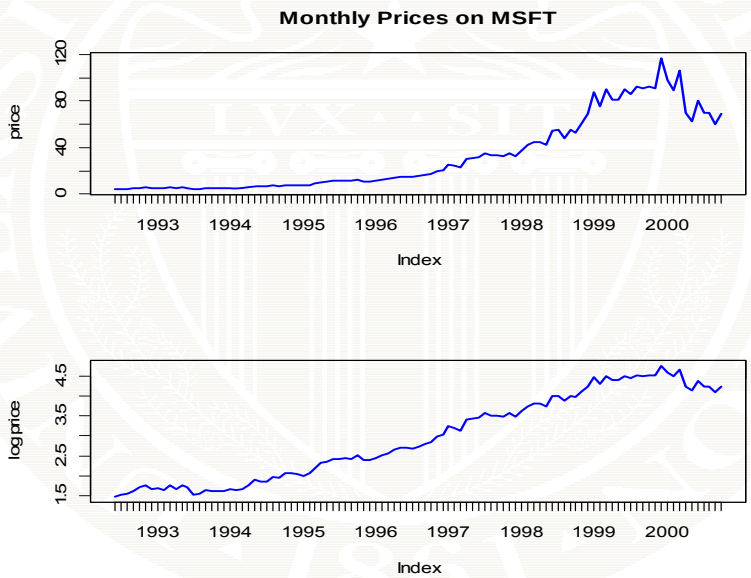
Constant Expected Return Model

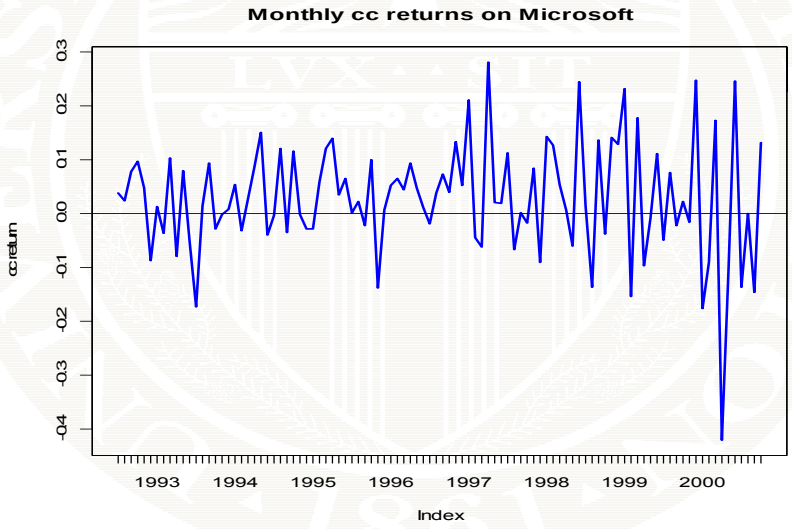
Econ 424/Amath 540
Eric Zivot
Summer 2012
Updated: July 12, 2012

Monthly Closing Prices of Microsoft Stock

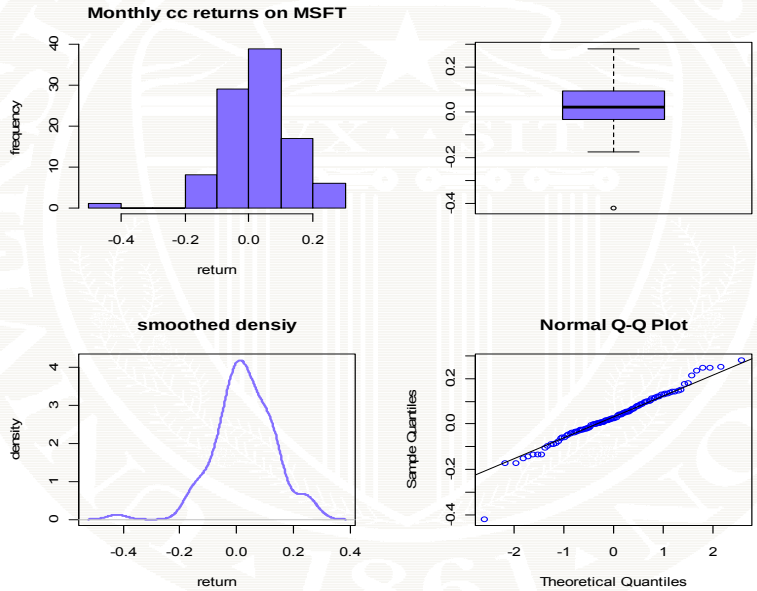


Conjecture: Monthly CC Returns follow CER Model

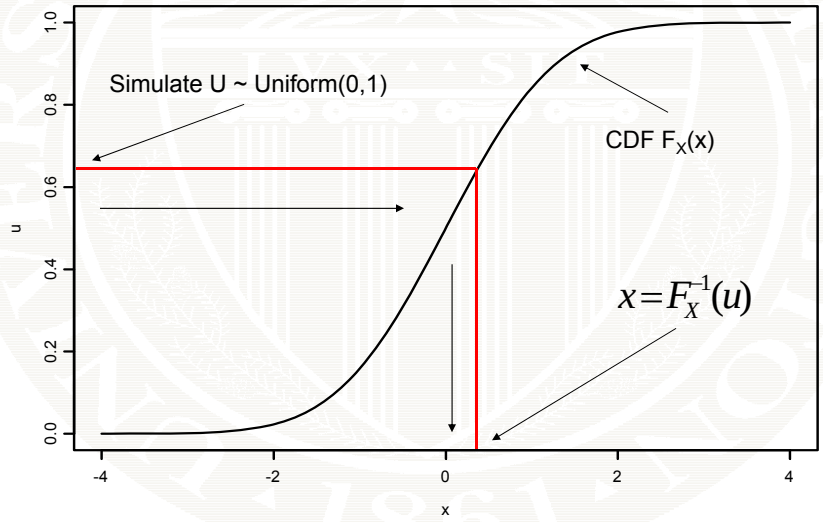
$r_t = \mu + \varepsilon_t$, $\varepsilon_t \sim$ iid $N(0,\sigma^2)$, t=July 1992,…,Oct 2000



Distribution Summary for Monthly CC Returns on MSFT



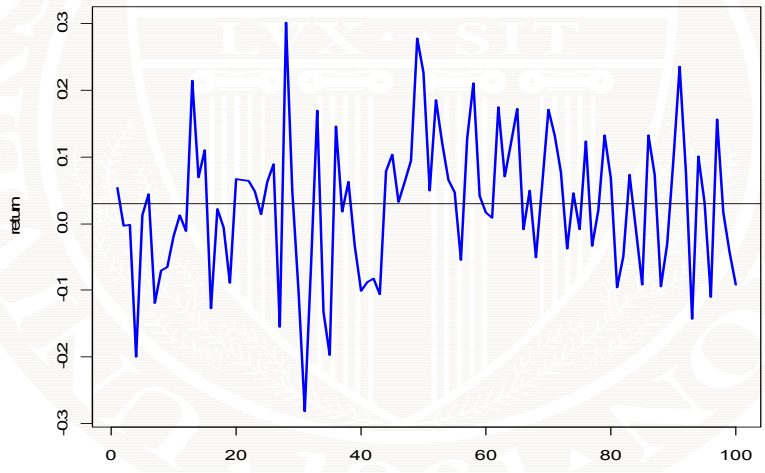
Simulating Random Data for X with CDF F_X



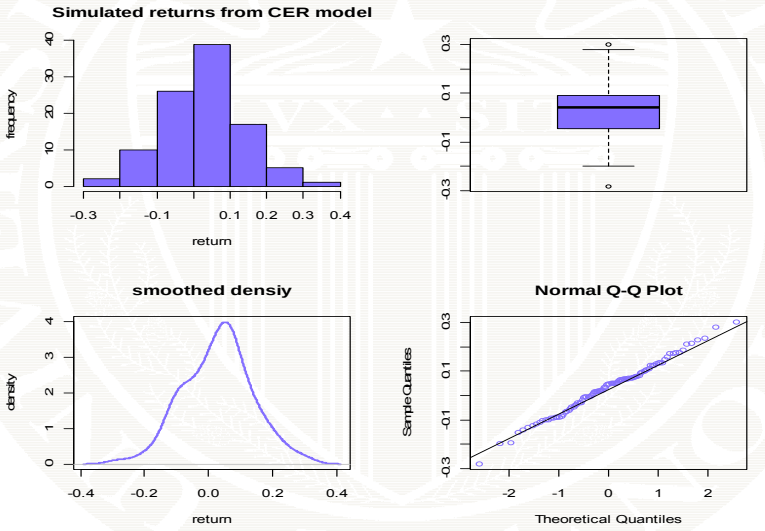
R Code for MC Simulation of CER Model

```
# set model parameters
> mu = 0.03
> sd.e = 0.10
> nobs = 100

# generate random numbers for errors
> set.seed(111)
> sim.e = rnorm(nobs, mean=0, sd=sd.e)

# simulate cc returns
> sim.ret = mu + sim.e
```

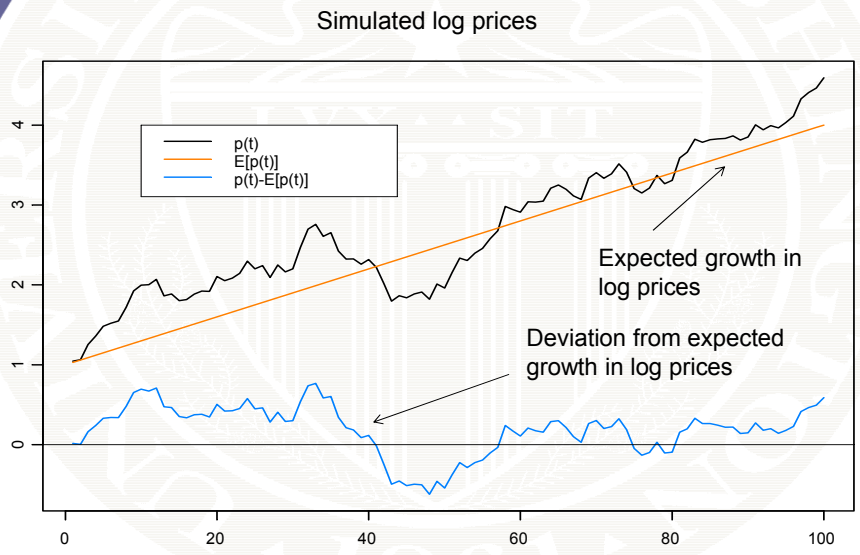
Simulated Returns from CER Model with Same Mean and SD as Microsoft



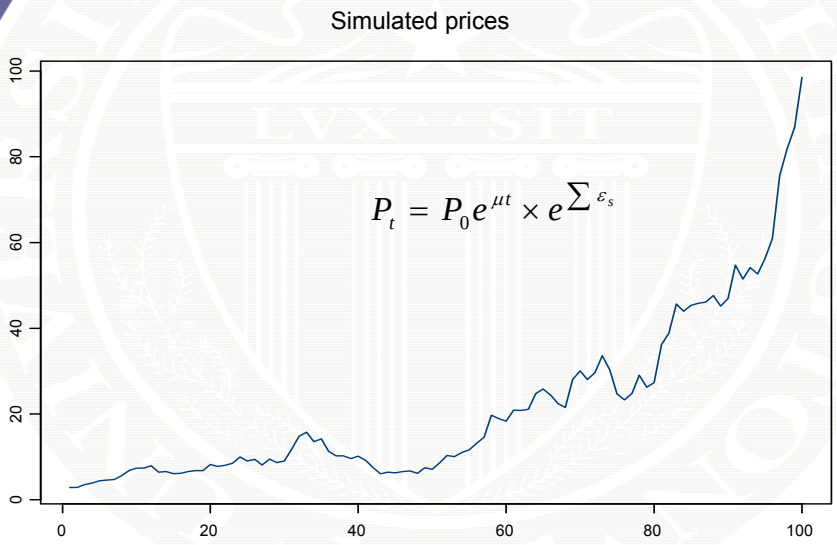
Distribution Summary for Simulated Data from CER Model



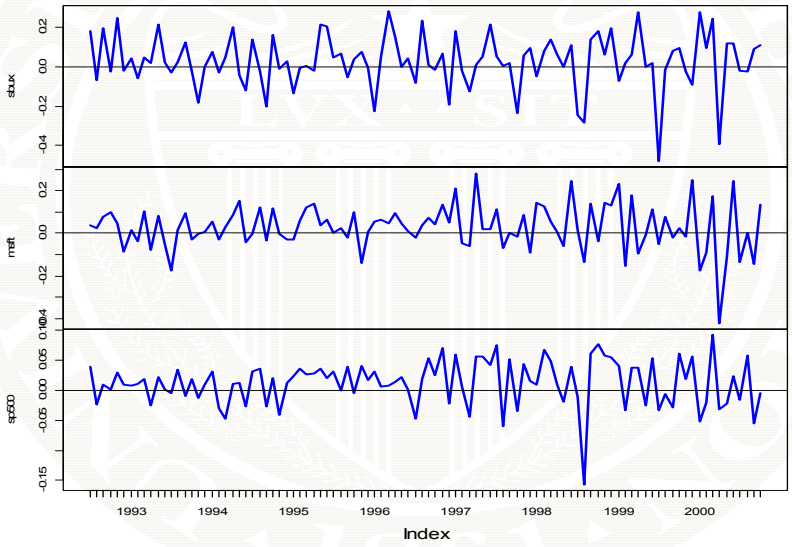
Simulated Data from Random Walk Model for Log Prices



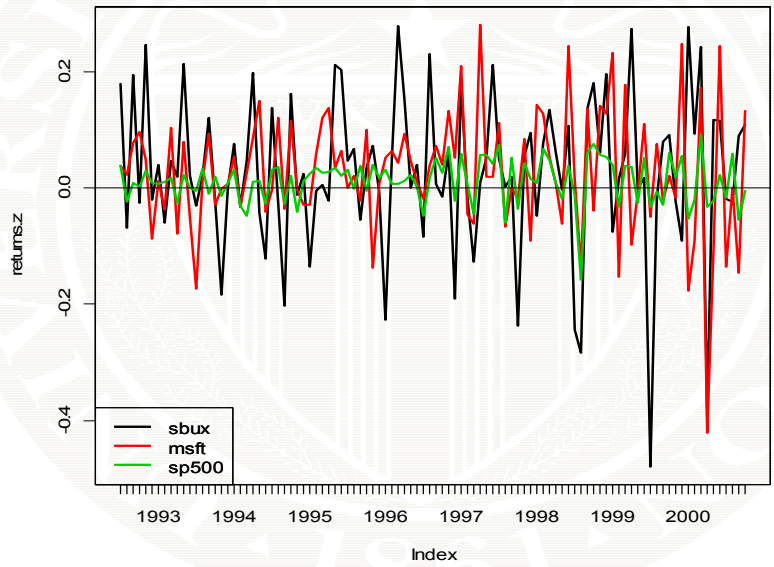
Simulated Prices from Random Walk Model



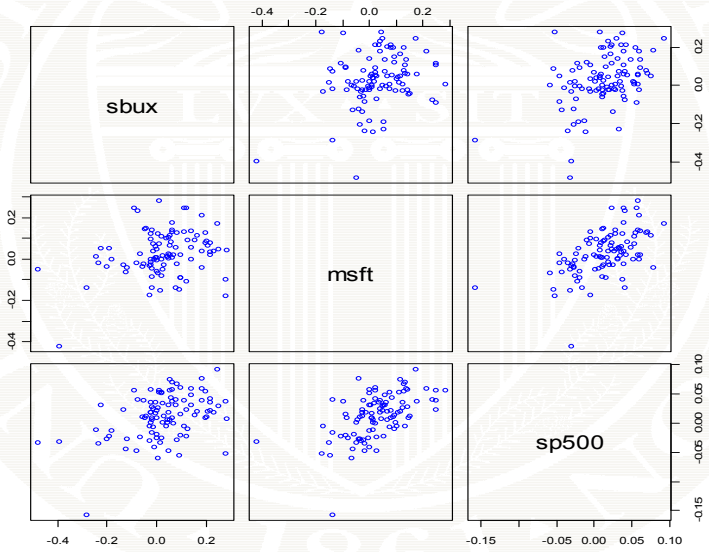
Monthly Returns on MSFT, SBUX and S&P 500



Monthly Returns on SBUX, MSFT and SP500



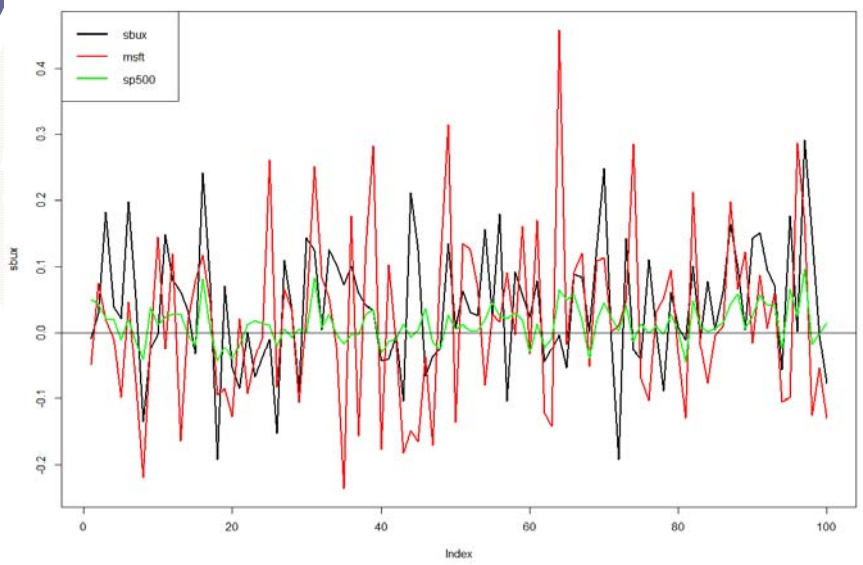
Monthly Returns on SBUX, MSFT and SP500



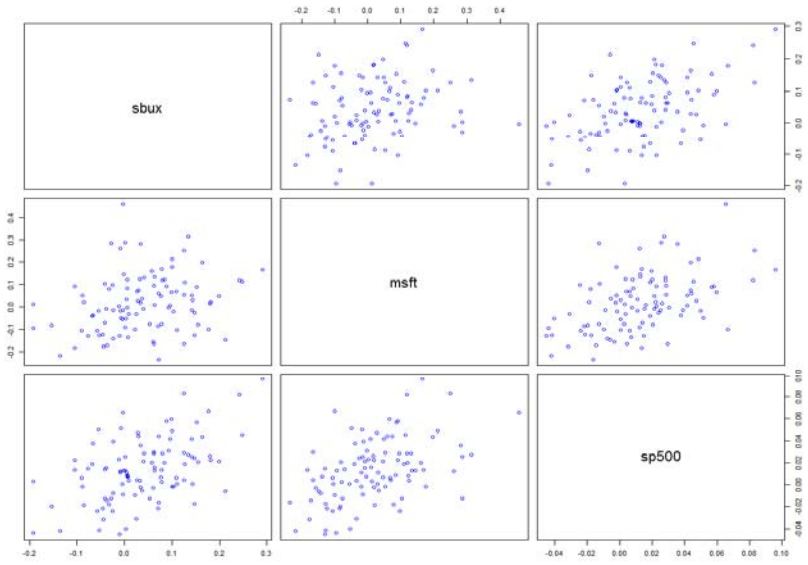
R Code for MC Simulation for Multiple Assets

```
# multivariate simulation
> library("mvtnorm")
> mu = c(0.03,0.03,0.01)
> sig2.msft = 0.018
> sig2.sbux = 0.011
> sig2.sp500 = 0.001
> sig.msft.sbux = 0.004
> sig.msft.sp500 = 0.002
> sig.sbux.sp500 = 0.002
> Sigma = matrix(c(sig2.sbux, sig.msft.sbux, sig.sbux.sp500,
+                  sig.msft.sbux, sig2.msft, sig.msft.sp500,
+                  sig.sbux.sp500, sig.msft.sp500, sig2.sp500),
+                  nrow=3, ncol=3, byrow=TRUE)> nobs = 100
> set.seed(123)
> returns.sim = rmvnorm(nobs, mean=mu, sigma=Sigma)
```

Simulated Return Data



Simulated Returns



CER Model Estimates

```
> muhat.vals = apply(returns.mat,2,mean)
> muhat.vals
     sbux     msft    sp500
  0.02777  0.02756  0.01253
```

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} r_t$$

```
> sigma2hat.vals = apply(returns.mat,2,var)
> sigma2hat.vals
     sbux     msft     sp500
  0.01846  0.01141  0.001432
```

$$\hat{\sigma}^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \hat{\mu})^2$$

```
> sigmahat.vals = apply(returns.mat,2,sd)
> sigmahat.vals
    sbux    msft    sp500
  0.1359  0.1068  0.03785
```

$$\hat{\sigma} = \sqrt{\hat{\sigma}^2}$$

CER Model Estimates

```
> cov.mat = var(returns.ts)
> cor.mat = cor(returns.ts)
```

$$\hat{\sigma}_{ij} = \frac{1}{T-1}\sum_{t=1}^{T}(r_{it} - \hat{\mu}_i)(r_{jt} - \hat{\mu}_j)$$

$$\hat{\rho}_{ij} = \frac{\hat{\sigma}_{ij}}{\hat{\sigma}_i \hat{\sigma}_j}$$

```
> covhat.vals = cov.mat[lower.tri(cov.mat)]
> rhohat.vals = cor.mat[lower.tri(cor.mat)]
> names(covhat.vals) = names(rhohat.vals) =
+ c("sbux,msft","sbux,sp500","msft,sp500")

> covhat.vals
 sbux,msft sbux,sp500 msft,sp500
 0.00403   0.00215    0.00224

> rhohat.vals
 sbux,msft sbux,sp500 msft,sp500
   0.2777    0.4197     0.5551
```

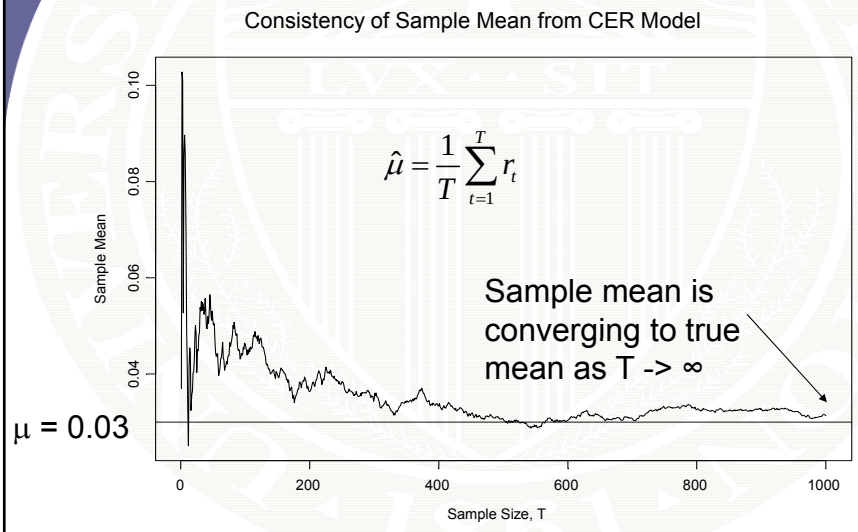
Estimated Standard Errors

```
> se.muhat = sigmahat.vals/sqrt(nobs)
> rbind(muhat.vals,se.muhat)
              sbux     msft     sp500
muhat.vals  0.0277   0.0275   0.01253
  se.muhat  0.0135   0.0106   0.00378
> se.sigma2hat = sigma2hat.vals/sqrt(nobs/2)
> rbind(sigma2hat.vals,se.sigma2hat)
                 sbux      msft      sp500
sigma2hat.vals  0.01845   0.01141   0.00143
  se.sigma2hat  0.00261   0.00161   0.00020
> se.sigmahat = sigmahat.vals/sqrt(2*nobs)
> rbind(sigmahat.vals,se.sigmahat)
                sbux     msft     sp500
sigmahat.vals  0.1358   0.1068   0.0378
  se.sigmahat  0.0096   0.0075   0.0026
```


Estimated Standard Errors

```
> se.rhohat = (1-rhohat.vals^2)/sqrt(nobs)
> rbind(rhohat.vals,se.rhohat)
             sbux,msft sbux,sp500 msft,sp500
rhohat.vals  0.2777    0.4197     0.5551
  se.rhohat  0.0922    0.0823     0.0691
```

Sample means computed with increasing sample sizes



95% Confidence Intervals For μ

```
> mu.lower = muhat.vals - 2*se.muhat
> mu.upper = muhat.vals + 2*se.muhat
> mu.width = mu.upper - mu.lower

> cbind(mu.lower,mu.upper,mu.width)
       mu.lower mu.upper mu.width
 sbux    0.0006   0.0549   0.0543
 msft    0.0061   0.0489   0.0427
sp500    0.0049   0.0201   0.0151
```

Wide 95% confidence intervals for the mean => imprecise estimate. Note: width of CI is large relative to size of estimate for sbux and msft

95% Confidence Intervals for σ

```
> sigma.lower = sigmahat.vals - 2*se.sigmahat
> sigma.upper = sigmahat.vals + 2*se.sigmahat
> sigma.width = sigma.upper - sigma.lower
> cbind(sigma.lower,sigma.upper,sigma.width)
       sigma.lower sigma.upper sigma.width
 sbux 0.1166      0.1550      0.0384
 msft 0.0917      0.1219      0.0302
sp500 0.0324      0.0431      0.0107
```

Narrow 95% confidence intervals for the sd => precise estimate. Note: width of CI is small relative to value of estimate

95% Confidence Intervals for ρ

```
> rho.lower = rhohat.vals - 2*se.rhohat
> rho.upper = rhohat.vals + 2*se.rhohat
> rho.width = rho.upper - rho.lower

> cbind(rho.lower,rho.upper,rho.width)
             rho.lower rho.upper rho.width
 sbux,msft    0.0931    0.4622    0.3691
sbux,sp500    0.2549    0.5845    0.3295
msft,sp500    0.4167    0.6934    0.2767
```

95% confidence interval for rho is moderately large => somewhat imprecise estimate for rho.

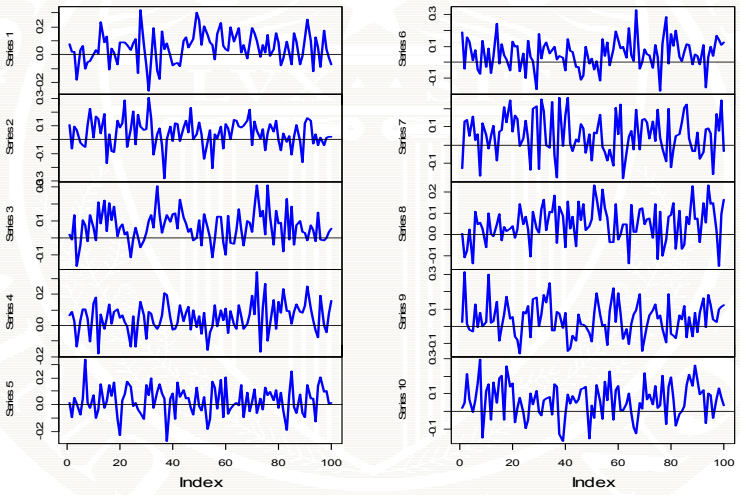
Stylized Facts for the Estimation of CER Model Parameters

- The expected return is not estimated very precisely
  - Large standard errors relative to size of mean estimates
- Standard deviations and correlations are estimated more precisely than the expected return

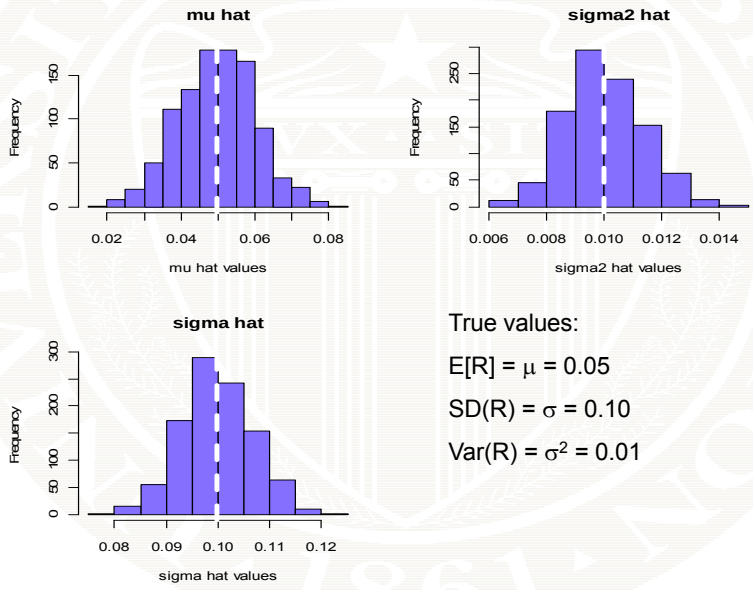
Monte Carlo Simulation Loop

```
mu = 0.05
sd = 0.10
n.obs = 100
n.sim = 1000
set.seed(111)
sim.means = rep(0,n.sim)
sim.vars = rep(0,n.sim)
sim.sds = rep(0,n.sim)
for (sim in 1:n.sim) {
    sim.ret = rnorm(n.obs,mean=mu,sd=sd)
    sim.means[sim] = mean(sim.ret)
    sim.vars[sim] = var(sim.ret)
    sim.sds[sim] = sqrt(sim.vars[sim])
}
```

10 simulated samples from CER model



Histograms of 1000 Monte Carlo Estimates



True values:

E[R] = μ = 0.05

SD(R) = σ = 0.10

Var(R) = σ^2 = 0.01

Monte Carlo Evaluation of Bias

```
> mean(sim.means)       # true mean = 0.05
[1] 0.04969
> mean(sim.means) - mu  # estimate of bias
[1] -0.0003105

> mean(sim.vars)        # true variance = 0.01
[1] 0.00999
> mean(sim.vars) - sd^2 # estimate of bias
[1] -9.865e-06

> mean(sim.sds)         # true SD = 0.10
[1] 0.09972
> mean(sim.sds) - sd    # estimate of bias
[1] -0.0002782
```


Monte Carlo Evaluation of Estimated Standard Error

```
> sd(sim.means)    # SD of mu estimates across 1000
[1] 0.01041        # Monte Carlo experiments
> sd/sqrt(nobs)
[1] 0.01           # true SE estimate from formula

> sd(sim.vars)     # SD of sigma^2 estimates across 1000
[1] 0.001352       # Monte Carlo experiments
> sd^2/sqrt(nobs/2)
[1] 0.001414       # approx SE estimate from formula

> sd(sim.sds)      # SD of sigma estimates across 1000
[1] 0.006764       # Monte Carlo experiments
> sd/sqrt(2*nobs)
[1] 0.007071       # approx SE estimate from formula
```

Monte Carlo Evaluation of 95% Confidence Interval Coverage

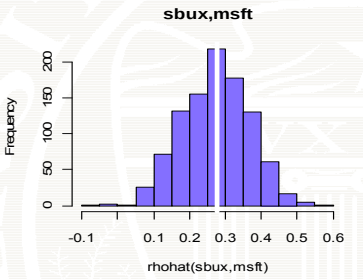
```
mu = 0.05
sd = 0.10
n.sim = 1000
set.seed(111)
mu.lower = rep(0,n.sim) # initialize vectors
mu.upper = rep(0,n.sim)
for (sim in 1:n.sim) {
    sim.ret = rnorm(n.obs,mean=mu,sd=sd)
    mu.hat = mean(sim.ret)
    se.muhat = sd(sim.ret)/sqrt(n.obs)
    mu.lower[sim] = mu.hat - 2*se.muhat
    mu.upper[sim] = mu.hat + 2*se.muhat
}
in.interval = (mu >= mu.lower) & (mu <= mu.upper)
sum(in.interval)/n.sim
```

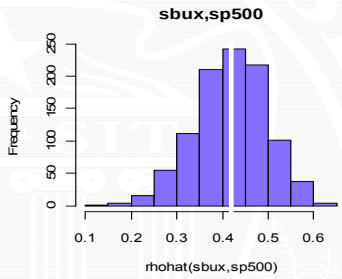
```
>  0.934 # coverage probability of 95% CI for mu
```

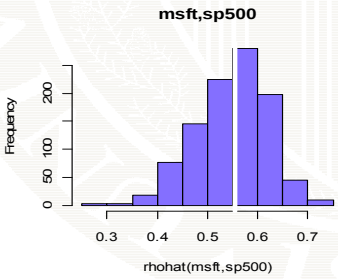

Monte Carlo Simulation Loop to Evaluate Distribution of correlation estimate

```
# generate 1000 samples from CER and compute correlations
# use estimated parameters as true parameters for MC
n.obs = 100
n.sim = 1000
set.seed(111)
sim.corrs = matrix(0,n.sim,3) # initialize vectors
colnames(sim.corrs) = c("sbux,msft", "sbux,sp500",
                        "msft,sp500")

for (sim in 1:n.sim) {
    sim.ret = rmvnorm(n.obs, mean=muhat.vals,
                      cov=cov.mat)
    cor.mat = cor(sim.ret)
    sim.corrs[sim,] = cor.mat[lower.tri(cor.mat)]
}
```

Histograms of 1000 Monte Carlo Estimates of Correlation







True values:

Corr(SBUX,MSFT) = 0.28

Corr(SBUX,SP500) = 0.42

Corr(MSFT,SP500) = 0.56

Monte Carlo Evaluation

```
# true correlation values
 sbux,msft sbux,sp500 msft,sp500
    0.2777    0.4198    0.5551
# Averages across 1000 Monte Carlos
> apply(sim.corrs,2,mean)
 sbux,msft sbux,sp500 msft,sp500
     0.277    0.4176    0.5505
# Monte Carlo Standard Deviations
> apply(sim.corrs,2,sd)
 sbux,msft sbux,sp500 msft,sp500
   0.09606   0.08148   0.07244
# Analytic SE values for rhohat
 sbux,msft sbux,sp500 msft,sp500
   0.09229   0.08238   0.06919
```

Estimating VaR in CER Model

```
# estimate quantiles from CER model
> qhat.05 = muhat.vals + sigmahat.vals*qnorm(0.05)
> qhat.05
     sbux      msft     sp500
 -0.19571 -0.14815 -0.049717

# estimate 5% VaR
> W0 = 100000
> VaRhat.05 = (exp(qhat.05)-1)*W0
> VaRhat.05
   sbux   msft   sp500
 -17775 -13769 -4850.1
```