Introduction to Portfolio Theory

Econ 424
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Portfolios of 2 Risky Assets

```
# Set asset return distribution parameters
> mu.A = 0.175
> sig.A = 0.258
> sig2.A = sig.A^2
> mu.B = 0.055
> sig.B = 0.115
> sig2.B = sig.B^2
> rho.AB = -0.164
> sig.AB = rho.AB*sig.A*sig.B
```

Example Portfolio 1: Long Only

```
# specify equally weighted portfolio
> x.A = 0.5
> x.B = 0.5

# compute portfolio mean, variance and sd
> mu.p1 = x.A*mu.A + x.B*mu.B
> sig2.p1 = x.A^2 * sig2.A + x.B^2 * sig2.B + 2*x.A*x.B*sig.AB
> sig.p1 = sqrt(sig2.p1)
> mu.p1
[1] 0.115

> sig2.p1
[1] 0.01751

> sig.p1
[1] 0.1323
```


Example Portfolio: Long-Short

```
# specify long-short portfolio
> x.A = 1.5
> x.B = -0.5
# compute portfolio mean, variance and sd
> mu.p2 = x.A*mu.A + x.B*mu.B
> sig2.p2 = x.A^2 * sig2.A + x.B^2 * sig2.B + 2*x.A*x.B*sig.AB
> sig.p2 = sqrt(sig2.p2)
> mu.p2
[1] 0.235

> sig2.p2
[1] 0.1604

> sig.p2
[1] 0.4005
```

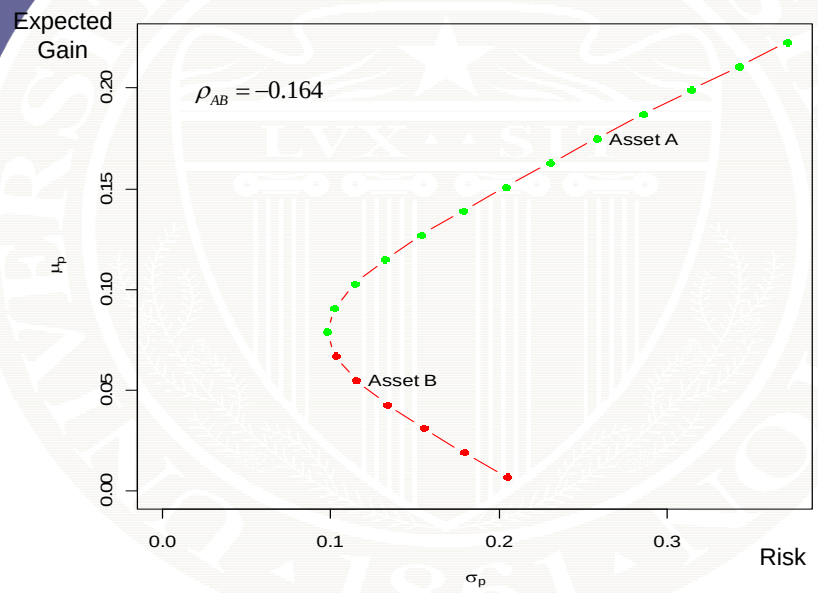
Asset and Portfolio VaR

```
> w0 = 100000
# Individual asset VaR
> VaR.A = (mu.A + sig.A*qnorm(0.05))*w0
> VaR.A
[1] -24937

> VaR.B = (mu.B + sig.B*qnorm(0.05))*w0
> VaR.B
[1] -13416

# Asset weighted VaR for equally weighted portfolio
> x.A*VaR.A + x.B*VaR.B
[1] -19177

# VaR for equally weighted portfolio
> VaR.p1 = (mu.p1 + sig.p1*qnorm(0.05))*w0
> VaR.p1
[1] -10268
```

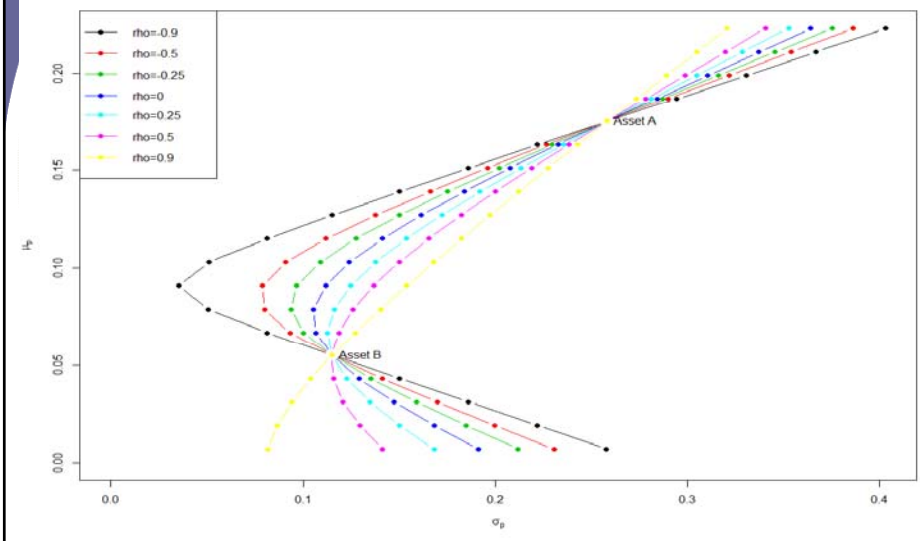

Create Portfolio Frontier

```
> x.A = seq(from=-0.4, to=1.4, by=0.1)
> x.B = 1 - x.A
> mu.p = x.A*mu.A + x.B*mu.B
> sig2.p = x.A^2 * sig2.A + x.B^2 * sig2.B +
+          2*x.A*x.B*sig.AB
> sig.p = sqrt(sig2.p)

> plot(sig.p, mu.p, type="b", pch=16,
+      ylim=c(0, max(mu.p)), xlim=c(0, max(sig.p)),
+      xlab=expression(sigma[p]),ylab=expression(mu[p]),
+      col=c(rep("red", 6), rep("green", 13)))
> text(x=sig.A, y=mu.A, labels="Asset A", pos=4)
> text(x=sig.B, y=mu.B, labels="Asset B", pos=4)
```

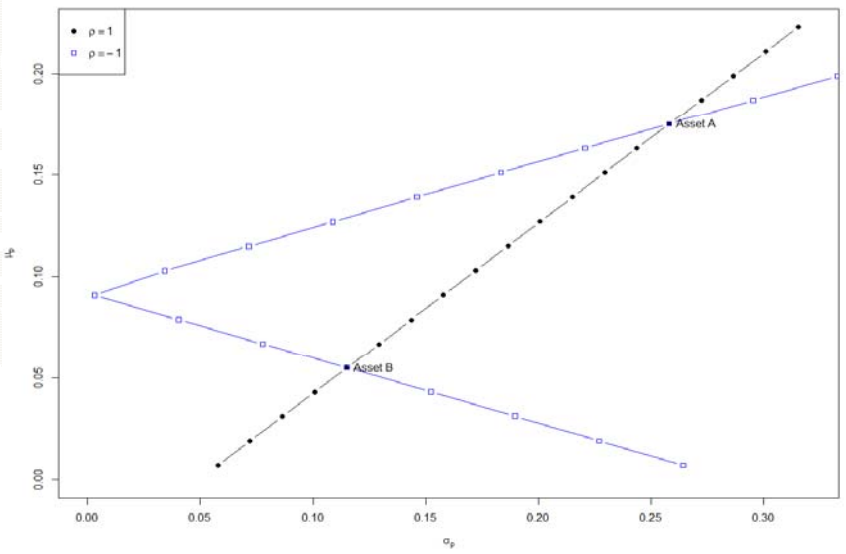
Portfolio Frontier



Portfolio Frontier as function of ρ



Portfolio Frontier as function of ρ

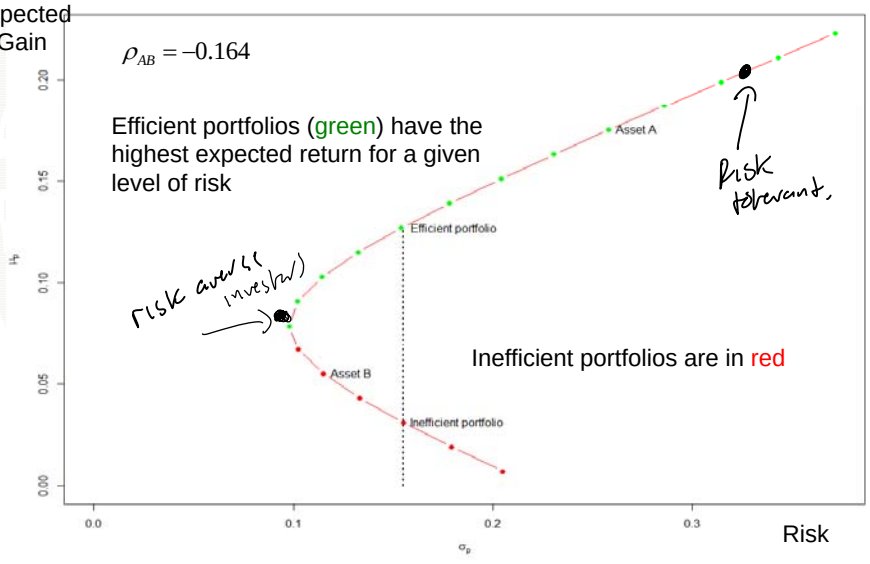


Efficient Portfolios

Expected Gain

$\rho_{AB} = -0.164$

Efficient portfolios (green) have the highest expected return for a given level of risk



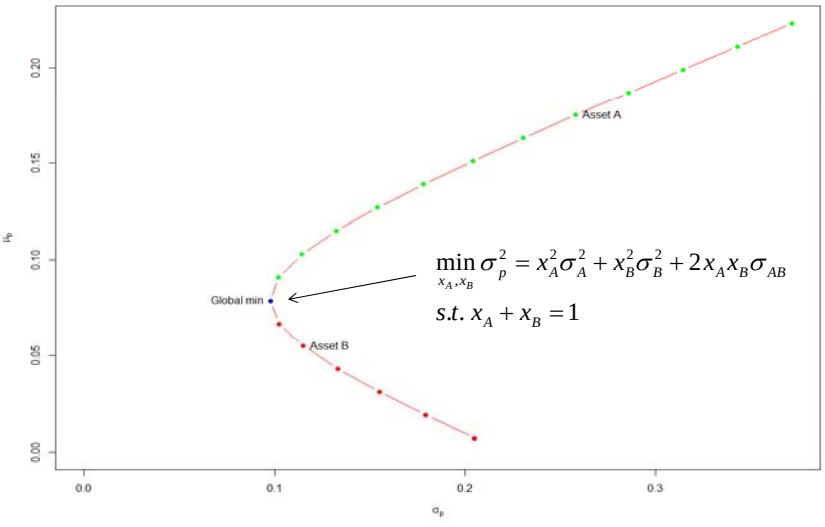
Inefficient portfolios are in red

Risk

Computing the Minimum Variance Portfolio

```
# minimum variance portfolio
> xA.min = (sig2.B - sig.AB)/(sig2.A + sig2.B - 2*sig.AB)
> xB.min = 1 - xA.min
> xA.min
[1] 0.2021

> xB.min
[1] 0.7979

# compute mean, variance and volatility
> mu.p.min = xA.min*mu.A + xB.min*mu.B
> sig2.p.min = xA.min^2 * sig2.A + xB.min^2 * sig2.B +
+              2*xA.min*xB.min*sig.AB
> sig.p.min = sqrt(sig2.p.min)
> mu.p.min
[1] 0.07925

> sig.p.min
[1] 0.09782
```


Global Minimum Variance Portfolio



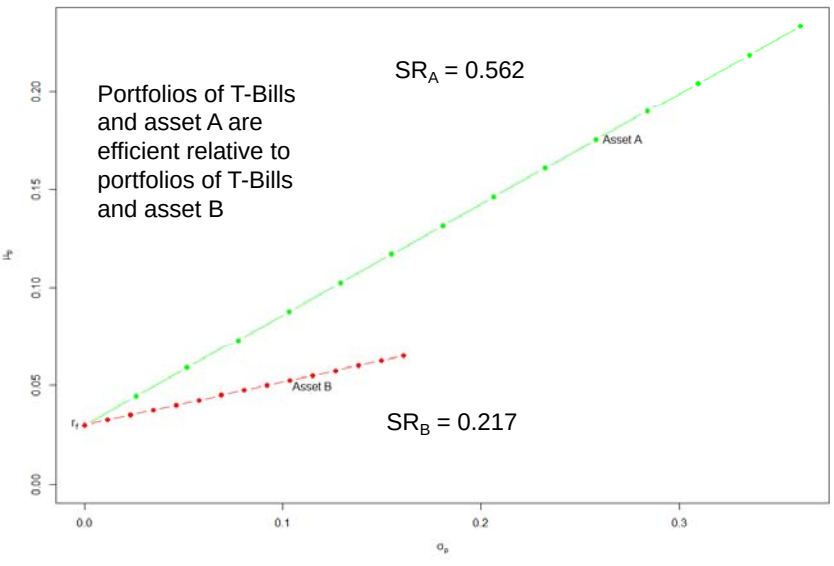
$$\min_{x_A, x_B} \sigma_p^2 = x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + 2x_A x_B \sigma_{AB}$$

$$s.t.\ x_A + x_B = 1$$

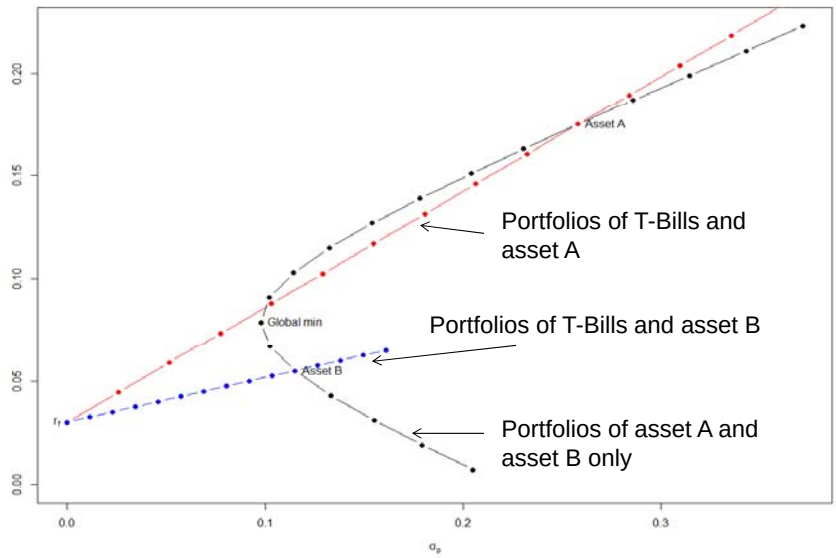
Portfolios of T-Bills and 1 Risky Asset

```
# Risk-free (T-Bill) rate
> r.f = 0.03

# T-bills + asset A
> x.A = seq(from=0, to=1.4, by=0.1)
> mu.p.A = r.f + x.A*(mu.A - r.f)
> sig.p.A = x.A*sig.A
> sharpe.A = (mu.A - r.f)/sig.A
> sharpe.A
[1] 0.562

# T-bills + asset B
> x.B = seq(from=0, to=1.4, by=0.1)
> mu.p.B = r.f + x.B*(mu.B - r.f)
> sig.p.B = x.B*sig.B
> sharpe.B = (mu.B - r.f)/sig.B
> sharpe.B
[1] 0.2174
```


Portfolios of T-Bills and 1 Risky Asset



Portfolios of T-Bills and 2 Risky Assets



Portfolios of T-Bills and asset A

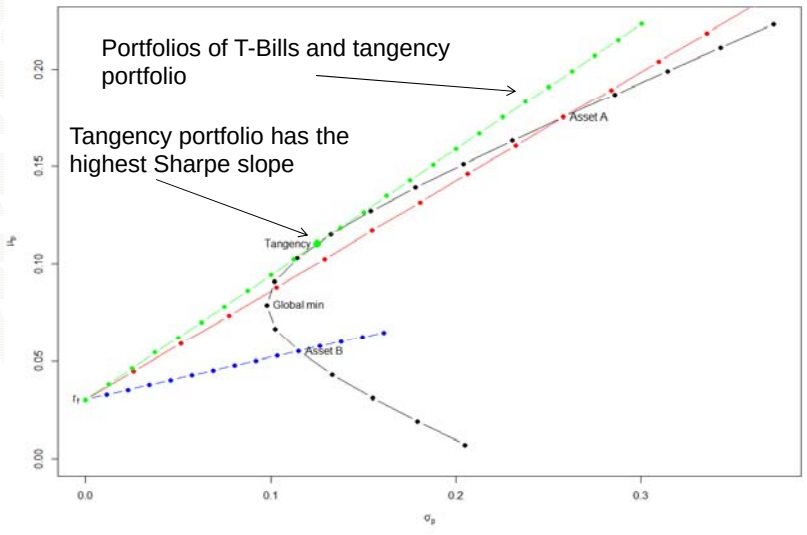
Portfolios of T-Bills and asset B

Portfolios of asset A and asset B only

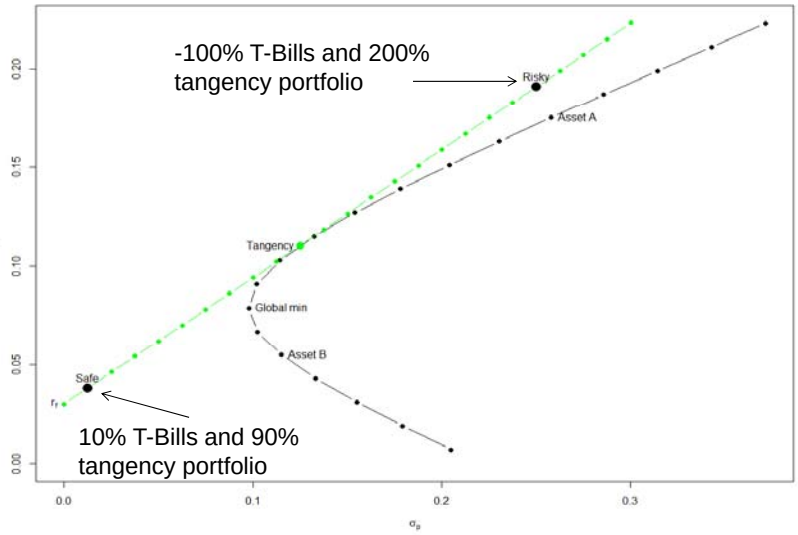
Portfolios of T-Bills and 2 Risky Assets

Portfolios of T-Bills and tangency portfolio

Tangency portfolio has the highest Sharpe slope



Efficient Portfolios and Risk Preferences



Interpreting Efficient Portfolios