MODULE :- 06

Note :- def^n → Imp → tick in resource book

Kinematics : The description of motion (position, velocity, acceleration, time) without regard to force

Kinetics : Determining the force associated with motion (based on $F=ma$)

The system of force acting on body in motion is in equilibrum with Interia force or reverse effective forces

KINETICS OF PARTICLES

D'Alembert's principle :

(i) Newton's second law :-

$$\sum F = ma$$

a → acceleration in m/s^2

m → mass of particle in kg

$\sum F$ → Algebraic sum of all force acting on the particle

(ii) Application of Newton's 2nd law :

For rectangular co-ordinate system

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$\sum F_z = ma_z$$

For curvilinear motion

$$\sum F_t = ma_t = m\frac{dv}{dt}$$

Work - Energy principle :-

- The principle of work - energy also known as work and kinetic energy principle

state that the work done by all force acting on a particle is equal to change in K.E of particle.

$$\boxed{U_{1-2} = K.E_2 - K.E_1}$$

$K.E_1$ → K.E at position 1

$K.E_2$ → K.E at position 2

U_{1-2} → algebraic sum of work done by all force from position 1 to 2

Enlist the different types of work done involved in work energy principle

(1) Translational works
(2) rotational work
(3) Gravity work
(4) Spring work
(5) Frictional work

State any two example of General plane motion

(1) **Projectile motion**

State :- when a practile is projected in space its motion is combination of horizontial and vertical motion

(2) **Rectilinear motion**

State :- the motion of the particle in the straight line

e.g car traveling along a straight line

(3) **Curvilinear motion**

State : It is motion in which particle travel along curved path

e.g car traveling along a curved road.

① Impulse (J)

↳ when a large force acts over a short period of time

that force is called as Impulsive force

$$J = \int_{t_1}^{t_2} F\,dt$$

The impulse of a force F acting over a time interval for t_1 to t_2

is definite by the integral

② momentum :

consider motion of the particle of mass m and Force F

acted upon by a Force F

The equation of motion of the particle in the x and y direction

$F_x = m\ddot{x}$ and $F_y = ma_y$

or or

$F_x = m\left(\frac{dv_x}{dt}\right)$ $F_y = m\left(\frac{dv_y}{dt}\right)$

unit $mv = kg\,(m/s) = kg\frac{m}{s^2}s = Ns$

$F_x = \frac{d}{dt}(mv_x)$ $F_y = \frac{d}{dt}(mv_y)$

$F = \frac{d}{dt}(mv)$

- Force F acting on a particle is equal to the rate of $\frac{d}{dt}$ charge of momentum of particle

is called momentum and linear momen

- It has the same direction as the velocity of particle

principle of Impulse (J) and momentum (p):

we know that by Newton's 2nd law of motion

$F = ma$ $F = m\frac{dv}{dt}$

$F\,dt = m\,dv$

Integrating both sides

$$\int_{t_1}^{t_2} F\,dt = m\int_{v_1}^{v_2} dv$$

impulse

$$F[t]_{t_1}^{t_2} = m[v]_{v_1}^{v_2}$$

$$F(t_2 - t_1) = m(v_2 - v_1)$$

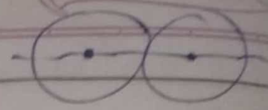
$$\boxed{F\,\Delta t = mv_2 - mv_1}$$

Impulse(J) = Final momentum − initial momentum

Impulse(J) = change in momentum

State that the impulse applied to an object will equal to the change in its momentum

$\frac{u_2}{u_1}$



Coefficient of restitution (e) :-

It is ratio of relative velocity of separation to the relative velocity of approach in (opposite direction)

$$e = \frac{\text{Impulse during restoration period}}{\text{Impulse during deformation period}}$$

$$e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}}$$

$$e = \frac{v_2 - v_1}{u_2 - u_1}$$ ← Final, ← Ini

where v_1, v_2 are the velocity of particle 1 and 2 immediately after impact

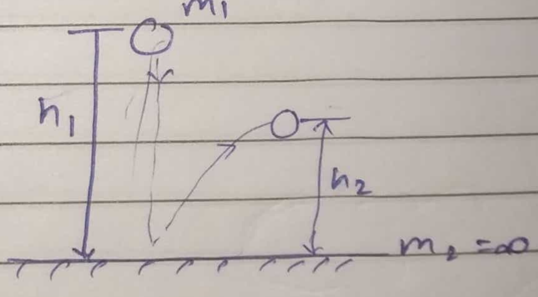
Impact with infinite mass

$$e = \sqrt{\frac{h_2}{h_1}}$$ → Height just after impact, → Height just before impact

u_1, u_2 are the velocity of particle 1 and 2 just before impact

$e = \sqrt{\frac{h_2}{h_1}}$

perfectly elastic impact $(e = 1)$



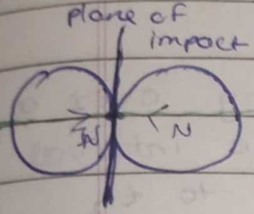
semi elastic impact $0 < e < 1$

plastic impact $(e = 0)$

$u_2 = v_2 = 0$

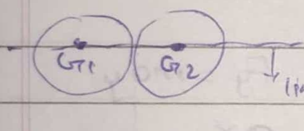
Impact :- A collision of two bodies which take place during very small interval of time and during which the colliding bodies exert relatively large force on each other is known as impact.

Line impact when two bodies collide the line joining the common normal of the colliding bodies is called line of impact



(1) **Central impact**

when the mass center of the colliding bodies lies on the line impact is calle. ___

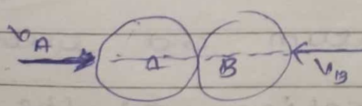


(2) **Eccentric impact**

when the mass center of the colliding bodies do not lie on the line impact

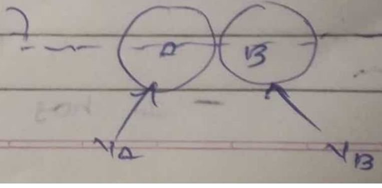
(3) **Direct ~~...~~ Central impact**

If the velocities of the two particle are along the the line of impact



(4) **Oblique Central impact**

If the velocities of of one or both particle are not directed along the line of impact

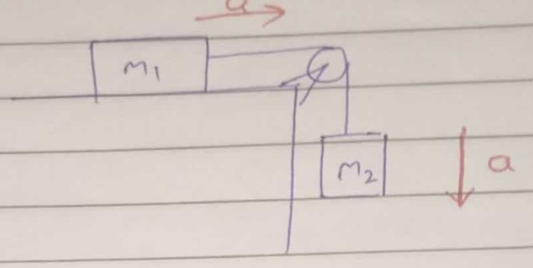


extra

part of

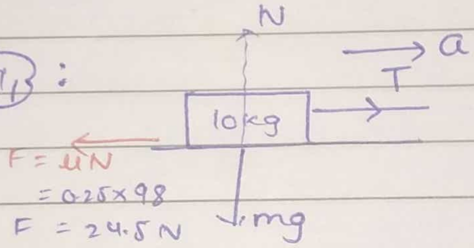
other module

(1) Two blocks of masses m_1 and m_2 are connected by a flexible but inextensible string as shown in the figure. Assuming the coefficient of friction b/w block m_1 and the horizontal surface to be μ. Find the acceleration? of the masses and tension? in the string. Assume $m_1 = 10kg$ and $m_2 = 5kg$ and $\mu = 0.25$



Sol: ⇒ given : $\mu = 0.25$ $m_1 = 10kg$
$m_2 = 5kg$

(a) Block m_1 :



(−),(+) sign calculated by where our body move

why $\Sigma F_y = 0$ because no move along y-axis → $\Sigma F_y = 0$

$a = 0$

$N - mg = 0$

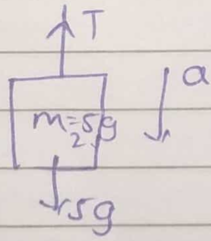
$N = mg$ $\boxed{N = 10 \times 9.8}$ $\boxed{N = 10g}$

$\Sigma F_x = m_1 a_x$

$T - F = 10a$

$\boxed{T - 24.5 = 10a}$ --(1)

(b) Block m_2 :



(−),(+) decide by direction of our body move.

↓ downword

all downword force will be (+ve)

$\Sigma F_y = m_2 a$

$\boxed{5g - T = 5a}$ ~~--(2)~~

$5 \times 9.8 - T = 5a$

$\boxed{49 - T = 5a}$ --(2)

Date
Page

Adding equation (1) and (2)

$$\cancel{T} - 24.5 + 49 - \cancel{T} = 10a + 5a$$

$$24.5 = 15a$$

$$a = \frac{24.5}{15}$$

$$a = 1.633 \ m/s^2$$

put value of a in eqn (2)

$$49 - T = 5a$$

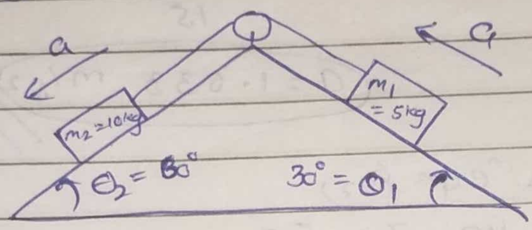
$$49 - T = 5 \times 1.633$$

$$49 - 8.165 = T$$

$$T = 40.835 N$$

$1kg = 1000g$
$=$

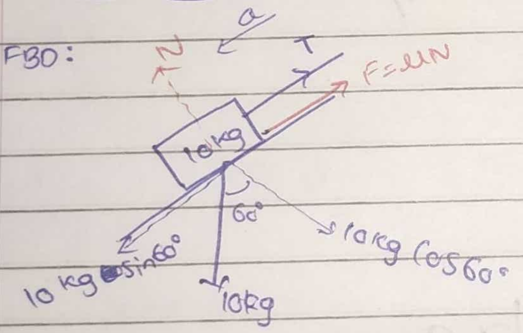
(2) Two blocks of masses M_1 and M_2 are placed on two inclined planes of elevation θ_1 and θ_2 are connected by a string as shown in the figure. Find acceleration of the masses. The coefficient of friction b/w the block and the plane is μ. Assume $M_1 = 5kg$, $M_2 = 10kg$, $\theta_1 = 30°$, $\theta_2 = 60°$, $\mu = 0.33$



Sol: ⟹ Given :- $\theta_2 = 60°$, $\theta_1 = 30°$, $\mu = 0.33$
$M_1 = 5kg$, $M_2 = 10kg$

(a) For Block M_2 :-

FBD:



$\Sigma F_y = 0$

$N - 10g \cos 60° = 0$

$N = 10kg \frac{1}{2}$

$\boxed{N = 5g}$

$\Sigma F_x = m_2 a$

$10kg \sin 60° - T - 0.33 \cdot 5kg = 10 \times a$

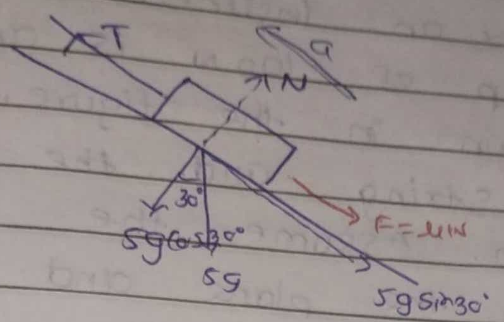
$10 \times 9.8 \frac{\sqrt{3}}{2} - T - 0.33 \times 5 \times 9.8 = 10 \times a$

$84.87 - T - 16.17 = 10a$

$-T + 68.77 = 10a$

$\boxed{10a + T = 68.77}$ ...(1)

For Block m_1 :-



$\Sigma F_y = 0$

$N - 5g\cos 30° = 0$

$N = 5 \times 9.8 \times \frac{\sqrt{3}}{2}$

$\boxed{N = 42.435}$

$\Sigma F_x = m_1 a$

$T - 0.33 \times 42.435 - \frac{5 \times 9.8}{2} = 5a$

$T - 14 - 24.5 = 5a$

$\boxed{T - 38.50 = 5a}$ ..(2)

$\boxed{5a - T = -38.50}$ ..(2)

Adding eq(1) and (2)

$10a + \cancel{T} + 5a - \cancel{T} = 68.77 - 38.50$

$15a = 30.26$

$\boxed{a = 2.017 m/s^2}$

Imp

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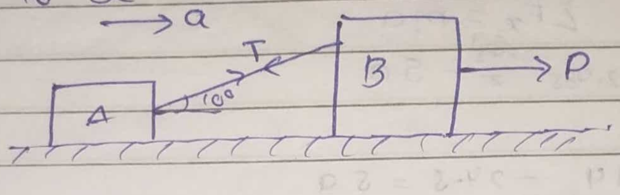
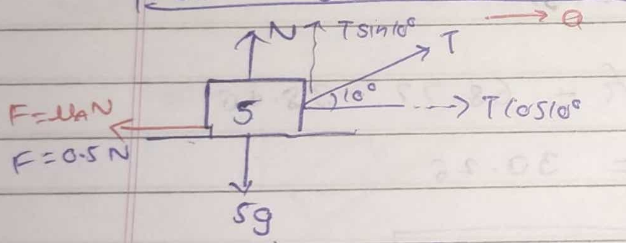
(1) Two Block A and B of masses 5 kg and 20 kg are connected by an inclined string. A Horizontal force P of 100 N is applied to the Block B as shown in the figure. Calculate the tension in the string and the acceleration of the system. Assume the coefficient of friction b/w the plane and the Block A and B to be 0.5 and 0.25 respectively.

⇒



Draw FBD of Block A and B in dynamic equilibrium and Apply condition of equilibrium

From FBD of Block A



$\Sigma F_y = 0$ (+ve ↑)

$N + T\sin 10° - 5g = 0$

$N_A + T\sin 10° = 5 \times 9.8$

$\boxed{N_A + T\sin 10° = 49}$ --(i)

$\Sigma F_x = ma$ (→ +ve)

$T\cos 10° - 0.5 N_A = m_A a$

$\boxed{T\cos 10° - 0.5 N_A = 5a}$ --(ii)

Put the value of N_A in eq^(ii)

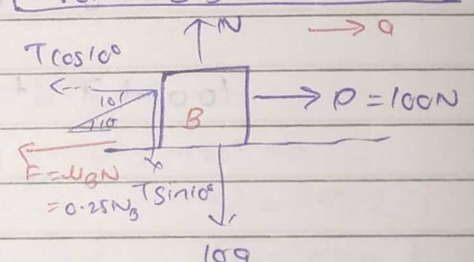
$T\cos 10° - 0.5(49 - T\sin 10°) = 5a$

$T(0.98) - 24.5 + T(0.17) = 5a$

$\boxed{(1.15)T - 24.5 = 5a}$ --(1)

$\boxed{(1.15)T - 5a = 24.5}$ --(1)

From FBD of Block B



$\Sigma F_y = 0$ (↑ +ve)

$N_B - T\sin 10° - 10g = 0$

$\boxed{N_B - T\sin 10° = 98}$ --(1)

$\Sigma F_x = ma$ (→ +ve)

$100 - T\cos 10° - 0.25 N_B$

$\boxed{100 - T\cos 10° - 0.25 N_B}$

Put the value of N_B in eq^(ii)

$100 - T\cos 10° - 0.25[98 + T\sin 10°] = 10a$

$100 - T(0.98) - 24.5 - T(0.17) = 10a$

$75.5 \quad -(1.15)T = 10a$

$\boxed{10a + (1.15\,T) = 75.5}$ --(2)

Adding eq^n (1) and (2)

$$\cancel{(1.15)T - 5a + 10a + (1.15)T = 24.5 + 75.5}$$

$$\cancel{\boxed{2.3T + 5a = 100}}$$

eq (1) multiply by 2

$$(2.3)T - 10a = 24.5 \quad \ldots(3)$$

$$(1.15)T + 10a = 75.5 \quad \ldots(2)$$

$$T = \frac{100}{3.45}$$

$$\boxed{T = 28.98N}$$

put the value T in eq (2)

$$\cancel{10a + (1.15) \times 28.98 = 75.5}$$

$$\cancel{a = \frac{75.5 - 33.327}{10}}$$

$$\cancel{a = \frac{42.173}{10}}$$

$$\cancel{\boxed{a = 4.2173 \ m/s^2}}$$

$m_B a$

$10a$. (ii)

$$(1.15 \times 28.98) - 5a = 24.5$$

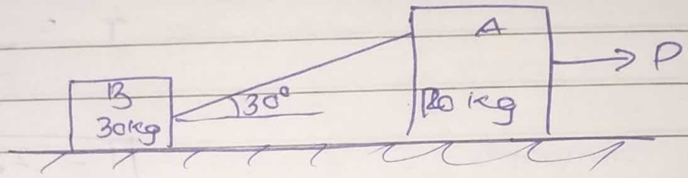
$$33.327 - 24.5 = 5a$$

$$\frac{8.827}{5} = a$$

$$m/s^2 \ 1.7654 = a$$

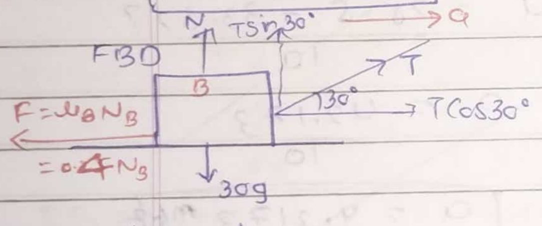
Imp ★★★

(2) A Horizontal Force $P = 600N$ is exterted on Block A of mass 120 kg as shawn in the figure. The coefficient of friction b/w block A and Horizontal plane is 0.25 Block B has a mass of 30kg and the coefficient of friction b/w it and the plane is 0.4. the wire b/w the two Blocks makes an angle 30° with the horizontal. Calculate the tension in the wire and acceleration of Blocks



⇒ Draw FBD of Block A and B in edyanmics equilibrium and apply condition of equilibrium

For Block B



$\Sigma F_y = 0$ (↑+ve)

$N_B + T\sin 30 = 30g$

$N_B + T(0.5) = 30 \times 9.8$

$N_B + (0.5)T = 294.3$

$\boxed{N_B = 294.3 - (0.5)T}$

$\Sigma F_x = ma$ (→+ve)

$T\cos 30° - 0.4 N_B = 30a$

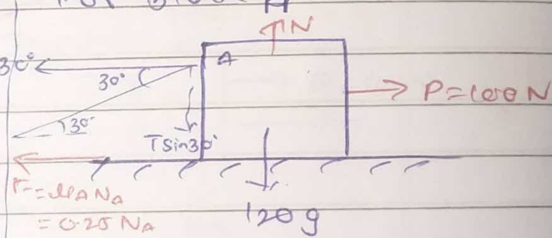
$T(0.86) - 0.4\left[294.3 - (0.5)T\right] = 30a$

$(0.86)T - 117.72 + 0.2T = 30a$

$\boxed{(1.06)T - 117.72 = 30a}$ ---(1)

$\boxed{1.06T - 30a = 117.72}$ — (1)

For Block A



$\Sigma F_y = 0$ (↑+ve)

$N_A - 120 \times 9.8 - T\sin 30° = 0$

$\boxed{N_A = (0.5)T + 1176}$

$\Sigma F_x = ma$ (→+ve)

$100 - T\cos 30° - 0.25 N_A = 120a$

$100 - T\cos 30° - 0.25(0.5T + 1176) = 120a$

$100 - T(0.86) - 0.125T - 294 = 120a$

$\boxed{-(0.9910)T - 194 = 120a}$ (2)

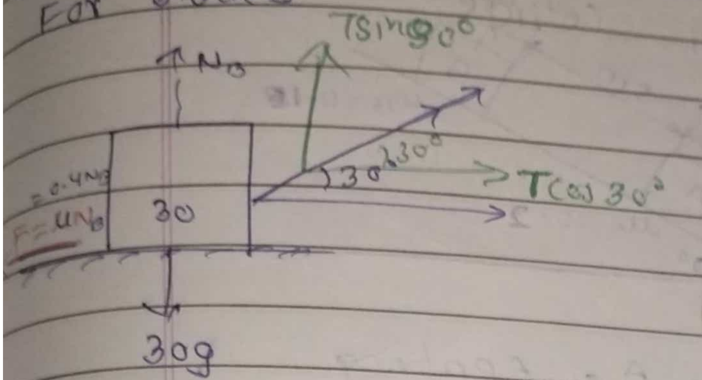
$-(0.9910)T - 120a = 194$ — (2)

$T = ?$ $a = ?$

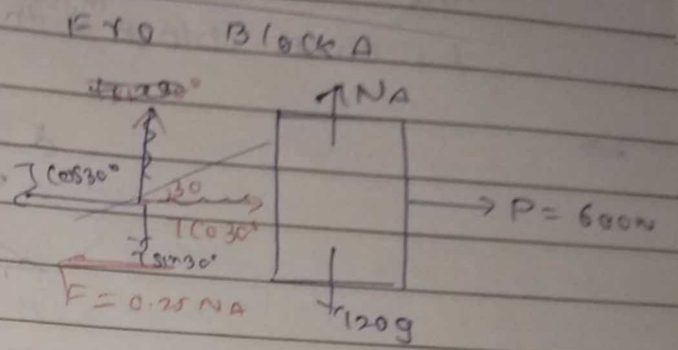
eq(1) and eq(2)

T = ?   a = ?

For Block B



For Block A



$\Sigma F_y = 0$ (+↑ve)

$N_B - 30g + T\sin 30° = 0$

$\boxed{N_B = 30 \times 9.8 - (0.5)T}$

$\boxed{N_B = 294 - (0.5)T}$ — (1)

$\Sigma F_x = ma$ (→ +ve)

$T\cos 30° - F \times 0.4 N_B = 30a$

$T(0.866) - 0.4(294 - 0.5(T)) = 30a$

$(0.86)T - (117.6) + 0.2T = 30a$

$\boxed{(1.06)T - 117.6 = 30a}$ — (1)

$1.06T - 30a - 117.6 = 0$ — (1)

$\Sigma F_y = 0$

$N_A = 120g - T\sin 30°$

$\boxed{N_A = 1176 - (0.5)T}$ — (1)

$\Sigma F_x = ma$

$600 - T\cos 30° - F = 120a$

$-(0.866)T - 0.25N_A + 600 = 120a$

$-(0.866)T - 0.25[1176 - (0.5)T] + 600 = 120a$

$-0.866T - 294 + 0.125T + 600 = 120a$

$-0.741T + 306 = 120a$

$-0.741T - 120a + 306 = 0$ — (2)

Solve simultaneously

By calculator

$T = 155.87\ N$ $a = 1.587$

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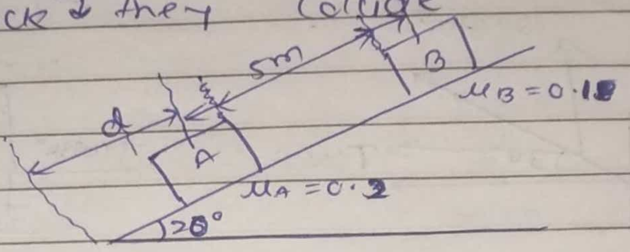
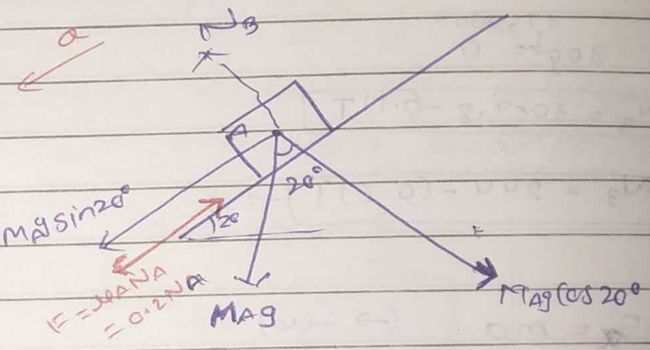
(1) Two Block A and B are placed 5m apart and are released simultaneously from rest. Calculate the time taken and the distance travelled by each block before they collide

⟹



Sol :- From FBD of Block A. Applying condition of dynamic equilibrium



$\Sigma F_x = m_A a$ (↙ +ve)

$M_A g \sin 20° - 0.2 N_A = M_A a_A$

$(3.35) M_A - 0.2 N_A = M_A a_A$

$N_A = \dfrac{M_A (3.35) - m_A a_A}{(0.2)}$

$\boxed{N_A = m_A (16.75 - 5a)}$ — (I)

$\Sigma F_y = 0$ (↑ +ve)

$N_A - m_A g \cos 20° = 0$

$\boxed{N_A = m_A (9.2)}$ — (II)

~~eq (I) and~~ (II) substituting value of N_A in eq (I)

$\cancel{m_A}(9.2) = \cancel{m_A}(16.75 - 5a_A)$

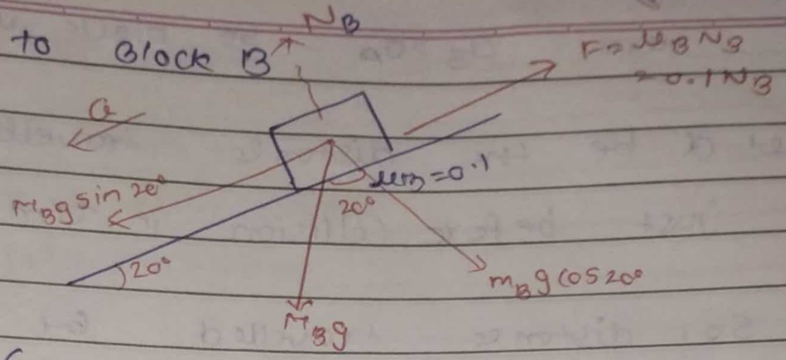
$9.2 = 16.75 - 5a_A$

$5a_A = 16.75 - 9.2$

$a_A = \dfrac{7.55}{5}\ m/s^2$

$a_A = 1.51\ m/s^2$

Similarly to Block B



$\Sigma F_x = m_B a_B$ ($\leftarrow$ +ve)

$$m_B g \sin 20° - 0.1 N_B = m_B a_B$$

$$(3.35)\, m_B - 0.1 N_B = m_B a_B$$

$$N_B = \frac{3.35 m_B - m_B a_B}{0.1}$$

$$\boxed{N_B = m_B (33.5 - 10 a_B)} \quad \text{— (III)}$$

$\Sigma F_y = 0$ ($\uparrow$ +ve)

$$N_B - m_B g \cos 20° =$$

$$N_B = m_B \times 9.8 \cos 20°$$

$$\boxed{N_B = 9.208 m_B} \quad \text{--- (IV)}$$

putting the value of N_B in eq (III)

$$(\cancel{m_B})\, 9.208 = \cancel{m_B} (33.5 - 10 a_B)$$

$$10 a_B = 33.5 - 9.208$$

$$= 24.29$$

$$\boxed{a_B = 2.429 \text{ m/s}^2}$$

Here $a_B > a_A$ So block will collide.

let 'd' be the distance travelled by Block A just before collision in time 't'

So, distance travelled by Block B will

For Block A:

$S_A = ut + \frac{1}{2}at^2$

$S_A = u_A t + \frac{1}{2}a_A t^2$

$d = 0 \times t \; (\frac{1}{2} \times 1.51 \times t^2)$

$\boxed{d = 0.755\,t^2}$ --- (i)

For Block B:

$S = ut + \frac{1}{2}at^2$

$S_B = u_B \times t + (\frac{1}{2} \times a_B t^2)$

$d+5 = 0 \times t + (\frac{1}{2} \times 2.43 \times t^2)$

$\boxed{d+5 = 1.215\,t^2}$ --- (ii)

Substituting d in eq (ii)

$0.755t^2 + 5 = 1.215t^2$

$5 = 1.215t^2 - 0.755t^2$

$5 = 0.46 \times t^2$

$10.869 = t^2$

$\sqrt{10.869} = t$

$\boxed{t = 3.296 \text{ sec}}$

putting the value t in eq (i)

$d = 0.755t^2$

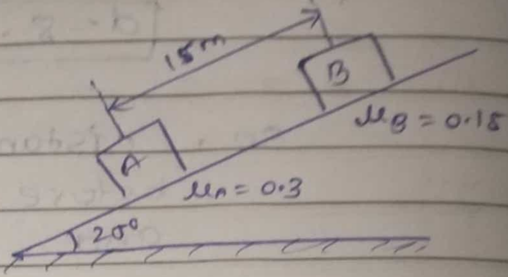
$d = 0.755 \times 10.869$

$\boxed{d = 8.206 m}$ ......

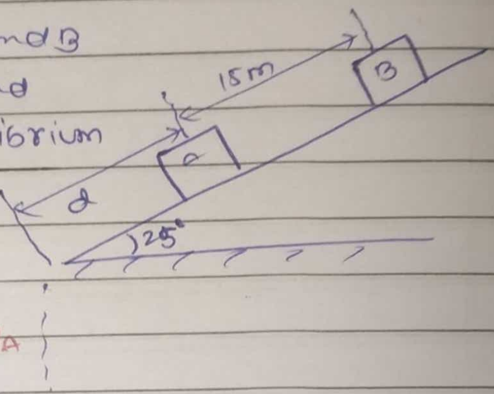
so, distance travelled by block A before collision is 8.206 m

and of block B is $(8.206 + 5) = 13.206$

(1) Two blocks A and B are placed 15 m apart and are released simultaneously from rest. Calculate the time taken and the distance travelled by each block before they collide

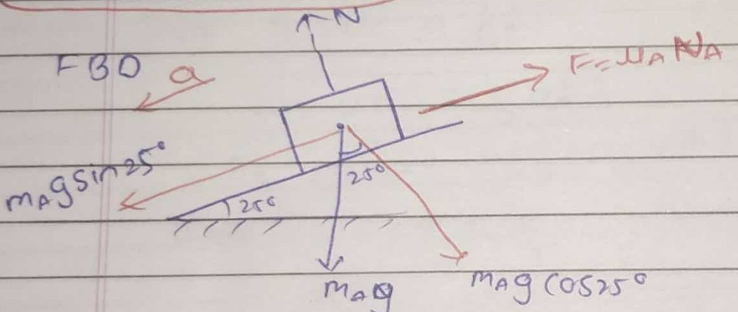


Sol: ⇒ Draw the FBD Block A and B in dynamic equilibrium and applying condition of equilibrium



From Block A

FBD



$\Sigma F_x = ma$ (← +ve)

$m_A g \sin 25° - 0.3 N_A = m_A a_A$

$m_A(4.14) - 0.3 N_A = m_A a_A$

$N_A = \dfrac{m_A(4.14 - a_A)}{0.3}$

$\boxed{N_A = m_A(13.8 - 3.33\, a_A)}$ — (I)

$\Sigma F_y = ma$ (↑+ve)

$\Sigma F_y = 0$ {No (a) in y direction}

$N_A - m_A g \cos 25° = 0$

$\boxed{N_A = m_A(8.88)}$ — (II)

Substituting the value N_A in eq (I)

$\cancel{m_A}(8.88) = \cancel{m_A}(13.8 - 3.33\, a_A)$

$3.33\, a_A = 13.8 - 8.88$

$a_A = \dfrac{4.92}{3.33}$

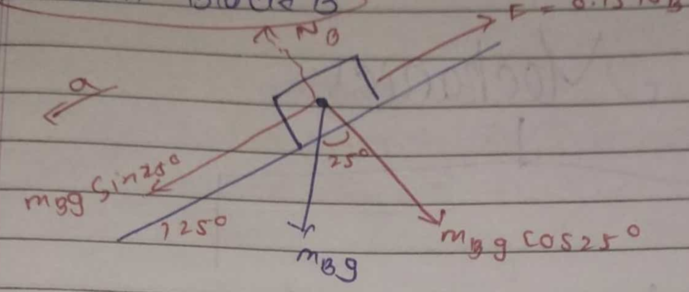
$a_A = 1.476\ m/s^2$

putting value t in eq (vi)

$d = 0.738 \times 22.15$

$d = 16.3467$

From Block B



$\Sigma F_x = ma$ ($\leftarrow$ +ve)

$m_B g \sin 25° - 0.15 N_B = m_B a_B$

$m_B (4.14) - m_B a_B = 0.15 N_B$

$N_B = m_B \dfrac{(4.14 - a_B)}{0.15}$

$\boxed{N_B = m_B (27.61 - 6.67 a_B)}$ --- (III)

($\uparrow$ +ve)

$\Sigma F_y = 0$ {No acceleration in y-direction}

$N_B - m_B g \cos 25° = 0$

$N_B = m_B (8.88)$ --- (IV)

Substituting the value N_B in equation (III)

$\cancel{m_B} (8.88) = \cancel{m_B} (27.61 - 6.67 a_B)$

$6.67 a_B = 27.6 - 8.88$

$a_B = \dfrac{18.71}{6.67}$ $\boxed{a_B = 2.83 \ m/s^2}$

Here $a_B > a_A$, so block will collide

Let 'd' be the distance travelled by block A just before collision in time 't'

So, distance travelled by block B will

For Block A

$S_A = u_A t + \frac{1}{2} a_A t^2$

$d = 0 + \frac{1}{2} 1.476 t^2$

$\boxed{d = 0.738 t^2 m}$ --- (vi)

For Block B

$S_B = u_B t + \frac{1}{2} a_B t^2$

$d + 15 = 0 + \frac{1}{2} 2.83 t^2$ --- (V)

putting the value (d) in eq (V)

$0.738 t^2 + 15 = (1.415) t^2$

$15 = 0.677 t^2$

$t^2 = 22.15$

$t = \sqrt{22.15}$

$\boxed{t = 4.706 \text{ sec}}$ $\rightarrow$ distance

So, distance travelled by Block A before collision is (16.34) m and distance travelled by Block B is $(16.34 + 15) = 31.34$ m

Module :- 6

(1) Define Kinetics

It is the branch of Dynamics, that deals with the study of motion ~~and~~ considering the forces causing the motion.

(2) Differentiate between kinematics and kinetics with the examples.

Kinetics	Kinematics.
It refers to the study of the forces that causes the motion	It refers to the study of the path and property of the object during motion
ex torque, gravity, friction	displacement, time, velocity

(3) State Newton's Second Law

The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object.

$$F = ma$$

$$\therefore a = \frac{F}{m}$$

(4) Explain D'Alembert's Principle
The reaction due to the inertia of an accelerated body is equal and opposite to the force causing the acceleration and results in a condition of dynamic equilibrium.

we know that $\Sigma F = ma$ NSL

Applicatn of NSL
$$\left.\begin{aligned} \Sigma F_x &= m a_x \\ \Sigma F_y &= m a_y \\ \Sigma F_z &= m a_z \end{aligned}\right\} \text{for rectangular co-ordinate system}$$

&
$$\left.\begin{aligned} \Sigma F_t &= m a_t \\ &= m \frac{dv}{dt} \\ \Sigma F_n &= m a_n \\ &= m \frac{v^2}{\rho} \end{aligned}\right\} \text{Curvilinear Motion}$$

(5) what is work energy principle.
The principle of work energy (also known as the work and kinetic energy principle) states that the work done by all the forces acting on a particle (the work of the resultant force) equals the change in the kinetic energy of the particle.

$$U_{1-2} = KE_2 - KE_1$$

(6) What is principle of impulse momentum
The principle of impulse momentum states that the impulse acting on an object equals the change in momentum of the object.

$\therefore$ Impulse $= mv_2 - mv_1$

(7) What is coefficient of restitution

$$e = \frac{\text{impulse during restoration period}}{\text{impulse during deformation period}}$$

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} \qquad e = \sqrt{\frac{h_2}{h_1}}$$

Where, v_1 & v_2 are the velocities of particle 1 and 2 immediately after impact
u_1 & u_2 are the velocities of particle 1 & 2 just before impact

(8) What is dynamic equilibrium
Dynamic equilibrium is a state of the body that continues to move with uniform velocity.
In this case, the net external force and torque on the body are equal to 0

(9) Define impulse & state its formula.
impulse:- when a large force acts over a short period of time, that force is called as impulsive force

I] $J = \int_{t_1}^{t_2} F \, dt$ (Ns)

II] $J = F_{average} \times \Delta t$ (Ns)

(10) what are the types of impact:
Impact:- A collision of two bodies which takes place during a very small interval of time & during which the colliding bodies exert relatively larger force on each other is known as impact

Types. (1) Central impact
(2) Eccentric impact
(3) Direct central impact
(4) oblique central impact.