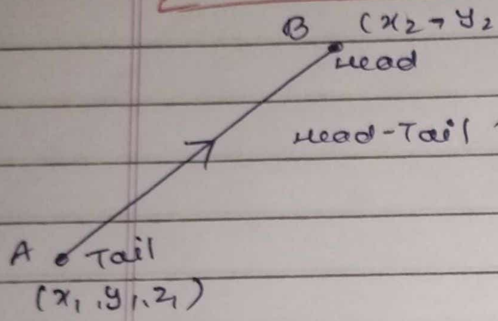
Module - 4

Forces in space

Force vector :-



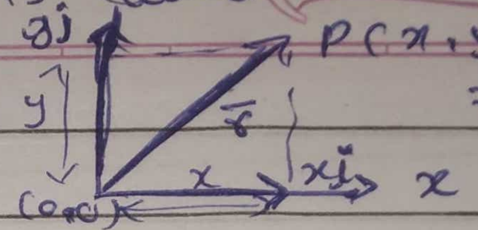
magnitude

unit vector ($\bar{e}_{AB}$)

$$\overline{F_{AB}} = F_{AB}\left(\frac{(x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}\right)$$

Force vector

(2m) Position vector - once you want to specify position of a point, you need to have a reference point. This reference point is called origin



$\bar{r} = x\hat{i} + y\hat{j}$

(2m)
① unit vector :- ~~It is the ratio~~ A vector whose magnitude is equal to ~~to~~ one.

and

which direction is along the original force is called unit vector

$$\bar{F} = F\,\bar{e}$$

$\bar{e}$ → unit vector

F → magnitude

$\bar{F}$ → Force

(2 M)

② Condition of Equilibrium for concurrent forces in space:

If a particle is in equilibrium the component of resultant must be equal to zero

$R_x = 0 \qquad \Sigma F_x = 0$

$R_y = 0 \qquad \Sigma F_y = 0$

$R_z = 0 \qquad \Sigma F_z = 0$

(2 M)

(3) State the conditions of equilibrium for a particle in space

A Body is said to be in equilibrium if the resultant force and the resultant momentum acting on a body is zero

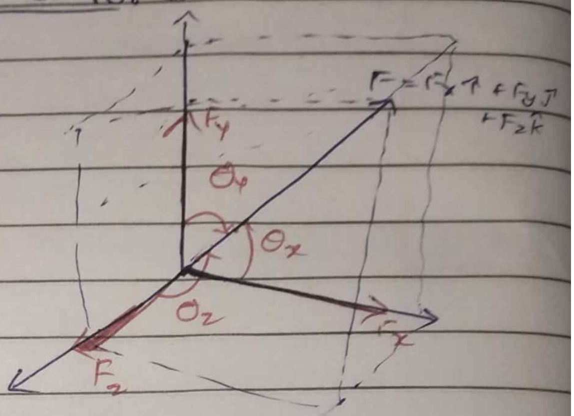
- For a $\boxed{\text{body in space}}$ to remain in equilibrium following condition must be satisfied

algebraic sum of the x-component of all force is zero, $\Sigma F_x = 0$

algebraic sum of the y-component of all force is zero $\Sigma F_y = 0$

algebraic sum of the z-component of all force is zero $\Sigma F_z = 0$

rectangular component of a force in space.



① $\bar{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

can be resolved into three components

② $\boxed{\begin{aligned} F_x &= F\cos\theta_x \\ F_y &= F\cos\theta_y \\ F_z &= F\cos\theta_z \end{aligned}}$

where $\underbrace{\theta_x, \theta_y, \theta_z}$ direction of force along x, y and z direction resp...

magnitude:

③ $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

$\frac{F_x}{F} = l = \cos\theta_x$

$\frac{F_y}{F} = m = \cos\theta_y$

$\frac{F_z}{F} = n = \cos\theta_z$

$\boxed{l^2 + m^2 + n^2 = 1}$

$\boxed{\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1}$

$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$

(1) If $F = (-238\hat{i} + 157\hat{j} + 312\hat{k})$ kN, determine the magnitude and direction of the force.

$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

$F = \sqrt{(-238)^2 + (157)^2 + (312)^2} = \sqrt{56644 + 24649 + 97344}$

$F = \sqrt{178637}$

$\boxed{F = 422.65 \text{ kN}}$ ... {magnitude of given force}

direction of the force

$F_x = F \cos\theta_x$ $\quad$ $F_y = F \cos\theta_y$ $\quad$ $F_z = F \cos\theta_z$

$\cos\theta_x = \frac{F}{F_x}$

$\theta_x = \cos^{-1}\left(\frac{F}{F_x}\right)$ $\quad$ $\theta_y = \cos^{-1}\left(\frac{F}{F_y}\right)$ $\quad$ $\theta_z = \cos^{-1}\left(\frac{F}{F_z}\right)$

$\theta_x = \cos^{-1}(-1.775)$ $\quad$ $\boxed{\theta_y = 68.19^\circ}$ $\quad$ $\boxed{\theta_z = 42.42^\circ}$

$\boxed{\theta_x = 124.27^\circ}$

(2) A Force of 1000N forms angle 60°, 45° and 120° with x, y and z axes respectively. write equation in the vector form

$F = 1000$ $\quad$ $F_x = F\cos\theta_x = 1000 \times \cos 60^\circ = \underline{500}$

$\theta_x = 60^\circ$

$\theta_y = 45^\circ$ $\quad$ $F_y = F\cos\theta_y = 1000 \times \frac{1}{\sqrt{2}} = 707.10$

$\theta_z = 120^\circ$

$F_z = F\cos\theta_z = 1000 \times \cos(90 + 30^\circ)$

$= 1000(-\sin 30^\circ)$

$= -500$

$\vec{F} = (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})$

$\boxed{F = 500\hat{i} + 707.10\hat{j} - 500\hat{k}}$

(3) A force act at origin in a direction defined by angle $\theta_y = 65^\circ$ and $\theta_z = 40^\circ$. knowing that x component of force is -750N Find
(I) The other components
(II) magnitude of force
(III) the value of θ_x

=>

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$
$$(\cos 65)^2 + (\cos 40)^2 = 1 - \cos^2\theta_x$$
$$(0.422)^2 + (0.766)^2 = \sin^2\theta_x$$
$$0.1780 + 0.5867 = \sin^2\theta_x$$
$$\sqrt{0.764} = \sin\theta_x$$
$$\sin^{-1}(0.874) = \theta_x$$
$$\boxed{\theta_x = 60.9^\circ}$$

$F_x = -750\,N$

$F_x = F\cos\theta_x$

$-750 = F\cos(60.9)$

$-750 = F(0.486)$

$|F| = -1543.2\,N$

$\boxed{F = 1543.2\,N}$

magnitude of force

$F_y = F\cos\theta_y = 1543.2 \times \cos 65^\circ$
$= 1543.2 \times 0.422$

$\boxed{F_y = 651.23\,N}$

$F_z = F\cos\theta_z = 1543.2 \times \cos 40^\circ$
$= 1543.2 \times 0.766$

$\boxed{F_z = 1182.09\,N}$

(4) A force f is directed along a line whose direction cosines are $l = 0.5$, $m = 0.5$ w.r.t x and y-axis respectively. Find the components of force F along x, y and z direction if its magnitude is 1000 N

=> $F = 1000\ N$

Direction cosines of line

$l = \cos\theta_x = 0.5$ $\quad \theta_x = \cos^{-1}(0.5)$ $\quad \theta_x = \cos^{-1}(\cos\frac{\pi}{3})$

$\boxed{\theta_x = \pi/3 = 60°}$

$m = \cos\theta_y = 0.5$ $\quad \theta_y = \cos^{-1}(0.5)$ $\quad \boxed{\theta_y = 60°}$

$n = \cos\theta_z = ?$

$F_x = F\cos\theta_z$

$\boxed{F_x = 1000 \cos\theta_z}$ --(1)

we have $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$(\frac{1}{2})^2 + (1/2)^2 + \cos^2\theta_z = 1$

$= 1 - \cos^2\theta_z$

$\frac{1}{4} + \frac{1}{4} = \sin^2\theta_z$

$\sqrt{\frac{1}{2}} = \sin\theta_z$ $\quad \theta_z = \sin^{-1}(\frac{1}{\sqrt{2}})$

$\boxed{\theta_z = 45°}$

Eq (1) putting the value θ_z

$F_x = 1000 \cos 45°$

$= \frac{1000}{\sqrt{2}}$ $\quad \boxed{F_x = 707.10N}$

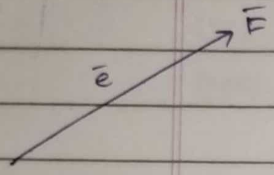
$\boxed{n = \cos\theta_y = \frac{1}{\sqrt{2}} = 0.7}$

$\boxed{F_y = F\cos\theta_y = \frac{1000}{2} = 500\ N}$

$\boxed{F_z = F\cos\theta_z = \frac{1000}{2} = 500N}$

(e)

unit vector : A vector whose magnitude is equal to 1 and which is direction along the original force is called unit vector.



magnitude

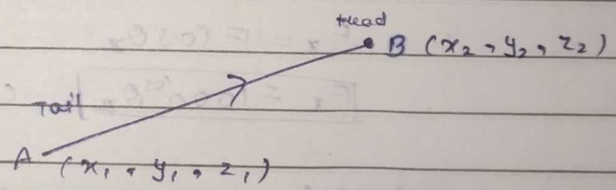
$$\boxed{\bar{F} = F\bar{e}} \rightarrow \text{unit vecter}$$

$$e = \frac{\bar{F}}{|F|} = \frac{\text{Force vector}}{\text{magnitude of the force vector}}$$

unit vector when force is speuficed by two vector

Force passing through two point $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$

$\bar{F} = F\bar{e}$



$$\bar{F} = F\left|\frac{(x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}\right|$$

e.g

(1) A 150kN force act at P(8, 12, 0) and passes through Q(2, 0, 4) put the force in vector form

=> $\bar{F} = F\bar{e}$ let $(x_1, y_1, z_1) \equiv (8, 12, 0)$

$(x_2, y_2, z_2) \equiv (2, 0, 4)$

$\cancel{\boxed{F = 150kN}}$

$$F = 150\left(\frac{(2-8)\hat{i} + (0-12)\hat{j} + (4-0)\hat{k}}{\sqrt{(-6)^2 + (-12)^2 + (4)^2}}\right)$$

$$= 150\left(\frac{-6\hat{i} - 12\hat{j} + 4\hat{k}}{\sqrt{36 + 144 + 16}}\right) = 150\,\frac{-6\hat{i} - 12\hat{j} + 4\hat{k})}{\sqrt{196}}$$

$$= \frac{150(-\cancel{2})(3\hat{i} + 6\hat{j} - 2\hat{k})}{\cancel{14}\,7}$$

$$= -21.42\,(3\hat{i} + 6\hat{j} - 2\hat{k})$$

$$\boxed{\bar{F} = (-64.2\hat{i} - 128.52\hat{j} + 42.84\hat{k})\,N}$$

(2) $|P| = 200N$. Coordinates of point are $A = (0, 15, 0)$km, $E = (30, 0, 0)$. Determine unit vector along line EA and express the force P in vector form

$$\underset{E}{(30,0,0)} \longrightarrow \underset{A}{(0,15,0)}$$

$$\bar{P} = |P|\,\bar{\lambda}_{EA}$$

$$= 200 \times \left(\frac{-30\hat{i} + 15\hat{j} + 0\hat{k}}{\sqrt{900 + 225}}\right)$$

$$= 200\,\frac{(-30\hat{i} + 15\hat{j})}{\sqrt{1125}} = \frac{200}{33.54}(-30\hat{i} + 15\hat{j})$$

$$= 5.96\,(-30\hat{i} + 15\hat{j})$$

$$\boxed{\bar{P} = -178.8\hat{i} + 89.4\hat{j}}$$

Force × ⊥ᵃʳ distance

$\boxed{\text{Moment of force about a point}}$:-

- moment of a force about a point O

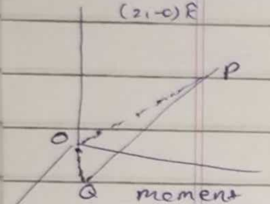
$$\boxed{\bar{M}_0 = \bar{r} \times \bar{F}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

position vector ($\bar{r}$)

Force vector ($\bar{F}$)

$\boxed{\text{moment of a force about origin}}$:

$OP = (x_1 - 0)\hat{i} + (y_1 - 0)\hat{j} + (z_1 - 0)\hat{k}$



moment about origin

$OQ = (x_2 - 0)\hat{i} + (y_2 - 0)\hat{j} + (z_2 - 0)\hat{k}$

$F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ passing through points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$

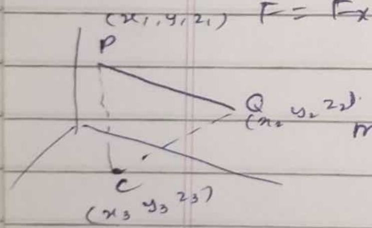
$\bar{M}_0 = \bar{OP} \times \bar{F}$

$$\boxed{\bar{M}_0 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ F_x & F_y & F_z \end{vmatrix}}$$

$\bar{M}_0 = \bar{OQ} \times \bar{F}$

$$\boxed{\bar{M}_0 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_2 & y_2 & z_2 \\ F_x & F_y & F_z \end{vmatrix}}$$

$\boxed{\text{moment of a force about any other point}}$:



$F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ passing through points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$

moment of this force about point $C(x_3, y_3, z_3)$

moment about point C

$$\boxed{\bar{M}_C = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (x_1 - x_3) & (y_1 - y_3) & (z_1 - z_3) \\ F_x & F_y & F_z \end{vmatrix}}$$

$$\boxed{\bar{M}_C = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (x_2 - x_3) & (y_2 - y_3) & (z_2 - z_3) \\ F_x & F_y & F_z \end{vmatrix}}$$

e.g

(1) A Force $\vec{F} = (3\hat{i} - 4\hat{j} + 12\hat{k})$ N acts at a point $P(1, -2, 3)$ m. Find

(a) moment of the force about origin

(b) moment of the force about point $Q(2, 1, 2)$ m

(a)

$$\vec{M}_O = \vec{r}_{PO} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (1-0) & (-2-0) & (3-0) \\ 3 & -4 & 12 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -4 & 12 \end{vmatrix}$$

$$= \hat{i}(-24 + 12) - \hat{j}(12 - 9) + \hat{k}(-4 + 6)$$

$$\boxed{\vec{M}_O = -12\hat{i} - 3\hat{j} + 2\hat{k} \text{ Nm}}$$

$O\,(x_1, y_1, z_1)$

$Q\,(x, y, z)$

(b)

$$\vec{M}_Q = \vec{r}_{PQ} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (1-2) & (-2-1) & (3-2) \\ 3 & -4 & +12 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 1 \\ 3 & -4 & +12 \end{vmatrix}$$

$$= \hat{i}(-36 + 4) - \hat{j}(-12 - 3) + \hat{k}(4 + 9)$$

$$\boxed{\vec{M}_Q \; -32\hat{i} + 15\hat{j} \quad 13\hat{k} \text{ Nm}}$$

(2) A Force of 1200N acts along PQ, P(4, 5, -2) and Q(-3, 1, 6) m. Calculate its moment about a point A(3, 2, 0) m

⟹

$$\bar{F}_{PQ} = F \times \left(\frac{(-3-4)\hat{i} + (1-5)\hat{j} + (6+2)\hat{k}}{\sqrt{(7)^2 + (4)^2 + (8)^2}} \right)$$

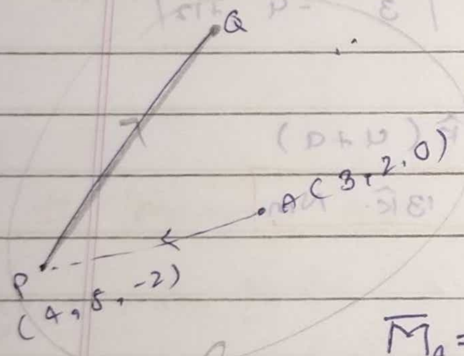
$$= 1200 \times \left(\frac{-7\hat{i} - 4\hat{j} + 8\hat{k}}{\sqrt{49 + 16 + 64}} \right)$$

$$= 1200 \times \left(\frac{-7\hat{i} - 4\hat{j} + 8\hat{k}}{\sqrt{129}} \right)$$

$$= \frac{1200}{11.35} (-7\hat{i} - 4\hat{j} + 8\hat{k})$$

?

$$\boxed{\bar{F} = 105.7 (-7\hat{i} - 4\hat{j} + 8\hat{k})}$$



$$\bar{r} = \overline{AP} = (4-3)\hat{i} + (5-2)\hat{j} - 2\hat{k}$$
$$= \hat{i} + 3\hat{j} - 2\hat{k}$$

$$\bar{M}_A = \bar{r} \times \bar{F}$$

$$= 105.7 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ -7 & -4 & 8 \end{vmatrix}$$

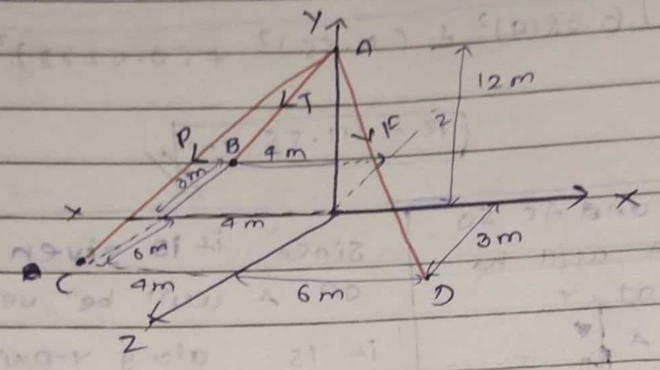
$$= 105.7 [\hat{i}(24 + (-8)) - \hat{j}(8 - 14) + \hat{k}(-4 + 21)$$

$$= 105.7 [16\hat{i} + 6\hat{j} + 17\hat{k}$$

$$\boxed{\bar{M}_A = 105.7 [16\hat{i} + 6\hat{j} + 17\hat{k}]}$$

Problems Q

(1) Find the co-ordinates of points A, B, C, D with respect to x - y - z - axis



A = (0, 12, 0)
B = (-4, 0, -3)
C = (-4, 0, +6)
D = (6, 0, 3)

(2) Determine the resultant of the con-current forces having following magnitude and passing through the indicated

Point P = 280N , (12, 6, -4)
T = 520N , (-3, -4, 12)
F = 270N , (6, -3, -6).

=> P = (12, 6, -4)
T = (-3, -4, 12)
F = (6, -3, -6)

(a) Force vector

$$\bar{P} = P\left(\frac{12\hat{i}+6\hat{j}-4\hat{k}}{\sqrt{144+36+16}}\right) = \frac{280}{\sqrt{196}}(12\hat{i}+6\hat{j}-4\hat{k})$$

$$= \frac{\cancel{280}^{40}}{\cancel{14}}(12\hat{i}+6\hat{j}-4\hat{k}) = 20(12\hat{i}+6\hat{j}-4\hat{k})$$

$$\boxed{\bar{P} = 240\hat{i}+120\hat{j}-80\hat{k}}$$

$$T = T\left(\frac{-3\hat{i}-4\hat{j}+12\hat{k}}{\sqrt{9+16+144}}\right) = \frac{520}{\sqrt{169}}(-3\hat{i}-4\hat{j}+12\hat{k})$$

$$= 40(-3\hat{i}-4\hat{j}+12\hat{k})$$

$$\boxed{\bar{T} = -120\hat{i}-160\hat{j}+480\hat{k}}$$

$$\bar{F} = F\left(\frac{6\hat{i} - 3\hat{j} - 6\hat{k}}{\sqrt{36+9+36}}\right) = \frac{\cancel{270}^{3}}{\cancel{9}}(6\hat{i} - 3\hat{j} - 6\hat{k})$$

$$= 30(6\hat{i} - 3\hat{j} - 6\hat{k})$$

$$\boxed{\bar{F} = 180\hat{i} - 90\hat{j} - 180\hat{k}}$$

(b) resultant

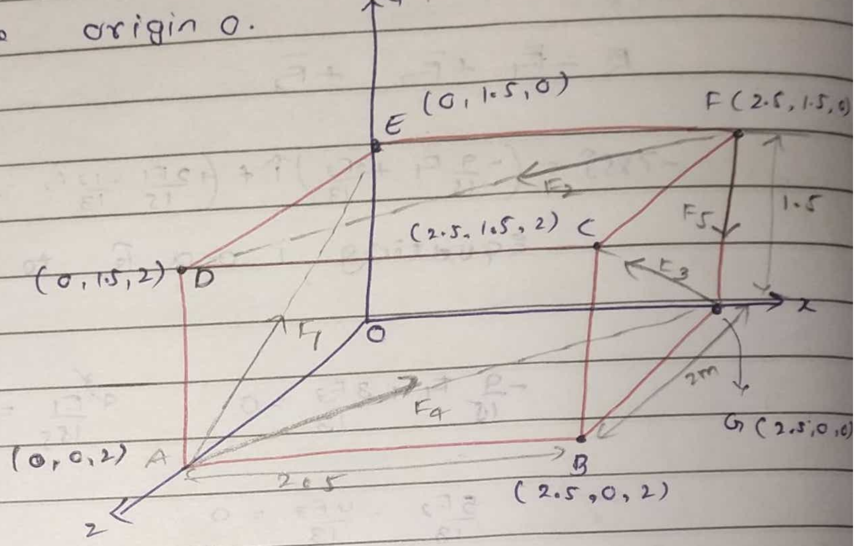
$$\bar{R} = \bar{P} + \bar{T} + \bar{F}$$

$$= 240\hat{i} + 120\hat{j} - 80\hat{k} - 120\hat{i} - 160\hat{j} + 480\hat{k} + 180\hat{i} - 90\hat{j} - 180\hat{k}$$

$$\boxed{\bar{R} = 300\hat{i} - 130\hat{j} + 220\hat{k}}$$

Resultant of General force system in space

(1) Force $F_1 = 1KN$, $F_2 = 3KN$, $F_3 = 2KN$, $F_4 = 5KN$ and $F_5 = 2KN$ act long the line joining the corner of the parallel piped whose sides are 2.5m, 2m and 1.5m as shown in figure. Find the resultant force and the moment of the resultant couple at the origin O.



(a) Coordinate :

A = (0, 0, 2)

B = (2.5, 0, 2)

C = (2.5, 1.5, 2)

D = (0, 1.5, 2)

E = (0, 1.5, 0)

F = (2.5, 1.5, 0)

(b) Force vector :

$$\bar{F_1} = F_1 \bar{e}_{AE} = 1 \times \left(\frac{1.5\hat{j} - 2\hat{k}}{\sqrt{2.25 + 4}} \right) = \frac{1.5\hat{j} - 2\hat{k}}{\sqrt{6.25}} = \frac{1.5}{2.5}\hat{j} - \frac{2}{2.5}\hat{k}$$

$$\boxed{F_1 = 0.6\hat{j} - 0.8\hat{k}}$$

$$\bar{F_2} = F_2 \bar{e}_{FO} = 3 \times \left(\frac{-2.5\hat{i} + 0\hat{j} + 2\hat{k}}{\sqrt{6.25 + 4}} \right) = \frac{3(-2.5\hat{i} + 2\hat{k})}{\sqrt{10.25}}$$

$$F_2 = \frac{3 \times -2.5)\hat{i}}{3.2} + \frac{6}{3.2}\hat{k}$$

$$\boxed{F_2 = -2.34\hat{i} + 1.87\hat{k}}$$

$$\bar{F_3} = F_3 \bar{e}_{(GD)} = 2 \times \left(\frac{0\hat{i} + 1.5\hat{j} + 2\hat{k}}{\sqrt{2.25 + 4}} \right) = \left(\frac{2 \times 1.5}{2.5}\right)\hat{j} + \frac{2 \times 2}{2.5}\hat{k}$$

$$\boxed{F_3 = 1.2\hat{j} + 1.6\hat{k}}$$

Resultant of force vector

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4 + \bar{F}_5$$

$$= 0.6\hat{j} - 0.8\hat{k} - 2.34\hat{i} + 1.87\hat{k} + 1.2\hat{j} + 1.6\hat{k}$$
$$3.9\hat{i} - 3.12\hat{k} - 2\hat{j}$$

$$\boxed{\bar{R} = 1.56\hat{i} - 0.2\hat{j} - 0.45\hat{k}}$$

$$\bar{F}_4 = F_4\, \bar{e}_{AG} = 5 \times \left(\frac{2.5\hat{i} - 2\hat{k}}{\sqrt{6.25 + 4}} \right)$$

$$= \left(\frac{5 \times 2.5}{3.2}\right)\hat{i} - \left(\frac{5 \times 2}{3.2}\right)\hat{k}$$

$$\boxed{F_4 = 3.9\hat{i} - 3.12\hat{k}}$$

$$\bar{F}_5 = F_5\, \bar{e}_{FG} = \underline{-2\hat{j}}$$

(b)

(c) position vector

$$\bar{r}_1 = 2\hat{k}, \quad \bar{r}_2 = 2.5\hat{i} + 1.5\hat{j}, \quad \bar{r}_3 = 2.5\hat{i}$$

$$\bar{r}_4 = 2k \qquad \bar{r}_5 = 2.5\hat{i}$$

(N) moment vector

$$\bar{M}_1 = \bar{r}_1 \times \bar{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 2 \\ 0 & 0.6 & -0.8 \end{vmatrix} \qquad \boxed{\bar{M}_1 = -1.2\hat{i}}$$

$$\bar{M}_2 = \bar{r}_2 \times \bar{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2.5 & 1.5 & 0 \\ -2.34 & 0 & 1.87 \end{vmatrix} \qquad \boxed{\bar{M}_2 = 2.8\hat{i} - 4.6\hat{j} + 3.5\hat{k}}$$

$$\bar{M}_3 = \bar{r}_3 \times \bar{F}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2.5 & 0 & 0 \\ 0 & 1.2 & 1.6 \end{vmatrix} \qquad \boxed{\bar{M}_3 = -4\hat{j} + 3\hat{k}}$$

$$\bar{M}_4 = \bar{r}_4 \times \bar{F}_4 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 2 \\ 3.9 & 0 & -3.12 \end{vmatrix} \qquad \boxed{\bar{M}_4 = +7.8\hat{j}}$$

$$\bar{M}_5 = \bar{r}_5 \times \bar{F}_5 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2.5 & 0 & 0 \\ 0 & -2 & 0 \end{vmatrix} \qquad \bar{M}_5 = -5\hat{k}$$

(d) Resultant moment vector

$$\Sigma \bar{M}_0 = \bar{M}_1 + \bar{M}_2 + \bar{M}_3 + \bar{M}_4 + \bar{M}_5$$

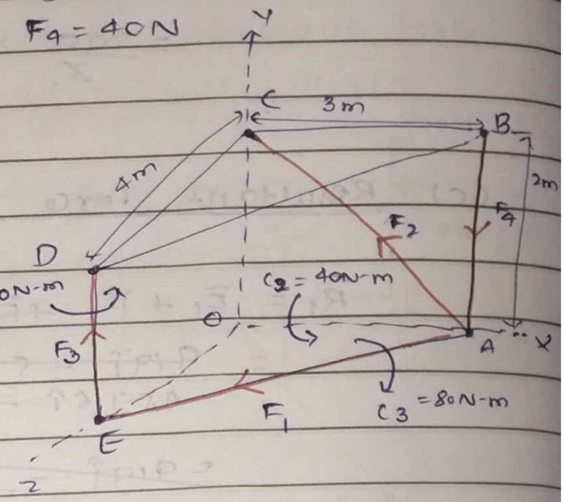
$$= -1.2\hat{i} + 2.8\hat{i} - 4.6\hat{j} + 3.5\hat{k} - 4\hat{j} + 3\hat{k} + 7.8\hat{j} - 5\hat{k}$$

$$\boxed{\Sigma \bar{M} = 1.6\hat{i} - 0.8\hat{j} + 1.5\hat{k}}$$

10 marks ★ ★ ★

Date ______
Page ______

(2) Determine the resultant force and the resultant couple of the force system shown in figure when $F_1 = 100\,N$, $F_3 = 40N$, $F_2 = 20N$, $F_4 = 40N$



(a) Co-ordinate of all points

A = (3, 0, 0)
B = (3, 2, 0)
C = (0, 2, 0)
D = (0, 2, 4)
E = (0, 0, 4)

(0,0,4) E ← A (3,0,0)

(b) Force vector :-

$$\bar{F}_1 = F_1 \lambda F_1 = 100\left(\frac{-3\hat{i} + 4\hat{k}}{\sqrt{9+16}}\right) = 20(-3\hat{i} + 4\hat{k})$$

$$\boxed{\bar{F}_1 = -60\hat{i} + 80\hat{k}}$$

C (0,2,0) ← A (3,0,0)

$$\bar{F}_2 = F_2 \lambda F_2 = 20\left(\frac{-3\hat{i} + 2\hat{j}}{\sqrt{9+4}}\right) = \frac{20}{\sqrt{3}}(-3\hat{i} + 2\hat{j})$$

$$= 5.54(-3\hat{i} + 2\hat{k})$$

$$\cancel{= 11.54(-3\hat{i} + 2\hat{j})}$$

$$\cancel{\bar{F}_2 = -34.64\hat{i} + 69.28\hat{j}}$$

$$\bar{F}_2 = -16.62\hat{i} + 11.08\hat{j}$$

(3, 2, 0) → A (3, 0, 0)

$$\bar{F}_4 = F_4 \lambda F_4 = 40\left(\frac{(3-3)\hat{i} - 2\hat{j} + 0\hat{k}}{\sqrt{4}}\right) = \frac{40}{2}(-2\hat{j}) \quad \boxed{\bar{F}_4 = -40\hat{j}}$$

D (0, 2, 4) ← E (0, 0, 4)

$$\bar{F}_3 = F_3 \lambda F_3 = 40\left(\frac{2\hat{j}}{\sqrt{(2)^2}}\right) = \frac{40}{2} 2\hat{j} = 40\hat{j}$$

$$\boxed{\bar{F}_3 = 40\hat{j}}$$

(c) Resultant of force

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4 = -60\hat{i} + 80\hat{k} - 34.64\hat{i} + 69.28\hat{j} + 40\hat{j} - 40\hat{j}$$

$$\boxed{\bar{R} = -94.64\hat{i} + 69.28\hat{j} + 80\hat{k}}$$

Position vector $\xleftarrow{}$ Tail ke co-ordinate

Head

(c) Resultant of force

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$$

$$= -60\hat{i} + 80\hat{k} - 16.62\hat{i} + 11.08\hat{j} + \cancel{40\hat{j}} - \cancel{40\hat{j}}$$

$$\boxed{\bar{R} = -76.62 + 11.08\hat{j} + 80\hat{k}}$$

Right Head thumb rule

(d) Couple vectors:

$$\bar{C}_3 = C_3 \lambda_{EA} = 80\left(\frac{+3\hat{i} + 4\hat{k}}{\sqrt{25}}\right) = \frac{\cancel{80}^{16}}{\cancel{5}}(+3\hat{i} + 4\hat{k})$$

$$\boxed{\bar{C}_3 = +48\hat{i} + 64\hat{k}}$$

D

E

$$\bar{C}_1 = C_1 \lambda_{DE} = 20\left(\frac{2\hat{j}}{\sqrt{(2)^2}}\right) = 20\frac{\cancel{2}\hat{j}}{\cancel{2}} = 20\hat{j}$$

O — A

$$\bar{C}_2 = C_2 \lambda_{OA} = 40\left(\frac{\cancel{3}\hat{i}}{\cancel{3}}\right) = 40\hat{i}$$

(e) Position vector

$$\bar{r}_1 = 3\hat{i} \qquad \bar{r}_2 = 3\hat{i} \qquad \bar{r}_3 = 4\hat{k} \qquad \bar{r}_4 = 3\hat{i} + 2\hat{j}$$

(f) moment vector

$$\bar{M}_1 = \bar{r}_1 \times \bar{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ -60 & 0 & 80 \end{vmatrix} = \hat{i}(0) + \hat{k}(0) - 240\hat{j}$$

$$\boxed{\bar{M}_1 = -240\hat{j}}$$

$$\bar{M}_2 = \bar{r}_2 \times \bar{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ -16.62 & 11.08 & 0 \end{vmatrix} = \hat{i}(0) - \hat{j}(0) + \hat{k}(33.24)$$

$$\boxed{\bar{M}_2 = 33.24\hat{k}}$$

$$\bar{M}_3 = \bar{r}_3 \times \bar{F}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 4 \\ 0 & 40 & 0 \end{vmatrix} = \hat{i}(-160) - \hat{j}(0) + \hat{k}(0)$$

$$\boxed{\bar{M}_3 = -160\hat{i}}$$

$$\bar{M}_4 = \bar{r}_4 \times \bar{F}_4 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 0 & -40 & 0 \end{vmatrix} = \hat{i}(0) - \hat{j}(0) + -120\hat{k}$$

$$\boxed{\bar{M}_4 = -120\hat{k}}$$

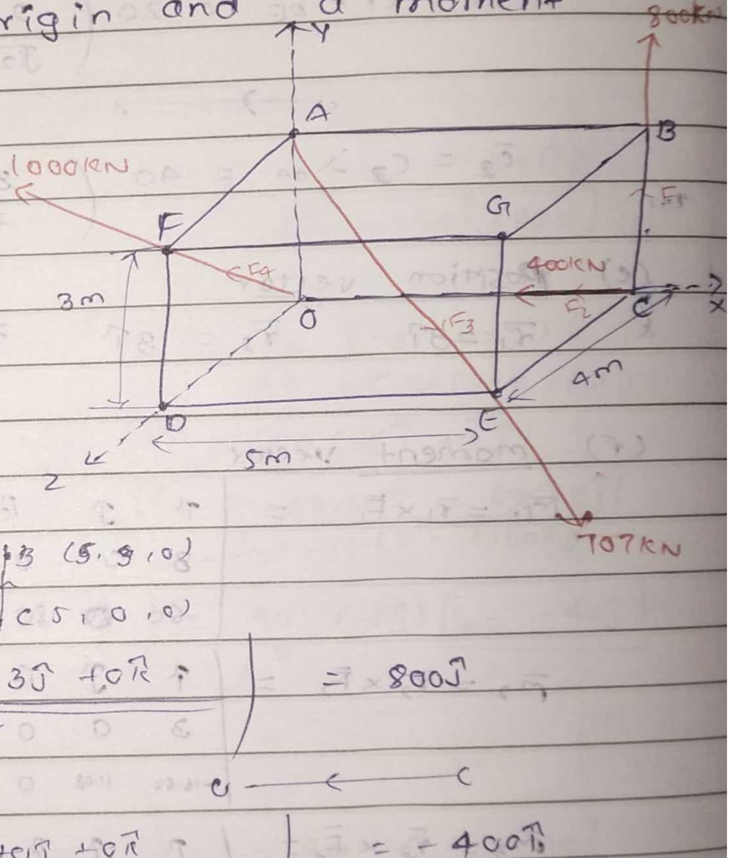
(g) <u>Resultant moment (couple):</u>

$$\sum M_o^R = \bar{M}_1 + \bar{M}_2 + \bar{M}_3 + \bar{M}_4 + \bar{C}_1 + \bar{C}_2 + \bar{C}_3$$

$$= -240\hat{j} + 33.24\hat{k} + (-160\hat{i}) - 120\hat{k} + 20\hat{j} + 40\hat{i}$$
$$48\hat{i} - 64\hat{k}$$

$$\boxed{\sum M_o^R = -72\hat{i} + 220\hat{j} - 150.76\hat{k}}$$

(3) A rectangular parallelopiped subjected to four force in direction. reduce them to <u>resultant force at the</u> origin and a moment



(a) <u>coordinate of all point</u>

A = (0 , 3 , 0)
B = (5 , 3 , 0)
C = (5 , 0 , 0)
D = (0 , 0 , 4)
E = (5 , 0 , 4)
F = (0 , 3 , 4)

B (5, 3, 0)
C (5, 0, 0)

(b) <u>Force vector</u>

$$\bar{F}_1 = F_1 \lambda F_1 = 800\left(\frac{0\hat{i} + 3\hat{j} + 0\hat{k}}{\sqrt{(3)^2}}\right) = 800\hat{j}$$

C ← C

$$\bar{F}_2 = F_2 \lambda F_2 = 400\left(\frac{-5\hat{i} + 0\hat{j} + 0\hat{k}}{\sqrt{(5)^2}}\right) = -400\hat{i}$$

$(0,3,0) \rightarrow (5,0,4)$
A → E

$$\bar{F}_3 = F_3 \lambda F_3 = 707\left(\frac{5\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{25 + 16 + 9}}\right)$$

$$= \frac{707}{5\times\sqrt{2}}(5\hat{i} - 3\hat{j} + 4\hat{k}) = \frac{707}{7.07}(5i - 3\hat{j} + 4\hat{k})$$
$$= 100(5\hat{i} - 3\hat{j} + 4\hat{k})$$
$$\bar{F}_3 = 500\hat{i} - 300\hat{j} + 400\hat{k}$$

$\bar{F}_4 = F_4 \lambda_4 = 1000 \times \left(\frac{3\hat{j} + 4\hat{k}}{\sqrt{25}} \right) = 200(3\hat{j} + 4\hat{k})$

$\boxed{\bar{F}_4 = 600\hat{j} + 800\hat{k}}$

(c) Resultant force

$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$

$= 800\hat{j} - 400\hat{i} + 500\hat{i} - 300\hat{j} + 400\hat{k} + 600\hat{j} + 800\hat{k}$

$= \cancel{1100\hat{i}}$

$\boxed{\bar{R} = 100\hat{i} + 1100\hat{j} + 1200\hat{k}}$

$R = \sqrt{(100)^2 + (1100)^2 + (1200)^2} = 1630.95 \text{ kN}$

(d) Position vector

$\bar{r}_1 = 5\hat{i}$ $\quad \bar{r}_2 = 5\hat{i}$ $\quad \bar{r}_3 = 3\hat{j}$, $\quad \bar{r}_4 = 0\hat{i} + 0\hat{j} + 0\hat{k}$

(e) moment vector

$$\bar{M}_1 = \bar{r}_1 \times \bar{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 0 & 0 \\ 0 & 800 & 0 \end{vmatrix} = 4000\hat{k}$$

$$\bar{M}_2 = \bar{r}_2 \times \bar{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 0 & 0 \\ -400 & 0 & 0 \end{vmatrix} = 0\hat{j} + 0\hat{i} + 0\hat{k}$$

$$\bar{M}_3 = \bar{r}_3 \times \bar{F}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 0 \\ 500 & -300 & 400 \end{vmatrix} = \hat{i}(1200) - \hat{j}(0) + \hat{k}\,-(1500)$$

$= 1200\hat{i} - 1500\hat{k}$

$$\bar{M}_4 = \bar{r}_4 \times \bar{F}_4 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0 \\ 0 & 600 & 800 \end{vmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

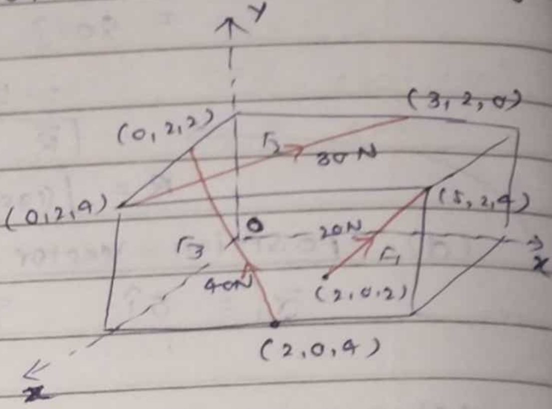
(f) Resultant moment

$\Sigma M_O^R = \bar{M}_1 + \bar{M}_2 + \bar{M}_3 + \bar{M}_4$

$= 4000\hat{k} + 0 + 1200\hat{i} - 1500\hat{k} + 0$

$= 1200\hat{i} + 2500\hat{k}$

(4) Determine the resultant of the non-concurrent, non-parallel system of forces as shown in fig



(a) <u>Force vector</u>

$$\bar{F}_1 = F_1 \lambda F_1 = 20\left(\frac{3\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{9+4+4}}\right)$$

$$= \frac{20}{\sqrt{17}}(3\hat{i}+2\hat{j}+2\hat{k})$$

$$= 4.85(3\hat{i}+2\hat{j}+2\hat{k})$$

$$\boxed{\bar{F}_1 = 14.55\hat{i} + 9.7\hat{j} + 9.7\hat{k}}$$

$$\bar{F}_2 = F_2 \lambda F_2 = 30\left(\frac{(3)\hat{i}+(2-2)\hat{j}-4\hat{k}}{\sqrt{9+16}}\right)$$

$$= \frac{\cancel{30}^{\,6}}{\cancel{5}}\,(3)\hat{i} - 4\hat{k}$$

$$\boxed{\bar{F}_2 = 18\hat{i} - 24\hat{k}}$$

$$\bar{F}_3 = F_3 \lambda F_3 = 40\left(\frac{-2\hat{i}+2\hat{j}-2\hat{k}}{\sqrt{4+4+4}}\right)$$

$$= 40 \times \frac{(-2\hat{i}+2\hat{j}-2\hat{k})}{\sqrt{4\times 3}} = \frac{20}{\sqrt{3}}(-2\hat{i}+2\hat{j}+2\hat{k})$$

$$= 11.54(-2\hat{i}+2\hat{j}+2\hat{k})$$

$$\boxed{\bar{F}_3 = -23.09\hat{i} + 23.09\hat{j} - 23.09\hat{k}}$$

(b) <u>resultant force</u>

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$$

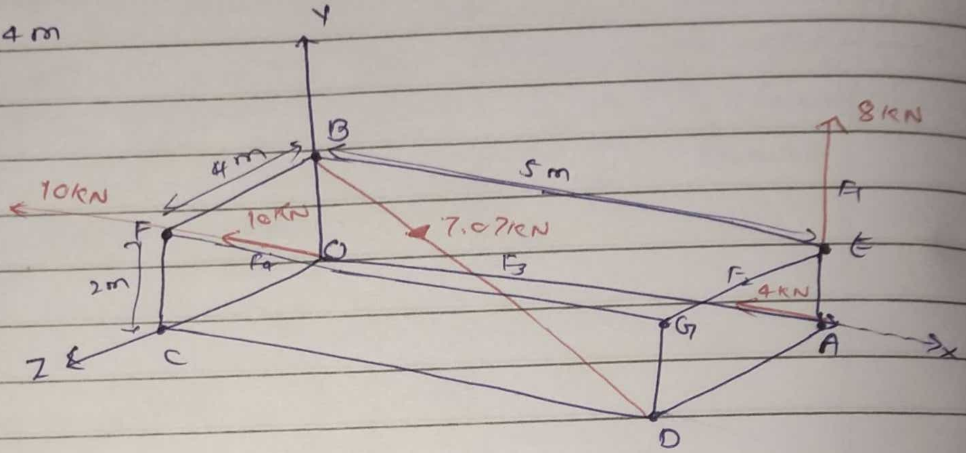
$$= 14.55\hat{i} + 9.7\hat{j} + 9.7\hat{k} + 18\hat{i} - 24\hat{k} - 23.09\hat{i} + 23.09\hat{j} - 23.09\hat{k}$$

$$\boxed{\bar{R} = 9.46\hat{i} + 32.79\hat{j} - 37.89\hat{k}}\ ?$$

Imp

*(5) A rectangular parallelepiped carries four forces a shown in fig ~~X-A8~~. Reduce the force system to a resultant force applied at the origin and a moment about origin.

OA = 5m OB = 2m
OC = 4m



(a) Co-ordinate of all point

A = (5, 0, 0)
B = (0, 2, 0)
D = (5, 0, 4)
E = (5, 2, 0)
F = (0, 2, 4)

E (5, 2, 0)
A (5, 0, 0)

(b) Force vector

$$\bar{F}_1 = F_1 \lambda F_1 = 8\left(\frac{0\hat{i} + 2\hat{j} + 0\hat{k}}{\sqrt{(2)^2}}\right) = \frac{8}{\cancel{2}}(\cancel{2}\hat{j}) = 8\hat{j}$$

O ← A (5, 0, 0)

$$\bar{F}_2 = F_2 \lambda F_2 = 4\left(\frac{-5\hat{i}}{\sqrt{(5)^2}}\right) = \frac{4(-\cancel{5}\hat{i})}{\cancel{5}} = -4\hat{i}$$

(0, 2, 0) B → D (5, 0, 4)

$$\bar{F}_3 = 7.07\left(\frac{5\hat{i} - 2\hat{j} + 4\hat{k}}{\sqrt{25 + 4 + 16}}\right) = \frac{7.07}{6.7}(5\hat{i} - 2\hat{j} + 4\hat{k}) = (5.27)\hat{i} - 2.1\hat{j} + 4.2\hat{k}$$

F (0, 2, 4) ← O (0, 0)

$$\bar{F}_4 = 10\left(\frac{2\hat{j} + 4\hat{k}}{\sqrt{4 + 16}}\right) = \frac{10}{\sqrt{5}}(\hat{j} + 2\hat{k}) = \frac{10(\hat{j} + 2\hat{k})}{2.23}$$

$$= 4.48\hat{j} + 8.96\hat{k}$$

$\bar{R} = 5.27\hat{i} - 2.1\hat{j} + 4.2\hat{k} + 8\hat{j} - 4\hat{i} + 4.48\hat{j} + 8.96\hat{k}$

$\boxed{\bar{R} = 1.27\hat{i} + 10.38\hat{j} + 13.16\hat{k}}$

(c) Position vector

$\bar{r}_1 = 5\hat{i}$ $\quad$ $\bar{r}_3 = 2\hat{j}$

$\bar{r}_2 = 5\hat{i}$ $\quad$ $\bar{r}_4 = 0\hat{i} + 0\hat{j} + 0\hat{k}$

(d) moment of force

$$\bar{M}_1 = \bar{r}_1 \times F_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 0 & 0 \\ 0 & 8 & 0 \end{vmatrix}$$

$$= 0\hat{i} - \hat{j}(0) + \hat{k}(40) = 40\hat{k}$$

$$\bar{M}_2 = \bar{r}_2 \times F_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 0 & 0 \\ -4 & 0 & 0 \end{vmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

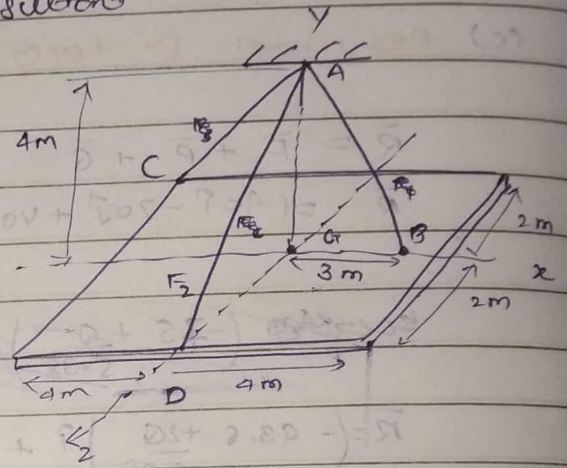
$$\bar{M}_3 = \bar{r}_3 \times F_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 0 \\ 5.27 & -2.1 & 4.2 \end{vmatrix} = 8.4\hat{i} + 0\hat{j} - 10.54\hat{k}$$

$$\Sigma M_o^R = \bar{M}_1 + \bar{M}_2 + \bar{M}_3$$

$$= 40\hat{k} + 0 + 8.4\hat{i} - 10.54\hat{k}$$

$$\boxed{\Sigma M_o^R = 8.4\hat{i} + 29.46\hat{k}}$$

(6) The lines of action of three forces concurrent at origin 'O' pass respectively through A, B, C having coordinates of G (0, 0, 0). The magnitudes of the forces are $\bar{F}_{AB} = 40N$, $\bar{F}_{AC} = 30N$, $\bar{F}_{AD} = 40N$. Find the magnitude and direction of the resultant



(a) co-ordinates of all points

$A = (0, 4, 0)$

$B = (3, 0, 0)$

$C = (-4, 0, -2)$

$D = (0, 0, 2)$

A (0,4,0) → B (3,0,0)

(b) Force vector

~~$F_{AB} = F_1$~~

~~$F_{AD} = F_2$~~

~~$F_{AC} = F_3$~~

$$\bar{F}_{AB} = F_{AB} \times \lambda_{AB} = 40\left(\frac{3\hat{i} - 4\hat{j}}{\sqrt{9+16}}\right)$$

$$= \frac{40}{5}(3\hat{i} - 4\hat{j}) = 24\hat{i} - 32\hat{j}$$

$$\boxed{\bar{F}_{AB} = 24\hat{i} - 32\hat{j}}$$

A (0,4,0) → D (0,0,2)

$$\bar{F}_{AD} = F_{AD}\lambda_{AD} = 40\left(\frac{-4\hat{j} + 2\hat{k}}{\sqrt{16+4}}\right)$$

$$= \frac{20}{\sqrt{5}}(-4\hat{j} + 2\hat{k}) = -35.77\hat{j} + 17.88\hat{k}$$

(0,4,0) → (-4,0,-2)

$$\bar{F}_{AC} = F_{AC}\lambda_{AC} = 30\left(\frac{-4\hat{i} - 4\hat{j} - 2\hat{k}}{\sqrt{16+16+4}}\right)$$

$$= 5(-4\hat{i} - 4\hat{j} + 2\hat{k}) \quad \boxed{F_{AC} = -20\hat{i} - 20\hat{j} + 10\hat{k}}$$

(c) Resultant of force

$\bar{R} = \bar{F}_{AB} + \bar{F}_{AC} + \bar{F}_{AD}$

$= 24\hat{i} - 32\hat{j} + -35.77\hat{j} + 17.88\hat{k} - 20\hat{i} - 20\hat{j} + 10\hat{k}$

$\boxed{\bar{R} = 4\hat{i} - 87.77\hat{j} + 27.88\hat{k}}$

$|\bar{R}| = \sqrt{(4)^2 + (-87.77)^2 + (27.88)^2}$ $\boxed{|\bar{R}| = 8\cancel{7}.86N}$

Imp **

(1) A force $P_1 = 10N$ in magnitude acts along direction AB whose coordinates of points A and B are (3, 2, -1) m and (8, 5, 3) m respectively. another force $P_2 = 5N$ in magnitude acts along BC where C has coordinates (-2, 11, -5) m
determine (a) the resultant of P_1 and P_2 in its vector form
(b) moment of the resultant about D(1, 1, 1) m

Sol:- $P_1 = 10N$ $P_2 = 5N$

(a) $\boxed{\text{Coordinates}}$

A = (3, 2, -1), B (8, 5, 3), C (-2, 11, -5)

A (3, 2, -1) ⟶ B (8, 5, 3)

(b) $\boxed{\text{Force vector}}$ $\bar{P_1} = P_1 \bar{e}_{AB}$

$$= 10 \left(\frac{(8-3)\hat{i} + (5-2)\hat{j} + 4\hat{k}}{\sqrt{(5)^2 + (3)^2 + (4)^2}} \right)$$

$$= \frac{10^2}{5\sqrt{2}} \left(5\hat{i} + 3\hat{j} + 4\hat{k} \right)$$

(8, 5, 3) B ⟶ C (-2, 11, -5)

$$= \sqrt{2} \left(5\hat{i} + 3\hat{j} + 4\hat{k} \right)$$

$$= (7.07)\hat{i} + (4.24)\hat{j} + (5.65)\hat{k}$$

$\bar{P_2} = P_2 \bar{e}_{BC}$

$$= 5 \left(\frac{-10\hat{i} + 6\hat{j} - 8\hat{k}}{\sqrt{100 + 36 + 64}} \right) = \frac{5}{10\sqrt{2}} (-10\hat{i} + 6\hat{j} - 8\hat{k})$$

$$= \frac{1}{2\sqrt{2}} \cancel{2} (-5\hat{i} + 3\hat{j} - 4\hat{k})$$

$$= \frac{1}{\sqrt{2}} (-5\hat{i} + 3\hat{j} - 4\hat{k})$$

$$= (-3.53)\hat{i} + (2.12)\hat{j} (2.82)$$

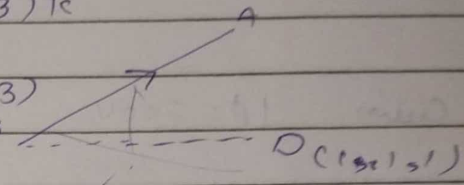
(c) resultant of force

$$\bar{R} = \bar{P}_1 + \bar{P}_2$$

$$= (7.07 - 3.53)\hat{i} + (4.24 + 2.12)\hat{j} + (5.65 - 2.82)\hat{k}$$

$$= (3.54)\hat{i} + (6.36)\hat{j} + (2.83)\hat{k}$$

(d) position vector



$$\bar{r}_{DB} = (8-1)\hat{i} + (5-1)\hat{j} + (3-1)\hat{k}$$

$$= 7\hat{i} + 4\hat{j} + 2\hat{k}$$

(e) moment of resultant about point D

$$\bar{M}_D = \bar{r}_{DB} \times \bar{R} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 4 & 2 \\ (3.54) & (6.36) & (2.83) \end{vmatrix}$$

$$= \hat{i}(11.32 - 12.72) - \hat{j}(19.81 - 7.08) + \hat{k}(44.52 - 14.16)$$

$$(-1.4\hat{i} - 12.73\hat{k} + 30.36\hat{k})\,Nm$$