Exercise 52

① Find values of (i) Δe^{ax} (ii) $\Delta^2 e^{x}$ (iii) $\Delta \log x$

→ (i) $\Delta f(x) = f(x+h) - f(x)$

$\therefore \Delta e^{ax} = e^{a(x+h)} - e^{ax}$

$= e^{ax} \cdot e^{ah} - e^{ax}$

$\Delta e^{ax} = e^{ax} [e^{ah} - 1]$

(ii) $\Delta^2 e^{x} = \Delta(\Delta e^{x})$

$\Delta e^{x} = e^{x+h} - e^{x} = e^{x}[e^{h} - 1]$

$\Delta^2 (e^{x}) = \Delta [e^{x}(e^{h}-1)]$

$= (e^{h}-1)[\Delta e^{x}]$

$= (e^{h}-1)[e^{x+h} - e^{x}]$

$= (e^{h}-1)^2 \cdot e^{x}$

(iii) $\Delta \log x = \log(x+h) - \log x$

$= \log \frac{(x+h)}{x}$

$= \log \left(1 + \frac{h}{x}\right)$

② We have

$(1+\Delta)(1-\nabla) f(x) = (1+\Delta)\{(1-\nabla) f(x)\}$

$= (1+\Delta)[f(x) - \{f(x) - f(x-h)\}]$

$= (1+\Delta) f(x-h) = E f(x-h) \quad \because E = 1+\Delta$

$\therefore (1+\Delta)(1-\nabla) f(x) = 1 \cdot f(x)$

Exercise 53

① Evaluate $\int_0^{10} \frac{1}{1+x^2}\,dx$ by using ~~Bo~~ Rectangle rule

→ ~~$h=\frac{b-a}{}$~~ Taking $h=1$, divide the interval $[0,10]$ into ten subintervals

$$h = \frac{10-0}{10} = 1$$

The midpoint ~~of~~ values of each 10 subintervals are 0.5, 1.5, 2.5, 3.5, 4.5, 5.5, 6.5, 7.5, 8.5, 9.5

Here $y = f(x) = \frac{1}{1+x^2}$

x	$y = \frac{1}{1+x^2}$
$x_1^* = 0.5$	$y_1 = 0.8$
$x_2^* = 1.5$	$y_2 = 0.307692$
$x_3^* = 2.5$	$y_3 = 0.379310$
$x_4^* = 3.5$	$y_4 = 0.075471$
$x_5^* = 4.5$	$y_5 = 0.047058$
$x_6^* = 5.5$	$y_6 = 0.032$
$x_7^* = 6.5$	$y_7 = 0.023121$
$x_8^* = 7.5$	$y_8 = 0.017467$
$x_9^* = 8.5$	$y_9 = 0.013651$
$x_{10}^* = 9.5$	$y_{10} = 0.010958$

$$\therefore \int_0^{10} \frac{1}{1+x^2}\,dx$$

$$= h[y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10}]$$

$$= 1[0.8 + 0.307692 + 0.379310 + 0.075471 + 0.047058 + 0.032 + 0.023121 + 0.017467 + 0.013651 + 0.010958]$$

$$= \cancel{1.707148}$$

$$\approx 1.707149$$

Ex. 3 Evaluate $\int_0^2 x^2 \, dx$ using rectangle rule with four subintervals

→ Here $h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{2}{4} = 0.5$

The midpoint of subintervals are 0.25, 0.75, 1.25 & 1.75

x	$x_1^* = 0.25$	$x_2^* = 0.75$	$x_3^* = 1.25$	$x_4^* = 1.75$
$y = x^2$	$y_1 = 0.0625$	$y_2 = 0.5625$	$y_3 = 1.5625$	$y_4 = 3.0625$

$$\int_0^2 x^2 \, dx = h[y_1 + y_2 + y_3 + y_4] = 0.5[0.0625 + 0.5625 + 1.5625 + 3.0625]$$
$$\approx 2.625$$

Exercise 54

1) Evaluate $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\,dx$ by using Trapezoidal rule.

→ $I = \int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\,dx$

We have $a = 0.2$, $b = 1.4$

consider $n = 6$ $\therefore h = \frac{b-a}{n} = \frac{1.4 - 0.2}{6} = 0.2$

$\therefore h = 0.2$

let $y = \sin x - \log_e x + e^x$, consider following table

	x_0	x_1	x_2	x_3	x_4	x_5	x_6
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y	3.0295	2.7975	2.8976	3.1660	3.5597	4.0698	4.4042
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By Trapezoidal rule

$I = \frac{h}{2}[X + 2R]$ where X - extreme values, R - Remaining value

$= \frac{h}{2}[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$

$I = \frac{0.2}{2}\left[(3.029 + 4.4042) + 2[2.7975 + 2.8976 + 3.1660 + 3.5597 + 4.0698]\right]$

$I = 6.04815$

Exercise 54

Ex. ③ $\int_0^{\pi/2} \frac{\sin x}{x}\,dx$ by using Trapezoidal Rule

→ Let $I = \int_0^{\pi/2} \frac{\sin x}{x}\,dx$, we have $a=0$, $b=\frac{\pi}{2}$

$n=6$

$$h = \frac{b-a}{n} = \frac{\pi/2 - 0}{6} = \frac{\pi}{12}$$

$\therefore\ y = \frac{\sin x}{x}$

consider following table

	x_0	x_1	x_2	x_3	x_4	x_5	x_6
x	0	$\pi/12$	$\frac{2\pi}{12}$	$\frac{3\pi}{12}$	$\frac{4\pi}{12}$	$\frac{5\pi}{12}$	$\pi/2$
y	1	0.9886	0.9579	0.9003	0.8270	0.7379	0.6366
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By Trapezoidal Rule

$I = \frac{h}{2}\left[X + 2R\right]$ where X - Extreme points

R - Remaining points

$$= \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{\pi}{24}\left[(1.0000 + 0.6366) + 2(0.9886 + 0.9579 + 0.9003 + 0.7379 + 0.8270)\right]$$

$I = 1.3692$

Exercise 56

① Evaluate $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)dx$ by using Simpson's $\frac{1}{3}$rd rule

→ Let $I = \int_{0.2}^{1.4} (\sin x - \log_e x + e^x)dx$

$a = 0.2$, $b = 1.4$

consider $n = 6$, $\therefore h = \frac{b-a}{n} = \frac{1.4 - 0.2}{6} = 0.2$

consider table

x	x_0 0.2	x_1 0.4	x_2 0.6	x_3 0.8	x_4 1.0	x_5 1.2	x_6 1.4
y	3.0295 y_0	2.7975 y_1	2.8976 y_2	3.1660 y_3	3.5597 y_4	4.0698 y_5	4.4042 y_6

By Simpson's $\frac{1}{3}$rd rule

$$I = \frac{h}{3}[X + 4O + 2E]$$

where
X - Extreme terms
O - odd terms
E - Even terms

$$I = \frac{0.2}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{0.2}{3}\left[(3.0295+4.4042) + 4(2.7975+3.1660+4.0698) + 2(2.8976+3.5597)\right]$$

$$= 0.066[7.4337 + 4(10.0333) + 2(6.4573)]$$

$$= 0.0666[7.4337 + 40.1332 + 12.9146]$$

$$= 0.0666[60.4815]$$

$$= 4.0280$$

Exercise 56

(2) Evaluate $\int_{4}^{5.2} \log x \, dx$ by simpson's $\frac{1}{3}$rd rule

→ Let $I = \int_{4}^{5.4} \log x \, dx$, $a=4$, $b=5.2$.

Let $n=6$ [Minimum value of n which is applicable for all 3 rules is 6.
[Trape., simpson's $\frac{1}{3}$rd, simpson's $\frac{3}{8}$th)

$$h = \frac{b-a}{n} = \frac{5.2-4}{6} = \frac{1.2}{6} = 0.2$$

x	x_0 4.0	x_1 4.2	x_2 4.4	x_3 4.6	x_4 4.8	x_5 5.0	x_6 5.2
$y=\log x$	1.3863 y_0	1.4351 y_1	1.4816 y_2	1.5261 y_3	1.5686 y_4	1.6094 y_5	1.6487 y_6

By simpson's $\frac{1}{3}$rd rule

$$I = \frac{h}{3}[X + 4O + 2E]$$, where X - Extreme point
O - odd terms
E - Even term

$$I = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \frac{0.2}{3}\left[(1.3863 + 1.6487) + 4(1.4351 + 1.5261 + 1.6094) + 2(1.4816 + 1.5686)\right]$$

$$= 1.8278472$$

Exercise 57

1) The velocity of train starts from rest is given by the following table, the time being reckoned in minutes from the start and speed in km/hour

Time	3	6	9	12	15	18
Velocity	22	29	31	20	4	0

Estimate approximately the distance covered in 18 minutes by simpson's $\frac{1}{3}$rd rule.

→ We know that $\frac{ds}{dt} = v$ i.e. $S = \int v\,dt$

~~i.e. s = ∫~~

∴ To use simpson's rule we need even number of ordinates. We take one more ordinate at $t=0$, by data when $t=0$, $v=0$

let the ordinates be denoted as

y_0	y_1	y_2	y_3	y_4	y_5	y_6
0	22	29	31	20	4	0

By simpson's $\frac{1}{3}$rd rule

$$S = \frac{h}{3}\left[X + 2E + 4.O\right]$$

We have $h = \frac{3}{60}$ hrs $= \frac{1}{20}$ hrs

$X = 0+0$, $E = 29 + 20 = 49$, $O = 22 + 31 + 4$, $O = 57$

$$S = \frac{1}{60}\left[0 + 98 + 4(57)\right] = \frac{1}{60}[326] \approx 5.433 \text{ Kms}$$

Exercise 58

1. Evaluate $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\, dx$ using simpson's $\frac{3}{8}$th rule

→ Let $I = \int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\, dx$,

We have $a = 0.2$, $b = 1.4$

consider $n = 6$, $h = \frac{1.4 - 0.2}{6} = 0.2$

consider the table for $f(x)$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
Y	3.0295	2.7975	2.8976	3.1660	3.5597	4.0698	4.4042
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By simpson's $\frac{3}{8}$th rule

$$I = \frac{3h}{8}[X + 2T + 3R]$$

Where X = Extreme terms, T - Multiple of 3 terms
R - Remaining terms

$$I = \frac{3h}{8}[(y_0 + y_6) + 2y_3 + 3(y_1 + y_2 + y_4 + y_5)]$$

$$= \frac{3(0.2)}{8}[(3.0295 + 4.4042) + 2(3.1660) + 3(2.7975 + 2.8976 + 3.5597 + 4.0698)]$$

$$\boxed{I = 4.05297}$$

Exercise 58

(3) Evaluate $\int_0^{\pi/2} \frac{\sin x}{x} dx$ by simpson's $\frac{3}{8}$th rule

→ We take $h = \frac{\pi}{12}$ & prepare table

x	0	$\frac{\pi}{12}$	$\frac{2\pi}{12}$	$\frac{3\pi}{12}$	$\frac{4\pi}{12}$	$\frac{5\pi}{12}$	$\frac{6\pi}{12}$
y	1 y_0	0.9886 y_1	0.9579 y_2	0.9003 y_3	0.8270 y_4	0.7379 y_5	0.6366 y_6

By simpson's $\frac{3}{8}$th rule

$$S = \frac{3h}{8}\left[X + 2T + 3R\right] = \frac{3h}{8}\left[(y_0 + y_6) + 2y_3 + 3(y_1 + y_2 + y_4 + y_5)\right]$$

where X - Extreme points, T - multiple of 3 terms
R - Remained terms

$$\therefore S = \frac{3}{8} \cdot \frac{\pi}{12}\left[1.6366 + 2(0.9003) + 3(3.5084)\right]$$

$$= 1.37075$$

Exercise 59

Exercise 59

① Find volume of solid of revolution formed by rotating about x-axis, the area bounded by the lines $x=0, x=1.5, y=0$ and the curve passing through

x :	0.00	0.25	0.50	0.75	1.0	1.25	1.50
y :	1.00	0.9655	0.9195	0.8215	0.7081	0.5812	0.5759

→ since volume is given by $\int \pi y^2 dx$, we first prepare table of y^2

x :	0.00	0.25	0.50	0.75	1.0	1.25	1.50
y :	1.00 y_0	0.9655 y_1	0.9195 y_2	0.8261 y_3	0.7081 y_4	0.5812 y_5	0.5759 y_6

By simpson's $\frac{3}{8}$ th rule

$$\int y^2 dx = \frac{3h}{8}[X + 2T + 3R]$$

Where $X = y_0 + y_6$, $T = y_3$, $R = y_1 + y_2 + y_4 + y_5$

$$\therefore \int y^2 dx = \frac{3(0.25)}{8}[1.5759 + 2(0.8261) + 3(3.1703)]$$

$$= 1.19428$$

$$\therefore \text{volume} = \pi \int y^2 dx = \pi(1.19428)$$

$$= 3.7519$$

Exercise 55

Ex (3) The velocity of train which starts from rest is given by following table the time being reckoned in minutes from the start & speed in km/hr

Time	3	6	9	12	15	18
Velocity	22	29	31	20	4	0

Estimate approximately the distance covered in 18 minutes by Trapezoidal rule.

→ We know that $\frac{ds}{dt} = v \Rightarrow \int \frac{ds}{dt} = \int v$

$$\Rightarrow \int ds = \int v dt$$

$$\Rightarrow s = \int v dt$$

∴ To use trapezoidal rule we need even number of ordinates. We can take one more ordinate at $t=0$.

By data at $t=0$, $v=0$ we have

y_0	y_1	y_2	y_3	y_4	y_5	y_6
0	22	29	31	20	4	0

By trapezoidal rule

$$s = \frac{h}{2}[X + 2R]$$

We have $h = \frac{3}{60}$ hrs $= \frac{1}{20}$ hrs, $X = 0+0$, $R = 22+29+31+20+4$ $=$

$$s = \frac{1}{40}[2(106)]$$

$$s \approx \frac{212}{40} \approx 5.3 \text{ kms.}$$