Practical attacks on the round-reduced PRINCE

ISSN 1751-8709
Received on 25th September 2015
Revised 1st June 2016
Accepted on 22nd June 2016
E-First on 4th August 2016
doi: 10.1049/iet-ifs.2015.0432
www.ietdl.org

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Abstract: The PRINCE cipher is the result of a cooperation between the Technical University of Denmark, NXP Semiconductors and the Ruhr University Bochum. The cipher was designed to reach an extremely low-latency encryption and instant response time. PRINCE has already gained a lot of attention from the academic community, however, most of the attacks are theoretical, usually with very high time or data complexity. This work helps to fill the gap in more practically oriented attacks, with more realistic scenarios and complexities. New attacks are presented, up to seven rounds, relying on integral and higher-order differential cryptanalysis.

1 Introduction

A need of low-cost cryptosystems for several fast-growing applications, such as radio-frequency identification tags, sensor networks or Internet of Things, has drawn great attention to the area of lightweight cryptographic primitives over the last decade. It has been a vibrant research area, where a good trade-off between security and efficiency is a particularly challenging task. Some well-established algorithms (e.g. AES [1]) may not meet the basic requirements of constrained devices – low cost hardware implementation, low power usage and latency.

Recently, at the Asiacrypt 2012 conference, a new lightweight block cipher called PRINCE has been proposed [2]. PRINCE is the result of a cooperation between the Technical University of Denmark (DTU), NXP Semiconductors and the Ruhr University Bochum. The cipher was designed to reach an extremely low-latency encryption and instant response time. These requirements are highly desirable for applications such as instant authentication or block-wise read/write access to memory devices, e.g. in solid-state hard disks.

For PRINCE – a serious proposal with a clear motivation from industry – it is very important to estimate the security margin, particularly for practical settings, regarding a future deployment of the cipher. Too conservative design (e.g. too many rounds) might result in the algorithm being below the industry expectations. On the other hand, insufficient level of security will make the users and customers reluctant to deploy and use the algorithm.

Table 1 Practical key recovery attacks

Rounds	Time	Data	Technique	Reference
4	$5 \cdot 2^{13}$	$5 \cdot 2^{5}$	integral	Section 3.1[a]
4	2^{19}	14	subspace	Grassi and Rechberger[a,b]
4	$2^{3.1}$	2^{6}	integral	Raddum and Rasoolzadeh[a,b]
4	2^{28}	$6 \cdot 2^{3}$	bit-pattern integral	Section 4
5	2^{29}	$6 \cdot 2^{4}$	integral	Section 3.2
6	2^{41}	$6 \cdot 2^{16}$	integral	Section 3.3
6	$2^{33.7}$	2^{16}	MitM	[8][a]
6	$2^{14.6}$	2^{37}	integral	[9][a]
6	2^{13}	2^{13}	integral	Raddum and Rasoolzadeh[a,b]
7	2^{57}	$6 \cdot 2^{32}$	higher-order differential	Section 5

a
This result has been given the PRINCE challenge prize.
b
This result was announced at Eurocrypt 2016 Rump Session, not published yet.

PRINCE has already gained a lot of attention from the academic community and some interesting cryptanalysis has been published [3–6]. However, most of the attacks are theoretical, usually with very high time or data complexity. To spur on more practically oriented research, PRINCE designers launched ‘PRINCE challenge’ [7] – a competition where cryptanalysts are encouraged to find key recovery attacks with time complexity below 2^{64} and a number of plaintexts set to a more realistic scenario.

Findings presented in the paper help to fill the gap in the practical attacks on PRINCE, giving a better estimation of the security margin. Table 1 [8, 9] summarises our results with reference cryptanalysis with practical complexity. (Practicality criteria taken from the PRINCE challenge, that is time and memory complexity below 2^{64} and 2^{48}, respectively.)

Regarding the attacks on a higher number of rounds, these are only theoretical ones. The up-to-date survey of these attacks is given by designers and posted on their website [7].

2 Description of PRINCE

In this section, a description of the PRINCE cipher is given with all the details needed to follow the attacks. For a complete specification and design rationale of the cipher, a reader is referred to [2].

PRINCE is the 64-bit block cipher which uses 128-bit key k. First, k is divided into two subkeys $k_0 \parallel k_1$ and then is expanded into 192 bits with a simple linear transformation L.

$$k = (k_0 \parallel k_1) \rightarrow (k_0 \parallel k_0' \parallel k_1), \quad \text{where} \quad k_0' = L(k_0) = (k_0 >\!>\!> 1) \oplus (k_0 \gg 63)$$

Fig. 1 shows the complete scheme of the PRINCE cipher. The 64-bit subkeys k_0 and $k_0{}'$ can be treated as the input and output whitening keys to the underlying block cipher named PRINCE$_{\text{core}}$ with its internal 64-bit key k_1.

The PRINCE cipher is the substitution-permutation network composed of 12 rounds. The 64-bit state can be organised as the 4 × 4 array of nibbles and this convention is used throughout the

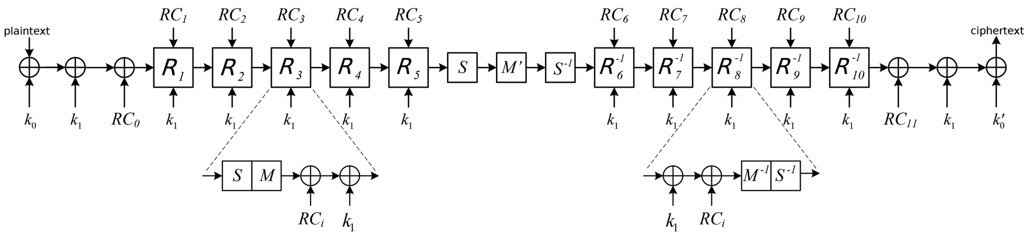
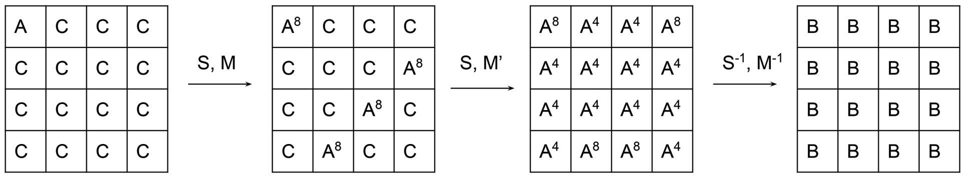

plaintext
ciphertext
RC_1 RC_2 RC_3 RC_4 RC_5
R_1 R_2 R_3 R_4 R_5 S M' S^{-1} R_6^{-1} R_7^{-1} R_8^{-1} R_9^{-1} R_{10}^{-1}
RC_6 RC_7 RC_8 RC_9 RC_{10}
k_0 k_1 RC_0 k_1 k_1 k_1 k_1 k_1 k_1 k_1 k_1 k_1 k_1 RC_{11} k_1 k'_0
S M RC_i k_1
M^{-1} S^{-1} k_1 RC_i

Fig. 1 *Scheme of the PRINCE cipher*


A C C C / C C C C / C C C C / C C C C
S, M
A⁸ C C C / C C C A⁸ / C C A⁸ C / C A⁸ C C
S, M'
A⁴ ... A⁸ A⁴ A⁴ A⁸ / A⁴ A⁴ A⁴ A⁴ / A⁴ A⁴ A⁴ A⁴ / A⁴ A⁸ A⁸ A⁴
S⁻¹, M⁻¹
B B B B / B B B B / B B B B / B B B B

A - active nibble (all 16 distinct values taken)
A^n - quasi-active nibble (n distinct values, each taken 16/n times)
C - constant nibble
B - balanced nibble (XOR sum equals 0)

Fig. 2 *3.5-round integral distinguisher for PRINCE*

paper. To specify a given nibble, we use a notation $[x,y]$. Four nibbles sharing the same x coordinate are called a *column*.

The 4-bit Sbox S can be specified in the hexadecimal notation $S = [B, F, 3, 2, A, C, 9, 1, 6, 7, 8, 0, E, 5, D, 4]$. In each round the Sbox S (or its inverse S^{-1}) is applied to all 16 nibbles. For one of the attacks, we need an explicit form of the Sbox equations given in the algebraic normal form. Four Sbox outputs $y_0 \dots y_3$ are described as follows:

$$y_3 = x_1 \oplus x_0 \oplus x_3x_2 \oplus x_3x_0 \oplus x_2x_1 \oplus x_1x_0 \oplus x_3x_2x_1 \oplus 1$$
$$y_2 = x_3x_1 \oplus x_2x_1 \oplus x_2x_0 \oplus x_3x_2x_1 \oplus x_2x_1x_0 \oplus 1$$
$$y_1 = x_3x_2 \oplus x_3x_0 \oplus x_2x_0 \oplus x_3x_2x_0 \oplus x_2x_1x_0 \oplus x_3 \oplus x_0$$
$$y_0 = x_2x_1 \oplus x_1x_0 \oplus x_3x_2x_1 \oplus x_3x_2x_0 \oplus x_3x_1x_0 \oplus x_2 \oplus x_0 \oplus 1$$

The linear step M consists of the linear matrix M' and the nibble shifting SR (similar to ShiftRows in AES); $M = \mathrm{SR} \circ M'$. The M' is an involutive, linear transformation, a kind of equivalent of MixColumns in AES. The bitwise equations of M' are given in Appendix. For a detailed algebraic description and design rationale of M, please see [2].

In the first five rounds, the order of steps is as follows. First, the subkey k_1 and the round constant RC_i are added to the state. Then, the Sbox layer is applied, followed by the linear transformation M. In the last five rounds, the inverse transformations S^{-1} and M^{-1} are used, moreover the order of them is reversed, as shown in Fig. 1. The middle rounds consist of only S, M', and S^{-1}.

We attack the round-reduced variants of PRINCE. In case of an even number of rounds, we keep the symmetry of the cipher, that is the same number of rounds are before and after the middle rounds. In case of an odd number of rounds, one extra round is added at the beginning of the cipher. (If an extra round would be added at the end, all the reported attacks still work.)

3 Integral attacks

Integral cryptanalysis was originally designed as a dedicated attack against the square cipher [10]. This cryptanalytic attack is particularly applicable to block ciphers based on substitution-permutation networks and PRINCE falls into this category.

Unlike differential cryptanalysis, where we usually trace the XOR difference between a pair of plaintexts, integral cryptanalysis uses bigger sets, e.g. 256 chosen plaintexts. Typically, most part of the plaintexts is set to a constant and some words vary through all possibilities. (These words are called *active*.) Then, we study how the XOR sum in given words changes through the subsequent steps of a cipher. We hope that after a few steps/rounds, some words still sum up to zero. (These words are called *balanced*.) Such a property would distinguish a given cipher from a random permutation and often leads to a key recovery attack.

The base for all our integral attacks is the 3.5-round integral distinguisher. We start from one active nibble (the position of the nibble is arbitrary) and after 3.5 rounds, all nibbles are still balanced, that is, their XOR sum is zero. A subsequent Sbox layer destroys the property. An integral distinguisher is very similar to the one presented originally for the square cipher and AES. Fig. 2 shows the 3.5-round integral distinguisher.

What is interesting is how the linear M' affects a single column when one nibble is active and the rest are constant. In the classic square attack (also applied to AES) if a byte is active and other three bytes are constant, then applying MixColumn operation gives you four active bytes (in that column). However, in PRINCE, the M' step transforms such column in a way that each nibble (in that column) takes exactly eight distinct values (rather than 16, as one might expect). Each of these eight values is present two times, hence they balance each other and a nibble is still balanced (sums up to zero) and behaves as it were an active nibble. This property of M' becomes clear when one studies its bitwise equations.

Bitwise Equations of M' can be expressed as the parallel application of two independent transformations: $\hat{M}^{(0)}$ and $\hat{M}^{(1)}$. $\hat{M}^{(0)}$ is applied to columns 0 and 3, whereas $\hat{M}^{(1)}$ is applied to columns 1 and 2. Hence, four nibbles $(x_3 \dots x_0)$ are transformed into other four nibbles $(y_3 \dots y_0)$ by these transformations. Bitwise equations behind $\hat{M}^{(0)}$ and $\hat{M}^{(1)}$ are as follows. (A nibble in a column and a particular bit in a nibble are denoted by a lower and an upper index, respectively.) (see equation below) (see equation below) These equations are helpful to analyse how M' affects the column with one active nibble and other three constant. It was stated that M' step transforms such a column in a way that each nibble (in that column) takes exactly eight distinct values (rather than 16, as one might expect). For example, let us assume that $\hat{M}^{(0)}$ is applied to a column and the nibble x_0 is active; x_0 consists of four bits $(x_0^0, x_0^1, x_0^2, x_0^3)$. Now, if we look carefully, for all four output nibbles (y_3, y_2, y_1, y_0), there is one output bit $(y_3^3, y_2^2, y_1^1, y_0^0$, respectively) which does not depend on any of these active bits $(x_0^0, x_0^1, x_0^2, x_0^3)$. Hence, such a bit has to be constant. Consequently, a nibble cannot take all 16 values (but takes eight), as one bit is constant. Reasoning for other situations (e.g. three constant nibbles and one quasi-active A^8) can be done in a similar way.

$\hat{M}^{(0)}$:

$$\begin{array}{llll}
y_0^0 = x_1^0 \oplus x_2^0 \oplus x_3^0 & y_1^0 = x_0^0 \oplus x_1^0 \oplus x_2^0 & y_2^0 = x_0^0 \oplus x_1^0 \oplus x_3^0 & y_3^0 = x_0^0 \oplus x_2^0 \oplus x_3^0 \\
y_0^1 = x_0^1 \oplus x_2^1 \oplus x_3^1 & y_1^1 = x_1^1 \oplus x_2^1 \oplus x_3^1 & y_2^1 = x_0^1 \oplus x_1^1 \oplus x_2^1 & y_3^1 = x_0^1 \oplus x_1^1 \oplus x_3^1 \\
y_0^2 = x_0^2 \oplus x_1^2 \oplus x_3^2 & y_1^2 = x_0^2 \oplus x_2^2 \oplus x_3^2 & y_2^2 = x_1^2 \oplus x_2^2 \oplus x_3^2 & y_3^2 = x_0^2 \oplus x_1^2 \oplus x_2^2 \\
y_0^3 = x_0^3 \oplus x_1^3 \oplus x_2^3 & y_1^3 = x_0^3 \oplus x_1^3 \oplus x_3^3 & y_2^3 = x_0^3 \oplus x_2^3 \oplus x_3^3 & y_3^3 = x_1^3 \oplus x_2^3 \oplus x_3^3
\end{array}$$

$\hat{M}^{(1)}$:

$$\begin{array}{llll}
y_1^0 = x_1^0 \oplus x_2^0 \oplus x_3^0 & y_0^0 = x_0^0 \oplus x_1^0 \oplus x_2^0 & y_1^0 = x_0^0 \oplus x_1^0 \oplus x_3^0 & y_2^0 = x_0^0 \oplus x_2^0 \oplus x_3^0 \\
y_1^1 = x_0^1 \oplus x_2^1 \oplus x_3^1 & y_0^1 = x_1^1 \oplus x_2^1 \oplus x_3^1 & y_1^1 = x_0^1 \oplus x_1^1 \oplus x_2^1 & y_2^1 = x_0^1 \oplus x_1^1 \oplus x_3^1 \\
y_1^2 = x_0^2 \oplus x_1^2 \oplus x_3^2 & y_0^2 = x_0^2 \oplus x_2^2 \oplus x_3^2 & y_1^2 = x_1^2 \oplus x_2^2 \oplus x_3^2 & y_2^2 = x_0^2 \oplus x_1^2 \oplus x_2^2 \\
y_1^3 = x_0^3 \oplus x_1^3 \oplus x_2^3 & y_0^3 = x_0^3 \oplus x_1^3 \oplus x_3^3 & y_1^3 = x_0^3 \oplus x_2^3 \oplus x_3^3 & y_2^3 = x_1^3 \oplus x_2^3 \oplus x_3^3
\end{array}$$

```
Encrypt 5 sets of $2^4$ plaintexts with one active nibble;
foreach 16 nibbles do
    foreach values of $k_1 \oplus k_0'$ nibble do
        foreach 5 sets of plaintexts do
            Decrypt all ciphertexts for a given set through the Sbox;
            Sum the decrypted nibbles;
            If the sum is zero, then the guess is a candidate for correct $k_1 \oplus k_0'$;
        end
    end
    Identify a candidate of $k_1 \oplus k_0'$ which appears in all 5 sets for a given nibble;
end
Encrypt another 5 sets of $2^4$ plaintexts with four active nibbles (placed as in the
second diagram in Figure 2);
Peel off the 4th round with the recovered $k_1 \oplus k_0'$;
foreach 16 nibbles do
    foreach values of $k_1$ nibble do
        foreach 5 sets of plaintexts do
            Decrypt all ciphertexts for a given set through the Sbox;
            Sum the decrypted nibbles;
            If the sum is zero, then the guess is a candidate for correct $k_1$;
        end
    end
    Identify a candidate of $k_1$ which appears in all 5 sets for a given nibble;
end
Recover 64 bits of $k_0$ by solving a set of linear equations of the form $L(k_0) = k_0'$;
```

Fig. 3 *Algorithm 1: 4-round integral attack*

3.1 4-round attack

First, we encrypt a set of 2^4 plaintexts, with one active nibble. We guess a value of $k_1 \oplus k_0'$, partially decrypt ciphertexts through the last Sbox and check whether the given nibble is balanced or not. With a correct guess, a nibble must be balanced (all bits sum up to zero). This is repeated for all 16 nibbles. For a given nibble, there are 2^4 possible sums and one may expect that only for the correct key nibble, we get the balanced nibble. However, our implementation shows that there are some false positives and it is better to use more sets of plaintexts, making the filter stronger. (Five sets are sufficient to eliminate all false positives.)

Then, the standard procedure would be to peel off the fourth round (as $k_1 \oplus k_0'$ is already known) and next recover k_1, nibble by nibble. However, this is not possible since at the end of the third round all nibbles are still balanced, then any k_1 guess would be 'correct', hence there is no filter. To deal with that, the attacker may exhaustively search k_1 (or k_0) and recover the whole key with time complexity 2^{64} (as it was done in [4]). However, we can do better by using an extra set of plaintexts.

Instead of using the 3.5-round distinguisher, we now use the 2.5-round one, starting from four active nibbles (placed as in the second diagram in Fig. 2). We peel off the fourth round and then recover k_1 key, on nibble by nibble basis. Now a filter works as at the end of the third round a nibble is not guaranteed to be balanced.

Once we know k_1 and $(k_1 \oplus k_0')$, k_0' is also known. Therefore, we can easily calculate k_0 from a set of linear equations $L(k_0) = k_0'$. The pseudo-code of the attack is given below.

Data complexity of the attack is $5 \cdot 2^4 + 5 \cdot 2^4 \cong 2^7$ chosen plaintexts. Time complexity is dominated by the Sbox calls and is equal to 16(nibbles) $\cdot$ 5(sets)$\cdot$ 2^4 $\cdot$ 2^4(possible keys) = $5 \cdot 2^{13}$. We implemented the attack on a desktop PC and the full key recovery takes a fraction of a second (see Fig. 3).

3.2 5-round attack

An extension of the attack to five rounds is done by guessing a complete column (four nibbles) of $k_1 \oplus k_0'$, rather than a single nibble. Consequently, we can partially decrypt a column of ciphertexts through the Sbox layer and M-layer. Then, we can guess a single nibble of $k_1[0, 0]$ to pass through the subsequent Sbox. If the guesses are correct, the balance property should hold (according to the 3.5-round distinguisher).

The attack implementation indicates that six sets of plaintexts make the filter strong enough. Thus, data complexity is $6 \cdot 2^4$ chosen plaintexts. Time complexity is dominated by the Sbox calls

$$\begin{array}{cccc|cccc|cccc|cccc}
a_0 & c & c & c & c & a_1 & c & c & c & c & a_2 & c & c & c & c & c \\
c & c & c & c & c & c & c & c & c & c & c & c & c & c & c & c \\
c & c & c & c & c & c & c & c & c & c & c & c & c & c & c & c \\
c & c & c & c & c & c & c & c & c & c & c & c & c & c & c & c
\end{array}$$

$$\downarrow$$
S-Layer
$$\downarrow$$

$$\begin{array}{cccc|cccc|cccc|cccc}
d_0 & d_0 & d_0 & d_0 & d_1 & d_1 & d_1 & d_1 & d_2 & d_2 & d_2 & d_2 & c & c & c & c \\
c & c & c & c & c & c & c & c & c & c & c & c & c & c & c & c \\
c & c & c & c & c & c & c & c & c & c & c & c & c & c & c & c \\
c & c & c & c & c & c & c & c & c & c & c & c & c & c & c & c
\end{array}$$

$$\downarrow$$
M-Layer
$$\downarrow$$

$$\begin{array}{cccc|cccc|cccc|cccc}
c & d_0 & d_0 & d_0 & d_1 & c & d_1 & d_1 & c & d_2 & d_2 & d_2 & c & c & c & c \\
d_1 & d_1 & c & d_1 & d_2 & c & d_2 & d_2 & c & c & c & c & d_0 & c & d_0 & d_0 \\
d_2 & d_2 & c & d_2 & c & c & c & c & d_0 & d_0 & c & d_0 & d_1 & d_1 & d_1 & c \\
c & c & c & c & d_0 & d_0 & d_0 & c & c & d_1 & d_1 & d_1 & d_2 & d_2 & d_2 & c
\end{array}$$

$$\downarrow$$
S-Layer
$$\downarrow$$

$$\begin{array}{cccc|cccc|cccc|cccc}
d_0 & d_0 & d_0 & d_0 & d_1 & d_1 & d_1 & d_1 & d_2 & d_2 & d_2 & d_2 & c & c & c & c \\
d_1 & d_1 & d_1 & d_1 & d_2 & d_2 & d_2 & d_2 & c & c & c & c & d_0 & d_0 & d_0 & d_0 \\
d_2 & d_2 & d_2 & d_2 & c & c & c & c & d_0 & d_0 & d_0 & d_0 & d_1 & d_1 & d_1 & d_1 \\
c & c & c & c & d_0 & d_0 & d_0 & d_0 & d_1 & d_1 & d_1 & d_1 & d_2 & d_2 & d_2 & d_2
\end{array}$$

$$\downarrow$$
M'-Layer
$$\downarrow$$

$$\begin{array}{cccc|cccc|cccc|cccc}
b_1 & b_0 & b_0 & b_0 & b_1 & b_0 & b_0 & b_0 & b_0 & b_0 & b_1 & b_0 & b_0 & b_0 & b_1 & b_0 \\
b_0 & b_1 & b_0 & b_0 & b_0 & b_1 & b_0 & b_0 & b_0 & b_0 & b_0 & b_1 & b_0 & b_0 & b_0 & b_1 \\
b_0 & b_0 & b_1 & b_0 & b_0 & b_0 & b_1 & b_0 & b_1 & b_0 & b_0 & b_0 & b_1 & b_0 & b_0 & b_0 \\
b_0 & b_0 & b_0 & b_1 & b_0 & b_0 & b_0 & b_1 & b_0 & b_1 & b_0 & b_0 & b_0 & b_1 & b_0 & b_0
\end{array}$$

$$\downarrow$$
S-Layer
$$\downarrow$$

$$\begin{array}{cccc|cccc|cccc|cccc}
? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? \\
? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? \\
? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? \\
? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ? & ?
\end{array}$$

Fig. 4 *Evolution of patterns through the first three rounds. After the third Sbox layer the balance property does not hold any more for any of bits*

and is equal to $6 \cdot 2^4$ (plaintexts) $\cdot 4$(columns) $\cdot\ 2^{16+4} \cong 2^{29}$. A secret key is recovered in about 3 minutes on a desktop PC.

3.3 6-round attack

The 6-round attack is exactly the same as the 5-round variant except that we start with a larger structure of 2^{16} plaintexts. We use the same idea which allows to add one more round (at the beginning) for the integral attack against square or AES [10]. Four nibbles from the same column takes all possible 2^{16} values and the remaining nibbles are set to some arbitrary constant. After the first round, we have, in fact, 2^{12} sets of 2^4 plaintexts ready for the 5-round attack. Hence, data and time complexity have to be multiplied by a factor of 2^{12} (in comparison to the 5-round attack). Thus, required data is $6 \cdot 2^{16}$ chosen plaintexts and time is 2^{41}. The 6-round attack would take a couple of days on a single PC, but it can be easily parallelised and 2^{41} complexity is not an obstacle to get the very practical time. We experimented with the limited pool of keys (including the correct key) to confirm that indeed the correct key is recovered and false ones are discarded.

4 Bit-pattern based integral attack

The integral attack naturally fits to the primitives with the word-oriented structure such as square, AES and also PRINCE. However, it has been shown [11], that a modified variant of the technique (called bit-pattern based integral attack) can be applied to the bit-oriented algorithms. The classic integral attack starts with (at least) one active s-bit word, hence a number of chosen plaintexts (data complexity) has a lower bound 2^s. The bit-pattern variant of the integral attack allows to work with fewer chosen plaintexts. For practical attacks, where the adversary might have very limited power to harvest chosen plaintexts, we believe it is important to push data complexity to the lowest possible value. Hence our motivation to mount the bit-pattern based integral attack against PRINCE.

Let us first briefly describe how the technique works. There are two features which differ the bit-pattern based attack from the classic integral attack. First, we trace single bits (their patterns) rather than the whole words (such as nibbles in PRINCE or bytes in AES). Second, we care about the order of plaintexts. In a given structure (e.g. eight plaintexts), each bit position holds a specific sequence of 0's and/or 1's. According to the notation introduced in [11], we have the following patters:

- The constant pattern c means that all bits within the structure are either all 0's or all 1's.
- The pattern b_i is built with smaller, 2^i-bit constant blocks.
- The active pattern a_i is a special case of the b_i pattern. If the first 2^i-bit block is, say, all 0's, then next block is all 1's, then again all 0's and so on.
- The dual pattern d_i means the pattern is either constant or a_i.

Each pattern carries some information about the bits in a given pattern. Clearly, from the constant pattern c we can deduce the most, that is all bits are either 0's or all bits are 1's. In the opposite direction, there is b_0, which carries no information as bits take arbitrary value. For clarity, we denote this unknown pattern by ? and for b_0 we assume it is balanced (an XOR sum is 0).

To trace how bit patters change through the subsequent rounds in PRINCE, first we need the bitwise description of the cipher. The linear M/M'-layer involves only XORs of nibbles, hence the bit-level description is straightforward. For the Sbox layer we use four ANF equations given in Section 2. Therefore, the whole cipher can be expressed with only the bitwise AND and XOR. Now we have to figure out how to operate on patterns with these two operators. Some operations are pretty obvious, for example, the constant pattern c XORed with any other pattern p gives p. A complete list of operations on the patterns can be found in [11]. Below we give some examples (Table 2).

A key recovery attack with the bit-pattern variant of the integral attack works the same as for the classic integral attack. Once we have the bit-pattern integral distinguisher, we guess a part of the key from the last round, partially decrypt ciphertexts through the Sbox layer and check the balance property of certain bits. If the property holds (according to a distinguisher), then our key guess, most likely, is correct.

Fig. 4 shows an evolution of patterns through the first three rounds. We start with three active bits, hence 2^3 chosen plaintexts are needed. Please note that three active bits are placed in separate nibbles. This way they do not interact in the first round and the b_0 pattern does not appear right after the first S-layer. Addition of secret key and round constants can be omitted as these operations do not bring any changes to patterns of bits (XORing with the the constant pattern c). The balance property holds up to 2.5 rounds and the third layer of the Sboxes destroys the property.

Table 2 Example operations on patterns

a_1 00110011	c 00000000	b_2 00001111	a_1 00110011
$\oplus$ a_2 11110000	$\oplus$ a_2 11110000.	& b_1 11000011	& b_0 11100100
b_1 11000011	a_2 11110000	b_1 00000011	? 00100000

```
Encrypt 6 sets of $2^3$ plaintexts where active bits $a_0$, $a_1$ and $a_2$ are placed in
separate columns (an example placement shown in Figure 3);
foreach 4 columns of nibbles do
    foreach values of $(k_1 \oplus k_0')$ column and $k_1[0,0]$ nibble do
        foreach 6 sets of plaintexts do
            Decrypt the column through S-Layer and M-Layer;
            Decrypt a nibble [0,0] through the Sbox;
            Sum the decrypted nibbles;
            If the sum is zero, then the guess is a candidate for correct $k_1 \oplus k_0'$;
        end
    end
    Identify a candidate of $k_1 \oplus k_0'$ which appears in all 6 sets for a given column;
end
Peel off the 4th round with the recovered $k_1 \oplus k_0'$;
foreach 16 nibbles do
    foreach values of $k_1$ nibble do
        foreach 6 sets of plaintexts do
            Decrypt all ciphertexts for a given set through the Sbox;
            Sum the decrypted nibbles;
            If the sum is zero, then the guess is a candidate for correct $k_1$;
        end
    end
    Identify a candidate of $k_1$ which appears in all 6 sets for a given nibble;
end
Recover 64 bits of $k_0$ by solving a set of linear equations of the form $L(k_0) = k_0'$;
```

Fig. 5 *Algorithm 2: 4-round bit-pattern based integral attack*

We experimented with other placement of active bits (e.g. all three active bits in a single nibble) but it did not lead to better results than three rounds.

One may ask why, instead of three active bits (a_0, a_1, a_2), we do not take only two active bits and limit a number of chosen plaintexts even further. The problem is that with two active bits (structure of four plaintexts) the symbolic operations do not work in the same way as for bigger structures. For example, (a_1 XOR a_0) AND c is no longer guaranteed to be balanced, while for bigger structures the outcome is balanced, namely b_0. Consequently, the '?' symbol would appear earlier. (It becomes even more clear with a trivial structure of two plaintexts, with only a_0 bit.)

4.1 4-round attack

The scheme of the attack is very similar to the 5-round classic integral attack described earlier. Six sets of plaintexts are enough to identify the correct guess of a column of $k_1 \oplus k_0'$ and eliminate all false positives. Then, the same sets of plaintexts are used to recover k_1, on a nibble by nibble basis. Thus, data complexity is $6 \cdot 2^3$ chosen plaintexts, fewer than $10 \cdot 2^4$ needed for the classic integral attack from Section 3.1. Time complexity is dominated by the Sbox calls in the first big loop, that is, $6 \cdot 2^3$ (plaintexts) $\cdot$ 4 (columns) $\cdot$ $2^{16+4} \cong 2^{28}$ operations. We implemented the attack on a desktop PC (2.2 GHz, single core) and it takes a minute to recover the key. We observed that for some nibbles there are more than one suggestion of k_1. To identify the correct 64-bit k_1, we run trial encryptions with all suggested combinations of k_1 nibbles and then compare the obtained ciphertexts to the real ciphertexts from the data set. We mounted the attack for 100 randomly chosen keys and every time a number of those extra cipher calls was negligible and did not affect time complexity.

Here is a pseudo-code of the attack (see Fig. 5).

5 7-round higher-order differential attack

Higher order differential attack is applicable to ciphers which can be represented as Boolean polynomials of a low algebraic degree [12]. In PRINCE the only non-linear step is the Sbox layer, hence an algebraic degree of a single round is the same as the degree of the Sbox, which is 3. We can take advantage of this relatively small degree to reach seven rounds.

For the standard differential cryptanalysis we operate on differences between a pair of plaintexts. Higher-order differential cryptanalysis is a natural extension, where we trace differences between a larger set of plaintexts. In our attack, we are interested in calculating ith derivative at some selected points. To do so, we need to form a set of 2^i plaintexts, where i plaintext variables change through all possible values, while the rest of the state is set to an arbitrary constant.

A ciphertext variable (expressed as a polynomial in plaintext and key variables) of the 3-round PRINCE has an algebraic degree (at most) $3^3 = 27$. Therefore, any 28-th order derivative (or higher) must be 0, regardless of the actual key values. This simple observation could lead us to the 5-round attack (similar to the integral attack), but we would have to use 2^{28} chosen plaintexts (to calculate a derivative), hence it would not bring a better result than that obtained with the integral attack. However, we can get the first few steps for 'free' and start the actual attack after the second S-layer.

We form a structure of 2^{32} chosen plaintexts, where two columns (eight nibbles) take all possible values. As the PRINCE Sbox is a bijection, the first Sbox layer preserves the property, that is, two selected columns still take all possible values. The next step M' works on columns independently (see Appendix), thus we still have 32 state bits taking all possible combinations. Then, the SR step only shifts nibbles in the state. The second Sbox layer keeps the desired state property and eventually M' in the second round destroys the property.

Therefore, we get first 1.5 rounds (two S-layers) for free and then we can launch the higher order differential attacks which covers another three S-layers plus. It is not possible to cover four S-layers as the algebraic degree $3^4 = 81$ would be greater than 32 and hence the 32-th derivative is not guaranteed to be 0. Please also note that the linear steps do not matter here as they do not change the algebraic degree. Once we have a distinguisher for 5.5 rounds the remaining 1.5 round is peeled off (key guessing) as in our previous attacks.

The scheme of the attack (taking the attack 'interface' point of view) is basically the same as the one given for the 5-round integral attack in Section 3.2. The only difference is that we start with a larger structure of 2^{32} plaintexts, hence time and data complexities are higher. Time complexity is $6 \cdot 2^{32}$ (plaintexts) $\cdot$ 4(columns) $\cdot$ $2^{16+4} \cong 2^{57}$ operations. (It is assumed, that as in the previous attacks, six sets of plaintexts make the filter strong enough.)

6 Conclusion

A few new attacks on the round-reduced (up to seven rounds) variants of PRINCE are presented. We focused on the practical attacks, most of them implemented and verified on a single desktop PC. Such analysis should help to evaluate the security margin of the cipher, especially regarding real-life scenarios and potential deployment of the algorithm. Using integral cryptanalysis we managed to reach six rounds with low data complexity. We also mounted the 7-round attack with an aid of higher-order differential cryptanalysis. We conclude that the full, 12-round PRINCE has sufficient security margin against the attacks which exploit a low algebraic degree in a cryptosystem.

7 Acknowledgment

This project was financed by National Science Centre, Poland, project registration number DEC-2014/15/B/ST6/05130.

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