Exercise 6: Suppose a and b are integers that divide the integer c. If a and b are relatively prime, show that ab divides c. Show, by example, that if a and b are not relatively prime, then ab need not divide c.
By the fundamental theorem of algebra we can create a prime factorization for a,b, and c. Note that $a|c$ and $b|c$ implies that c contains the primes of a and b. Since a and b are relatively prime we know that they share no common prime. Thus there multiple contains all of the primes of a and those of b to the power of the original in eater a or b. Thus the primes in c are in ab and so $ab|c$.
Consider $a = 4$ and $b = 6$ and $c = 12$. Note that a and b divide c however $ab = 24 > c$ and thus ab does not divide c

Exercise 7: If a and b are integers and n is a positive integer, prove that $a \mod n = b \mod n$ if and only if n divides $a - b$.
Suppose $a \mod n = b \mod n$. Note that there exists integer i, such that, $a = in + (a \mod n)$, also there exists integer j such that $b = jn + (b \mod n)$. Note that $a - b = in + (a \mod n) - jn - (b \mod n) = in - jn = (i - j)n$ thus n divides $a - b$.
Suppose n divides $a - b$. Note that there exists integer i, such that, $a = in + (a \mod n)$, also there exists integer j such that $b = jn + (b \mod n)$. Note that $a - b = in + (a \mod n) - jn - (b \mod n) = in - jn + (a \mod n) - (b \mod n) = (i - j)n + (a \mod n) - (b \mod n)$. Note that $(i - j)n + (a \mod n) - (b \mod n) = kn$ for some k. Thus we know that $(a \mod n) - (b \mod n) = vn$ for some integer v. However we also know $-n < (a \mod n) - (b \mod n) < n$, thus $(a \mod n) - (b \mod n) = 0$ or $(a \mod n) = (b \mod n)$.

Exercise 9: Let n be a fixed positive integer greater than 1. If $a \mod n = a'$ and $b \mod n = b'$, prove that $(a - b) \mod n = (a' - b') \mod n$ and $(ab) \mod n = (a'b') \mod n$. (This exercise is referred to in Chapters 6, 8, 10, and 15.)
Note there exists j and k such that $a = jn + a'$ and $b = kn + b'$. Note $(a - b) \mod n = (jn + a' - kn - b') \mod n = (a' - b') \mod n$. Note that $(ab) \mod n = (jn + a')(kn + b') \mod n = (jnkn + kna' + jnb' + a'b') \mod n = (a'b') \mod n$.

Exercise 11: Let n and a be positive integers and let $d = \gcd(a, n)$. Show that the equation $ax \mod n = 1$ has a solution if and only if $d = 1$. (This exercise is referred to in Chapter 2.)
Suppose there exists a x such that $ax \mod n = 1$. Note that there exists some j such that $ax = jn + ax \mod n$. Note that $ax - jn = 1$ thus $\gcd(a, n) = 1 = d$.

Exercise 12: Show that $5n + 3$ and $7n + 4$ are relatively prime for all n.
Note that $7(5n + 3) - 5(7n + 4) = 7 \cdot 5n + 21 - 5 \cdot 7n - 20 = 1$ thus $5n + 3$ and $7n + 4$ are relatively prime.

Exercise 13: Suppose that m and n are relatively prime and r is any integer. Show that there are integers x and y such that $mx + ny = r$.
Since m and n are relatively prime there exists integers j and k such that $jm + kn = 1$. Let

$x = rj$ and $y = rk$. Note that $mx + ny = mrj + nrk = r(jm + kn) = r$.