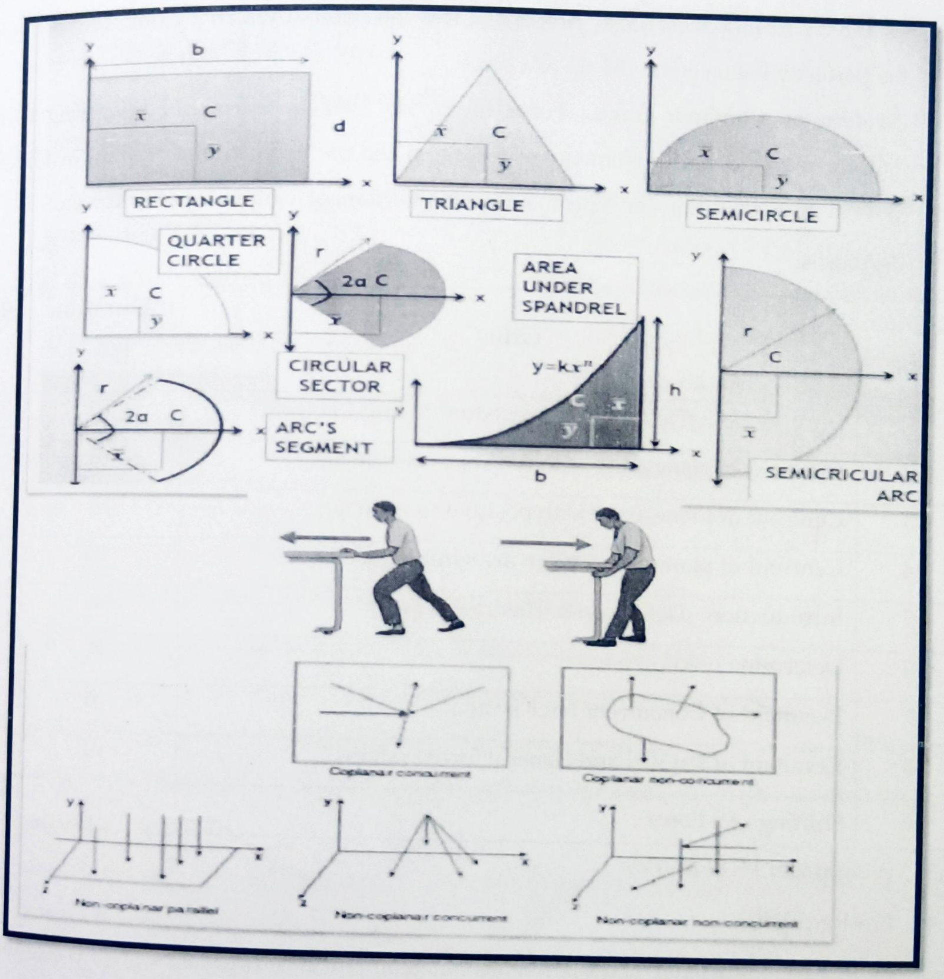
# Module 1: Introduction to Co-Planar System of Forces

## *Infographics*



# Lecture: 1

## 1.1 *Introduction to Centroid of Plane Laminas*

### 1.1.1 Motivation:

**Centroid**: In mathematics and physics, the centroid or geometric center of a plane figure is the arithmetic mean (average) position of all the points in the shape. The definition extends to any object in n-dimensional space: its centroid is the mean position of all the points in all of the coordinate directions. Informally, it is the point at which a cutout of the shape could be perfectly balanced on the tip of a pin.

**System of Coplanar forces:** Force has a key role in learning Engineering Mechanics. Different types of arrangements of forces on the body constitutes System of forces which aids in understanding concepts like Resultant, Moment, couple, equilibrium etc.

### 1.1.2 Syllabus:

| Lecture No. | Title | Duration (Hrs.) | Self-Study (Hrs.) |
|---|---|---|---|
| L | Introduction: (Definition & Procedure to find Centroid) | 01 | 02 |
| 2 | Centroid of plane areas with negative coordinates | 01 | 02 |
| 3 | Centroid of plane areas with positive coordinates | 01 | 02 |
| 4 | Centroid of plane areas which are symmetric | 01 | 02 |
| 5 | Introduction: (Definition & Theory of Force) | 01 | 02 |
| 6 | Determine unknown force | 01 | 02 |
| 7 | Resultant of Concurrent force systems | 01 | 02 |
| 8 | Resultant of Parallel and General force systems | 01 | 02 |
| 9 | Shifting of a Force | 01 | 02 |

### 1.1.3 Weightage: 15-18 Marks

### 1.1.4 Pre-Requisite:

1) A basic knowledge of a 2- Dimensional geometrical co-ordinate system is required.
2) Knowledge of formulas for calculating areas of basic shapes like circle, rectangle, triangle, semicircle, quarter circle is required.
3) Knowledge of location of centroid of the above-mentioned shapes is required.
4) Knowledge of fundamentals of physics (forces) and mathematical formulation learnt at higher secondary level of education (trigonometry).

### 1.1.5 Learning Objectives: Learners shall be able to

1) Locate and place forces such that the body remains balanced.
2) Identify all the standard shapes of the given composite shape and find the individual centroids.
3) Calculate the position of centroid of the complete composite body using the Centroid formula for areas.
4) Understand various systems of forces; Calculate and find the effect forces exerted on them.
5) Find resultant of two forces by Parallelogram law of Forces& Resultant of three of more forces by method of resolution
6) Locate of resultant by Varignon's Theorem

### 1.1.6 Key Notations:

m = meter

km = kilometer

kg = kilogram

t = for ton or tons

s = for second

min = minute

N = Newton

N-m = Newton × meters

kN-m = kilo-Newton × meters

rad = radian or radians

rev = revolution or revolutions

CG = Centre of gravity

$\bar{X}$= x-co-ordinate of the CG

$\bar{Y}$= y-co-ordinate of the CG

A = area of given lamina

R = Resultant.

$\sum F_x$ = Summation of all horizontal components of forces.

$\sum F_y$ = Summation of all vertical components of forces.

$\sum M$ = Summation of Moments of all forces taken about a point.

$\theta$ = Angle between two forces P and Q for Parallelogram Law of forces.

$\alpha$ = Angle made by the resultant with the horizontal force.

d = Perpendicular distance between line of action of force and point about which moment is required to be taken.

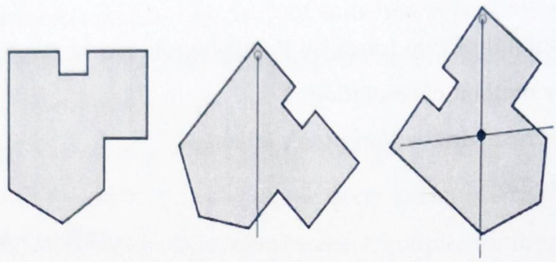
x = Perpendicular distance between $\sum F_y$ i.e. R and point about which moment is required to be taken for vertical parallel force system.

y = Perpendicular distance between $\sum F_x$ i.e. R and point about which moment is required to be taken for horizontal parallel force system.

### 1.1.7 Theoretical Background:

**Locating the centroid:**

- **Plumb line method** - The centroid of a uniform planar lamina, such as (a) below, may be determined, experimentally, by using a plumb line and a pin to find the center of mass of a thin body of uniform density having the same shape. The body is held by the pin inserted at a point near the body's perimeter, in such a way that it can freely rotate around the pin; and the plumb line is dropped from the pin (b). The position of the plumb line is traced on the body. The experiment is repeated with the pin inserted at a different point of the object. The intersection of the two lines is the centroid of the figure (c). This method can be extended (in theory) to concave shapes where the centroid lies outside the shape, and to solids (of uniform density), but the positions of the plumb lines need to be recorded by means other than drawing.



(a) (b) (c)

**Fig. 1.1**

- **Balancing method** - For convex two-dimensional shapes, the centroid can be found by balancing the shape on a smaller shape, such as the top of a narrow cylinder. The centroid occurs somewhere within the range of contact between the two shapes. In principle, progressively narrower cylinders can be used to find the centroid to arbitrary precision. In practice air currents make this unfeasible. However, by marking the overlap range from multiple balances, one can achieve a considerable level of accuracy.

### 1.1.8 Formulae:

1. Centroid for areas,

$$\bar{X} = \frac{\sum A_i X_{Gi}}{\sum A_i};\ \bar{Y} = \frac{\sum A_i Y_{Gi}}{\sum A_i};$$

2. Parallelogram law of forces,

   Magnitude:

   $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

   Direction:

   $\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$

3. Lami's Theorem:

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

4. Resolution of forces,

   Magnitude:

   $\Sigma F_x$= Forces along X-direction,

   $\Sigma F_y$= Forces along Y-direction

   $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

   Direction: $\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right)$

5. Moment: $M_O = F \times d$ (Force x Perpendicular distance)

### 1.1.9 *Introduction: (General-Mechanics, Definition & Procedure to find Centroid)*

**Learning Objective:** Learner will be able to understand the key concepts related to Engineering Mechanics

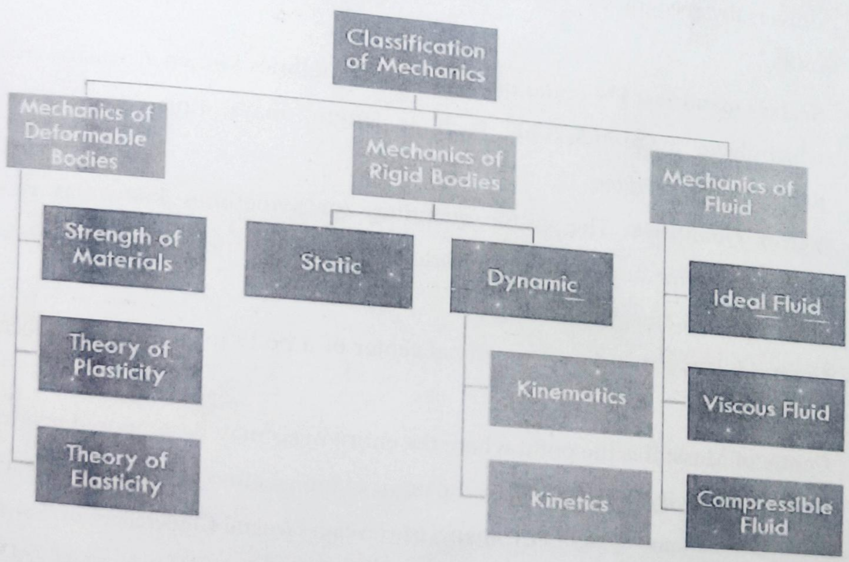
### 1.1.10 Theory:

- **Newtonian mechanics:** A branch of mechanics that deals with concepts of Newton's law of motion as distance, time, and mass in a period of time are known as Newtonian mechanics. This Newtonian mechanics describes the motion of objects in routine life affected by the forces. Newtonian mechanics is a very straightforward formulation theory that deals with Newton's second law of motion.

  Generally, as for the theoretical approach, mechanics is split into three parts Newtonian, Lagrangian and Hamiltonian mechanics, but our routine work deals in Newtonian mechanics.

- **Engineering Mechanics:** The subject of Engineering Mechanics is that branch of Applied Science, which deals with the laws and principles of Mechanics, along with their applications to engineering problems. As a matter of fact, knowledge of Engineering Mechanics is very essential for an engineer in planning, designing and construction of his various types of structures and machines. To take up the job more skillfully, an engineer must pursue the study of Engineering Mechanics in a most systematic and scientific manner.

- **Divisions of Engineering Mechanics:** The subject of Engineering Mechanics may be divided into the following groups:




- **Fundamental Units:** The measurement of physical quantities is one of the most important operations in engineering. Every quantity is measured in terms of some arbitrary, but internationally accepted units, called fundamental units. All the physical quantities, met with in Engineering Mechanics, are expressed in terms of three fundamental quantities, i.e. Length, Mass, Time.

- **METRE (Length):** The international 'metre' may be defined as the shortest distance (at 0°C) between two parallel lines engraved upon the polished surface of the Platinum-Iridium bar, kept at the International Bureau of Weights & Measures at Sevres near Paris.
- **KILOGRAM (Mass):** The international kilogram may be defined as the mass of the Platinum-Iridium cylinder, which is also kept at the International Bureau of Weights and Measures at Sevres near Paris.
- **SECOND (Time):** The fundamental unit of time for all the four systems is second, which is 1/ (24 × 60 × 60) = 1/86 400th of the mean solar day. A solar day may be defined as the interval of time between the instants at which the sun crosses the meridian on two consecutive days. This value varies throughout the year. The average of all the solar days, of one year, is called the mean solar day.

➤ **Derived Units:** Sometimes, the units are also expressed in other units (which are derived from fundamental units) known as derived units e.g. units of area, velocity, acceleration, pressure etc.

➤ **Systems of Units:** There are only four systems of units, which are commonly used and universally recognized. These are known as: S.I. units, M.K.S. units, C.G.S. units, sssF.P.S. units

➤ **Scalar Quantities:** The scalar quantities (or sometimes known as scalars) are those quantities which have magnitude only such as length, mass, time, distance, volume, density, temperature, speed etc.

➤ **Vector Quantities:** The vector quantities (or sometimes known as vectors) are those quantities which have both magnitude and direction such as force, displacement, velocity, acceleration, momentum etc.

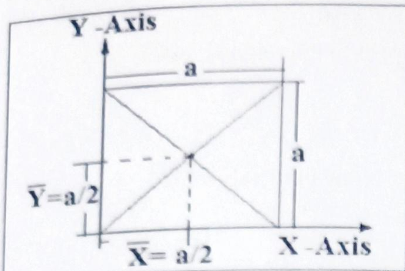
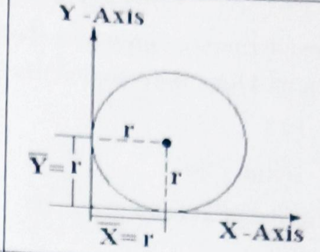
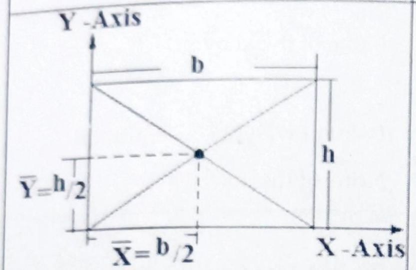
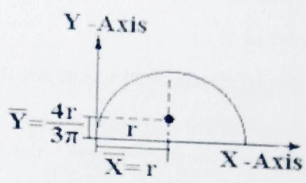
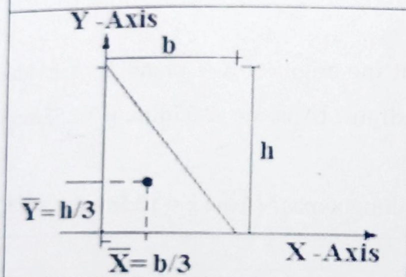
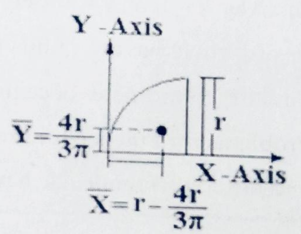
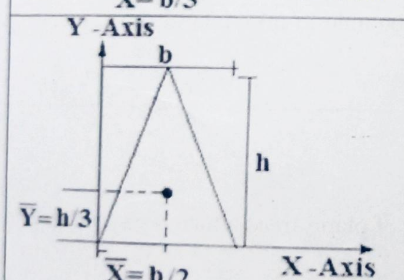
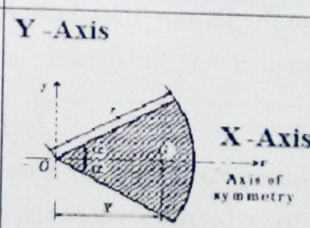
➤ **Centroid:** It is defined as geometrical center of a body (e.g., center of a rectangle, center of triangle etc.).

➤ **Centre of Mass:** It is the point where the entire mass may be supposed to be concentrated.

➤ **Center of Gravity:** It is defined as the point of intersection of all the gravity axes of the body.

**S.I. Units (International System of Units):** The eleventh General Conference* of Weights and Measures has recommended a unified and systematically constituted system of fundamental and derived units for international use. This system of units is now being used in many countries. In India, the Standards of Weights and Measures Act of 1956 (vide which we switched over to M.K.S. units) has been revised to recognize all the S.I. units in industry and commerce. In this system of units, the fundamental units are meter (m), kilogram (kg) and second (s) respectively. But there is a slight variation in their derived units. Density (Mass density) - kg/m$^3$, Force - N (Newton), Pressure - N/mm$^2$ or N/m$^2$, Work done (in joules) J = N-m, Power in watts W = J/s

## Centroid of Standard Shapes

| | | | |
|---|---|---|---|
|  | **Square:** $\overline{X} = \frac{a}{2}$ $\overline{Y} = \frac{a}{2}$ |  | **Circle** $\overline{X} = r$ $\overline{Y} = r$ |
|  | **Rectangle** $\overline{X} = \frac{b}{2}$ $\overline{Y} = \frac{h}{2}$ |  | **Semi - Circle** $\overline{X} = r$ $\overline{Y} = \frac{4r}{3\pi}$ |
|  | **Right Angle Triangle** $\overline{X} = \frac{b}{3}$ $\overline{Y} = \frac{h}{3}$ |  | **Quarter - Circle** $\overline{X} = r - \frac{4r}{3\pi}$ $\overline{Y} = \frac{4r}{3\pi}$ |
|  | **Equilateral Triangle** $\overline{X} = \frac{b}{2}$ $\overline{Y} = \frac{h}{3}$ |  | **Sector of Circle** $\overline{X} =$ 2rsinα / 3α |

- **Procedure to solve problems of centroid of a given respective figure:**
  a) From a given composite figure, consider each figure separately in the form of triangle, circle, semicircle, etc.
  b) Specify the reference axis as x-axis and y-axis, if not specified.
  c) Determine the area of each figure as $A_1$, $A_2$, $A_3$, $A_4$, etc. and find the addition of all areas considering the shape to be subtracted.
  d) Determine $x_1$, $x_2$, $x_3$, $x_4$, etc. i.e. distance between centroid of the figure and references y-axis.
  e) Similarly, $y_1$, $y_2$, $y_3$, $y_4$, etc. i.e. distance between centroid of the figure and references x-axis.
  f) Adding the product of area and distance ($A_i x_i$) for plane figure whereas for hollow figure required figure is to be added and remaining part is to be deducted.
  g) By using formula,

$$\bar{X} = (A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4) / (A_1 + A_2 + A_3 + A_4)$$
$$\bar{Y} = (A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4) / (A_1 + A_2 + A_3 + A_4)$$

we can determine co-ordinates of centroid with respect to the reference axis.

1. The Centroid for a right-angled triangle is
   - i) h/3 ii) 4r/3π
   - iii) 2rsinα / 3α iv) None of the above
2. The Centroid of a Sector of a circle
   - i) h/3 ii) 4r/3π
   - iii) 2rsinα / 3α iv) None of the above
3. The Centroid is
   - i) Point of mass concentration ii) Point of weight concentration
   - iii) Geometric Center iv) None of the above

**Exercise:**

1) Determine the centroid of a right-angled triangle with height 4cm (along y-axis) and breadth 3cm (along x-axis). (**Ans: x = 1cm, y = 1.33cm)**

2) The center of a circle having radius 6mm is placed at the origin of X-Y plane. It is divided in IV quadrants. Find the co-ordinates of centroid of the II quadrant? **(Ans: x = -2.55mm, y = 2.55mm)**

**Questions/Problems for Practice for the day:**

1) Determine centroid of a semicircle having dia. 25mm along x-axis? **(Ans: x = 12.5mm, y = 5.30mm)**

**Learning from the lecture 'Introduction'**: Learner will be able to understand the key concepts and apply the concept of Centroid to basic shapes

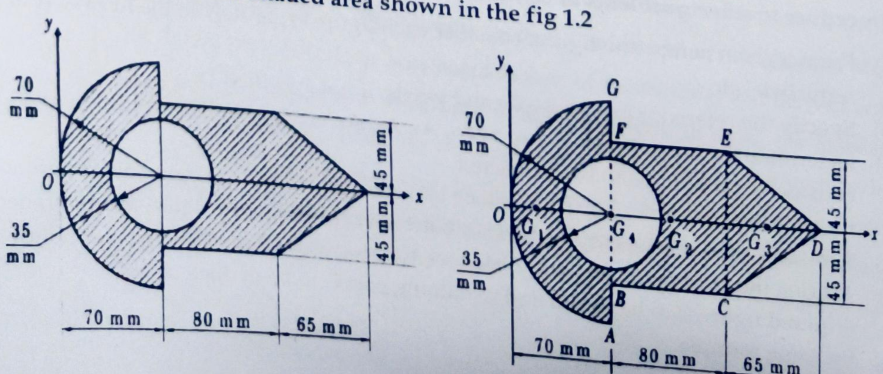
## Lecture: 2

***Centroid of plane areas which are Symmetric***

**Learning Objectives:** Learners will be able to find centroid of plane areas which are symmetric in shape

**Solved Problems:**

1) **Find the centroid of the shaded area shown in the fig 1.2**




g 1.2

**Solution:**

| Component | Area $A_i$ (mm²) | Co-ordinates $X_i$(mm) | $A_iX_i$ (mm³) |
|---|---|---|---|
| Triangle | $\frac{1}{2} \times 90 \times 65 = 2925$ | $150 + \frac{b}{3} = 171.67$ | $502.13 \times 10^3$ |
| Semi-circle | $\frac{\Pi}{2}r^2 = \frac{\Pi}{2}[70^2] = 7697$ | $70 - \frac{4r}{3\pi} = 70 - \frac{4(70)}{3\pi} = 40.3$ | $310.115 \times 10^3$ |
| Rectangle | $80 \times 90 = 7200$ | $70 + \frac{b}{2} = 110$ | $792 \times 10^3$ |
| Circle | $-\Pi r^2 = -\Pi[35^2] = -3848.45$ | $35 + r = 70$ | $269.4 \times 10^3$ |
| | $\sum A_i = 13.97 \times 10^3$ | | $\sum A_iX_{Gi} = 1334.85 \times 10^3$ |
| $\bar{X} = \frac{\sum A_iX_i}{\sum A_i} = \frac{1334.85 \times 10^3}{13.97 \times 10^3} = 95.53$mm, Centroid [X, Y] = [95.53, 0] mm | | | |

2) **Find the centroid of the shaded area shown in the given figure 1.3**

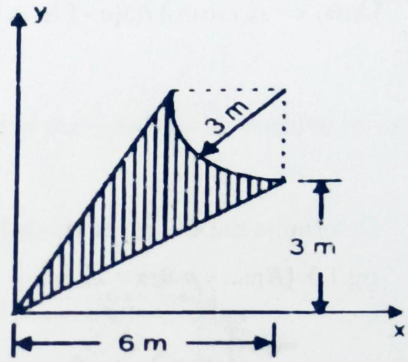



**Fig 1.3**

| Component | Area $A_i$ (m²) | Co-ordinates $X_i$(m) | Co-ordinates $Y_i$(m) | $A_iX_i$ (m³) | $A_iY_i$ (m³) |
|---|---|---|---|---|---|
| Triangle (along Y) | $-\frac{1}{2} \times 3 \times 6 = -9$ | $\frac{b}{3} = 1$ | $\frac{2h}{3} = 4$ | -9 | -36 |
| Triangle (along X) | $-\frac{1}{2} \times 6 \times 3 = -9$ | $= \frac{2b}{3} = 4$ | $\frac{h}{3} = 1$ | -36 | -9 |
| Square | $6 \times 6 = 36$ | $\frac{b}{2} = 3$ | $\frac{b}{2} = 3$ | 108 | 108 |
| Quarter circle | $-\frac{\Pi}{4}r^2 = -\frac{\Pi}{4}[3^2] = -7.07$ | $4r/3\Pi = 4[3]/3\Pi = 4.727$ | $4r/3\Pi = 4[3]/3\Pi = 4.727$ | -33.42 | -33.42 |
| $\sum$ | $\sum A_i = 10.93$ | | | $\sum A_iX_{Gi} = 29.58$ | $\sum A_iY_{Gi} = 29.58$ |
| $\bar{X} = \frac{\sum A_iX_{Gi}}{\sum A_i} = \frac{29.58}{10.93} = 2.706m$, $\bar{Y} = \frac{\sum A_iY_{Gi}}{\sum A_i} = \frac{29.58}{10.93} = 2.706m$ *Centroid [X, Y] = [2.706, 2.706]m* | | | | | |

1. If the given section is symmetrical about y-y axis, then we must calculate for

   i) X coordinate  ii) Y coordinate

   iii) both X & Y coordinates  iv) None of the above

2. If the given section is symmetrical about x-x axis, then we must calculate for

   i) X coordinate  ii) Y coordinate

   iii) both X & Y coordinates  iv) None of the above

**Exercise:**

1. Determine the X-co-ordinate of the centroid of the portion of a circular segment in terms of radius r and angle a (for shaded area only) figure. 1.4 **[Ans. $x = (2 r\sin^3\alpha) / (3(\alpha - \sin\alpha . \cos\alpha)]$**

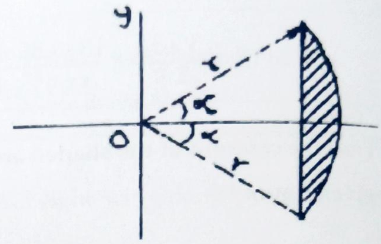



Fig 1.4

2. Determine the CG of the shaded area as shown in fig 1.5. **[Ans.: y = 0, x = 29.6cm]**

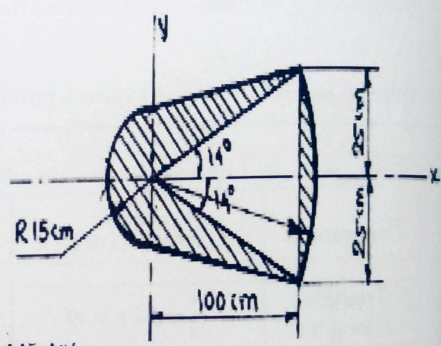



Fig 1.5

**Questions/Problems for Practice for the day:**

1. An isosceles triangle is cut from a square plate as shown in figure. The plate remains in the equilibrium in any position when suspended from point E (apex of the triangle). Determine height of the removed portion of the triangle for fig 1.6. **[Ans: h= 0.634m]**

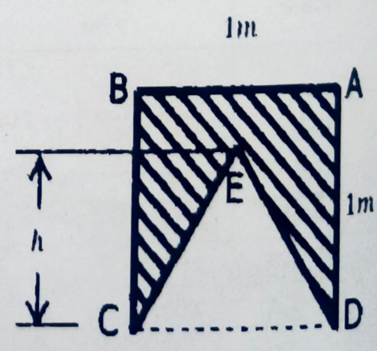



Fig 1.6

2. Centroid of T-section shown in figure 1.7 is on line AB. Find depth 'h' of the web. **[Ans: h=58.14mm]**

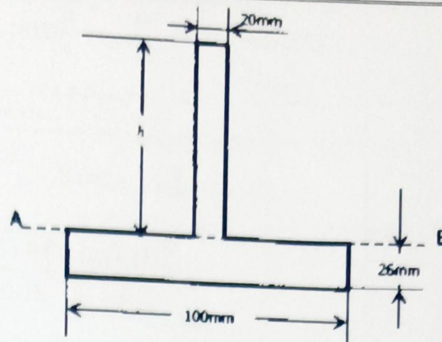


Fig 1.7

**Learning from the lecture 'Centroid for Symmetric Areas'**: Learner will be able to Apply the formulae for basics shapes which are symmetric in shape

## Lecture: 3

*Centroid of plane areas with positive coordinates*

**Learning Objectives:** Learners will be able to find centroid of plane areas with positive co-ordinates

**Solved Problems:**

3) **Find centroid of plane area for given fig 1.8.**

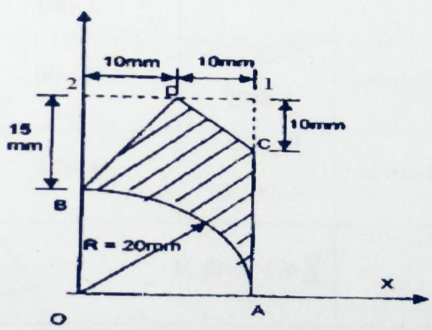


Fig 1.8

**Solution:**

| Component | Area $A_i$ (m$^2$) | Co-ordinates $X_i$(m) | Co-ordinates $Y_i$(m) | $A_iX_i$ (m$^3$) | $A_iY_i$ (m$^3$) |
|---|---|---|---|---|---|
| Triangle B2D | $\frac{1}{2}$ x 10 x 15 = -75 | $\frac{b}{3}$ = 3.33 | 20 + $\frac{2h}{3}$ = 30 | - 249.7 | - 2250 |
| Rectangle OA12 | 20 x 35 = 700 | $\frac{b}{2}$= 10 | $\frac{h}{2}$ = 17.5 | 7000 | 2250 |
| Triangle C1D | $\frac{1}{2}$ x 10 x 10 = -50 | 20 + $\frac{2b}{3}$ = 16.67 | 25 + $\frac{2h}{3}$ = 31.67 | - 833.5 | - 1583.5 |

| | | | | | |
|---|---|---|---|---|---|
| Quarter circle | $-\frac{\Pi}{4}r^2 = -\frac{\Pi}{4}[20^2]$ = - 314.15 | 4r/3 Π = 4[20]/3 Π = 8.5 | 4r/3 Π = 4[20]/3 Π = 8.5 | - 2667.2 | - 2667.221 |
| | $\sum A_i$ = 260.8 | | | $\sum A_iX_{Gi}$ = 3429.5 | $\sum A_iY_{Gi}$ = 5749.3 |

$\bar{X} = \frac{\sum A_i X_{Gi}}{\sum A_i} = \frac{3429.5}{260.8} = 12.45mm$, $\bar{Y} = \frac{\sum A_i Y_{Gi}}{\sum A_i} = \frac{5749.3}{260.8} = 22.04mm$,

Centroid [$\bar{X}$, $\bar{Y}$] = [12.45, 22.04] mm

4) **Find the centroid of shaded area of the semicircle of dia. 100cm for fig 1.9.**

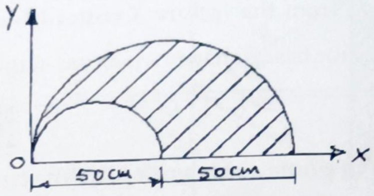



Fig 1.9

**Solution:**

| Component | Area $A_i$ ($m^2$) | Co-ordinates $X_i$(m) | Co-ordinates $Y_i$(m) | $A_iX_i$ ($m^3$) | $A_iY_i$ ($m^3$) |
|---|---|---|---|---|---|
| Semi-circle | Π x50²/2 = 3927 | R = 50 | =4R/3Π = 21.22 | 196.35 x $10^3$ | 83.33 x $10^3$ |
| Semi-circle · | Πx25²/2 = -981.75 | r = 25 | =4r/3Π = 10.61 | -24.5 x $10^3$ | -10.41 x $10^3$ |
| | $\sum A_i$ = 2945.24 | | | $\sum A_iX_{Gi}$ = 171.8 x $10^3$ | $\sum A_iY_{Gi}$ = 72.91 x $10^3$ |

$\bar{X} = \frac{\sum A_i X_{Gi}}{\sum A_i} = \frac{171.8 \times 10^3}{2945.24}$ =58.33cm, $\bar{Y} = \frac{\sum A_i Y_{Gi}}{\sum A_i} = \frac{72.91 \times 10^3}{2945.24}$ =24.76cm

*Centroid [$\bar{X}$, $\bar{Y}$] = [58.33, 24.76] cm*

5) **Determine the coordinates Xc and Yc of the center of a 100 mm diameter circular hole cut in a thin plate so that this point will be the centroid of the remaining shaded area shown in Fig 1.10. (All dimensions are in mm)**

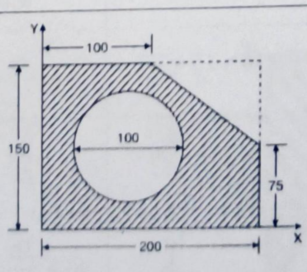



**Fig. 1.10**

**Solution:** If Xc and Yc are the coordinates of the center of the circle, centroid also must have the coordinates Xc and Yc as per the condition laid down in the problem. The shaded area may be considered as a rectangle of size 200 mm × 150 mm minus a triangle of sides 100 mm × 75 mm and a circle of diameter 100 mm.

| Component | Area $A_i$ ($m^2$) | Co-ordinates $X_i$(m) | Co-ordinates $Y_i$(m) | $A_iX_i$ ($m^3$) | $A_iY_i$ ($m^3$) |
|---|---|---|---|---|---|
| Circle | $\Pi$ x$50^2$ =- 7855 | Xc | Yc | -7855* Xc | -7855* Yc |
| Rectangle | b*d = 30000 | 100 | 75 | 3 x $10^6$ | 22.5 x $10^4$ |
| Triangle | -100*75/2=-3750 | 200-(100/3) =166.67 | 150-25 =125 | -62.5 x $10^4$ | 46.875 x $10^4$ |
| | $\sum A_i$ = 18393 | | | $\sum A_iX_{Gi}$ = -7855* Xc +3 x $10^6$-62.5 x $10^4$ | $\sum A_iY_{Gi}$ = -7855* Yc+22.5 x $10^4$+46.875 x $10^4$ |

X= Xc = $\frac{\sum A_i X_{Gi}}{\sum A_i}$ =*90.48mm*, Y= Yc = $\frac{\sum A_i Y_{Gi}}{\sum A_i}$ =*67.86mm, Centroid [X, Y] = [90.4867.86]sssss*

1. Where will the C.G of this plane area will lie

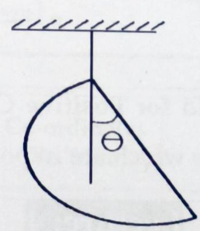


Fig 1.11

i) On the circumference of circle
ii) On the diametrical line of the circle
iii) On the vertical line passing through the point of suspension
iv) Outside the Semicircle

2. For a Semi-Circle having its center on the origin and diameter horizontal along it, its x-coordinate will be ______________

i) On the circumference of circle
ii) R
iii) 0
iv) -r

**Exercise:**

1. Determine the co-ordinates of centroid of the shaded portion as shown in figure 1.12 **[Ans: x= 53.21mm, y= 38.54mm]**

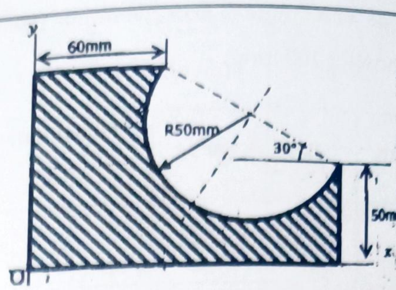


Fig 1.12

**Questions/Problems for Practice for the day:**

1. Find the distance 'y' so that C.G. of given area in the figure 1.13 has co-ordinates (25, 20) mm **[Ans: y= 25.625mm]**

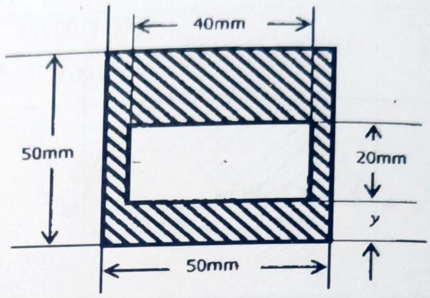


Fig 1.13

2. A plane lamina is hung freely from point D. Find the angle made by BD with the vertical for given fig 1.14. **[Ans.: θ=29.62°]**

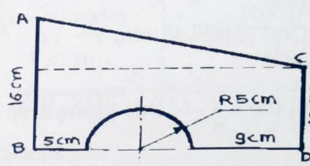


Fig 1.14

**Learning from the lecture 'Centroid for Positive Coordinates'**: Learner will be able to Apply the formulae for basics shapes which are in positive coordinates.

## Lecture: 4

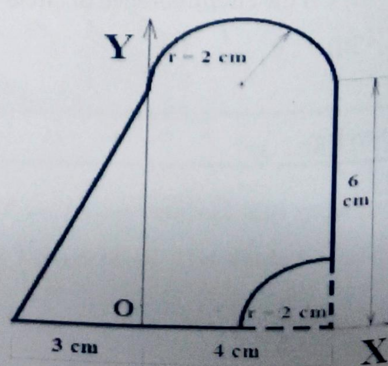
*Centroid of plane areas with Negative coordinates*

**Learning Objectives:** Learners will be able to find centroid of plane areas with positive co-ordinates

**Solved Problems**

6) **Find the centroid of the given shape for a given fig. 1.15**

**Solution:**



| Component | Area '$A_i$' ($cm^2$) | Co-ordinates '$X_{Gi}$'(cm) | Co-ordinates '$Y_{Gi}$'(cm) | '$A_iX_{Gi}$' ($cm^3$) | '$A_iY_{Gi}$' ($cm^3$) |
|---|---|---|---|---|---|
| Semi-circle | $\Pi r^2/2 = 6.28$ | r = 2 | $= 6 + 4r/3\Pi = 6.84$ | 12.56 | 42.95 |
| Rectangle | 6 x 4 = 24 | $\frac{b}{2} = 2$ | $\frac{h}{2} = 3$ | 48 | 72 |
| Triangle | $\frac{1}{2} \times 3 \times 6 = 9$ | $-\frac{b}{3} = -1$ | $\frac{h}{3} = 2$ | -9 | 18 |
| Quarter circle | $-\frac{\Pi}{4}r^2 = -3.14$ | $4 - 4r/3\Pi = 3.15$ | $4r/3\Pi = 0.85$ | -9.89 | -2.67 |
| | $\sum A_i = 36.14$ | | | $\sum A_iX_{Gi} = 41.67$ | $\sum A_iY_{Gi} = 130.28$ |

$$\bar{X} = \frac{\sum A_i X_{Gi}}{\sum A_i} = \frac{41.67}{36.14} = 1.15\ cm,\quad \bar{Y} = \frac{\sum A_i Y_{Gi}}{\sum A_i} = \frac{130.28}{36.14} = 3.60\ cm$$

*Centroid* $[\bar{X}, \bar{Y}] = [1.15, 3.60]\ cm$

7) **Determine the location of the centroid of the plane area shown in fig. 1.16 shaded on sketch.**

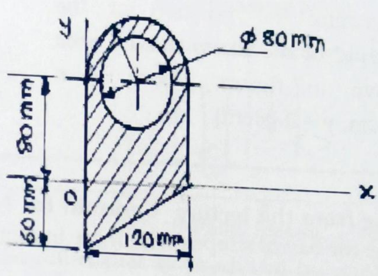



**Fig 1.16**

**Solution:**

| Component | Area '$A_i$' ($mm^2$) | Co-ordinates '$X_{Gi}$'(mm) | Co-ordinates '$Y_{Gi}$'(mm) | '$A_iX_{Gi}$' ($mm^3$) | '$A_iY_{Gi}$' ($mm^3$) |
|---|---|---|---|---|---|
| Semi-circle | $\frac{\pi r^2}{2} = \frac{\pi(60)^2}{2} = 5.65 \times 10^3$ | r = 60 | $= 80 + \frac{4r}{3\pi}$ $= 80 + \frac{4(60)}{3\pi} = 105.46$ | $576 \times 10^3$ | $384 \times 10^3$ |
| Rectangle | $120 \times 80 = 9.6 \times 10^3$ | $\frac{b}{2} = 60$ | $\frac{h}{2} = 40$ | $144 \times 10^3$ | $-72 \times 10^3$ |
| Triangle | $\frac{1}{2} \times 120 \times 60 = 3.6 \times 10^3$ | $\frac{b}{3} = 40$ | $-\frac{h}{3} = -20$ | $339.3 \times 10^3$ | $596.4 \times 10^3$ |
| Circle | $-\Pi r^2 = -\Pi [40^2] = -5.65 \times 10^3$ | 20 + r = 60 | 40 + r = 80 | $-301.6 \times 10^3$ | $-402.2 \times 10^3$ |
| | $\sum A_i = 13.828 \times 10^3$ | | | $\sum \sum A_iX_{Gi} = 757.7 \times 10^3$ | $\sum A_iY_{Gi} = 506.2 \times 10^3$ |

$$\bar{X} = \frac{\sum A_i X_{Gi}}{\sum A_i} = \frac{757.7 \times 10^3}{13.828 \times 10^3} = 54.8\ mm,\quad \bar{Y} = \frac{\sum A_i Y_{Gi}}{\sum A_i} = \frac{506.2 \times 10^3}{13.828 \times 10^3} = 36.6\ mm$$

*Centroid* $[\bar{X}, \bar{Y}] = [54.8, 36.6]\ mm$

1. **What should be taken as a negative value for a shape not to be included in centroid**

**calculations?**

i) X-coordinate
ii) Y-coordinate
iii) Area
iv) None of the above

**2. What does the negative sign indicate?**

i) Removal of area
ii) Negative area
iii) Removal of negative coordinate
iv) None of the above

**Exercise:**

1. Determine the centroid of the following plane area shown in fig. 1.17. **[Ans.: 54.867mm, 18.454mm]**

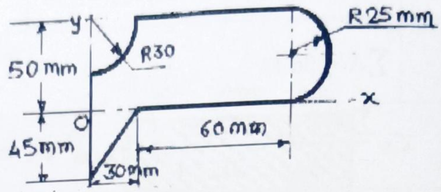



Fig 1.17

**Questions/Problems for Practice for the day:**

2. Determine the position of the centroid of the plane-shaded area shown in figure. **[Ans.: x = 1.59cm, y = 2.08cm]**

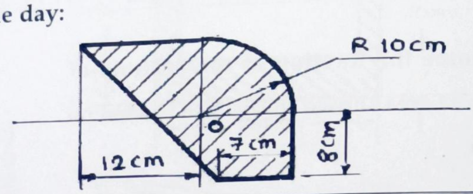



Fig 1.18

**Learning from the lecture 'Centroid for Negative Coordinates'**: Learner will able to Apply the formulae for basics shapes which are in negative coordinates.

## Lecture: 5

### 1.2 *System of Coplanar Forces*

- **Force:** It is defined as an external agency which produces or tends to produce, destroys or tends to destroy the motion. It is characterized by Magnitude, Direction, Sense and Point of application. It is a vector quantity and S.I unit is Newton (N).1 Newton force is defined as force required to produce unit acceleration on unit mass. Therefore, 1 Kg = 9.81 N
- **System of forces:** There are mainly seven types of system of forces:
  - **Co-planar forces:** The forces which are acting in the same plane are known as co-planer forces. **(Fig 1.19)**
  - **Non-Coplanar forces:** The force system in which the forces acting in the different planes is called as non-coplanar forces. **(Fig 1.20)**

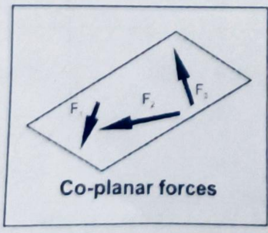



Fig 1.19

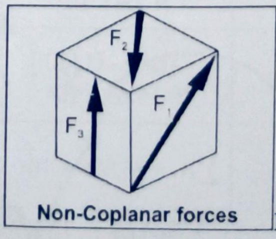



Fig 1.20

  - **Collinear forces:** The forces which are acting along the same straight line are
  - **Non-collinear forces:** The forces which are not acting along the straight line are called as

called as collinear forces. (**Fig 1.21**)

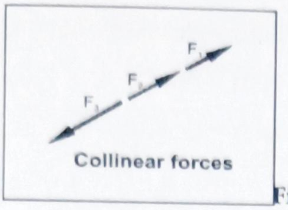



Fig 1.21

Non-collinear forces. (**Fig 1.22**)

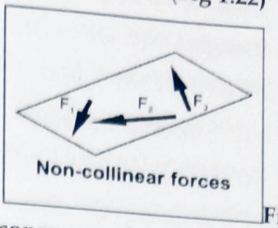



Fig 1.22

- **Concurrent forces:** The forces which are passing through a common point are called concurrent forces. (**Fig 1.23**)

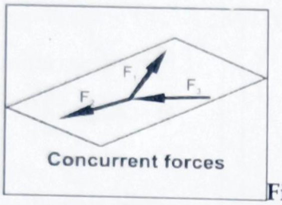



Fig 1.23

- **Non-concurrent forces:** The forces which are not passing through a common point are called as non-concurrent forces. (**Fig 1.24**)

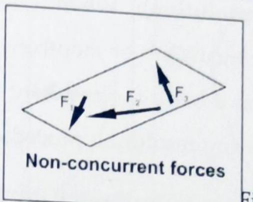



Fig 1.24

- **Parallel forces:** The forces whose lines of action are parallel to each other are known as parallel forces. (**Fig 1.25**)

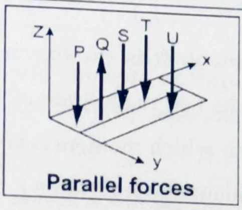



Fig 1.25

➢ **Resultant:** A single force producing the same effect that as produced by number of forces when acting together. It is denoted by 'R'

**Methods of composition: (to find R): -**

- **Resultant of two concurrent forces: (Law of parallelogram of forces):** It states that "if two forces simultaneously acting at a point be represented in magnitude and direction by two adjacent sides of a parallelogram, the diagonal will represent resultant in magnitude and direction, but passing through the point of intersection of two forces"

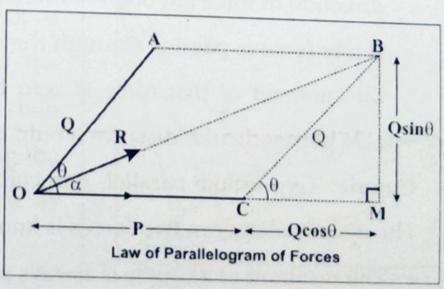



Fig 1.26

Consider two forces P and Q acting at a point represented by two sides OA and OC of a parallelogram OABC. Let θ be the angle between two forces P and Q, α be the angle between P and R. Draw perpendicular BM and produce QC.

In triangle CMB, BM= Q sinθ and CM = Q cosθ

- **Magnitude of R:**

In triangle OMB, $OB^2 = OM^2 + BM^2$

$OB^2 = (OC + CM)^2 + BM^2$

$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$

$R^2 = (P^2+2PQ\cos\theta + Q^2\cos^2\theta)+(Q^2\sin^2\theta)$

$R^2 = P^2+2PQ\cos\theta+Q^2 (\sin^2\theta + \cos^2\theta)$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

- **Direction of R:**

In triangle OMB, $\tan\alpha = BM/OM$

$\tan\alpha = BM/ (OC + CM)$

$\tan\alpha = Q \sin\theta / (P+ Q \cos\theta)$

If $\theta = 90°$, $R = \sqrt{P^2 + Q^2}$, $\tan\alpha = \frac{Q}{P}$

- **Resultant of two or more forces: (Method of resolution):** When two or more coplanar concurrent or non-concurrent forces acting on a body the resultant can be found out by using resolution procedure.

  Magnitude of resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

Direction: $\theta = \tan^{-1}(\frac{Fy}{Fx})$

Where, $\Sigma F_x$= Forces along X-direction,

$\Sigma F_y$= Forces along Y-direction

$\theta$ = Angle of 'R' with x-axis

- **Moment:** It is the turning effect produced by a force. of a force about any point is the product of magnitude of the force and perpendicular distance about that point. The point about which moment is taken is called as moment center.

  Moment about O, $M_O = F \times d$. S.I unit: N-m.

  - While taking moment of any force do not observe direction of force but observe direction of rotation.
  - If any force is passing through the moment center, the moment of that force is zero because for the case perpendicular distance would become zero

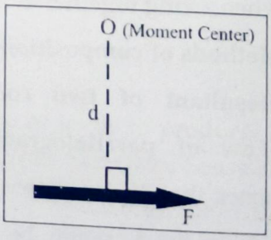



Fig 1.27

- **Couple:** Two unlike parallel, non-collinear forces having same magnitude form a Couple. The distance between two forces is known as arm or lever of the couple.
  - The resultant of a couple is always zero.
  - The moment of a couple is product of forces and lever arm of the couple. Therefore, $M= F \times d$.
  - A couple cannot be balanced by a single force.It can be balanced only by another couple of opposite nature.
  - The moment of couple is independent of the moment center.

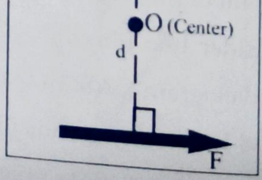



Fig 1.28

- **Composition of forces:** The process of addition of forces is called as composition of forces.

- **Resolution of forces:** It is the procedure of splitting up a single force into number of components without changing the effects of the same.
- **Principle of transmissibility:** The point of application of a force can be transmitted anywhere along its line of action, but within the body. It is only applicable to rigid bodies. The principle is neither applicable from the point of view of internal resistances nor internal forces developed in the body nor to deformable bodies under any circumstances.

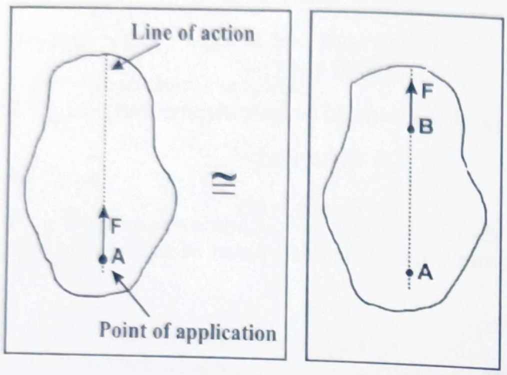



Fig 1.29 Principle of Transmissibility

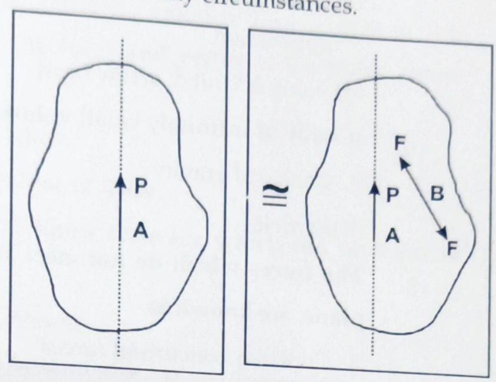



**Fig 1.30 Principle of Superposition**

- **Principle of Superposition:** The effect of a force on a body remains unaltered if we add or subtract any system which is in equilibrium.
- **Varignon's theorem:** The sum of the moment of all the forces about a point is equal to the moment of their resultant about the same point. $\Sigma M_o^F = M_o^R$

Consider a force F acting at a point A and having component $F_1$ and $F_2$ in ant two directions. Let us choose any point O, lying in the plane of the forces, as a moment center. Attach at A two rectangular axes such that the y-axis is along the line AO and the x-axis is perpendicular to it, as shown in the figure 1.31.

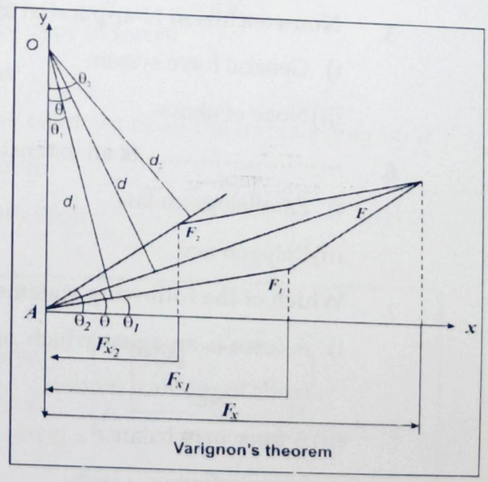



**Fig 1.31**

Moment of the force F about O

$F \times d = F \times OA \cos\theta = OA \times F\cos\theta$

$F \times d = OA \times F_x$ .................... (1)

Moment of the force $F_1$ about O,

$F_1 \times d_1 = F_1 \times OA \cos\theta_1 = OA \times F_1\cos\theta_1$

$F_1 \times d_1 = OA \times F_{x1}$ .................. (2)

Moment of the force $F_2$ about O,

$F_2 \times d_2 = F_2 \times OA \cos\theta_2 = OA \times F_2\cos\theta_2$

$F_2 \times d_2 = OA \times F_{x2}$ .................. (3)

$F_1 \times d_1 + F_2 \times d_2 = OA \times F_x$......... [Sum of the x-components of the forces $F_1$ and $F_2$ = x-components of the resultant force F; $F_x = F_{x1} + F_{x2}$]

$F_1 \times d_1 + F_2 \times d_2 = F \times d$ .......... from (1) &

Adding (2) and (3)

$F_1 \times d_1 + F_2 \times d_2 = OA \times (F_{x1} + F_{x2})$ ……… (4)

(4)

i.e. $\Sigma M_o^F = M_o^R$

1. **Force can be completely defined by it's**
   i) Magnitude, Direction & point of application
   ii) Unit & sense
   iii) Value & Unit & arrow head
   iv) All above
2. **A body of infinitely small volume and is assumed to be concentrated point is known as**
   i) Centre of gravity
   ii) Rigid body
   iii) Particle
   iv) Plastic body
3. **The forces which do not meet at one point, but their lines of actions lie on the same plane, are known as**
   i) Coplanar concurrent forces
   ii) Coplanar Non-concurrent forces
   iii) Non-Coplanar Non-concurrent forces
   iv) Non-coplanar Concurrent
4. **The process of finding out the resultant force, of a few given forces, is called**
   i) Composition
   ii) Resolution
   iii) Equilibrant
   iv) None of these
5. **Non-concurrent Non-parallel, coplanar forces are called as**
   i) General force system
   ii) Space forces
   iii) None of above
   iv) Both i& ii
6. **……………………..is an extension of Triangle law of forces for more than two forces**
   i) Parallelogram law
   ii) Coulomb's law
   iii) Polygon law
   iv) Sine Rule
7. **Which of the following statement is correct?**
   i) A force is an agent which produces or tends to produce motion
   ii) A force is an agent which stops or tends to stop motion
   iii) A force may balance a given number of forces acting on a body
   iv) Both a & b
8. **To determine the effects of force acting on a body, we must know**
   i) Its magnitude and direction of the line along which it acts
   ii) Its nature (where push or pull)
   iii) Point through which it acts on the body
   iv) All of the above

9. **If number of forces acting simultaneously on a particle, then the resultant of these forces will have the same effect as produced by the all the forces, this is known as**

   i) Principle of physical independences of forces
   ii) Principle of transmissibility of forces
   iii) Principle of resolution of forces
   iv) None of the above

10. **The moment of a force about any point is geometrically equal to ………. area of the triangle whose base is the line representing the force and vertex is the point about which the moment is taken**

    i) Half
    ii) Same
    iii) Twice
    iv) None of these

11. **In a clockwise moment, we use wall clock to know time for which the moment is applied**

    i) Right
    ii) Wrong
    iii) Can't Say
    iv) Partially Correct

12. **According to the Lami's theorem, the three forces**

    i) Must be equal
    ii) Must be at 120° to each other
    iii) Must be both of above
    iv) May not be any of the two

13. **The Lami's theorem is applicable only for**

    i) Coplanar forces
    ii) Concurrent forces
    iii) Coplanar and concurrent forces
    iv) Any type of forces

14. **If a body is in equilibrium, we may conclude that**

    i) The moment of all the forces about any point is zero
    ii) The resultant of all the forces acting on it is zero
    iii) No force is acting on the body
    iv) Both i & ii

**Exercise:**

1. What do you mean by resolution of a force into a force and a couple? Convert the given force into a force couple at point B.

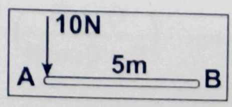


**Fig 1.32**

2. How many types of forces can exist?

**Questions/Problems for Practice for the day:**

1. List out all the Force Systems.
2. Where will the resultant of a Concurrent Force System pass from?

Learning from the lecture 'Definition & Theory of Force': Learner will be able to know the definitions of forces and different other concepts & procedures related to analysis of forces.

# Lecture: 6

## 1.1.1 *Determine Unknown Force*

**Learning Objectives:** Learners will be able to find the unknown forces when resultant is given

## 1.1.2 Solved Problems

8) **R=800N is the resultant of 4 concurrent forces. Find the fourth force $F_4$.**

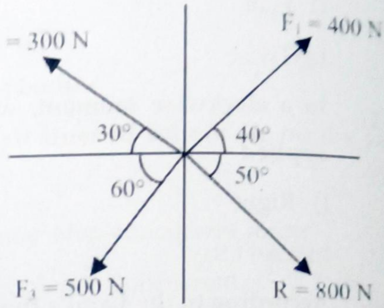


Fig 1.3?

**Solution:** This is a concurrent system of four forces.

R=800N at θ=50°; Rx= R cos50°= 800 cos50°;

Ry= R sin50°= 800 sin50°

$\Sigma$ Fx= Rx(→ +ve); 400 cos45°–300 cos30°-500 cos60°+ ($F_{4x}$) = 800 cos50°

$F_{4x}$ = 741.2N (→)

$\Sigma$ Fy= Ry(↑ +ve)

400 sin45° – 300 sin30° -500 sin60° + ($F_{4y}$) = 800 cos50°

$F_{4y}$ = -612.6N = 612.6N (↓)

$$F_4 = \sqrt{(F_{4x})^2 + (F_{4y})^2} = 961.6N$$

$$\theta_4 = \tan^{-1}\left(\frac{F_{4y}}{F_{4x}}\right) = \tan^{-1}\left(\frac{612.6}{741.2}\right) = 39.6°$$

9) **The sum of two concurrent forces P and Q is 270 N and their resultant is of 180 N. If resultant is perpendicular to P. Find P & Q.**

**Solution:** Let θ be the angle between two forces P and Q.

Here, P + Q = 270N, R = 180N and α=90°

Using $\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$

$\therefore 90° = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right) = \infty$

∴ P + Q cosθ = 0

∴ Q cosθ = - P

Also, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

∴ $(180)^2 = P^2 + Q^2 + 2P(-P)$

∴ $(180)^2 = Q^2 - P^2$

∴ $(180)^2 = (Q – P)(Q + P)$

∴ $(180)^2 = (Q – P)(270)$

∴ *(Q – P) = 120*

As (Q + P) = 270 & (Q – P) = 120,

Solving, we get, **P = 75N & Q = 195N**

10) **For two forces P and Q acting at a point, maximum resultant is 2000N and minimum magnitude of resultant is 800N. Find values of P and Q.**

**Solution:**

We know that, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

For maximum value of R, θ = 0.

$\therefore R_{max} = \sqrt{P^2 + Q^2 + 2PQ}$

$\therefore R_{max} = \sqrt{(P+Q)^2} = (P + Q)$

$\therefore (P + Q) = 2000N$.................... (1)

For minimum value of R, $\theta = 180°$

$\therefore R_{max} = \sqrt{P^2 + Q^2 - 2PQ}$

$\therefore R_{max} = \sqrt{(P-Q)^2} = (P - Q)$

$\therefore (P + Q) = 800N$............... (2)

Solving we get, **P = 1400N & Q = 600N**

1. **If the resultant of a force system is vertical, then**
   i) $\Sigma F_y = R_y$
   ii) $\Sigma F_x = R_x$
   iii) Both
   iv) None
2. **If the resultant of a force system is horizontal, then**
   i) $\Sigma F_y = R_y$
   ii) $\Sigma F_x = R_x$
   iii) Both
   iv) None

Exercise:

1. A force R = 25N has components $F_a$, $F_b$ and $F_c$ as shown in figure 1.34. If $F_c$=20N, find $F_a$ and $F_b$. **[Ans: $F_a$=33.9N, $F_b$= 35.09N]**

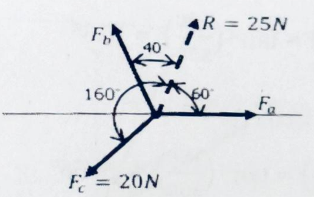



Fig 1.34

2. Three forces act on the bracket. Determine the magnitude and the direction of the force $F_1$ so that the resultant forces are directed along the line-x and has magnitude of 800N for given fig 1.35. **[Ans: $F_1$= 193.8N, θ = 24.63°]**

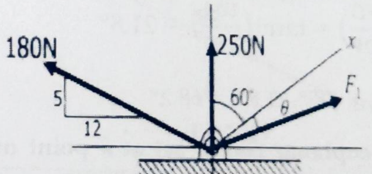



Fig 1.35

**Learning from the lecture 'Determine unknown force'**: Learner will able to find the unknown forces when magnitude & nature of resultant is given.

## Lecture: 7

### 1.1.3 *Resultant of Concurrent force systems*

**Learning Objectives:** Learners will be able to find the resultant of Concurrent force systems

**Solved Problems**

**11) The striker of carom board lying on the board is being pulled by four players as shown in given fig 1.36. The players are sitting exactly at the center of the four sides. Find the resultant forces in magnitude & direction.**

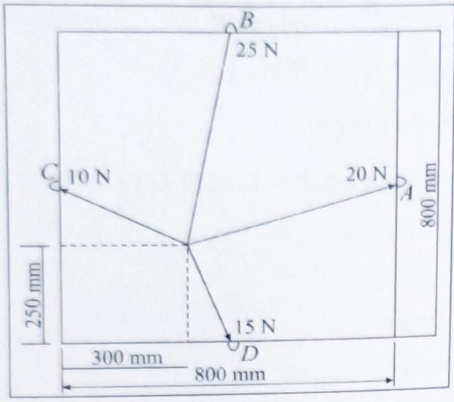



Fig 1.36

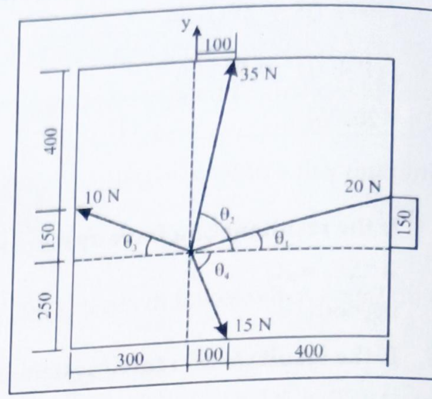



Fig 1.37

**Solution:**

$\theta_1 = \tan^{-1}\left(\frac{AG}{OG}\right) = \tan^{-1}\left(\frac{150}{500}\right) = 16.7°$

$\alpha = \tan^{-1}\left(\frac{EB}{OE}\right) = \tan^{-1}\left(\frac{100}{550}\right) = 10.3°$

$\theta_2 = (90° - \alpha) = (90° - 10.3°) = 79.7°$

$\theta_3 = \tan^{-1}\left(\frac{CH}{OH}\right) = \tan^{-1}\left(\frac{150}{300}\right) = 26.56°$

$\beta = \tan^{-1}\left(\frac{FD}{OF}\right) = \tan^{-1}\left(\frac{100}{250}\right) = 21.8°$

$\theta_4 = (90° - \beta) = (90° - 21.8°) = 68.2°$

$\Sigma F_x$ (→+ve) $= 25\cos\theta_2 + 20\cos\theta_1 - 10\cos\theta_3 + 15\cos\theta_4 = 20.25N$ (→)

$\Sigma F_y$ (↑+ve) $= 25\sin\theta_2 + 20\sin\theta_1 + 10\sin\theta_3 - 15\sin\theta_4 = 20.89N$ (↑)

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(20.25)^2 + (20.89)^2} = 29.1N$

$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right) = \tan^{-1}\left(\frac{20.89}{20.25}\right) = 45.89°$

**12) Three coplanar forces act at a point on a bracket as shown in fig 1.38. Determine the value of the angle ά such that the resultant of the three forces will be vertical. Also find the magnitude of the resultant.**

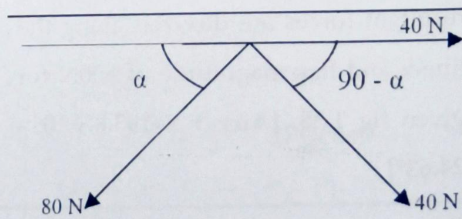



Fig 1.38

**Solution:**

Resultant of three forces will be vertical.

$\Sigma F_y$ (↑+ve) $= R_y$

$\therefore -80\sin a - 40\sin(90-a) = R$ ......(I)

$\Sigma F_x$ (→+ve) $= 0$

$\therefore -80\cos\alpha + 40\cos(90-\alpha) + 40 = 0$

$\therefore -2\cos\alpha + \cos(90-\alpha) + 1 = 0$

$\therefore -2\cos\alpha + \sin\alpha + 1 = 0$ ($\because \cos(90-\alpha) = \sin\alpha$)

Or, $2\cos\alpha = 1 + \sin\alpha$

$\therefore 2\left(\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}\right) = \left(\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}\right) + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$\therefore 2\left(\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\right)\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right) = \left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right)^2$

$\therefore 2\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right) = \left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right)$

Divide by $\cos\frac{\alpha}{2}$

$\therefore 2\left(1-\tan\frac{\alpha}{2}\right) = \left(\tan\frac{\alpha}{2}+1\right)$

$\therefore 3\tan\frac{\alpha}{2} = 1$

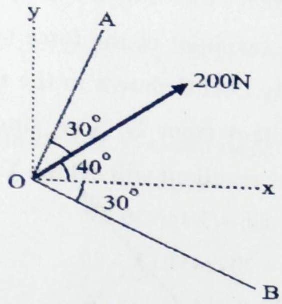
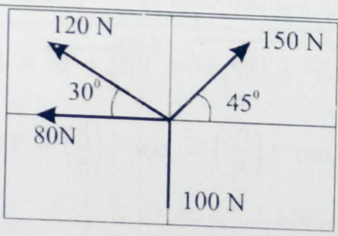
$\therefore \alpha = 2\tan^{-1}\left(\frac{1}{3}\right) = 36.86°$ ......(II)

From (I) & (II),

$-80\sin 36.86° - 40\sin(90° - 36.86°) = R$

$\therefore$ **R=-80N**

1. **The Lami's theorem is applicable only for .............. forces**
   i) One ii) Two
   iii) Three iv) Four
2. **The Lami's theorem is applicable only for ..............**
   i) Coplanar forces ii) Concurrent forces
   iii) Coplanar and concurrent forces iv) Any type of forces

| Exercise: | |
|---|---|
| 1) Resolve 200N force into components along A & B directions. Refer fig. 1.39 **[Ans: 190.84N, 101.54N]** | <br>**Fig1.39** |
| 2) Find the resultant system of four concurrent forces as shown in the figure analytically.**[Ans: R=100.7 N, $\alpha$ = 143.6°]** | <br>**Fig 1.40** |

**Learning from the lecture 'Resultant of Concurrent force systems'**: Learner will able to find the resultant of Concurrent force systems where all forces meet at a common point.

# Lecture: 8

## 1.1.4 *Resultant of Parallel & General force systems*

**Learning Objectives:** Learners will be able to find the resultant of Parallel & General force systems

### Solved Problems

**13) Determine the magnitude and position of the resultant with respect to point A of the parallel forces shown in fig 1.41**

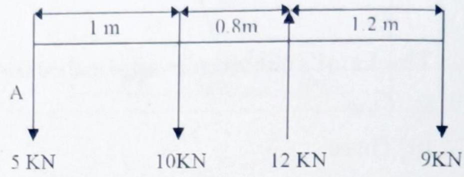


Fig 1.41

**Solution:** $\Sigma Fy\ (\uparrow +ve) = R$

$\therefore -5 - 10 + 12 - 9 = R$

$\therefore R = 12N\ (\downarrow)$

Using Varignon's theorem

$\Sigma M_A{}^F = M_A{}^R\ (\circlearrowleft +ve)$

$\therefore -(10 \times 1) + (12 \times 1.8) - (9 \times 3) = -R \times d$

$\therefore -15.4 = -12x\ d, \qquad \therefore d = 1.3m$

**14) Find the resultant of the force system acting on a body OABC shown in the fig 1.42. Find the resultant from O. Also find the points where the resultant will cut the X & Y axis.**

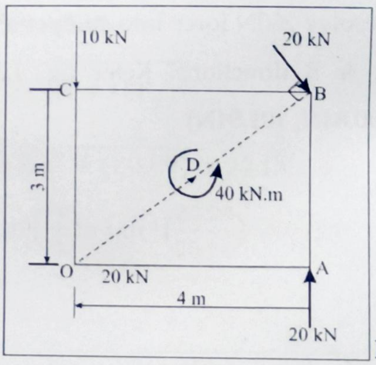


Fig 1.42

**Solution:** $\alpha = \tan\text{-}1(3/4) = 36.86°$

$\Sigma Fx\ (\rightarrow +ve) = 20\cos 53.13° - 20$

$\therefore \Sigma Fx = -8\ KN = 8kN\ (\leftarrow)$

$\Sigma Fy\ (\uparrow +ve) = -10 - 20\sin 53.13 + 20 = -6KN = 6KN(\downarrow)$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{8^2 + 6^2} = 10kN$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{6}{8}\right) = 36.86°$$

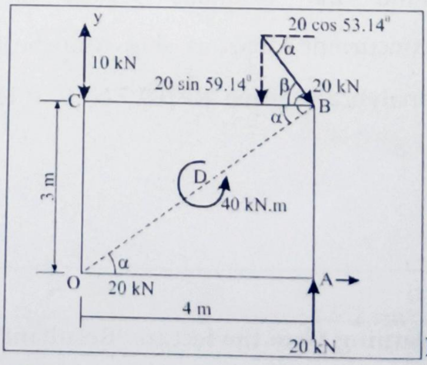


Fig 1.43

**15) A system of forces acting on a bell crank as shown in fig 1.44. Determine the magnitude, direction and point of application of the resultant with respect to O.**

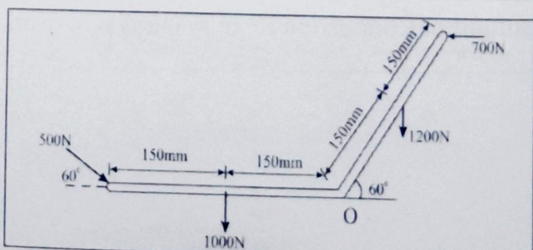


Fig 1.44

**Solution:**

$\Sigma F_x$ (→+ve) = 500 cos60° - 700

$\therefore \Sigma F_x$= - 450N = 450N (←)

$\Sigma F_y$ (↑+ve) = - 500 sin60° -1000 -1200

$\therefore \Sigma F_y$= -2633N = 2633N (↓)

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\therefore R = \sqrt{450^2 + 2633^2} = 2671.2N$$

$$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right) = \tan^{-1}\left(\frac{2633}{450}\right) = 80.3°$$

**16) Determine completely the resultant of the four coplanar forces shown in the fig 1.45. Locate the line of action of the resultant with respect to 'A'**

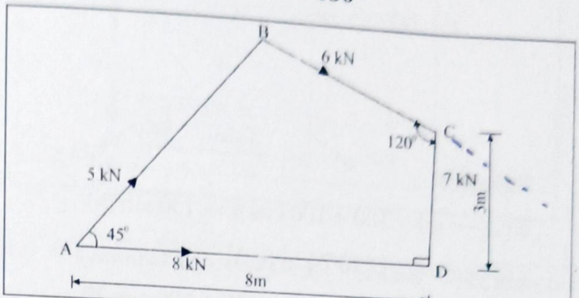



**Fig 1.45**

6 sin 60

6 cos 60

**Solution:**

$\Sigma F_x$ (→+ve) = 5 cos45° +8+6 cos30° =16.73KN (→)

$\Sigma F_y$ (↑+ve) = 5 sin45° - 7 - 6 sin30° = - 6.46KN

= 6.46KN (↓)

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{16.73^2 + 6.46^2}$$

$\therefore$ *R = 17.93KN*

$$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right) = \tan^{-1}\left(\frac{6.46}{16.73}\right) = 21.1°$$

Using Varignon's theorem,

$\Sigma M_A{}^F = M_A{}^R$ (↺) +ve

$\therefore$(7 x 8) + (6 x 6 sin30°) – (4.15 x 6 cos30°) = 17.93 x d

**d = 5.33m (∵ d is +ve, moment will be ↺)**

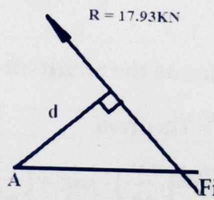



**Fig 1.46**

**17) Determine the resultant of general coplanar force system shown in fig 1.47.**

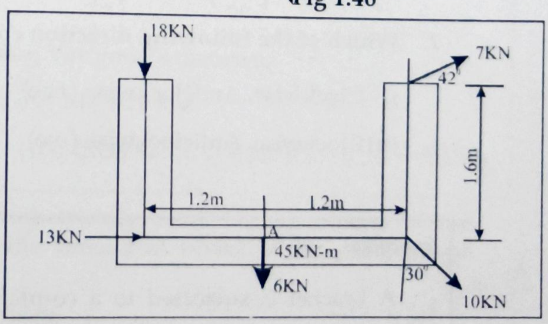



**Fig 1.47**

**Solution:** $\Sigma$Fx(→+ve) = 7 cos42° +13+10 sin30°= 23.2KN (→); $\Sigma$Fy (↑+ve) = -18 -6 -10cos30 + 7sin42 = -27.98KN = 27.98KN (↓)

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \therefore R = \sqrt{23.2^2 + 27.98^2}$$

= 36.3KN

$$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right) = \tan^{-1}\left(\frac{27.98}{23.2}\right) = 50.33°$$

Moment about point A,

Using Varignon's theorem,

$\Sigma M_A{}^F = M_A{}^R$ (↺) +ve

$\therefore$(18 x 1.2) – (1.6 x 7 cos42°) + (1.2 x 7 sin42°) = 36.3 x d ; d = 1.474m

**18) Determine the resultant of four forces tangential to the circle of radius 4 cm as shown. What will be the location of the resultant with respect to the center of the circle?**

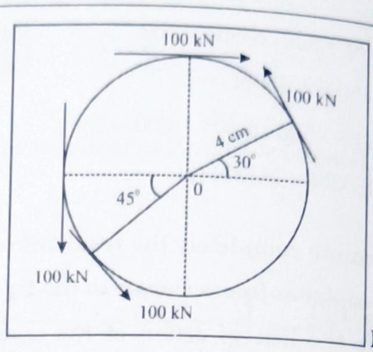



Fig 1.48

**Solution:**

$\Sigma F_x (\rightarrow +ve) = 100 + 100 \cos 45 - 100 \sin 30$

$\therefore \Sigma F_x (\rightarrow +ve) = 120.7$ kN $(\rightarrow)$

$\Sigma F_y (\uparrow +ve) = -100 - 100 \sin 45 + 100 \cos 30$

$\therefore \Sigma F_y (\uparrow +ve) = -84.1$ kN $= 84.1$ kN $(\downarrow)$

Using $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\therefore R = \sqrt{120.72^2 + 84.1^2} = 147.1$KN

$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right) = \tan^{-1}\left(\frac{84.1}{120.7}\right) = 34.86°$

Location of resultant forcé,

Using Varignon's theorem

$\Sigma M_o^F = M_o^R$ (↺) +ve

$\therefore -100 \times 4 + 100 \times 4 + 100 \times 4 + 100 \times 4 = -147.1 \times d$

**d = - 5.44 mor d = 5.44 m (left of 'O')**

1. **How to locate the resultant of a given force system?**

   i) Lami's Theorem　　ii) Varignon's Theorem

   iii) Sine Rule　　iv) Ohm's Law

2. **Which of the following direction convention of moment of a force is correct?**

   i) Clockwise, Anticlockwise (+ve)　　ii) Clockwise (-ve), Anticlockwise (+ve)

   iii) Clockwise, Anticlockwise (-ve)　　iv) None of the above

**Exercise:**

1. A bracket is subjected to a co-planer force system as shown in figure Determine the magnitude and line of action of the resultant. **[Ans. R = 252.18 N, $\theta$ = 37.52°, x = 350 m]**

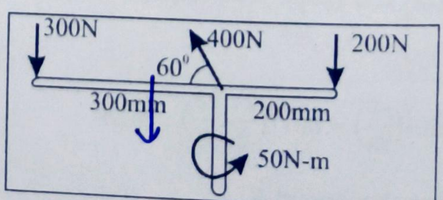



Fig 1.49

**Questions/Problems for Practice for the day:**

1. Find the resultant of coplanar force system given below and locate the same on AB with consideration of applied moment of 4800N-mm **[Ans: R= 510N, $\theta$ = 66.95 °, passing through point A]**

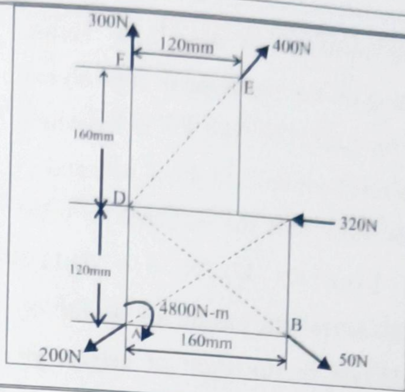



**Fig 1.50**

**Learning from the lecture 'Resultant of Parallel & General force systems'**: Learner will able to find the resultant of Parallel & General force systems and position of the resultant

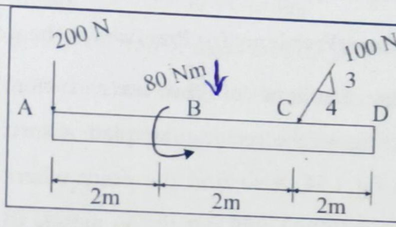
# Lecture: 9

### 1.1.5 *Shifting of a Force*

**Learning Objectives:** Learners will be able to know shifting of a force from one point to another.

**Solved Problems**

**19) Resolve the system of forces shown in fig. 1.51 into a force & couple at point A.**




**ig 1.51**

**Solution:** $\theta = \tan^{-1}(3/4) = 36.87°$

$\Sigma F_x$ (→+ve) = -100 cos36.87°

$\therefore \Sigma F_x$ = - 80N = 80N (←)

$\Sigma F_y$ (↑+ve) = - 200 -100 sin36.87°

$\therefore \Sigma F_y$ = -260N = 260N (↓)

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\therefore R = \sqrt{80^2 + 260^2} = 272.03\text{KN}$$

$$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right) = \tan^{-1}\left(\frac{260}{80}\right) = 72.89°$$

Using Varignon's theorem,

$\Sigma M_A{}^F = M_A{}^R$(↺) +ve

80 - (100 sin36.87° x 4) = 2671.2 x d ; **d = 0.59m**

1. **The moment of a force about any point is geometrically equal to .......... area of the triangle whose base is the line representing the force and vertex is the point about which the moment is taken**

   0.59

   i) Half
   ii) Same
   iii) Twice
   iv) Zero

2. **Which of the following statement is correct?**
   i) A force is an agent which produces or tends to produce motion
   ii) A force is an agent which stops or tends to stop motion

**Exercise:**

1. The resultant of the force system acting on the rectangular plate shown in fig 1.52. Also find the point where the resultant will cut the x- axis and y-axis. Also shift the resultant to point 'A' **[Ans: R= 471.7N, $\theta$ = 21.12 °, 3.217m to the right of point B, x=13.24m to the right of origin, y= 5.11m above the origin]**

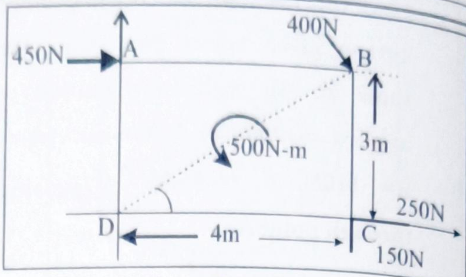



Fig 1.52

2. Resolve the force 'F' equal to 900N acting at B, as shown in fig 1.53 into
   (i) Parallel components at A & 0,
   (ii) A couple and force at O.

   **[Ans. $F_A$= 2700 N (↓) $F_o$ = 1800 N (↓),F =900 N (↓) M=2700 N-m]**

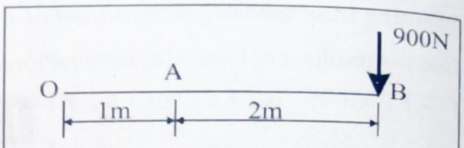



Fig 1.53

**Questions/Problems for Practice for the day:**

1. The resultant of the force system acting on the rectangular plate shown in fig 1.54. Also find the point where the resultant will cut the x- axis & y-axis. Also shift the resultant to point 'B' **[Ans: R=201.4N, $\theta$ = 59.87 °, x= 0.3866m to the left of origin, Y= 0.666m above the origin]**

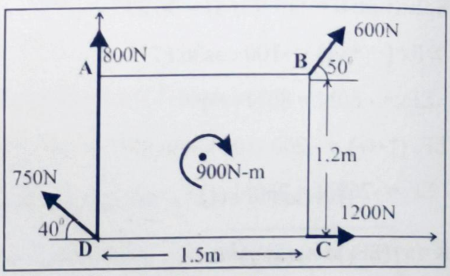



Fig 1.54

**Learning from the lecture 'Shifting of a force'**: Learner will be able to know shifting of a force from one point to another

## 1.3 Conclusion:

**Learning Outcomes:** Learners should be able to

➢ **Know, Comprehend**

1. Define different types of forces and fundamental parameters such as tensile, compressive, point of application, etc.
2. Identify and locate the forces in a figure with respect to different axis.

➢ **Apply, Analyze**

3. Locate different types of forces in a figure by considering position of resultant and the moment of forces.
4. Find the equilibrant of a system, which can bring a system into equilibrium.

➢ **Synthesize**

5. Analyze fundamental parameters for different forces and centroid for plane lamina.

## 1.4 Add to Knowledge (Content Beyond Syllabus)

- Centroids indicates centre of mass of a uniform solid. Stick a pivot at a centroid and the object will be in perfect balance.
- Lots of construction applications & engineering applications to design things so that minimal stress and energy is used to stabilize a component.
- In stress and deflection analysis of a beam, the location of centroid is very important.
- Recent Research: https://calcresource.com/centroid-how-to-find.html

## 1.5 Set of Multiple-Choice Questions:

**1)** Density is best given by _______

a) Product of volume and density   b) Ratio of mass to Volume

c) Addition of mass and density   d) Subtraction of mass and density

**2)** If solving the question in 3D calculations is difficult, then use the 2D system and then equate the ratio of the product of the centroid of the section to its mass to the total mass of the body to the centroid.

a) True   b) False

**3)** One of the uses of the centroid is as in the simplification of the loading system the net force acts at the ___________ of the loading body.

a) Centroid   b) The centre axis   c) The corner   d) The base

**4)** The use of centroid comes in picture as if the non-Uniform loading is of the type of parabola then what will be the best suited answer among the following?

a) The net load will not be formed as all the forces will be cancelled

b) The net force will act the centre of the parabola

c) The net force will act on the base of the loading horizontally

d) The net force will act at the centroid of the parabola

**5)** The x axis coordinate and the y axis coordinate of the centroid are having different types of calculations to calculate them.

a) True   b) False

**6)** The centre of ____________ is the ratio of the product of centroid and volume to the total volume.

a) Centroid axis   b) Density   c) Mass   d) Volume

7) If the force vector F acting along the centroid is having its x-axis component being equal to Z N, y-axis component be X N and z-axis component be Y N then vector F is best represented by?

a) Xi + Yj + Zk b) Yi + Xj + Zk c) Zi + Yj + Xk d) Zi + Xj + Yk

8) Centroid of a body does depends upon the small weights of tiny particles. Which statement is right for force acting by the small particles of the body having it's vector form as = Ai + Bj + Ck?

a) In rectangular components representation of any vector we have vector F = Ai + Bj + Ck

b) In rectangular components representation of any vector we have vector F = Ax + By + Cz

c) In rectangular components representation of any vector we have vector F = Fx + Fy + Fz

d) In rectangular components representation of any vector we have vector F = Fi + Fj + Fk

9) Centroid determination involves the calculations of various forces. In that forces are having various properties. That is force is developed by a support that does not allow the _______ of its attached member.

a) Translation b) Rotation c) Addition d) Subtraction

10) Determine the y coordinate of centroid of the area in the shape of circle as shown.

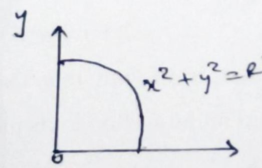



Fig 1.55

a) 4R/п b) 2/п c) 2R/3 d) 2R/5

11) What is not the condition for the equilibrium for the calculations used for the determination of the centroid in three dimensional system of axis?

a) ∑Fx=0 b) ∑Fy=0 c) ∑Fz=0 d) ∑F≠0

12) Determine the x coordinate of centroid of the area in the shape of circle as shown.

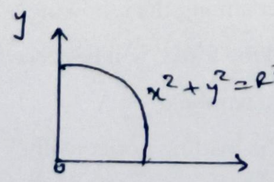



Fig 1.56

a) 4R/п b) 2/п c) 2R/3 d) 2R/5

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1) B | 2) B | 3) A | 4) D | 5) B | 6) D | 7) D | 8) C | 9) A | 10) A | 11) D | 12) A |

### 1.6 Short Answer Questions:

1) Find the difference between centroid, centre of gravity & centre of mass.

2) Determine centroid of an equilateral triangle having base & height on x-axis & y-axis

respectively.

3) Locate the centroid of a semicircle having center O at origin and diameter on y-axis.

4) Locate the centroid of an area as shown in fig 1.57.

5) Locate the centroid of a L section having base of 10cm on x-axis and 8cm height on y-axis.

6) Locate the centroid of an area as shown in fig 1.58. having radius R

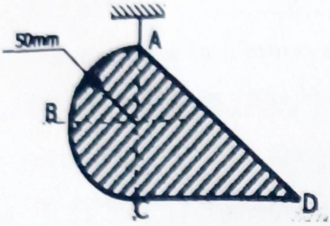



Fig 1.57

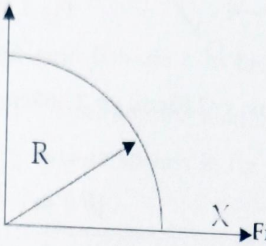



Fig 1.58

7) Forces act on the plate ABCD as shown in the fig 1.59. The distance AB is 4m. Given that the plate is in equilibrium find: Force F, Angle α [Ans: F= 13N, α= 67.4°]

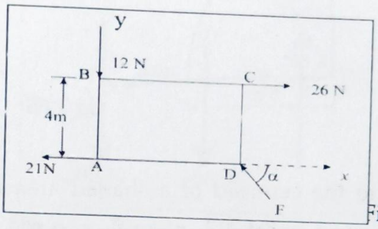



Fig 1.59

8) State and prove Varignon's Theorem

9) Define Lami's Theorem and explain it with neat diagram

10) Determine the magnitude and direction of the resultant of the forces shown in the given fig 1.60. **[Ans.: 178 N, 355°]**

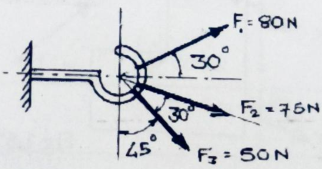



Fig 1.60

11) Determine the equilibrant of the co-planer concurrent forces shown in fig 1.61. **[Ans: R= E = 97.95N, $\theta$ =26.11°]**

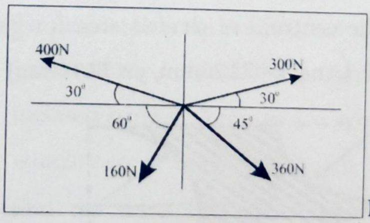



Fig 1.61

12) Determine the resultant of the following parallel forces. **[Ans: R= 80N]**

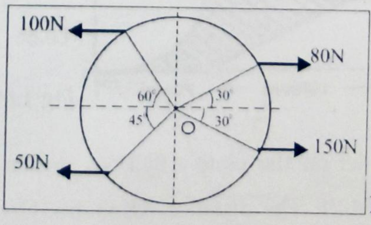



Fig 1.62

## 1.7 Long Answer Questions:

13) Locate the centroid of a plane lamina for

14) Locate the centroid of a shaded area for

given fig 1.63(**Ans: x= 1.532 m, y= 1.721 m)**

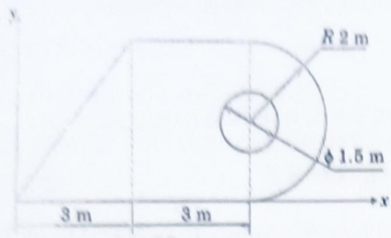



Fig 1.63

given fig 1.62 (**Ans: x=0.349a, y= 0.637a)**

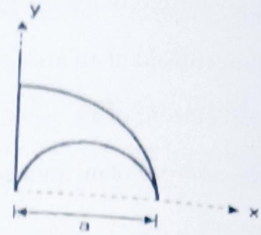



Fig 1.64

**15)** Locate the centroid of a shaded area for a given fig 1.65 (**Ans: x=2.63cm, y= 2.365cm)**

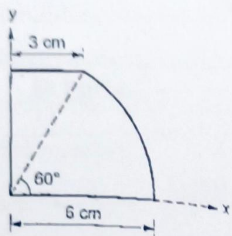



Fig 1.65

**16)** Locate the centroid of a shaded area for a given fig 1.66 (**Ans: x=4.625cm, y= 1cm)**

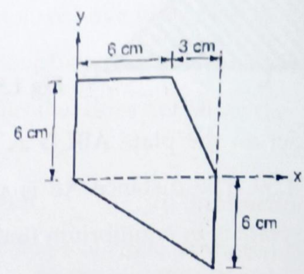



Fig 1.66

**17)** Locate the centroid of a shaded area for a given fig 1.67 (**Ans: x=5a/12 y= 5a/12)**

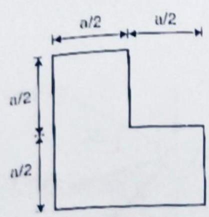



Fig 1.67

**18)** Find the centroid of the shaded area as shown in fig 1.68. **[Ans.: x = 17.673cm, y = 11.835cm]**

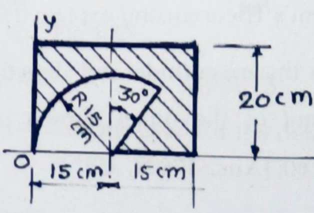



Fig 1.68

**19)** Find the centroid of shaded area for given fig 1.69. **[Ans: x= 72.26mm, y= 77.16mm]**

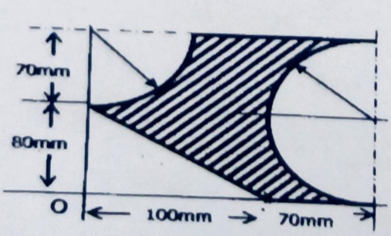



Fig 1.69

**20)** Determine the CG of the shaded area for given fig. 1.70. **[Ans: x= 125cm, y= 31.81cm]**

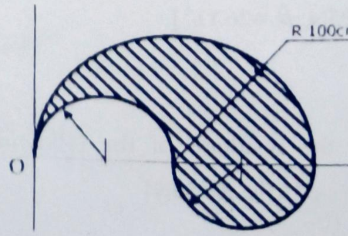



Fig 1.70

**21)** Forces act on the plate ABCD as shown in the fig. 1.71. The distance AB is 4m. Given that the plate is in equilibrium find - Force F, Angle α, Distance AD **[Ans: F= 13N, α=**

**22)** Two concurrent forces P & Q acts at O such that their resultant acts along x-axis for given fig 1.72. Determine the magnitude of Q and hence resultant. **[Ans: Q=300N, R=**

**67.4°, AD= 8.66m]**

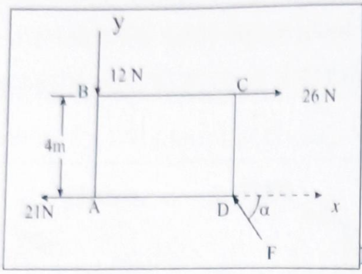



**Fig 1.71**

**595.69N]**

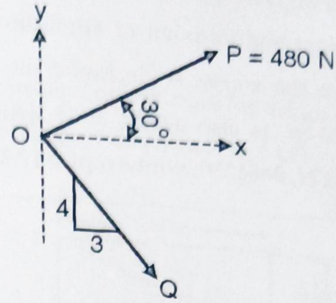



**Fig 1.72**

23) Determine the resultant of the forces as given in the fig. 1.73. Find the angle which the resultant makes with positive x-axis. **[Ans: R= 100.919 N, θ=30.45°]**

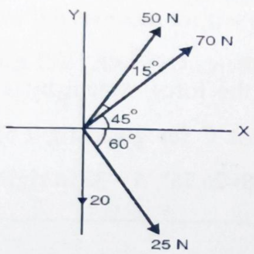



**Fig 1.73**

24) Determine the resultant of the vertical force system shown in fig 1.74. **[Ans: M=4500 N-m (↺)]**

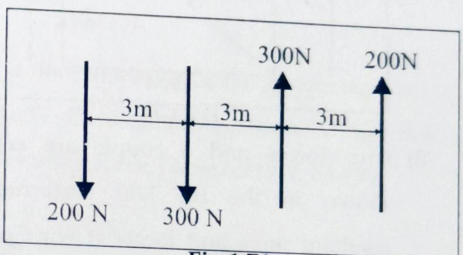



**Fig 1.74**

25) Determine the resultant of the non-concurrent, non-parallel system of forces as shown in fig 1.75. **[Ans: R= 175N, ϴ = 55°, 1.317m to the right of point O.]**

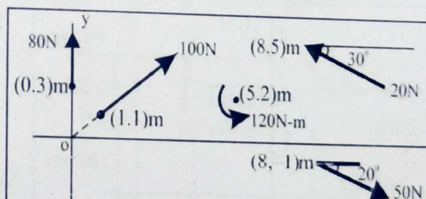



**Fig 1.75**

26) For the system shown, determine (a) The required value of if resultant of three forces is to be vertical, (b) The corresponding magnitude of resultant **[Ans: 21.8°, 229.29N (↓)]**

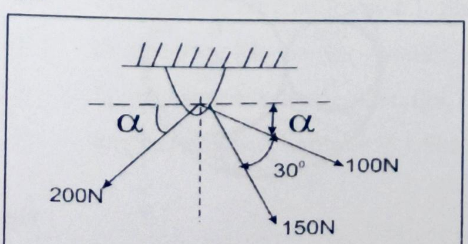



**Fig 1.76**

27) To pull hook from an inclined surface, total pull required is 800N perpendicular to inclined surface. 3 forces are applied out of which two of them are shown in fig 1.77, find the third force.**[Ans: R= 426N, $\theta$ = 58.83°]**

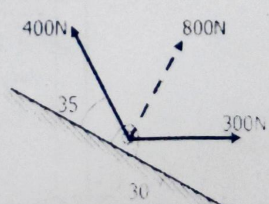



**Fig 1.77**

28) For the given system of fig 1.78, find the resultant and its point of application w.r.t. 'O' on the x-axis. A clockwise moment of 5000N-cm is also acting at O. **[Ans: R= 341.42 N, θ=64.79°, x-intercept = 17.23 cm]**

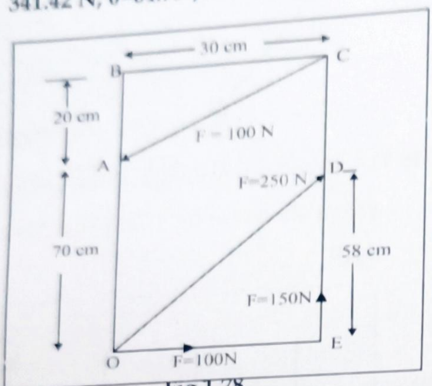


Fig 1.78

29) Replace the force system given in the fig 1.79 by a single force w.r.t point C. **[Ans: R= 78.115 k N, θ=20.22°, d = 10.763 m right of C]**

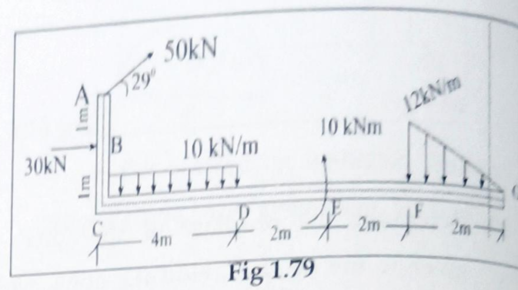


Fig 1.79

30) Four forces and a couple are acting as shown in the fig 1.80. Determine the resultant force and locate it w.r.t point A.. **[Ans: R= 305.28 N, θ=31.6°, d = 248.95m]**

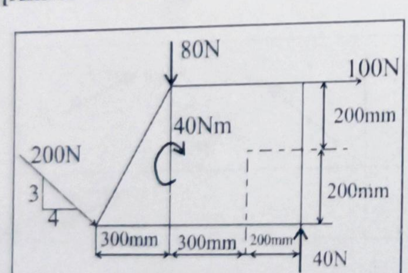


Fig 1.80

31) Replace the force system by a single force w.r.t point C for given fig 1.81. **[Ans: R= 22.36 N, θ=26.56°, d = 35 m right of C]**

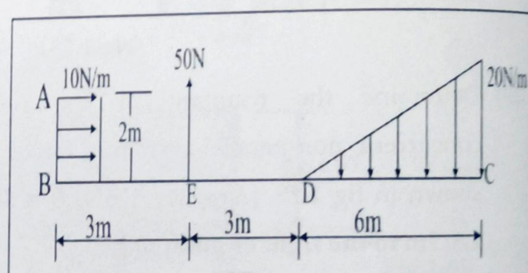


Fig 1.81

32) Replace loading on frame fig 1.82, by a force & moment at point A. **[Ans: R= 921.9N, d= 3.47m]**

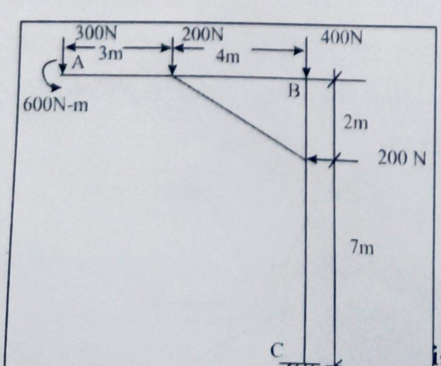


Fig 1.82

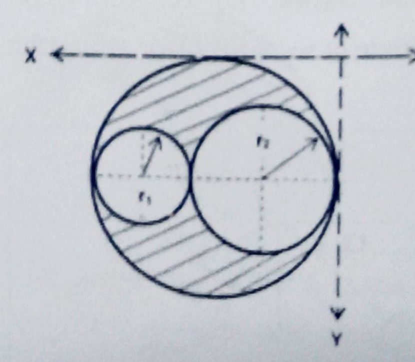


Fig 1.83

33) Determine the centroid of the shaded area shown in fig 1.83 with respect to given reference axes (Given: $r_1$ = 13cm, $r_2$ = 23cm) **(Ans: -40.99cm, -36cm)**

34) Determine the centroid for the given shaded area for fig 1.84 **(Ans: 107.19mm,107.19mm)**

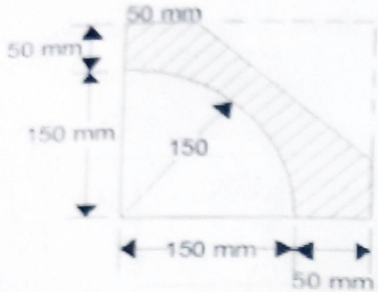



Fig 1.84

35) Determine the centroid for the given shaded area for fig 1.85 **(Ans: 7.50, 5.08)**

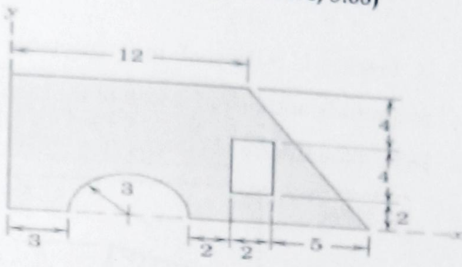



**Fig 1.85**

36) Determine the centroid for the given shaded area for fig 1.86 **(Ans: -10mm, 87.12mm)**

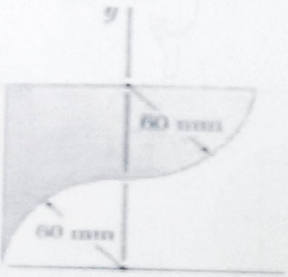



**Fig 1.86**

37) For a non-concurrent force system shown in fig 1.87. Determine magnitude & direction of resultant force. **(Ans: 6.71kN, 63.49°)**

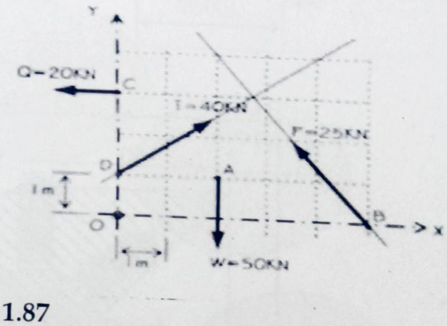


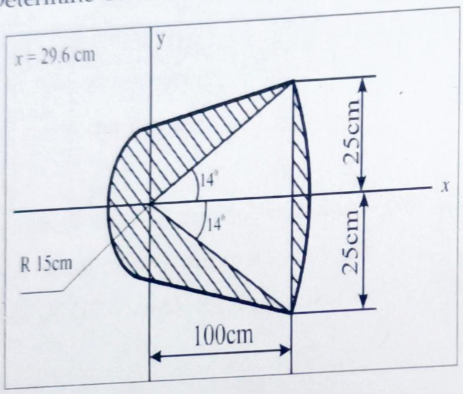

**Fig 1.87**

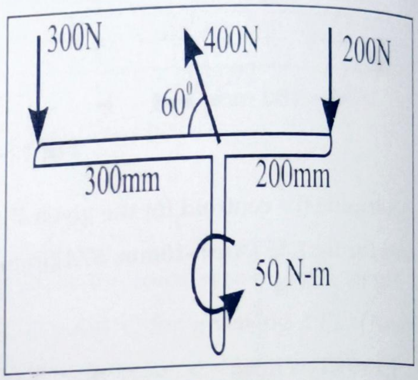
## 1.8 References:

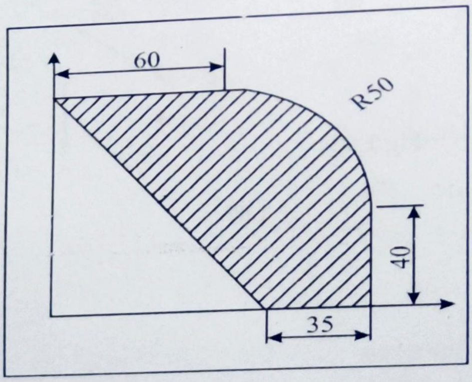

1) Engineering Mechanics byTayal, Umesh Publication
2) Engineering Mechanics by Beer & Johnson, Tata McGraw Hill
3) Engineering Mechanics by F.L. Singer by Harper
4) Engineering Mechanics – Statics, R. C. Hibbler
3) Engineering Mechanics – Statics, J. L. Merium, I. G. Kraig
5) Engineering Mechanics – P. J. Shah, R. Bade

**Self-Assessment**

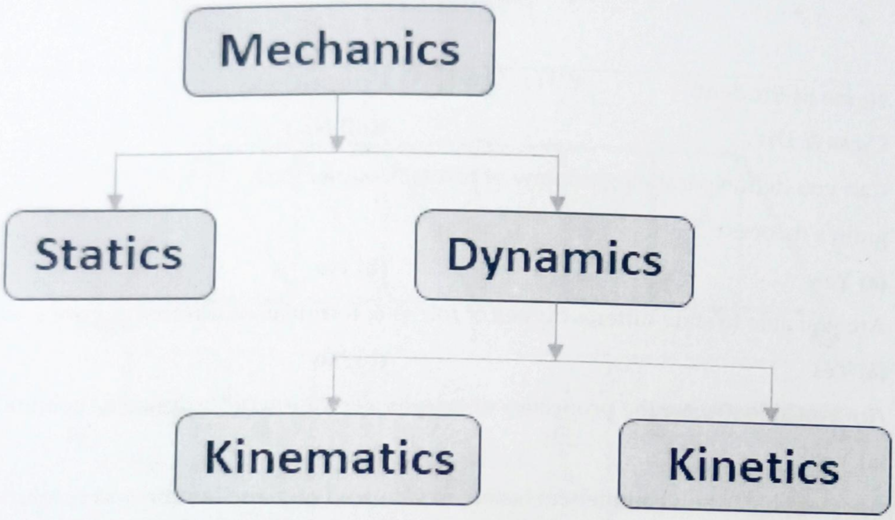
1. What is the difference between moment & couple? **(Level 1)**
2. Why is force called a sliding vector? **(Level 2)**
3. Can you guess where will the centroid of this figure will lie? **(Level 3)**
4. A bracket is subjected to a co-planer force system as shown in fig. Determine the magnitude and line of action of the resultant. **(Level 4)**
5. Determine Centre of Gravity of shaded area. **(Level 5)**

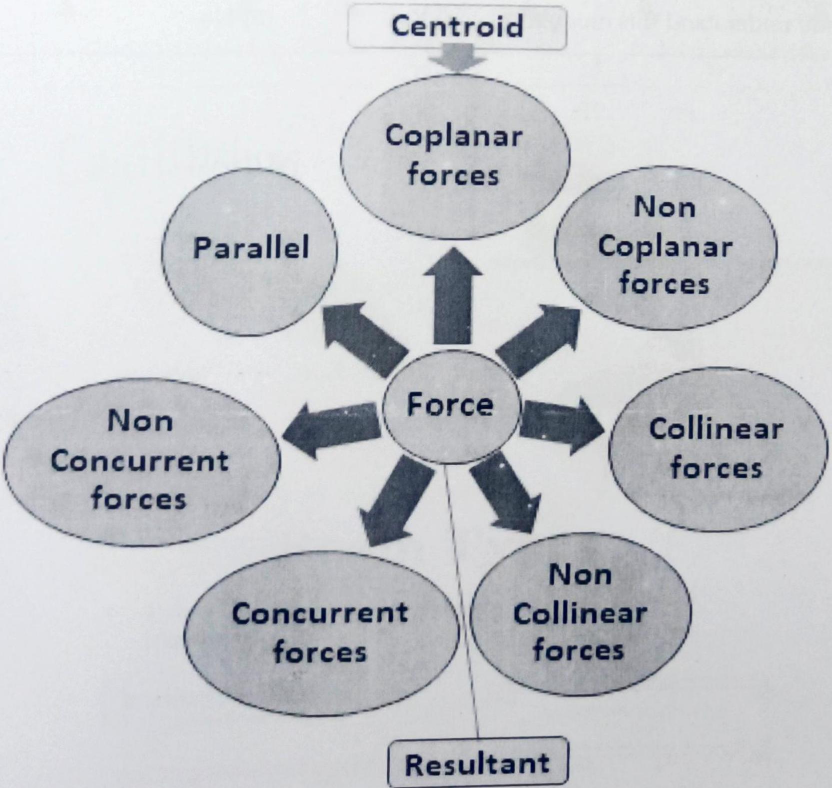


## Concept Mapping



## Self-Evaluation

**Name of Student:** **Course Code:**

**Class & Div.:** **Roll No.:**

1. Can you define parallelogram law of forces, Couple, Vari gnon's theorem?

   (a) Yes (b) No

2. Are you able to state different types of forces & formulae of different figure in centroid?

   (a) Yes (b) No

3. Are you able to state the properties of couple, derivation of Varignon's theorem?

   (a) Yes (b) No

4. Are you able to solve numerical based on Centroid of plane lamina and resolution of forces?

   (a) Yes (b) No

5. Do you understand this module? (a) Yes (b) No