Numerical Analysis 1 – Class 3

Thursday, January 25, 2018

Subjects covered

- Sampled data and time series.
- Taking numeric derivatives.
- Complexity – big-O notation.
- Fourier series and Fourier transform.
- Filtering and processing of signals in the time and frequency domain.

Readings

- Fundamentals of Signal Analysis – HP app note 243 (on Blackboard). Provides a very clear explanation of signal analysis from the electrical engineering perspective. You don't need to read the whole thing, but it's worthwhile to skim the document and read the sections relevant to this evening's class.
- "An Intuitive but Not-All-That-Mathematically-Sound Explanation of the Fourier Transform" by D. Morris (on Blackboard).
- Kutz, Sections 4.1, 10.1, 13.1 – 13.3.

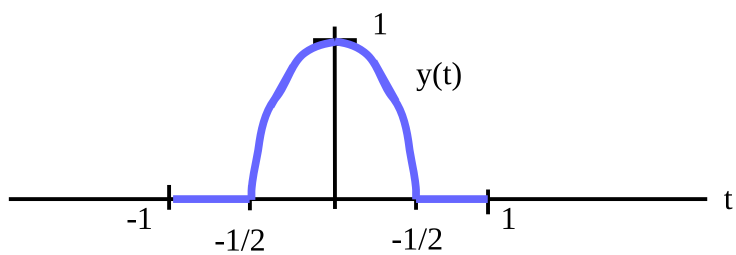
Problems

Most of the following problems require you to write a program. When writing the program, please use your preferred programming language from the list of languages given in class. As always, please make sure you also write a test harness which validates your program's implementation. You will be graded on both your program as well as on your test. E-mail your answers to our TA: Mohamed Elbehiry, elbehiry.m@husky.neu.edu.

1. Consider the inverted parabola y(t) shown below.

$$y(t)=1-\left(\frac{t}{(1/2)}\right)^2$$

The relevant part of the function is defined over the domain -1 < t < 1. The peak of the parabola occurs at t = 0. Assume the function y(t) is zero for abs(y) > 1/2.

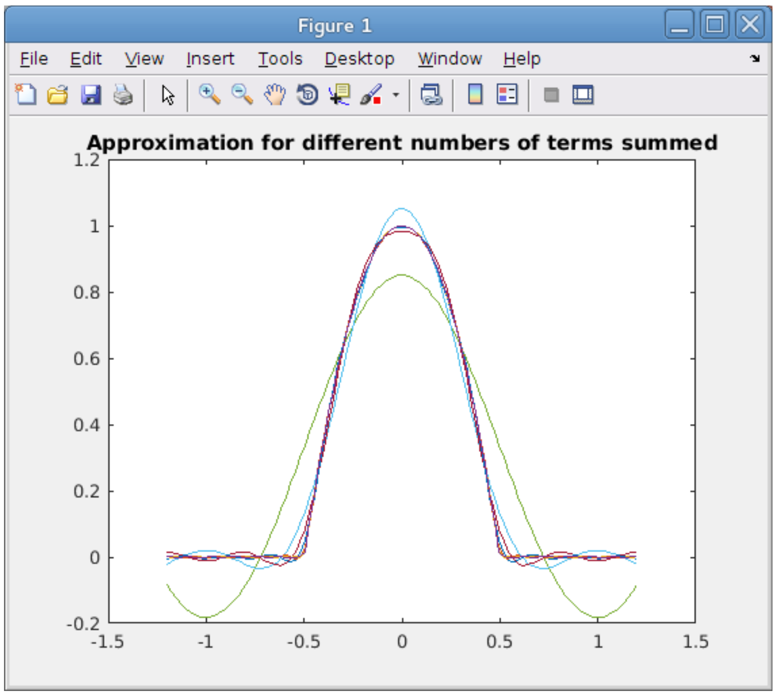


Do the following:

- Using pencil/paper, derive the coefficients of its Fourier series expansion. (You can exploit the fact that the function is anti-symmetric about origin to deduce the basis functions for the expansion.) To perform the integrals, I suggest you use the Wolfram Alpha integral calculator, available online at

  http://www.wolframalpha.com/calculators/integral-calculator/

- Take N to be the highest order term in your series expansion. Write a program which loops over increasing N and makes a plot similar to that below showing how the series converges to the inverted parabola. (My result is shown below.)



- Make a plot of the RMS error between your Fourier series expansion and the original function y(t) as a function of highest order term summed N. The RMS error is defined as

$$err = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(y_{true}(x_i) - y_{app}(x_i)\right)^2}$$

  (For more information, please consult the note "On Computing RMS Error" available under "Course Materials" on Blackboard.)

- Hand in your derivation of the Fourier coefficients, as well as your code which makes the two plots.

2. When you dial a touchtone phone, the telephone generates a so-called DTMF signal, which is composed of two sine waves at different, incommensurate frequencies. This signal is used to command the telephone company's switching equipment which number you want to dial. Different digits are represented by different pairs of frequencies. The frequency assignments are shown in the table below.

	1209 Hz	1336 Hz	1477 Hz
697 Hz	1	2	3
770 Hz	4	5	7
852 Hz	7	8	9
941 Hz	*	0	#

Under the class 3 materials section of Blackboard I have placed sound samples corresponding to a 7 digit telephone number. The samples are labeled “A”, “B”, ... “G”. Your assignment: Download the samples and write a program which deduces the telephone number being dialed. Obviously, this is an exercise in using the FFT to process the sound signals. Use “audioread” to read in the signals. The difficult part is that you need to pay careful attention to the time and the frequency axes in order to correctly identify the frequencies of the tones. Review the materials in lecture session 3 if you need help with this.

Hand in the telephone number being dialed (“A” first, then “B”, etc.) along with your code.