2y^2)$$

$$y\frac{dx}{dy} = -x + 3x^3y^2$$

$$\frac{dx}{dy} = -\frac{x}{y} + 3x^3y$$

$$\boxed{\frac{dx}{dy} + \left(\frac{1}{y}\right)x = 3yx^3}$$

dividing by x^3

$$\boxed{\frac{1}{x^3}\frac{dx}{dy} + \frac{1}{y}\left(\frac{1}{x^2}\right) = 3y} \quad \text{----(1)}$$

let $\boxed{\frac{1}{x^2} = t}$

$$-\frac{2}{x^3}\frac{dx}{dy} = \frac{dt}{dy}$$

$$\frac{1}{x^3}\frac{dx}{dy} = -\frac{1}{2}\frac{dt}{dy}$$

Substituting above the equation

$$-\frac{1}{2}\frac{dt}{dy} + \frac{t}{y} = 3y$$

$$\frac{dt}{dy} + \left(-\frac{2}{y}\right)t = -6y$$

This is in form of $\boxed{\frac{dt}{dy} + Pt = Q}$

$P = -2/y \qquad Q = -6y$

$$IF = e^{\int P\,dy} = e^{\int 2/y\,dy} = e^{-2\int \frac{1}{y}dy}$$

$$IF = e^{-2\log y} = e^{\log_e (y)^{-2}}$$

$$IF = y^{-2} = 1/y^2$$

$\therefore$ solution of L.D.E is

$$t(IF) = \int Q(IF)\,dy + c$$

$$t\,\frac{1}{y^2} = -\int 6y \cdot \frac{1}{y^2}\,dy + c$$

$$\frac{1}{x^2}\frac{1}{y^2} = -6\int 1/y\,dy + c$$

$$\boxed{\frac{1}{x^2y^2} = -6\log y + c}$$

This is the required General solution

$\frac{dy}{dx} + Py = Qy^n$

(3) $\frac{dy}{dx} + y\cos x = y^4 \sin 2x$

$\Rightarrow$ dividing by y^4, we get

$$\frac{1}{y^4}\frac{dy}{dx} + \cos x \frac{1}{y^3} = \sin 2x$$

let $\frac{1}{y^3} = t$

$$-\frac{3}{y^4}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{1}{y^3}\frac{dy}{dx} = -\frac{1}{3}\frac{dt}{dx}$$

Substituting above the equation

$$-\frac{1}{3}\frac{dt}{dx} + \cos x\, t = \sin 2x$$

$$\frac{dt}{dx} + (-3\cos x)t = -3\sin 2x$$

This is in form of $\frac{dt}{dx} + Pt = Q$

$P = -3\cos x \qquad Q = -3\sin 2x$

$$IF = e^{\int P dx} = e^{\int -3\cos x\, dx} = e^{-3\sin x}$$

$\therefore$ solution of L.D.E is

$$t(IF) = \int Q(IF)\,dx + C$$

$$te^{-3\sin x} = \int -3\sin 2x\; e^{-3\sin x} + c$$

$$te^{-3\sin x} = -3\int 2\sin x \cos x\; e^{-3\sin x}\, dx + c$$

put $-3\sin x = u$

$-3\cos x\, dx = du$

$\cos x\, dx = -\frac{1}{3} du$

$$= \int -\frac{2}{3}\, u \cdot e^{+u}\, du$$

$$= -\frac{2}{3}\int u \cdot e^{+u}\, du$$

$$te^{-3\sin x} = -\frac{2}{3}\left[u\int e^{+u}du - \int\left[\frac{du}{du}\int e^{+u}du\right]du\right] + c$$

$$= -\frac{2}{3}\left[u + e^{+u} - \int + e^{+u}\, du\right] + c$$

$$= -\frac{2}{3}\left[+ue^{+u} - \int e^{+u}\, du\right] + c$$

$$= -\frac{2}{3}\left[+ue^{+u} - e^{+u}\right] + c$$

$$te^{-3\sin x} = -\frac{2}{3}ue^{+u} + \frac{2}{3}e^{+u} + C$$

$$\frac{1}{y^3}e^{-3\sin x} = \frac{2}{3}\left[+3\sin x\, e^{-3\sin x} + e^{-3\sin x}\right] + c$$

$$\frac{1}{y^3}e^{-3\sin x} = +\frac{2}{3}(e^{-3\sin x})\left[3\sin x + 1\right] + c$$

$$\boxed{y^{-3} = \frac{2}{3}\left[1 + 3\sin x\right] + Ce^{3\sin x}}$$

This is the required General Solution

$$\frac{dy}{dx} + Py = Qy^n \qquad IF = e^{\int P\,dx}$$

(2) $$\frac{dy}{dx} + y = y^2(\cos x - \sin x)$$

~~Given DE of the form~~ $\frac{dy}{dx} + Py = Qy^n$ i.e Bernoulli's DE $y^n(y)$

⇒ dividing by y^2

$$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{y} = \cos x - \sin x$$

let $\frac{1}{y} = t$

$$-\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{1}{y^2}\frac{dy}{dx} = -\frac{dt}{dx}$$

Substituting above the equation

$$-\frac{dt}{dx} + t = \cos x - \sin x$$

$$\frac{dt}{dx} - t = \sin x - \cos x$$

This is in from of $\frac{dt}{dx} + Pt = Q$

$P = (-1)$ $\qquad Q = \sin x - \cos x$

$$IF = e^{\int -1\,dx} = e^{-x}$$

∴ The Solution of L.D.E is

$$t\,(IF) = \int Q\,(IF)\,dx$$

$$t\,e^{-x} = \int (\sin x - \cos x)\,e^{-x}\,dx$$

$$t\,e^{-x} = \int \sin x \cdot e^{-x} - \int \cos x\, e^{-x}\,dx$$

or

this solve by integration formula or U.V rule

1st keep as it is and 2nd used U.V rule

$\frac{dx}{dy} + Px = Qx^n$

P — function of y or constant

(3) $xy(1+xy^2)\frac{dy}{dx} = 1$

$\Rightarrow$ rearranging the given equation

$\frac{1}{xy(1+xy^2)} \frac{dx}{dy} = \frac{1}{1}$

$\frac{dx}{dy} = xy(1+xy^2)$

$\frac{dx}{dy} = xy + x^2y^3$

$\frac{dx}{dy} - xy = x^2y^3$

$\frac{dx}{dy} - (y)x = (y^3)x^2$

Given D.E of the form $\frac{dx}{dy} + Px = Qx^n$

i.e Bernaulli's DE in $\boxed{x}$

dividing by x^2

$\boxed{\frac{1}{x^2}\frac{dx}{dy} - \frac{y}{x} = y^3}$ ---(1)

let $-\frac{1}{x} = t$

$\frac{1}{x^2}\frac{dx}{dy} = \frac{dt}{dy}$

Substituting the above equation

$\int e^{f(x)} f'(x)\, dx = f(x)$

$\frac{dt}{dy} + yt = y^3$

This is in form of $\frac{dt}{dy} + Pt = Q$

$P = y \qquad Q = y^3$

$I.F = e^{\int P dy} = e^{\int y dy} = e^{y^2/2}$

$\therefore$ The solution of L.D.E is

$t(IF) = \int Q(IF)\, dy + C$

$t(IF) = \int y^3 e^{y^2/2}\, dy + C$

$-\frac{1}{x} e^{y^2/2} = 2\int y \cdot \frac{y^2}{2} e^{y^2/2}\, dy + C$

put $\frac{y^2}{2} = u$

$\frac{2y}{2}\, dy = du$

$= 2\int u \cdot e^u\, du + C$

$= 2\left[u \int e^u du - \int\left(\frac{d(u)}{du}\int e^u du\right) du\right] + C$

$= 2\left[u e^u - \int (1) e^u\, du\right] + C$

$-\frac{1}{x} e^{y^2/2} = 2\left[u e^u - e^u\right] + C$

$-\frac{1}{x} e^{y^2/2} = 2\left[\frac{y^2}{2} e^{y^2/2} - e^{y^2/2}\right] + C$

$-\frac{1}{x} e^{y^2/2} = y^2 e^{y^2/2} - 2e^{y^2/2} + C$

$-\frac{1}{x} = \left[y^2 - 2\right] + \frac{C}{e^{y^2/2}}$

$\boxed{\frac{1}{x} = \left[2 - y^2\right] + Ce^{-y^2/2}}$ > This is constant either -ve or +ve (coz both n...)

This is the required General solution

(4) $y^4\,dx = \left(x^{-3/4} - y^3 x\right) dy$

$\Rightarrow$ rearranging the given D.E

$$\frac{dx}{dy} = \frac{\left(x^{-3/4} - y^3 x\right)}{y^4}$$

$$\frac{dx}{dy} = \frac{x^{-3/4}}{y^4} - \frac{1}{y}x$$

$$\frac{dx}{dy} + \left(\frac{1}{y}\right)x = \left(\frac{1}{y^4}\right)x^{-3/4}$$

Given D.E is in form of $\frac{dx}{dy} + Px = Qx^n$ ~~divided~~

i.e Bernoulli's D.E in $\boxed{x}$

dividing by $x^{-3/4}$ throughout the eq^n

$x^{3/4-1}$

$x^{-1/4}$

$x^{1/4}$

$\frac{1}{4}x^{-3/4}$

$$\frac{1}{x^{-3/4}}\frac{dx}{dy} + \frac{1}{y}\frac{x}{x^{-3/4}} = \frac{1}{y^4}$$

$$\frac{1}{x^{-3/4}}\frac{dx}{dy} + \frac{1}{y}\frac{1}{x^{-7/4}} = \frac{1}{y^4}$$

let $\frac{1}{x^{-7/4}} = t$

$$\frac{7/4}{x^{3/4}}\frac{dx}{dy} = \frac{dt}{dy}$$

$$\frac{1}{x^{3/4}}\frac{dx}{dy} = \frac{4}{7}\frac{dt}{dy}$$

Substituting in above equation

$$\frac{4}{7}\frac{dt}{dy} + \frac{1}{y}t = \frac{1}{y^4}$$

$$\frac{dt}{dy} + \frac{7}{4y}t = \frac{7}{4y^4}$$

This is in form of $\frac{dt}{dy} + Pt = Q$

$P = \frac{7}{4y}$ $\qquad Q = \frac{7}{4y^4}$

$$IF = e^{\int \frac{7}{4y}dy} = e^{\frac{7}{4}\int \frac{1}{y}dy} = e^{\frac{7}{4}\log y}$$

$$IF = e^{\log_e (y)^{7/4}}$$

$$IF = y^{7/4}$$

$\therefore$ The solution of L.D.E is

$$t\,(IF) = \int Q\,(IF)\,dy + C$$

$$= \int \cancel{\tfrac{7}{4y^4}}$$

$$t\,y^{7/4} = \int \frac{7}{4}\frac{1}{y^4} \times y^{7/4} + C$$

$$= \frac{7}{4}\int y^{7/4}\,y^{-4} + C$$

$$= \frac{7}{4}\int y^{(7/4 - 4)} + C$$

$$= \frac{7}{4}\int y^{\left(\frac{7-16}{4}\right)} + C$$

$$= \frac{7}{4}\int y^{-9/4} + C$$

$$= \frac{7}{4}\left(\frac{y^{-9/4+1}}{-9/4+1}\right) + C$$

$$= \frac{7}{4}\,\frac{y^{-5/4}}{-5/4}$$

$$t\,y^{7/4} = -\frac{7}{5}y^{-5/4} + C$$

$$\frac{y^{+7/4}}{x^{-7/4}} = -\frac{7}{5}y^{-5/4} + C$$

$$\boxed{x^{7/4}\,y^{7/4} = -\frac{7}{5}\,y^{-5/4} + C}$$

This is the required General solution.

(5) $xy(1+x^2y^2)\frac{dy}{dx} = 1$

rearranging the Given DE

$$\frac{1}{(xy+x^3y^3)}\frac{dx}{dy} = 1$$

$$\frac{dx}{dy} = xy + x^3y^3$$

$$\frac{dx}{dy} - (y)x = (y^3)\,x^3$$

Given D.E is in the form of $\frac{dx}{dy} + Px = Qx^n$

i.e Bernaulli's D.E in $\boxed{x}$

dividing by x^3

$$\frac{1}{x^3}\frac{dx}{dy} - \frac{y}{x^2} = y^3$$

let $-\frac{1}{x^2} = t$

$$\frac{+2}{x^3}\frac{dx}{dy} = \frac{dt}{dy}$$

x^{-2}

x^{-2-1}

$2/x^3$

substituting the above equation

$$\boxed{\frac{1}{2}\frac{dt}{dy} + yt = y^3}$$

Note Common mistake multiply 2 ~~dividing~~ me

$$\frac{dt}{dy} + (2y)t = 2y^3$$

→ R.H.S me nhi kar rona hai sab Question me

$P = 2y$ $\quad Q = 2y^3$

$$IF = e^{\int P dy} = e^{\int 2y dy} = e^{y^2}$$

∴ The solution of L.D.E is

$$t(IF) = \int Q\,(IF)\,dy + C$$

$$t\,e^{y^2} = \int 2y^3 e^{y^2} dy + C$$

$$= \cancel{\tfrac{2}{2}}\int y^2\, 2y\, e^{y^2} dy + C$$

put $y^2 = u$

$2y\,dy = du$

$$t\,e^{y^2} = \cancel{\tfrac{2}{2}}\int u.e^u\,du + C$$

$$-\frac{1}{x^2}e^{y^2} = \cancel{\tfrac{2}{2}}\left[u.e^u - e^u\right] + C$$

$$-\frac{1}{x^2}e^{y^2} = \cancel{\tfrac{2}{2}}\left[y^2e^{y^2} - e^{y^2}\right] + C$$

$$-\frac{1}{x^2} = \cancel{\tfrac{2}{2}}\left[y^2 - 1\right] + \frac{C}{e^{y^2}}$$

$$\frac{1}{x^2} = -[y-1] + Ce^{-y^2}$$

$$\frac{1}{x^2} = -y + 1 + Ce^{-y^2}$$

$$\boxed{\frac{1}{x^2} = Ce^{-y^2} - y + 1}$$

This is the required General Solution

LIATE

(6) $y \frac{dx}{dy} = x - yx^2 \sin y$

$\Rightarrow$ Rerranging the given D.E

$$\frac{dx}{dy} = \frac{x}{y} - x^2 \sin y$$

$$\frac{dx}{dy} - \left(\frac{1}{y}\right)x = (-\sin y)\, x^2$$

Given D.E is in form of $\boxed{\frac{dx}{dy} + Px = Qx^n}$

i.e Bernaulli's D.E of in $\boxed{x}$

dividing by x^2

$$\frac{1}{x^2}\frac{dx}{dy} - \left(\frac{1}{y}\right)\frac{1}{x} = -\sin y$$

let $-\frac{1}{x} = t$

$$\frac{1}{x^2}\frac{dx}{dy} = \frac{dt}{dy}$$

Substituting in above equation

DE is Becomes, $\frac{dt}{dy} + \left(\frac{1}{y}\right)t = -\sin y$

This is in form $\boxed{\frac{dt}{dy} + Pt = Q}$

ie which is linear in t

$P = \frac{1}{y}$ $\quad Q = -\sin y$

$$IF = e^{\int 1/y\, dy} = e^{\log y} = y$$

Substituting in above equation

$\therefore$ The Solution of LDE is

$$t(IF) = \int Q(IF)\, dy + c$$

$$t(y) = \int -\sin y \cdot y + c$$

$$-\frac{y}{x} = -\int y \cdot \sin y + c$$

$$= -\left[y\int \sin y\, dy - \int\left(\frac{d(y)}{dy}\int \sin y\, dy\right)dy\right] + c$$

$$= -\left[y(-\cos y) - \int (1)(-\cos y)\, dy\right] + c$$

$$= -\left[-y\cos y + \int \cos y\, dy\right] + c$$

$$= -\left[-y\cos y + \sin y\right] + c$$

$$\boxed{-\frac{y}{x} = y\cos y - \sin y + c}$$

This is the required General Solution

(7) $\frac{dy}{dx} + \frac{y}{x} = y^2$

=> The Given D.E is $\boxed{\frac{dy}{dx} + Py = Qy^n}$

i.e Bernoulli's D.E in y

dividing by y^2

$$\frac{1}{y^2}\frac{dy}{dx} + \left(\frac{1}{x}\right)\frac{1}{y} = \quad (1)$$

put $\frac{1}{y} = t$

$$-\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{1}{y^2}\frac{dy}{dx} = -\frac{dt}{dx}$$

Substituting in above equation

$$-\frac{dt}{dx} + \left(\frac{1}{x}\right)t = +1$$

$$\frac{dt}{dx} + \left(-\frac{1}{x}\right)t = -1$$

This is in form of $\boxed{\frac{dt}{dx} + Pt = Q}$

i.e which is linear in t

$P = -1/x \qquad Q = -1$

$$IF = e^{\int P dx} = e^{\int -1/x\, dx} = e^{-\int 1/x\, dx}$$

$$IF = e^{-\log x} = e^{\log(x)^{-1}}$$

$$\boxed{IF = \frac{1}{x}}$$

∴ The Solution of L.D.E is

$$t(IF) = \int Q(IF)\,dx + C_1$$

$$\frac{1}{y}\frac{1}{x} = \int -1\,\frac{1}{x}\,dx + C_1$$

$$\frac{1}{xy} = -\int \frac{1}{x}\,dx + C_1$$

$$\frac{1}{xy} = -\log x + C_1$$

$$1/xy = -\log x + \log c$$

$$1/xy = \log c - \log x \quad \because \left\{\log a - \log b = \log(a/b)\right\}$$

$$1/xy = \log(c/x)$$

$$\boxed{1 = xy \log(c/x)}$$

This is the required General solution.

(8) $y(2xy + e^x)\,dx - e^x dy = 0$

$\Rightarrow$ Rerranging the given D.E is

$$y(2xy + e^x) = e^x \frac{dy}{dx}$$

$$\left(\frac{2xy^2}{e^x} + y\right) = \frac{dy}{dx}$$

$$\frac{dy}{dx} - y = \left(\frac{2x}{e^x}\right) y^2$$

The Given D.E is in form $\boxed{\frac{dy}{dx} + Py = Qy^n}$

i.e Bernaulli's D.E in y

dividing by y^2

$$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y} = \left(\frac{2x}{e^x}\right)$$

let $-1/y = t$

$$\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$$

Substituting ~~in~~ in above the equation

$$\frac{dt}{dx} + t = \left(\frac{2x}{e^x}\right)$$

$P = 1$ $\quad$ $Q = 2x/e^x$

$$IF = e^{\int P dx} = e^{\int 1 dx} = e^x$$

$\therefore$ The Solution of L.D.E is

$$t(IF) = \int Q(IF)\,dx + C$$

$$-\frac{1}{y} e^x = \int \frac{2x}{\cancel{e^x}} \cancel{e^x} + C$$

$$e^x = 2\int x\,dx + C$$

$$\boxed{-\frac{1}{y} e^x = \frac{\cancel{2} x^2}{\cancel{2}} + C}$$ Right hai ke nhi nhi pata, Resource me alag ans hai

This is the required General Solution

1.4 # Exponential delay model (5)

let $y(t)$ = amount of radioactive substance present at time t.

$\frac{dy}{dt}$ = rate of change of radioactive substance w.r.t time t

$\boxed{\frac{dy}{dt} \propto y(t)}$ $\boxed{\frac{dy}{dt} = k\,y(t)}$ where k = proportionality constant of depending upon ~~variable~~ various radioactive substance

as time t (↑ increases), $y(t)$ amount of substance decreases

and rate of change is negative i.e (k is negative)

$$\frac{dy}{dt} = k\,y(t)$$

$$\frac{1}{y(t)}\,dy = k\,dt$$ .... Variable separable form

Solution ..

$$\int \frac{1}{y}\,dy = \int k\,dt + c_1$$

$$\log y = kt + c_1$$

$$y = e^{kt+c}$$

$$y = e^{kt}\,e^{c}$$

(1) $\boxed{y = e^{kt}\,c_1}$ $(c_1 = e^{c})$

is amount of radioactive substance at time t

At time $t=0$

Initial amount of substance is is $\boxed{y(t_0)}$ **OR** $\boxed{y_0}$

at $t=0$

$$y(t) = c_1 e^{kt}$$

$$y(0) = c_1 e^{k(0)}$$

$$\boxed{y_0 = c_1}$$

$\frac{dy}{dt} \propto y(t)$

$y(t) = e^{kt}c$

$t_{1/2} = \frac{\log(1/2)}{k}$

Solution become

(2) $\boxed{y(t) = y_0\,e^{kt}}$ amount of radioactive substance a time t

But Knowing initial amount y_0

Note For half life of radioactive substance

$$t \longrightarrow t_{1/2}$$

$$y(t) \longrightarrow \frac{y_0}{2} \longrightarrow \text{initial amount}$$

Solution become

$$y(t) = y_0\,e^{kt}$$

$$\frac{y_0}{2} = y_0\,e^{kt_{1/2}}$$

$$\frac{1}{2} = e^{kt_{1/2}}$$

$$\log(1/2) = k\,t_{1/2}$$

(3) $\boxed{t_{1/2} = \frac{\log(1/2)}{k}}$

is half-life of radioactive substance

Imp Note: ✗ not use $\log(1/2)$ ✓ use $\ln(1/2)$ in calculator most

~~log~~ ~~not log(1/2)~~

$\boxed{y(t) = c_1 e^{kt}}$ ↳ amount of radioactive subs. at time t

$\boxed{y(t) = y_0 e^{kt}}$ ↳ amount of radioactive sub. at time t But knowing initial amount

$\boxed{\frac{\log(1/2)}{k} = t_{1/2}}$ half life

(1) Given an amount of a radioactive substance say 0.5 gm. Find the amount present a any later time t.

OR

⇒ Given

$y(t)$ = amount of radioactive substance at time t

y_0 = initial amount of radioactive substance = 0.5

$$y(t) = y_0 e^{kt}$$

$$\boxed{y(t) = 0.5 e^{kt}}$$

∴ The amount of present at time t.

(2) Half-life of Cobalt-60 is 5.3 years. If an old sample of 10 grams has now decayed to 1 gram, how much time has passed.

y_0 ... $y(t)$

⇒ $y(t)$ = amount of Cobalt-60 at time t

Half life of Cobalt-60 is 5.3 years

$$\boxed{t_{\frac{1}{2}} = 5.3 \text{ years}}$$

$y_0 =$

for half-life

$$\boxed{t_{1/2} = \frac{\log(1/2)}{k}}$$

$$k = \frac{1}{t_{1/2}} \times \log(1/2)$$

$$k = \frac{1}{5.3} \times \log(\tfrac{1}{2}) = \frac{\ln(1/2)}{5.3}$$

$$\boxed{k = -}$$ $= -0.1307$

$t = ?$

Initial/old Sample of 10 gram is decayed 1 gram

y_0 = Initial amount of Cobalt-60
= 10 gram

$y(t)$ = amount of Cobalt-60 at time t
= 1 gram

Solution become

$$\boxed{y(t) = y_0 e^{kt}}$$

$$1 = 10\, e^{(-0.1307)t} \Rightarrow \frac{1}{10} = e^{(-0.1307)t}$$

$$\log\left(\frac{1}{10}\right) = (-0.1307)t$$

$$t = -\frac{\log(1/10)}{0.1307} = \cancel{\frac{\ln(1/10)}{0.1307}}$$

$$\boxed{t = 17.61 \approx 17.6 \text{ years}}$$

17.6 years will required to reduce amount of Cobalt-60 from 10 gram to 1 gram

(2) Half-life of uranium-232 is 68.9 years. How much of a 100-gram sample is present after 250 years

$\Rightarrow$ $y(t)$ = amount of uranium-232 at time t

Half-life of uranium-232 is 68.9 years

$$\boxed{t_{\frac{1}{2}} = 68.9} \text{ years}$$

For Half-life

$$\boxed{t_{\frac{1}{2}} = \frac{\log(1/2)}{k}}$$

$$68.9 = \frac{\log(1/2)}{k}$$

$$\cancel{k = 68.9 \times \log(1/2)}$$

$k =$

$$k = \frac{\log(1/2)}{68.9}$$

$$k = -\frac{0.6931}{68.9}$$

$$\boxed{k = -0.01}$$

Initial amount of uranium-232 is 100-gram

$y_0 = 100$

$t = 250$ years

Solution become .. $y(t) = y_0 e^{kt}$

$$= 100 \times e^{(-0.01 \times 250)}$$

$$y(t) = 100 \times e^{-2.5}$$

$$y(t) = 100 \times 0.0820$$

$$\boxed{y(t) = 8.2}$$

$\therefore$ In 100-gram of sample 8.2 sample is remaining after 250 years

~~Revious~~ pervious Q (2) to (3) Q imp

(3)

(3) A 30kg sample of plutonium - 239 will decay by one kg in 1180 years. What is the half life of plutonium - 239?

$\Rightarrow$ $y(t)$ = ~~sample~~ amount of plutonium - 239 after 1180 year

$\boxed{y(1180) = 29\text{ kg}}$ $\longrightarrow$ matlab 30kg me se 1 decay ho gaya or bacha kitna 29 kg to at time 1180 29 kg bacha

y_0 = initial sample of plutonium

$\boxed{y_0 = 30\text{ kg}}$

$t = (1180)$

Solution become

$y(t) = y_0 e^{kt}$

$\frac{29}{30} = e^{kt}$

$\frac{\log\left(\frac{29}{30}\right)}{1180} = k$

-0.0147

-0.0000124

$(k = ?)$ $t =$

$y(1180) = 30 \times e^{k \times (1180)}$

$29 = 30 \; e^{k \times 1180}$

$\frac{29}{30} = e^{k \times 1180}$

$\log\left(\frac{29}{30}\right) = k \times 1180$

$k = \frac{\log\left(\frac{29}{30}\right)}{1180}$

$t_{1/2} = \frac{\log(1/2)}{k}$

$k = \frac{-0.0147}{1180}$

~~$k = -0.0000124$~~

$\boxed{k = -2.873 \times 10^{-5}}$

for half life of plutonium - 239

$t_{1/2} = \frac{\log(1/2)}{k} = -\frac{\log(1/2)}{0.0000\cancel{1247}} = \frac{\ln(1/2)}{-2.873 \times 10^{-5}}$

$t_{1/2} = \cancel{24140.336}$ 24126.25

$t_{1/2} = 24126.25$ years

24126.25

24140 years half life of plutonium - 239

1.5 Newtoon's law of cooling (6)
(Heating problem)

$T(t)$ — temperature of an object at time t

T_A — temp. of surrounding / ambient temp.

$\frac{dT}{dt}$ — time rate of change of temp. of object

$$\boxed{\frac{dT}{dt} \propto (T - T_A)}$$ --- model of cooling

$$\boxed{\frac{dT}{dt} = K\,(T - T_A)}$$

where

k — proportionality constant

$$\frac{1}{(T-T_A)}\,dT = k\,dt$$ .... variable seperable form

$$\int \frac{1}{(T-T_A)}\,dt = \int k\,dt + c$$

(1) $$\boxed{\log(T-T_A) = kt + c}$$ ----

Imp

(1) A cup of coffee at 190°F is left in room of 70°F. at time $t=0$, the coffee is cooling at 15°F per time minute. find the function that models the cooling of the coffee and solve it. How ~~many~~ much time will it takes the temperature to reach 143°F?

⇒ Let

$T(t)$ = temp. of coffeee a time t

T_A = (temp. surrounding)

$T(t) = 190°F$ at time $t=0$

$T_A = 70°F$

$\therefore \boxed{\dfrac{dT}{dt} = k(T-T_A)}$ .... model of cooling of coffee

$$\frac{dT}{dt} = k(T-70)$$

$$\int \frac{1}{(T-70)} dT = \int k\,dt + C_1$$

$$\log(T-70) = kt + C_1$$

$$= e^{kt+C_1}$$

$$(T-70) = e^{kt} e^{C_1}$$

$$T-70 = e^{kt} C$$

$$T = 70 + e^{kt} \times C$$

$\boxed{T(t) = 70 + e^{kt} C}$ ---- (1)

↳ temp. of coffee at t

at time $t=0$ $T(t) = 190°$

$$T(t) = 70 + e^{kt} \times C$$

$$190 = 70 + e^{kt} \times C$$

$$120 = C \times e^{k(0)}$$

$$\boxed{120 = C.}$$

Substituting C in eq^n (1)

$\boxed{T(t) = 70 + e^{kt} 120}$ ---(2)

temp. of coffee at time t knowing initial temp.

Since coffee is cooling at 15°F per time

$$\frac{dT}{dt} = -15°F$$

$$k(T-T_A) = -15$$

$$k(190-70) = -15$$

$$k(120) = -15$$

$$= \frac{-15}{120}$$

$$\boxed{k = -0.125}$$

$T(t) = 143°F$ then $t = ?$

$$T(t) = 70 + 120\, e^{(-0.125)t}$$

$$143 = 70 + 120\, e^{(-0.125)t}$$

$$73 = 120\, e^{(-0.125)t}$$

$$\ln\left(\frac{73}{120}\right) = -0.125t \qquad t = \frac{\ln\left(\frac{73}{120}\right)}{-0.125}$$

$\boxed{t = 3.97 \text{ minutes}}$

3.97 minutes is required to reach the temperature at 143°F.

(2) The brewing pot temperature of coffee is 180°F and the room temperature is 76°F. After 5 minutes, the temperature of the coffee is 168°F.

(a) **Find** an exponential equation to **represent** this situation.

(b) How long will it take for the coffee to reach a serving temperature of 155°F?

⇒ $T(t)$ = temperature brewing pot of coffee

$T(t) = 180°F$

T_A = room temp. = 76°F

~~After t=5 minutes~~ ~~T(t) = 180°F~~

At time $t=0$ $\quad T(t) = 180°F$

$$\therefore \boxed{\frac{dT}{dt} = k(T - T_A)}$$

$$\frac{dT}{dt} = k(T-76)$$

$$\int \frac{1}{(T-76°)} dT = \int k\,dt + c$$

$$\log(T-76°) = kt + C_1$$

$$(T-76°) = e^{(kt+C_1)}$$

$$T - 76 = e^{kt} \cdot e^{C_1}$$

$$T(t) - 76 = e^{kt}\, C \quad \ldots (1) \qquad e^{C_1} = C$$

At time $t=0$; Temperature $T(t) = 180°$

From equation (1)

$$T(t) - 76 = e^{kt} C$$

$$180 - 76 = e^{k(0)} \times C$$

$$\boxed{104 = C}$$

Substituting the C in equation (1)

$$\boxed{T(t) = 76 + e^{kt}(104)} \quad \ldots (2)$$

At time $t = 5$ minutes $\quad T(t) = 168°F$

$$T(t) = 76 + e^{kt}(104) \quad \ldots \{\text{from eq (1)}\}$$

$$168 = 76 + 104\, e^{k(5)}$$

$$\frac{92}{104} = e^{k(5)} \qquad \log(92/104) = 5k$$

$$k = \frac{\log(92/104)}{5}$$

$$\boxed{k = -0.0245}$$

equation 2 become

(a) $\boxed{T(t) = 76 + 104\, e^{(-0.0245)t}}$

(b) $T(t) = 155°$ $\qquad t = ?$

$$T(t) = 76 + 104\, e^{(-0.0245)t}$$

$$155 = 76 + 104\, e^{(-0.0245)t}$$

$$\frac{79}{104} = e^{(-0.0245)t} \qquad \frac{\log\left(\frac{79}{104}\right)}{-0.0245} = t$$

$\boxed{t = 11.22}$ minutes

1.5 RC Circuit, RL Circuit

(1) A resistance of 100Ω and inductance of 0.5 henries are connected in series with battery of 20 V Find the current at any instant give the relation b/w L, R, E is given by

$$L\frac{di}{dt} + Ri = E$$

$i(t) = ?$

$\Rightarrow$ Given :

$R = 100\Omega$

$L = 0.5\,H$

$E = 20v$

$$L\frac{di}{dt} + Ri = E$$

$$(0.5)\frac{di}{dt} + 100i = 20$$

$$\frac{1}{2}\frac{di}{dt} + 100i = 20$$

$$\boxed{\frac{di}{dt} + 200i = 40}$$

linear D.E in $\boxed{i}$ with

$P = 200$ $\quad$ $Q = 40$

$$IF = e^{\int P dt} = e^{\int 200 dt} = e^{200t}$$

$\therefore$ solution of linear differential equation is

$$i(IF) = \int Q\, IF\, dt + c$$

$$i \cdot e^{200t} = \int 40\, e^{200t}\, dt + c$$

$$i \cdot e^{200t} = \cancel{40}\frac{e^{200t}}{\cancel{200}_{5}} + c$$

$$i e^{200t} = \frac{1}{5} e^{200t} + c$$

$$i = \frac{1}{5} + ce^{-200t}$$

$$\boxed{i(t) = \frac{1}{5} + ce^{-200t}}$$

current at any instant.

Imp

(2) In a single closed circuit, the current 'i' at any time 't' is given by $Ri + L\frac{di}{dt} = E$,

Find the current i at any time t, given that t=0, i=0 and L, R, E are constant.

=> $\frac{dy}{dx} + Py = Q$

$$\boxed{L\frac{di}{dt} + Ri = E}$$ ... linear D.E in i with

$$\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}$$ (i)

$P = \frac{R}{L}$ $Q = \frac{E}{L}$

$$IF = e^{\int P dt} = e^{\int R/L \, dt}$$

$$IF = e^{\frac{R}{L}t}$$

∴ Solution of linear differential equation is

$$i(IF) = \int Q(IF)\, dt + c$$

$$i e^{R/L t} = \int \frac{E}{L} e^{R/L t} dt + c$$

$$= \frac{E}{\cancel{L}} \frac{e^{R/L t}}{R/\cancel{L}} + c$$

$$\boxed{i e^{R/L t} = \frac{E}{R} e^{R/L t} + c} \quad \text{--(1)}$$

at t=0 i=0

$$0 \times e^{R/L \times (0)} = \frac{E}{R} e^0 + c$$

$$\boxed{c = -E/R}$$

Substituting c value in equation (1)

$$i e^{R/L t} = \frac{E}{R} e^{R/L t} - \frac{E}{R}$$

$$\boxed{i = E/R - \frac{E}{R} e^{-R/L t}}$$

$$\boxed{i = E/R \left[1 - e^{-\frac{R}{L}t}\right]}$$

Imp

(3) The current i is a circuit containing an inductance L, Resistance R and voltage $E \sin\omega t$ is given by $\boxed{L\frac{di}{dt} + Ri = E\sin\omega t}$

for $t=0$, $i=0$ find i

$\Rightarrow$

$$L\frac{di}{dt} + Ri = E\sin\omega t$$

$$\boxed{\frac{di}{dt} + \left(\frac{R}{L}\right) i = \left(\frac{E}{L}\sin\omega t\right)}$$ + linear D.E in i with

$$P = \frac{R}{L} \qquad Q = \frac{E}{L}\sin\omega t$$

$$IF = e^{\int \frac{R}{L} dt} = e^{R/L\, t}$$

$\therefore$ Solution of linear differential eq^n is

$$i(IF) = \int Q(IF)\, dt + C$$

$$i e^{\frac{R}{L}t} = \int \frac{E}{L}\sin\omega t \times e^{R/L\, t} + C$$

$$i e^{\frac{R}{L}t} = \frac{E}{L}\int \sin\omega t \times e^{R/L\, t} + C$$

$$i e^{R/L\, t} = \frac{E}{L}\int e^{R/L\, t} \times \sin\omega t + C$$

using $\int e^{ax}\sin bx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]$

$$i e^{R/L\, t} = \frac{E}{L}\left[\frac{e^{R/L\, t}}{\frac{R^2}{L^2}+\omega^2}\left[\frac{R}{L}\sin\omega t - \omega\cos\omega t\right]\right] + C$$

$$\cancel{i e^{\frac{R}{L}t} = \frac{E}{L}\left[\frac{e^{R/L\, t}}{\frac{R^2+L^2\omega^2}{L^2}} \sin\omega t \quad R\cos\omega t\right]}$$

$$i e^{\frac{R}{L}t} = \frac{E}{L}\left[\frac{e^{R/L\, t}}{\frac{R^2+L^2\omega^2}{L^2}}\left(\frac{R}{L}\sin\omega t - \omega\cos\omega t\right)\right] + C$$

$$= \frac{E}{\cancel{L}}\left[\frac{e^{R/L\, t}}{\frac{R^2+L^2\omega^2}{\cancel{L^2}}}\frac{(R\sin\omega t - \omega L\cos\omega t)}{\cancel{L}}\right] + C$$

$$\boxed{i e^{R/L\, t} = \frac{E\, e^{R/L\, t}}{R^2+L^2\omega^2}\left[R\sin\omega t - \omega L\cos\omega t\right] + C}$$

$\boxed{t=0,\ i=0}$

$$0 \times e^{\frac{R}{L}(0)} = \frac{E\, e^{(0)}}{R^2+L^2\omega^2}\left[R\sin(0) - \omega L\cos(0)\right] + C$$

$$\boxed{\frac{+E \times \omega L}{R^2+L^2\omega^2} = C}$$

Substituting in above equation

$$i e^{R/L\, t} = \frac{E\, e^{R/L\, t}}{R^2+L^2\omega^2}\left(R\sin\omega t - \omega L\cos\omega t\right) + \frac{E\omega L}{R^2+L^2\omega^2}$$

$$\boxed{i = \frac{E}{R^2+L^2\omega^2}\left(R\sin\omega t - \omega L\cos\omega t + \omega L e^{-\frac{R}{L}t}\right)}$$

(4) when a resistance R ohms is connected in series with an inductance L henries, on e.m.f of E volts, the current i amperes ~~at~~ time t is given by $L\frac{di}{dt} + Ri = E$

If $\boxed{E = 10\sin t \text{ volt}}$ and $i=0$ when $t=0$, find i as a function of t.

$\Rightarrow$

$$L\frac{di}{dt} + Ri = E$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}$$ linear D.E in i with $P = R/L$, $Q = E/L$

$$IF = e^{\int P dx} \quad IF = e^{\int \frac{R}{L} dx} \quad IF = e^{R/L\, t}$$

∴ Solution of linear D.E is

$$i(I.F) = \int Q(IF)\,dt + c$$

$$i\, e^{R/L\, t} = \int \frac{E}{L} e^{R/L\, t} + C$$

(E = 10 sin t)

~~$= \frac{E}{L} \frac{e^{R/L\, t}}{R/L} + c$~~

~~$i\, e^{R/L\, t} = \frac{E}{R} e^{R/L\, t} + c \quad \text{-- (1)}$~~

~~If $E = 10 \sin t$, $i=0$, $t=0$;~~

~~$0 \times e^{R/L \times 0} = \frac{10 \sin t}{R} e^{R/L(0)} + c$~~

~~$-\frac{10 \sin t}{R} = c$~~

~~Substituting the c value in eq (1)~~

~~$i\, e^{R/L\, t} =$~~

$$i\, e^{R/L\, t} = \frac{10E}{L} \int \sin t\, e^{R/L\, t} + c$$

$$i\, e^{R/L\, t} = \frac{10E}{L} \int e^{R/L\, t} \sin t + c$$

$$= \frac{10E}{L}\left[\frac{e^{R/L\, t}}{\frac{R^2}{L^2}+1}\left(\frac{R}{L}\sin t - \cos t\right)\right] + c$$

$$= \frac{10}{L}\left(\frac{e^{R/L\, t}}{\frac{R^2+L^2}{L^2}}\left(\frac{R \times \sin t - L\cos t}{L}\right)\right) + c$$

$$i\, e^{R/L\, t} = \frac{10\, e^{R/L\, t}}{R^2+L^2}\left(R\sin t - L\cos t\right) + c$$

$i=0$, $t=0$

$$0 \times e^{R/L(0)} = \frac{10\, e^{R/L(0)}}{R^2+L^2}\left(R\sin 0 - L\cos 0\right) + c$$

$$\boxed{\frac{10L}{R^2+L^2} = c}$$

Substituting c in above eqn

$$i\, e^{R/L\, t} = \frac{10\, e^{R/L\, t}}{R^2+L^2}\left(R\sin t - L\cos t\right) + \frac{10L}{R^2+L^2}$$

$$\boxed{i = \frac{10}{R^2+L^2}\left(R\sin t - L\cos t + L e^{-R/L\, t}\right)}$$