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Abstract

High-temperature superconductivity (HTSC) in cuprate materials emerges from a complex interplay of multiple electronic and lattice degrees of freedom. We present an **Adaptonic framework** for HTSC, treating the superconductor as an adaptive system with **multi-channel thermodynamic response constants**. We derive a generalized free energy formalism $F = E - \Theta_{\text{total}}\cdot S$ from first principles, where a *tensorial temperature* Θ_{total} encapsulates contributions from thermal, spin, charge, orbital, lattice (phonon), mixing, and external field channels. Each channel i is characterized by an adaptive temperature Θ_i (the conjugate force to the entropy S_i of that mode) and associated coupling terms. In this framework, competition and cooperation between superconducting order and the enigmatic pseudogap are quantified by a **mixing angle** θ_{mix} between their order parameters. We show that the cross-channel *mixing temperature* Θ_{mix} is determined by θ_{mix} via $\Theta_{\text{mix}} = \Theta_0\,\sin(2\,\theta_{\text{mix}})$, introducing a *configurational entropy of mixing* maximal at equal admixture. Real-world cuprates (YBa$*2$Cu$_3$O$*$$, Bi$*2$Sr$_2$CaCu$_2$O$*$$, HgBa$*2$CuO$*$$ peaks near optimal doping. We outline experimental protocols to extract each Θ_i from measurements – }$) are analyzed in terms of channel-specific energy scales: antiferromagnetic spin exchange, charge-density-wave order, loop-current orbital order, phonon modes, etc. We provide tables of Θ_i values (in Kelvin or meV units) and their doping dependence, demonstrating, for example, how Θ_{spin} decreases with doping while $\Theta_{\text{mix}*e.g.* neutron scattering for Θ_{spin}, X-ray/STM for Θ_{charge}, ARPES for Θ_{mix} – and we predict testable relationships such as the doping dependence of T_c vs. Θ_{mix} and total entropy capacity $\Theta_{\text{total}}(p)$. Figures include a phase diagram of hole-doped cuprates and a schematic tensor network of Θ illustrating channel coupling. By replacing ad hoc model parameters with a systematic set of adaptive response constants, the Adaptonic framework offers a predictive, computation-ready paradigm for HTSC materials design.

Theoretical Foundations

Multi-Channel Thermodynamics of Cuprates: Cuprate superconductors are layered copper-oxide compounds that currently hold the record for highest superconducting transition temperatures at ambient pressure [1]. Their undoped "parent" compounds are Mott insulating antiferromagnets [2], but upon carrier doping they develop high-T_c superconductivity. This dual nature implies that multiple degrees of freedom – spins, charges, orbitals, lattice vibrations – are strongly coupled and adapt as the system evolves from an ordered insulator to a superconductor. In an adaptonic view, we treat each *physical channel* χ_i (with i = thermal, spin, charge, etc.) as contributing an entropy S_i and having an associated *adaptive temperature* (or *intensity*) Θ_i governing its fluctuations. The **total thermodynamic "temperature"** of the system is a tensor (or vector in a simplified model) composed of all channel contributions: $\Theta_{\text{total}} = (\Theta_{\text{thermal}},\, \Theta_{\text{spin}},\,\Theta_{\text{charge}},\,\Theta_{\text{orbital}},\,\Theta_{\text{phonon}},\, \Theta_{\text{mix}},\,\Theta_{\text{field}},\dots)$. Physically, Θ_i generalizes the concept of temperature to *non-thermal channels* – for example, Θ_{spin} quantifies the effective "noise" or agitation in the spin system (related to spin fluctuation strength), Θ_{charge} does likewise for charge density modulations, etc. The system's free energy is then written as a generalized **free energy functional**:

$$F = E - \Theta_{\text{total}} \cdot S \ ,$$

where E is the internal energy and $\Theta_{\text{total}}\cdot S = \sum_{ij}\Theta_{ij}\,S_{ij}$ in the most general tensorial form (accounting for cross-correlations ${i!j}$) [3] [4]. In a simplified decoupled approximation, one can write $F \approx E - \sum_i \Theta_i S_i$, analogous to $F = E - TS$ in standard thermodynamics but with T replaced by a sum of channel-specific "temperatures." Each Θ_i serves as a thermodynamic force tending to maximize the entropy S_i of channel i, subject to the energetic costs E of excitations in that channel.

Key Adaptive Response Constants: In our formalism, Θ_i are *adaptive response constants* that measure the system's ability to absorb energy into channel i as entropy. High Θ_i means channel i can contribute a lot of entropy at little energy cost (analogous to a high temperature reservoir), whereas low Θ_i indicates that channel is "stiff" or not easily excited. For example, a large Θ_{spin} in underdoped cuprates reflects strong spin fluctuations (the spin system readily contributes entropy), whereas a low Θ_{thermal} at low ambient temperatures simply reflects that the lattice temperature is low. The adaptonic hypothesis is that the cuprate finds a **balanced distribution** of Θ_i that maximizes stability: as external conditions (doping p, temperature T, field H, etc.) change, the system redistributes entropy among channels to minimize free energy. This perspective explains why cuprates exhibit multiple crossovers and phase transitions – the system shifts between spin order, pseudogap, superconductivity, charge order, etc. as needed to adapt to the thermodynamic constraints.

Entropy and Free Energy in Multi-Channel Form: We formalize the entropy contributions as $S = \sum_i S_i + \sum_{i<j} S_{ij} + \cdots$, where S_i is the self-entropy of channel i and S_{ij} are cross-channel (configurational) entropies coupling channels i and j [4]. For instance, a state with coexisting superconductivity and pseudogap might have an extra entropy term $S_{\text{SC-PG}}$ reflecting the number of ways the two orders can spatially or temporally coexist. In general, S_{ij} can be related to the mutual information or cross-correlation between fluctuations in i and j [5] [6]. The generalized free energy then reads:

$$F \;=\; E \;-\; \frac{1}{2}\sum_{ij}\Theta_{ij}\,S_{ij}\;,$$

with the factor $1/2$ ensuring each pair is counted once [4]. In matrix form, Θ_{ij} is a symmetric positive-definite **temperature tensor** coupling the entropy of mode i with mode j [3]. The **diagonal elements** Θ_{ii} correspond to the intrinsic temperature of channel i (fluctuation intensity of mode i itself), while **off-diagonals** Θ_{ij} ($i\neq j$) represent how much channels i and j *correlate* or mix [5]. A positive off-diagonal $\Theta_{ij}>0$ means modes i and j fluctuate in unison (correlated entropy production), whereas $\Theta_{ij}<0$ would indicate they counter-fluctuate (one's ordering suppresses the other's entropy) [7]. In cuprates, we expect significant positive off-diagonals between, say, spin and charge channels (since spin and charge fluctuations often coexist in stripe-like correlations) and negative off-diagonals between, for example, superconductivity and certain normal-state fluctuations (since the onset of superconducting order reduces low-energy spin/charge entropy).

Equations of State: Extremizing F with respect to an order parameter of a given channel yields an equation of state involving Θ_i. For a channel order parameter X_i (e.g. staggered magnetization for spin, superconducting gap Δ for SC, etc.), one generally gets:

$$\frac{\partial F}{\partial X_i} = \frac{\partial E}{\partial X_i} - \sum_j \Theta_{ij}\,\frac{\partial S_j}{\partial X_i} = 0\;,$$

at equilibrium. This indicates that at a phase transition, an increase in order X_i (which typically lowers E) must be compensated by the entropy cost $\Theta_{ij}\partial S_j$ it imposes on all channels j. In other words, Θ_{ij} act like "generalized temperatures" that oppose ordering by rewarding entropy. A high Θ_{ij} linking to channel j means that ordering in i will be strongly penalized if it reduces entropy in channel j. For instance, if Θ_{spin} is large, the system resists any ordering (like superconductivity) that would significantly reduce spin fluctuations entropy. Conversely, if Θ_{spin} decreases (e.g. upon doping, weakening AF correlations), it becomes easier for another order (like SC) to set in, as there is less entropic penalty from the spin sector.

Adaptive Equilibrium: The adaptonic principle asserts that the stable state at given external conditions minimizes F by *redistributing Θ and S among channels*. Unlike a single-order-parameter Landau theory, here the "competitors" (spin, charge, SC, etc.) are not just fighting for electronic states but also exchanging entropy. One can think in terms of **partial effective temperatures**: each channel can have its own effective temperature in a non-equilibrium sense, but at true equilibrium all channels settle into a *consistent Θ_{total}*. In practice, when a channel orders (reduces its entropy), the released entropy must go into other channels or into the heat bath. The Θ framework tracks this flow quantitatively.

Derivations of Central Formalism

Free Energy Derivation (First Principles): The form $F = E - \Theta_{\text{total}}\cdot S$ can be derived from the **maximum entropy principle**. Consider the total system as a collection of subsystems (channels) in weak contact, exchanging generalized "heat." In equilibrium, the condition for maximum total entropy at fixed total energy leads to *equalization of generalized temperatures* across subsystems. For a simple case, let channel i have internal energy E_i and entropy $S_i(E_i)$ (with S_i' denoting $\partial S_i/\partial E_i = 1/T_i$ in conventional thermodynamics). Equating $\partial S_{\text{total}}/\partial E_i = 0$ for energy redistribution gives $1/\Theta_i = \partial S_i/\partial E_i$ equal for all i. Thus Θ_i plays the role of temperature for channel i. The total free energy differential is $dF = \sum_i (dE_i - \Theta_i\,dS_i)$. Assuming a single energy reservoir and summing yields $dF = dE_{\text{total}} - \sum_i \Theta_i\,dS_i$. Integrating gives $F = E_{\text{total}} - \sum_i \Theta_i S_i + \text{const}$. In the presence of correlations S_{ij}, one generalizes this reasoning using the Gibbs entropy for joint distributions, $S_{\text{total}} = -k_B\langle \ln p({X_i})\rangle$, and finds cross terms in dF leading to the tensor Θ_{ij} coupling dS_{ij} [8]. This yields the more general $F = E - \frac{1}{2}\sum_{ij}\Theta_{ij}S_{ij}$ given earlier.

Decomposition of Θ_{total}:

We now *fully decompose* Θ_{total} into the contributions of each physical channel and their couplings. In matrix form:

$$\Theta_{\text{total}} \equiv \begin{pmatrix} \Theta_{\text{thermal}} & \Theta_{\text{thermal-spin}} & \Theta_{\text{thermal-charge}} & \cdots \\ \Theta_{\text{thermal-spin}} & \Theta_{\text{spin}} & \Theta_{\text{spin-charge}} & \cdots \\ \Theta_{\text{thermal-charge}} & \Theta_{\text{spin-charge}} & \Theta_{\text{charge}} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

For clarity, we denote Θ_{mix} specifically as the coupling between the **superconducting (SC) channel** and the **pseudogap (PG) channel**, since these two orders are central to cuprates. Thus if we label SC as channel s and PG as channel p, then $\Theta_{\text{mix}} \equiv \Theta_{sp} = \Theta_{ps}$ is the 2×2 off-diagonal element coupling superconductivity and pseudogap. Similarly, one could have $\Theta_{\text{spin-charge}}$ for spin–charge coupling, $\Theta_{\text{spin-phonon}}$ for spin–lattice coupling, etc. In total, Θ_{total} for a cuprate might be expressed as:

- **Thermal:** $\Theta_{\text{thermal}} = T$ (the ordinary temperature of the lattice and electrons). This sets the baseline for random thermal excitations in all channels.

- **Spin:** Θ_{spin} quantifies the spin system's effective temperature. It is related to the energy scale of spin fluctuations (paramagnons, magnons). In a Mott insulator, Θ_{spin} is on the order of the superexchange J (hundreds of meV); as doping increases and long-range antiferromagnetic order disappears, low-energy Θ_{spin} drops, though high-energy spin excitations persist (see §Empirical Applications).

- **Charge:** Θ_{charge} relates to charge density wave (CDW) or charge fluctuations. It can be associated with the energy scale of charge ordering tendencies (typically a few meV to tens of meV in cuprates) or the temperature at which charge ordering onsets.

- **Orbital:** Θ_{orbital} addresses orbital currents or other orbital degrees (such as loop-current order proposed for the pseudogap). If the pseudogap involves circulating currents breaking time-reversal symmetry, Θ_{orbital} corresponds to the onset temperature of that orbital magnetism (the pseudogap T^* itself in some theories).

- **Phonon:** Θ_{phonon} characterizes the lattice's contribution. A rough scale is the Debye temperature or prominent optical phonon energy. Cuprate Debye temperatures are a few hundred K; specific phonon modes (e.g. Cu–O bond-stretching modes ~70 meV) would correspond to ~800 K in energy units.

- **Field:** Θ_{field} represents external field effects (magnetic field, pressure, etc.). For a magnetic field H, one can define $\Theta_{\text{field}} = \mu_0 H\, / (\partial S/\partial H)$ in analogy to a magnetic entropy contribution. In practice, applying a magnetic field introduces vortex excitations and Zeeman effects, effectively injecting entropy. We include Θ_{field} to incorporate how an external field "heats" certain modes (e.g. vortices in SC act as localized hot spots of entropy).

- **Mixing (SC–PG coupling):** Θ_{mix} couples the SC order parameter and the pseudogap order parameter. This term is crucial for modeling the *competition or cooperation* between superconductivity and the pseudogap state.

We emphasize that **Θ_{total} is not a simple sum of scalars** but a composite object capturing both independent channel "temperatures" and their interactions. However, in many cases one can simplify to an *effective scalar* by writing $\Theta_{\text{total}} = \Theta_{\text{thermal}} + \Theta_{\text{spin}} + \Theta_{\text{charge}} + \cdots$ if cross-correlations are weak or if one is interested in an overall energy scale. In such an approximation, one could think of Θ_{total} as an *effective temperature* that is higher than the actual lattice temperature due to additional fluctuation channels (for example, even at low lattice temperature T, the presence of strong spin fluctuations acts like an "internal heat" keeping the electronic system disordered, analogous to an elevated Θ_{spin}).

Mixing Angle Between Superconducting and Pseudogap Orders: A central new concept is the **mixing angle** θ_{mix}. We introduce a two-component order parameter that combines the superconducting (SC) gap Δ and the pseudogap (PG) magnitude W (whatever quantity characterizes the pseudogap state, e.g. an energy gap in the anti-nodal quasiparticle spectrum). We can represent the combined order as a vector in a two-dimensional order parameter space:

$$\mathbf{\Psi} = \begin{pmatrix} \Delta \\ W \end{pmatrix} = M \begin{pmatrix} \cos\theta_{\text{mix}} \\ \sin\theta_{\text{mix}} \end{pmatrix},$$

where $M = \sqrt{\Delta^2 + W^2}$ is an overall magnitude and θ_{mix} is the angle delineating how much of the order is in the SC vs PG direction. Two limiting cases are: $\theta_{\text{mix}} = 0^\circ$ (pure superconductivity, $W=0$, $\Delta=M$) and $\theta_{\text{mix}} = 90^\circ$ (pure pseudogap with no superconductivity). In general, θ_{mix} quantifies the *mixture* of the two orders in the system's ground state or fluctuating state.

Now, how does θ_{mix} determine Θ_{mix}? To derive this, consider how the **entropy of the two-component system** depends on the mixture. If the SC and PG orders compete or share a common "order parameter pool," there is a *configurational entropy* associated with distributing the order between them. A simple analogy is a two-phase mixture: if a fraction x of the system is in phase A and $(1-x)$ in phase B, there is a mixing entropy $S_{\rm mix} = -k_B[x\ln x + (1-x)\ln(1-x)]$. This is maximized at $x=0.5$ (equal mixture) and zero at $x=0$ or 1 (pure phases). In our case, $\cos^2\theta_{\text{mix}}$ could play the role of x (fraction of "order parameter weight" in SC) and $\sin^2\theta_{\text{mix}}$ the fraction in PG. The analogy suggests that *entropy is maximized at intermediate $\theta_{\text{mix}} = 45^\circ$*.

We propose that Θ_{mix}, the conjugate force to the SC–PG cross entropy, is proportional to the *product* of the SC and PG order components. Geometrically, the product ΔW is $\Delta W = M^2 \cos\theta_{\text{mix}}\sin\theta_{\text{mix}} = \frac{M^2}{2}\sin(2\theta_{\text{mix}})$. Thus it is natural to let:

$$\Theta_{\text{mix}} = \Theta_0 \, \sin(2\,\theta_{\text{mix}}) \ ,$$

where Θ_0 is a characteristic *maximal mixing temperature* (with dimensions of energy or temperature) when the two orders are equally mixed ($\theta_{\text{mix}}=45^\circ$, $\sin2\theta_{\text{mix}}=1$). This form ensures that Θ_{mix} is **zero when $\theta_{\text{mix}} = 0^\circ$ or 90°** (no coupling if only one order is present) and **peaks at $\theta_{\text{mix}} = 45^\circ$** (maximal entropic interaction when SC and PG are of equal weight). Essentially, Θ_{mix} is an off-diagonal term Θ_{sp} linking SC (s) and pseudogap (p) channels, and we model it as $\Theta_{sp} \propto \langle s\,p \rangle$. If the SC and PG order parameters can be considered roughly complementary (as some theories suggest – the pseudogap consumes some density of states that would otherwise form Cooper pairs [9] [10]), then *when both orders coexist, there are more possible microscopic configurations (domains, fluctuations) than if one order rigidly excludes the other*. This extra freedom is captured in a positive Θ_{mix} promoting fluctuations where SC and PG locally interconvert.

To derive the impact on entropy more formally: define an entropy S_{sp} associated with distributing a fixed order parameter magnitude between SC and PG. For small fluctuations, one can show S_{sp} is maximized at intermediate angles, and a quadratic expansion yields $S_{sp} \propto \sin^2(2\theta_{\text{mix}})$ to leading order. The thermodynamic conjugate is $\Theta_{sp} = \partial E/\partial S_{sp}$, which then is proportional to $\sin(2\theta_{\text{mix}})$ (since $\partial S_{sp}/\partial\theta_{\text{mix}} \propto \sin(4\theta_{\text{mix}})$, zero at extremes and central). Thus Θ_{mix} takes the form $\Theta_0 \sin(2\theta_{\text{mix}})$ as the simplest representation consistent with symmetry (note that $\sin(2\theta)$ is 2π-periodic and invariant under exchanging SC and PG since $\sin[2(90^\circ-\theta)] = \sin(2\theta)$).

Modulation of Configurational Entropy: Because Θ_{mix} multiplies the SC–PG entropy S_{sp} in F, it effectively modulates how favorable mixed configurations are. When θ_{mix} is near 45° (optimal mixing), Θ_{mix} is high, meaning there is a strong "entropic drive" favoring states that explore both SC and PG configurations (analogous to a high temperature that favors disorder). This can be interpreted as *frustration* between orders: neither order can fully freeze out the other, leading to many fluctuating states (which is entropically advantageous). At the ends ($\theta_{\text{mix}}\to 0$ or 90°), $\Theta_{\text{mix}}\to 0$, indicating no extra entropy from mixing – the system is either purely SC or purely pseudogap with no ambiguity, hence fewer available microstates. In essence, $\Theta_{\text{mix}}(\theta_{\text{mix}})$ acts like an *effective temperature for phase configurations*: it is highest when the system is in a heterogeneous state (many configurations) and zero when the system is locked into a single phase configuration.

Entropy Content of Competing Orders: It is instructive to consider the entropy balance when SC and pseudogap coexist. The pseudogap phase by itself has significant entropy reduction relative to a metal (since it removes states near E_F). Superconductivity at $T=0$ has even lower entropy (gap opening and coherence). If both orders exist partially, one might naïvely expect an even larger entropy loss – yet experiments (specific heat, NMR) indicate that the total entropy in the SC state of underdoped cuprates is not as low as one might get if pseudogap and SC effects were simply additive [11]. This hints that the two orders share a common "entropy budget." Our framework accounts for this by the cross-entropy S_{sp}: some fraction of the lost entropy from one order can be compensated by fluctuations in the other. Θ_{mix} quantifies the energy scale associated with these compensating fluctuations. In practical terms, this means the presence of a pseudogap above T_c provides a reservoir of fluctuations (pre-formed pairs, etc.) that reduce the entropy cost of forming the superconducting state – essentially an adaptive pathway to SC. Conversely, once SC sets in, the residual pseudogap (if any) is not as thermodynamically costly as it would be without SC, because the system can continuously interchange spectral weight between the two gaps.

Empirical Applications to HTSC Materials

Figure 1: Schematic phase diagram of a hole-doped cuprate superconductor as a function of temperature and doping (hole concentration p). At low doping ($p\to0$), long-range antiferromagnetic (AF) order persists up to a Néel temperature T_N of a few hundred K (green region). Upon increasing p, AF order vanishes beyond $p\approx0.05$, and a pseudogap phase (purple) opens at a temperature $T^*(p)$ that decreases with doping [12]. Superconductivity (SC, red dome) emerges around $p\approx0.05$ and T_c rises to a maximum at **optimal doping** $p_{\rm opt}\approx0.16$ [13], then falls to zero near $p\approx0.27$. The pseudogap phase terminates at a critical doping $p^*\approx0.19$ *independent of temperature (vertical line), beyond which a more conventional Fermi-liquid metal is recovered [12]. The overlapping of the pseudogap and SC regions in underdoped samples gives rise to competition and mixing between the two orders (blue hashed area). In this regime, the Adaptonic framework decomposes the total "thermal" energy Θ_{total} into multiple channels: thermal (lattice) temperature T, spin fluctuations (green), charge order (orange, appears around $p\sim0.1$), orbital loop currents (if any, associated with pseudogap, purple), and the SC–PG mixing (blue). The balance of these contributions varies with doping and temperature.* [1] [12]

To demonstrate the Adaptonic framework in action, we analyze three representative cuprate families – **Y123** (YBa_2Cu_3O), **Bi2212** ($Bi_2Sr_2CaCu_2O$), and **Hg1201** ($HgBa_2CuO$) – highlighting the physical scales of each Θ_i channel and how they can be extracted from experiments. These materials span a range of structures (double-layer vs single-layer, presence/absence of CuO chains, etc.) yet all realize the universal phase diagram of Fig. 1.

Channel Decomposition and Scales

We summarize typical energy/temperature scales for each channel in Table 1, for an optimally or underdoped cuprate:

- **Thermal (Lattice) Channel:** Simply the bath temperature T. In our context, $ \Theta_{\text{thermal}} = T$ sets the base for $F = E - TS$ portion. At optimal doping, T_c itself is on the order of 90 K (Y123), 95 K (Hg1201) or 85 K (Bi2212) [14]. Thus around T_c, thermal energy k_BT is ~8 meV. This is much smaller than many internal scales (spin exchange, pseudogap magnitude), indicating that at T_c the thermal channel alone is insufficient to overcome ordering tendencies – additional effective "temperature" from other channels must contribute to suppress order above T_c.

- **Spin Channel (Magnetic):** An undoped cuprate like La$*2$CuO$_4$ or YBa$_2$Cu$_3$O$_6$ has an antiferromagnetic exchange $J\approx 100$–150 meV (roughly 1200–1700 K) and a Néel temperature $T_N\sim300$–400 K. We therefore identify a high intrinsic $\Theta*)$ in the parent compound (reflecting robust spin entropy at room temperature) [10]. Doping rapidly reduces static AF order, so the }} \sim \mathcal{O}(10^3~{\rm K*low-frequency* spin entropy (e.g. scattering at $\omega\approx0$) vanishes by $p\approx0.05$. However, *dynamic* spin fluctuations persist to high energies even in doped superconductors. Notably, in Y123 at optimal doping, neutron scattering reveals a **resonant magnetic mode** at $\hbar\omega_{\text{res}} \approx 41$ meV in the superconducting state [15]. This resonance (often considered a spin-triplet exciton) corresponds to an energy of ~475 K, which we can take as an effective $\Theta_{\text{spin}} $ scale for the resonant spin fluctuation channel. More generally, resonant and high-energy spin excitations in cuprates carry substantial spectral weight; RIXS (Resonant Inelastic X-ray Scattering) studies have shown that the *integrated spin spectral weight changes little with doping*, remaining comparable to the undoped magnon spectrum [16]. This implies that $ \Theta_{\text{spin}}$ at high energy remains large (~1000 K) across the phase diagram, even though at low energy the spin channel is gapped out in the superconducting state (a spin gap of tens of meV opens). In adaptonic terms, doping shifts Θ_{spin} from low-frequency (order-parameter-like) to high-frequency (fluctuation) modes, rather than eliminating the spin channel. The spin channel thus provides a reservoir of entropy (through fast fluctuations) that can impede or assist ordering in other channels.

- **Charge Channel:** Cuprates are doped by adding or removing oxygen or cations, which introduces hole or electron charge carriers. In addition to uniform charge flow, many cuprates exhibit **charge density wave (CDW)** order at moderate doping. In Y123 (YBa$*2$Cu$_3$O$*$ with this ordering scale, on the order of 100–150 K in underdoped Y123. Similarly, La-based cuprates (La$}$), a short-range incommensurate CDW sets in around $p\approx0.12$ and can be enhanced by a high magnetic field once superconductivity is partially suppressed. X-ray scattering detects this charge order emerging below $T_{\text{co}}\sim 50$–70 K at zero field, but under a 15–18 T field (which weakens SC), the charge order can persist up to ~150 K [17]. We identify $\Theta_{\text{charge}*{2-x}Sr_xCuO_4$) show stripe order (combined spin and charge modulation) with onset $T*$ in cuprates is in the }}\sim50$ K near $x=1/8$. Bi2212 and Hg1201 also show charge modulation in spectroscopic imaging STM and X-ray scattering, though often short-range and peaking around 100 K or lower. As a broad statement, $ \Theta_{\text{charge}**tens to few hundreds of Kelvin** range, lower than the spin exchange scale. Doping dependence: Θ_{charge} is negligible at very low doping (no mobile charges), rises to a maximum in the underdoped regime (where charge order is strongest, e.g. $p\sim0.1$–0.15 for CDW in Y123), then likely drops in overdoped where charge fluctuations become more conventional (Fermi liquid with no CDW). The adaptive role of the charge channel is evident in the competition between CDW and SC: when SC is weakened (by underdoping or magnetic field), the charge channel “heats up” (fluctuations grow, order appears), indicating an **anti-correlation** $\Theta_{\text{SC-charge}} < 0$. Conversely, when superconductivity dominates (optimal doping, zero field), charge order is mostly absent, implying the charge entropy is absorbed into the SC condensate.

- **Orbital Channel:** By “orbital” we refer to intra-unit-cell currents or any ordering involving the orbital motion of electrons that could be linked to the pseudogap. A prominent proposal is **loop current order** (Varma’s theory), in which circulating currents in the CuO$*2$ unit cell produce a magnetic moment and break time-reversal symmetry without affecting lattice translation. Experimentally, **polarized neutron diffraction** on YBCO and Hg1201 detected a novel magnetic signal in the pseudogap phase [18], and Kerr effect measurements on YBCO found the onset of a weak spontaneous magnetization (time-reversal symmetry breaking) near $T^$ [19]. These observations support an orbital magnetic order setting in at $T^$, which would be the $\Theta*$ scale. In underdoped Y123, $T^}*$ can be as high as ~300 K at $p=0.1$ [10], and it decreases to ~0 at $p*^\approx0.19$ [12]. Hg1201, with $T_c^{\max}\approx95$ K, has a pseudogap onset around $T^*sim350$ K at $p\sim0.1$ as well. We thus assign $\Theta_{\text{orbital}} \sim T*^$ in the underdoped regime, of order a few hundred Kelvin. Doping dependence: $\Theta_{\text{orbital}}(p)$ drops toward zero at the critical doping p^* where the pseudogap phase disappears [12]. Beyond $p*^$, we consider $\Theta_{\text{orbital}}=0$ (no orbital order/fluctuations of that type). In the adaptonic picture, the orbital channel is active only in the pseudogap regime, and it competes with superconductivity (since orbital order like loop currents vanish when the pseudogap closes under optimal doping). This competition is likely reflected by a negative coupling $\Theta_{\text{orbital-SC}}$: the development of SC tends to weaken the orbital order parameter (as seen by the Kerr signal dropping below T_c).

- **Phonon Channel:** Lattice vibrations contribute to thermodynamic entropy and can interact with the electronic orders. In conventional BCS theory, a phonon mode of frequency Ω contributes a Bose-Einstein entropy and can mediate pairing. In cuprates, phonons are generally not the primary pairing glue, but certain modes (e.g. the B$*{1g}$ oxygen bond-buckling mode) show renormalization at T_c. We take $\Theta*$ since lattice acts as a heat bath. They become crucial if one considers non-equilibrium situations or strong electron-phonon resonance (which we will not delve into here).}}$ roughly as a Debye temperature Θ_D. For Y123, $\Theta_D\sim 400$ K; for Bi2212, similar order. A more relevant specific mode energy might be ~ 40–80 meV (320–640 K) for Cu–O stretch modes that couple to charge density waves and electrons. These numbers suggest $\Theta_{\text{phonon}}\sim \mathcal{O}(10^2~{\rm K})$. Doping typically softens some phonon modes (e.g. the half-breathing mode softens near charge order wavevector at certain doping). Thus Θ_{phonon} might slightly decrease with doping due to softer lattice (more polarizable with more carriers). However, phonon entropy is mostly a monotonic function of temperature and not strongly sensitive to the electronic phases except near structural instabilities. In our framework, we primarily include phonons as part of $ \Theta_{\text{thermal}

- **SC–PG Mixing:** Θ_{mix} is not an independent “physical” energy scale like the above, but rather a derived coupling. Its magnitude Θ_0 can be estimated by considering that at optimal doping, the system has both SC and PG characteristics and this mixed state is in some sense the most fluctuating. If we assume Θ_{mix} peaks around optimal doping $p_{\rm opt}\approx0.16$ where $\theta_{\text{mix}}\approx45^\circ$, then $\Theta_0

\approx \Theta_{\text{mix}}(45^\circ)$. What should that be? One approach: consider the entropy difference between having two separate gaps vs one combined gap. Near optimal doping, $T^*approx T_c$ (the pseudogap onset merges with T_c), so one could argue the system almost has one order parameter instead of two distinct ones. In underdoped (two orders) vs optimal (one effective order), the difference in entropy at T_c could be a measure of mixing entropy. Empirically, the electronic specific heat jump at T_c is smaller in underdoped cuprates than what one would expect by extrapolating from overdoped (where no pseudogap)* [11] *. This is because some entropy is already lost to the pseudogap above T_c. However, Tallon and Loram* [12] *find that one can define a* mean-field $T_c^*(p)$* (parapairing temperature) that runs above T_c across the dome, distinct from pseudogap T^*. This parapairing may be viewed as a consequence of mixing: even above T_c, superconducting correlations (paracoherence) extend up to this $T_c*^(p)$ line. We can surmise that $ \Theta_{\text{mix}}$ is related to the *paracoherent pairing energy*. Taking optimal doping, $T_c^{\rm opt}\approx93$ K (Y123) and pseudogap $T^*approx120$ K (if any) – the fact they are close suggests maximal mixing. So one might set $\Theta_{\text{mix}}^{\max} \sim 100$ K scale. This is admittedly a rough estimate. A more microscopic estimate could use the GL free energy coupling term: write a free energy density $f_{\text{mix}} = -\gamma |\Psi_{\text{SC}}|^2 |\Psi_{\text{PG}}|^2$ for a bi-order parameter theory (with γ a coupling). Such a term yields a contribution to entropy when both orders fluctuate. Converting γ to an energy scale, one expects it to be of order of the smaller gap magnitude. The SC gap $\Delta_{\rm sc}\approx20$–30 meV at optimal doping, pseudogap magnitude $E_{\rm pg}\approx30$–50 meV (in anti-nodal spectra). If $ \Theta_{\text{mix}}$ was on order of these, that's 200–500 K. However, it's unlikely to be so large because Θ_{mix} specifically multiplies* configurational *entropy (which is a smaller subset of total entropy). We will use $\Theta_0 \sim 200$ K as a ballpark and refine by noting doping trends: $\Theta_{\text{mix}}(p)$ should form a dome peaking at $p_{\rm opt}$. At $p\to0$ or $p>p^*$, $\theta_{\text{mix}}\to 0$ or 90° (only one order present), so $\Theta_{\text{mix}}\to0$. Near $p_{\rm opt}$, $\theta_{\text{mix}}\approx45^\circ$, so $\Theta_{\text{mix}}\approx \Theta_0$. Thus *qualitatively, $\Theta_{\text{mix}}(p)$ will track the superconducting $T_c(p)$ dome*. This is one of the **predictions** of the adaptonic model: we expect a close correlation between $T_c(p)$ and $\Theta_{\text{mix}}(p)$, since optimal coexistence of SC and PG (which maximizes Θ_{mix}) also yields maximal T_c. We shall return to this as a testable relationship.

Table 1 below compiles these estimates and their doping dependence:

Channel (i)	Physical Meaning	Θ_i Scale (Optimal/Underdoped)	Doping Trend (qualitative)
Thermal ($ \Theta_{\text{thermal}} $)	Lattice temperature (environment)	– (set by external T) – e.g. $T_c\sim90$ K at opt.	N/A (external control parameter)
Spin ($ \Theta_{\text{spin}}$)	AF spin fluctuations	~ 1000 K (high-energy magnons); ~ 500 K (resonance) [15]	High at low p (AF order), shifts to high-$ \omega$ fluctuations; remains substantial even overdoped [16], low-ω part suppressed by SC

Channel (i)	Physical Meaning	Θ_i Scale (Optimal/Underdoped)	Doping Trend (qualitative)
Charge ($ \Theta_{\text{charge}}$)	CDW/stripe fluctuations/ order	~ 100 K (Y123 CDW onset under field) [17] ; $~50$ K (no field)	Rises from 0 at $p=0$ to max around $p\sim0.1$–0.15, then decreases (negligible by $p\to0.2$+)
Orbital ($ \Theta_{\text{orbital}}$)	Loop-current (PG order)	$\sim T^*$, e.g. 200–300 K at $p=0.1$* [10] *; 0 at $p*^\approx0.19$ [12]	Decreases with p, vanishes at p^* (critical doping where PG ends)
Phonon ($ \Theta_{\text{phonon}} $)	Lattice vibrations	~ 300–500 K (Debye temp or optical mode)	Mostly constant; slight decrease with doping (softening modes)
Mixing ($ \Theta_{\text{mix}}$)	SC–PG coupling (config. entropy)	$\sim \Theta_0$; est. $ \sim 100$–200 K (peak near opt. doping)	Forms a dome peaking at $p_{\rm opt} \approx0.16$; 0 at low and high doping (only one order present)
Field ($ \Theta_{\text{field}}$)	External field-induced entropy	e.g. 15 T field induces CDW up to 150 K ⇒ $ \Theta_{\text{field}}$ analogous to that effect	Increases with applied field; couples to spin (Zeeman) and vortex excitations in SC (roughly $\propto H^2/8\pi$ for vortex energy density)

YBa$*2$Cu$_3$O$*$ (Y123): Y123 is a two-CuO$*2$-layer cuprate with a maximum $T_c\approx92$ K at $ \delta\approx0$ (optimally doped $p\approx0.15$)* [20] *. It famously has **Cu–O chain layers** that provide additional carriers and anisotropy. These chains also stabilize charge order at certain dopings. From an adaptonic perspective, Y123 has rich interplay: a spin resonance at 41 meV (present for $p\approx0.1$–0.2, $ \Theta*\sim475$ K), a }*pseudogap* up to $T^*\sim300$ K at $p=0.1$ (hence $\Theta_{\text{orbital}}*\sim300$ K *dropping to 0 by $p=0.19$), and a* field-enhanced CDW *ordering around 50–150 K at $p\approx0.12$ (so $ \Theta_{\text{charge}}\sim100$ K scale)* [17] *. Y123 also shows an onset of loop-current-like order via neutron polarimetry at $T*^$ (~240 K for $p=0.12$) [18] . All these numbers fit the multi-channel table above. Y123's Θ_{mix} is maximal near $p\approx0.15$–0.16, where pseudogap T^* and T_c nearly coincide (around 100–120 K) – indeed optimal Y123 samples show very little pseudogap above T_c, suggesting $\theta_{\text{mix}}\to 0$ or 45°? Actually, near optimal doping the pseudogap collapses or possibly merges with SC, implying the system chooses a single-channel route (pure SC) at low T. In our angle language, as $p\to p_{\rm opt}$ from below, θ_{mix} decreases from 45° toward maybe 30°, meaning SC dominates pseudogap. But just above T_c, the pseudogap reappears (e.g. ARPES sees partial gap open above T_c even at optimal doping in some cuprates). This dynamic behavior could be described by a temperature-dependent $\theta_{\text{mix}}(T)$: at low T in the SC state, θ_{mix} is smaller (more SC), whereas approaching T_c from below, θ_{mix} rotates toward equal mixture as the SC amplitude diminishes and PG reappears. This underscores that Θ_{mix} can itself be temperature-dependent in a fixed-doping sample – rising near T_c. We might associate $\Theta_{\text{mix}}(T)$ with the strength of SC–PG phase fluctuations near T_c, which are known to be significant (fluctuation diamagnetism, Nernst effect, etc., are observed above T_c in underdoped cuprates, indicating the mixing of orders). Y123 is an excellent testbed for the framework because all channels are active: spin (neutron, NMR), charge (X-ray), orbital (neutron/Kerr), SC (obviously, with known gap values), and mixing (manifest in competition phenomena). Model calculations within this framework could predict, for example, how T_c is suppressed when Θ_{charge} (CDW) is elevated by a magnetic field – consistent with the empirical observation that at $p=0.12$, T_c is lowered by field as CDW grows [17]. In our terms, increasing Θ_{field} adds entropy to the system via vortices and also effectively raises Θ_{charge} (since field activates CDW fluctuations), thus reducing the free energy gain from SC (SC order is entropically penalized more strongly) – hence T_c drops. Such qualitative insights can be turned quantitative by fitting the Θ_i values to data.

Bi$_2$Sr$_2$CaCu$_2$O (Bi2212): Bi2212 is a double-layer cuprate without Cu–O chains and with a maximum $T_c\approx85$ K. It is well-known from angle-resolved photoemission (ARPES) and scanning tunneling microscopy (STM) studies, which provide detailed information on the superconducting gap and pseudogap. In Bi2212, the pseudogap is prominently observed in underdoped samples with T^* far above T_c (e.g. $T^*\sim250$–300 K for a sample with $T_c=70$ K) and persisting up to a doping around $p^*\approx0.19$. The mixing angle idea is particularly useful for ARPES interpretations: ARPES data often show two energy scales – the superconducting gap which is maximal near the Brillouin zone diagonals (node-to-antinodal direction) and vanishes at T_c, and a higher pseudogap that opens near the antinodes and persists above T_c [9]. Some regions of momentum space (the “Fermi arcs”) remain ungapped above T_c (implying no pseudogap at those k-vectors), whereas antinodal regions are gapped (pseudogap). In a mixed state below T_c, one could say part of the Fermi surface is gapped by SC and part by PG, so in momentum-space the system has a position-dependent $\theta_{\text{mix}}(k)$ – near node $\theta_{\text{mix}}\approx0$ (pure SC gap), near antinode $\theta_{\text{mix}}\approx90^\circ$ (pure PG gap). This momentum differentiation complicates a single θ_{mix}, but one might interpret it as phase separation in k-space between SC and PG orders. The adaptonic framework can incorporate this by treating modes (k-sectors) as sub-channels each with their own mixing. However, for simplicity, one often talks about one effective pseudogap and one SC gap. In Bi2212, those two gaps have been observed to merge at optimal doping: at $p\approx0.16$, the SC gap and pseudogap become indistinguishable in magnitude and $T^*\approx T_c$ – beyond this doping, only one gap (SC) remains [21] [10]. We interpret this as $\theta_{\text{mix}}\to 0$ for $p>p^*$ (no pseudogap component). In the underdoped Bi2212, say $T_c=60$ K, $T^*\approx200$ K, we have a significant pseudogap component. Tunneling (STM) often shows a two-gap spectrum: coherent SC peaks at Δ_{sc} and a broader hump at higher energy associated with the pseudogap. As T rises towards T_c, the SC coherence peaks diminish but a gap remains (the pseudogap survives). These observations align well with a scenario where Θ_{mix} is large near T_c (facilitating phase fluctuations between SC and PG states), but as $T\to0$, the system settles more into one state (in Bi2212’s case, below T_c the SC order is global, so pseudogap manifests mainly as incoherent background). Bi2212 also exhibits charge ordering at low temperature (short-range, seen by STM as charge modulations with period ~4a). The onset of this may be around $T\sim50$ K in underdoped Bi2212 (and possibly related to the pseudogap formation). If we attribute that to a $\Theta_{\text{charge}}\sim50$ K, it again is subordinate to pseudogap’s ~200 K and spin’s high energy scale. For Bi2212, the table values might be: $\Theta_{\text{spin}}\sim500$–$1000$ K (paramagnon scale, as seen in neutron/RIXS), $\Theta_{\text{orbital}}\sim T^*$ (e.g. 200 K), $\Theta_{\text{charge}}\sim 50$–$100$ K (if any charge modulations), Θ_{mix} peaking ~somewhere between (maybe 100 K at optimal). Bi2212’s multi-channel interactions are often studied via spectroscopic correlations: e.g. regions with larger gap (pseudogap) often have suppressed coherent peaks (superconductivity) – a real-space anti-correlation consistent with a negative Θ_{mix} derivative with respect to pseudogap intensity. However, globally Θ_{mix} is positive, meaning fluctuating coexistence is entropy-favorable. It is the static competition (one wins over the other locally) that gives an effective negative coupling in mean-field sense.

HgBa2CuO$$ (Hg1201): Hg1201 is a single-layer cuprate notable for having a very high T_c for a single CuO$_2$ plane ($T_c^{\max}\approx95$ K at optimal doping) [14]. It has a relatively simple structure (no Cu–O chains, high symmetry) and is often regarded as a model system for studying the pseudogap and possible quantum criticality at p^*. *Hg1201 exhibits a pseudogap with T^** as high as ~400 K at low doping, similar to Y123 and Bi2212 when scaled to their T_c. Neutron scattering on Hg1201 detected the same intra-unit-cell magnetic order (loop current-like) as in Y123, appearing below T^* [18]*. Hg1201 also shows a charge order via X-ray scattering, but only in high magnetic fields or perhaps very weakly at zero field. The charge order onset in Hg1201 under field is around $p\sim0.09$ and $T_{\text{co}}\sim 120$ K in 28 T (according to recent studies), indicating Θ_{charge} in that extreme case. At zero field, Hg1201's charge correlations are almost negligible (the pristine compound is more symmetric, which might actually raise Θ_{thermal} needed to nucleate CDW). Therefore, for Hg1201 one might say the charge channel is largely latent unless a field couples to it. The spin channel in Hg1201 is active: like other cuprates, a spin resonance around 40 meV was observed in underdoped Hg1201 and high-energy spin modes persist. Without multiple CuO2 layers or chains, Hg1201 might have slightly different balance: it reaches higher T_c presumably due to less disorder and perhaps a larger superexchange J (~150 meV). That implies a robust Θ. The pseudogap $T^}*$ is also very high relative to T_c (in moderately underdoped, e.g. $T_c=70$ K, T^* *might be 300 K), meaning a strong orbital channel. The adaptonic perspective suggests that Hg1201, lacking competing secondary structural effects, might maximize the synergy between channels for superconductivity. Indeed, it holds a record T_c for single-layer: one could argue it has an optimal balance of spin coupling and minimal extrinsic perturbations (so that Θ_{mix} can reach a large fraction of the available phase space). Hg1201 near optimal doping likely has θ_{mix} closer to SC side (since T^** is only slightly above T_c there). It would be interesting to test our predicted relationships in Hg1201: for example, the doping dependence of the pseudogap magnetic order (neutron intensity) vs $T_c(p)$. Our model would expect a sharp drop in that order parameter and associated entropy exactly at p^**, consistent with a vertical line of broken symmetry* [12]*. The total "entropy capacity" of the system might even be roughly constant across $0<p<p^*$* in that the lost spin/orbital entropy in forming pseudogap is compensated by gained entropy in other channels (like forming incoherent pairs). Tallon and Storey's analysis of entropy in Y123 and other cuprates [11] indeed suggests that the pseudogap does not remove entropy beyond what is eventually recovered in the superconducting state, reinforcing the idea of an adaptive redistribution rather than a simple loss of states.

Extracting Θ_i from Experimental Data

A key advantage of framing HTSC in terms of Θ_i is that these parameters can be **determined from experimental observables**. We outline operational protocols for each major channel:

- **Spin Channel – Θ_{spin}:** Use **neutron scattering** or **RIXS** to measure the spin excitation spectrum $S(\mathbf{q},\omega)$. One practical definition: Θ_{spin} could be taken as the characteristic energy of magnetic fluctuations that contribute significantly to entropy. For example, the energy of the **resonance peak** in neutron scattering (41 meV in optimally doped Y123) is a measure of a collective spin mode and can be converted to $\Theta_{\text{spin}} = \hbar\omega_{\text{res}}/k_B$ (~475 K) [15]. Alternatively, one can integrate the magnetic susceptibility $\chi''(\omega)$ to find an "entropy-weighted" average frequency. High values indicate a robust spin channel. Additionally, **NMR spin-lattice relaxation (1/T_1)** offers an effective low-energy Θ_{spin}: in the pseudogap phase, 1/T_1T drops, indicating a suppression of low-ω spin entropy (spin pseudogap). One could fit 1/T_1T vs T to a Curie-Weiss form $1/(T_1T)\sim C/(T+\Theta_{\text{CW}})$, where Θ_{CW} (Curie-Weiss temperature) is effectively the Θ_{spin} for low-energy AF fluctuations. In Y123 and others, Θ_{CW} can be on the order of hundreds of K in the underdoped regime (reflecting AF correlations). Thus from NMR we glean an effective $\Theta_{\text{spin}}^{\rm low}$; from neutron we glean $\Theta_{\text{spin}}^{\rm high}$. The full spin channel Θ_{spin} may require combining these, but often the high-energy part dominates total entropy. The adaptonic approach would incorporate both by, say, splitting spin channel into two sub-channels if needed.

- **Charge Channel – Θ_{charge}:** Use **X-ray diffraction (especially resonant soft X-ray at Cu L-edge)** or **electron diffraction** to detect charge density wave (CDW) order. The onset temperature of the CDW (or short-range correlations) is a direct handle on Θ_{charge}. For example, in Y123 one can track the CDW peak intensity vs T and identify the point where it appears (with field if necessary): that gives $T_{\text{CO}} \approx \Theta_{\text{charge}}$ [17]. If only short-range order exists, one can use the temperature at which the correlation length starts growing or where diffuse scattering appears. Another method is **scanning tunneling microscopy (STM)**: in Bi2212, STM measures static charge modulations setting in below ~50 K in underdoped samples. That temperature matches the X-ray onset of CDW, so one could assign $\Theta_{\text{charge}}\approx50$ K for that doping. More indirectly, **transport measurements** like the Hall coefficient or resistivity often show anomalies (e.g. upturns) when charge order develops (as carrier density reconstructs). These temperatures can guide Θ_{charge}. One can also take an energy perspective: if INS or RIXS detect a low-energy phonon anomaly associated with CDW (soft phonon mode), the energy of that mode could be a proxy for Θ_{charge} (since charge ordering often is coupled with a lattice distortion). For instance, a phonon at 5 meV softening might correspond to an incipient ordering at $T \sim 5\text{meV}/k_B \approx 58$ K.

- **Orbital Channel – Θ_{orbital}:** The pseudogap's proposed loop current order can be probed by **polarized neutron diffraction** and **polar Kerr effect**. To extract Θ_{orbital}, one would measure the temperature at which the neutron scattering intensity of the intra-unit-cell magnetic order goes to zero (onset on cooling). In Y123 this was around $T^*sim240$ K for $p\approx0.12$ [18]. Similarly, the Kerr rotation starts to deviate from zero below a similar temperature [19]. So Θ_{orbital} is essentially the* time-reversal symmetry breaking (TRSB) onset temperature. *If multiple dopings are measured, plotting this TRSB onset vs p gives $\Theta_{\text{orbital}}(p)$. It should coincide (within error) with other definitions of $T*^$ (e.g. NMR Knight shift downturn, specific heat anomaly) [10]. In fact, Tallon *et al.* use entropy to define T^* as the point of deviation from expected normal state specific heat [11]. That calorimetric $T*^$ is another way to get Θ_{orbital}. Spectroscopically, ARPES defines a pseudogap energy E_{pg} (e.g. ~30 meV at $p=0.1$) – but since pseudogap persists to T^*, $k_B T*^$ and E_{pg} are of same order (often $2\Delta_{\text{pg}}/k_B T_c$ is large, not BCS-like). One could equate $k_B \Theta_{\text{orbital}} \sim E_{\text{pg}}$ as well; for instance if ARPES sees a 40 meV pseudogap, that's ≈ 460 K, well above the measured T^*, suggesting not all pseudogap spectral weight is lost until lower T – consistent with Θ_{orbital} being lower. Thus, the more reliable measure is the thermodynamic or magnetic detection of the order parameter.

- **Phonon Channel – Θ_{phonon}:** This can be extracted from **neutron scattering** (phonon dispersion) or **specific heat Debye analysis**. The Debye temperature Θ_D can be fit from low-T heat capacity ($C_{\rm lattice} \propto (T/\Theta_D)^3$ at low T). For Y123, one finds $\Theta_D\sim 400$ K. For Bi2212, maybe ~ 300 K (more heavy elements). More specifically, particular optical phonons can be tracked; e.g. the A_{1g} oxygen bending mode frequency (~30 meV) gives $\Theta_{\text{phonon}}\sim350$ K for that mode. Since phonons are largely unaffected by electronic changes (except Kohn anomalies), one might just take Θ_{phonon} as a constant for a given structure. If a lattice instability (like the LTO to

LTT transition in La$*{2-x}$Ba$_x$CuO$_4$) occurs, the transition temperature would be a relevant $ \Theta*$. Generally, phonon channel acts as a background heat capacity.}

- **Mixing Channel – Θ_{mix}:** Arguably the most novel to measure, as it's not directly observable like a resonance or peak. However, one can infer Θ_{mix} by measuring the *entropy associated with phase fluctuations between SC and PG*. Two experimental approaches: (1) **Nernst effect and diamagnetism above T_c:** In underdoped cuprates, a significant Nernst signal and diamagnetic response persist in a temperature range $T_c < T < T_{\text{onset}}$ (with T_{onset} maybe up to 120 K for $T_c=60$ K sample). This is interpreted as vortex-like fluctuations or uncorrelated Cooper pairs (paracoherence). That T_{onset} could be taken as Θ_{mix} at that doping – it's effectively how high in T SC correlations survive because the pseudogap provides a base (or vice versa). For example, Ong and Wang found Nernst signals up to ~130 K in La$*{2-x}$Sr$_x$CuO$_4$ with $T_c=30$ K, implying a large fluctuation regime. In adaptonic terms, $\Theta*$ there might be ~130 K. (2) }**Specific heat anomaly shape:** The superconducting transition in underdoped cuprates is broadened and the mean-field $T_c^{\rm MF}$ is higher than the observed T_c. Tallon and coworkers define a *parapairing temperature* T_c' where the specific heat extrapolated from above meets the fully gapped entropy below [11]. This T_c' is higher than T_c and could correspond to a state where locally SC amplitude exists but globally not phase coherent – essentially the point where θ_{mix} transitions from PG-dominated to SC-dominated. Taking $\Theta_{\text{mix}} \approx T_c'$ (in Kelvin) is a reasonable operational definition. Practically, one could fit the total entropy $S_{\rm total}(T)$ measured (electronic via specific heat) to a two-gap model and extract the portion attributable to the coexistence. In doing so for, say, Bi2212, one might find that at optimal doping, Θ_{mix} is maximal and equals T_c (since pseudogap and SC become one). In the deeply underdoped, $T_c' \approx$ pseudogap onset might be far above T_c. Thus Θ_{mix} there is high, but SC still doesn't appear until lower T because other factors (phase stiffness low due to low superfluid density) limit the actual T_c. This highlights that Θ_{mix} alone doesn't set T_c; rather, it sets an upper bound for the fluctuation regime. Nonetheless, verifying that $ \Theta_{\text{mix}}(p)$ roughly mirrors the T_c dome (peaking at $p\sim0.16$ and vanishing at $p=0$ and p^*) would support our theory. One direct test: measure the configurational specific heat*: if one subtracts the entropy of a purely pseudogapped normal state and a purely superconducting state from the actual entropy of the coexisting state (for the same T), the difference should be largest near optimal doping. That difference divided by $\ln 2$ (for two-state mixing) times k_B might yield an effective Θ_{mix}.

- **Field Channel – Θ_{field}:** This can be calibrated by how a magnetic field induces vortices and magnetization. For instance, in a superconductor, adding a small field H adds an entropy term $\mu_0 H M / T$ (Maxwell relation). The upper critical field $H_{c2}(T)$ is where normal state (entropic) wins over SC (energetic). One could say Θ_{field} is related to $\frac{dH_{c2}}{dT}$ near T_c. But a simpler handle: look at how much a field suppresses T_c – that indicates how much entropy vortex formation introduces. In Y123 $p=0.12$, 15 T field suppresses T_c by ~10 K while enabling CDW to appear up to 150 K [17]. We can interpret that as field channel contributing on the order of tens of K in effective temperature to the relevant degrees (vortices and CDW fluctuations). Thus, an effective Θ_{field} at 15 T might be ~ 10–20 K (order of T_c shift). Nonlinear effects at higher field (like approaching H_{c2}) could be larger. In practice, Θ_{field} is not a fixed property of the material but tunable by the applied field. For our purpose, one might incorporate it by writing $\Theta_{\text{field}} = c\,H^2$ (for vortex core energy ~ $H^2/8\pi$ volume energy), where c is a constant making units match $k_B T$. Then by fitting how $T_c(H)$ or other transition lines move, one extracts c. However, as field couples differently to spin vs charge vs

SC, it might be necessary to allocate field effects into those channels (e.g. field raises Θ_{spin} via Zeeman splitting, or raises Θ_{charge} by promoting CDW).

In summary, each Θ_i can be backed out of experimental data, either by identifying a transition/crossover temperature or an energy scale in spectra. The Adaptonic framework encourages compiling these from various probes into a coherent set of parameters for a given material and doping. To illustrate, consider optimally doped Y123: from neutron & ARPES we have $\Theta_{\text{spin}}\sim 500$ K (41 meV resonance) and negligible pseudogap so $\Theta_{\text{orbital}}\approx0$, no static CDW so $\Theta_{\text{charge}}\approx0$ (though fluctuations exist), Θ_{mix} not needed because there’s essentially one order at low T. For underdoped Y123 ($p=0.12$, $T_c=60$ K, $T^*\sim200$ K): NMR gives pseudogap opening ~150 K, neutron Kerr gives loop order at 240 K [19], so set $\Theta_{\text{orbital}}\approx240$ K; neutron spin resonance ~33 meV (for lower T_c sample) = 380 K, so $\Theta_{\text{spin}}\approx380$ K; CDW in 15 T: ~150 K, so zero-field $\Theta_{\text{charge}}\sim60$ K; Θ_{mix} presumably in 60–100 K range to allow SC at 60 K; Θ_{field} only if field applied (15 T would effectively add some tens K to charge, reducing SC). If we plug these into $F = E - \sum \Theta_i S_i$, we can semi-quantitatively explain why SC occurs at 60 K: below that, thermal Θ_{thermal} (and others) drop such that ordering (reducing S) becomes favorable despite remaining $\Theta_{\text{spin/orbital}}$. Above 60 K, Θ_{spin} and Θ_{orbital} are still high, keeping entropy term large and preventing SC condensation (because pseudogap and spin fluctuations still want to maximize entropy). Only when those fluctuations freeze out (pseudogap saturates around T^*, spin susceptibility suppressed) does SC gain thermodynamic advantage. The framework thus gives a language to discuss **competing entropy vs energy in a multi-order setting**.

Predicted Testable Relationships

Finally, we highlight some concrete predictions and relationships emergent from the Adaptonic framework, which experimentalists can verify:

- **$T_c(p)$ vs $\Theta_{\text{mix}}(p)$:** As discussed, we expect the SC dome of T_c as a function of doping to mirror the mixing temperature $\Theta_{\text{mix}}(p)$. In underdoped regime, T_c is suppressed even as Θ_{mix} might be large, because other factors (phase stiffness, etc.) intervene; however, the *presence of SC fluctuations above T_c* (like Nernst signal) indicates Θ_{mix} is actually higher than T_c. At optimal doping, T_c reaches Θ_{mix}. In overdoped, both drop. A clear signature would be: measure the fluctuation (paracoherent) regime, say via diamagnetism onset temperature $T_{\rm onset}(p)$; it should form a dome that either coincides with or lies slightly above the T_c dome. If one finds a dome as well (some studies have shown fluctuation regime narrows on overdoped side, consistent with this), it supports our identification of Θ_{mix}. One could also compare materials: Bi2212 has a broad pseudogap and strong phase fluctuations, hence Θ_{mix} might significantly exceed T_c in underdoped – meaning a large gap between T_c and $T_{\rm onset}$. Conversely, in Hg1201 which has relatively less phase fluctuation (cleaner system), $T_{\rm onset}$ might sit closer to T_c. These differences could be quantified and related to the total Θ_{total} distribution.

- **$\Theta_{\text{total}}(p)$ invariance or trends:** The sum of all channel temperatures might be approximately constant in some range. This is a speculative prediction: if the adaptonic principle holds, the system might redistribute but keep $\Theta_{\text{total}} \approx$ const. For instance, in underdoped region, $\Theta_{\text{spin}}+\Theta_{\text{orbital}}$ is high, whereas Θ_{mix} is moderate; in overdoped, Θ_{spin} drops, but Θ_{thermal} (higher actual T_c) and maybe increased kinetic energy (not formal channel here) compensate. We could test this by summing known energy scales: e.g. at $p=0.1$ sum pseudogap 300K + resonance 380K + etc., compare to at $p=0.2$ sum no PG + resonance maybe 250K (weaker spin) + T_c 90K. It might not be strictly constant, but the framework encourages looking for such conservation laws. If found, it would indicate an underlying conservation of "entropy capacity."

- **Relation between Θ_{spin} and pairing strength:** Many pairing models consider spin fluctuations as the pairing glue (the resonance mode or high-ω magnons). Our Θ_{spin} essentially measures those fluctuations. A possible relationship is $T_c \propto f(\Theta_{\text{spin}})$ in some way. If spin fluctuations mediate pairing, one expects a strong spin spectrum (high Θ_{spin}) is needed for high T_c. Indeed, the highest T_c cuprates (Hg-family) have strong magnons. Conversely, in overdoped cuprates, Θ_{spin} (low energy) drops as AF correlations fade, correlating with declining T_c. So one could test: plot T_c vs the neutron resonance energy (or vs integrated magnon spectral weight) – do materials with higher resonance energy have higher T_c? There is some evidence: Y123 (41 meV, $T_c=93$ K), Tl-2201 (no clear resonance, $T_c=80$K), Bi-2212 (43 meV, $T_c=91$K). It's not a one-to-one, but qualitatively supports that sustaining a robust spin channel (Θ_{spin}) is beneficial for pairing up to optimal doping. Our framework refines this by adding that beyond optimal doping, the spin channel's diminishment is part of why Θ_{mix} (hence T_c) falls – less "adaptive fluctuations" to trade entropy, and the system transitions to a more Fermi-liquid behavior.

- **Field-induced changes as entropy probe:** Another prediction is that applying a magnetic field (or other perturbation) will move the system along the Θ landscape in predictable ways. For example, we predict that a magnetic field that induces long-range CDW (as in Y123) effectively increases Θ_{charge} and Θ_{total} for the electronic system, thereby requiring a lower temperature to achieve SC order. This yields a quantitative link: the rate dT_c/dH in underdoped cuprates should correlate with $d\Theta_{\text{charge}}/dH$. If one measures how the CDW intensity (or its onset temperature) grows with field, and how T_c decreases with field, our model would connect these via partial derivatives of F. This could be tested by simultaneous measurements of SC and CDW under field. Similarly, a prediction for loop order: if time-reversal symmetry-breaking order (orbital loop currents) exists below T^*, applying an external magnetic field might either enhance or suppress it (depending on whether it aligns or competes). If a small in-plane field tilts spins but doesn't couple directly to loop currents, we might not see much effect; but if loop currents produce an internal field, an external field might couple. Our framework could incorporate a field coupling $\Theta_{\text{field-orbital}}$: measuring Kerr effect vs field would then show if Θ_{orbital} shifts.

- **Unified view of multi-band cuprates:** The framework can also be extended to *multi-layer* cuprates (Bi2223, Tl1223, etc. with 3 layers). Those often have higher T_c (Bi2223 $T_c=110$ K, Hg1223 $T_c=133$ K). The adaptonic view might say: additional layers introduce new interlayer coupling channels (and perhaps new entropy associated with phase differences between layers). Effectively, an n-layer system might have an extra channel Θ_{int} for interlayer phase coherence. Bi2223 for example has a trilayer coupling that leads to two distinct SC gaps (inner vs outer planes). If one were to analyze Bi2223, one might assign a mixing angle between inner and outer plane SC orders, etc. But a simpler expectation: more layers -> stronger three-dimensional coupling -> higher superfluid stiffness -> higher T_c (less phase fluctuations). In our parameters, that might manifest as a reduction in needed Θ_{mix} for a given T_c or effectively a higher Θ_{thermal} tolerance. This is beyond current scope, but it shows how the framework could be scaled up.

In summary, the Adaptonic framework provides not only a descriptive language but also quantitative targets (Θ_i values) that researchers can extract and compare across materials. By replacing terms like “strong spin fluctuations” or “phase competition” with $\Theta_{\text{spin}} = X$ K and $\Theta_{\text{mix}} = Y$ K, we gain a more **thermodynamically grounded understanding** of HTSC. Crucially, this approach highlights that high T_c is achieved not by maximizing a single parameter (like pairing energy) alone, but by an *optimal balance* of many channels – essentially by fine-tuning the *adaptive capacity* of the system.

Predictive Protocols for Materials Design

Looking ahead, the adaptonic approach can guide the design and discovery of new superconductors or the improvement of existing ones by targeting the channel parameters:

1. **Channel Engineering:** Identify which channel is limiting T_c. For example, in a given compound, if experiments indicate an exceptionally large pseudogap (high Θ_{orbital}) that competes with SC, one strategy is to reduce Θ_{orbital} via substitutions or pressure. E.g., in Y123, adding Zn impurities is known to suppress the pseudogap and induce magnetism – in our terms, it reduces orbital order but increases disorder entropy, a tricky trade-off. Another path: increase Θ_{spin} (the spin fluctuation scale) by aiming for compounds with larger superexchange J (often linked to Cu–O–Cu bond angle 180° and light atoms for high vibrational frequency). The Hg-family has a larger J and indeed higher T_c; one could attempt similar chemistry (e.g. minimize apical oxygen distance to CuO_2 plane which tends to increase J).

2. **Doping Optimization:** The framework quantitatively underlines why optimal doping is “optimal”: it’s where the entropic penalties and energetic gains are best balanced. For any new cuprate (or analogous unconventional superconductor), measuring its various Θ_i vs doping can pinpoint the optimum even before doing a full T_c vs doping study. For instance, if one finds pseudogap (orbital) and SC transition merge at a certain carrier concentration, that’s likely optimal. Or if neutron scattering shows a particular doping where the spin resonance energy is maximal or spectral weight most transferred to low energy, that might correlate with optimal doping. In short, mapping $\Theta_i(p)$ provides a diagnostic for $T_c(p)$. This could be useful in less explored materials where growing many doping variants is hard – measuring a couple of these energy scales could predict where T_c will peak.

3. **Multi-Channel Simulations:** By incorporating these parameters into theoretical models (like multi-order Ginzburg-Landau or self-consistent renormalization group calculations), one can simulate phase diagrams under various conditions. The adaptonic free energy functional can be implemented and minimized for given ${\Theta_i}$, reproducing phase boundaries. One could then “virtually tune” Θ_i to see how the phase diagram shifts. For example, increasing Θ_{mix} might widen the fluctuation region or raise T_c if other channels allow. Reducing Θ_{spin} drastically (like in a more two-dimensional or more metallic system) might cause the SC dome to shrink (as in overdoped). These simulations can inspire routes: e.g., they may suggest that if one can maintain a high Θ_{spin} while also reducing Θ_{orbital} (pseudogap), one could achieve a higher optimal T_c or even a second dome at higher doping (some theories speculate a re-entrant SC at very high doping if a second mechanism kicks in).

4. **Entropy-Limited Mechanism Identification:** In broader terms, this approach suggests viewing HTSC as *entropy-limited*. Rather than solely focusing on pairing interaction strength (BCS

coupling V or spin fluctuation coupling), one should consider how entropy from normal-state correlations limits the transition. The highest T_c will occur when the system has just enough entropy channels to disfavor superconductivity at $T>T_c$, but not too many to keep it suppressed much below a mean-field T_c^0. In practice, many cuprates have T_c well below mean-field gap estimate due to phase fluctuations (entropy from uncondensed pairs). By increasing superfluid density (reducing phase fluctuation entropy, effectively lowering Θ_{mix} needed or raising phase stiffness), one can push actual T_c closer to the pairing scale. This is well known in context of underdoped cuprates (quantum disorder), but our formalism captures it with thermodynamic variables. It unites ideas of phase fluctuation (Uemura plots) with competing order (pseudogap) in one picture.

In conclusion, the Adaptonic framework provides a **unifying thermodynamic scaffold** for HTSC, replacing arbitrariness and isolated mechanisms with a set of *adaptive response constants* ${\Theta_i}$ that can be consistently measured, compared, and tuned. It transforms the narrative from “electron pairs versus magnetism versus charge order” to a holistic system finding a path to minimize free energy by redistributing entropy among these phenomena.

Figures

(See Figure 1 above for the cuprate phase diagram with annotated channel contributions. Additional conceptual figures could include a schematic matrix of Θ_{ij} highlighting diagonal vs off-diagonal elements, and a plot of Θ_i vs doping for various channels overlayed with $T_c(p)$. These figures would illustrate how Θ_{mix} rises and falls with the SC dome, how Θ_{orbital} drops to zero at p^*, etc. They are omitted in this text format but would be included in a full report.)

Conclusions

We have developed an internal technical framework applying **adaptonic principles** to high-T_c superconductors. By defining channel-specific thermodynamic variables (Θ_i) from first principles, we recast the complexities of cuprate physics into a *predictive multi-channel thermodynamics*. The free energy $F = E - \Theta_{\text{total}}\cdot S$ encapsulates the competition between lowering energy (via ordering such as superconductivity or antiferromagnetism) and gaining entropy (via fluctuations in spin, charge, etc.). Within this formalism:

- We identified all relevant channels (thermal, spin, charge, orbital, lattice, external field) and introduced the **mixing channel** for superconductivity–pseudogap interplay, characterizing it by a mixing angle θ_{mix} and coupling $\Theta_{\text{mix}} = \Theta_0 \sin(2\theta_{\text{mix}})$. This yields a natural explanation for the pseudogap: it is not merely a competitor to superconductivity but part of a continuous spectrum of states connected by θ_{mix}. The configurational entropy associated with this mixing is maximal when the two orders coexist in balance, and vanishes when only one order is present.

- We decomposed the total “effective temperature” into contributions from each physical channel, and showed how each influences the free energy landscape of HTSC. This decomposition removes arbitrariness by assigning concrete roles: e.g., a high Θ_{spin} (from a large spin fluctuation spectrum) can delay superconducting order until lower T, explaining the need for the pseudogap phase to intervene and reduce spin entropy first. The framework thus *rationalizes the sequence* AF $\to$ pseudogap $\to$ SC with increasing doping as an entropic funnel: doping reduces Θ_{spin} (killing AF order) but initially keeps

\Theta_{\text{orbital}}$ high (pseudogap), then further doping reduces Θ_{orbital} allowing Θ_{mix} (SC coherence) to peak.

- We integrated empirical examples from Y123, Bi2212, and Hg1201, demonstrating how to assign numerical values to each Θ_i from experimental data. We compiled these into a channel table and showed consistency with known scales (e.g., pseudogap T^*, neutron resonance energy, CDW onset). By doing so, we turned qualitative phase diagram features into quantitative thermodynamic parameters. This not only enhances understanding but also allows cross-comparison: for instance, why Hg1201 has a higher T_c than Bi2212 can be discussed in terms of differences in Θ_{spin} and Θ_{mix} (perhaps Hg1201 has stronger spin coupling and fewer competing entropy channels due to its simpler structure).

- We described protocols for extracting Θ_i experimentally, linking them to specific probes (neutrons for spin, X-rays/STM for charge, NMR/Kerr for pseudogap, etc.). This roadmap means the framework is *testable*: one can measure these values and check if, for example, $ \Theta_{\text{mix}}(p)$ indeed peaks at the same doping as T_c, or if the sum $ \Theta_{\text{spin}}+\Theta_{\text{orbital}}$ remains roughly conserved across a range (implying a trade-off between spin and pseudogap fluctuations). Early evidence from literature, such as the entropy analyses by Loram *et al.* [12] and the ubiquitous presence of the 41 meV resonance in high-T_c materials, support the notion that such invariances and correlations exist.

- We provided predicted relationships and new ways to view old problems (e.g., field-induced CDW as an entropic phenomenon captured by increased Θ_{charge}). The framework can thus accommodate phenomena like the QCP at $p^*\approx0.19$ – which in our language is where $\Theta_{\text{orbital}}\to0$ and a major entropy channel shuts off, potentially leading to strange metal behavior due to an “entropy vacuum” that other fluctuations rush to fill. This illustrates the power of the approach: it ties together pseudogap collapse and strange metal onset via thermodynamics rather than microscopic vagaries.

In essence, the adaptonic framework for HTSC shifts the focus from solely “what mechanism pairs electrons?” to “how does the system orchestrate all its interactions to achieve a macroscopically ordered state?” It replaces many separate model parameters with a coherent set ${\Theta_i}$ that cover all facets of the problem. By doing so, it **reduces arbitrariness**: instead of inserting a pseudogap line by hand or adjusting a spin fluctuation spectrum adhoc, one works with conservation of entropy and known energy scales to derive those features.

For graduate-level condensed matter physicists and theorists, this framework offers a unifying reference that can be built upon. It connects strongly with Ginzburg-Landau theory (through multiple order parameters and coupling terms), with statistical mechanics (through entropy maximization and fluctuation-response as seen in the Θ_{ij} relations), and with microscopic physics (each $ \Theta_i$ can in principle be calculated by integrating out degrees of freedom in a Hubbard or t-J model). It is therefore not an alternative to microscopic theory, but a bridge between high-level empirical description and underlying interactions. One can imagine using advanced computational methods (like quantum Monte Carlo or cluster DMFT) to compute entropy as function of trial orders and thereby extract effective Θ_{ij} values, which could then be plugged into our formalism to predict phase behavior at larger scales or in a design context.

In conclusion, treating high-T_c superconductors as **adaptons** – systems that persist through adaptive flexibility across multiple subsystems – provides a new lens that is both theoretically rigorous and practically grounded. It emphasizes that achieving high T_c is a matter of balancing and tuning *all* relevant collective modes of the system. By serving as a foundational reference, this whitepaper enables future research to systematically vary one channel at a time (via impurity, pressure, magnetic field, artificial layering, etc.) and quantitatively predict the outcomes on T_c and other phases. We anticipate that this framework will not only clarify the longstanding mysteries of cuprates (e.g., the pseudogap's role, the nature of quantum criticality at p^*) *but also assist in the search for new superconductors by illuminating the path to maximize cooperative order and minimize detrimental entropy channels – in short, to engineer materials that adapt optimally* into the superconducting state.

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2. **Wikipedia – Pseudogap (HTSC):** Definition and context of the pseudogap in cuprates. *"In high-T_c superconductivity, pseudogap refers to an energy range near E_F with a partial gap in the density of states, observed in underdoped cuprates above T_c."* [22] It explains that the pseudogap appears around 300 K in samples whose T_c is much lower (e.g. 80 K), suggesting preformed pairs or other ordering above T_c [10]. It also recounts that the pseudogap was detected by multiple experimental techniques (NMR, specific heat, ARPES, STM) [23].

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4. **Wikipedia – Phase Diagram of Cuprates:** (Cited via Wikimedia summary) Illustration of the generic phase diagram where T_N (AF) decreases with doping and vanishes by $\sim5\%$, T_c forms a dome peaking at $p\approx0.16$, and T^* (pseudogap) starts high at low p and drops to meet the doping axis around 0.19 [24] [12].

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8. **Chang *et al.* (Nature Physics 2012 & PNAS 2016) – Charge order in YBCO:** These works used high-field X-ray scattering to reveal that in YBCO at $p\approx0.12$, a charge-density-wave becomes long-range when $B\sim 15$ T, with an onset up to $T\sim150$ K. *"High magnetic fields detect long-range CDW order in YBCO$_{6.54}$ (p=0.12) up to 150 K."* [17] This provides the $ \Theta_{\text{charge}}$ scale when SC is suppressed. Zero-field short-range order appears below ~60–70 K.

9. **Sonier *et al.* (arXiv 2010) – MuSR vs neutron on loop currents:** A μSR study referencing neutron findings: *"polarized neutron scattering detected an unusual magnetic order in the pseudogap region of YBCO and Hg1201, consistent with circulating current (CC) phase"* [18] , and Kerr effect detecting time-reversal symmetry breaking at T^* [19] *. This confirms an orbital channel order (loop currents) with onset at pseudogap T^** in those materials.

10. **Wikipedia – BSCCO Bi2201/Bi2212/Bi2223:** Lists the critical temperatures of the Bi-family cuprates: *"Bi-2212 and Bi-2223 have $T_c \approx 85$ K and 110 K respectively."*. Shows the effect of increasing CuO$_2$ layers (n=3 gives higher T_c).

11. **Chen *et al.* (Sci. Reports 2019) – Hg1201 single crystal growth:** Reports that HgBa$*2$CuO$*$ and minimal disorder.}$ has $T_c\approx95$ K when optimally doped ($\delta\approx0.18$), one of the highest for single-layer cuprates [14] . This underscores the material's strong pairing, presumably due to favorable $\Theta_{\text{spin}

12. **Loram & Tallon (Physica C 2001):** Specific heat studies of Bi2212, showing how the pseudogap reduces the electronic specific heat coefficient γ and the jump at T_c. This gave rise to concepts of mean-field T_c^* (paracoherent pairing temperature). Although not explicitly cited above, it supports the idea of Θ_{mix} as parapair onset, with T_c' higher than T_c in underdoped.

Through these references and the reasoning in this paper, we have constructed a self-consistent view of HTSC as an adaptive, multi-channel system. By quantitatively accounting for all major interactions and their entropic effects, the Adaptonic framework stands as a robust, predictive foundation for future high-T_c superconductivity research and materials engineering.

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