Hypothesis Testing in the CER Model

Econ 424/Amath 540
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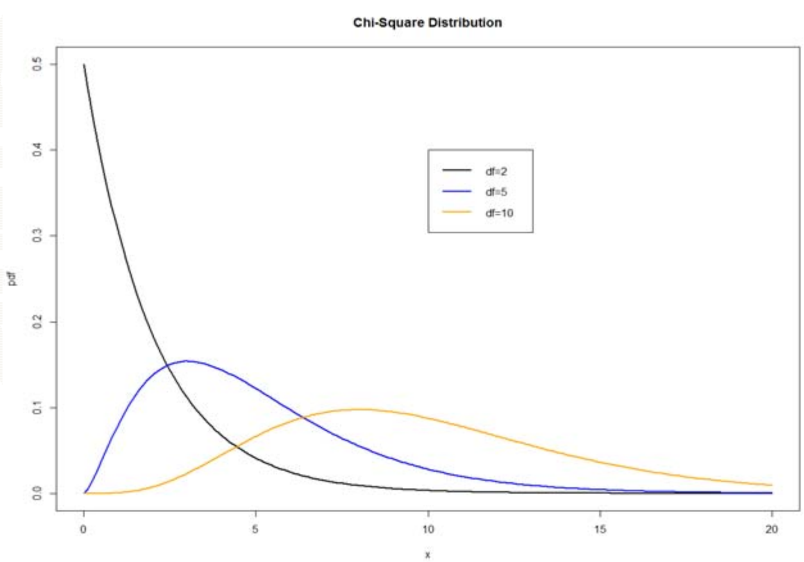
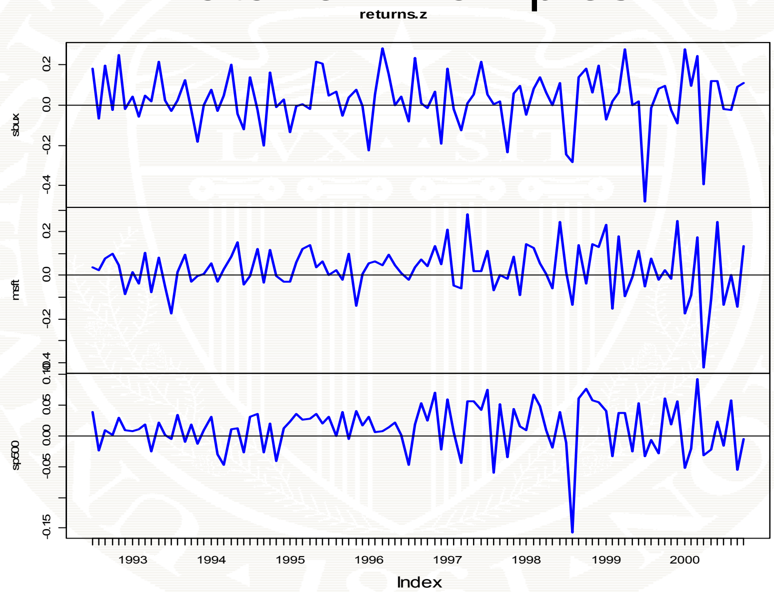
Chi-Square Distribution



Data for Examples



T = 100 months

H_0: $\mu = 0$ vs. H_1: $\mu \neq 0$

```
# construct test by brute force
> nobs = nrow(returns.z)
> muhat.vals = apply(returns.z,2,mean)
> muhat.vals
   sbux    msft   sp500
0.02777 0.02756 0.01253

> sigmahat.vals = apply(returns.z,2,sd)
> se.muhat = sigmahat.vals/sqrt(nobs)
> se.muhat
   sbux    msft    sp500
0.01359 0.01068 0.003785
> t.stats = muhat.vals/se.muhat
> abs(t.stats)
 sbux msft sp500
2.044 2.58 3.312
```

|t-stats| > 2 => we should reject H0: $\mu = 0$

H_0: $\mu = 0$ vs. H_1: $\mu \neq 0$

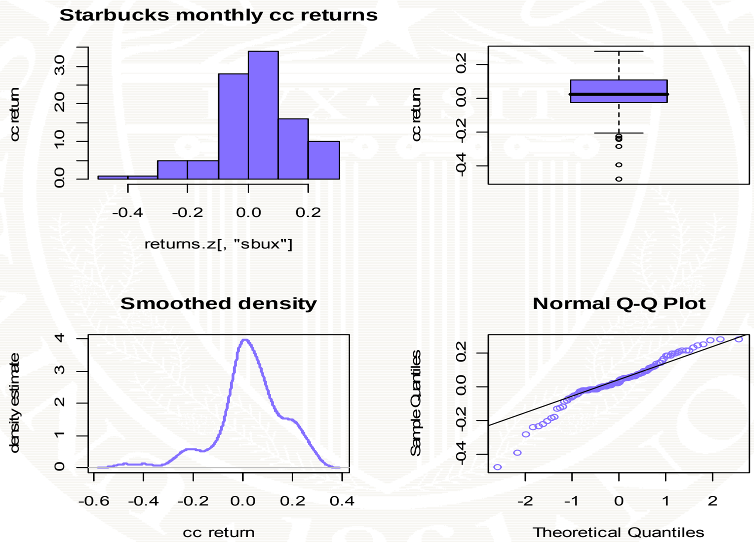
```
# compute 2-sided 5% critical values
> cv.2sided = qt(0.975, df=nobs-1)
> cv.2sided
[1] 1.984
> abs(t.stats) > cv.2sided
 sbux msft sp500
    T    T     T

# compute 2-sided p-values
> 2*(1-pt(abs(t.stats),df=nobs-1))
    sbux    msft    sp500
 0.04363 0.01134 0.001295
```


R `t.test()` function

```
# Test H0: mu = 0 for msft
> t.test.msft = t.test(returns.z[,"msft"],
+                      alternative="two.sided",
+                      mu=0, conf.level=0.95)
> class(t.test.msft)
[1] "htest"
> t.test.msft
        One Sample t-test
data:  returns.z[, "msft"]
t = 2.580, df = 99, p-value = 0.01134
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.006368 0.048760
sample estimates:
mean of x
  0.02756
```

μ = 0 does not lie in 95% CI so we reject H0 μ=0 at 5% level

Test for Normal Distribution



Jarque-Bera Test for Normality

```
> sbux.skew = skewness(returns.z[,"sbux"])
> sbux.ekurt= kurtosis(returns.z[,"sbux"])
> sbux.skew
[1] -0.8272737
> sbux.ekurt
[1] 1.761706
> JB = nobs*(sbux.skew^2 + 0.25*sbux.ekurt^2)/6
> JB
[1] 24.33806
```

JB = 24.34 > 6 so we reject H0: returns on sbux are normally distributed at the 5% level

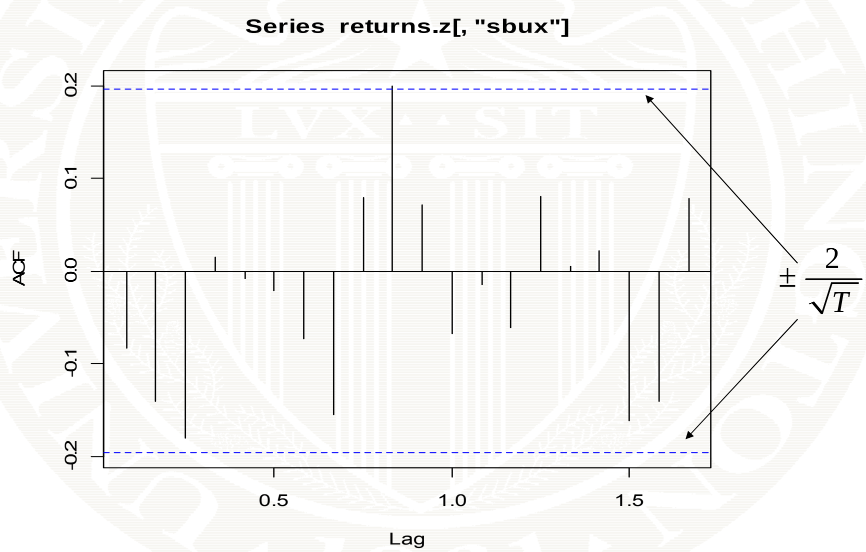
```
> p.value = 1 - pchisq(JB, df = 2)
> p.value
[1] 5.188691e-06
```

tseries function `jarque.bera.test()`

```
> library(tseries)
> jarque.bera.test(returns.z[,"sbux"])

        Jarque Bera Test

data:  returns.z[, "sbux"]
X-squared = 24.34, df = 2, p-value = 5.189e-06
```

JB statistic

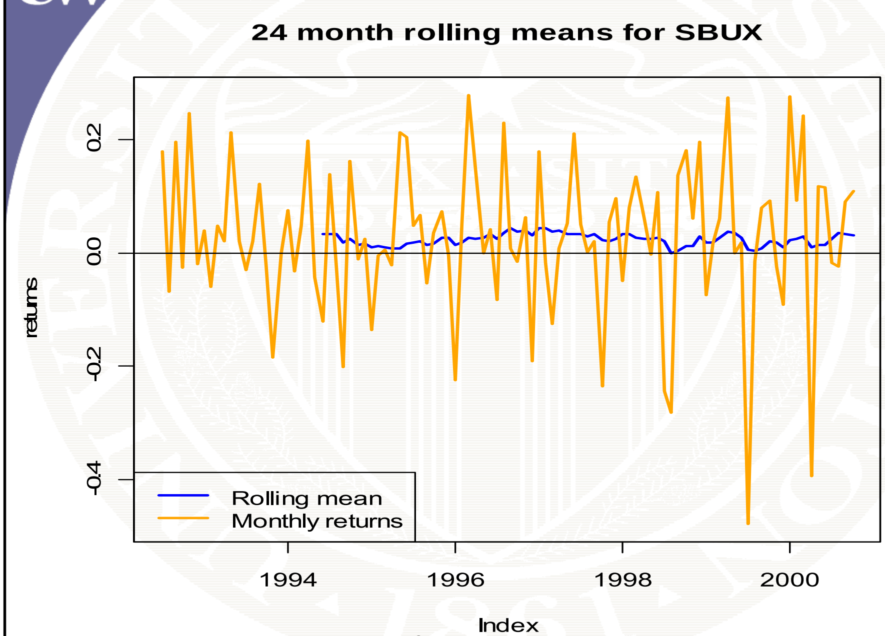
Testing for Serial Correlation

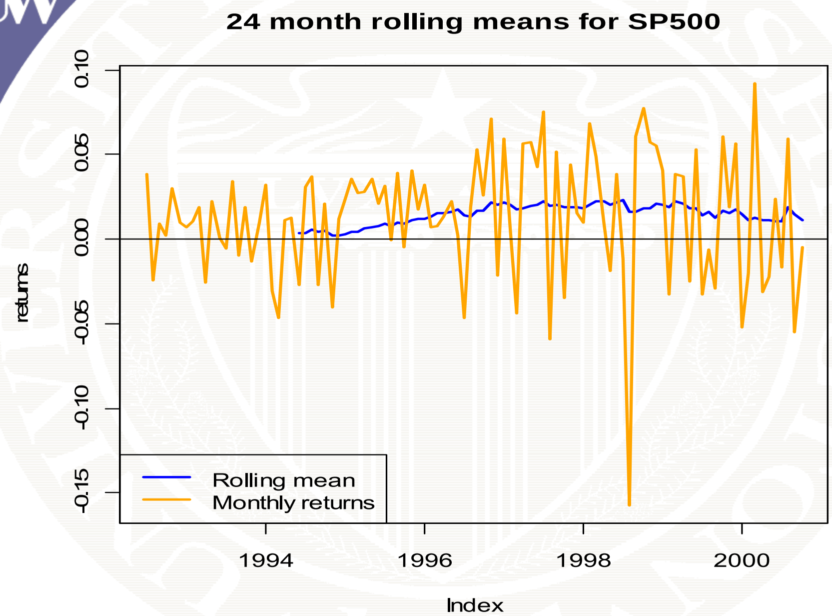



```
> acf(returns.ts[,"sbux"]
```

Compute Rolling Means using zoo function `rollapply()`

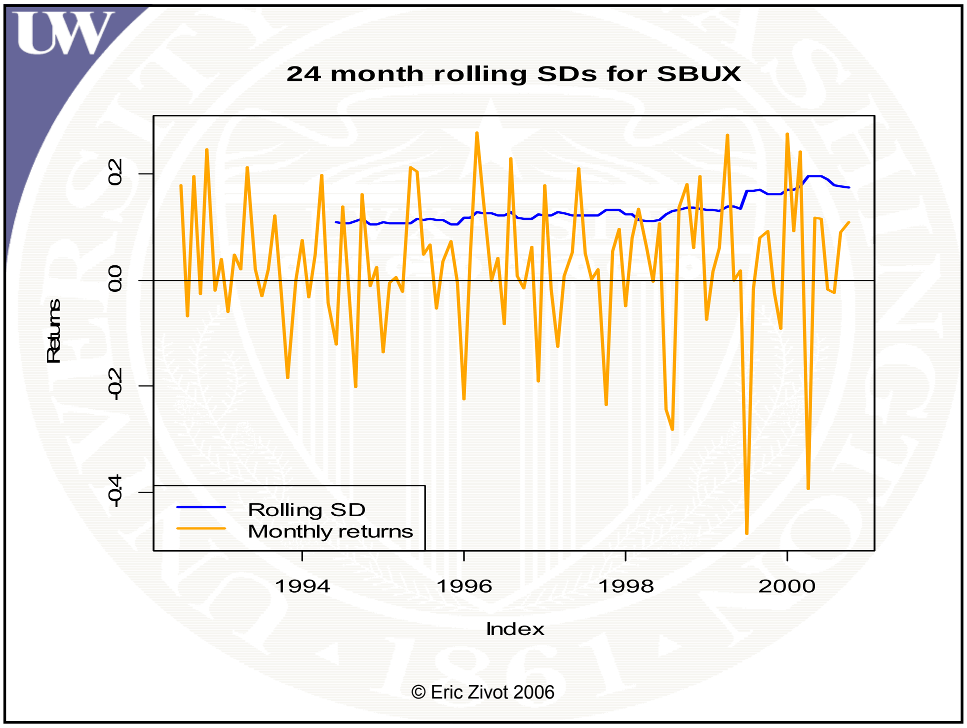
```
# 24-month rolling means incremented by 1 month
> roll.muhat = rollapply(returns.z[,"sbux"], width=24,
+                         FUN=mean, align="right")
> class(roll.muhat)
[1] "zoo"

> roll.muhat[1:5]
Jun 1994 Jul 1994 Aug 1994 Sep 1994 Oct 1994
 0.03415  0.03244  0.03418  0.01758  0.02538
```



Compute Rolling SDs Using zoo Function `rollapply()`

```
# 24-month rolling SD incremented by 1 month
> roll.sigmahat = rollapply(returns.z[,"sbux"], width=24,
+                           FUN=sd, align="right")
> class(roll.sigmahat)
[1] "zooreg" "zoo"

> roll.sigmahat[1:5]
Jun 1994 Jul 1994 Aug 1994 Sep 1994 Oct 1994
  0.1101   0.1080   0.1067   0.1114   0.1148
```


UW
24 month rolling SDs for SBUX
Returns
0.2
0.0
-0.2
-0.4
Rolling SD
Monthly returns
1994
1996
1998
2000
Index
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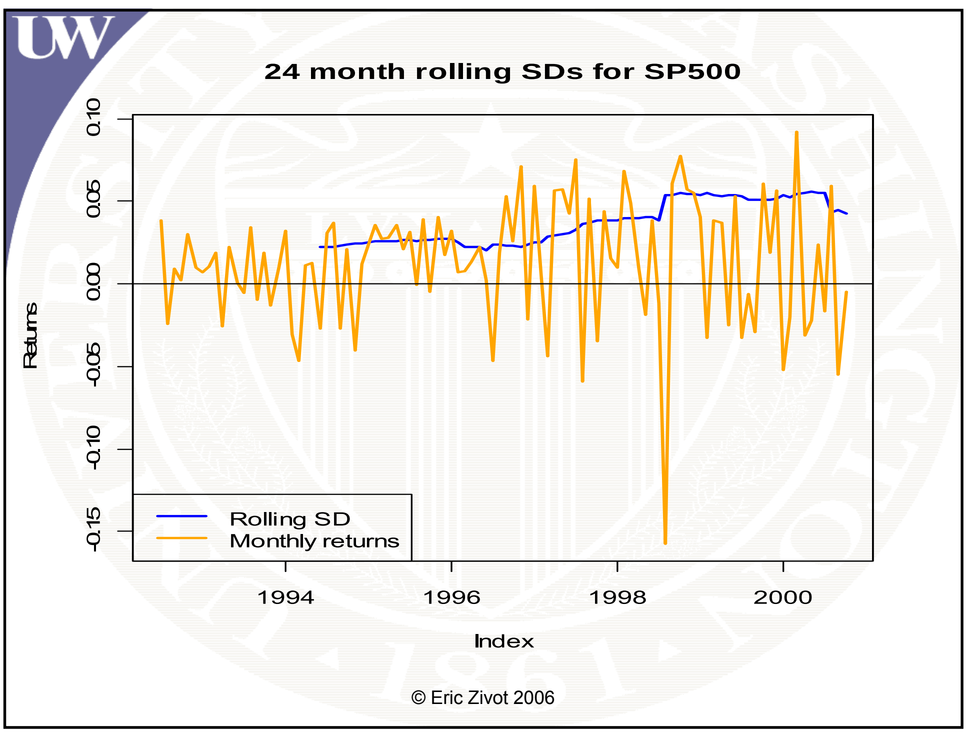

UW
24 month rolling SDs for SP500
Returns
0.10
0.05
0.00
-0.05
-0.10
-0.15
Rolling SD
Monthly returns
1994
1996
1998
2000
Index
© Eric Zivot 2006

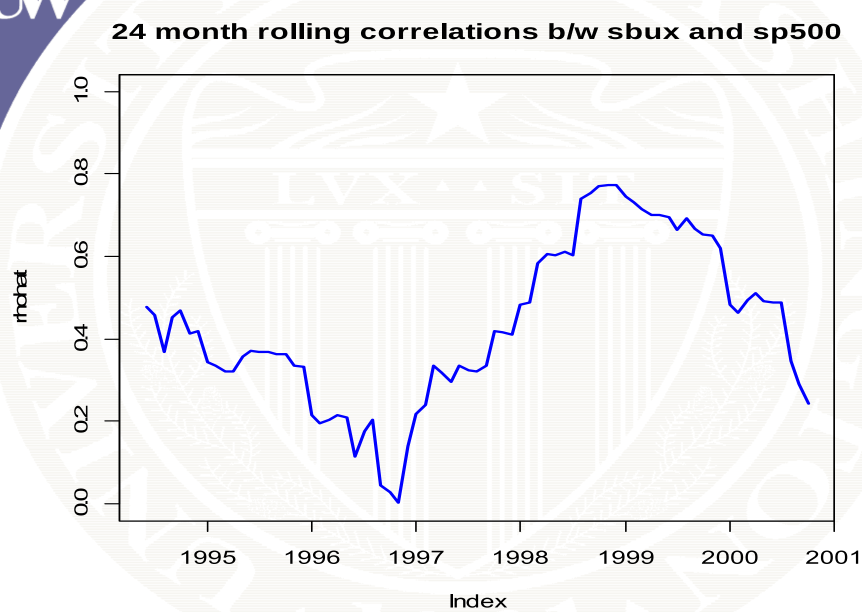
Compute Rolling Correlations Using zoo Function `rollapply()`

```
# compute 24-month rolling correlations between
# sp500 and sbux

# function to compute pairwise correlation
rhohat = function(x) {
      cor(x)[1,2]
}

> roll.rhohat = rollapply(returns.z[,c("sp500","sbux")],
+                         width=24,FUN=rhohat,
+                         by.column=FALSE, align="right")
> class(roll.rhohat)
[1] "zoo"

> roll.rhohat[1:5]
  Jun 1994 Jul 1994 Aug 1994 Sep 1994 Oct 1994
  0.4786   0.4570   0.3694   0.4515   0.4683
```

Summary of Hypothesis Testing in CER model

- Hypothesis tests about μ are not very powerful because $SE(\hat{\mu})$ is typically very large
- Can often reject hypothesis that monthly returns are normally distribution
- Typically cannot reject hypothesis that monthly returns are uncorrelated over time
- Rolling window estimates indicate that μ, σ and ρ_{ij} are typically not constant over time
  - Assumption of covariance stationarity is suspect!