

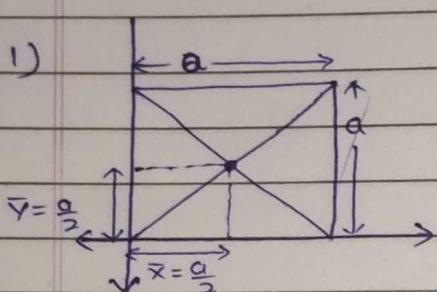
module - I

CENTROID OF PLANE LAMINAS

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A

Centroid of standard shapes

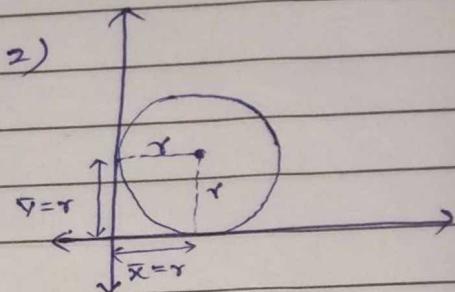


Square

$$\bar{x} = \frac{a}{2}$$

$$\bar{y} = \frac{a}{2}$$

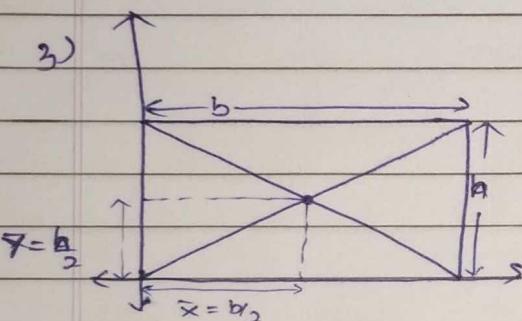
$$A = a^2$$



Circle

$$\bar{x} = \bar{y} = r$$

$$A = \pi r^2$$

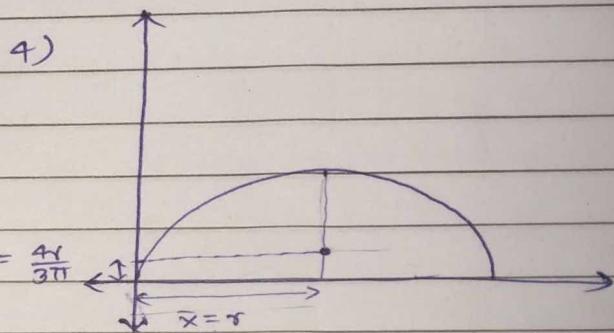


rectangle

$$\bar{y} = \frac{h}{2}$$

$$\bar{x} = \frac{b}{2}$$

$$A = b \times h$$

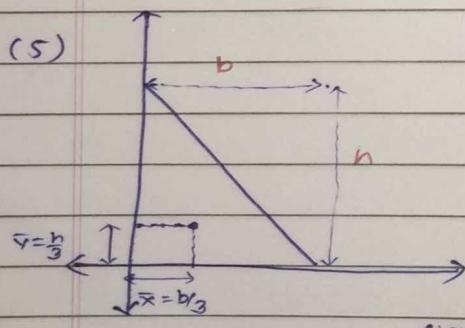


Semi-circle

$$\bar{x} = r$$

$$\bar{y} = \frac{4r}{3\pi}$$

$$A = \frac{\pi r^2}{2}$$

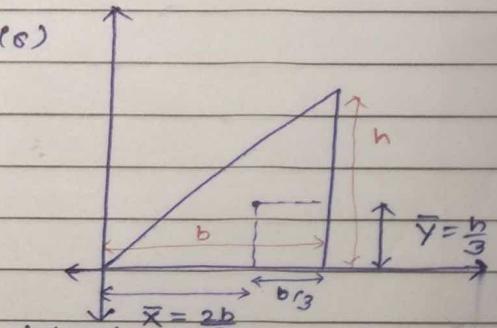


right angle triangle

$$\bar{x} = \frac{b}{3}$$

$$\bar{y} = \frac{h}{3}$$

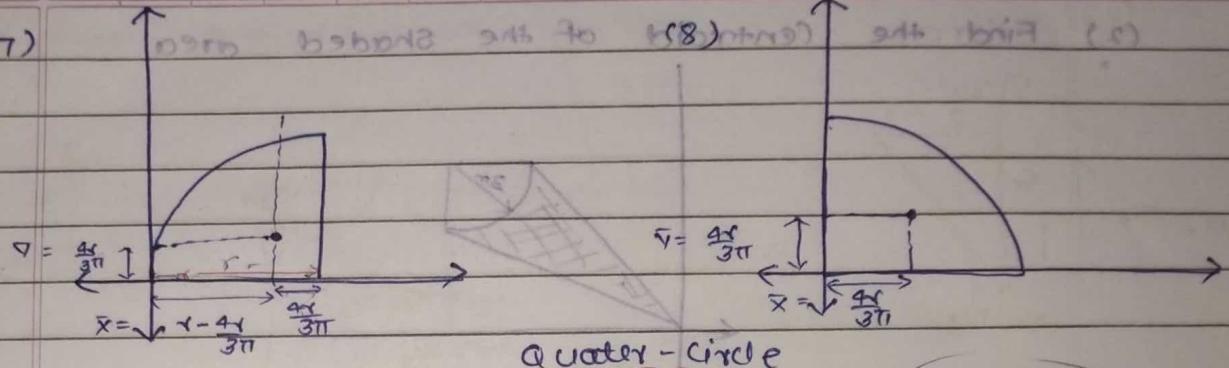
$$A = \frac{b \times h}{2}$$



$$\bar{x} = \frac{2b}{3}$$

$$\bar{y} = \frac{h}{3}$$

(7) Norm husband 2nd to 1(8) Maths 9th Unit (S)



$$\bar{x} = r - \frac{4r}{3\pi}$$

$$\bar{y} = \frac{4r}{3\pi}$$

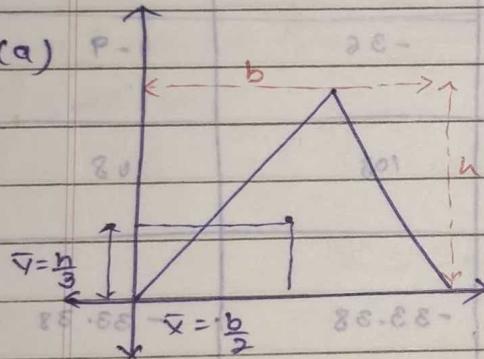
Quarter-Circle

$$A = \frac{\pi r^2}{4}$$

$$\bar{x} = \bar{y} = \frac{4r}{3\pi}$$

$$A = \frac{\pi r^2}{4}$$

(a)



Equilateral triangle

$$\bar{x} = \frac{b}{2}$$

$$\bar{y} = \frac{h}{3}$$

$$A = \frac{b \times h}{2}$$

Sector of Circle

$$\bar{x} = \frac{2r \sin \alpha}{3\alpha}$$

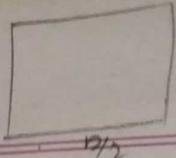
$$A = \frac{\pi r^2 \theta}{360}$$

$\angle AOB = \frac{1}{2} \angle AOC = \frac{1}{2} \angle AOD = \frac{1}{2} \angle AOE = \frac{1}{2} \angle AOF$ in radian

$$\bar{x} = \frac{2r \sin \alpha}{3\alpha}$$

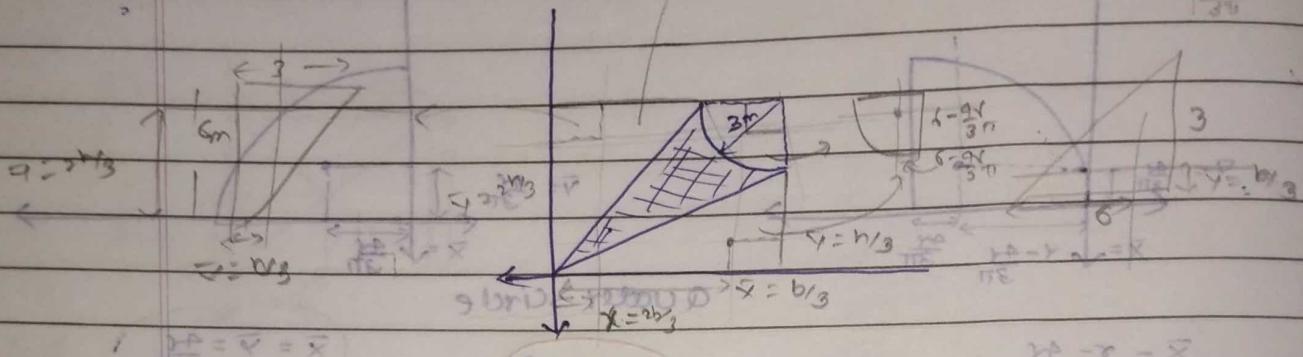
$$A = \frac{\pi r^2 \theta}{360}$$

(-1) sign in Area because get required Area



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(2) Find the centroid of the shaded area



$\text{Area} = A$	x	y	Ax	Ay	
Triangle along (y)	$A = \frac{b \times h}{2} = \frac{3 \times 3}{2} = 9$	$\bar{x} = b/3 = \frac{3}{3} = 1$	$\bar{y} = \frac{2h}{3} = \frac{2 \times 3}{3} = 2$	-9	-3.6
Triangle along (x)	$A = \frac{b \times h}{2} = \frac{3 \times 3}{2} = 9$	$\bar{x} = 2b/3 = \frac{2 \times 3}{3} = 2$	$\bar{y} = \frac{h}{3} = \frac{3}{3} = 1$	-3.6	-9 (D)
Square	$A = b \times h = 6 \times 6 = 36$	$\bar{x} = b/2 = 3$	$\bar{y} = h/2 = 3$	108	108
Quarter Circle	$A = \frac{\pi r^2}{4} = \frac{\pi \times 9}{4} = -7.0625$	$\bar{x} = \frac{6-4r}{3\pi} = \frac{6-4 \times 3}{3\pi} = 4.727$	$\bar{y} = \frac{6-4r}{3\pi} = \frac{6-4 \times 3}{3\pi} = 4.727$	-33.38	-33.38

$\Sigma A = 10.93$

$\Sigma Ax = 29.62$ $\Sigma Ay = 29.62$

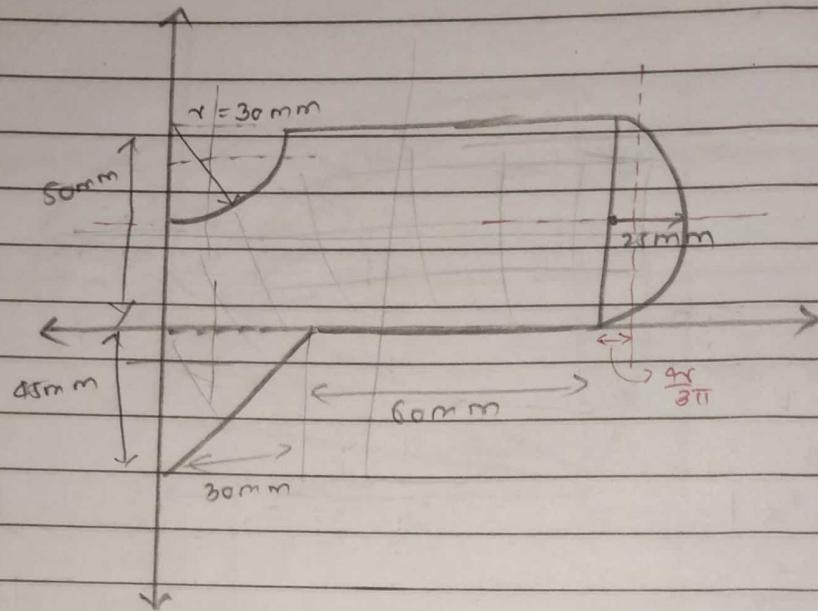
$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{29.62}{10.93}$$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{29.62}{10.93}$$

= 2.709 m

= 2.709 m

(2) Determine the centroid of the following plane area shown in figure

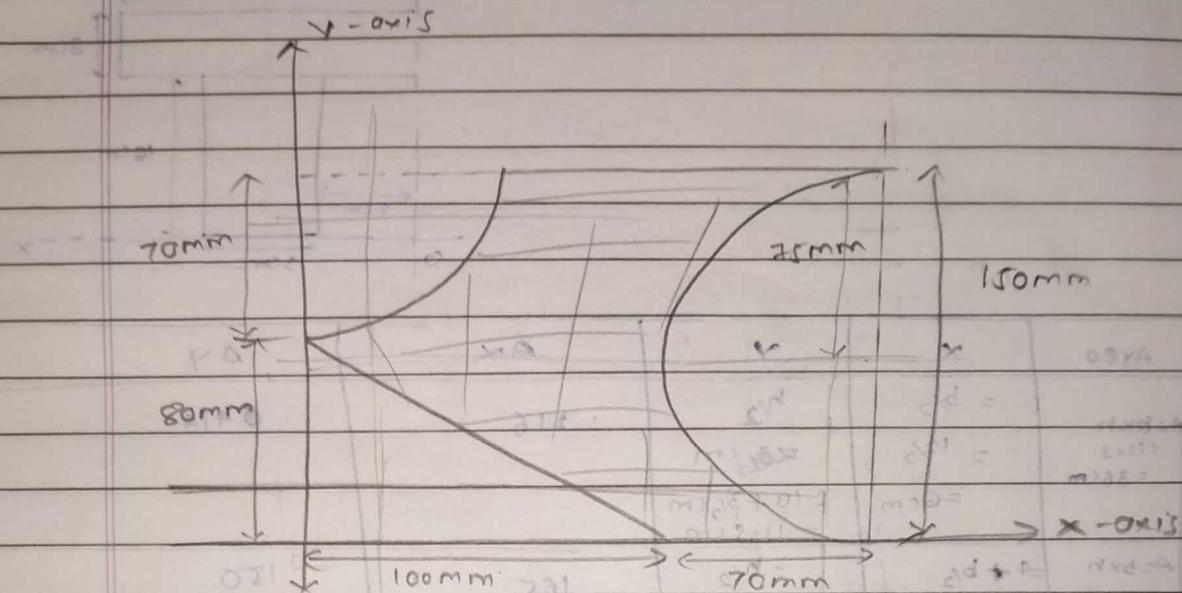


	Area	X	Y	AX	AY
Semicircle	$A = \frac{\pi}{2} r^2 = \frac{\pi}{2} \times (25)^2 = 981.74$	$= 90 + \frac{25}{3\pi} = 100.61$	25	98772.86	24543.5
Quarter - Circle	$A = \frac{\pi}{4} r^2 = \frac{\pi}{4} \times (30)^2 = 706.85$	$= \frac{25}{3\pi} = 12.73$	$50 - \frac{25}{3\pi} = 37.267$	-6998.2	-26342.17
Rectangle	$A = b \times h = 60 \times 50 = 3000$	45	25	202000	112500
Right angled triangle	$A = \frac{1}{2} b \times h = \frac{1}{2} \times 60 \times 15 = 450$	$\frac{30}{2} = 15$	$-\frac{15}{3} = -5$	6750	-10125
	$\Sigma A = 5449.89$			$\Sigma AX = 299020.66$	$\Sigma AY = 100576.33$

$$\bar{x} = \frac{\Sigma A x}{\Sigma A} = \frac{299020.66}{5449.89} = 54.86$$

$$\bar{y} = \frac{\Sigma A y}{\Sigma A} = \frac{100576.33}{5449.89} = 18.45$$

* (3) Determine the centroid of the following plane area shown in figure.



	$Ax = A$	x	y	$Ay = A$	y
Semicircle	$A = \frac{\pi r^2}{2} = \frac{\pi \times 70^2}{2}$ $= 1220743.07$	$170 - \frac{9\pi}{3\pi} = 100$ $= 138.16$	$= 75$	-1220743.07	-662679
Quadrant-circle	$A = \frac{\pi r^2}{4} = \frac{\pi \times 70^2}{4}$ $= 3848.95$	$150 - \frac{9\pi}{3\pi} = 120$ $= 29.768$	$= 120.292$	-114329.75	-962937.74
Right angled triangle	$A = \frac{1}{2}bh = \frac{1}{2} \times 100 \times 80$ $= 4000$	$\frac{b}{3} = \frac{100}{3}$ $= 33.33$	$\frac{h}{3} = \frac{80}{3}$ $= 26.66$	-133320	-106640
rectangle	$A = b \times h = 150 \times 70$ $= 25500$	85	75	2167500	1912500
	$\Sigma A = 8815.83$			$\Sigma Ax = 609107.18$	$\Sigma Ay = 680293.26$

$$\bar{x} = \frac{609107.18}{8815.83}$$

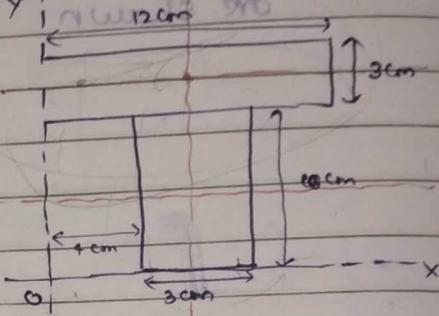
$$[\bar{x} = 79.3]$$

$$\bar{y} = \frac{680293.26}{8815.83}$$

$$[\bar{y} = 77.16]$$

Ques. Find all the co-ordinates of centroid of the section

Setup in mm



rectangle	Area	x	y	Ax	Ay
	$A = b \times h$ $= 12 \times 3$ $= 36 \text{ cm}^2$	$b/2$ $= 12/2$ $= 6 \text{ cm}$	$h/2$ $\approx 8.6 \text{ cm}$ $= 10 + 3/2 \text{ cm}$ $= 11.5 \text{ cm}$	216	6414
	$A = b \times h$ $= 10 \times 3$ $= 30$	$4 + b/2$ $= 4 + 3/2$ $= 5.5 \text{ cm}$	$h/2$ $= 5 \text{ cm}$	165	150
	$\Sigma A = 66$			$\Sigma Ax = 381$	204

$$X = \frac{\Sigma Ax}{\Sigma A} = \frac{381}{66} = 5.77 \text{ cm}$$

$$Y = \frac{\Sigma Ay}{\Sigma A} = \frac{204}{66} = 3.06 \text{ cm}$$

$$Y = 8.595 \text{ cm}$$

B
✓ #

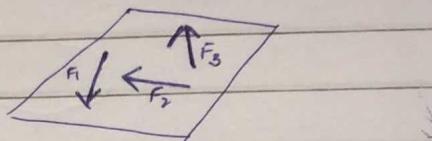
(Coplanar System of force.)

(#) **System of force**: There are mainly seven types of system forces.

Give **Types of force**:

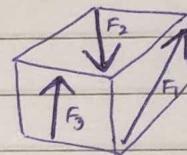
(1) **Co-planar forces**: (1)

- The forces which are acting in the same plane.



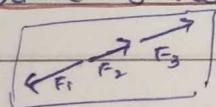
(2) **Non-Coplanar forces**: (2)

- The forces which are acting in the different plane.



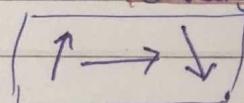
(3) **Collinear forces**: (3)

- The force which are acting along the same straight line.



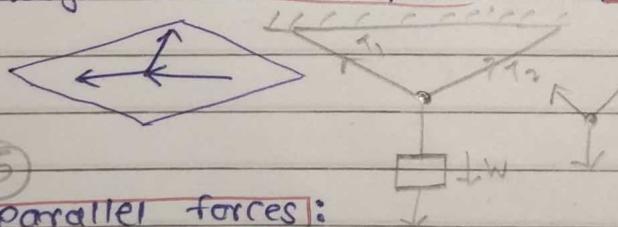
(4) **Non-Collinear forces**:

- The force which are not acting along the straight line.



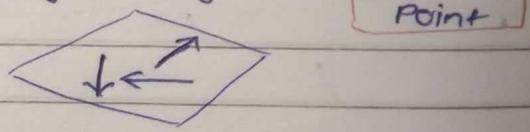
(5) **Concurrent forces**: (1)

- The forces which are passing through a common point.



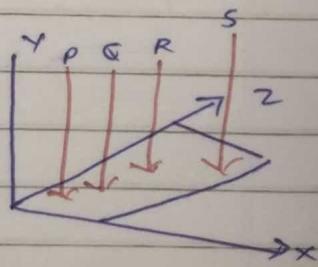
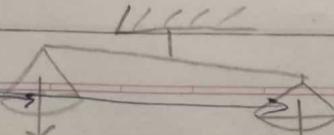
(6) **Non-Concurrent forces**:

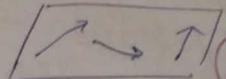
- The forces which are not passing through a common point.



(7) **parallel forces**: (5)

- The forces whose line of action are parallel to each other.





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✓ #

Resultant of two or more forces:

- When two or more coplanar concurrent or non-concurrent forces acting on a body the resultant can be found out by using resolution procedure.

magnitude of : $R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$

force along
y-direction
force along
x-direction

direction :

$$\theta = \tan^{-1} \left(\frac{F_y}{F_x} \right)$$

angle of "R" with
x-axis

when a Force acting on a body produces turning effect or rotational effect is called moment of force

✓ #

Moment : S.I unit N-m

- It is the turning effect produced by a force.

- Force about any point is the

product of magnitude of force and for distance about that point.

Couple

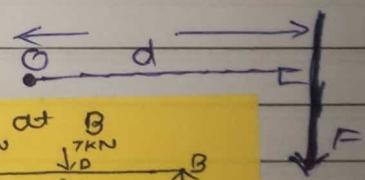
Two non-collinear parallel forces

having same magnitude

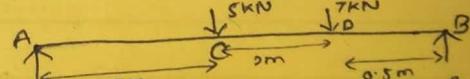
But opposite direction forms a couple.

$$M_o = F \times d$$

moment at
Point O



Find reaction at B



$$\sum M_A = 0 \quad \text{--- FBD}$$

$$(R_A \times 1.5) + (5 \times 1.5) + (3 \times 7) - (R_B \times 4) = 0$$

$$7.5 + 24.5 - R_B \times 4 = 0$$

$$4 R_B = 32 \quad \boxed{R_B = 8 \text{ kN}}$$

Note:

The sum of all moment should be zero

$$\sum M = 0$$

↓
of all forces

How to allocate the resultant of a given force system

Vorignon's theorem: ro out to #

ro (moment) from ro outwards -

The sum of the moment of all the force about a point is equal to the moment of their resultant about the same point

Q7
The sum of the moment of all the force about a point is equal to the moment of their resultant about the same point.

$$\sum M_o F = M_o R$$

$$\sum M_o F = M_o R$$

Summation of moment of all forces w.r.t any point is

is equal to

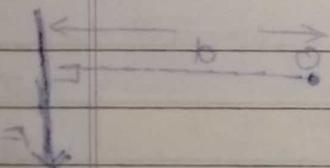
moment of resultant of that force

$$\sum M_a F = M_a R$$

about a point printed with same point

about a point to obtain to print

• thing cont. two



$$b \times f = M$$

to moment
of force

translational out
and rotation

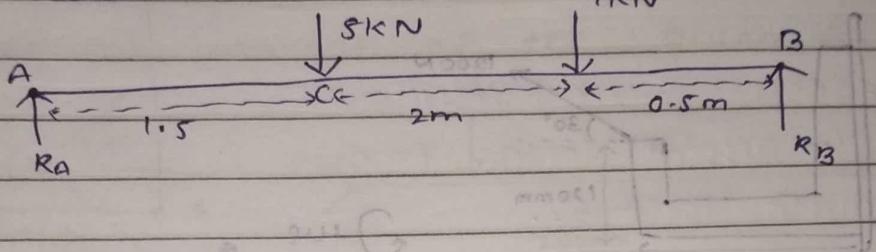
about origin and print

about origin of force

signs

Ans (6m)

Find reaction at B



$$\sum F_x = 0$$

$$\Rightarrow x \quad (\sum F_y = -R_A + 8 + 7 + R_B = 0)$$

$$22.50 \text{ kN} + (R_A + R_B) + 15 = 0$$

$$22.50 + 15 + 8 = 0$$

$$R_A =$$

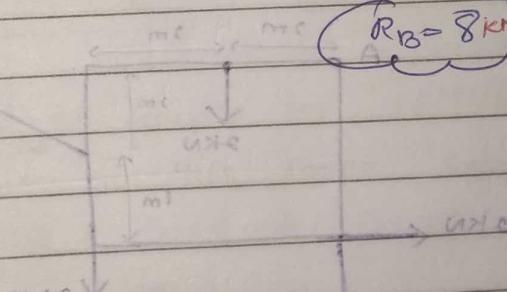
$$[22.50 + 15 = 0]$$

$$+ \textcircled{1} \quad M_A = 0 \cdot R_A + 8 \cdot 1.5 + 7 \cdot 2.5 - R_B \cdot 4 = 0$$

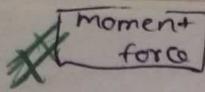
$$7.50 + 17.50 - R_B \cdot 4 = 0$$

$$R_B = \frac{32.50}{4}$$

$$R_B = 8 \text{ kN}$$



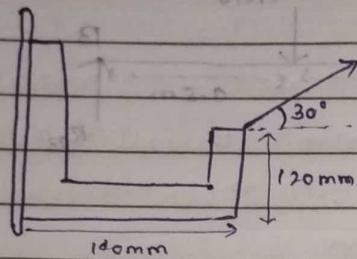
moment question wale



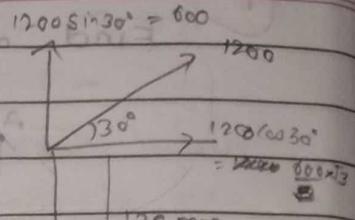
; the force when act on a body produces the turning effect or rotational effect is called moment force

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(1) Find moment about point A



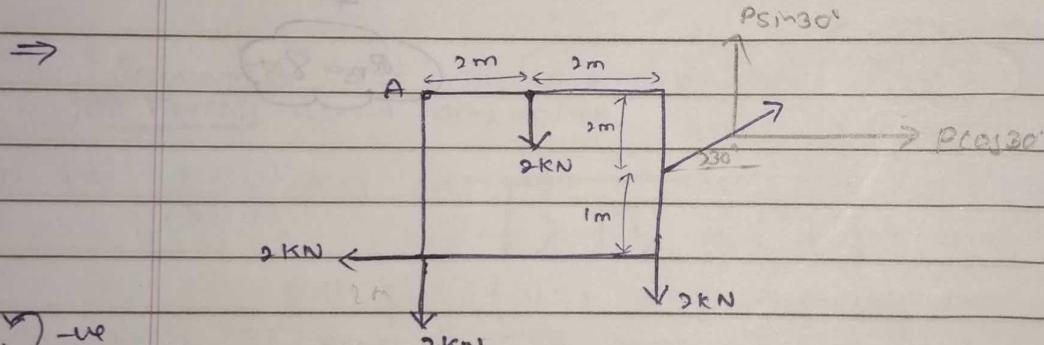
C-wise G-ue A 140mm



$$\begin{aligned}\Sigma M_A &= -600(1200 \sin 30^\circ) \times 140 + (1200 \times \frac{\sqrt{3}}{2}) \times 120 \\ &= -600 \times 140 + 600\sqrt{3} \times 120 \\ &= -84000 + 127707.65\end{aligned}$$

$$\boxed{\Sigma M_A = 40707.65}$$

(2) Find the value of force P, so that moment about point A is zero as shown in figure



G-ue

C-wise

Moment of all the force about pt. A = 0

$$\boxed{\Sigma M_A = 0} (2 \times 2) + (2 \times 0) + (2 \times 3) + (2 \times 4) - (P \cos 30^\circ \times 2) - P \sin 30^\circ = 0$$

$$2 + 0 + 6 + 8 + 0 - P \times \sqrt{3}$$

$$\frac{2}{x} P = 0$$

$$- (3 \cdot 732) P + 18 = 0$$

$$\boxed{P = \frac{18}{3.732}}$$

$$18 - P = 0$$

$$P = 18$$

$$\boxed{P = 3.732}$$

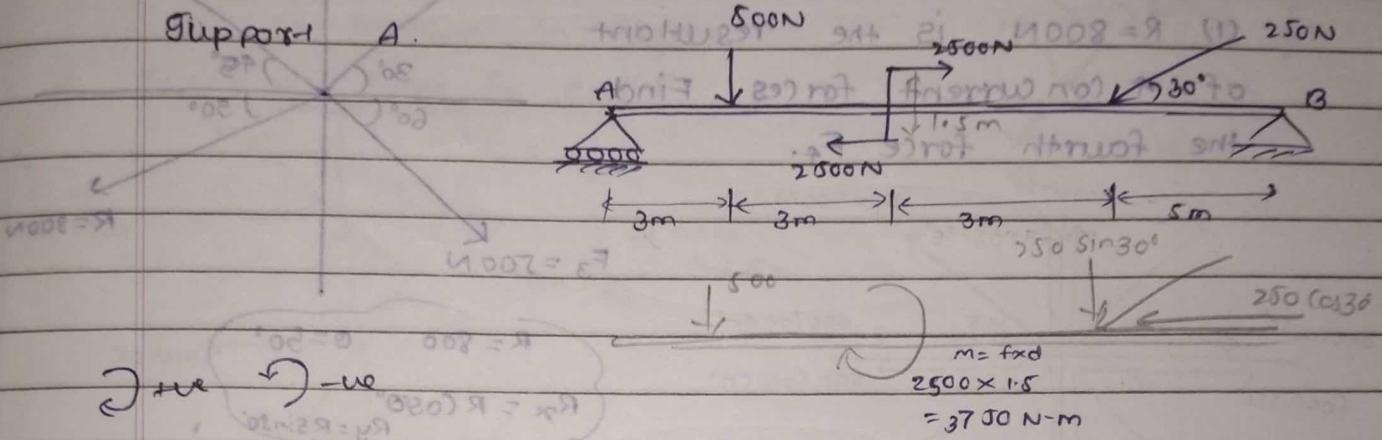
~~$$-P(3.732) = 18$$~~

$$\boxed{P = 4.825 \text{ kN}}$$

Couple: when two equal & like parallel, non-collinear force acts on a body that couple will form.

(1) Compute the moment of the given force about

~~Support A~~

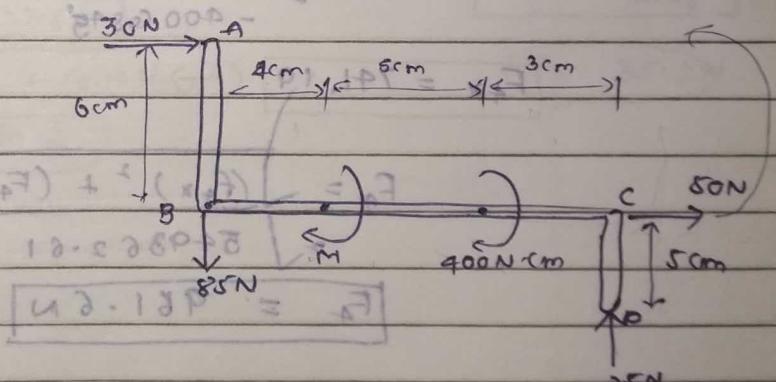


$$\begin{aligned} \text{IM}_A &= (500 \times 3) + 3750 + (250 \cos 30^\circ \times 0) + (650 \sin 30^\circ \times 9) \\ &= 1500 + 3750 + 0 + \frac{250 \times 9}{2} \\ &= 5250 + 1125 \end{aligned}$$

$$\Sigma M_p = 6375 \text{ N-m}$$

(2) Find the moment of couple M_1 , if the line of action of resultant of this force system is to pass through point A.

$$\begin{aligned}
 \Sigma M_A &= (30 \times 0) + (85 \times 0) + M + 400 \\
 &\quad - (6 \times 50) - (25 \times 12) \\
 &= M - 300 + 400 - 300 \\
 &= M - 600 \\
 &= M - 200
 \end{aligned}$$



$$\sum M_{\text{ext}} = 0$$

$$M - 200d = 0 \quad M = 200 \text{ Nm}$$

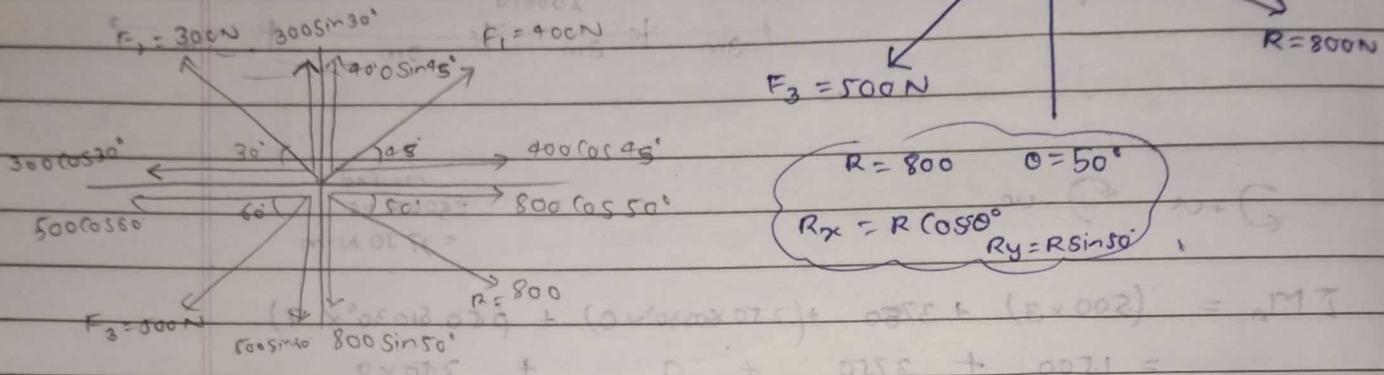
+ 2.95 = 7.9

Note: $\sum F_x = R_x$ $\sum F_y = R_y$

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① # Determine unknown forces :

Note (1) $R = 800\text{N}$ is the resultant of 4 concurrent forces. Find the fourth force F_4 .



$$\sum F_x = R_x$$

$$\sum F_y = R_y$$

$$R = 800 \quad \theta = 50^\circ$$

$$R_x = R \cos 50^\circ$$

$$R_y = R \sin 50^\circ$$

$$R_x = 800 \cos 50^\circ = 612.6 \text{ N}$$

$$R_y = 800 \sin 50^\circ = 612.7 \text{ N}$$

$$400 \cos 45^\circ + 300 \cos 30^\circ - 500 \cos 60^\circ + (F_{4x}) = 800 \cos 50^\circ$$

$$300 \sin 30^\circ + 400 \sin 45^\circ - 500 \sin 60^\circ + (F_{4y}) = -800 \sin 50^\circ$$

$$F_{4x} = 250 + 300 \times \frac{\sqrt{3}}{2} - 400 \cos 45^\circ$$

$$F_{4y} = 500 \sin 60^\circ - 300 \sin 30^\circ - 400 \sin 45^\circ$$

$$F_{4x} = 741.19$$

$$F_{4y} = -612.6$$

$$F_4 = \sqrt{(F_{4x})^2 + (F_{4y})^2}$$

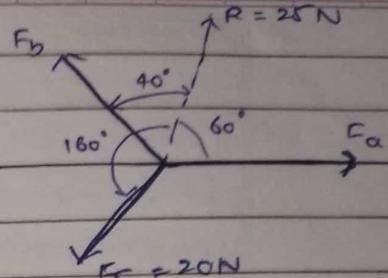
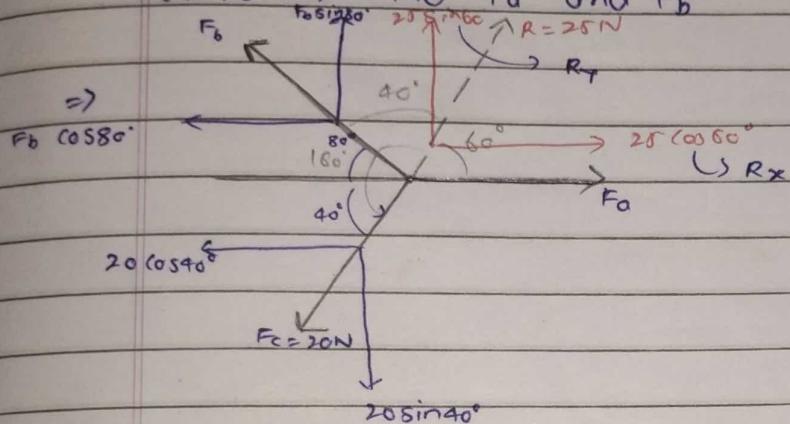
$$F_4 = 961.6 \text{ N}$$

$$\theta_4 = \tan^{-1} \left(\frac{F_{4y}}{F_{4x}} \right) = \tan^{-1} \left(\frac{-612.6}{741.19} \right)$$

$$\theta_4 = 39.57$$

(2) A force $R = 25\text{ N}$ has components F_a , F_b and F_c as shown in figure.

$F_c = 20\text{ N}$, Find F_a and F_b



$$\sum F_x = R_x$$

$$= 25 \cos 60^\circ$$

$$\sum F_y = R_y$$

$$F_b \sin 80^\circ - 20 \sin 40^\circ = 25 \sin 60^\circ$$

$$-F_b \cos 80^\circ - 20 \cos 40^\circ$$

$$+ F_a$$

$$F_a - F_b(0.17) - 18.32 = \frac{25}{2}$$

$$F_a - 0.17F_b = 12.5 + 15.32$$

$$\boxed{F_a - 0.17F_b = 27.82}$$

$$F_a = 27.82 + (0.17)35.2$$

$$\boxed{F_a = 33.80\text{ N}}$$

$$(0.98)F_b - 20 \sin 40^\circ$$

$$(0.98)F_b - 12.85 = 21.65$$

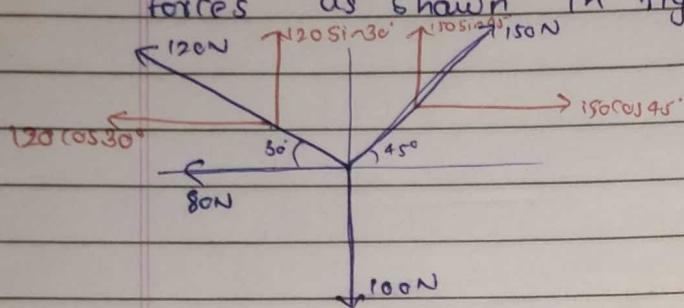
$$\boxed{F_b = \frac{34.5}{0.98}}$$

$$\boxed{F_b = 35.2\text{ N}}$$

Q2

Resultant of Concurrent force Systems:

- * (ii) Find the resultant System of Four Concurrent forces as shown in figure

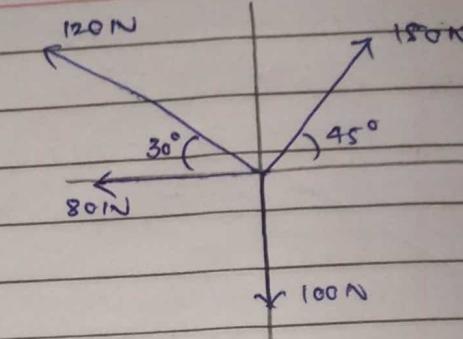


$$\sum F_x = -80 - 120 \cos 30^\circ + 150 \cos 45^\circ$$

$$= -80 - 60\sqrt{3} + 106.06$$

$$= -80 - 103.9 + 106.06$$

$$[\sum F_x = -77.86] \quad (\Leftarrow)$$



$$\begin{aligned} \sum F_y &= -100 + 120 \sin 30^\circ + 150 \sin 45^\circ \\ &= -100 + 60 + 106.06 \\ [\sum F_y &= 66.06] \quad (\uparrow) \end{aligned}$$

$$\begin{aligned} R &= \sqrt{(\sum F_x)^2 + (\sum F_y)^2} \\ &= \sqrt{(6062.41) + (4363.92)} \\ &= \boxed{R = 102.109} \end{aligned}$$

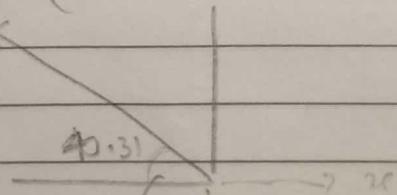
$$\text{Inclination } y \text{-axis } \theta = \tan^{-1} \left(\frac{66.06}{77.86} \right)$$

$$77.86 = -w$$

which quadrant
 $(-w)$,

$$\theta = 40.31$$

$$R = 102.109$$



θ with respect
to $x = -w$

Q3

(1) (3) The force system shown in fig

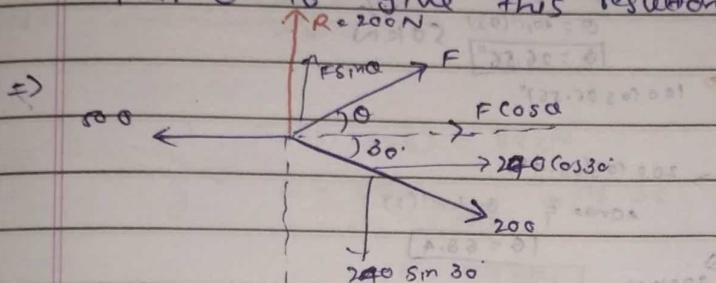
has a resultant of 200N

point up along the Y-axis

Compute the value of

force F and angle θ

required to give this resultant



$$\sum F_x = 0$$

$$\sum F_y = R$$

$$\sum F_y = 200N$$

$$\sum F_x = 0$$

$$\sum F_y = 200$$

$$-500 + F \cos \theta + 200 \cos 30^\circ = 0$$

$$F \sin \theta - 200 \times L = 200$$

$$-500 + F \cos \theta + 200 \cos 30^\circ = 0 \quad + 207.84$$

$$F \sin \theta = 320$$

$$F \cos \theta = 672.15$$

$$\frac{F \sin \theta}{F \cos \theta} = \frac{320}{672.15}$$

$$\tan \theta = 1.095$$

$$\theta = \tan^{-1}(1.095)$$

$$\theta = 47.59^\circ$$

$$F \sin(47.59^\circ) = 320$$

$$F(0.738) = 320$$

$$F = \frac{320}{0.738}$$

$$F = 433.6N$$

$$\left(\frac{320}{433.6}\right)^{\text{rad}} = \left(\frac{0.738}{1}\right)^{\text{rad}} = 0$$

$$(0.738)^{\text{rad}} < 0$$

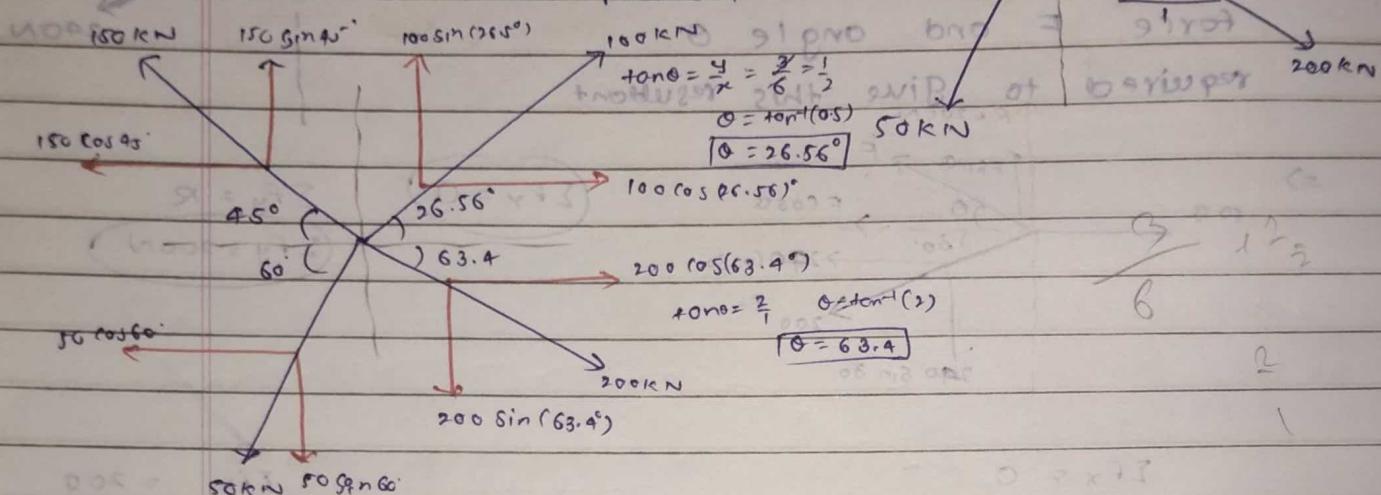
$$PP \cdot \pi / 2 = 0$$

through
find
slope
angle

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100 kN

- ② Find the resultant force for the given system - Also find the direction and position.



$$\begin{aligned}\Sigma F_x &= 100 \cos(26.56^\circ) + 200 \cos(63.4^\circ) - 150 \cos 45^\circ - 50 \cos 60^\circ \\ &= 84.94 + 89.55 - 150 \frac{1}{\sqrt{2}} - 25 \\ &= 148.99 - 106.06 \\ &= 42.92\end{aligned}$$

$$\begin{aligned}\Sigma F_y &= 150 \sin(45^\circ) + 100 \sin(26.56^\circ) - 50 \sin 60^\circ - 200 \sin(63.4^\circ) \\ &= 106.06 - 43.3 + 44.61 - 178.8 \\ &= -71.46\end{aligned}$$

$$\theta = \tan^{-1} \left(\frac{\Sigma F_y}{\Sigma F_x} \right) = \tan^{-1} \left(\frac{-71.46}{42.92} \right)$$

$$\theta = \tan^{-1}(1.664)$$

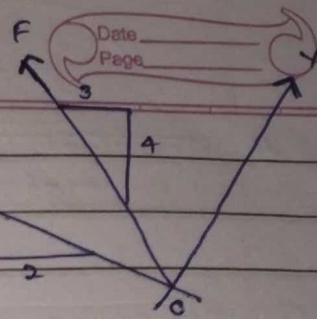
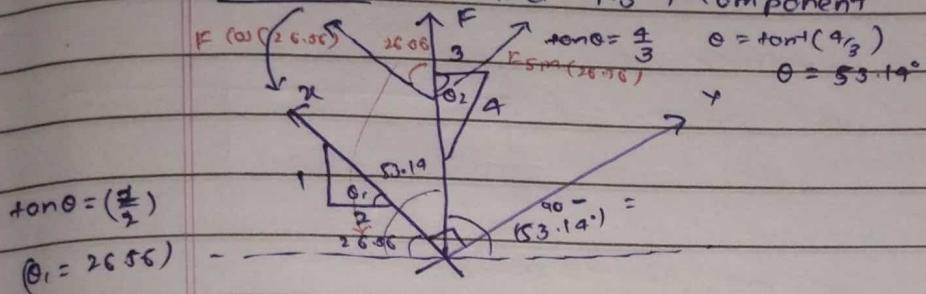
$$\boxed{\theta = 58.99}$$

Graph

③ In the given figure,

X component of force F

is 893N. Find its Y-component



$$F \cos(26.56) = 893 \text{ N}$$

$$F = \frac{893}{\cos(26.56)} = 998.36$$

$$\boxed{F = 998.36 \text{ N}}$$

Y-component = $F \sin(26.56)$

of F

$$= 998.36 \times \sin(26.56)$$

$$\boxed{\text{Y-component} = 496.498 \text{ N}}$$

(4)

Note :-

$$\begin{aligned}\Sigma F_x &= R_x \\ \Sigma F_y &= R_y\end{aligned}$$

Vorrigors theorem

$$\Sigma M_A F = M_A R$$

used whenever you have to find the exact position of resultant in non-concurrent coplanar system

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(3)

Resultant of parallel and General force System

- (1) Determine the magnitude and position of the resultant with respect

\Leftrightarrow

$\Sigma F_x = 0$ ← no any force in x-direction

$$\Sigma F_y = R$$

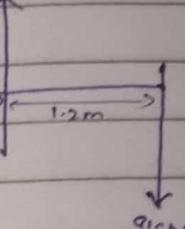
$$-5 - 10 - 9 + 12 = R$$

$$R = -12$$

$$R = 12 N \downarrow$$

Ans) (5 - ve)

12 kN



At point A force 5kN distance 2m
using V.T

$$\Sigma M_A F = M_A R$$

$$(-5 \times 0) - (10 \times 1) + (12 \times 1.8) - (9 \times 3) = -R \times d$$

$$0 - 10 + 21.6 - 27 = -12 \times d$$

$$-3.4 + 21.6 = -12 \times d$$

$$+ 15.4 = 12 \times d$$

$$d = \frac{15.4}{12} = 1.283 \text{ m}$$

Imp Note:

(1) If resultant is Horizontal

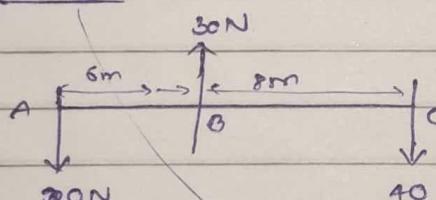
then $\Sigma F_x = R$, $\Sigma F_y = 0$

(2) If resultant is vertical
then $\Sigma F_x = 0$, $\Sigma F_y = R$

(3) If resultant is zero
then $\Sigma F_x = 0$, $\Sigma F_y = 0$

(2) A bar ABC carrying forces 20N at A downward, 30N at B upward and 40N at downward.

Compute the resultant force and locate its position from point A. IF AB = 6m and BC = 8m



$$\Sigma F_x = 0$$

using Vorrigors theorem

$$\Sigma F_y = -20 + 30 - 40$$

$$\Sigma M_A F = M_A R$$

$$\Sigma F_y = -30 N$$

$$-(20 \times 0) + (30 \times 6) - (40 \times 14) = -R \times d$$

$$180 - 560 = -30 \times d$$

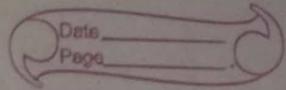
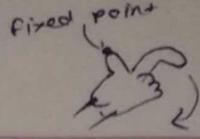
$$\begin{aligned}R &= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \\ &= \sqrt{0 + (30)^2}\end{aligned}$$

$$\therefore \frac{300}{30} = d$$

$$R = 30 N \downarrow$$

$$d = 12.66 \text{ m}$$

Resultant is acting vertically downward

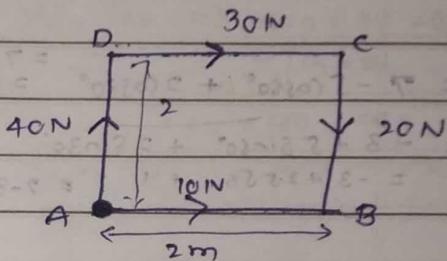


(3)

ABCD is a square of 2m side. Along sides AB, CB, DC and AD the forces of 10N, 20N, 30N and 40N are acting respectively. Find the magnitude, direction and the position of the resultant of the forces from point A.

$$\rightarrow \sum F_x = 30 + 10 = 40 \text{ N} \Rightarrow \text{to the right}$$

$$\uparrow \sum F_y = 40 - 20 = 20 \text{ N} \Rightarrow \text{up}$$



$$R = \sqrt{(40)^2 + (20)^2}$$

$$= \sqrt{1600 + 400}$$

$$= \sqrt{2000}$$

$$R = 44.72 \text{ N}$$

B + Varignon's theorem

$$(\Sigma M_A) = R \times d$$

$$(40 \times 0) + (10 \times 0) + (30 \times 2) + (20 \times 2) = 44.72 \times d$$

$$0 + 0 + 60 + 40 = 44.72 \times d$$

$$\theta = \tan^{-1} \left(\frac{20}{40} \right)$$

$$\theta = 26.56^\circ$$

$$\frac{10}{44.72} = d$$

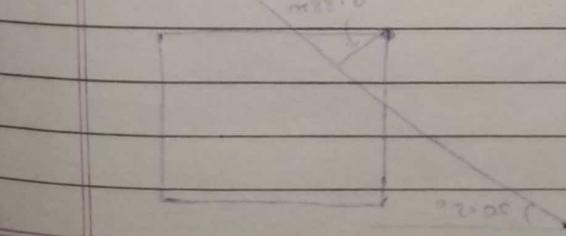
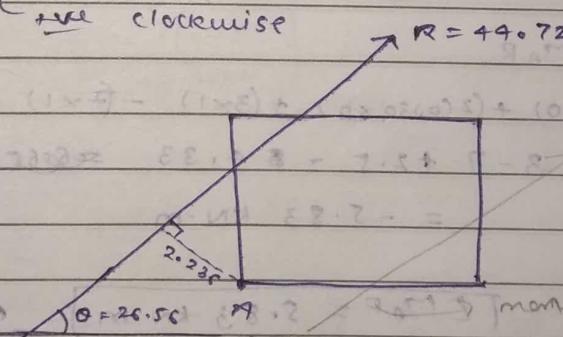
$$d = 2.236$$

$$(\Sigma M_A) = 100 \text{ N-m}$$

clockwise

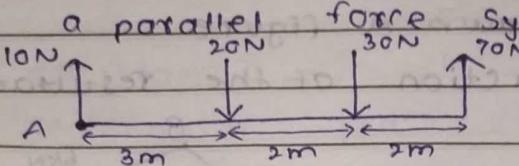
Resultant in
1st quadrant

because
 $\sum F_x > 0$



(5) Locate the resultant with magnitude and direction

For a parallel force system shown in fig.



using Varignon's theorem



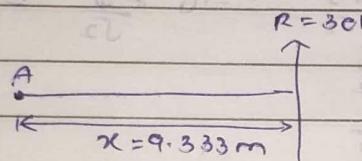
$$\sum F_x = 0$$

$$\begin{aligned}\sum F_y &= 10 - 20 - 30 + 70 \\ &= 30 \text{ N}\end{aligned}$$

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$R = \sqrt{(0)^2 + (30)^2}$$

$$R = 30 \text{ N} \quad (\uparrow) \text{ due}$$



$$\sum M_A F = R \times d$$

consider point A in 10N force

$$\begin{aligned}\sum M_A F &= (10 \times 0) + (20 \times 3) + (30 \times 5) \\ &\quad - (70 \times 7)\end{aligned}$$

$$= 0 + 60 + 150 - 490$$

$$[\sum M_A = 280 \text{ Nm}] \rightarrow \text{anti-clockwise}$$

$$[\sum M_A = 28 \text{ N-m}]$$

$$280 = R \times d$$

$$280 = 30 \times d$$

$$d = \frac{280}{30}$$

$$[d = 2.83 \text{ m}] \quad [d = 9.33 \text{ m}]$$

(6) A system of force acting on a bell crank as shown in fig. - determine the magnitude, direction and point of application of the resultant with respect to O.

$$\sum F_x = \frac{200}{50\sin 60^\circ} - 700 = -450 \text{ N} \quad (\leftarrow)$$

$$\begin{aligned}\sum F_y &= -500 \times \frac{1}{\sin 60^\circ} - 1200 - 1000 \\ &= -2633\end{aligned}$$

$$[\sum F_y = 2633 \text{ (down)}]$$

$$R = \sqrt{(450)^2 + (2633)^2}$$

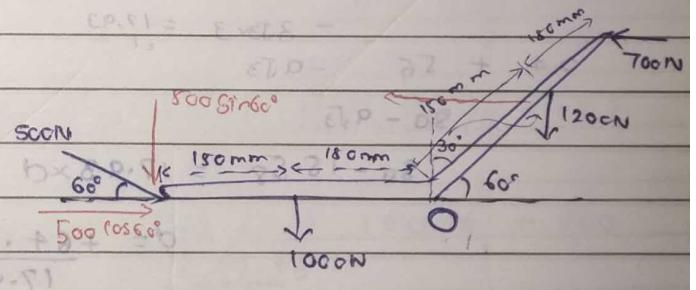
$$R = \sqrt{202500 + 6932689}$$

$$[R = 2671.1 \text{ N}]$$

$$\theta = \tan^{-1} \left(\frac{2633}{450} \right)$$

$$\theta = \tan^{-1} (5.85)$$

$$[\theta = 80.29^\circ]$$



~~5 marks
for marking~~

(Note): ① Varignon's theorem

d → for distance b/w resultant and moment centre

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The

②

$$\bar{x} = \frac{\sum M}{\sum F_y}$$

$$\bar{y} = \frac{\sum M}{\sum F_x}$$

The resultant of the force system acting on the rectangular plate shown in fig. Also find the point where the resultant will meet the x-axis / y-axis

also find shift the resultant to point B

$$\sum F_x = -750 \cos 40^\circ + 1200 + 600 \cos 50^\circ$$

$$\sum F_x = -574.53 + 1200 + 385.67$$

$$[\sum F_x = 1011.14 \text{ N}] \rightarrow$$

$$\sum F_y = 800 + 750 \sin 40^\circ + 600 \sin 50^\circ$$

$$[\sum F_y = 1741.71 \text{ N}] \uparrow$$

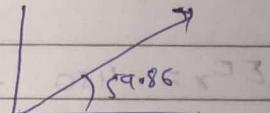
$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$= \sqrt{(1741.7)^2 + (1011.14)^2}$$

$$= 2013.94 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{\sum F_y}{\sum F_x} \right) = \tan^{-1} \left(\frac{1741.71}{1011.14} \right)$$

$$\theta = 59.86^\circ$$



using Varignon's theorem

$$\sum M_B = M_B^R \quad \dots (1)$$

$$\sum M_B^R = -(1200 \times 1.2) + (750 \cos 40^\circ \times 1.2) + (750 \sin 40^\circ \times 1.5) + (800 \times 1.5) + 900$$

$$= -1440 + 689.43 + 723.13 + 1200 + 900$$

$$\sum M_B^R = 1172.56 + 900 = 2072.56$$

$$(M_B^R = R \times d)$$

$$M_B^R = 2013.94 \times d$$

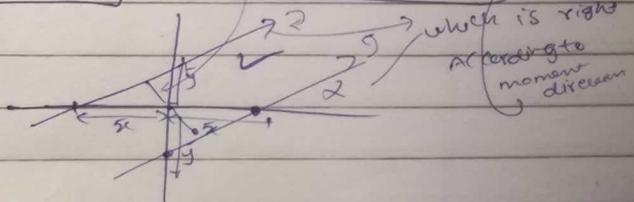
$$2072.56 = 2013.94 \times d$$

$$d = \frac{2072.56}{2013.94} = 1.049$$

$$d = 0.582 \text{ m}$$

$$(d = 1.02107 \text{ m})$$

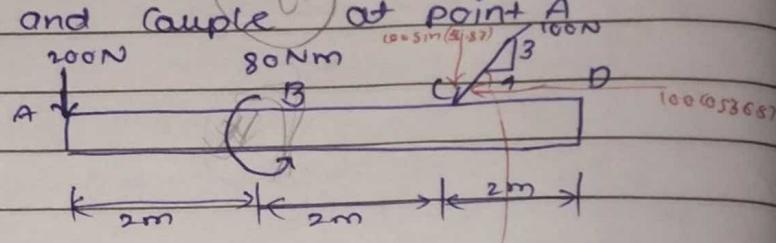
$$\bar{x} = \frac{\sum M}{\sum F_y} = \frac{2072.56}{1741.71} = 1.01899$$



A

Shifting of a force :

- (1) Resolve the system of forces shown in fig. into a force and couple at point A.

 \Rightarrow 

$$\sum F_x = -100 \cos(36.87^\circ)$$

$$[\sum F_x = -80 \text{ N}]$$

$$[\sum F_x = 80 \text{ N}] (\leftarrow)$$

$$\theta = \tan^{-1} \left(\frac{3}{4} \right)$$

$$\sum F_y = -200 - 100 \sin(36.87^\circ)$$

$$\theta = 36.87^\circ$$

$$[\sum F_y = -260 \text{ N}] \quad [\sum F_y = 260 \text{ N}] (\downarrow)$$

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$= \sqrt{(80)^2 + (260)^2} = 272.03 \text{ kN}$$

$$\theta = \tan^{-1} \left(\frac{F_y}{F_x} \right) = \tan^{-1} \left(\frac{260}{80} \right)$$

$$\theta = 72.89^\circ$$

Using Varignon's theorem

$$\sum M_A F = M_A R ; \quad \sum M_A F = R \times d$$

$$\sum M_A F = (200 \times 0) + [100 \cos(36.87^\circ) \times 0] + [100 \sin(36.87^\circ) \times 4]$$

\star

$$= 100 \sin(36.87^\circ) \times 4 - 80$$

$$= 240 - 80$$

$$[\sum M_A F = 160 \text{ N-m}]$$

$$\therefore 160 = R \times d$$

$$160 = 272.03 \times d$$

$$d = \frac{160}{272.03}$$

$$[d = 0.588 \text{ m}]$$