Optimization Methods

```
import numpy as np
import matplotlib.pyplot as plt
import scipy.io
import math
import sklearn
import sklearn.datasets

from opt_utils import load_params_and_grads, initialize_parameters, forward_propagation, backward_propagation
from opt_utils import compute_cost, predict, predict_dec, plot_decision_boundary, load_dataset
from testCases import *

%matplotlib inline
plt.rcParams['figure.figsize'] = (7.0, 4.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'
```

```
/content/opt_utils.py:76: SyntaxWarning: assertion is always true, perhaps remove parentheses?
  assert(parameters['W' + str(l)].shape == layer_dims[l], layer_dims[l-1])
/content/opt_utils.py:77: SyntaxWarning: assertion is always true, perhaps remove parentheses?
  assert(parameters['W' + str(l)].shape == layer_dims[l], 1)
```

1 - Gradient Descent

```
# GRADED FUNCTION: update_parameters_with_gd

def update_parameters_with_gd(parameters, grads, learning_rate):
    """
    Update parameters using one step of gradient descent

    Arguments:
    parameters -- python dictionary containing your parameters to be updated:
                    parameters['W' + str(l)] = Wl
                    parameters['b' + str(l)] = bl
    grads -- python dictionary containing your gradients to update each parameters:
                    grads['dW' + str(l)] = dWl
                    grads['db' + str(l)] = dbl
    learning_rate -- the learning rate, scalar.

    Returns:
    parameters -- python dictionary containing your updated parameters
    """

    L = len(parameters) // 2 # number of layers in the neural networks

    # Update rule for each parameter
    for l in range(L):
        ### START CODE HERE ### (approx. 2 lines)
        parameters["W" + str(l+1)] = parameters["W" + str(l+1)] - learning_rate*grads['dW' + str(l+1)]

        parameters["b" + str(l+1)] = parameters["b" + str(l+1)] - learning_rate*grads['db' + str(l+1)]
        ### END CODE HERE ###

    return parameters
```

```
parameters, grads, learning_rate = update_parameters_with_gd_test_case()

parameters = update_parameters_with_gd(parameters, grads, learning_rate)
print("W1 = " + str(parameters["W1"]))
print("b1 = " + str(parameters["b1"]))
print("W2 = " + str(parameters["W2"]))
print("b2 = " + str(parameters["b2"]))
```

```
W1 = [[ 1.63535156 -0.62320365 -0.53718766]
 [-1.07799357  0.85639907 -2.29470142]]
b1 = [[ 1.74604067]
 [-0.75184921]]
W2 = [[ 0.32171798 -0.25467393  1.46902454]
 [-2.05617317 -0.31554548 -0.3756023 ]
 [ 1.1404819  -1.09976462 -0.1612551 ]]
```

```
b2 = [[-0.88020257]
 [ 0.02561572]
 [ 0.57539477]]
```

2 - Mini-Batch Gradient descent

```
# GRADED FUNCTION: random_mini_batches

def random_mini_batches(X, Y, mini_batch_size = 64, seed = 0):
    """
    Creates a list of random minibatches from (X, Y)

    Arguments:
    X -- input data, of shape (input size, number of examples)
    Y -- true "label" vector (1 for blue dot / 0 for red dot), of shape (1, number of examples)
    mini_batch_size -- size of the mini-batches, integer

    Returns:
    mini_batches -- list of synchronous (mini_batch_X, mini_batch_Y)
    """

    np.random.seed(seed)            # To make your "random" minibatches the same as ours
    m = X.shape[1]                  # number of training examples
    mini_batches = []

    # Step 1: Shuffle (X, Y)
    permutation = list(np.random.permutation(m))
    shuffled_X = X[:, permutation]
    shuffled_Y = Y[:, permutation].reshape((1,m))

    # Step 2: Partition (shuffled_X, shuffled_Y). Minus the end case.
    num_complete_minibatches = math.floor(m/mini_batch_size) # number of mini batches of size mini_batch_size in your partitionning
    for k in range(0, num_complete_minibatches):
        ### START CODE HERE ### (approx. 2 lines)
        mini_batch_X = shuffled_X[:, mini_batch_size*k:mini_batch_size*(k+1)]

        mini_batch_Y = shuffled_Y[:, mini_batch_size*k:mini_batch_size*(k+1)]
        ### END CODE HERE ###
        mini_batch = (mini_batch_X, mini_batch_Y)
        mini_batches.append(mini_batch)

    # Handling the end case (last mini-batch < mini_batch_size)
    if m % mini_batch_size != 0:
        ### START CODE HERE ### (approx. 2 lines)
        mini_batch_X = shuffled_X[:,num_complete_minibatches *mini_batch_size : m]

        mini_batch_Y = shuffled_Y[:,num_complete_minibatches *mini_batch_size : m]

        ### END CODE HERE ###
        mini_batch = (mini_batch_X, mini_batch_Y)
        mini_batches.append(mini_batch)

    return mini_batches
```

```
X_assess, Y_assess, mini_batch_size = random_mini_batches_test_case()
mini_batches = random_mini_batches(X_assess, Y_assess, mini_batch_size)

print ("shape of the 1st mini_batch_X: " + str(mini_batches[0][0].shape))
print ("shape of the 2nd mini_batch_X: " + str(mini_batches[1][0].shape))
print ("shape of the 3rd mini_batch_X: " + str(mini_batches[2][0].shape))
print ("shape of the 1st mini_batch_Y: " + str(mini_batches[0][1].shape))
print ("shape of the 2nd mini_batch_Y: " + str(mini_batches[1][1].shape))
print ("shape of the 3rd mini_batch_Y: " + str(mini_batches[2][1].shape))
print ("mini batch sanity check: " + str(mini_batches[0][0][0][0:3]))
```

```
shape of the 1st mini_batch_X: (12288, 64)
shape of the 2nd mini_batch_X: (12288, 64)
shape of the 3rd mini_batch_X: (12288, 20)
shape of the 1st mini_batch_Y: (1, 64)
shape of the 2nd mini_batch_Y: (1, 64)
shape of the 3rd mini_batch_Y: (1, 20)
mini batch sanity check: [ 0.90085595 -0.7612069   0.2344157 ]
```

3 - Momentum

```
# GRADED FUNCTION: initialize_velocity

def initialize_velocity(parameters):
    """
    Initializes the velocity as a python dictionary with:
                - keys: "dW1", "db1", ..., "dWL", "dbL"
                - values: numpy arrays of zeros of the same shape as the corresponding gradients/parameters.
    Arguments:
    parameters -- python dictionary containing your parameters.
                    parameters['W' + str(l)] = Wl
                    parameters['b' + str(l)] = bl

    Returns:
    v -- python dictionary containing the current velocity.
                    v['dW' + str(l)] = velocity of dWl
                    v['db' + str(l)] = velocity of dbl
    """

    L = len(parameters) // 2 # number of layers in the neural networks
    v = {}

    # Initialize velocity
    for l in range(L):
        ### START CODE HERE ### (approx. 2 lines)
        v["dW" + str(l+1)] = np.zeros(parameters['W' + str(l+1)].shape)

        v["db" + str(l+1)] = np.zeros(parameters['b' + str(l+1)].shape)

        ### END CODE HERE ###

    return v
```

```
parameters = initialize_velocity_test_case()

v = initialize_velocity(parameters)
print("v[\"dW1\"] = " + str(v["dW1"]))
print("v[\"db1\"] = " + str(v["db1"]))
print("v[\"dW2\"] = " + str(v["dW2"]))
print("v[\"db2\"] = " + str(v["db2"]))
```

```
v["dW1"] = [[0. 0. 0.]
 [0. 0. 0.]]
v["db1"] = [[0.]
 [0.]]
v["dW2"] = [[0. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]]
v["db2"] = [[0.]
 [0.]
 [0.]]
```

```
# GRADED FUNCTION: update_parameters_with_momentum

def update_parameters_with_momentum(parameters, grads, v, beta, learning_rate):
    """
    Update parameters using Momentum

    Arguments:
    parameters -- python dictionary containing your parameters:
                    parameters['W' + str(l)] = Wl
                    parameters['b' + str(l)] = bl
    grads -- python dictionary containing your gradients for each parameters:
                    grads['dW' + str(l)] = dWl
                    grads['db' + str(l)] = dbl
    v -- python dictionary containing the current velocity:
                    v['dW' + str(l)] = ...
                    v['db' + str(l)] = ...
    beta -- the momentum hyperparameter, scalar
    learning_rate -- the learning rate, scalar

    Returns:
    parameters -- python dictionary containing your updated parameters
```

```
    v -- python dictionary containing your updated velocities
    """

    L = len(parameters) // 2 # number of layers in the neural networks

    # Momentum update for each parameter
    for l in range(L):

        ### START CODE HERE ### (approx. 4 lines)
        # compute velocities
        v["dW" + str(l+1)] = beta * v["dW" + str(l+1)] +(1- beta )*grads['dW' + str(l+1)]

        v["db" + str(l+1)] =  beta * v["db" + str(l+1)] +(1- beta )*grads['db' + str(l+1)]
        # update parameters
        parameters["W" + str(l+1)] = parameters["W" + str(l+1)]-learning_rate * v["dW" + str(l+1)]
        parameters["b" + str(l+1)] = parameters["b" + str(l+1)]- learning_rate * v["db" + str(l+1)]


        ### END CODE HERE ###

    return parameters, v
```

```
parameters, grads, v = update_parameters_with_momentum_test_case()

parameters, v = update_parameters_with_momentum(parameters, grads, v, beta = 0.9, learning_rate = 0.01)
print("W1 = " + str(parameters["W1"]))
print("b1 = " + str(parameters["b1"]))
print("W2 = " + str(parameters["W2"]))
print("b2 = " + str(parameters["b2"]))
print("v[\"dW1\"] = " + str(v["dW1"]))
print("v[\"db1\"] = " + str(v["db1"]))
print("v[\"dW2\"] = " + str(v["dW2"]))
print("v[\"db2\"] = " + str(v["db2"]))
```

```
W1 = [[ 1.62544598 -0.61290114 -0.52907334]
 [-1.07347112  0.86450677 -2.30085497]]
b1 = [[ 1.74493465]
 [-0.76027113]]
W2 = [[ 0.31930698 -0.24990073  1.4627996 ]
 [-2.05974396 -0.32173003 -0.38320915]
 [ 1.13444069 -1.0998786  -0.1713109 ]]
b2 = [[-0.87809283]
 [ 0.04055394]
 [ 0.58207317]]
v["dW1"] = [[-0.11006192  0.11447237  0.09015907]
 [ 0.05024943  0.09008559 -0.06837279]]
v["db1"] = [[-0.01228902]
 [-0.09357694]]
v["dW2"] = [[-0.02678881  0.05303555 -0.06916608]
 [-0.03967535 -0.06871727 -0.08452056]
 [-0.06712461 -0.00126646 -0.11173103]]
v["db2"] = [[0.02344157]
 [0.16598022]
 [0.07420442]]
```

4 - Adam

Adam is one of the most effective optimization algorithms for training neural networks. It combines ideas from RMSProp (described in lecture) and Momentum.

```
# GRADED FUNCTION: initialize_adam

def initialize_adam(parameters) :
    """
    Initializes v and s as two python dictionaries with:
                - keys: "dW1", "db1", ..., "dWL", "dbL"
                - values: numpy arrays of zeros of the same shape as the corresponding gradients/parameters.

    Arguments:
    parameters -- python dictionary containing your parameters.
                    parameters["W" + str(l)] = Wl
                    parameters["b" + str(l)] = bl

    Returns:
    v -- python dictionary that will contain the exponentially weighted average of the gradient.
```

```
                    v["dW" + str(l)] = ...
                    v["db" + str(l)] = ...
    s -- python dictionary that will contain the exponentially weighted average of the squared gradient.
                    s["dW" + str(l)] = ...
                    s["db" + str(l)] = ...

    """

    L = len(parameters) // 2 # number of layers in the neural networks
    v = {}
    s = {}

    # Initialize v, s. Input: "parameters". Outputs: "v, s".
    for l in range(L):
    ### START CODE HERE ### (approx. 4 lines)
        v["dW" + str(l+1)] = np.zeros(parameters["W" + str(l+1)].shape)
        v["db" + str(l+1)] =  np.zeros(parameters["b" + str(l+1)].shape)
        s["dW" + str(l+1)] =  np.zeros(parameters["W" + str(l+1)].shape)
        s["db" + str(l+1)] = np.zeros(parameters["b" + str(l+1)].shape)
    ### END CODE HERE ###

    return v, s
```

```
parameters = initialize_adam_test_case()

v, s = initialize_adam(parameters)
print("v[\"dW1\"] = " + str(v["dW1"]))
print("v[\"db1\"] = " + str(v["db1"]))
print("v[\"dW2\"] = " + str(v["dW2"]))
print("v[\"db2\"] = " + str(v["db2"]))
print("s[\"dW1\"] = " + str(s["dW1"]))
print("s[\"db1\"] = " + str(s["db1"]))
print("s[\"dW2\"] = " + str(s["dW2"]))
print("s[\"db2\"] = " + str(s["db2"]))
```

```
v["dW1"] = [[0. 0. 0.]
 [0. 0. 0.]]
v["db1"] = [[0.]
 [0.]]
v["dW2"] = [[0. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]]
v["db2"] = [[0.]
 [0.]
 [0.]]
s["dW1"] = [[0. 0. 0.]
 [0. 0. 0.]]
s["db1"] = [[0.]
 [0.]]
s["dW2"] = [[0. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]]
s["db2"] = [[0.]
 [0.]
 [0.]]
```

```
# GRADED FUNCTION: update_parameters_with_adam

def update_parameters_with_adam(parameters, grads, v, s, t, learning_rate = 0.01,
                                beta1 = 0.9, beta2 = 0.999,  epsilon = 1e-8):
    """
    Update parameters using Adam

    Arguments:
    parameters -- python dictionary containing your parameters:
                    parameters['W' + str(l)] = Wl
                    parameters['b' + str(l)] = bl
    grads -- python dictionary containing your gradients for each parameters:
                    grads['dW' + str(l)] = dWl
                    grads['db' + str(l)] = dbl
    v -- Adam variable, moving average of the first gradient, python dictionary
    s -- Adam variable, moving average of the squared gradient, python dictionary
    learning_rate -- the learning rate, scalar.
    beta1 -- Exponential decay hyperparameter for the first moment estimates
    beta2 -- Exponential decay hyperparameter for the second moment estimates
    epsilon -- hyperparameter preventing division by zero in Adam updates
```

```
    Returns:
    parameters -- python dictionary containing your updated parameters
    v -- Adam variable, moving average of the first gradient, python dictionary
    s -- Adam variable, moving average of the squared gradient, python dictionary
    """

    L = len(parameters) // 2                 # number of layers in the neural networks
    v_corrected = {}                         # Initializing first moment estimate, python dictionary
    s_corrected = {}                         # Initializing second moment estimate, python dictionary

    # Perform Adam update on all parameters
    for l in range(L):
        # Moving average of the gradients. Inputs: "v, grads, beta1". Output: "v".
        ### START CODE HERE ### (approx. 2 lines)
        v["dW" + str(l+1)] = beta1*v["dW" +str(l+1)]+(1-beta1)*grads['dW' + str(l+1)]
        v["db" + str(l+1)] = beta1*v["db" +str(l+1)]+(1-beta1)*grads['db' + str(l+1)]
        ### END CODE HERE ###

        # Compute bias-corrected first moment estimate. Inputs: "v, beta1, t". Output: "v_corrected".
        ### START CODE HERE ### (approx. 2 lines)
        v_corrected["dW" + str(l+1)] = v["dW" +str(l+1)]/(1-math.pow(beta1,t))

        v_corrected["db" + str(l+1)] = v["db" +str(l+1)]/(1-math.pow(beta1,t))
        ### END CODE HERE ###

        # Moving average of the squared gradients. Inputs: "s, grads, beta2". Output: "s".
        ### START CODE HERE ### (approx. 2 lines)
        s["dW" + str(l+1)] = beta2*s["dW" +str(l+1)]+(1-beta2)*(grads['dW' + str(l+1)]**2)

        s["db" + str(l+1)] = beta2*s["db" +str(l+1)]+(1-beta2)*(grads['db' + str(l+1)]**2)
        ### END CODE HERE ###

        # Compute bias-corrected second raw moment estimate. Inputs: "s, beta2, t". Output: "s_corrected".
        ### START CODE HERE ### (approx. 2 lines)
        s_corrected["dW" + str(l+1)] = s["dW" +str(l+1)]/(1-math.pow(beta2,t))
        s_corrected["db" + str(l+1)] = s["db" +str(l+1)]/(1-math.pow(beta2,t))
        ### END CODE HERE ###

        # Update parameters. Inputs: "parameters, learning_rate, v_corrected, s_corrected, epsilon". Output: "parameters".
        ### START CODE HERE ### (approx. 2 lines)
        parameters["W" + str(l+1)] = parameters["W" +str(l+1)]-learning_rate * v_corrected["dW" +str(l+1)]/(np.sqrt(s_corrected["dW" + str(l+

        parameters["b" + str(l+1)] = parameters["b" +str(l+1)]-learning_rate * v_corrected["db" +str(l+1)]/(np.sqrt(s_corrected["db" + str(l+


        ### END CODE HERE ###

    return parameters, v, s
```

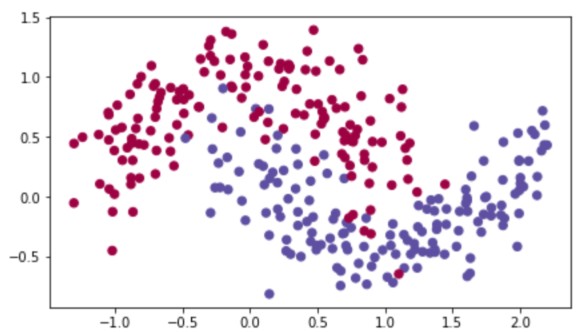
```
parameters, grads, v, s = update_parameters_with_adam_test_case()
parameters, v, s  = update_parameters_with_adam(parameters, grads, v, s, t = 2)

print("W1 = " + str(parameters["W1"]))
print("b1 = " + str(parameters["b1"]))
print("W2 = " + str(parameters["W2"]))
print("b2 = " + str(parameters["b2"]))
print("v[\"dW1\"] = " + str(v["dW1"]))
print("v[\"db1\"] = " + str(v["db1"]))
print("v[\"dW2\"] = " + str(v["dW2"]))
print("v[\"db2\"] = " + str(v["db2"]))
print("s[\"dW1\"] = " + str(s["dW1"]))
print("s[\"db1\"] = " + str(s["db1"]))
print("s[\"dW2\"] = " + str(s["dW2"]))
print("s[\"db2\"] = " + str(s["db2"]))
```

```
W1 = [[ 1.63178673 -0.61919778 -0.53561312]
 [-1.08040999  0.85796626 -2.29409733]]
b1 = [[ 1.75225313]
 [-0.75376553]]
W2 = [[ 0.32648046 -0.25681174  1.46954931]
 [-2.05269934 -0.31497584 -0.37661299]
 [ 1.14121081 -1.09244991 -0.16498684]]
b2 = [[-0.88529979]
 [ 0.03477238]
 [ 0.57537385]]
v["dW1"] = [[-0.11006192  0.11447237  0.09015907]
```

```
 [ 0.05024943  0.09008559 -0.06837279]]
v["db1"] = [[-0.01228902]
 [-0.09357694]]
v["dW2"] = [[-0.02678881  0.05303555 -0.06916608]
 [-0.03967535 -0.06871727 -0.08452056]
 [-0.06712461 -0.00126646 -0.11173103]]
v["db2"] = [[0.02344157]
 [0.16598022]
 [0.07420442]]
s["dW1"] = [[0.00121136 0.00131039 0.00081287]
 [0.0002525  0.00081154 0.00046748]]
s["db1"] = [[1.51020075e-05]
 [8.75664434e-04]]
s["dW2"] = [[7.17640232e-05 2.81276921e-04 4.78394595e-04]
 [1.57413361e-04 4.72206320e-04 7.14372576e-04]
 [4.50571368e-04 1.60392066e-07 1.24838242e-03]]
s["db2"] = [[5.49507194e-05]
 [2.75494327e-03]
 [5.50629536e-04]]
```

5 - Model with different optimization algorithms

```
train_X, train_Y = load_dataset()
```



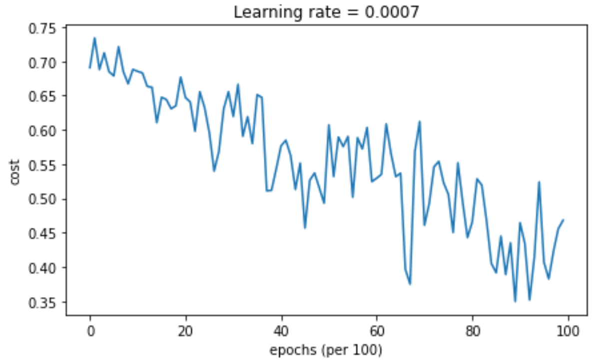
```
def model(X, Y, layers_dims, optimizer, learning_rate = 0.0007, mini_batch_size = 64, beta = 0.9,
          beta1 = 0.9, beta2 = 0.999,  epsilon = 1e-8, num_epochs = 10000, print_cost = True):
    """
    3-layer neural network model which can be run in different optimizer modes.

    Arguments:
    X -- input data, of shape (2, number of examples)
    Y -- true "label" vector (1 for blue dot / 0 for red dot), of shape (1, number of examples)
    layers_dims -- python list, containing the size of each layer
    learning_rate -- the learning rate, scalar.
    mini_batch_size -- the size of a mini batch
    beta -- Momentum hyperparameter
    beta1 -- Exponential decay hyperparameter for the past gradients estimates
    beta2 -- Exponential decay hyperparameter for the past squared gradients estimates
    epsilon -- hyperparameter preventing division by zero in Adam updates
    num_epochs -- number of epochs
    print_cost -- True to print the cost every 1000 epochs

    Returns:
    parameters -- python dictionary containing your updated parameters
    """

    L = len(layers_dims)             # number of layers in the neural networks
    costs = []                       # to keep track of the cost
    t = 0                            # initializing the counter required for Adam update
    seed = 10                        # For grading purposes, so that your "random" minibatches are the same as ours

    # Initialize parameters
    parameters = initialize_parameters(layers_dims)

    # Initialize the optimizer
    if optimizer == "gd":
        pass # no initialization required for gradient descent
    elif optimizer == "momentum":
        v = initialize_velocity(parameters)
    elif optimizer == "adam":
        v, s = initialize_adam(parameters)
```

```
    # Optimization loop
    for i in range(num_epochs):

        # Define the random minibatches. We increment the seed to reshuffle differently the dataset after each epoch
        seed = seed + 1
        minibatches = random_mini_batches(X, Y, mini_batch_size, seed)

        for minibatch in minibatches:

            # Select a minibatch
            (minibatch_X, minibatch_Y) = minibatch

            # Forward propagation
            a3, caches = forward_propagation(minibatch_X, parameters)

            # Compute cost
            cost = compute_cost(a3, minibatch_Y)

            # Backward propagation
            grads = backward_propagation(minibatch_X, minibatch_Y, caches)

            # Update parameters
            if optimizer == "gd":
                parameters = update_parameters_with_gd(parameters, grads, learning_rate)
            elif optimizer == "momentum":
                parameters, v = update_parameters_with_momentum(parameters, grads, v, beta, learning_rate)
            elif optimizer == "adam":
                t = t + 1 # Adam counter
                parameters, v, s = update_parameters_with_adam(parameters, grads, v, s,
                                                               t, learning_rate, beta1, beta2,  epsilon)

        # Print the cost every 1000 epoch
        if print_cost and i % 1000 == 0:
            print ("Cost after epoch %i: %f" %(i, cost))
        if print_cost and i % 100 == 0:
            costs.append(cost)

    # plot the cost
    plt.plot(costs)
    plt.ylabel('cost')
    plt.xlabel('epochs (per 100)')
    plt.title("Learning rate = " + str(learning_rate))
    plt.show()

    return parameters
```

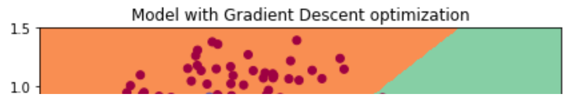
5.1 - Mini-batch Gradient descent

```
# train 3-layer model
layers_dims = [train_X.shape[0], 5, 2, 1]
parameters = model(train_X, train_Y, layers_dims, optimizer = "gd")

# Predict
predictions = predict(train_X, train_Y, parameters)

# Plot decision boundary
plt.title("Model with Gradient Descent optimization")
axes = plt.gca()
axes.set_xlim([-1.5,2.5])
axes.set_ylim([-1,1.5])
plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X, train_Y)
```

```
Cost after epoch 0: 0.690736
Cost after epoch 1000: 0.685273
Cost after epoch 2000: 0.647072
Cost after epoch 3000: 0.619525
Cost after epoch 4000: 0.576584
Cost after epoch 5000: 0.607243
Cost after epoch 6000: 0.529403
Cost after epoch 7000: 0.460768
Cost after epoch 8000: 0.465586
Cost after epoch 9000: 0.464518
```




```
Accuracy: 0.7966666666666666
```



5.2 - Mini-batch gradient descent with momentum

```
# train 3-layer model
layers_dims = [train_X.shape[0], 5, 2, 1]
parameters = model(train_X, train_Y, layers_dims, beta = 0.9, optimizer = "momentum")

# Predict
predictions = predict(train_X, train_Y, parameters)

# Plot decision boundary
plt.title("Model with Momentum optimization")
axes = plt.gca()
axes.set_xlim([-1.5,2.5])
axes.set_ylim([-1,1.5])
plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X, train_Y)
```

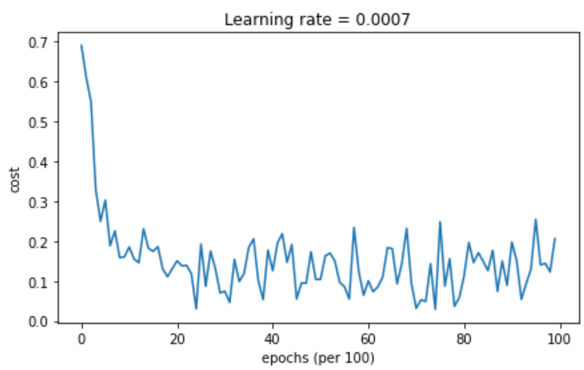
```
Cost after epoch 0: 0.690741
Cost after epoch 1000: 0.685341
Cost after epoch 2000: 0.647145
Cost after epoch 3000: 0.619594
Cost after epoch 4000: 0.576665
Cost after epoch 5000: 0.607324
Cost after epoch 6000: 0.529476
Cost after epoch 7000: 0.460936
Cost after epoch 8000: 0.465780
Cost after epoch 9000: 0.464740
```

Learning rate = 0.0007

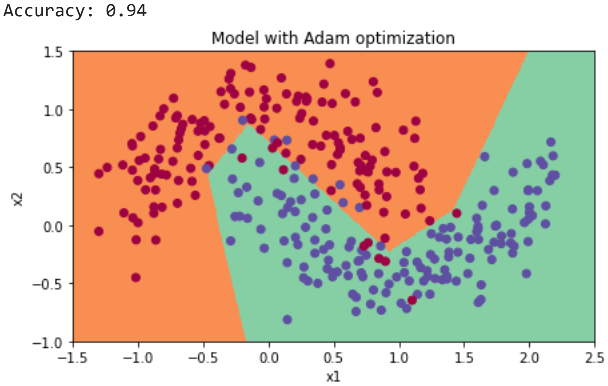
5.3 - Mini-batch with Adam mode

```
# train 3-layer model
layers_dims = [train_X.shape[0], 5, 2, 1]
parameters = model(train_X, train_Y, layers_dims, optimizer = "adam")

# Predict
predictions = predict(train_X, train_Y, parameters)

# Plot decision boundary
plt.title("Model with Adam optimization")
axes = plt.gca()
axes.set_xlim([-1.5,2.5])
axes.set_ylim([-1,1.5])
plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X, train_Y)
```

```
Cost after epoch 0: 0.690552
Cost after epoch 1000: 0.185567
Cost after epoch 2000: 0.150852
Cost after epoch 3000: 0.074454
Cost after epoch 4000: 0.125936
Cost after epoch 5000: 0.104235
Cost after epoch 6000: 0.100552
Cost after epoch 7000: 0.031601
Cost after epoch 8000: 0.111709
Cost after epoch 9000: 0.197648
```




```
Accuracy: 0.94
```



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