Replication of:

Making combination vaccines more accessible to low-income countries: The antigen bundle pricing problem

Ruben A. Proano, Sheldon H. Jacobson, Wendo Zhang

Pawel Bogdanowicz

December 11, 2015

OPR 624

Advanced Mathematical Programming

Dr. Benson

Drexel University

Contents

[Abstract 2](#_Toc437614791)

[Problem 3](#_Toc437614792)

[Markets 3](#_Toc437614793)

[Bundles 3](#_Toc437614794)

[Model Definition 4](#_Toc437614795)

[Objective Function 4](#_Toc437614796)

[Constraints 5](#_Toc437614797)

[Results 7](#_Toc437614798)

[Replication 7](#_Toc437614799)

[Comparison 8](#_Toc437614800)

[Discussion 9](#_Toc437614801)

[Numerical Results 10](#_Toc437614802)

# Abstract

Combinational vaccines, or Antigen Bundles, combine two or more vaccines in one shot. These vaccines are preferred in high-income countries but are prohibitively expensive for middle and low-income countries. Manufacturers often prefer or are pressured to produce bundle vaccines for developed countries years before producing for low-income countries. The optimization problem tries to create a model for a segmented pricing strategy that maximizes social utility, which the study calls social welfare.

This paper strives to partially replicate the work done in “Making combination vaccines more accessible to low-income countries: The antigen bundle pricing problem.”

# Problem

The ultimate goal of this model is to determine the optimal pricing and bundle combinations to serve each market in order to satisfy all immunization requirements.

## Markets

There are four markets that are taken into account. These markets include countries that fall into GNI PPP (gross national income per capita based on purchasing power parity).

|  |  |
| --- | --- |
| **Market Segment** | **Income** |
| High income | GNI PPP >= $12,000 |
| Middle income | GNI PPP: $3,500 - $12,000 |
| Low income | GNI PPP: $1,000 - $3,500 |
| Poorest Countries | GNI PPP < $1,000 |

## Bundles

There are four antigens that make up the fifteen possible combinations of bundles. The four antigens are 1) DTaP, 2) HBV, 3) HiB, 4) IPV. Each of the bundles also have reservation prices which indicate the most each market is willing to pay for that bundle.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Antigen** | | | | **Market Segment** | | | |
| **Bundle** | **1** | **2** | **3** | **4** | **M1** | **M2** | **M3** | **M4** |
| **1** | X |  |  |  | 24.00 | 6.00 | 2.40 | 0.80 |
| **2** |  | X |  |  | 19.00 | 4.75 | 1.90 | 0.63 |
| **3** |  |  | X |  | 19.00 | 4.75 | 1.90 | 0.63 |
| **4** |  |  |  | X | 22.00 | 5.50 | 2.20 | 0.73 |
| **5** | X | X |  |  | 55.00 | 13.75 | 5.50 | 1.83 |
| **6** | X |  | X |  | 55.00 | 13.75 | 5.50 | 1.83 |
| **7** | X |  |  | X | 60.00 | 15.00 | 6.00 | 2.00 |
| **8** |  | X | X |  | 50.00 | 12.50 | 5.00 | 1.67 |
| **9** |  | X |  | X | 55.00 | 13.75 | 5.50 | 1.83 |
| **10** |  |  | X | X | 55.00 | 13.75 | 5.50 | 1.83 |
| **11** | X | X | X |  | 95.00 | 23.75 | 9.50 | 3.17 |
| **12** | X | X |  | X | 99.00 | 24.75 | 9.90 | 3.30 |
| **13** | X |  | X | X | 100.00 | 25.00 | 10.00 | 3.33 |
| **14** |  | X | X | X | 95.00 | 23.75 | 9.50 | 3.17 |
| **15** | X | X | X | X | 140.00 | 35.00 | 14.00 | 4.67 |

# Model Definition

This problem is a mixed-integer nonlinear problem. As in the paper, the Knitro solver was used to achieve a solution.

## Objective Function

The objective function is defined as:

where:

The above objective function provides the optimal solution which is the summation of total profit and total customer surplus. Total profit is defined as:

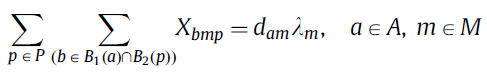
where:

Total customer surplus is defined as:

When describing the results, the upper bound solution is the maximization of equation (2). The lower bound solution is the maximization of equation (3). The summation of equations (2) and (3) results in the optimal solution which is the maximization of equation (1).

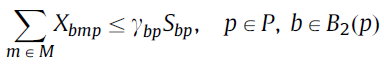
## Constraints

**Constraint 3**



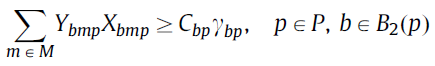
Constraint 3 maintains that the number of doses that are supplied to each market satisfy the need for all children to be vaccinated. The demand is calculated by multiplying the number of doses of each antigen needed to vaccinate each child by the average number of children born in each market segment each year.

**Constraint 4**



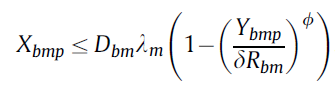
Constraint 4 makes sure that the number of bundles decided to be supplied is less than the production capacity of each producer.

**Constraint 5**



Constraint 5 ensures that producers only engage in production if the product is profitable. The left had side of the equation calculate the total profit and the right hand side estimates the cost.

**Constraint 6**



Constraint 6 captures the elastic demand for vaccines. The objective of the elastic demand is to better model the social importance of vaccines. Meaning that consumers are likely to purchase the vaccine over a much broader price range than if it were linear demand.

**Constraint 8**



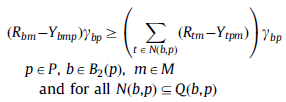
Constraint 8 simply states that the price to be offered is non-negative and it is less than the maximum consumers are willing to pay in each market.

**Constraint 9**



Constraint 9 is a non-negativity constraint in that only positive numbers of bundles can be supplied.

**Constraint 2**



Constraint 2 was omitted from the replication. This constraint states that the bundles that are selected are more optimal than any other possible combination of bundles to satisfy the antigen requirements. The way this constraint was interpreted is that the solution of the optimization problem is the optimal solution.

**Constraint 7**



Constraint 7 was also omitted from the replication. This constraint states that if a bundle is offered in multiple markets, the price of the bundle in a market with a higher reservation price, is higher than in a market with a lower reservation price. Given constraint 8 captures that the price of any bundle is non-negative and less than the reservation price, in addition to the objective function, this did not seem necessary. This was interpreted similar to constraint 2 in that the price that is chosen for a bundle in any market is optimal.

# Results

This section shows the results of the model. The first graphs show the output of the replication model and the later part compares the replication model to the solution in the paper. Numerical results are shown at the end of the report.

## Replication

The chart below shows the lower bound price, upper bound price, and the optimal price for each selected bundle in market 1. Market 1 was chosen because it had the most prominent separation of the upper and lower bound.

The objective value for the 3 models is shown below:

|  |  |
| --- | --- |
| **Model** | **Objective Function Value** |
| Upper Bound (MAX Profit) | 2299 |
| Optimal (MAX Profit+Customer Surplus | 3280 |
| Lower Bound (MAX Customer Surplus) | 3279 |

## Comparison

The chart below shows the upper and lower bounds for the optimal solution as well as the solutions in the paper. Because there is no tabulated presentation of the results in the paper, and because the results are plotted on a logarithmic scale, the paper values are approximate values. These graphs use market three as for the other markets, the paper selects only a handful of bundles whereas the replicated model typically chooses most of the bundles.

The following chart shows the approximate upper and lower bounds from the paper and the optimal solution from the replicated model.

The chart below shows the number of doses supplied for the replicated lower, upper, and optimal model as well as the solution from the paper.

# Discussion

Overall, the replicated model was able to provide a solution that met the constraints even though it did not mimic the papers results. The results show that to meet the demand for the four different markets, the producers do not have to sacrifice profits in order to serve the less developed countries at a discounted price point.

For the consumers, a tiered pricing strategy, if implemented by the producers, would allow for vaccines to reach the poorer countries faster. The producers of the vaccines can use this model to work out forecasting and pricing strategies in order to plan for future years. Finally, if an oversight organizations wants to bring a new vaccine to an underserved market, it can use this model to provide incentives for producers. For example, the capital recover cost could be manipulated to simulate the effect of an incentive and if it would encourage producers.

One additional consideration for this problem may be the number of bundles in order to satisfy the immunization requirement. This problem understood the markets from a macro perspective in that it did not consider the implications on individuals being vaccinated. For example, the paper solution suggests 12 bundles should be produced for market 3. However, it does not consider how many combinations of vaccines need to be administered to the person. A proposal to consider the micro implications would be to design an optimization schedule based on what bundles are needed per individual. In that, there would be a maximum number of bundles allowed to immunize.

# Numerical Results

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Bundle** | **Market** | **Lower Price** | **Upper Price** | **Opt Doses** | **Opt Price** | **Paper Doses** | **Paper Lower** | **Paper Upper** |
| 1 | 1 | 0.06 | 18.74 | 70.00 | 0.26 | 16.00 | 8.00 | 20.00 |
| 2 | 1 | 7.86 | 9.11 | 15.43 | 10.12 | - | - | - |
| 3 | 1 | 7.24 | 10.22 | 27.16 | 10.03 | - | - | - |
| 4 | 1 | 7.63 | 11.58 | 27.81 | 11.60 | - | - | - |
| 5 | 1 | 7.87 | 30.70 | 0.00 | 25.05 | - | - | - |
| 6 | 1 | 8.47 | 33.80 | 0.00 | 25.00 | - | - | - |
| 7 | 1 | 8.76 | 28.75 | 0.00 | 27.14 | - | - | - |
| 8 | 1 | 9.16 | 28.15 | 9.65 | 28.57 | - | - | - |
| 9 | 1 | 14.64 | 31.62 | 8.99 | 31.74 | - | - | - |
| 10 | 1 | 13.58 | 26.56 | 11.26 | 32.10 | - | - | - |
| 11 | 1 | 10.48 | 51.75 | 0.00 | 43.16 | - | - | - |
| 12 | 1 | 10.74 | 54.13 | 0.00 | 44.89 | - | - | - |
| 13 | 1 | 10.32 | 47.77 | 0.00 | 45.71 | 13.00 | 3.00 | 100.00 |
| 14 | 1 | 14.76 | 54.66 | 7.93 | 53.66 | - | - | - |
| 15 | 1 | 10.90 | 79.63 | 0.00 | 55.37 | 40.00 | 2.00 | 100.00 |
| 1 | 2 | 0.03 | 4.93 | 230.00 | 0.08 | - | - | - |
| 2 | 2 | 2.45 | 2.28 | 84.76 | 2.53 | - | - | - |
| 3 | 2 | 2.34 | 2.47 | 132.26 | 2.43 | - | - | - |
| 4 | 2 | 2.77 | 2.88 | 136.66 | 2.77 | - | - | - |
| 5 | 2 | 4.58 | 7.69 | 0.00 | 6.32 | - | - | - |
| 6 | 2 | 4.53 | 6.13 | 0.00 | 6.32 | - | - | - |
| 7 | 2 | 4.83 | 7.19 | 0.00 | 6.90 | 50.00 | 0.90 | 15.00 |
| 8 | 2 | 6.80 | 6.64 | 19.49 | 6.66 | - | - | - |
| 9 | 2 | 6.91 | 7.25 | 15.09 | 7.22 | - | - | - |
| 10 | 2 | 5.99 | 6.59 | 13.59 | 7.08 | - | - | - |
| 11 | 2 | 6.19 | 13.28 | 0.00 | 10.97 | 80.00 | 1.00 | 20.00 |
| 12 | 2 | 6.20 | 13.80 | 0.00 | 11.42 | 20.00 | 1.10 | 25.00 |
| 13 | 2 | 6.68 | 11.24 | 0.00 | 11.56 | 60.00 | 0.80 | 25.00 |
| 14 | 2 | 10.08 | 12.62 | 18.67 | 12.98 | 30.00 | 1.10 | 25.00 |
| 15 | 2 | 7.26 | 19.28 | 0.00 | 15.81 | 5.00 | 0.90 | 50.00 |
| 1 | 3 | 0.08 | 2.25 | 100.00 | 1.50 | 180.00 | 0.05 | 1.10 |
| 2 | 3 | 0.91 | 0.91 | 36.89 | 0.93 | 5.00 | 1.10 | 1.10 |
| 3 | 3 | 0.95 | 0.93 | 91.72 | 0.93 | 60.00 | 0.20 | 1.10 |
| **Bundle** | **Market** | **Lower Price** | **Upper Price** | **Opt Doses** | **Opt Price** | **Paper Doses** | **Paper Lower** | **Paper Upper** |
| 4 | 3 | 1.06 | 1.09 | 93.01 | 1.08 | 58.00 | 0.20 | 1.10 |
| 5 | 3 | 3.58 | 3.16 | 22.75 | 3.79 | - | - | - |
| 6 | 3 | 3.67 | 2.41 | 22.49 | 3.71 | 55.00 | 0.50 | 8.00 |
| 7 | 3 | 3.98 | 2.88 | 22.42 | 4.04 | 5.00 | 0.20 | 9.00 |
| 8 | 3 | 2.41 | 2.32 | 10.07 | 2.46 | 43.00 | 0.80 | 8.00 |
| 9 | 3 | 2.62 | 2.56 | 8.58 | 2.71 | 45.00 | 0.80 | 8.00 |
| 10 | 3 | 2.77 | 2.64 | 9.09 | 2.70 | 45.00 | 0.80 | 8.00 |
| 11 | 3 | 6.11 | 5.55 | 33.68 | 6.57 | 5.00 | 0.70 | 10.00 |
| 12 | 3 | 6.43 | 5.98 | 33.95 | 6.83 | 50.00 | 0.80 | 10.00 |
| 13 | 3 | 6.86 | 4.79 | 38.88 | 6.85 | 10.00 | 0.90 | 10.00 |
| 14 | 3 | 4.93 | 4.51 | 8.24 | 4.68 | - | - | - |
| 15 | 3 | 7.72 | 7.99 | 25.83 | 9.83 | - | - | - |
| 1 | 4 | 0.45 | 0.80 | 0.00 | 0.38 | 45.00 | 0.01 | 0.80 |
| 2 | 4 | 0.30 | 0.30 | 5.03 | 0.30 | 20.00 | 0.80 | 0.80 |
| 3 | 4 | 0.30 | 0.29 | 7.39 | 0.30 | 30.00 | 0.09 | 0.80 |
| 4 | 4 | 0.34 | 0.34 | 7.39 | 0.35 | 40.00 | 0.05 | 0.80 |
| 5 | 4 | 0.92 | 0.93 | 7.72 | 1.00 | - | - | - |
| 6 | 4 | 1.05 | 1.61 | 13.76 | 1.05 | - | - | - |
| 7 | 4 | 1.15 | 0.96 | 13.87 | 1.15 | - | - | - |
| 8 | 4 | 0.81 | 0.80 | 2.76 | 0.81 | 10.00 | 0.15 | 1.20 |
| 9 | 4 | 0.90 | 0.88 | 2.64 | 0.89 | - | - | - |
| 10 | 4 | 0.86 | 0.88 | 3.07 | 0.88 | - | - | - |
| 11 | 4 | 1.52 | 1.47 | 4.83 | 1.60 | - | - | - |
| 12 | 4 | 1.68 | 1.72 | 4.85 | 1.66 | - | - | - |
| 13 | 4 | 1.77 | 1.63 | 7.02 | 1.69 | - | - | - |
| 14 | 4 | 1.55 | 1.52 | 2.22 | 1.54 | - | - | - |
| 15 | 4 | 2.19 | 2.17 | 2.95 | 2.31 | - | - | - |