Dynamic law of PPC (curvature sourcing)

Einstein Field Equations in Energy Density–Pressure Language → Dynamic law of PPC (curvature sourcing)

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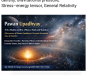


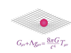
Abstract

Einstein's Field Equations relate spacetime curvature to the distribution of matter and energy through the stress–energy tensor. Conventionally, this tensor is expressed in terms of mass density and pressure. In this work, the Einstein Field Equations are reformulated explicitly in terms of **energy density** E_d and **gravitational pressure** P_g. A general equation of state $P_g = wE_d$ is introduced, allowing a unified description of known physical regimes. A special pressure-dominated regime $w = 1$, corresponding to $P_g = E_d$, is shown to yield a direct proportionality between scalar curvature and energy density. This reformulation does not alter General Relativity but provides a pressure-centric interpretation of gravitational dynamics.

1. Introduction

General Relativity, formulated by Albert Einstein, describes gravitation as the curvature of spacetime produced by matter and energy. The Einstein Field Equations encode this relationship through the stress–energy tensor $T_{\mu\nu}$.

In relativistic gravitation, pressure contributes to curvature on the same footing as energy density. This paper rewrites the Einstein Field Equations entirely in terms of **energy density** E_d and **gravitational pressure** P_g, making the role of pressure explicit and transparent.

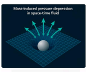


Using PPC definitions:

$$T_{\mu\nu} = (E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}$$

The equation becomes:

$$G_{\mu\nu} = \frac{8\pi G}{c^4}\left[(E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}\right] - \Lambda g_{\mu\nu}$$



2. Definitions and Notation

We define the following quantities:

- ρ : mass density
- $E_d = \rho c^2$: energy density
- P_g : gravitational pressure
- w : equation-of-state parameter
- $g_{\mu\nu}$: spacetime metric
- u^μ : four-velocity
- $T_{\mu\nu}$: stress–energy tensor
- $T = g^{\mu\nu}T_{\mu\nu}$: trace of the stress–energy tensor

The **base equation of state** is assumed as:

$$P_g = wE_d$$

3. Einstein Field Equations

The Einstein Field Equations are:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

These equations are exact and remain unchanged in this reformulation.

4. Stress–Energy Tensor in Energy–Pressure Form

For an isotropic gravitational medium, the stress–energy tensor is written as:

$$T_{\mu\nu} = (E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}$$

This expression shows explicitly that **both energy density and pressure act as sources of spacetime curvature.**

5. Einstein Equations in Energy Density–Pressure Language

Substituting $T_{\mu\nu}$ into the field equations yields:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}\left[(E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}\right]$$

This is the Einstein Field Equations written entirely in energy–pressure variables.

6. Trace Equation and Scalar Curvature

Taking the trace of the Einstein Field Equations:

$$-R = \frac{8\pi G}{c^4}T$$

The trace of the stress–energy tensor is:

$$T = E_d - 3P_g$$

Therefore, scalar curvature is:

$$R = \frac{8\pi G}{c^4}(3P_g - E_d)$$

This equation demonstrates that scalar curvature is governed by the **balance between gravitational pressure and energy density.**

7. Ricci Tensor in Energy–Pressure Form

The Ricci tensor can be expressed as:

$$R_{\mu\nu} = \frac{8\pi G}{c^4}\left[(E_d + P_g)u_\mu u_\nu + \frac{1}{2}(E_d - P_g)g_{\mu\nu}\right]$$

This form separates curvature into:

- time-like contributions from energy density,
- spatial contributions from pressure.

8. Conservation Law

Einstein's equations imply covariant conservation:

$$\nabla_\mu T^{\mu\nu} = 0$$

In energy–pressure language:

$$\nabla_\mu\left[(E_d + P_g)u^\mu u^\nu\right] + \nabla^\nu P_g = 0$$

This represents **energy-density transport and pressure balance** in curved spacetime.

9. Special Regime: $w = 1$ (Pressure = Energy Density)

In the special case:

$$P_g = E_d$$

the stress–energy tensor becomes:

$$T_{\mu\nu} = 2E_d u_\mu u_\nu + E_d g_{\mu\nu}$$

The trace reduces to:

$$T = -2E_d$$

Scalar curvature is then:

$$R = \frac{16\pi G}{c^4}E_d$$

Thus, scalar curvature is directly proportional to energy density alone.

10. Physical Interpretation

- Energy density manifests as gravitational pressure
- Pressure contributes equally to spacetime curvature
- In the $w = 1$ regime, pressure fully converts energy density into curvature

Gravity emerges as a pressure-structured energy phenomenon rather than a force.

11. Comparison with Other w-Regimes

w	Physical interpretation	Trace T
0	Dust / matter	E_d
$\frac{1}{3}$	Radiation	0
-1	Vacuum energy	$4E_d$
< 1	Sub-stiff matter	$(1 - 3w)$
1	Stiff gravitational pressure	$-2E_d$

Only $w = 1$ yields direct proportionality between curvature and energy density.

12. Conclusion

Einstein Field Equations have been reformulated in explicit energy density–pressure language. While the mathematical structure of General Relativity remains unchanged, this representation highlights the fundamental role of gravitational pressure. A special pressure-dominated regime $w = 1$, corresponding to $P_g = E_d$, leads to a simple and direct relation between scalar curvature and energy density. This framework provides a unified and physically transparent interpretation of gravitation within General Relativity.

Declaration

This paper presents a theoretical reformulation within established General Relativity and does not claim experimental verification.

PPC Law Statement:

> *PPC Law asserts that gravitational phenomena arise from energy density and gravitational pressure, where pressure acts as an active geometric agent. In the special regime $P_g = E_d$, gravitational pressure is fully determined by energy density, and spacetime curvature and geodesic motion are governed entirely by this energy-density–pressure equivalence, yielding a pressure-dominated realization of General Relativity.*

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