Exercise 14.6: Find all maximal ideals in

a. Z_8
$\{0, 2, 4, 6\}$

b. Z_{10}
$\{0, 5\}, \{0, 2, 4, 6, 8\}$

c. Z_{12}
$\{0, 2, 4, 6, 8, 10\}, \{0, 3, 6, 9\}$

d. Z_n
If $n = p_1^{k_1} p_2^{k_2} \cdots$ in terms of prime factorization then $< p_1 >_+, < p_2 >_+, \cdots$ are all maximal ideals in Z_n.

Exercise 14.8: Prove that the intersection of any set of ideals of a ring is an ideal.
Suppose ring R with ideals A and B. Choose some arbitrary element $r \in R$ and choose $x \in A \cap B$. Note that $rx \in A$ and $rx \in B$ thus $rx \in A \cap B$ and thus $A \cap B$ is an ideal.

Exercise 14.11: In the ring of integers, find a positive integer a such that

a. $< a >=< 2 > + < 3 >$
$a = 1$

b. $< a >=< 6 > + < 8 >$
$a = 2$

c. $< a >=< m > + < n >$
$a = \gcd(m, n)$

Exercise 14.13: Find a positive integer a such that

a. $< a >=< 3 >< 4 >$
$a = 12$

b. $< a >=< 6 >< 8 >$
$a = 48$

c. $< a >=< m >< n >$
$a = mn$

Exercise 14.18: Suppose that in the ring Z, the ideal $< 35 >$ is a proper ideal of J and J is a proper ideal of I. What are the possibilities for J? What are the possibilities for I?
$J =< 35a >$ where a is some natural bigger than 1 and $I =< 35ab >$ where b is some natural bigger than 1.

Exercise 14.22: Let $I =< 2 >$. Prove that $I[x]$ is not a maximal ideal of $Z[x]$ even though I is a maximal ideal of Z.
If we were to add anything to I to try to find a ideal J containing I it must be a odd integer, thus if we simply subtract off the previous integer,witch is even and thus in I and J we conclude that $1 \in J$ and thus $J = Z$. We can now note that I is maximal.
Note that $I[x]$ is all polynomials with even coefficients. Recall that the set of all polynomials where the constant term is even is a ideal of $Z[x]$ and since $I[x]$ is a subset of that set witch is itself not $Z[x]$ we conclude that $I[x]$ is not a maximal ideal of $Z[x]$.

Exercise 14.35: In $R = Z \bigoplus Z$, let $I = \{(a, 0) \mid a \in Z\}$. Show that I is a prime ideal but not a maximal ideal.
Suppose $(a, b) \in R$ and $(c, 0) \in I$. Note that $(a, b)(c, 0) = (ac, 0) \in I$ thus I is ideal.
Suppose $(a, b), (c, d) \in R$. Further suppose that $(a, b)(c, d) \in I$. Note that ether $b = 0$ or $d = 0$. WLoG take $b = 0$. Note that $(a, b) = (a, 0) \in I$. Thus I is prime.
Note that $J = \{(a, 2b) \mid a, b \in Z\}$ is ideal and that $I \subset J \subset R$. Note that I is not maximal.