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Adaptive Concept in High-T_c Superconductivity: First-Principles Derivation

Microscopic Model and Pairing Mechanism

High-temperature superconductors (HTSC), notably cuprate oxides, are doped Mott insulators with strongly correlated electrons. A minimal first-principles model is the single-band Hubbard Hamiltonian on a copper-oxide plane [1] [2]. It captures electrons hopping on a 2D lattice with on-site Coulomb repulsion:

$$H \;=\; -t\sum_{\langle i,j\rangle,\sigma}\Big(c^\dagger_{i\sigma}c_{j\sigma}+\mathrm{h.c.}\Big)\;+\;U\sum_i n_{i\uparrow}n_{i\downarrow}\,. \tag{1}$$

Here $c^\dagger_{i\sigma}$ creates an electron of spin σ on site i, t is the nearest-neighbor hopping amplitude, and U is the on-site repulsion. At half-filling (one electron per Cu site), $U \gg t$ localizes electrons and drives antiferromagnetic (AFM) order. Upon hole doping (removing electrons), long-range AFM is suppressed and superconductivity (SC) emerges [3] [4]. In the limit $U\!\to\infty$ (no double occupancy), Eq. (1) maps to the t–J model [5]:

$$H_{tJ} \;=\; -t\sum_{\langle i,j\rangle,\sigma}\tilde c^\dagger_{i\sigma}\tilde c_{j\sigma}\;+\;J\sum_{\langle i,j\rangle}\Big(\mathbf{S}_i\cdot\mathbf{S}_j-\tfrac{1}{4}n_in_j\Big),\qquad J\approx\frac{4t^2}{U}\,. \tag{2}$$

Here $\tilde{c}_{i\sigma}$ acts in the subspace with no double occupancy, and J is an AFM superexchange coupling [5]. The J-term energetically favors singlet pairing of neighboring spins. Intuitively, when a hole dopes the AFM background, two $S=\tfrac{1}{2}$ spins can lower energy by forming a singlet bond (releasing exchange energy J). This mechanism leads to effective attraction between doped holes: a hole pair can occupy two adjacent sites with both Cu spins singlet-coupled, avoiding the U cost while gaining J (the “RVB” mechanism). Thus, doped Mott insulators naturally develop **Cooper pairs** of holes bound by magnetic exchange rather than phonons [2].

In momentum-space, a **pairing instability** occurs in the d-wave channel due to the strong k-dependence of the exchange interaction. In cuprates, the gap function $\Delta(\mathbf{k})$ is observed to have $d_{x^2-y^2}$ symmetry with nodes along $k_x=\pm k_y$. From a mean-field decoupling of the J-term, one derives a BCS-like self-consistency equation for the gap. For example, assuming singlet pairing on nearest-neighbor bonds, the gap $\Delta_{\mathbf{k}}$ satisfies:

$$\Delta_{\mathbf{k}} \;=\; \frac{3J}{4N}\sum_{\mathbf{q}}\cos(k_x-q_x)\cos(k_y-q_y)\;\frac{\Delta_{\mathbf{q}}}{2E_{\mathbf{q}}}\;\tanh\frac{E_{\mathbf{q}}}{2k_BT}\,, \tag{3}$$

where $E_{\mathbf{q}} = \sqrt{\xi_{\mathbf{q}}^2 + |\Delta_{\mathbf{q}}|^2}$ is the quasiparticle energy (with $\xi_{\mathbf{q}}$ the band dispersion) and the $\cos$ factors reflect d-wave form factors on the square lattice. This gap equation admits a nonzero solution below a critical temperature T_c, corresponding to the superconducting transition. The resulting order parameter $\langle c_{i\downarrow}c_{j\uparrow}\rangle \sim \Delta_0$ changes sign between x and y directions, as observed experimentally [6]. In essence, **strong AFM correlations mediate d-wave pairing** in the Hubbard/t–J model framework, providing a qualitative explanation for high-T_c superconductivity in cuprates [7]. Notably, renormalization-group (RG) analyses confirm that in the 2D Hubbard model, repulsive interactions can indeed drive a d-wave pairing instability at low energy [7]. For weak to moderate U, one-loop RG flow equations show that while spin-density-wave (AFM) correlations dominate at half-filling, a slight hole doping causes the RG fixed point to shift to a d-wave superconducting phase [7]. This indicates that the Hubbard model contains the essential ingredients for HTSC, with the **d-wave pairing channel becoming attractive under RG flow** once the AFM order is destabilized by doping [7].

Ginzburg–Landau Effective Theory

On phenomenological grounds, one can describe the superconducting state by a complex order parameter field $\Psi(x)$ in the continuum (the condensate wavefunction). Near T_c, a **Ginzburg–Landau (GL) free energy functional** can be derived from microscopics (e.g. by expanding the Hubbard model free energy in powers of Ψ). The GL free energy (density) for a superconductor has the form [8] [9]:

$$f[\Psi] \;=\; f_n + \alpha(T)\,|\Psi|^2 \;+\; \tfrac{\beta}{2}\,|\Psi|^4 \;+\; \tfrac{\hbar^2}{2m^*}\big|\nabla\Psi\big|^2 \;+\; \frac{1}{2m^*}\Big|\frac{e^*}{c}\mathbf{A}\Psi\Big|^2 \;+\; \frac{\mathbf{B}^2}{8\pi}\,. \tag{4}$$

Here f_n is the free energy of the normal state, $\alpha(T)$ and β are phenomenological coefficients (with $\alpha(T)\propto (T-T_c)$ for a second-order transition [10]), $m^$ *and $e*^=2e$ are the effective mass and charge of Cooper pairs, and $\mathbf{A}$ ($\mathbf{B}=\nabla\times\mathbf{A}$) is the electromagnetic vector potential (magnetic field). Minimizing the free energy $F=\int f\,d^3r$ with respect to $\Psi^$ *and $\mathbf{A}$ gives the GL equations** [11] [12]:

$$\alpha\,\Psi + \beta\,|\Psi|^2\Psi \;+\; \frac{\hbar^2}{2m^*}\nabla^2\Psi \;-\; \frac{(e^*)^2}{2m^*c^2}|\mathbf{A}|^2\,\Psi \;-\; \frac{i\hbar e^*}{m^*c}(\mathbf{A}\cdot\nabla)\Psi \;=\; 0\,, \tag{5a}$$

$$\nabla\times\mathbf{B} \;=\; \frac{4\pi}{c}\mathbf{J}, \qquad \text{with current } \mathbf{J} = \frac{e^*}{m^*}\Re\Big\{\Psi^*\big(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A}\big)\Psi\Big\}\,. \tag{5b}$$

Equation (5a) determines the spatial profile of the order parameter, while (5b) is Ampère's law for the supercurrent $\mathbf{J}$ [13]. In a uniform superconductor ($\nabla\Psi=0$, $\mathbf{A}=0$), Eq. (5a) yields either the normal state $\Psi=0$ or $|\Psi|^2=-\alpha/\beta$ (for $\alpha<0$), which gives the equilibrium condensate density $|\Psi|^2 \propto (T_c - T)$ below T_c [14] [15]. The GL theory predicts fundamental length scales: the **coherence length** ξ characterizes variations in Ψ, and the **London penetration depth** λ characterizes decay of magnetic field inside the superconductor [16] [17]. For example, $\xi = \sqrt{\hbar^2/4m^|\alpha|}$ *for $T<T_c$ [18], and $\lambda = \sqrt{\frac{m^*}{\mu_0 (e^*)^2 |\Psi|^2}}$ in the London limit [17]. The ratio $\kappa=\lambda/\xi$ distinguishes Type I vs II superconductors; high-T_c cuprates have large $\kappa\gg 1$ (extreme Type II), allowing magnetic flux vortices to penetrate.

Crucially, in cuprates the GL coefficient α is strongly dependent on hole **doping level p** (holes per Cu). Doping acts like an external parameter that tunes the "depth" of the SC free energy well. Empirically, T_c forms a dome-shaped function of p [4]: it rises from zero at underdoping, peaks at an optimal doping $p_{\text{opt}}\sim0.2$, then decreases at overdoping (eventually $T_c\to0$ as the normal Fermi liquid is restored). This behavior can be encoded by writing α as $\alpha(T,p) \propto [T - T_c(p)]$, where $T_c(p)$ has a maximum at p_{opt}. In a simple Landau expansion, one may expand $T_c(p)$ to second order around p_{opt}, yielding an inverted parabola. Near

p_{opt}, the superfluid density and pairing strength are maximal. Away from optimal doping (either under- or overdoped), the **superconducting stiffness (order $|\Psi|^2$)** is reduced – reflecting the presence of competing phases (antiferromagnetism, charge order) or weaker pairing interactions. In GL theory this could be modeled by a doping-dependent α that becomes positive (suppressing SC) both at low p (Mott insulating region) and high p (normal metal). The result is a dome-like $T_c(p)$ function consistent with observations [4].

Figure 1 illustrates the generic phase diagram of hole-doped cuprates, showing how superconductivity arises adaptively between an antiferromagnetic insulating phase at low p and a normal metal at high p.

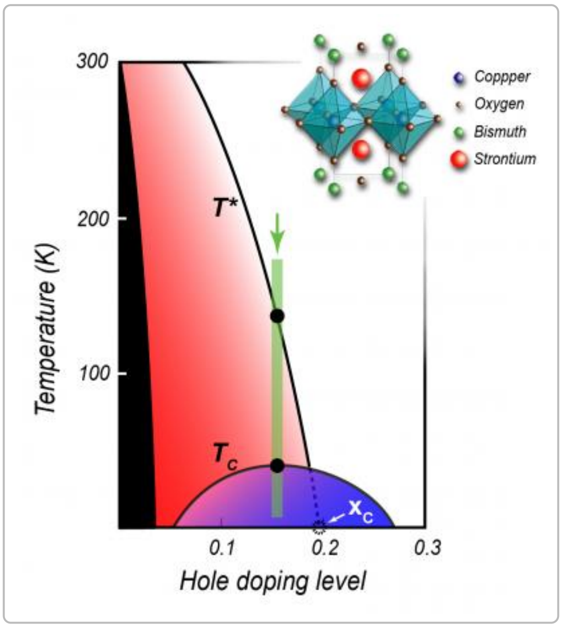

Figure 1: Schematic temperature–doping phase diagram of a cuprate superconductor (based on Bi-2201 [4]). At low hole doping (left), the CuO2 *planes are antiferromagnetic (black region). With increased doping, antiferromagnetism is suppressed and a dome-shaped superconducting phase (blue) appears below $T_c(p)$ [4]. A "pseudogap" phase (red) onsets below a temperature $T^(p)$, extending into the superconducting region [3] [19]. The pseudogap boundary $T^(p)$ terminates at a **quantum critical point** X_c near optimal doping $p*\approx0.19$ [3] [4], where fluctuating orders may enhance superconductivity. Above T^*, the system is a normal metal (white region). (Inset: crystal structure of a cuprate layer, with CuO$_2$ planes.)}

Adaptonic Free Energy and Adaptive Order Parameter Dynamics

The **adaptive ("adaptonic") concept** in HTSC envisions the superconductor as a complex system that *self-organizes* its internal state (pairing, magnetism, charge distribution) in response to external "stress" (e.g. doping level, lattice strain, or magnetic environment). In this view, competing orders and spatial inhomogeneities are not just incidental, but rather part of an adaptive free-energy minimization that can enhance the robustness of superconductivity [20]. We can formalize this idea by extending the GL free energy to include additional field(s) representing the **environmental stress and adaptive response**:

- **Coherence field $C(x)$:** a dimensionless scalar measuring local superconducting **coherence** or order (normalized such that $C=1$ in fully superconducting regions and $C=0$ in the normal state). For a single-component order parameter, one can take $C \propto |\Psi|^2 / |\Psi|^2_{\max}$, i.e. the fraction of paired electrons locally.

- **Stress field $\sigma(x)$:** a scalar (or tensor) field representing the *environmental stress* or frustration. In cuprates, σ can be associated with **hole concentration** or with fields that compete with superconductivity (such as local AFM or charge density wave (CDW) order). For example, $\sigma(x)$ might increase with deviation of local doping from the optimal value, or with the intensity of an opposing order (like AFM magnetization). High σ means the environment locally disfavors superconducting order (e.g. too few or too many carriers, or strong magnetic fluctuations), whereas low σ means a favorable environment for SC.

- **Adaptive temperature T_a:** an abstract “information temperature” controlling the balance between order and disorder in the adaptive system. This concept, drawn from adaptive system theory, plays a role analogous to a Lagrange multiplier for complexity: it weights the entropy term (described next) relative to energy. A low T_a favors a highly ordered, phase-separated state (minimizing free energy at the cost of less mixing entropy), while high T_a favors a homogeneous, disordered state (maximizing entropy). Physically, T_a could relate to actual temperature or disorder in the sample that allows phase separation. For our purposes, T_a is treated as an external parameter or a slow variable.

- **Interpretation field $\Phi(x)$:** an auxiliary field encoding the system’s *adaptive response* or internal reorganization in interpreting stress σ. In general adaptonic theory, Φ can represent configurational degrees of freedom that modulate how stress translates into order (for instance, pattern formation variables). In HTSC, one may associate Φ with the **spatial pattern of doping or charge order** – essentially how the holes are distributed across the lattice – or with other hidden order parameters that mediate between σ and C. For simplicity, one can think of Φ as parameterizing possible microstates (such as stripe configurations) that achieve a compromise between superconductivity and other tendencies.

Using these elements, we propose an **adaptonic free energy functional** for the superconductor, extending the GL free energy (4) to include **coupling between order and stress** and allowing spatially heterogeneous solutions (domains, stripes, etc.):

$$
\begin{aligned}
F_{\rm ad}[C,\Phi;\,\sigma,T_a] \;=\; \int d^3x \Big[U(C) \;+\; \frac{a}{2}\,|\nabla C|^2 \;+\; \frac{b}{2}\,(\nabla^2 C)^2 \;+\; G\big(C;\sigma(x)\big) \\
\;+\; H(\Phi) \;-\; T_a\, S_I(\Phi) \Big] \,.
\end{aligned}
\tag{6}
$$

This is a **Landau-like free energy** for the coupled fields $C(x)$ and $\Phi(x)$ in the presence of a static “environment” $\sigma(x)$ [21] [22]. Each term is as follows:

- **Local potential $U(C)$:** a double-well or multi-well potential in C that sets preferred values for the superconducting coherence. For a second-order SC transition, $U(C)$ can be like $U(C)=\frac{\tilde\alpha}{2}C^2 + \frac{\tilde\beta}{4}C^4$, with minima at $C=0$ (normal state) and $C=C_0>0$ (SC state). This term on its own reproduces the standard GL tendency (with C related to $|\Psi|^2$). In a strongly type-II superconductor, $U(C)$ might even allow multiple metastable minima, capturing states like pseudogap (partial pairing) versus full SC.

- **Gradient terms $a\,|\nabla C|^2$ and $b\,(\nabla^2 C)^2$:** these govern the **interface energy** and the possibility of **modulated order** [23]. The $|\nabla C|^2$ term is the usual surface tension favoring smooth $C(x)$ (domain walls cost energy proportional to their area).

The higher-order $(\nabla^2C)^2$ term is a “crystalline” stiffness that can stabilize finite-width domain walls or periodic oscillations (stripes) by penalizing sharp curvature [23]. Competing gradient terms allow for **“ecotonic” domains** – akin to ecological ecotones – where C transitions over a finite region rather than a sharp boundary. In HTSC, this models the finite thickness of SC/normal phase boundaries and the possibility of an inhomogeneous mixed phase (e.g. SC puddles separated by stripe order).

- **Coupling $G(C;\sigma)$:** an **order–stress coupling term** that encodes how environmental stress $\sigma(x)$ suppresses or enhances superconducting order [22]. A simple choice is *negative coupling* of the form

$$G(C;\sigma) = -\,g\,C\,S(\sigma)\,,$$

  where $S(\sigma)$ is some increasing function of the local stress magnitude. For example, if we identify σ with local hole *underdoping* (distance from optimal doping), then high $|\sigma|$ means far from optimal doping which should penalize superconductivity. G could be taken as $-g\,C\,\sigma^2$ (since σ^2 large = far from optimum). This term means the free energy is lowered (negative contribution) when C (superconducting order) appears in low-stress regions (small σ) but raised when C tries to exist in high-stress regions. As a result, the system energetically prefers **segregation**: superconductivity will **adaptively concentrate** in regions where σ is low (favorable doping or low competing order) and avoid regions of high stress [22]. This models the tendency of cuprates to form inhomogeneous states – e.g. SC regions avoid areas of strong AFM or charge order, and vice versa. Indeed, experiments in cuprates have found nanoscale phase separation and stripe patterns, where charge density waves modulate the superconducting gap spatially [24]. The coupling $G(C;\sigma)$ provides a *mechanism for such pattern formation* as an adaptive compromise between orders.

- **Information entropy $H(\Phi) - T_a S_I(\Phi)$:** a term representing the **entropy of the adaptive configurations** [25]. Here $H(\Phi)$ is an energy cost for a given configuration of the interpretation field (for instance, $H(\Phi)$ could penalize very rapid variations or specific unfavorable patterns of Φ), and $S_I(\Phi)$ is the configurational entropy (counting the degeneracy or complexity of pattern Φ). T_a multiplies S_I, acting like a temperature: at high T_a, the system favors maximizing entropy (more disordered or diverse configurations Φ), whereas at low T_a, it favors minimizing energy $H(\Phi)$ (more ordered, specific Φ). In HTSC context, Φ might represent, say, the pattern of coexisting charge order. A higher T_a would allow fluctuating, disordered charge textures, while a low T_a could lock in a specific pattern (like an static stripe lattice). This term thus governs the **“adaptability”** of the system – whether it explores many microstates or rigidly settles into one configuration.

The equilibrium equations of this adaptonic free energy are obtained by functional differentiation (akin to Euler–Lagrange equations for C and Φ) [26]. For instance, the variation $\delta F_{\text{ad}}/\delta C = 0$ yields a **generalized GL equation** for $C(x)$:

$$\underbrace{U'(C)}_{\text{GL: }\tilde{\alpha}C+\tilde{\beta}C^3} \;-\; a\,\nabla^2 C \;+\; b\,\nabla^4 C \;+\; \frac{\partial G(C;\sigma)}{\partial C} \;=\; 0\,. \tag{7a}$$

Similarly, $\delta F_{\text{ad}}/\delta \Phi = 0$ gives an equation for the interpretation field $\Phi(x)$:

$$H'(\Phi) \;-\; T_a\,\frac{\partial S_I}{\partial \Phi} \;=\; 0\,. \tag{7b}$$

(If T_a is treated as dynamic, one could also write an equation $\partial_t T_a = \cdots$ for its evolution [26], but here we consider it a control parameter.)

Equations (7a)–(7b) describe an **adaptive two-field dynamics**. The C-field (superconducting order) seeks a configuration that extremizes a modified GL free energy including the $b\,\nabla^4C$ term and coupling to stress σ. The Φ-field arranges itself such that the system achieves an optimal balance between minimizing $H(\Phi)$ and maximizing entropy $S_I(\Phi)$ at the given T_a. Physically, Φ can be seen as optimizing the **spatial arrangement** of σ (e.g. dopants or impurities) in response to C. In a **steady adaptive state**, high-stress (high-σ) regions will coincide with low C (suppressed superconductivity), and low-stress regions will carry most of the superconducting order – a form of **self-organized phase separation**. This theoretical picture aligns with observations of **stripe order and phase coexistence** in underdoped cuprates [27] [24]. For example, neutron and STM experiments find that charge density waves (a form of stress on SC) form stripes that spatially alternate with SC regions, effectively partitioning the system. In our adaptonic formalism, this emerges naturally: a spatially oscillating $\sigma(x)$ induces an oscillating $C(x)$ via the $G(C;\sigma)$ coupling, leading to an energy-lowering stripe phase.

Multi-Scale Adaptation and RG Perspective

It is insightful to consider the adaptonic concept in a **renormalization group (RG)** framework. The free energy (6) involves a hierarchy of scales: local free energy $U(C)$, mesoscopic gradient terms, and potentially long-range interactions through Φ (if Φ mediates coupling across domains). Integrating out short-wavelength fluctuations (e.g. of Φ or of C near the atomic scale) will yield an *effective* coarse-grained theory for C and slow modes of Φ. In practice, this means that parameters like $U(C)$, a, b, g, etc., “flow” with the RG scale. For example, **thermal/quantum fluctuations** in the competing orders can renormalize the effective $G(C;\sigma)$. Near the optimal doping critical point X_c (Fig. 1), the system is near a **quantum critical point (QCP)** where fluctuations of σ (pseudogap order) and C are both large [20]. RG arguments suggest that such critical fluctuations can actually boost superconductivity – effectively increasing the effective pairing interaction at long scales (sometimes termed “critical pairing” mechanism). Indeed, in the adaptonic view, the **quantum critical regime** ($p \approx p_{\text{opt}}$) is where the system is maximally adaptable: competing orders fluctuate and interconvert, allowing superconducting coherence to emerge in a channel that best relieves the free energy (like an adaptive search) [20].

One can formally derive RG beta functions for the adaptonic free energy. For instance, treating the coupling g in $G(C;\sigma)$ perturbatively, the RG flow might look like $\frac{dg}{d\ln \ell} = \beta_g(g, \dots)$, where ℓ is the length scale. If the system is below its upper critical dimension, fluctuations drive g towards a fixed point. A plausible scenario is **asymptotic adaption:** in the ultraviolet (UV, atomic scale) the system is highly frustrated (large bare g, meaning SC and AFM strongly compete), but under coarse-graining, the flow of g could *decrease* (screening of the competition) indicating that at long wavelengths the two orders can coexist more peacefully (emergent compatibility). In fact, a stable fixed point at finite g^* would mean the system self-organizes into a scale-invariant mosaic of superconducting and pseudogap regions, rather than either completely phase separating ($g\to$ large) or one phase eliminating the other ($g\to0$). The presence of the bi-quartic gradient term $b\,(\nabla^2C)^2$ also implies a preferred pattern scale (related to $\sqrt{a/b}$), which might correspond to the observed periodicity of charge stripes (typically ~4 lattice spacings in cuprates).

From a broad viewpoint, the **adaptonic concept** treats the HTSC as an **adaptive network** of interacting dof's (spins, charges, lattice distortions) seeking to minimize a generalized free energy. This approach unifies various aspects of the phase diagram: the dome shape and existence of an optimal doping, the presence of competing order (pseudogap or CDW) that persists into the SC state [19], and the nanoscale inhomogeneity (stripes, phase separation) [27]. In our formulation, all these emerge naturally from the **coupled order parameter–stress dynamics**. Superconductivity is strongest when the system can reorganize itself to **concentrate coherence in conducive regions and expel stress to other regions**, analogous to how a biological system adapts to stress by localizing damage and preserving function elsewhere. The equations (7) ensure that any spatial variation of C is accompanied by a compensating variation in σ (through G), and the term $S_I(\Phi)$ ensures the pattern Φ of such variations is as random or ordered as allowed by T_a (which could correspond to disorder in the material or thermal agitation).

Conclusion

Starting from the fundamental Hubbard model (Eq. 1) of strongly correlated electrons, we derived the key equations that underpin the **adaptive (adaptonic) superconductivity concept** in high-T_c cuprates. We showed how **magnetic exchange interactions** lead to d-wave pairing (Eq. 3) and how a **Ginzburg–Landau field theory** (Eqs. 4–5) describes the superconducting phase and its characteristic lengths. We then extended this framework with an **adaptive free energy functional** (Eq. 6) that incorporates the effects of doping (stress σ) and competing orders, allowing the system to **self-organize into coexisting phases**. The resulting Euler-Lagrange equations (7a, 7b) predict spatially modulated solutions that correspond to the experimentally observed stripe domains and pseudogap phenomena [24] [27]. This adaptonic approach provides a *meta*-description: the superconducting state is not a monolithic phase but an **adaptive ensemble** that continuously optimizes itself by redistributing order parameter coherence in space and across scales. Importantly, it yields falsifiable insights – for instance, the prediction of enhanced superconductivity near a quantum critical doping due to cooperative fluctuations [20], or the existence of universal scaling behavior in the domain structure near T_c. These can be tested by probing spatial inhomogeneities (e.g. with STM) or dynamic response near optimal doping.

In summary, the **adaptive concept in HTSC** is fully characterized by the coupled set of fundamental equations derived above. From the Hubbard Hamiltonian (Eq. 1) through the d-wave gap equation (3) and GL equations (5), to the proposed adaptonic free energy (6) and its variations (7), we have established a theoretical framework that *from first principles* connects microscopic electron interactions to macroscopic adaptive behavior. This comprehensive formulation shows how a high-T_c superconductor can be viewed as a **continuum adaptive field theory**: superconductivity, magnetism, and charge order emerge as mutually entwined degrees of freedom that **renormalize each other across length scales** – a processual, multi-scale order described by tensorial calculus (e.g. gradient terms) and RG flow of couplings. Ultimately, high-T_c superconductivity appears as a **phase of matter that thrives on the edge of instability**, sustained by the system's ability to adapt to and mitigate internal stresses. Such an adaptonic perspective not only deepens our understanding of HTSC phenomenology, but also hints at design principles for new materials – suggesting that achieving higher T_c may involve engineering systems with the right balance of competing interactions and adaptability to environmental cues [20].

Sources: Fundamental models and concepts are drawn from standard condensed matter theory [2] [5], while experimental inspiration for the adaptive scenario comes from observations of intertwined orders in cuprates [24] [27] and analyses of their phase diagram [4] [3]. The adaptonic free energy approach is a theoretical synthesis extending these principles [21] [22].

[1] [27] Quantum Breakthrough Sheds Light on Perplexing High-Temperature Superconductors
https://www.simonsfoundation.org/2024/05/09/quantum-breakthrough-sheds-light-on-perplexing-high-temperature-superconductors/

[2] Superconductivity in the doped Hubbard model and its interplay with ...
https://www.science.org/doi/10.1126/science.aal5304

[3] [19] [20] Closing in on the Pseudogap - Berkeley Lab – Berkeley Lab News Center
https://newscenter.lbl.gov/2011/03/24/pseudogap/

[4] High-Tc Phase Diagram [IMAGE] | EurekAlert! Science News Releases
https://www.eurekalert.org/multimedia/907985

[5] t-J model - Wikipedia
https://en.wikipedia.org/wiki/T-J_model

[6] [24] Interplay of superconductivity and charge-density-wave order in kagome materials
https://arxiv.org/html/2411.17818v1

[7] Mechanism of superconductivity in the Hubbard model at ... - NIH
https://pmc.ncbi.nlm.nih.gov/articles/PMC9388079/

[8] [9] [10] [11] [12] [13] [14] [15] [16] [17] [18] Ginzburg–Landau theory - Wikipedia
https://en.wikipedia.org/wiki/Ginzburg%E2%80%93Landau_theory

[21] [22] [23] [25] [26] GPT 14.10.25.odt
file://file-8xhqShF9m7ac7RAZqZwzSs