Exercise 8.26: The group $S_3 \bigoplus Z_2$ is isomorphic to one of the following groups: Z_{12} , $Z_6 \bigoplus Z_2$, A_4 , D_6 . Determine which one by elimination.
It cannot be isomorphic to Z_{12} since Z_{12} is cyclic and S_3 is not. Since S_3 is not abelian we know that it is not isomorphic to $Z_6 \bigoplus Z_2$. Note that A_4 has 8 elements of order 3 and $S_3 \bigoplus Z_2$ has 2 thus they are not isomorphic. By elimination $S_3 \bigoplus Z_2$
$approx D_6$.

Exercise 8.32: What is the order of the largest cyclic subgroup of $Z_6 \bigoplus Z_{10} \bigoplus Z_{15}$? What is the order of the largest cyclic subgroup of $Z_{n_1} \bigoplus Z_{n_2} \bigoplus \cdots \bigoplus Z_{n_k}$?
Noting that all of these Z_n are cyclic we see that the largest cyclic sub group will be generated by $(g_1, g_2, \cdots, g_k)$ where g_i is the generator for ith group in the product. Note that $|(g_1, g_2, \cdots, g_k)| = \text{lcm}(|g_1|, |g_2|, \cdots, |g_n|) = \text{lcm}(n_1, |n_2|, \cdots, |n_3|)$. Thus for $Z_6 \bigoplus Z_{10} \bigoplus Z_{15}$ the largest cyclic sub group has order $\text{lcm}(2 * 3, 2 * 5, 3 * 5) = 2 * 3 * 5 = 30$.

Exercise 8.55: How many isomorphisms are there from Z_{12} to $Z_4 \bigoplus Z_3$?
Note that $Z_4 \bigoplus Z_3$ is cyclic and has twelve elements thus there is at least one isomorphisim. Deciding where to map a generator uniquely defines a isomorphisim, thus we need only examine what we map to the generator $(1, 1) \in Z_4 \bigoplus Z_3$. We know we can only map generators to that element and the generators in Z_{12} are 1,5,7,11. Thus we conclude there are exactly 4 isomorphic

Exercise 8.58: Prove that $Z_5 \bigoplus Z_5$ has exactly six subgroups of order 5.
Note that

$$< (1, 0) >= \{(1, 0), (2, 0), (3, 0), (4, 0), (0, 0)\}$$

$$< (1, 1) >= \{(1, 1), (2, 2), (3, 3), (4, 4), (0, 0)\}$$

$$< (1, 2) >= \{(1, 2), (2, 4), (3, 1), (4, 3), (0, 0)\}$$

$$< (1, 3) >= \{(1, 3), (2, 1), (3, 4), (4, 2), (0, 0)\}$$

$$< (1, 4) >= \{(1, 4), (2, 3), (3, 2), (4, 1), (0, 0)\}$$

$$< (0, 1) >= \{(0, 1), (0, 2), (0, 3), (0, 4), (0, 0)\}$$

thus there are at least 6 sub groups of order 5.
Suppose $Z_5 \bigoplus Z_5$ had 7 subgroups of order 5. Note that each of these sub groups must be cyclic, since 5 is prime. Note that each of the non identity elements are generators of there sub group since 5 is prime. Thus we see that each sub group has the identity and 4 elements in no other sub group thus there are at least $1 + 5 * 7 = 36$ elements, a contradiction since the group only has 25 elements, thus there are exactly 6 sub groups of order 5.

Exercise 8.66: Express $U(165)$ as an external direct product of cyclic groups of the form Z_n.
Note that $U(165) = U(5 * 3 * 11) \approx U(3) \bigoplus U(5) \bigoplus U(11) \approx Z_2 \bigoplus Z_4 \bigoplus Z_{10}$. (see pg 167)

Exercise 8.67: Express $U(165)$ as an external direct product of U-groups in four different ways.

$$U(165) = U(3 * 55) \approx U(3) \bigoplus U(55)$$

$$U(165) = U(5 * 33) \approx U(5) \bigoplus U(33)$$

$$U(165) = U(11 * 15) \approx U(11) \bigoplus U(15)$$

$$U(165) = U(5 * 3 * 11) \approx U(3) \bigoplus U(5) \bigoplus U(11)$$

Exercise 8.68: Without doing any calculations in Aut(Z_{20}), determine how many elements of Aut(Z_{20}) have order 4. How many have order 2?
Note that Aut(Z_{20}) $\approx U(20) \approx U(2^2) \bigoplus U(5) \approx Z_2 \bigoplus Z_4$. Note that $Z_2 \bigoplus Z_4$ has 4 elements of order 4 and 3 elements of order 2, thus Aut(Z_{20}) has 4 elements of order 4 and 3 elements of order 2.

Exercise 8.78: Find a subgroup of order 6 in $U(700)$. (see appendix)
From the table below $< 51 >$ is a subgroup of order 6. $< 51 >= \{51, 501, 351, 401, 151, 1\}$

table

```
>> x=[1:1:699];
>> isDiv=(gcd(x,700)==1);
>> pow6=(mod(x.^6,700)==1);
>> not23=(mod(x.^3,700)~=1 & mod(x.^2,700)~=1);
>> [x;(isDiv & pow6 & not23)]'
ans =

     1     0
     2     0
     3     0
     4     0
     5     0
     6     0
     7     0
     8     0
     9     0
    10     0
    11     0
    12     0
    13     0
    14     0
    15     0
    16     0
    17     0
    18     0
    19     0
    20     0
    21     0
    22     0
    23     0
    24     0
    25     0
    26     0
    27     0
    28     0
    29     0
    30     0
    31     0
    32     0
    33     0
    34     0
    35     0
    36     0
    37     0
    38     0
    39     0
    40     0
    41     0
    42     0
    43     0
    44     0
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45    0
46    0
47    0
48    0
49    0
50    0
51    1
52    0
53    0
54    0
55    0
56    0
57    0
58    0
59    0
60    0
61    0
62    0
63    0
64    0
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 99     0
100     0
101     1
102     0
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149     1
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151     1
152     0
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    697      0
    698      0
    699      0

>> diary off
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