Kinematic Law of PPC (Motion)

Geodesic Motion Equation in the Form of Energy Density and Gravitational Pressure (Kinematic Law of Pawan Upadhyay's Pressure-Curvature Law of Gravity (PPC Law of Gravity))

Author: Pawan Upadhyay
Affiliation: Independent Researcher
Email: pawanupadhyay28@hotmail.com
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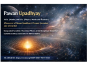

Abstract

In General Relativity, gravitational motion is described through geodesics in curved spacetime. This curvature is sourced by the stress–energy tensor, traditionally expressed in terms of mass density and pressure. In this work, geodesic motion is reformulated explicitly in terms of **energy density** E_d and **gravitational pressure** P_g. A general proportional relation $P_g = wE_d$ is introduced. By imposing the trace condition $T = -2E_d$, the equation-of-state parameter is uniquely fixed as $w = 1$, leading to $P_g = E_d$. Other physically relevant cases ($w < 1, w = -1, w = 0, w = \frac{1}{3}$) are examined for comparison. The analysis shows that the proposed single master equation is valid exclusively in the $w = 1$ regime.

1. Introduction

In the geometric theory of gravitation developed by Albert Einstein, free particles follow geodesics of a curved spacetime rather than trajectories determined by a force. The curvature of spacetime is governed by the stress–energy tensor, which includes both energy density and pressure as gravitational sources.

This paper reformulates the geodesic motion equation using energy density E_d and **gravitational pressure** P_g, with particular emphasis on a pressure-dominated gravitational regime.

2. Definitions and Notation

We define the following quantities:

- ρ : mass density
- $E_d = \rho c^2$: energy density
- P_g : gravitational pressure
- w : equation-of-state parameter
- $g_{\mu\nu}$: spacetime metric
- u^μ : four-velocity
- $T_{\mu\nu}$: stress–energy tensor
- $T = g^{\mu\nu}T_{\mu\nu}$: trace of the stress–energy tensor

The **base equation of state** is assumed as:

$$\boxed{P_g = wE_d}$$

3. Trace of the Stress–Energy Tensor

For an isotropic gravitational medium, the trace is:

$$\boxed{T = \rho c^2 - 3P_g}$$

Using $E_d = \rho c^2$:

$$\boxed{T = E_d - 3P_g}$$

4. Fundamental Trace Condition

We impose the defining relation:

$$\boxed{T = -2E_d}$$

Substituting:

$$E_d - 3P_g = -2E_d$$

$$-3P_g = -3E_d$$

$$\boxed{P_g = E_d} \quad \Rightarrow \quad \boxed{w = 1}$$

Thus, the trace condition uniquely selects the **stiff gravitational regime**.

5. Curvature–Energy Relation

Einstein's field equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Taking the trace:

$$-R = \frac{8\pi G}{c^4} T$$

Using $T = -2E_d$:

$$\boxed{R = \frac{16\pi G}{c^4} E_d}$$

Hence, scalar curvature is directly proportional to energy density.

6. Comparison of Different w -Regimes

Using

$$T = E_d - 3wE_d = (1 - 3w)E_d$$

we analyze key cases:

6.1 $w = 1$ (Stiff gravitational pressure)

$$P_g = E_d, \quad T = -2E_d$$

- Pressure equals energy density
- Trace condition satisfied
- **Master equation valid**
- Strong curvature sourcing

6.2 $w = \frac{1}{3}$ (Radiation)

$$P_g = \frac{1}{3}E_d, \quad T = 0$$

- Conformal matter
- No scalar curvature sourcing
- Master equation **not applicable**

6.3 $w = 0$ (Dust / Matter)

$$P_g = 0, \quad T = E_d$$

- Newtonian-like matter
- Pressureless gravity
- Incompatible with $T = -2E_d$

6.4 $w = -1$ (Vacuum / Dark Energy)

$$P_g = -E_d, \quad T = 4E_d$$

- Negative pressure
- Accelerated expansion
- Opposite sign curvature response

6.5 $w < 1$ (General sub-stiff regime)

$$T = (1 - 3w)E_d$$

- Partial pressure contribution
- Energy density does not fully convert to curvature
- Geodesic motion not governed solely by E_d

7. Geodesic Motion Equation

The geodesic equation remains:

$$\boxed{\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0}$$

In the $w = 1$ regime, the Christoffel symbols depend implicitly only on E_d, making geodesic motion **energy-density governed**.

8. Single Master Equation (Validity Condition Explicit)

$$\boxed{\begin{aligned} &P_g = wE_d, \quad w = 1 \\ &T = -2E_d \\ &R = \frac{16\pi G}{c^4}E_d \\ &\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}(E_d)\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0 \end{aligned}}$$

9. Physical Interpretation

> **Only in the $w = 1$ regime does gravitational pressure fully convert energy density into spacetime curvature, making geodesic motion a direct manifestation of energy-density structure.**

10. Conclusion

This work presents a reformulation of geodesic motion using energy density and gravitational pressure. By assuming a general equation of state $P_g = wE_d$ and imposing the trace condition $T = -2E_d$, the theory uniquely selects $w = 1$. Other physically relevant regimes are shown to fall outside the validity of the proposed master equation. The results identify a special pressure-dominated gravitational state within General Relativity.

Declaration

This paper presents a theoretical framework within the mathematical structure of General Relativity and does not claim experimental confirmation.

> *"The master equation derived in this work is valid exclusively for the stiff gravitational equation of state $w = 1$."*

PPC Law Statement:

> *PPC Law asserts that gravitational phenomena arise from energy density and gravitational pressure, where pressure acts as an active geometric agent. In the special regime $P_g = E_d$, gravitational pressure is fully determined by energy density, and spacetime curvature and geodesic motion are governed entirely by this energy-density–pressure equivalence, yielding a pressure-dominated realization of General Relativity.*

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