Exercise 2.11: Find the inverse of the element $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ in $GL(2, Z_{11})$.

The inverse to this matrix would be $\frac{1}{3}\begin{bmatrix} 5 & -6 \\ -3 & 2 \end{bmatrix} \mod 11 = 4\begin{bmatrix} 5 & 5 \\ 8 & 2 \end{bmatrix} \mod 11 = \begin{bmatrix} 9 & 9 \\ 10 & 8 \end{bmatrix}$

Exercise 2.13: Translate each of the following multiplicative expressions into its additive counterpart. Assume that the operation is commutative.

(a) a^2b^3 translates to $2a + 3b$.

(b) $a^{-2}(b^{-1}c)^2$ translates to $-2a + 2(-b + c)$.

(c) $(ab^2)^{-3}c^2 = e$ translates to $-3(a + 2b) + 2c = e$.

Exercise 2.19: Prove that the set of all 2×2 matrices with entries from $\mathbb{R}$ and determinant $+1$ is a group under matrix multiplication.
I will now go through the definition of a group:

(1) Associativity
We have previously discussed that all matrices have this property under multiplication.

(2) Identity
Note that $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ acts as Identity on this set.

(3) Inverse
Note that given a arbitrary matrix with determinate of 1, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(4) Closure
Suppose that the matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\begin{bmatrix} e & f \\ g & h \end{bmatrix}$ both are determinate one. Note that there multiple $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$, has a determinant of $(ae + bg)(cf + dh) - (af + bh)(ce + dg) = aecf + bgcf + aedh + bgdh - afce - bhce - afdg - bhdg = bgcf + aedh - bhce - afdg = bc(gf - he) + ad(eh - fg) = bc(-1) + ad(1) = 1$, thus closure.

Exercise 2.22: Let G be a group with the property that for any x, y, z in the group, $xy = zx$ implies $y = z$. Prove that G is Abelian. (Left-right cancellation implies commutativity.)
Note that $b \cdot ab = ba \cdot b$. By left-right cancellation $ab = ba$.

Exercise 2.25: Prove that a group G is Abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all a and b in G.
Suppose G is Abelian. Consider a and b, elements of G. Note that a^{-1} and b^{-1} are in G.

Note that $(ab)^{-1} = b^{-1}a^{-1} = a^{-1}b^{-1}$.
Suppose $(ab)^{-1} = a^{-1}b^{-1}$, for all a and b in G. Consider a and b, elements of G. Note that a^{-1} and b^{-1} are in G. Note that $ab = (a^{-1})^{-1}(b^{-1})^{-1} = (b^{-1}a^{-1})^{-1} = (b^{-1})^{-1}(a^{-1})^{-1} = ba$, thus G is Abelian.