Module - 6

Numerical Integration

~~Trapezoidal~~

Forward difference operator $\Delta : \Delta f(x) = f(x+h) - f(x)$

Backward difference operator $\nabla : \nabla f(x) = f(x) - f(x-h)$

ex:- (1) $\Delta^2(\sin x) = \Delta(\Delta \sin x)$

$= \Delta(\sin(x+h) - \sin x)$

$= \Delta(\sin(x+h)) - \Delta \sin x)$

$= \sin(x+h+h) - \sin(x+h) - (\sin(x+h) - \sin x)$

$= \sin(x+2h) - \sin(x+h) - \sin(x+h) - \sin x$

$= \sin(x+2h) - 2\sin(x+h) - \sin x$

(2) $\Delta e^{ax} = e^{a(x+h)} - e^{ax}$

$= e^{ax+ah} - e^{ax}$

$= e^{ax} \cdot e^{ah} - e^{ax}$

$= e^{ax}(e^{ah} - 1)$

(3) $\Delta^2 e^{x} = \Delta(\Delta e^{x})$

$= \Delta(e^{(x+h)} - e^{x})$

$= \Delta e^{(x+h)} - \Delta e^{x}$

$= \Delta e^{(x+2h)} - \Delta e^{(x+h)} - (\Delta e^{(x+h)} - \Delta e^{x})$

$= e^{(x+2h)} - 2e^{(x+h)} - e^{x}$

Imp

(3) $\Delta\nabla = \nabla\Delta$

consider $\Delta\nabla y_n = \Delta(y_n - y_{n-1})$

$= \Delta y_n - \Delta y_{(n-1)}$

$= y_{(n+1)} - y_n - (y_n - y_{(n-1)})$

$= y_{(n+1)} - 2y_n - y_{(n-1)}$ ... (i)

consider $\nabla\Delta y_n = \nabla(\Delta y_n)$

$= \nabla(y_{(n+1)} - y_n)$

$= \nabla\{y_{(n+1)}\} - \nabla y_n)$

$= y_{(n+1)} - y_n - (y_n - y_{(n-1)})$ ... (ii)

$= y_{(n+1)} - 2y_n - y_{(n-1)}$ ... (ii)

from (i) and (ii) we get $\Delta\nabla = \nabla\Delta$

(4) Show that $(1+\Delta)(1-\nabla)=1$

To prove $(1+\Delta)(1-\nabla)=1$

enough to prove $\boxed{(1+\Delta)(1-\nabla)f(x)=f(x)}$

consider $(1+\Delta)(1-\nabla)f(x)$

$= (1+\Delta)[f(x) - \nabla f(x)]$

$= (1+\Delta)[f(x) - (f(x) - f(x-h))]$

$= (1+\Delta)[\cancel{f(x)} - \cancel{f(x)} + f(x-h)]$

$= (1+\Delta) f(x-h)$

$= f(x-h) + \Delta f(x-h)$

$= \cancel{f(x-h)} + f(x) - \cancel{f(x-h)}$

$= f(x)$

(5) $\Delta^2(1/x) = \Delta\Delta(1/x)$

$= \Delta[\Delta(1/x)]$

$= \Delta\left[\frac{1}{(x+h)} - \frac{1}{x}\right]$ ...where $h=1$

$= \Delta\left[\frac{1}{x+1} - \frac{1}{x}\right]$

$= \Delta\left[\frac{1}{x+1} - \frac{1}{x}\right]$

$= \Delta\left(\frac{1}{x+1}\right) - \Delta\left(\frac{1}{x}\right)$

$= \left[\frac{1}{((x+1)+1)} - \frac{1}{(x+1)}\right] - \left[\frac{1}{x+1} - \frac{1}{x}\right]$ where $h=1$

$= \frac{1}{(x+2)} - \frac{1}{(x+1)} - \frac{1}{(x+1)} - \frac{1}{x}$

$= \frac{1}{(x+2)} - \frac{2}{(x+1)} + \frac{1}{x}$

8)

$$\Delta^2(\cos 2x)$$

$$= \Delta(\Delta\cos 2x)$$

$$= \Delta[\cos 2(x+h) - \cos 2x]$$

$$= \Delta\cos 2(x+h) - \Delta\cos 2x$$

$$= \Delta\cos 2(x+2h) - \cos 2(x+h) - [\cos 2(x+h) - \cos 2x]$$

$$= \cos 2(x+2h) - 2\cos 2(x+h) - \cos 2x$$ //

(5 m)

Rectangular Rule :-

To evaluate $\int_a^b f(x)\,dx$ approximately

(1) partitian $[a, b]$ into n-subintervals

(2) Find $h = \frac{b-a}{n}$ → step-size or interval of differenting

(3) Find midpoint for each sub interval

starting point → $a = x_0$

end point → $b = x_n$

$x_1 = x_0 + h$, $x_2 = x_1 + h$, ... and so on.

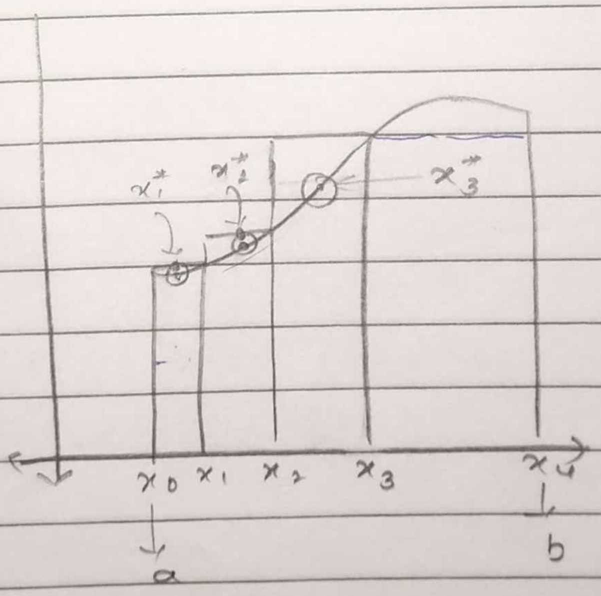
$[x_0, x_1]$; $[x_1, x_2]$; $[x_2, x_3]$ .... $[x_{n-1}\ x_n]$

Say x_1^* - mid point of $[x_0, x_1]$

x_2^* - mid point of $[x_1\ x_2]$

~~(4) Find funct~~

(4) Graph



$$\int_0^4 x^2 = \int_0^1 x^2 + \int_1^2 x^2 + \int_2^4 x^2 + \int_3^4 x^2$$

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(5) Find functional value of each of the midpoint i.e Find $f(x_1^*)$, $f(x_2^*)$ ---- $f(x_n^*)$

$\boxed{\text{rectangular rule}}$ says

$$\int_a^b f(x)\,dx \approx h\left[f(x_1^*) + f(x_2^*) + \cdots\cdots + f(x_n^*) \right]$$

$$\approx h \sum_{i=1}^{n} f(x_i^*)$$

eg.
(1) Evaluate integer $\int_{a \to 0}^{b \to 10} \frac{1}{1+x^2}\, dx$ using rectongular rule

$\Rightarrow$ here $a = 0$ $b = 10$, $f(x) = \frac{1}{1+x^2}$ ~~$f(x^*) = \frac{1}{1+x^2}$~~ ~~$f(x^*) = \frac{1}{1+(x^*)^2}$~~ $\frac{1}{1+(0.5)^2}$

calculate, $h = \frac{b-a}{n} = \frac{10-0}{10} = \underline{1}$, where $n = 10$ $\downarrow$ no of subinterval

$\int^{11}$

$\frac{1}{}$

x —	x_0	$x_1 = x_0 + h$	$x_2 = x_1 + h$	....	x_n
	0	1	2 $\frac{0+1}{2}$	....	10

$\int_{6}^{16}$ $\frac{8+9}{2}$

$\frac{0+1}{2} = 0.5$

$\frac{1+2}{2} = 1.5$

<u>midpoint</u>

$x_1^* =$ midpoint of $[0, 1] = 0.5$

$x_2^* =$ midpoint of $[1, 2] = 1.5$

$x_3^* =$ midpoint of $[2, 3] = 2.5$

$\vdots$

$x_{10}^* =$ midpoint of $[9, 10] = 9.5$

x_1^* x_2^*

x_i^*	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5
$f(x_i^*)$	0.8	0.3076	0.1379	0.0754	0.0470	0.032	0.0236	0.0174	0.0136	0.0109

upto 4 digit

Now,

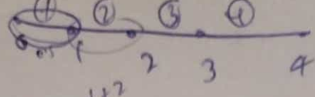
applying rectangle rule we have

$$\int_0^{10} \frac{1}{1+x^2}\, dx \approx h\left[f(x_1^*) + f(x_2^*) + \cdots + f(x_{10}^*)\right]$$

$$\approx 1 \times \left[\begin{array}{l} 0.8 + 0.3076 + 0.1379 + 0.0754 + 0.047 \\ 0.0321 + 0.023 + 0.0174 + 0.0136 + 0.0109 \end{array}\right]$$

$$\approx 1.4646$$

$\frac{0+1}{2} = 0.5$



(2) Evaluate $\int_0^4 x^3 \, dx$ by using rectangle rule. divide the interval into four equal parts

$\Rightarrow$ Here $a = 0$ $b = 4$, $f(x) = x^3$

Calculate, $h = \frac{b-a}{n} = \frac{4-0}{4} = 1$, whereen $n = 4$ (no. of subinterval)

$x \rightarrow x_0$	$x_1 = x_0 + h$	$x_2 = x_1 + h$	---	x_n
0	1	2		4

midpoint $x_1^* =$ midpoint of $[0, 1] = 0.5$

$x_2^* =$ mid pont of $[1, 2] = 1.5$

$x_3^* =$ — " — $[2, 3] = 2.5$

$x_4^* =$ — " — $[3, 4] = 3.5$

Now

x_i^*	x_1^*	x_2^*	x_3^*	x_4^*
$f(x_i^*)$	0.125	3.375	15.625	42.875

Now, apply rectangle rule we have,

$$\int_0^4 x^3 \, dx \approx h \left[f(x_1^*) + f(x_2^*) + f(x_3^*) + f(x_4^*) \right]$$

$$\approx 1 \left[0.125 + 3.375 + 15.625 + 42.875 \right]$$

$$\approx 1 (62)$$

$$\approx \underline{62}$$

3) Evaluate $\int_0^8 x^3\,dx$ by rectangle rule

→ Here $a=0$, $b=8$, $f(x)=x^3$

Calculate $h = \frac{b-a}{n} = \frac{8-0}{8} = 1$

<u>midpoint</u> $x_1^* =$ midpoint of $[0,1] = 0.5$

$x_2^* =$ — " — $[1,2] = 1.5$

$x_3^* =$ — " — $[2,3] = 2.5$

$\vdots$

$x_8^* =$ — " — $[7,8] = 7.5$

x_i^*	$x_1^* = 0.5$	$x_2^* = 1.5$	$x_3^* = 2.5$	$x_4^* = 3.5$	$x_5^* = 4.5$	$x_6^* = 5.5$	$x_7^* = 6.5$	$x_8^* = 7.5$
$f(x_i^*)$	0.125	3.375	15.625	42.875	91.125	168.375	274.625	421.875

$y=f(x)$

1016

Now, applying rectangle rule

$$\int_0^8 x^3\,dx = h\left[f(x_1^*) + f(x_2^*) + f(x_3^*) + f(x_4^*) + \cdots f(x_8^*)\right]$$

$$= 1\times\left[0.125 + 3.375 + 15.625 + 42.875 + 91.125 + 166.375 + 274.625 + 421.875\right]$$

$$= \underline{\underline{1016}}$$

4) Evaluate $\int_0^2 x^2\,dx$ using rectangle rule with four subinterval

⇒ Here $a=0$ $b=2$, $f(x)=x^2$, $n=4$

Calculate $h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$.

midpoint $x_1^* =$ mid point of $[0, 0.5] = 0.25$

$x_2^* =$ — 11 — of $[0.5, 1] = 0.75$

$x_3^* =$ — 11 — of $[1, 1.5] = 1.25$

$x_4^* =$ — 11 — of $[1.5, 2] = 1.75$

x_i^*	0.25	0.75	1.25	1.75
$f(x_i^*)$	0.0625	0.5625	1.5625	3.0625

5.25

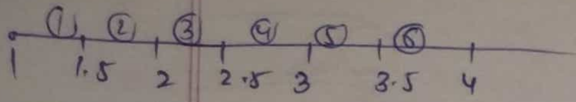
Now, applying rectangle rule

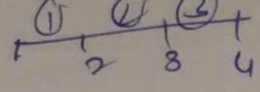
$$\int_0^2 x^2\,dx = h\left[f(x_1^*) \;\cdots\; + f(x_4^*)\right]$$

$$= 0.5\left[0.0625 + 0.5625 + 1.5625 + 3.0625\right]$$

$$= 0.5\,[5.25]$$

$$= 2.625$$





(5) Evaluate $\int_1^4 (3x+1)\,dx$ by using rectangle rule

Here $a = 1$, $b = 4$, $f(x) = (3x+1)$

$h = \frac{b-a}{n} = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2} = 0.5$ where $n = 6$

The midpoint values for the six subinterval

$x_1^* =$ midpoint of $[1, 1.5] = 1.25$

$x_2^* =$ —11— $[1.5, 2] = 1.75$

$x_3^* =$ —11— $[2, 2.5] = 2.25$

$x_4^* =$ —11— $[2.5, 3] = 2.75$

$x_5^* =$ —11— $[3, 3.5] = 3.25$

$x_6^* =$ —11— $[3.5, 4] = 3.75$

	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*
x_i^*	1.25	1.75	2.25	2.75	3.25	3.75
$f(x_i^*)$	(4.75)	(6.25)	(7.75)	(9.25)	10.75	12.25
	$f(x_1^*)$	$f(x_2^*)$	$f(x_3^*)$	$f(x_4^*)$	$f(x_5^*)$	$f(x_6^*)$

Now applying Rectangle rule we have

$$\int_0^4 (3x+1)\,dx \approx h\left[f(x_1^*) + f(x_2^*) + f(x_3^*) + f(x_4^*) + f(x_5^*) + f(x_6^*)\right]$$

$$= 0.5\left[(4.75) + (6.25) + (7.75) + (9.25) + (10.75) + (12.25)\right]$$

$$= \frac{51}{2}$$

$$= 25.5$$

0 — 30 — 60 — 90 — 120° — 150 — 180

① ② ③ ④ ⑤ ⑥

(2) Estimate the area under the curve of the function $h(x) = \sin x$ from $x=0$ to $x=\pi$ using the rectangle method with six rectangles

=> Here $a=0$, $b=\pi$, $h(x) = \sin x$

$$h = \frac{b-a}{n} = \frac{\pi - 0}{6} = \pi/6 = 30^\circ \quad \text{where } n=6$$

The midpoint values for the six subinterval

$x_1^* =$ midpoint of $[0, \pi/6] = \pi/12$
$x_2^* =$ —11— $[\pi/6, \pi/3] = 3\pi/12$
$x_3^* =$ —11— $[\pi/3, \pi/2] = \frac{5\pi}{12}$
$x_4^* =$ —11— $[\pi/2, \frac{2\pi}{3}] = 7\pi/12$
$x_5^* =$ —11— $[\frac{2\pi}{3}, \frac{5}{6}\pi] = 9\pi/12$
$x_6^* =$ —11— $[\frac{5\pi}{6}, \pi] = \frac{11\pi}{12}$

x_i^*	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*
	$\pi/12$	$3\pi/12$	$5\pi/12$	$7\pi/12$	$9\pi/12$	$11\pi/12$
$f(x_i^*)$	$f(x_1^*)$	$f(x_2^*)$	$f(x_3^*)$	$f(x_4^*)$	$f(x_5^*)$	$f(x_6^*)$
	4.559×10^{-3} 0.00456	0.0137	0.0228	0.0319	0.0411	0.0502

we know applying rectangle rule, we have

$$\int_0^\pi \sin x \, dx \approx h\left[f(x_1^*) + f(x_2^*) \cdots f(x_6^*)\right]$$

$$= \frac{\pi}{6}\left[\begin{matrix}0.0045 + 0.0137 + 0.0228 \\ +0.0319 + 0.0411 + 0.0502\end{matrix}\right]$$

$$= \frac{\pi}{6}(0.1642) = 4.926$$

Trapezoidal rule

To evaluate $\int_a^b f(x)\,dx$

(1) Divide interval $[a, b]$ into $\boxed{n}$ sub-intervals

(2) and stepsize $\boxed{h = \frac{b-a}{n}}$

(3) let's set $x_0 = a$; $b = x_n$

x_0	x_1	x_2 ----	x_n
y_0	y_1	y_2 ----	y_n

(4) integeral of x_0 to x_1

$$\int_{x_0}^{x_1} f(x)\,dx = h\left(y_0 + \frac{1}{2}\Delta y_0\right)$$

$$= h\left(y_0 + \frac{1}{2}(y_1 - y_0)\right)$$

$$= hy_0 + \frac{hy_1}{2} - \frac{hy_0}{2}$$

$$= \frac{hy_0}{2} + \frac{hy_1}{2}$$

$$= \frac{h}{2}(y_0 + y_1)$$ → area of first trapezium

(5) similary $\int_{x_1}^{x_2} f(x)\,dx = \frac{h}{2}(y_1 + y_2)$ → area of 2^{nd} trapezium

In general $\int_{x_{n-1}}^{x_n} f(x)\,dx = \frac{h}{2}(y_{n-1} + y_n)$ → n^{th} trapezium

$\therefore$ adding all the area of trapezium

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[(y_0+y_1)+(y_1+y_2)+\cdots+(y_{n-1}+y_n)\right]$$

$$\boxed{\int_a^b f(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2\,(y_1+y_2+\cdots\cdots y_{n-1})\right]}$$

$$\boxed{=\frac{h}{2}\left[\underbrace{X}_{\text{sum of extereme ordinates}} + \underbrace{2R}_{\text{sum of remaining ordinates}}\right]}$$

(y_0+y_6) $\quad 2(x\cdots\cdots x)$

$$\int_a^b f(x) = \frac{h}{2}\left[X + 2R\right]$$

0.2 0.4 0.6 0.8 1.0 1.2 1.4

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(1) Evaluate

$$\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\, dx$$ by using Trapezoidal rule.

$\Rightarrow$ Here $a = 0.2$, $b = 1.4$, $f(x) = (\sin x - \log_e x + e^x)$

let $n = 6$ = no. of subinterval

$$h = \frac{b-a}{n} = \frac{1.4 - 0.2}{6} = 0.2$$

x	$x_0 = 0.2$	$x_1 = 0.4$	$x_2 = 0.6$	$x_3 = 0.8$	$x_4 = 1.0$	$x_5 = 1.2$	$x_6 = 1.4$
y	$y_0 = 2.8343$	$y_1 = 2.4150$	$y_2 = 2.3434$	$y_3 = 2.4626$	$y_4 = 2.7357$	$y_5 = 3.1587$	$y_6 = 3.7431$

$\therefore$ By Trapezoidal rule

x_0

x_n

$$\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\, dx = \frac{h}{2}\left[X + 2R\right]$$

$$= \frac{0.2}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$= 0.1\left[(2.8343 + 3.7431) + 2(2.4150 + 2.3434 + 2.4626 + 2.7357 + 3.1587)\right]$$

$$= 0.1\left[6.5774 + 2(13.1154)\right]$$

$$= 0.1\left[6.5774 + 26.2308\right]$$

$$= \cancel{(19.6928) \times 0.1} = (32.8082) \times 0.1$$

$$\cancel{= 1.969} \qquad = 3.28082$$

(2) Evaluate $\int_{-1}^{1} \frac{1}{1+x^2} dx$ using Trapezoidal rule

=> Here $a = -1$ $b = 1$ $f(x) = \frac{1}{1+x^2}$

let $n = 2$ = no. of subinterval

$h = \frac{b-a}{n} = \frac{1+1}{2} = \frac{\cancel{2}}{\cancel{2}} = 1$

x	$x_0 = -1$	$x_1 = 0$	$x_2 = 1$
y	$y_0 = 0.5$	$y_1 = 1$	$y_2 = 0.5$

$\therefore$ By trapezoidal rule

$$\int_{-1}^{1} \frac{1}{1+x^2} dx = \frac{h}{2}\left[X + 2R\right]$$

$$= \frac{1}{2}\left[(y_0 + y_2) + 2(y_1)\right]$$

$$= 0.5\left[1 + 2(1)\right] = 0.5\ (3)$$

$$= 1.5$$

0 ①, ② , ③ , 4 , ⑤ ⑥ 180

30 60 90 120 150

$\frac{\pi}{6}$

③ $\int_0^{\pi/2} \frac{\sin x}{x}\, dx$ by using trapezoidal rule

$\frac{\pi - 0}{6}$

$\Rightarrow$ Here $a = 0$ $b = \pi/2$; $f(x) = \frac{1}{1+x^2}$

$h = 20/6$

let $\boxed{n = 6}$

step size

In degree

$h = \frac{b-a}{n} = \frac{\pi/2 - 0}{6} = \frac{\pi}{12} = {}^{\circ}$

In radian

15°

x	0	π/12	2π/12	3π/12	4π/12	5π/12	6π/12
Y	1	0.0172	0.0166	0.0157	0.0144	0.0128	0.0111

calculator not give answer

↳ manual find

as $\lim_{x \to 0} \frac{\sin x}{x} = 1$

By using Trapezoidal rule

$$\int_0^{\pi/2} \frac{\sin x}{x}\, dx = \frac{h}{2}\left[X + 2R \right]$$

$$= \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5) \right]$$

$$= \frac{\pi/12}{2}\left[(1 + 0.0111) + 2\left(0.0172 + 0.0166 + 0.0157 + 0.0144 + 0.0128 \right) \right]$$

$$= \frac{\pi}{24}\left[1.0011 + 2(0.0767) \right]$$

$$= \frac{\pi}{24}\left[1.0111 \quad 0.1534 \right]$$

$$= \frac{\pi}{24}\left[1.1645 \right]$$

$$= 0.15243 \quad ?$$

0.1 0.25

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(4) $\int_0^1 \frac{x^2}{1+x^3} dx$ by using trapezoidal rule

$\Rightarrow$ Here $a=0$, $b=1$, $f(x) = \frac{x^2}{1+x^3}$

let $\boxed{n=4}$ = no. of subinterval

$h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$

we prepare the following table

	x_0	x_1	x_2	x_3	x_4
x	0	0.25	0.50	0.75	1.0
y	0	0.0615	0.2222	0.3956	0.5
	y_0	y_1	y_2	y_3	y_4

By Trapezoidal rule

$$\int_0^1 \frac{x^2}{1+x^3} dx \approx \frac{h}{2}\left[x + 2R \right]$$

$$\approx \frac{h}{2}\left[(y_0+y_4) + 2(y_1+y_2+y_3) \right]$$

$$= \frac{0.25}{2}\left[0.5 + 2\left(\frac{0.0615+0.2222+0.3956}{\cancel{0.25+0.5+0.75}} \right) \right]$$

$$\cancel{= \frac{0.25}{2}(0.5 + 3)}$$

$$\cancel{= \frac{0.25}{2}3.5}$$

$$= \frac{0.25}{2}\left[0.5 + 2(0.6793) \right]$$

$$= \frac{0.25}{2}\left[0.5 + (1.3586) \right]$$

$$= \frac{0.25 \times 1.8586}{2}$$

$$= \frac{0.46465}{2}$$

$$= 0.23233$$

$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$

(4) $\int_a^b$ $\int_{-3}^{3} x^4 \, dx$ by using Trapezoidal rule

$\Rightarrow$ Here $a = -3$, $b = 3$, $f(x) = x^4$

$h = \frac{b-a}{n} = \frac{3-(-3)}{6} = \frac{6}{6} = 1$ where $n = 6$

we prepare the following table

X	x_0	x_1	x_2	x_3	x_4	x_5	x_6
	-3	-2	-1	0	1	2	3
Y	81	16	1	0	1	16	81
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

we now, applying Trapezodial rule

$$\int_{-3}^{3} x^4 \, dx \approx \frac{h}{2}\left[\cancel{f(x)(4f(x)} \quad x+2R \right]$$

$$= \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5) \right]$$

$$= \frac{1}{2}\left[(81+81) + 2(16+1+0+1+16 \right]$$

$$= \frac{1}{2}\left[162 + 2(34) \right]$$

$$= \frac{1}{2}\left[162 + 68 \right]$$

$$= \frac{(230)}{2} = 115$$

① Find using the trapezoidal rule from the following table the area bounded by the curves and x-axis from $x = 7.47$ to $x = 7.52$.

x	7.47	7.48	7.49	7.50	7.51	7.52
F(x)	1.93	1.95	1.98	2.01	2.03	2.06

=> Here $a = 7.47$ $b = 7.52$

$$h = \frac{b-a}{6} = \frac{7.52 - 7.47}{6} = \frac{1}{120}$$

we applying Trapezoidal rule

$$\int_{7.47}^{7.52} f(x)\,dx = \frac{h}{2}\left[X + 2R\right]$$

$$= \frac{1}{240}\left[(2.06 + 1.93) + 2(1.95 + 1.98 + 2.01 + 2.03)\right]$$

$$= \frac{1}{240}\left[3.99 + 2(6.97)\right]$$

$$= \frac{1}{240}\left[3.99 + 13.94\right]$$

$$= \frac{21.83}{240}$$

$$= 0.0995$$

$1 ft = 30.48 cm$

(2) ~~Find~~ the velocity of a particle which starts from rest is given by the following table

t/sec	0	2	4	6	8	10	12	14	16	18	20
v (ft/sec)	0	16	29	40	46	51	32	18	8	3	0

Evaluate using Trapezium rule, the total distance travelled in 20 seconds.

$\Rightarrow$ Here $a = 0$ $b = 20$ $S = \int_0^{20} dv\,dt$

$h = \frac{b-a}{n} = \frac{20-0}{10} = 2$ where $\boxed{n = 10}$

we prepare the following table (ft/sec) to (cm/sec)

t/sec	0	2	4	6	8	10	12	14	16	18	20
v (cm/sec)	0	487.68	883.92	182.88	243.84	304.8	365.76	548.64	243.84	91.44	0
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

~~By~~ Trapezium rule

$$\int_0^{20} dv\,dt = \frac{h}{2}[X + 2R]$$

$$= \frac{h}{2}\left[(y_0 + y_{10}) + 2\left(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9\right)\right]$$

$$= \frac{\cancel{2}}{\cancel{2}}\left[0 + 2\left(487.68 + 883.92 + 182.88 + 243.84 + 304.8 + 365.76 + 548.64 + 243.84 + 91.44\right)\right]$$

$$= 2(3352.8)$$

$$= 6705.6 \text{ cm/sec}$$

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Note :- for only (n) no. of subinterval are $\boxed{\text{even}}$

Simpson's 1/3rd Rule :-

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[X + 2E + 4O \right]$$

sum of even ordinates

sum of extreme ordinates

sum of odd ordinates

x_i

Example :

① Evalute $\int_{0.2}^{1.4} \underbrace{(6\sin x - \log_e x + e^x)}_{f(x)}\, dx$ by using simpson's $1/3^{rd}$ rule.

⇒ Here $a = 0.2$, $b = 1.4$

let $n = 6$ = no. of subinterval

$$h = \frac{b-a}{n} = \frac{1.4 - 0.2}{6} = \frac{1.2}{6} = 0.2$$

x	$x_0 = 0.2$	$x_1 = 0.4$	$x_2 = 0.6$	$x_3 = 0.8$	$x_4 = 1.0$	$x_5 = 1.2$	$x_6 = 1.4$
y	$y_0 = 2.8343$	$y_1 = 2.415$	$y_2 = 2.3484$	$y_3 = 2.4626$	$y_4 = 2.7357$	$y_5 = 3.1587$	$y_6 = 3.7431$

Now. apply simpson's $1/3^{rd}$ rule

$$\int_{0.2}^{1.4} (\sin x - \log_e x + e^x) = \frac{h}{3}\left[X + 2E + 4O\right]$$

even no. of y

odd no. of y

$$= \frac{h}{3}\left[(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)\right]$$

$$= \frac{0.2}{3}\left[\begin{pmatrix}2.8343 \\ + 3.7431\end{pmatrix} + 2\begin{pmatrix}2.3484 \\ + 2.7357\end{pmatrix} + 4\begin{pmatrix}2.415 + 2.4626 \\ + 3.1587\end{pmatrix}\right]$$

$$= \frac{0.2}{3}\left[6.5774 + \cancel{10.3014}\ 10.1582 + 32.1428\right]$$

$$= \frac{0.2}{3} \times 48.8784 = \frac{9.77568}{3}$$

$$= 3.25856$$

(3) The velocity of train which starts from rest is given by the following table, the time being reckoned in minutes from the start and speed in km/hr

Time	3	6	9	12	15	18
velocity	22	29	31	20	4	0

Estimate approximately the distance covered in 18 minutes by (i) Trapezoidal rule
(ii) Simpson's $\frac{1}{3}^{rd}$ rule
(iii) Simpson's $\frac{3}{8}^{th}$ rule

$\Rightarrow$

Time	0	3	6	9	12	15	18
velocity	0	22	29	31	20	4	0
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Since, train starts at rest, here, $h = \frac{b-a}{n} = \frac{18-0}{6} = 3$

$= 3$ min

imp $\boxed{\text{Convert into hour}}$

$= 3/60$

(i) By Trapezoidal rule

$$\int_0^{18} y\,dx = \frac{h}{2}\left[X + 2R\right]$$

Indicates the velocity distance

$$= \frac{3/60}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$\int$ velocity = distance $= \frac{3}{120}\left[(0+0) + 2(22+29+31+20+4)\right]$

$$= \frac{\cancel{3}}{120}\left[\quad 2(106)\right]$$

$$= \frac{1}{40} \times \cancel{2} \times (106) = \frac{106}{20} = \frac{53}{10} = \boxed{5.30000 \text{ km}}$$

velocity = $\frac{\text{distance}}{\text{time}}$ ~~distance = velocity × time~~

~~$= 5.3 \times \frac{3}{60}$~~

~~$= 0.$~~

Since, speed = $\frac{\text{distance}}{\text{time}}$

i.e $\frac{dv}{dt}$ = acceleration

$\boxed{\int v\,dt = \text{distance}}$

Notes
velocity $= \frac{d}{dt}(\text{distance})$
$\int v\,dt =$ distance

(ii) By Simpson $\frac{1}{3}^{rd}$ rule

$$\int_0^{18} y\,dx = \frac{h}{3}\left[X + 2E + 4O\right]$$

$$= \frac{3}{60 \times 3}\left[(y_0+y_6) + 2(y_2+y_4) + 4(y_1+y_3+y_5)\right]$$

$$= \frac{1}{60} \times \left[(0+0) + 2(29+20) + 4(22+31+4)\right]$$

$$= \frac{1}{60} \times \left[2(49) + 4(\quad 57)\right]$$

$$= \frac{1}{60}\left[98 \quad + 228\right]$$

$$= \frac{326}{60}$$

$= 5.433$ km

(iii) By Simpson $3/8^{th}$ rule

$$\int_0^{y} y\,dx = \frac{3h}{8}\left[X + 2T + 3R\right]$$

$$= \frac{3 \times 3}{8\,60}\left[(y_0+y_6) + 2(y_3) + 3\left(\frac{y_1+y_2+y_4}{y_5}\right)\right]$$

$$= \frac{9}{6 \times 80}\left[(0+0) + 2(31) + 3(22+29+20+4)\right]$$

$$= \frac{9}{6 \times 80}(62 + 225) = \frac{9 \times 287}{6 \times 80}$$

$=$ ~~3.925~~ 5.38125 km

(4) Calculate $\int_2^{10} \frac{dx}{1+x}$ upto 4 decimal places by dividing the range into 8 places equal parts by Simpson's one third rule

=> Here $a=2$, $b=10$, $f(x) = \frac{1}{1+x}$

let $n=8$ = no. of sub-interval --{even}

$h = \frac{b-a}{n} = \frac{10-2}{8} = \frac{8}{8} = 1$

we prepare the table

X	2	3	4	5	6	7	8	9	10
Y	0.3333	0.2500	0.2000	0.1666	0.1428	0.125	0.1111	0.1000	0.0909
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

By simpson's $\frac{1}{3}$rd rule

$$\int_2^{10} \frac{dx}{1+x} = \frac{h}{3}\left[X + 2E + 4O\right]$$

$$= \frac{1}{3}\left[(y_0+y_8) + 2(y_2+y_4+y_6) + 4(y_1+y_3+y_5+y_7)\right]$$

$$= \frac{1}{3}\left[\begin{pmatrix}0.3333 + \\ 0.0909\end{pmatrix} + 2\begin{pmatrix}0.20 + 0.1428 \\ + 0.1111\end{pmatrix} + 4\begin{pmatrix}0.25 + 0.1666 + \\ 0.125 + 0.1\end{pmatrix}\right]$$

$$= \frac{1}{3}\left[0.4242 + 0.9078 + 2.5664\right]$$

$$= \frac{3.8984}{3} = \underline{\underline{1.29946}}$$

(5) Evaluate $\int_4^{5.2} \log x \, dx$ by simpson's $\frac{1}{3}$rd rule

$\Rightarrow$ Here $a = 4$ $b = 5.2$, $f(x) = \log x$

let $\boxed{n = 12}$ = no. of Sub-interval ---{even}

$$h = \frac{b-a}{n} = \frac{5.2-4}{12} = \frac{1.2}{12} = 0.1$$

we prepare the following table

x	4	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9
y	0.602	0.6127	0.6232	0.6334	0.6434	0.6532	0.6627	0.672	0.6812	0.6901
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9

x	5.0	5.1	5.2
y	0.6989	0.7075	0.716
	y_{10}	y_{11}	y_{12}

By simpson's $\frac{1}{3}$rd rule

$$\int_4^{5.2} \log x \, dx \simeq \frac{h}{3}\left[x + 2E + 4O \right]$$

$$= \frac{0.1}{3}\left[(y_0 + y_{12}) + 2\begin{pmatrix} y_2 + y_4 + y_6 \\ y_8 + y_{10} \end{pmatrix} + 4\begin{pmatrix} y_1 + y_3 + y_5 \\ y_7 + y_9 + y_{11} \end{pmatrix} \right]$$

$$= \frac{0.1}{3}\left[\begin{pmatrix} 0.602 \\ + \\ 0.716 \end{pmatrix} + 2\begin{pmatrix} 0.6232 + 0.6434 + 0.6627 + \\ 0.6812 + 0.6989 \end{pmatrix} + 4\begin{pmatrix} 0.6127 + 0.6334 \\ + 0.6532 + 0.672 \\ + 0.6901 + 0.7075 \end{pmatrix} \right]$$

$$= \frac{0.1}{3}\left[1.318 + 2(3.3094) + 4(3.9689) \right]$$

$$= \frac{0.1}{3}\left\{ 1.318 + 6.6188 + 15.8756 \right\}$$

$$= \frac{2.38124}{3} = \underline{0.7937}$$

$m \to \frac{m}{10^{-2}}$

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⑥ A rocket is launched from ground. Its acceleration is registered during first 80 seconds and is given below

T sec	0	10	20	30	40	50	60	70	80
A(m/s)	30.00	31.63	33.44	35.47	37.75	40.33	43.25	46.69	50.67

Using Simpson's one-third rule, Find the velocity of the rocket at time t=80 sec.

⇒ Here $a=0$ $b=80$. we know that $\int_0^{80} v\,dt$

$h = \frac{b-a}{n} = \frac{80}{8} = 10$ where $\boxed{n=10}$

we prepare following table (m/s to cm/s)

T/sec	0	10	20	30	40	50	60	70	80
A cm/s	3000	3163	3344	3547	3775	4033	4325	4669	5067
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

By Simpson's $\frac{1}{3}$rd rule

$$\int_0^{80} v\,dt \approx \frac{h}{3}\left[X + 2E + 4O\right]$$

$$\approx \frac{10}{3}\left[(y_0+y_8) + 2(y_2+y_4+y_6) + 4(y_1+y_3+y_5+y_7)\right]$$

$$\approx \frac{10}{3}\left[(3000+5067) + 2(3344+3775+4325) + 4(3163+3547+4033+4669)\right]$$

$$= \frac{10 \times (89603+3000)}{3} = \frac{926030}{3}$$

$$= 30867.66 \text{ cm/sec}$$

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$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[\underbrace{X}_{\text{initial + final ordinates}} + 2\underbrace{T}_{\text{sum of multiples of three ordinates}} + 3\underbrace{R}_{\text{remaining ordinates}}\right]$$

Simpson's $3/8^{th}$ Rule :

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[X + 2T + 3R\right]$$

X → sum of extreme ordinates

T → sum of ordinates which are multiples of 3

R → sum of remaining ordinates

e.g

$$\int_{x_0}^{x_6} f(x)\,dx = \frac{3h}{8}\left[(y_0 + y_6) + 2(y_3) + 3(y_1 + y_2 + y_4 + y_5)\right]$$

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① Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}$th rule

⇒ we divide the interval into 6 equal parts
$n=6$

$$h = \frac{b-a}{n} = \frac{6-0}{6} = 1$$

Calculated by manual

prepare the following table

x	0	1	2	3	4	5	6
y	1	0.5	0.2	0.1	0.0588	0.0384	0.027
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By simpson's 3/8th rule:

$$\int_0^6 \frac{1}{1+x^2}\,dx = \frac{3h}{8}\left[X + 2I + 3R\right]$$

$$= \frac{3\times1}{8}\left[(y_0+y_6) + 2(y_3) + 3(y_1+y_2+y_4+y_5)\right]$$

$$= \frac{3}{8}\left[(1.027) + 2\times(0.1) + 3\left(0.5 + 0.2 + 0.0588 + 0.0384\right)\right]$$

$$= \frac{3}{8}\left[(1.027) + 0.2 + 3(0.7972)\right]$$

$$= \frac{3}{8}(1.227 + 2.3916)$$

$$= \frac{3\times3.6186}{8}$$

$$= \frac{10.8558}{8}$$

$$= 1.3569$$

② Evaluate $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\, dx$ by using Simpson's $3/8^{th}$ rule

→ we divide interval into 6 parts $n = 6$

$$h = \frac{b-a}{n} = \frac{1.4 - 0.2}{6} = 0.2$$

we prepare the table

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
Y	2.8343	2.415	2.3434	2.4626	2.7357	3.1587	3.7431
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By Simpson's $3/8^{th}$ rule

$$\int_{0.2}^{1.4} (\sin x - \log_e x + e^x) = \frac{3h}{8}\left[x + 2T + 3R \right]$$

$$= \frac{3 \times 0.2}{8}\left[(y_0 + y_6) + 2(y_3) + (y_1 + y_2 + y_4 + y_5) \right]$$

$$= \frac{0.6}{8}\left[\begin{pmatrix} 2.8343 + \\ 3.7431 \end{pmatrix} + 2\left(2.4626\right) + \begin{pmatrix} 2.415 + 2.3434 \\ + 2.7357 + 3.1587 \end{pmatrix} \right]$$

$$= \frac{0.6}{8}\left[6.5774 + 4.9652 + 10.6528 \right]$$

$$= \frac{22.1954 \times 0.6}{8} = \frac{13.31724}{8}$$

$$= 1.6646$$

? 3.2595

0 — 0.2 — 0.4 — 0.6 — 0.8 — 1 (① ② ③ ④ ⑤)

$\log e^{x} = \ln(x)$

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⑤ $\int_0^1 \frac{\sin x}{x}\, dx$ by using simpson's $\frac{3}{8}$th rule

$\Rightarrow$ we divide ~~into~~ interval into 5 parts $n=5$

$$h = \frac{b-a}{n} = \frac{1-0}{5} = \frac{1}{5} = 0.2$$

we prepare the following table

x	0.0	0.2	0.4	0.6	0.8	1
y	1	0.0174	0.0174	0.0174	0.0174	0.0174
	y_0	y_1	y_2	y_3	y_4	y_5

- not calculated by calculator
- calculate manual $\lim_{x \to 0} \frac{\sin x}{x} = 1$

By simpson's rule

$$\int_0^1 \frac{\sin x}{x}\, dx = \frac{3h}{8}\left[x + 2T + 3R\right]$$

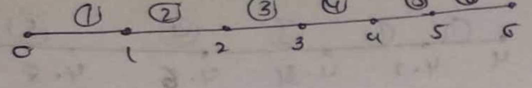
$$= \frac{3 \times 0.2}{8}\left[(y_0 + y_5) + 2(y_3) + 3(y_1 + y_2 + y_4)\right]$$

$$= \frac{0.6}{8}\left[(1 + 0.0174) + 2(0.0174) + 3(0.0174 + 0.0174 + 0.0174)\right]$$

$$= \frac{0.3}{4}\left[1.0174 + 0.0348 + 0.1566\right]$$

$$= \frac{\cancel{0.108792}}{\cancel{4}} \quad \frac{0.36264}{4}$$

$$= 0.09066$$



④ Find the approximate value of $\int_0^6 e^x\,dx$ by using Simpson's $3/8^{th}$ rule.

=> Here $a = 0$ $b = 6$; $f(x) = e^x$

let $n = 6$ = no. of sub interval

$h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

we prepare the following table

x	0	1	2	3	4	5	6
Y	1	2.7182	7.3890	20.0855	54.5981	148.4131	403.4287
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By Simpson's rule

$$\int_0^6 e^x\,dx = \frac{3h}{8}\left[X + 2T + 3R\right]$$

$$= \frac{3h}{8}\left[(y_0+y_6) + 2[y_3) + 3(y_1+y_2+y_4+y_5)\right]$$

$$= \frac{3h}{8}\left[(1+403.42) + 2(20.0855) + 3\underbrace{\begin{pmatrix}2.7182 + 7.3890 \\ + 54.5981 + 148.4131\end{pmatrix}}_{213.1184}\right]$$

$$= \frac{3}{8}\left[(404.42) + 40.171 + 639.3552\right]$$

$$= \frac{650.3677}{8}$$

$$= 81.2959$$

Simpson's $1/3^{rd}$ rule

① ② ③ ④ ⑤ ⑥

4   4.2   4.4   4.6   4.8   5   5.2

5) $\int_4^{5.2} \log x \, dx$ by simpson's $\frac{1}{3}^{rd}$ rule

$\Rightarrow$ Here $a = 4$ $\quad b = 5.2$ $\quad , f(x) = \log x$

$n = 8$ = no. of sub-interval ... (even)

$h = \frac{b-a}{n} = \frac{5.2-4}{8} = \frac{1.2}{8} = 0.15$

we perpare the following table

x.	$x_0 = 4$	$x_1 = 4.15$	$x_2 = 4.3$	$x_3 = 4.45$	$x_4 = 4.6$	$x_5 = 4.75$	$x_6 = 4.9$	$x_7 = 5.05$	$x_8 = 5.2$
Y	0.602	0.618	0.6334	0.6483	0.6627	0.6766	0.6901	0.7032	0.716
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

By simspon's $1/3^{rd}$ rule

$$\int_4^{5.2} \log x = \frac{h}{3}\left[X + 2E + 4O \right]$$

$$= \frac{0.15}{3}\left[\begin{pmatrix} 0.602 \\ + 0.716 \end{pmatrix} + 2\begin{pmatrix} 0.6334 + 0.6627 \\ + 0.6901 \end{pmatrix} + 4\begin{pmatrix} 0.618 + 0.6483 \\ 0.6766 + 0.7032 \end{pmatrix} \right]$$

$$= \frac{0.15}{3}\left[1.318 + 3.9724 + 10.5844 \right]$$

$$= \frac{0.15}{3}\left[15.8748 \right]$$

$$= 0.79374$$

Fixed for (10 M) in ESE

Newton - Cote Integration Formula:

① Trapezoidal rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[X + 2R\right]$$

X → sum of extreme ordinate

R → sum of remaining ordinates

Note: n → for 1/3rd is only even

② Simpson's 1/3rd rule

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[X + 2E + 4O\right]$$

E → sum of even ordinates

O → sum of odd ordinates

③ Simpson's 3/8th rule

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[X + 2T + 3R\right]$$

T → sum of ordinate which are multiple of 3

① ② ③ ④ ⑤

0 0.2 0.4 0.6 0.8 1

e.g

① Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using (i) Trapezoidal rule (ii) Simpson's $1/3^{rd}$ rule (iii) Simpson's $3/8^{th}$ rule

→ Here $a=0$, $b=1$, $f(x)=\frac{1}{1+x^2}$

let $n = 5$ = no. of subinterval

$h = \frac{b-a}{n} = \frac{1-0}{5} = \frac{1}{5} = 0.2$

we prepare following table

x	0	0.2	0.4	0.6	0.8	1.0
y	1	0.9615	0.862	0.7352	0.6097	0.5
	y_0	y_1	y_2	y_3	y_4	y_5

By Trapezoid rule, we get

$$\int_0^1 \frac{1}{1+x^2}dx = \frac{h}{2}\left[x+2R\right] = \frac{h}{2}\left[(y_0+y_5)+2(y_1+y_2+y_3+y_4)\right]$$

$$= \frac{0.2}{2}\left[(1+0.5)+2(0.9615+0.862+0.7352+0.6097)\right]$$

$$= 0.1[1.5 + 2(3.1684)]$$

$$= 0.1(1.5 + 6.3368)$$

$$= 0.1(7.8368)$$

$$= \underline{\underline{0.78368}}$$

By Simpson's $\frac{1}{3}$rd rule, we get

$$\int_0^1 \frac{1}{1+x^2}dx = \frac{h}{3}\left[X + 4O + 2E\right]$$

$$= \frac{h}{3}\left[(y_0+y_5)+4(y_1+y_3)+2(y_2+y_4)\right]$$

$$= \frac{h}{3}\left[(1.5)+4(0.9615+0.7352)+2(0.862+0.6097)\right]$$

$$= \frac{0.2}{3}\left[1.5+6.7868+2.9434\right]$$

$$= \frac{0.2\times 11.2302}{3} = 0.74868$$

By Simpson $3/8^{th}$, we get

$$\int_0^1 \frac{1}{1+x^2}dx = \frac{3h}{8}\left[X + 2T + 3R\right]$$

$$= \frac{3\times 0.2}{8}\left[(y_0+y_5)+2(y_3)+3(y_1+y_2+y_4)\right]$$

$$= \frac{0.6}{8}\left[(1+0.5)+2(0.7352)+3(0.9615+0.862+0.6097)\right]$$

$$= \frac{0.6}{8}\left[1.5 + 1.4704 + 7.2996\right]$$

$$\frac{10.27\times 0.6}{8}$$

$$= \underline{0.77025}$$

(2) Evaluate $\int_0^6 \frac{dx}{1+3x}$ by using (i) Trapezoidal rule
(ii) Simpson's $1/3^{rd}$ rule
(iii) Simpson's $3/8^{th}$ rule

$\Rightarrow$ ~~(i) Trapezo~~ Here $a=0$ $b=6$, $f(x) = \frac{1}{1+3x}$

$n = 6$ = no. of sub-interval ... {even}

step size $h = \frac{b-a}{n} = \frac{6-0}{6} = \underline{\underline{1}}$

~~(i) Trapezoidal rule~~ :-

we perpare the table

x	$x_0 = 0$	$x_1 = 1$	$x_2 = 2$	$x_3 = 3$	$x_4 = 4$	$x_5 = 5$	$x_6 = 6$
y	1	0.25	0.1428	0.1	0.0769	0.0625	0.0526
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

(i) Trapezoidal rule, we get

$$\int_0^6 \frac{1}{1+3x} dx = \frac{h}{2}\left[x + 2R\right] = \frac{1}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[(1+0.0526) + 2\begin{pmatrix}0.25+0.1428+0.1 \\ +0.0769+0.0625\end{pmatrix}\right]$$

$$= \frac{1}{2}\left[1.0526 + 1.2644\right]$$

$$= \frac{2.317}{2} = \underline{\underline{1.1585}}$$

(ii) Simpson's $\frac{1}{3}$rd rule, we get

$$\int_0^6 \frac{1}{1+3x} dx = \frac{h}{3}\left[x + 2E + 4O\right] = \frac{1}{3}\left[(y_0+y_6) + 2(y_2+y_4) + 4\begin{pmatrix}y_1+y_3 \\ +y_5\end{pmatrix}\right]$$

$$= \frac{1}{3}\left[(1.0526) + 2(0.1428 + 0.0769) + 4\begin{pmatrix}0.25+0.1 \\ +0.0625\end{pmatrix}\right]$$

$$= \frac{1}{3}\left[1.0526 + 0.4394 + 1.65\right]$$

$$= \frac{3.142}{3}$$

$$= 1.04733$$

(iii) Simpson's $\frac{3}{8}$th rule

$$\int_0^6 \frac{dx}{1+3x} = \frac{h \times 3}{8}\left[x + 2T + 3R\right]$$

$$= \frac{3 \times 1}{8}\left[(y_0+y_6) + 2(y_3) + 3(y_1+y_2+y_4+y_5)\right]$$

$$= \frac{3}{8}\left[(1.0526) + 2(0.1) + 3\begin{pmatrix}0.25+0.1428 \\ +0.0769 \\ +0.0625\end{pmatrix}\right]$$

$$= \frac{3}{8}\left(1.0526 + 0.2 + 1.5966\right)$$

$$= \frac{3 \times 2.8492}{8}$$

$$= 1.06845$$

③ Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using (i) Trapezoidal rule
(ii) Simpson's $1/3^{rd}$ rule
(iii) Simpson's $3/8^{th}$ rule

⟹ Here $a = 0$ $b = 1$, $f(x) = \frac{1}{1+x^2}$

& $n = 10$ = no. of sub-interval ∴ {even}

$h = \frac{b-a}{n} = \frac{1-0}{10} = 0.1$

we prepare the table

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
y	1	0.99	0.9615	0.9174	0.862	0.8	0.7352	0.6711	0.6097	0.5524	0.5
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

(i) Trapezoidal rule, we get

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[X + 2R\right] = \frac{0.1}{2}\left[(y_0+y_{10}) + 2\left(\begin{matrix} y_1+y_2+y_3+y_4+y_5 \\ +y_6+y_7+y_8+y_9 \end{matrix}\right)\right]$$

$$= \frac{0.1}{2}\left[(1+0.5) + 2\left(\begin{matrix} 0.99+0.9615+0.9174+0.862 \\ +0.8+0.7352+0.6711+0.6097 \\ +0.5524 \end{matrix}\right)\right]$$

$$= \frac{0.1}{2}\left[1.5 + 14.1986\right]$$

$$= \frac{(0.1)}{2} \times 15.6986 = \frac{1.56986}{2} = 0.78493$$

(ii) Simpson's $\frac{1}{3}^{rd}$ rule, we get

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{3}\left[X + 2E + 4O\right]$$

$$= \frac{0.1}{3}\left[(y_0+y_{10}) + 2\left(\begin{matrix} y_2+y_4+y_6 \\ +y_8 \end{matrix}\right) + 4\left(\begin{matrix} y_1+y_3+y_5+y_7 \\ +y_9 \end{matrix}\right)\right]$$

$$= \frac{0.1}{3}\left[(1.5) + 2\left(\begin{matrix} 0.9615+0.862 \\ +0.7352+0.6097 \end{matrix}\right) + 4\left(\begin{matrix} 0.99+0.9174+0.8 \\ +0.6711+0.5524 \end{matrix}\right)\right]$$

$$= \frac{0.1}{3}\left[1.5 + 6.3368 + 15.7236\right]$$

$$= \frac{2.35604}{3}$$

$$= 0.785346$$

(iii) Simpson $\frac{3}{8}^{th}$ rule, we get

$$\int_0^1 \frac{dx}{1+x^2} = \frac{3 \times h}{8}\left[X + 2T + 3R\right]$$

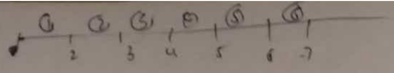
$$= \frac{3 \times 0.1}{8}\left[(y_0+y_{10}) + 2(y_3+y_6+y_9) + 3\left(\begin{matrix} y_1+y_2+y_4 \\ +y_5+y_7+y_8 \end{matrix}\right)\right]$$

$$= \frac{3 \times 0.1}{8}\left[1.5 + 2\left(\begin{matrix} 0.9174+0.7352 \\ +0.5524 \end{matrix}\right) + 3\left(\begin{matrix} 0.99+0.9615 \\ +0.862+0.8 \\ 0.6711+0.6097 \end{matrix}\right)\right]$$

$$= \frac{3 \times 0.1}{8}\left[1.5 + 4.41 + 14.6829\right]$$

$$= \frac{3 \times 20.5929 \times 0.1}{8} = \frac{6.17787}{8}$$

$$= 0.77223$$



(4) Evaluate $\int_1^7 y\,dx$ by using (i) Trapezoidal rule
(ii) Simpson's $1/3^{rd}$ rule
(iii) Simpson's $3/8^{th}$ rule

where y is given as

x	1	2	3	4	5	6	7
y	2.157	3.519	4.198	4.539	4.708	4.792	4.835
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$\Rightarrow$ Here $a = 1$ $\quad b = 7$, Here $f(x) = y$

let $n = 6$ = no. of sub-interval

$h = \frac{b-a}{n} = \frac{7-1}{6} = \frac{6}{6} = 1$

(1) Trapezoidal rule, we get

$$\int_1^7 y\,dx = \frac{h}{2}\left[X + 2R\right] = \frac{1}{2}\left[(y_0+y_6) + 2\left(y_1+y_2+y_3+y_4+y_5\right)\right]$$

$$= 0.5\left[(2.157 + 4.835) + 2\left(3.519 + 4.198 + 4.539 + 4.708 + 4.792\right)\right]$$

$$= 0.5\left[(6.992) + 43.512)\right]$$

$$= 0.5 \times 50.504 \quad = 25.252$$

(2) Simpson's $1/3^{rd}$ rule, we get

$$\int_1^7 y\,dx = \frac{h}{3}\left[X + 2E + 4O\right] = \frac{h}{3}\left[(y_0+y_6) + 2(y_2+y_4) + 4(y_1+y_3+y_5)\right]$$

$$= \frac{1}{3}\left[(2.157 + 4.835) + 2\left(4.198 + 4.708\right) + 4\left(3.519 + 4.539 + 4.792\right)\right]$$

$$= \frac{[(6.992) + (17.812) + 51.4)]}{3} = 25.4013$$

(3) Simpson's $3/8^{th}$ rule, we get

$$\int_1^7 y\,dx = \frac{3h}{8}\left[X + 2T + 3R\right]$$

$$= \frac{3}{8} \times 1\left[(y_0+y_6) + 2(y_3) + 3\left(y_1+y_2+y_4+y_5\right)\right]$$

$$= \frac{3}{8}\left[6.992 + 9.078 + 3\left(3.519 + 4.198 + 4.708 + 4.792\right)\right]$$

$$= \frac{3}{8}\left(6.992 + 9.078 + 51.651\right)$$

$$= \frac{67.721 \times 3}{8} = \frac{203.163}{8}$$

$$= 25.3953$$