Proof HW

Parker Whaley

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1 3.4.9(c)

Therm.
$\sum_{k=1}^{n}(2k-1) = n^2$ for $n \in \mathbb{N}$

Proof.
We will proceed with a proof by induction on n.

Consider the case that $n = 1$. Note $\sum_{k=1}^{n}(2k-1) = (2-1) = 1 = 1^2 = n^2$. We immediately see that the predicate holds in this base case.

Suppose $\sum_{k=1}^{n}(2k-1) = n^2$ holds for some $n \in \mathbb{N}$. Consider the sum $\sum_{k=1}^{n+1}(2k-1) = \sum_{k=1}^{n}(2k-1)+(2(n+1)-1) = n^2+(2n+2-1) = n^2+2n+1$. Note that $(n+1)^2 = n^2+2n+1$ by foiling out $(n+1)\cdot(n+1)$. We now see that $\sum_{k=1}^{n+1}(2k-1) = (n+1)^2$ thus if our predicate holds in the n case it must hold in the n+1 case.

By induction on n we conclude that $\sum_{k=1}^{n}(2k-1) = n^2$ for $n \in \mathbb{N}$.

2 3.5.20

Therm.
For $n \in \mathbb{W}$ such that $n \geq 30$ there exists a $j, k \in \mathbb{W}$ such that $n = 6j + 7k$.

Proof.
We will proceed with a six step proof by induction.

Consider the following cases, where we let j and k be the specified whole number values and calculate n:
$j = 5, k = 0 : 6j + 7k = 30$
$j = 4, k = 1 : 6j + 7k = 31$
$j = 3, k = 2 : 6j + 7k = 32$
$j = 2, k = 3 : 6j + 7k = 33$
$j = 1, k = 4 : 6j + 7k = 34$
$j = 0, k = 5 : 6j + 7k = 35$

So we can say that our predicate holds in the n=30,31,32,33,34,35 cases.

Suppose our predicate holds for some whole number n grater than 30. This means there exist two whole numbers, lets call them j and k, such that $n = 6j + 7k$. Consider $n + 6 = 6j + 7k + 6 = 6(j + 1) + 7k$. Note that j+1 and k are whole numbers. Thus if the predicate holds for n it must hold for $n + 6$.

Since we have shown that our predicate holds for the first six cases and that if it holds in case n it holds in case n+6, we conclude by induction that it holds in all cases $n \geq 30$.