Exercise 0.14: Let p, q, and r be primes other than 3. Show that 3 divides $p^2 + q^2 + r^2$.
Suppose x is some integer where $3 \nmid x$. We can use the division algorithm to conclude that $x = 3a + b$ where $a \in \mathbb{Z}$ and $b \in \{1, 2\}$. Note that $x^2 = 3(3a^2 + 2ab) + b^2$. Note that $x^2 \mod 3 = b^2 \mod 3$ witch is eater $1^2 \mod 3 = 1$ or $2^2 \mod 3 = 4 \mod 3 = 1$. We can now see that $x^2 = 3c + 1$ for some $c \in \mathbb{Z}$.
Noting that p, q, and r are primes other than 3 we conclude that 3 does not develop them, thus there exist a, b, c in the integers such that $p^2 = 3a + 1$, $q^2 = 3b + 1$, $r^2 = 3c + 1$. Note that $p^2 + q^2 + r^2 = 3(a + b + c + 1)$ thus 3 divides $p^2 + q^2 + r^2$.

Exercise 0.16: Determine $7^{1000} \mod 6$ and $6^{1001} \mod 7$.
Note that $7^{1000} \mod 6 = (7^2 \mod 6)^{500} \mod 6 = 1^{500} \mod 6 = 1$.
note that $6^{1001} \mod 7 = (6^{1000} \mod 7 \cdot 6) \mod 7 = ((6^2 \mod 7)^{500} \cdot 6) \mod 7 = (1^{500} \cdot 6) \mod 7 = 6$

Exercise 0.38: Prove that for every integer n, $n^3 \mod 6 = n \mod 6$.
Define $m = n \mod 6$. Note that $m \in \{0, 1, 2, 3, 4, 5\}$. Let's do some math with m in all cases. Note $m^3 \in \{0, 1, 8, 27, 64, 125\}$, thus $m^3 \mod 6 = \{0, 1, 2, 3, 4, 5\} = m$. Note that in all cases we have now shown $m^3 \mod 6 = m$. Note $n^3 \mod 6 = (n \mod 6)^3 \mod 6 = m^3 \mod 6 = m = n \mod 6$.

Exercise 1.10: If r_1 , r_2 , and r_3 represent rotations from D_n and f_1 , f_2 , and f_3 represent reflections from D_n , determine whether $r_1 r_2 f_1 r_3 f_2 f_3 r_3$ is a rotation or a reflection.
For any shape there are two orientations, one where the numbers assigned are increasing clockwise and another where they are increasing counter clockwise. By applying a reflection we transition between these two, thus if there are a odd number of reflections the end result will be in the reflected orientation and thus the entire operation is a reflection. in this case we have three reflections, thus the entire operation is a reflection.

Exercise 1.11: Find elements A, B, and C in D_4 such that $AB = BC$ but $A \neq C$.
$R_{270}H = D' = HR_{90}$

Exercise 1.13: Describe the symmetries of a non square rectangle. Construct the corresponding Cayley table.
The diagonal reflections no longer work as well as the rotations by 90 and 270, the rest of the symmetries work and the table is identical to the table appearing on page 33 except that the rows and columns associated with the forbidden symmetries are not present.