Proof HW

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1 Q1

Theorem.
The sum of two odd integers is even.
Proof.

Suppose A and B are odd integers. This means by definition $2 \nmid A$ and $2 \nmid B$. There exists a maximum integer k such that $k * 2 \leq A$. We note that since $2 \nmid A$ it must follow that $k * 2 \neq A$. Note that $k * 2 < A$ but that $(k+1) * 2 > A$ by our construction of k. We have thus bounded A as $k * 2 < A < k * 2 + 2$, note that there is only one integer in this range thus $A = k * 2 + 1$.

There exists a maximum integer j such that $j * 2 \leq B$. We note that since $2 \nmid B$ it must follow that $j * 2 \neq B$. Note that $j * 2 < B$ but that $(j+1) * 2 > B$ by our construction of j. We have thus bounded B as $j * 2 < B < j * 2 + 2$, note that there is only one integer in this range thus $B = j * 2 + 1$.

Now we note that $A + B = 2 * k + 1 + 2 * j + 1 = 2 * k + 2 * j + 2 = 2 * (k + j + 1)$. We immediately note that $(k + j + 1)$ is an integer and thus by definition $2 \mid (A + B)$. Thus by definition A+B is even.
□

2 Q2

Theorem.
If m and n are both divisible by k then m+n is divisible by k
Proof.

Suppose m and n are both divisible by k. Since m is divisible by k by definition there exists some integer j such that $j * k = m$. Since n is divisible by k by definition there exists some integer i such that $i * k = n$. We note that $m + n = j * k + i * k = k * (i + j)$. Noting that $(i + j)$ is an integer we can state by definition $m + n$ is divisible by k.
□

3 Q3

Theorem.
If n is a natural number then $15^n - 8^n$ is divisible by 7.
Proof.
We will proceed with a proof by induction.

Assume n=1. Note that $15^n - 8^n = 15 - 8 = 7 * 1$. Since 1 is an integer we conclude that $7 \mid 15^n - 8^n$ holds for this case.

Suppose $7 \mid 15^n - 8^n$. By definition there exist some integer, call it k, such that $7 * k = 15^n - 8^n$. Note that $15^{n+1} - 8^{n+1} = 15 * 15^n - 8 * 8^n = 14 * 15^n - 7 * 8^n + 15^n - 8^n$. We also know that $7 * k = 15^n - 8^n$ putting this in we see that $15^{n+1} - 8^{n+1} = 2 * 7 * 15^n - 7 * 8^n + 7 * k = 7 * (2 * 15^n - 8^n + k)$. Note that $(2 * 15^n - 8^n + k)$ is an integer and thus by definition $7 \mid 15^{n+1} - 8^{n+1}$.

By induction we conclude that for n being a natural number $15^n - 8^n$ is divisible by 7.
$\square$