Exercise 5.11: Determine whether the following permutations are even or odd.

(d) (12)(134)(152)
This would be made of 5 two cycles and thus is odd.

e (1243)(3521)
This would be made of 6 two cycles and thus is even.

Exercise 5.14: Find eight elements in S_6 that commute with (12)(34)(56). Do they form a subgroup of S_6 ?
Eight elements that commute are $A = \{e, (12), (12)(34), (12)(56), (12)(34)(56), (34), (34)(56), (56)\} \subseteq S_6$. Yes they do form a sub group. Note that every element is its own inverse. Define $D_1 = (12)$, $D_2 = (34)$, $D_3 = (56)$. Note that each of these Ds commute and are there own inverses. Note that every combination of Ds appear in A. Take two arbitrary elements of A, $D_1^a D_2^b D_3^c$ and $D_1^d D_2^e D_3^f$, note that combining them $D_1^{ad} D_2^{be} D_3^{cf}$ is a element in A, thus A is closed with respect to functional composition. We now can say A is a group.

Exercise 5.23: Show that if H is a subgroup of S_n, then either every member of H is an even permutation or exactly half of the members are even. (This exercise is referred to in Chapter 25.)
Suppose H is a subgroup of S_n.
Suppose that not every member of H is an even permutation and not exactly half of the members are even.
Suppose that there are more evens than odds. Note that there exists at least one odd permutation in H, call it a. Note that a applied to any even element produces a odd element of H. Noting that there are more evens than odds we can say there exist two distinct even permutations in H that go to the same odd permutation when a is applied to them (pigeonhole principal), call these elements b, c. Note that $b \neq c$ and yet $ab = ac$, witch via cancellation gives us $b = c$, a contradiction, conclude the negation of our supposition, there are more odds than evens(there can not be the same number by one of our above suppositions).
Note that a applied to any odd element produces a even element of H. Noting that there are more odds than evens we can say there exist two distinct odd permutations in H that go to the same even permutation when a is applied to them (pigeonhole principal), call these elements d, e. Note that $d \neq e$ and yet $ad = ae$, witch via cancellation gives us $d = e$, a contradiction, conclude the negation of our supposition, that either every member of H is an even permutation or exactly half of the members are even.

Exercise 5.27: Use Table 5.1 to compute the following.

a. The centralizer of $\alpha_3 = (13)(24)$
$C(\alpha_3) = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$

b. The centralizer of $a_1 2 = (124)$
$C(\alpha_{12}) = \{\alpha_1, \alpha_7, \alpha_{12}\}$

Exercise 5.32: Let $\beta = (123)(145)$. Write β^{99} in disjoint cycle form.
Note that $\beta = (14523)$. Note that $\beta^{99} = (\beta^5)^{19}\beta^4 = (e)^{19}(\beta^2)^2 = (15342)^2 = (13254)$

Exercise 5.38: Let $H = \{\beta \in S_5 \mid \beta(1) = 1 \text{ and } \beta(3) = 3\}$. Prove that H is a subgroup of S_5 . How many elements are in H? Is your argument valid when S_5 is replaced by S_n for $n \geq 3$? How many elements are in H when S_5 is replaced by A_n for $n \geq 4$?
To prove H is a subgroup we need only demonstrate closure. Take $\beta_1, \beta_2 \in H \subseteq S_5$. Note that $\beta_1\beta_2 \in S_5$ since S_5 has closure. Note that $\beta_1\beta_2(1) = \beta_1(1) = 1$ and $\beta_1\beta_2(3) = \beta_1(3) = 3$, thus $\beta_1\beta_2 \in H$. We conclude H is a subgroup.
Elements in H are allowed to permute all elements except the first and third, this would be 3 elements and so there are 3! ways to permute them, we conclude $|H| = 3! = 6$.
This argument holds as well if we replace S_5 with S_n for $n \geq 3$. In this case $|H| = (n-2)!$.
Noting that the elements in H if we replace S_5 with A_n for $n \geq 4$ are simply the even elements of H if we replace S_5 with S_n, and recalling exercise 5.23 (noting that there is at least one odd element-(24)), we conclude there are exactly half as many elements in H if we replace S_5 with A_n as we would have if we replace S_5 with S_n, that is $\frac{(n-2)!}{2}$.