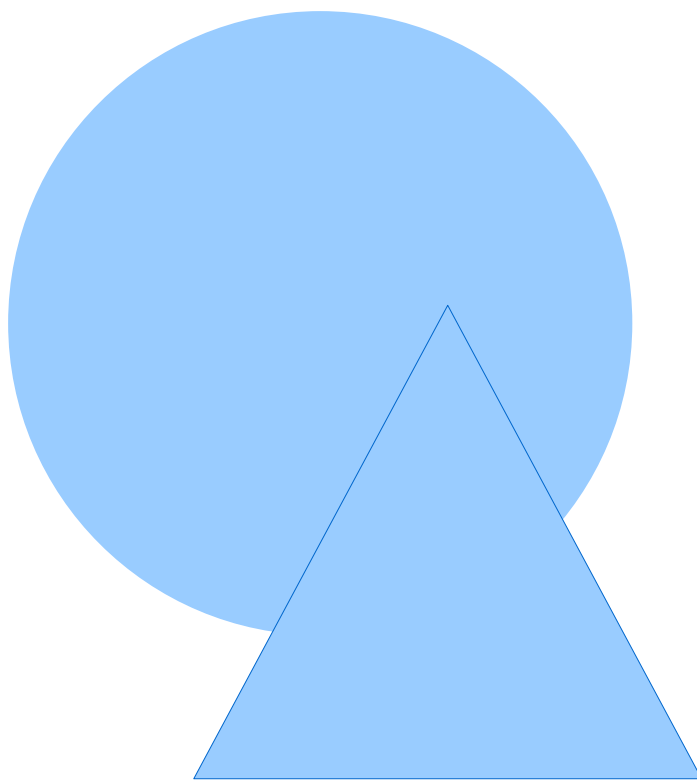



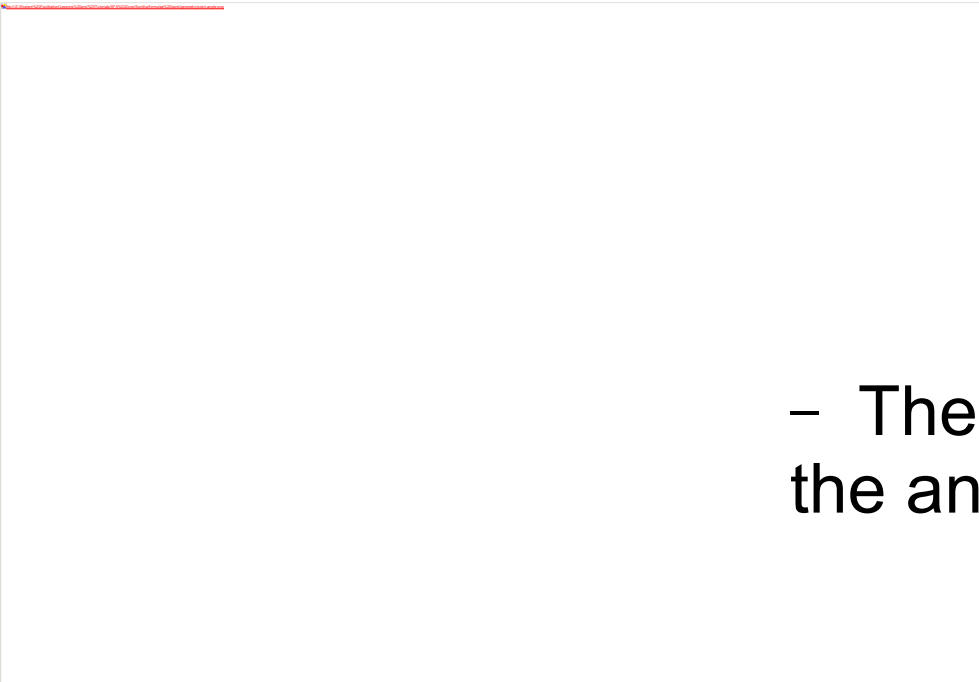
Geometry Formulae



Basic Geometry

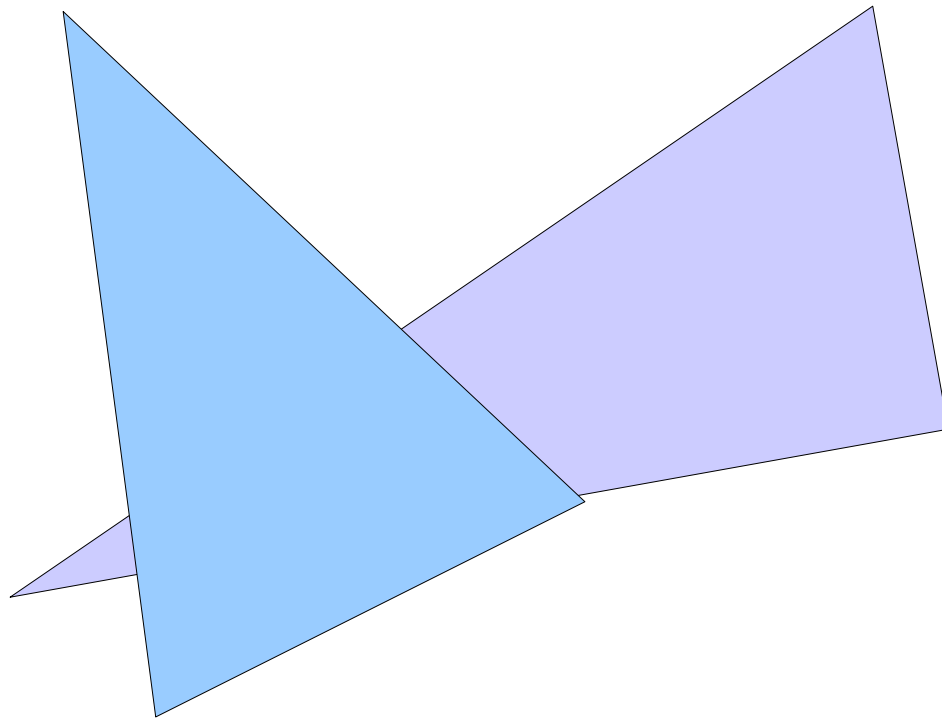
Basic Geometry

- 
- If two or more angles form a straight angle, the **sum** of their measures is 180°

- 
- The sum of all the measures of all the angles around **a point** is 360°

Parallel Lines

- If a pair of straight lines is cut by a transversal that is not perpendicular to the parallel lines, then
- **Vertically opposite angles** are $a=b$, $c=d$, $e=f$, $g=h$
- **Corresponding angles** are $a=e$, $c=g$, $d=h$, $b=f$
- **Alternate interior angles** are $c=h$, $e=b$
- **Alternate exterior angles** are $a=f$, $d=g$
- **Supplementary angle pairs** are $c+e = b+h = 180$



Triangles

Triangle

- In any triangle, the **sum** of the measures of the three angles is 180°
- The measure of the **exterior angle** of the triangle is equal to the sum of the measure of the **two opposite interior** angles
- In any **right triangle**, the sum of the measures of the two acute angles is 90°

30-60-90 Triangle

In most GRE geometry problems, you need not know Trigonometry.

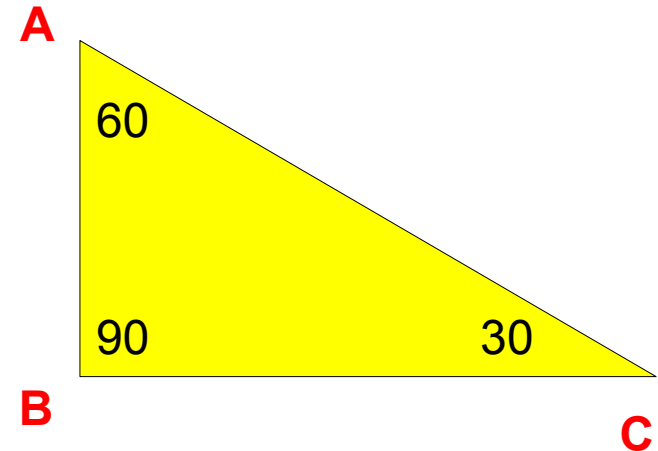
A few equivalent concepts can help you solve them.

In many problems, the triangles turn out to be a right triangle with one of the angles as 30 or 60. You have to immediately register that this is a

30-60-90 Triangle

and check if there is an opportunity for you to apply the following rule.

This rule helps you to determine length of 2 sides of the Triangle, if you know just one



The rule says that: In a 30-60-90 triangle (one shown above),
The ratio of the

The length of side opposite to 30° : The length of side opposite to 60° :
The length of side opposite to 90°

$$= 1 : \sqrt{3} : 2 = AB : BC : AC$$

45-45-90 Triangle

Sometimes, the triangles turn out to be an isosceles right triangle with one of the angles as 45.

You have to immediately register that this is a 45-45-90 Triangle

and check if there is an opportunity for you to apply the following rule.

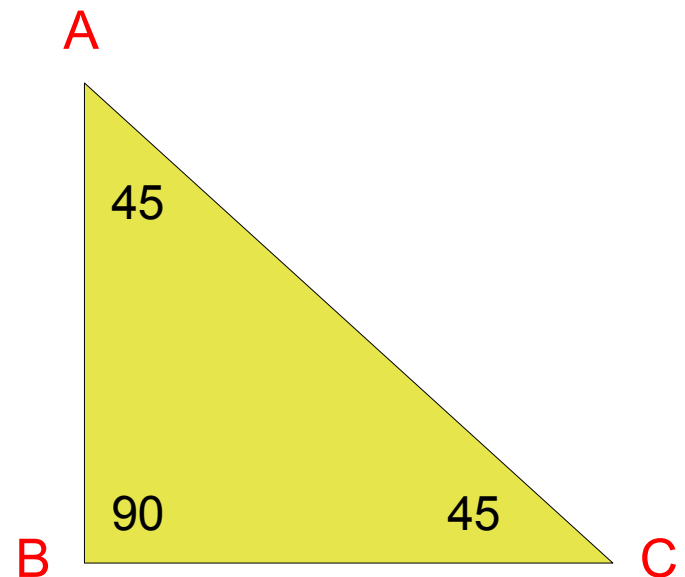
This rule helps you to determine length of the sides of the Triangle, if you know just one

The rule says that: In a 45-45-90 triangle (shown triangle),

The ratio of the

The length of side opposite to 45° : The length of side opposite to 45° : The length of side opposite to 90°

$$= 1 : 1 : \sqrt{2} = AB : BC : AC$$



Triangle

- An **altitude** divides an equilateral triangle into two 30-60-90 triangles
- The sum of the lengths of any two sides of a triangle is **greater** than the length of the third side
- The difference of the lengths of any two sides of a triangle is **less** than the length of the third side
- The area of the triangle is given by **$A = \frac{1}{2} b \cdot h$** , where b is the base of the triangle and h is the height of the triangle
- The area of an **equilateral triangle** with side s is given by

$$A = s^2 \sqrt{3}/4$$

Centroid

- Centroid is the **meeting point** of the medians drawn from the vertex to the mid-point of the opposite side of the triangle

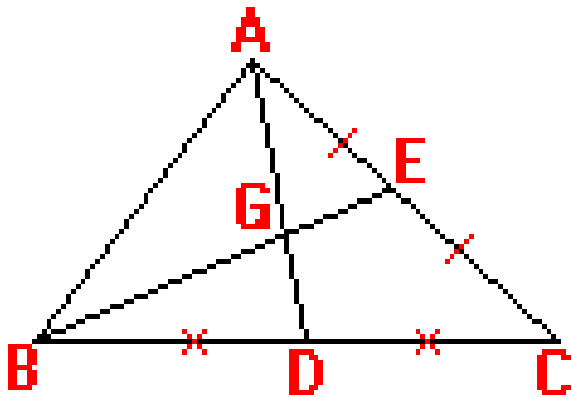
- Centroid divides the median in the ratio **1 : 2**

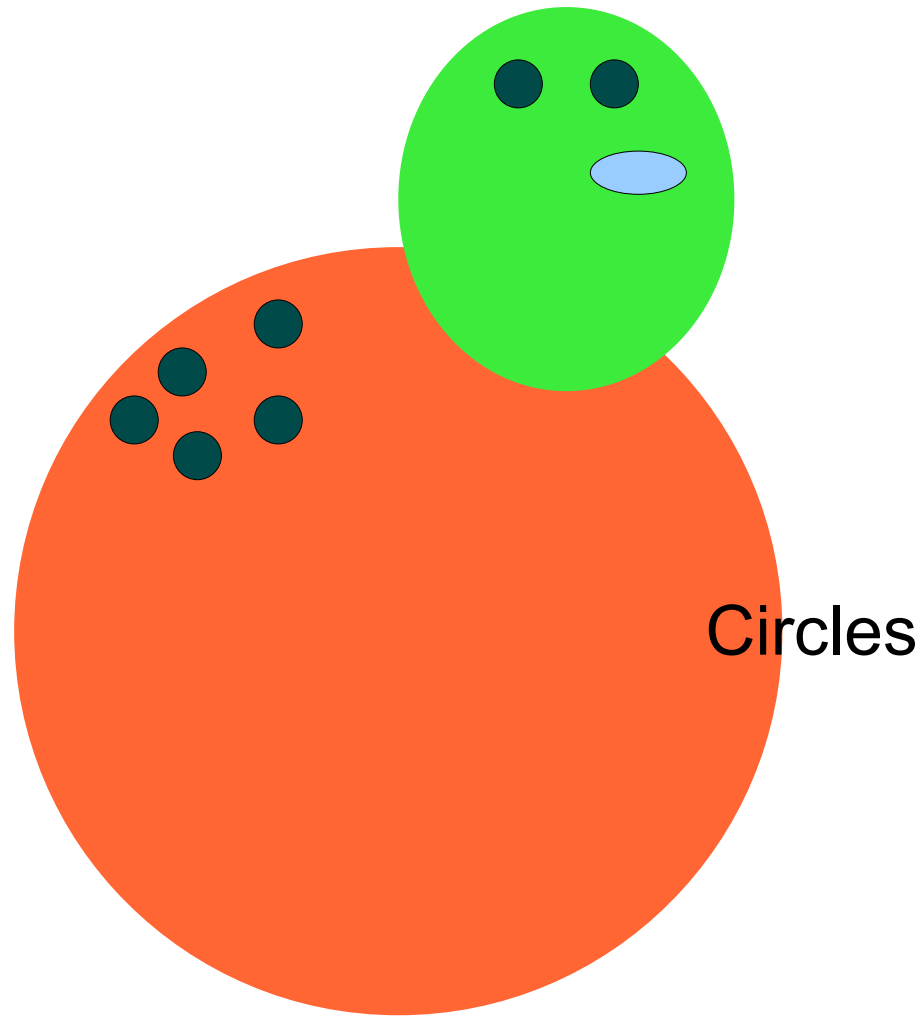
- Thus in the adjoining figure $GE/BE = 1/3$

- or $GE/BG = 1/2$

- Similarly, $GD/AD = 1/3$

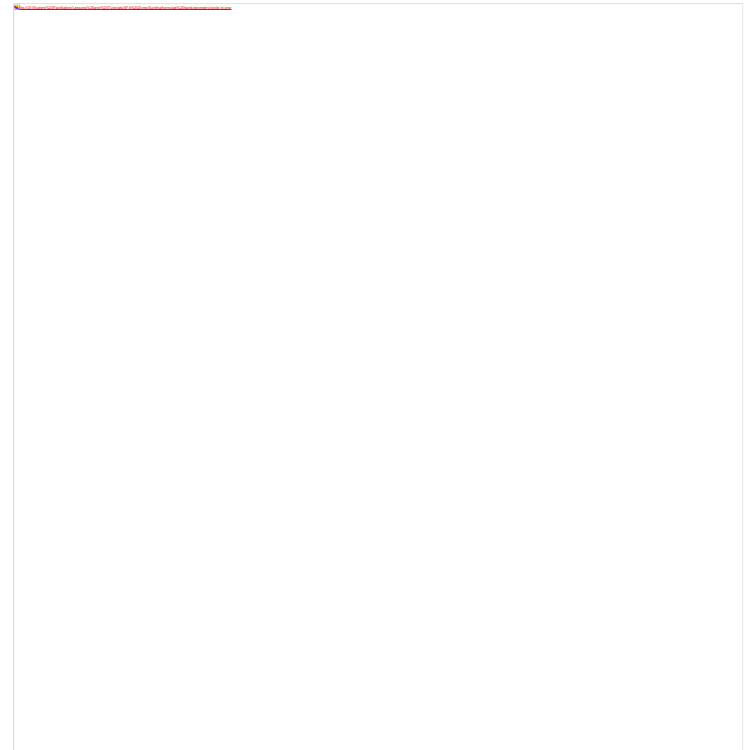
- or $GD/AG = 1/2$





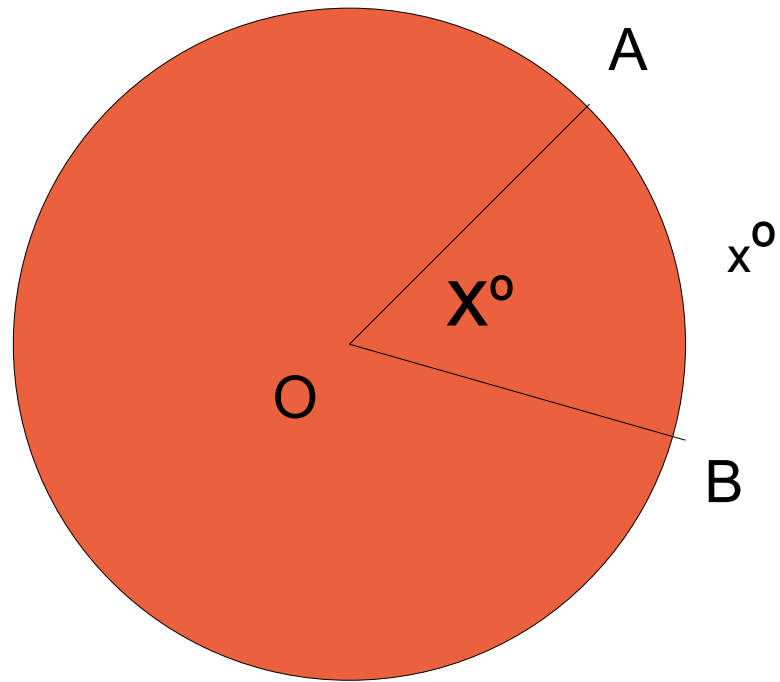
Circles

- Any triangle formed by connecting the endpoints of two radii, is **isosceles**. Here $\angle OPQ = \angle OQP$
- Circumference = $2\pi r = \pi d$, where d is the diameter of the circle
- Area = πr^2
- Circumference of a **semi circle** = $\pi r + d$



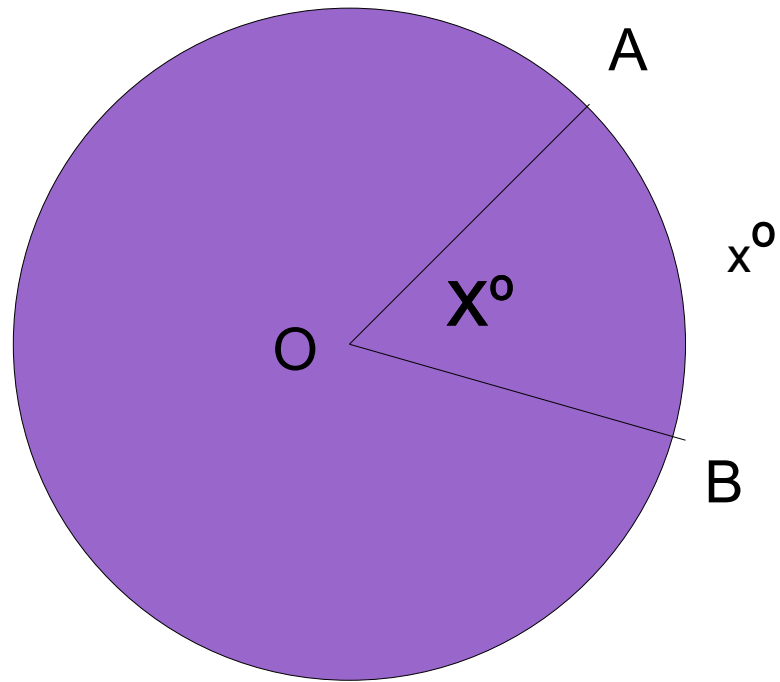
Arc of a circle

- Degree measure of a complete circle is 360°
- The degree measure of an arc $AB = x^\circ$



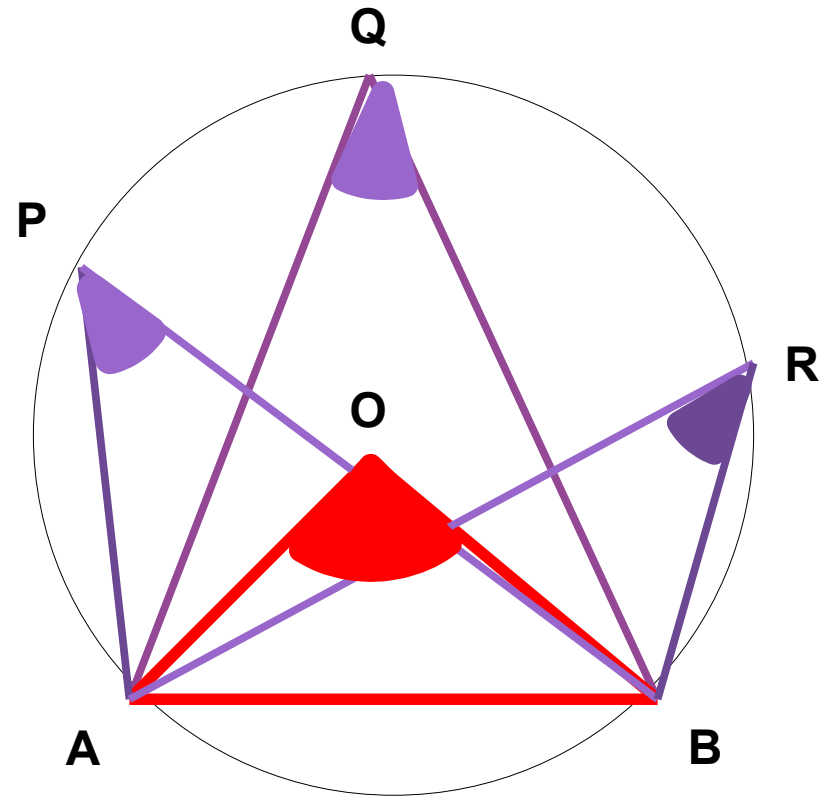
Arcs on a circle

- Length of a Arc AB / Circumference = $x^\circ/360^\circ$
- Area of a sector AOB / Area of the Circle = $x^\circ/360^\circ$



Angle Properties

- The angle subtended by a chord at the centre of the circle is **twice the angle** subtended by the Chord AB on the circle.
- Note that the angles (in violet) on the arc AQB are all equal angles
- If $\angle AOB = 2x^\circ$, then $\angle APB = x^\circ$;
 $\angle AQB = x^\circ$; $\angle ARB = x^\circ$



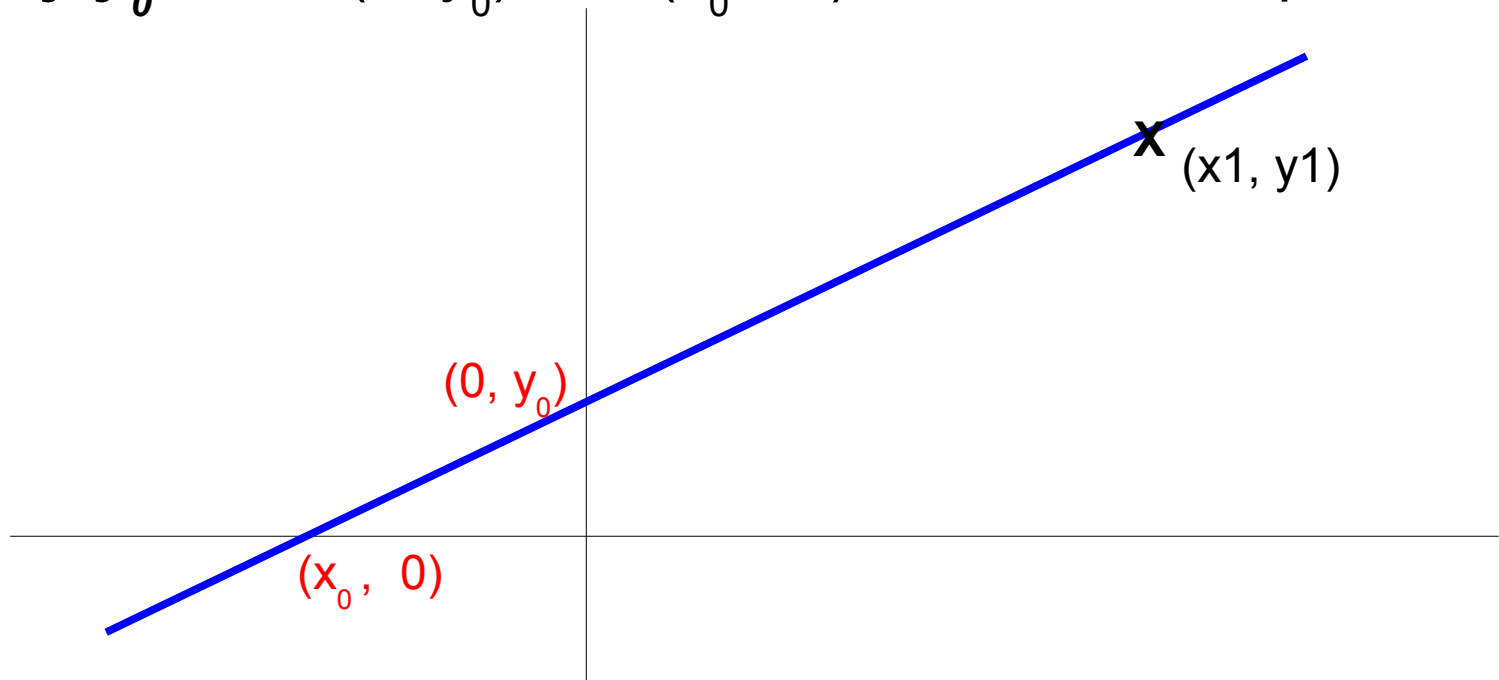
Coordinate Geometry

- The distance, d , between two given points, $A(x_1, y_1)$ and $B(x_2, y_2)$, can be calculated using the **distance formula**, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Vertical lines **do not** have slopes
- Slope of any horizontal line is **0**
- Slope of a line when **two points** are given is

$$m = (y_2 - y_1) / (x_2 - x_1)$$

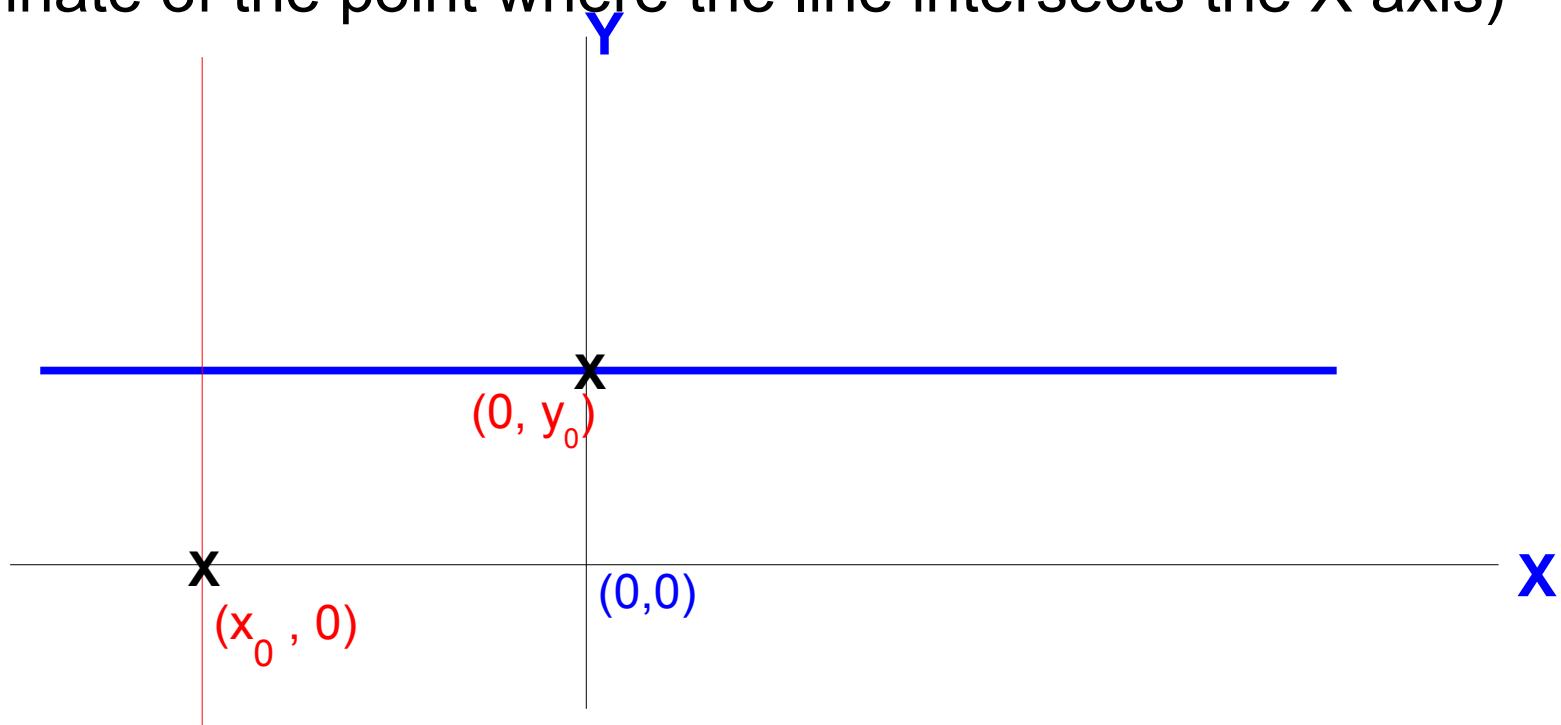
Equation of a line

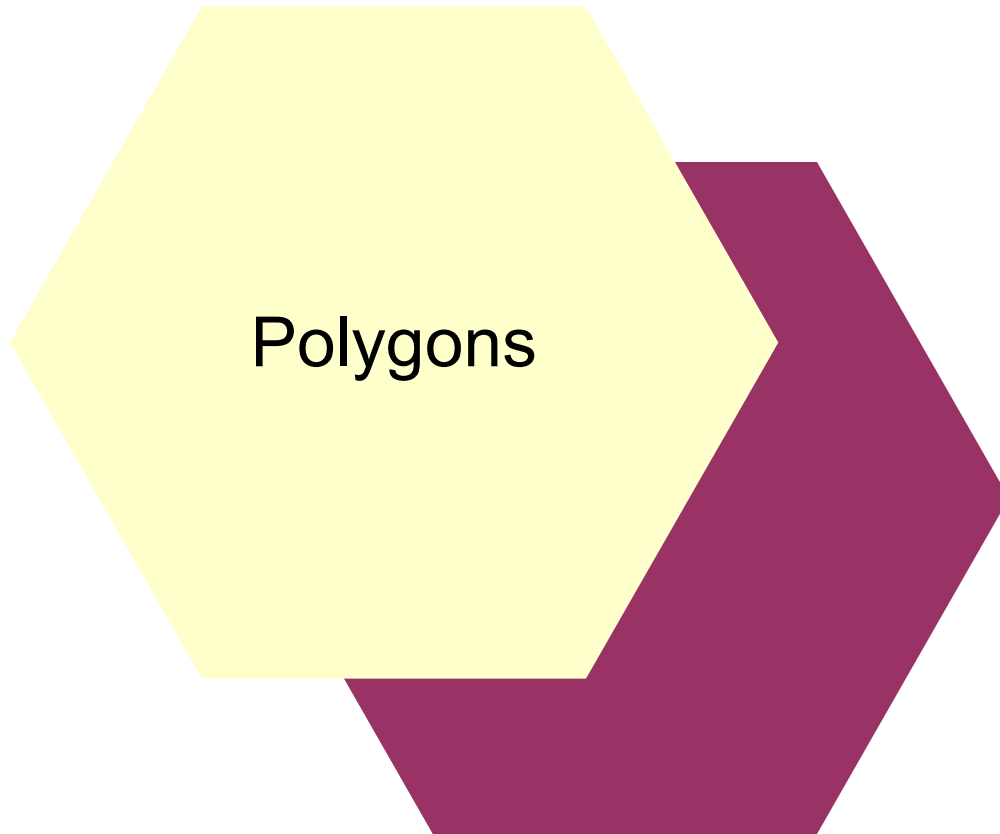
- Equation of line: $y = mx + b$
- $(y - y_1) / (x - x_1) = m$, if (x_1, y_1) is a point on the line
- $x/x_0 + y/y_0 = 1$, if $(0, y_0)$ and $(x_0, 0)$ are the intercepts



Equation of a line

- Equation of a line **parallel to X axis** is : $y = y_0$ (where y_0 is the Y co-ordinate of the point where the line intersects the Y axis)
- Equation of a line **parallel to Y axis** is : $x = x_0$ (where x_0 is the X co-ordinate of the point where the line intersects the X axis)





Polygon types

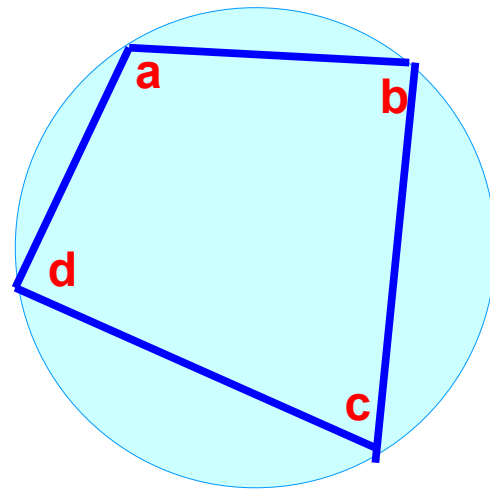
Name	Number of Sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nanogon	9
Decagon	10

Cyclic Polygon

- A convex Polygon is called a Cyclic Polygon, if all the vertices lie on a single circle
- Sum of opposite angles of a *Cyclic Quadrilateral* is 180°

$$a + c = 180$$

$$b + d = 180$$



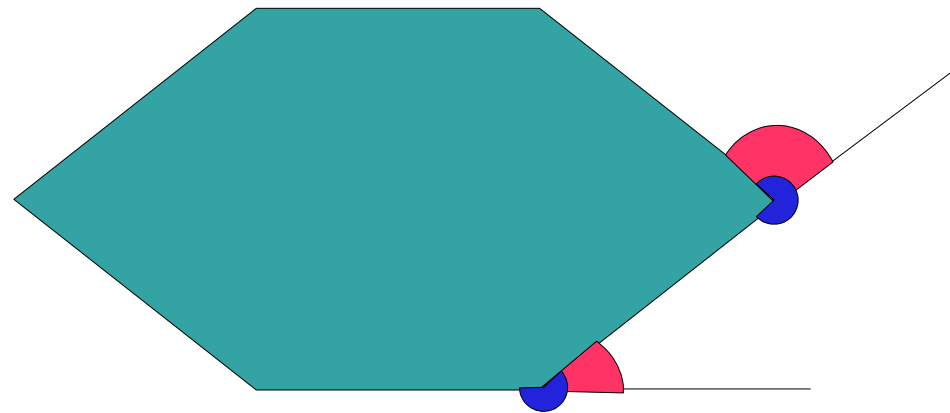
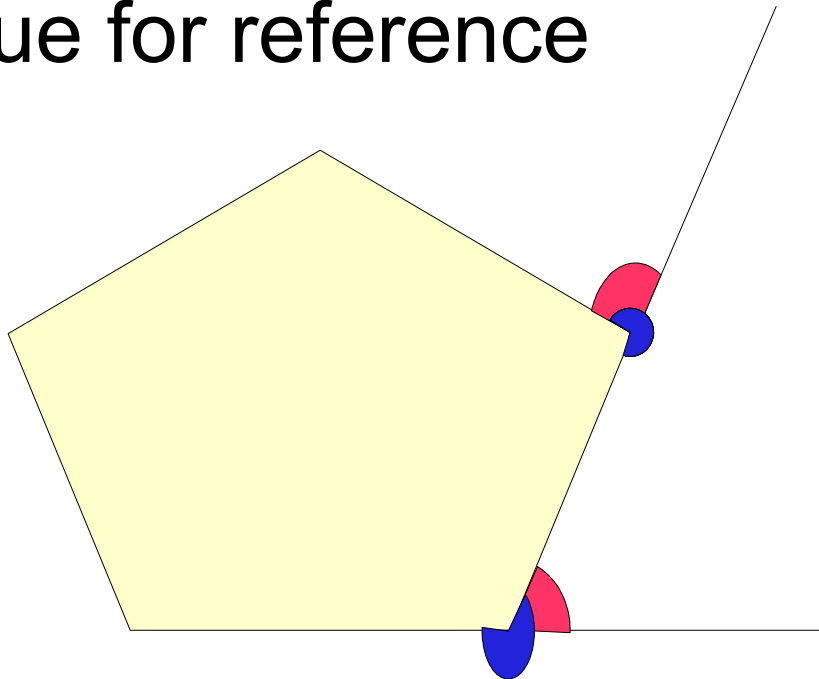
- A Regular polygon is a Cyclic Polygon whose sides are of Equal length

Formulae related to Polygons

- Sum of **Interior** Angles of a N sided Polygon =
 $(N - 2) \times 180^\circ$
- The interior angles of a Regular Polygon are **equal** to each other. The measure of an interior angle of a regular Polygon =
 $(N - 2) \times 180^\circ / N$
- Number of Diagonals of a N sides polygon =
 $N \times (N-3) / 2$

Angles

- Sum of **External** Angles of a N sided Polygon = $(n+2) * 180^\circ$
- The measure of each **Exterior** Angle is $360^\circ / N$
- The **External** angle is different and marked in blue for reference



Parallelogram

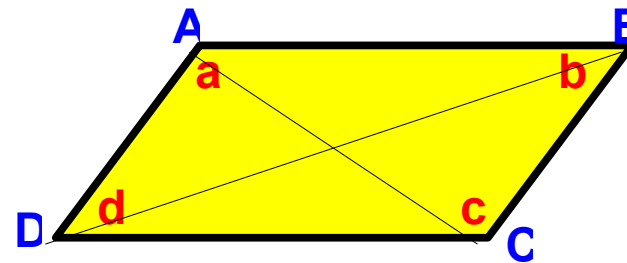
- $AB = DC$ and $AD = BC$

- $a = c$, $b = d$

- $a + b = 180^\circ$

$$c + d = 180^\circ$$

$$b + c = 180^\circ \text{ \& } a + d = 180^\circ$$

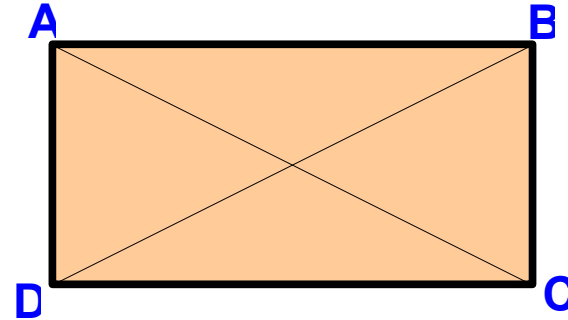


- Diagonals AC and BD bisect each other

- A diagonal divides the parallelogram into two **Congruent** triangles

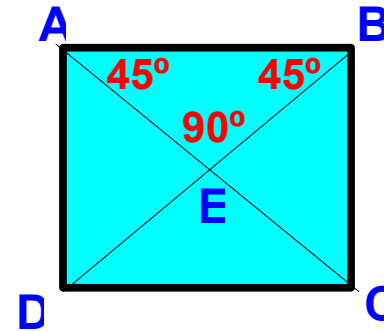
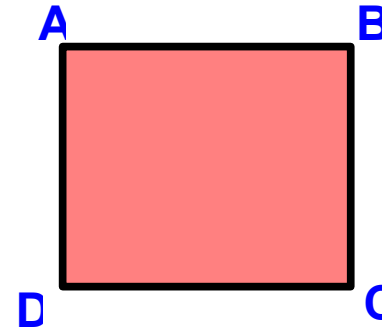
Rectangle

- $AB = DC$ and $AD = BC$
- Angles $a = c = b = d = 90^\circ$
- Diagonals AC and BD
bisect each other
- The diagonals of a rectangle have the **same length**, $AC = BD$



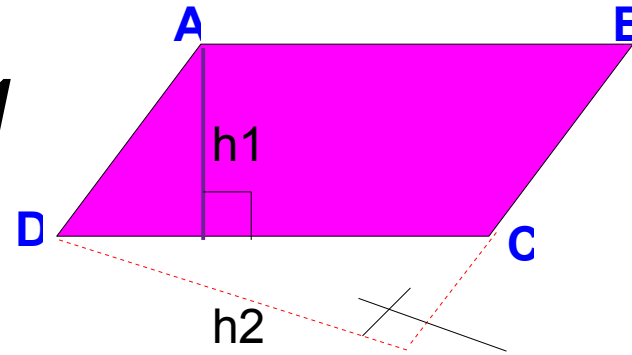
Square

- $AB = DC = AD = BC$
- $a = c = b = d = 90^\circ$
- Diagonals AC and BD bisect each other at right angles and are perpendicular to each other
- AEB, BEC, CED, DEA are $45^\circ-45^\circ-90^\circ$ triangles

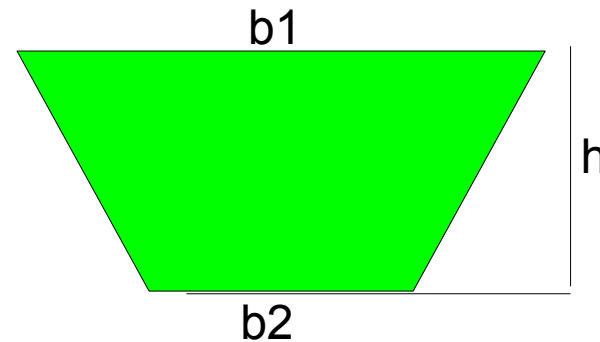


Area

- Parallelogram : $DC \times h1$
 $= BC \times h2$



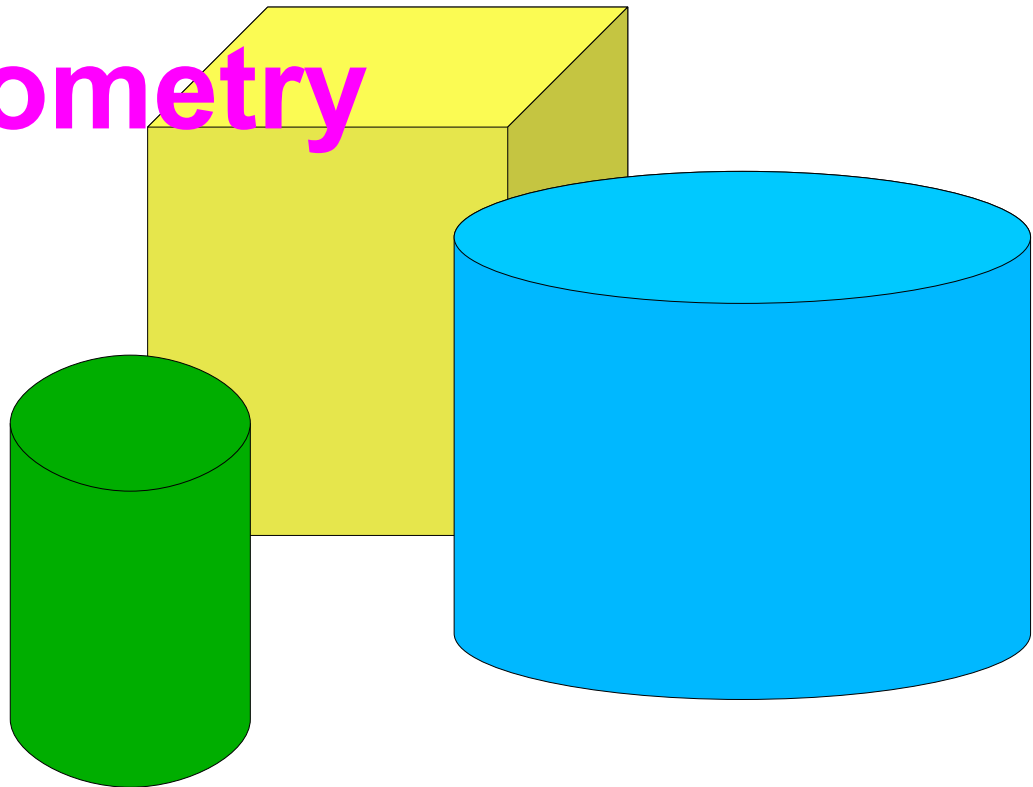
- Rectangle : $length \times breadth$
- Square : $(side)^2 = (diagonal)^2 / 2$
- Trapezium :
 $(\frac{1}{2}) (b1 + b2) \times h$



Some points to remember

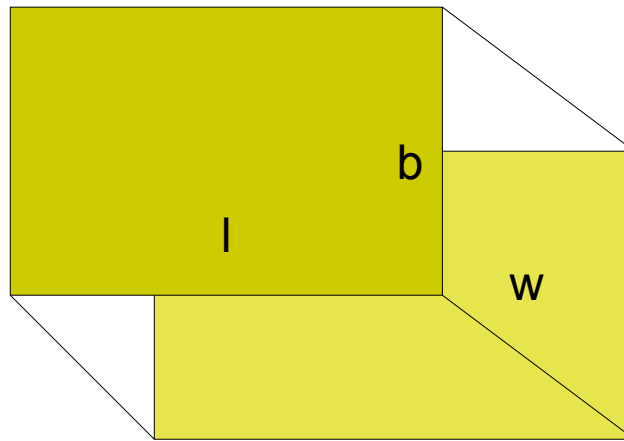
- For a given **Perimeter**, the rectangle with the **largest area** is a **square**.
- For a given area , the rectangle with the **smallest perimeter** is a **square**

Solid Geometry



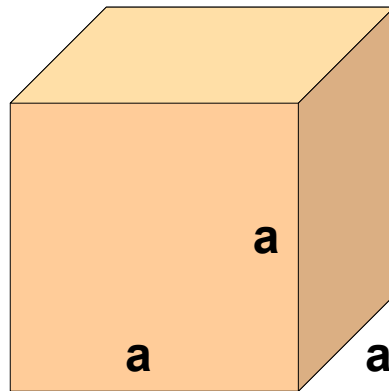
Cuboid

- The **volume** of a rectangular solid (cuboid), is $V = l.w.h$
- The **surface area** of a cuboid is $A = 2(lw + lh + wh)$



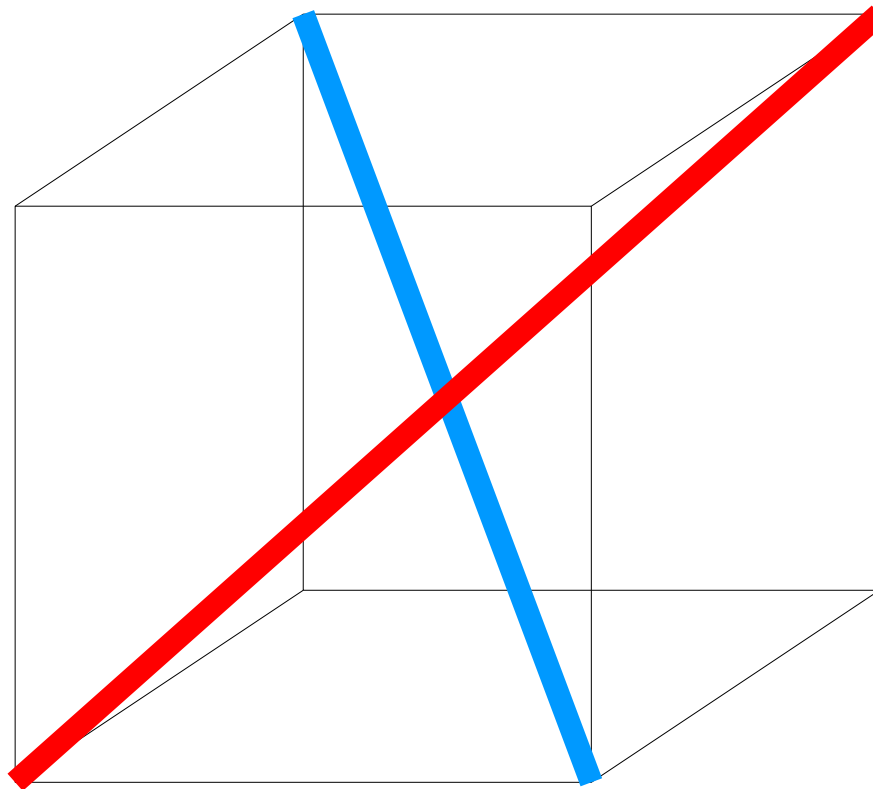
Cube

- The **volume** of the cube is $V = a.a.a = a^3$
- The **surface area** is $A = 6.a^2$



Diagonal

- A diagonal, d , of a box is the **longest line segment** that can be drawn between two points on the box, $d^2 = l^2 + w^2 + h^2$



Cylinder

- The **Volume of a Cylinder**, V whose circular base has radius r and height h is **$V = \pi r^2 h$**
- The surface area, A , of the cylinder is **$A = 2\pi rh$**

