PHYS 462 (optics) HW#2

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1 #54

What is the critical angle for total internal reflection in diamond? Looking on table 4.1 we see that diamond has a reflective index $n_i = 2.417$ we will assume outside of this diamond there is a vacuum or air or some other medium with a low index of reflection $n_t = 1$. We know total internal reflection occurs as the angle of the transmitted light with respect to the normal to the plane of incidence goes to $\pi/2$. Using the law of transmission we get

$$n_i \sin(\theta_i) = n_t \sin(\theta_t) \Rightarrow \arcsin[\frac{n_t}{n_i} sin(\theta_t)] = \theta_i = 28°$$

This is a very shallow angle of total internal reflection, so we would expect light to get into the diamond and then bounce around a lot before escaping. This would cause a well cut diamond to take a small bit of light and radiate it in all directions.

2 #57

See fig1

We are asked to consider a fish looking upwards towards the surface. Take the fish as looking at things in a cone of base angle θ the angle that the edge of that cone makes with the surface is then $\theta_i = \theta/2$. We can then calculate the angle of transmission as $\sin(\theta_t) = n_i \sin(\theta_i)$ and the apparent cone (the base angle of the sky cone that gets compressed into the fish's cone as $\theta' = \theta_t * 2$. Now we need only ask what angle the fish's cone must be for the entire sky to be viable:

$$\theta' = \pi \Rightarrow sin(\theta_t) = 1 = n_i \sin(\theta_i) \Rightarrow \theta = \theta_i * 2 = \arcsin(\frac{1}{n_i}) * 2 = 97°$$

So the fish sees the entire sky shrunk from a disk of 180° to a disk of 97° because of the angles of transmission.

3 #65

We are asked to compute the angle of total polarization of reflected light. We can re-state this as the angle at witch $r_{||} = 0$ or using equation(4.38)

$$0 = \frac{n_t}{\mu_t} \cos(\theta_i) - \frac{n_i}{\mu_i} \cos(\theta_t)$$

We also know that:

$$\sin(\theta_t) = \frac{n_i}{n_t} sin(\theta_i) \Rightarrow \cos(\theta_t) = \sqrt{1 - \frac{n_i^2 \sin^2(\theta_i)}{n_t^2}}$$

also recall:

$$\frac{c}{n} = \frac{1}{\sqrt{\mu\epsilon}} \Rightarrow (\frac{n_1}{n_2})^2 = \frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}$$

Now we can begin reducing:

$$0 = \frac{n_t}{\mu_t}\cos(\theta_i) - \frac{n_i}{\mu_i}\cos(\theta_t)$$

$$\frac{n_t}{\mu_t} = \frac{n_i}{\mu_i\cos(\theta_p)}\sqrt{1 - \frac{n_i^2\sin^2(\theta_p)}{n_t^2}}$$

$$(\frac{n_t\mu_i}{n_i\mu_t})^2 = \frac{1}{\cos^2(\theta_p)} - \frac{n_i^2\tan^2(\theta_p)}{n_t^2}$$

$$(\frac{n_t\mu_i}{n_i\mu_t})^2 = 1 + \tan^2(\theta_p) - \frac{n_i^2\tan^2(\theta_p)}{n_t^2}$$

$$\tan^2(\theta_p) = \frac{\frac{\epsilon_t\mu_i}{\epsilon_i\mu_t} - 1}{1 - \frac{\mu_i\epsilon_i}{\mu_t\epsilon_t}}$$

$$\tan(\theta_p) = \sqrt{\frac{\epsilon_t(\epsilon_t\mu_i - \epsilon_i\mu_t)}{\epsilon_i(\mu_t\epsilon_t - \mu_i\epsilon_i)}}$$

4 #70

Calculate the transmittance of normal and parallel polarized waves:
(4.62-rewriting)

$$T = \frac{\sin(\theta_i)\cos(\theta_t)}{\sin(\theta_t)\cos(\theta_i)}t^2$$

(4.44)

$$t_\perp = \frac{2\sin(\theta_t)\cos(\theta_i)}{\sin(\theta_i + \theta_t)}$$

so:

$$T_\perp = \frac{\sin(\theta_i)\cos(\theta_t)}{\sin(\theta_t)\cos(\theta_i)}\frac{4\sin^2(\theta_t)\cos^2(\theta_i)}{\sin^2(\theta_i + \theta_t)}$$

$$T_\perp = \frac{4sin(\theta_i)\cos(\theta_t)\sin(\theta_t)\cos(\theta_i)}{\sin^2(\theta_i + \theta_t)}$$

knowing:

$$2\sin(\theta)\cos(\theta) = sin(2\theta)$$

we get

$$T_\perp = \frac{sin(2\theta_i)\sin(2\theta_t)}{\sin^2(\theta_i + \theta_t)}$$

now compute $T_\parallel$

$$t_\parallel = \frac{2\sin(\theta_t)\cos(\theta_i)}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

$$T_\parallel = \frac{\sin(\theta_i)\cos(\theta_t)}{\sin(\theta_t)\cos(\theta_i)}\frac{4\sin^2(\theta_t)\cos^2(\theta_i)}{\sin^2(\theta_i + \theta_t)\cos^2(\theta_i - \theta_t)}$$

$$T_\parallel = \frac{4\sin(\theta_i)\cos(\theta_t)\sin(\theta_t)\cos(\theta_i)}{\sin^2(\theta_i + \theta_t)\cos^2(\theta_i - \theta_t)}$$

$$T_\parallel = \frac{sin(2\theta_i)\sin(2\theta_t)}{\sin^2(\theta_i + \theta_t)\cos^2(\theta_i - \theta_t)}$$

5 #71

Show $R_{\parallel} + T_{\parallel} = 1$ and $R_{\perp} + T_{\perp} = 1$.

$$R_{\parallel} + T_{\parallel} = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} + \frac{sin(2\theta_i)\sin(2\theta_t)}{\sin^2(\theta_i + \theta_t)\cos^2(\theta_i - \theta_t)}$$

$$\theta_i - \theta_t = \alpha$$

$$\theta_i + \theta_t = \beta$$

$$sin(2\theta_i)\sin(2\theta_t) = 4\sin(\theta_i)\cos(\theta_t)\sin(\theta_t)\cos(\theta_i) = (2\cos(\theta_i)\cos(\theta_t)) * (2\sin(\theta_t)\sin(\theta_i)) =$$

$$(cos(\alpha) + cos(\beta))(cos(\alpha) - cos(\beta)) = cos^2(\alpha) - cos^2(\beta)$$

$$T_{\parallel} = \frac{cos^2(\alpha) - cos^2(\beta)}{\sin^2(\beta)\cos^2(\alpha)}$$

$$cos^2(\alpha) - cos^2(\beta) = cos^2(\alpha) - cos^2(\beta) + cos^2(\alpha) * cos^2(\beta) - cos^2(\alpha) * cos^2(\beta) = (1 - cos^2(\beta))cos^2(\alpha) - (1 - cos^2(\alpha))cos^2(\beta) =$$

$$sin^2(\beta)cos^2(\alpha) - sin^2(\alpha)cos^2(\beta)$$

By combining the above identity and our equation for $T_{\parallel}$:

$$T_{\parallel} = 1 - \frac{\tan^2(\alpha)}{\tan^2(\beta)}$$

Witch if we observe the above sum of $R_{\parallel} + T_{\parallel}$ we see that the tangent ratios cancel out and we are left with 1.

Now examine $R_{\perp} + T_{\perp}$:

$$R_{\perp} + T_{\perp} = \frac{\sin^2(\alpha)}{\sin^2(\beta)} + \frac{cos^2(\alpha) - cos^2(\beta)}{\sin^2(\beta)}$$

$sin(2\theta_i)\sin(2\theta_t) = cos^2(\alpha) - cos^2(\beta)$ was obtained in the previous section.

$$R_{\perp} + T_{\perp} = \frac{\sin^2(\alpha) + cos^2(\alpha) - cos^2(\beta)}{\sin^2(\beta)} = \frac{1 - cos^2(\beta)}{\sin^2(\beta)} = \frac{sin^2(\beta)}{\sin^2(\beta)} = 1$$

6 #78

See fig2