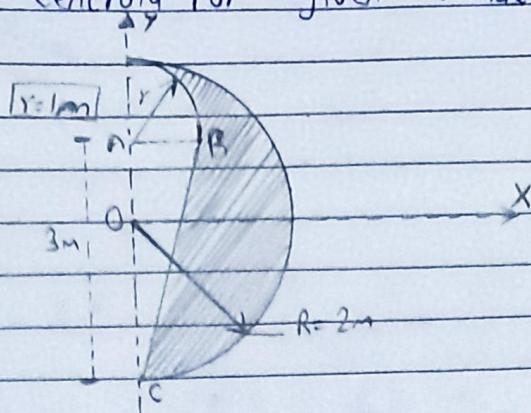


Q1

Determine the centroid for given shaded area for fig.



$$\text{- Area of Semicircle} = \frac{\pi r^2}{2}$$

Components	Area $A_i (m^2)$	x_i (m)	y_i (m)	$A_i x_i$ (m^3)	$A_i y_i$ (m^3)	\bar{x}_i
Semicircle	6.28	0.85	0	5.39	0	
Quarter Circle	-0.78	0.57	1.42	-0.45	-0.32	-1.10
Triangle	-1.5	0.33	1	-0.495	-1.5	
	$\sum A_i = 4$			$\sum A_i x_i = 4.395$	$\sum A_i y_i = -1.82$	$= -2.6$

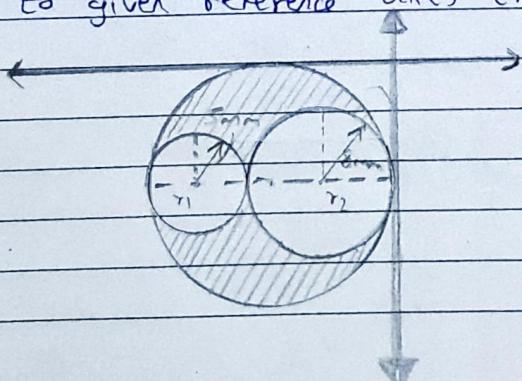
$$\therefore X = \frac{\sum A_i x_i}{\sum A_i} = \frac{4.395}{4} = 1.098$$

$$\therefore Y = \frac{\sum A_i y_i}{\sum A_i} = \frac{-1.82}{4} = -0.65 \Rightarrow -2.6 = 0.65$$

$$\therefore \text{Centroid } (X, Y) = (1.098, -0.65)$$

Q2

Determine the Centroid of the selected shaded area shown in fig with respect to given reference axes ($r_1 = 5mm$, $r_2 = 8mm$)



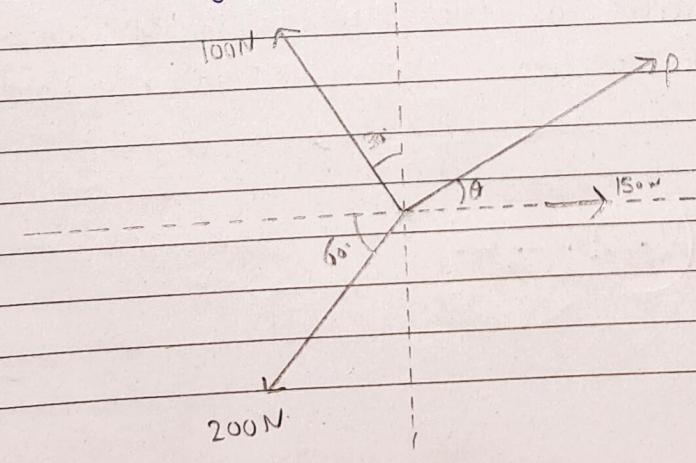
Components	A_{eq}	X_i	Y_i	$A_i X_i$	$A_i Y_i$	
	$A_i (\text{mm})$	(mm)	(mm)			0N.
Circle A	-78.5	-21	-13	1649.5	1251 1020.85	
Circle B	-200.96	-8	-13	1608.48	3215 2613.78	
Circle C	580.96	-16	-13	-12861.4	-12861.4 -6901.96	
	251.44			$\sum A_i X_i$	$\sum A_i Y_i$	
	$\sum A_i = 524.38$			= 3643.9	= -3268.67	

$$X = \frac{\sum A_i X_i}{\sum A_i} = \frac{-9605.2}{524.38} = -18.31 \text{ mm} \Rightarrow \frac{3643.9}{251.44} = -14.49$$

$$Y = \frac{\sum A_i Y_i}{\sum A_i} = \frac{-8890.1}{524.38} = -16 \text{ mm} \Rightarrow \frac{-3268.67}{251.44} = -13$$

Centroid $(X, Y) = (-18.31, -16) = (-14.49, -13)$.

- Q3 A system of four forces shown in figure has resultant 50N along the axis. Determine magnitude & direction of unknown force P.



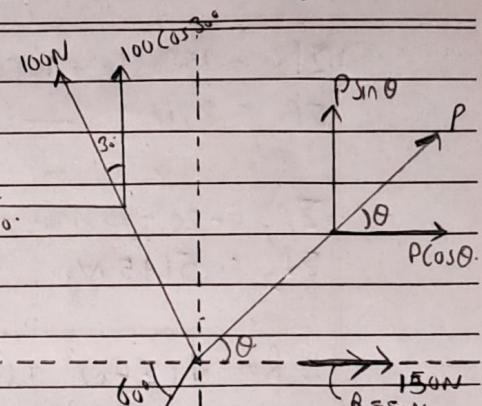
\therefore Resultant, $R = 50\text{N}$ is along the axis

$$\therefore R_x = 50\text{N} \text{ & } R_y = 0\text{N}$$

$$\therefore \sum F_x = R_x$$

$$\therefore 50 = 150 - 100 \sin 30^\circ + P \cos \theta - 200 \cos 60^\circ - 100 \sin 30^\circ$$

$$\therefore 50 = P \cos \theta \quad \text{--- (i)}$$

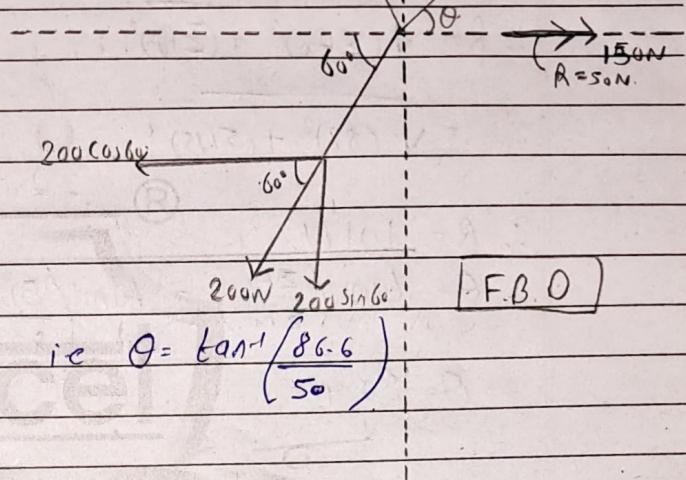


$$\therefore \sum F_y = R_y$$

$$0 = P \sin \theta + 100 \cos 30^\circ - 200 \sin 60^\circ$$

$$0 = P \sin \theta - 86.6$$

$$86.6 = P \sin \theta \quad \text{--- (ii)}$$



From (i) & (ii)

$$\frac{P \sin \theta}{P \cos \theta} = \frac{86.6}{50} \Rightarrow \therefore \tan \theta = \frac{86.6}{50} \text{ ie } \theta = \tan^{-1} \left(\frac{86.6}{50} \right)$$

$$\therefore \theta = 59.9^\circ$$

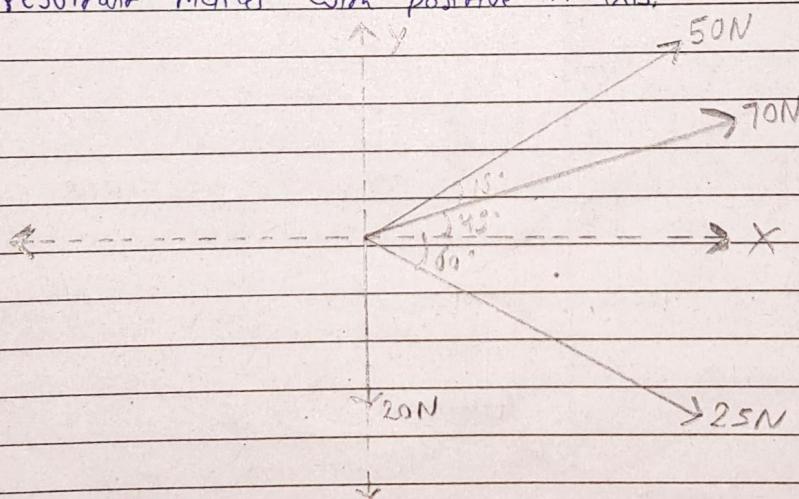
Now, Substituting $\theta = 59.9^\circ$ in eqn (i).

$$\therefore P = \frac{50}{\sin 59.9^\circ} = 99.7\text{N}$$

$$\cos 59.9^\circ$$

\therefore Magnitude of force P is 99.7N & Direction is 59.9° .

Q4 Determine the resultant of forces as given in fig. Find the angle which resultant makes with positive X-axis.



$$\sum F_x = 25 \cos 60^\circ + 70 \cos 45^\circ + 50 \sin 30^\circ$$

$$\sum F_x = 87 \text{ N} \quad \text{--- (1)}$$

$$\sum F_y = -20 - 25 \sin 60^\circ + 70 \sin 45^\circ + 50 \cos 30^\circ$$

$$\sum F_y = 51.15 \text{ N.} \quad \text{--- (2)}$$

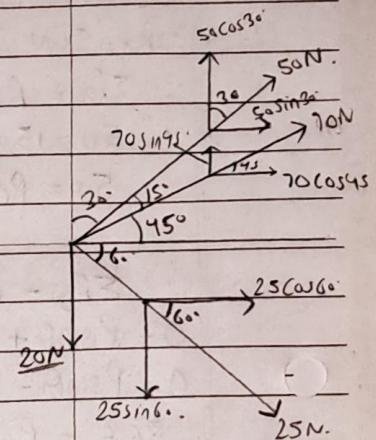
$$\therefore R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$= \sqrt{(87)^2 + (51.15)^2}$$

$$\therefore R = 101 \text{ N.}$$

$$\therefore \theta = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right) = \tan^{-1}\left(\frac{51.15}{87}\right) = 30.45^\circ$$

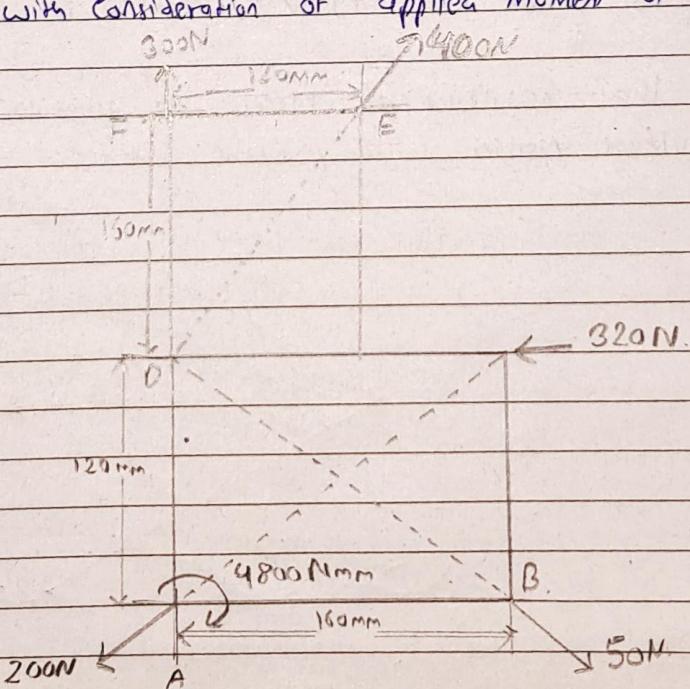
$$\therefore \theta = 30.45^\circ$$



FBD.

Q5

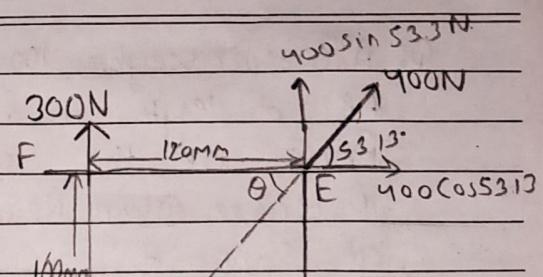
Find the Resultant of co-planar Forces system given below & Locate the same on AB with consideration of applied moment of 4800 Nm



For θ at point E,

$$\tan \theta = \frac{160}{120} \therefore \theta = \tan^{-1}\left(\frac{16}{12}\right) =$$

$$\therefore \theta = 53.13^\circ$$



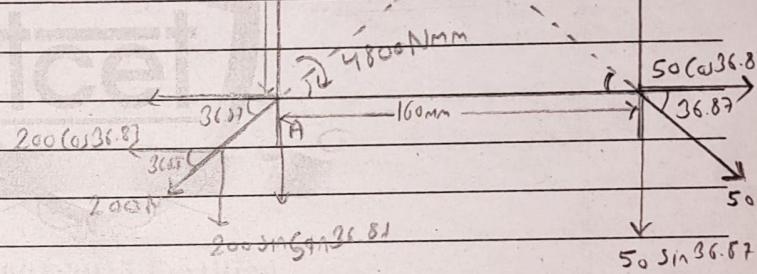
for θ at point A,

$$\tan \theta = \frac{120}{160} \therefore \theta = \tan^{-1}\left(\frac{12}{16}\right)$$

$$\theta = 36.87^\circ$$

$$\begin{aligned} \sum F_x &= -200\cos 36.87^\circ + 50\cos 36.87^\circ \\ &\quad - 320 + 400\cos 53.13 \\ &= -200N \text{ i.e } 200N (\leftarrow) \end{aligned}$$

$$\begin{aligned} \sum F_y &= -200\sin 36.87^\circ - 50\sin 36.87^\circ \\ &\quad + 400\sin 53.13 + 300 \\ &= 469.9N \text{ R } 470N (\uparrow) \end{aligned}$$



$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = \sqrt{(200)^2 + (470)^2} = 510.7N.$$

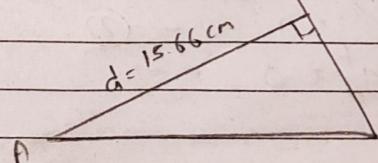
$$\therefore \theta = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right) = \tan^{-1}\left(\frac{470}{200}\right) = 66.94^\circ$$

Using Varignon's Theorem,

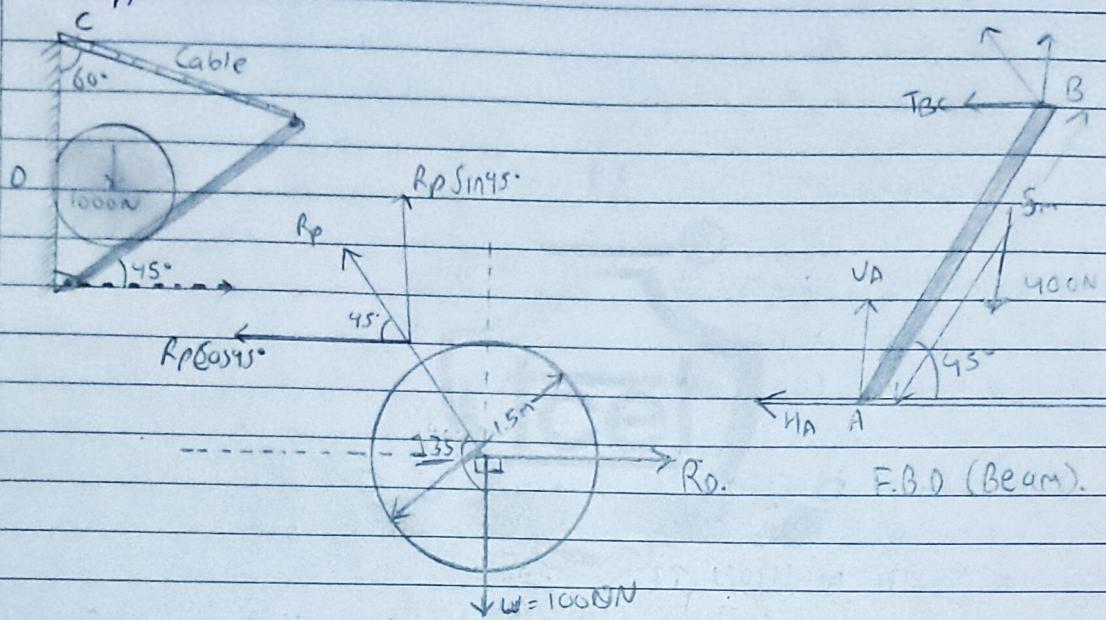
$$\sum M_A^F = \sum M_R^F = R \times d$$

$$\begin{aligned} \therefore -(160 \times -50\sin 36.87^\circ) + (320 \times 120) - 4800 + (120 \times 480\sin 53.13) &\rightarrow 510.7N \\ -(280 \times 400\cos 53.13) &= 510.7 \times d. \end{aligned}$$

$$\therefore d = 15.66 \text{ mm } (7)$$



Q6 A cylinder weighing 1000N & 1.5m diameter is supported by a beam AB of length 6m & weight 400N as shown in fig Neglecting friction at the surface of contact of the cylinder. Determine, (i) Wall Reaction at A, (ii) Tension on cable BC & (iii) Hinged Reaction at Support A.



F.B.D (Cylinder)

Applying Lami's Theorem

$$\frac{R_D}{1000} = \frac{R_p}{1419} = \frac{\sin 135^\circ}{\sin 135^\circ}$$

$$\therefore R_D = 1000\text{N} \quad \& \quad R_p = 1419\text{N}.$$

For Tension on Cable BC (T_{BC})

$$\begin{aligned} \sum F_x &= -R_p \cos 45^\circ - T_{BC} \sin 45^\circ - n_A \\ &= 0 \quad \text{---(1)} \end{aligned}$$

$$\begin{aligned} \sum F_y &= -R_p \sin 45^\circ + T_{BC} \cos 45^\circ - 400 + V_A \\ &= 0 \quad \text{---(2)} \end{aligned}$$

$$\sum n_A^F = 0 \quad (+ve)$$

$$\sum M_A^F = 0 \text{ (tue)}$$

$$= 1414(1.8) + 400(3\cos 45^\circ) - T_{BC}(6) = 0$$

$$\therefore T_{BC} = 568.04N$$

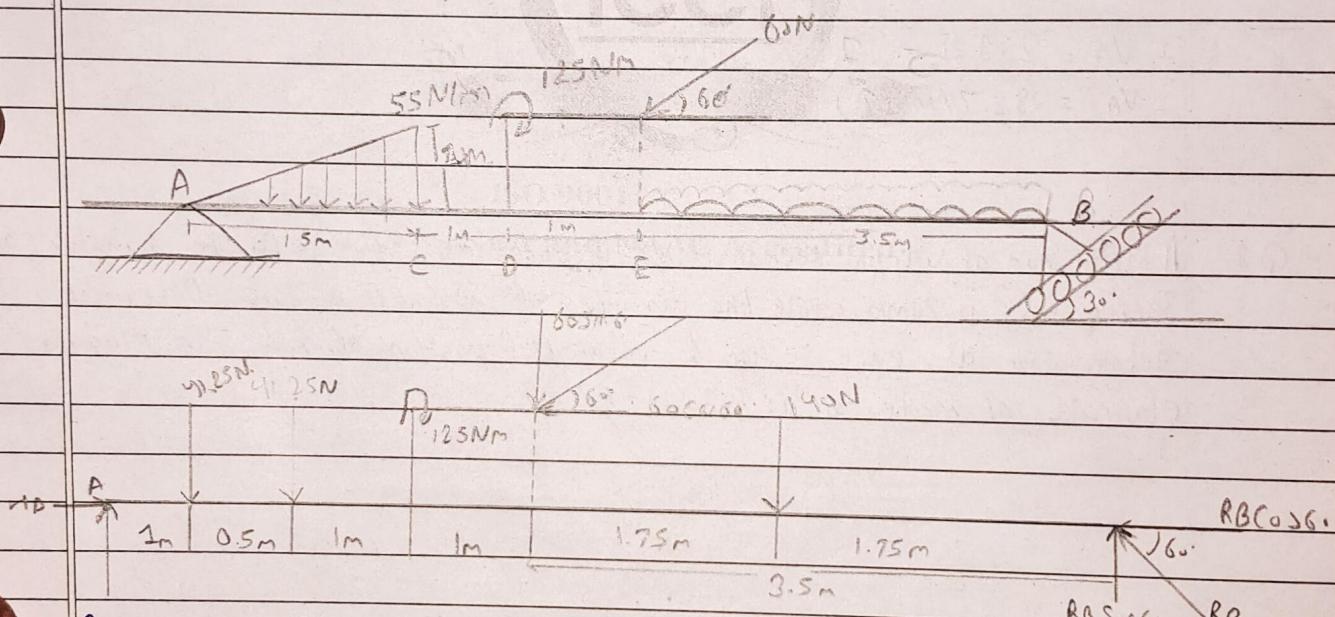
Putting T_{BC} in eqn ① & ②.

$$H_A = 598.33N \quad \& \quad V_A = 998.33N$$

$$\therefore R_A = 1163.9N$$

\therefore Reaction at D is 1000N, Tension on BC is 568.04N & Hinge Reaction at A is 1163N.

Q7 Find support reaction of beam as shown in figure



Applying Conditions of equilibrium,
 $\sum F_x = 0$.

$$\therefore H_A - 60\cos 60^\circ - R_B \cos 60^\circ = 0$$

$$\therefore H_A - 30 - R_B \cos 60^\circ = 0$$

$$\therefore H_A - R_B \cos 60^\circ = 30 \quad \text{---(1)}$$

$$\sum F_y = 0.$$

$$\therefore V_A + R_B \sin 60^\circ - 41.25 - 60 \sin 60^\circ - 140 = 0.$$

$$\therefore V_A + R_B \sin 60^\circ = 233.1 N \quad \text{--- (2)}$$

$$\sum M_A = 0$$

$$\therefore -(4.5 \times 60 \sin 60^\circ) - 6.25 \times 140 - 125 + (8 \times R_B \sin 60^\circ) = 0.$$

$$R_B = 154.2$$

$$\sin 60^\circ$$

$$R_B = 173.72 N.$$

Substituting R_B in eqn (1) & (2),

$$\therefore H_A = 30 + 173.72 \cos 60^\circ.$$

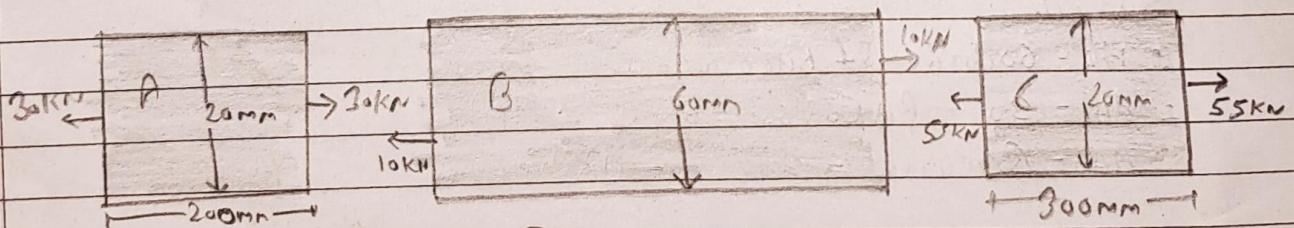
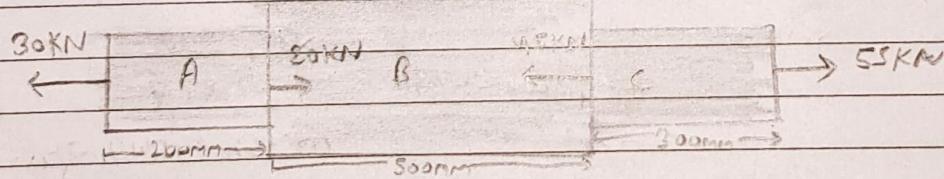
$$\therefore H_A = 116.86 N \quad (\rightarrow)$$

$$\therefore V_A = 233.21 - 173.72 \sin 60^\circ.$$

$$\therefore V_A = 82.77 N \quad (\uparrow)$$

Q8

A metal rod of varying section (A,B,C) is loaded as shown in fig. The diameter of Section A & C is 20mm, while the diameter of Section B is 60mm. Determine deformation at each section & total deformation of rod. The modulus of elasticity of material is $5 \times 10^3 N/mm^2$



$$\text{Area of section A, } A_1 = \frac{\pi}{4} \times (2a)^2 = 314 \text{ mm}^2$$

$$\text{Area of section B, } A_2 = \frac{\pi}{4} \times (4a)^2 = 2826 \text{ mm}^2$$

$$\text{Area of section C, } A_3 = \frac{\pi}{4} \times (2a)^2 = 314 \text{ mm}^2$$

Deformation at each section,

$$\therefore \Delta_1 = \frac{P_1 l_1}{A_1 E} = \frac{30 \times 1000 \times 200}{314 \times 5 \times 1000} = 3.821 \text{ N/mm}^2$$

$$\therefore \Delta_2 = \frac{P_2 l_2}{A_2 E} = \frac{10 \times 1000 \times 500}{2826 \times 5 \times 1000} = 0.3538 \text{ N/mm}^2$$

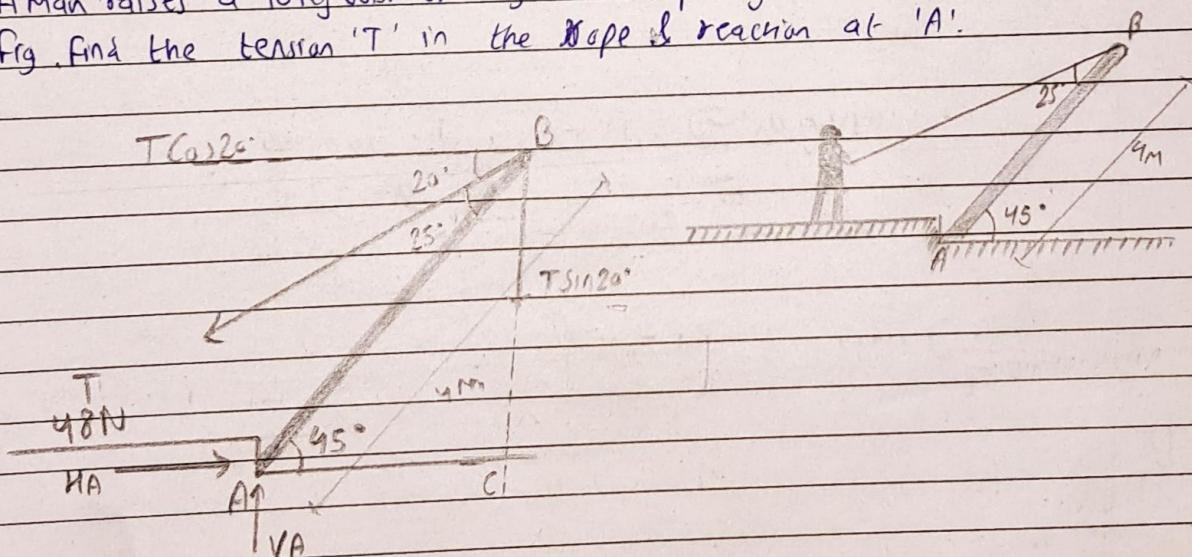
$$\therefore \Delta_3 = \frac{P_3 l_3}{A_3 E} = \frac{55 \times 1000 \times 300}{314 \times 5 \times 1000} = 10.5 \text{ N/mm}^2$$

Total Deformation of Rod,

$$\Delta_T = 3.821 + 0.3538 + 10.5$$

$$\therefore \Delta_T = 14.67 \text{ mm}$$

Q3. A man raises a 10kg joist of length 4m by pulling on a rope as shown in fig. Find the tension 'T' in the rope & reaction at 'A'.



Applying Conditions of Equilibrium,

$$\sum F_x = 0.$$

$$\therefore H_A - T \cos 20^\circ = 0 \quad \text{--- (1)}$$

$$\sum F_y = 0$$

$$\therefore V_A - T \sin 20^\circ - 98 = 0 \quad \text{--- (2)}$$

$$\sum M_A^P = 0.$$

$$[(T \cos 20^\circ) \times 2.83] - [T \sin 20^\circ \times 2.83] - (98 \times 1.41) = 0.$$

$$\therefore T(2.66 - 0.988) = 138.18.$$

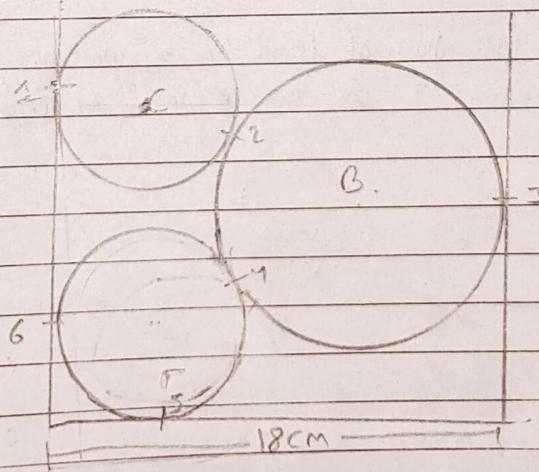
$$\therefore T = 81.67 \text{ N.}$$

Substituting 'T' value in eqn (1) & (2).

$$\therefore H_A = 76.19 \text{ N.}$$

$$\therefore V_A = 126 \text{ N.}$$

- Q10. 3 Cylinders are piled up as shown in fig. Determine the reaction at all the contact surfaces. (Cylinder A: radius = 4cm, m = 15kg ; Cylinder B: radius = 6cm, m = 40kg ; & Cylinder C: radius = 5cm., m = 20kg.)



Reaction R_1, R_3, R_5 & R_6 are
perpendicular to their respective
surfaces.

We have,

$$R_A = 4\text{cm} \quad \& \quad M_A = 15\text{kg}$$

$$R_C = 5\text{cm} \quad \& \quad M_C = 20\text{kg}$$

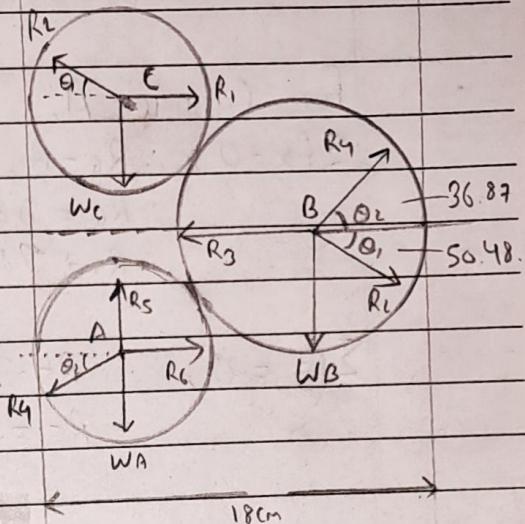
$$R_B = 6\text{cm} \quad \& \quad M_B = 40\text{kg}$$

$$AB = 10\text{cm}, \quad BC = 11\text{cm}$$

$$BP = 18 - R_C = R_B$$

$$BP = 18 - 5 - 6 = 7\text{cm}$$

$$AQ = 18 - 4 - 6 = 8\text{cm}$$



$$\text{In } \triangle BCP, \cos \theta_1 = \frac{BP}{BC} = \frac{7}{11}$$

$$\therefore \theta_1 = 50.48^\circ$$

$$\text{In } \triangle ABQ, \cos \theta_2 = \frac{AQ}{AB} = \frac{8}{10}$$

$$\therefore \theta_2 = 36.87^\circ$$

For Cylinder C:

Applying Lami's Theorem

$$\frac{R_1}{\sin(90 + 50.48)} = \frac{R_2}{\sin(180 - 50.48)} = \frac{R_6}{\sin 100}$$

$$\frac{R_1}{\sin(90 + 50.48)} = \frac{R_2}{\sin(180 - 50.48)} = \frac{R_6}{\sin 100}$$

$$\therefore R_1 = 161.89 \quad \& \quad R_2 = 254.34\text{N.}$$

For Cylinder B,

Applying COE.

$$\sum F_x = 0, \quad \therefore -R_3 + R_2 \cos \theta_1 + R_4 \cos \theta_2 = 0.$$

$$\therefore -R_3 + 254.34 \cos 50.48 + R_4 \cos 36.87^\circ = 0.$$

$$\sum F_y = 0, \therefore R_4 \sin \theta_2 - R_2 \sin \theta_1 - (g_0 \times 9.8) = 0.$$

$$\therefore R_4 \sin 36.87^\circ - 254.34 \sin 50.98^\circ - (g_0 \times 9.8) = 0.$$

$$\therefore R_2 = 946.99 \text{ N} \quad \& \quad R_4 = 981.22 \text{ N}.$$

For Cylinder A,

$$\sum F_x = 0, \therefore R_6 - R_4 \cos \theta_2 = 0.$$

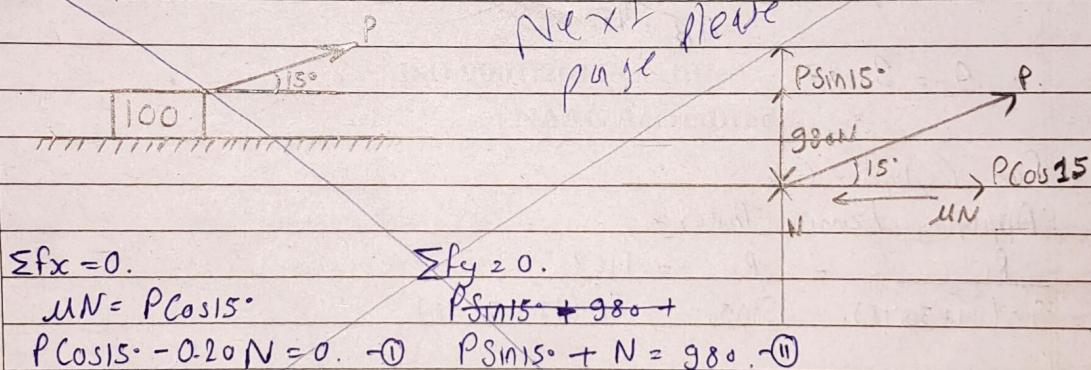
$$\therefore R_6 = 981.22 \cos 36.87^\circ$$

$$\therefore R_6 = 785 \text{ N}.$$

$$\sum F_y = 0, \therefore R_5 - R_4 \sin \theta_2 = 0.$$

$$\therefore R_5 = 732.74 \text{ N}.$$

Q11 Determine minimum value & the direction of a force P required to cause motion of a 100 kg block to impend up 30° plane. The coefficient of friction is 0.20.



$$\sum F_x = 0.$$

$$MN = P \cos 15^\circ$$

$$P \cos 15^\circ - 0.20 N = 0. \quad \text{---(1)}$$

$$\sum F_y = 0.$$

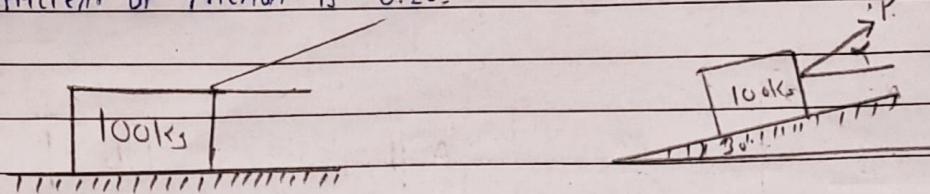
$$P \sin 15^\circ + 980 +$$

$$P \sin 15^\circ + N = 980. \quad \text{---(2)}$$

Solving eqn ① & ② simultaneously,

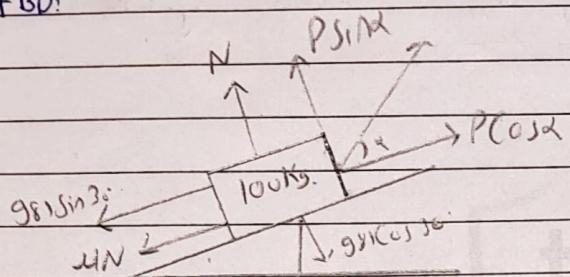
Q10

Determine minimum value & direction of a force P required to cause motion of an 100 kg block to impend upon 30° inclined plane. The coefficient of friction is 0.20.



Soln

FBD:



$$\text{Here } \tan \alpha = \mu$$

$$\alpha = \tan^{-1}(\mu)$$

$$= \tan^{-1}(0.2)$$

$$\alpha = 11.31^\circ$$

\therefore Because condition of just impeding condition of Equilibrium are applicable

$$\sum F_x = 0. P \cos 11.31 - 0.2N = 490.5 \quad \text{--- (i)}$$

$$0.98P - 0.2N = 490.5 \quad \text{--- (ii)}$$

$$\sum F_y = 0. N - P \sin 11.31 - 981 \cos 30^\circ = 0$$

$$0.19P - N = -849.57 \quad \text{--- (iii)}$$

From (i) & (ii)

$$0.98P = 0.2(0.19P + 849.57) + 490.5$$

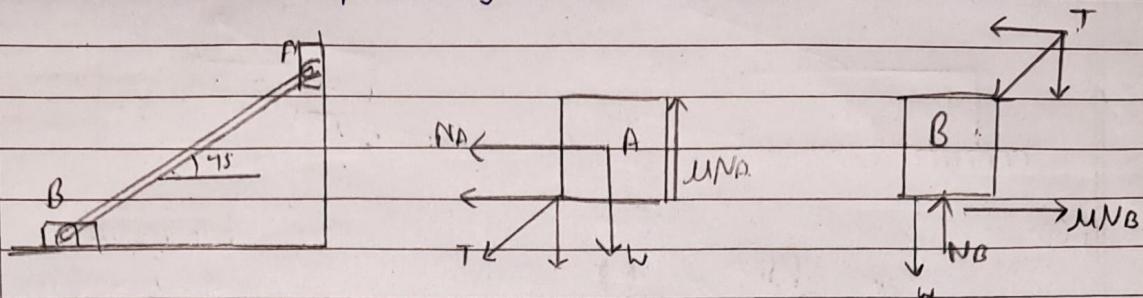
$$0.942P = 169.914 + 490.5$$

$$P = 701.07 \text{ N}$$

$\therefore P$'s minimum value is 660N & 701.07N & direction is 11.31°

Q12

Two Blocks A & B are identical 'w', each are supported by rod inclined at 45° to horizontal. If blocks are in limiting equilibrium. Find coefficient of friction, assuming it to same for wall & floor.



For Block A,

$$\sum F_x = 0$$

$$NA = T \cos 45^\circ = \frac{T}{\sqrt{2}} \quad \text{--- (1)}$$

$$\sum F_y = 0 : T \sin 45^\circ + \mu NA = w$$

$$\frac{T}{\sqrt{2}} + \frac{\mu T}{\sqrt{2}} = w \therefore \frac{T}{\sqrt{2}} (\mu + 1) = w \quad \text{--- (2)}$$

For Block B;

$$\sum F_x = 0, T \cos 45^\circ = \mu NB; \mu NB = \frac{T}{\sqrt{2}} = \text{--- (3)}$$

$$\sum F_y = 0, NB = w + \frac{T}{\sqrt{2}} \therefore \text{From (1) we get, } \mu \cdot \left(w + \frac{T}{\sqrt{2}}\right) = \frac{T}{\sqrt{2}}$$

$$\therefore \mu \left(w + \frac{T}{\sqrt{2}}\right) = \frac{T}{\sqrt{2}} \quad \therefore \text{By simplifying, } \mu^2 + 2\mu - 1 =$$

$$w = \frac{T}{\sqrt{2}} \left(\frac{1 - \mu}{\mu} \right) - \text{--- (4)} \quad \therefore \mu = \frac{-2 \pm \sqrt{8}}{2}$$

$$\text{from (2) \& (4).} \quad \therefore \boxed{\mu = 0.414}$$

$$\frac{T}{\sqrt{2}} (\mu + 1) = \frac{T}{\sqrt{2}} \left(1 + \frac{1}{\mu} \right)$$

Frictional Coefficient is 0.414.