PHYS 351 #4

Parker Whaley

February 21, 2016

1 Q1

Recall that the van der Waals equation of state is $(P + \frac{a}{v^2})(v - b) = RT$, where v is the molar volume and a and b depend only on the type of gas.

1.1 a

The coefficient of thermal expansion is defined as follows:

$$\beta \equiv \frac{1}{V}(\frac{\partial V}{\partial T})\Big|_P$$

Find β for a van der Waals gas. Show that this reduces to the ideal gas result, $\beta_{ideal} = \frac{1}{T}$, when a = 0 and b = 0.

Lets start by taking a $\frac{\partial}{\partial T}$ holding P as a constant. Noting that $v = V/n$ we begin taking a implicit derivative on boath sides.

$$\frac{\partial}{\partial T}(P + \frac{an^2}{V^2})(V/n - b) = \frac{\partial}{\partial T}RT$$

$$(\frac{-2an^2}{V^3}\frac{\partial V}{\partial T}\Big|_P)(V/n - b) + (P + \frac{an^2}{V^2})(1/n\frac{\partial V}{\partial T}\Big|_P) = R$$

$$\frac{\partial V}{\partial T}\Big|_P = \frac{R}{(\frac{-2an^2}{V^3})(V/n - b) + (P + \frac{an^2}{V^2})(1/n)}$$

$$\frac{\partial V}{\partial T}\Big|_P = \frac{Rv^2V}{(-2a)(v - b) + (Pv^2 + a)(v)} = \frac{Rv^2V}{2ab + (Pv^2 - a)v}$$

$$\beta = \frac{Rv^2}{2ab - av + Pv^3}$$

(Cool β is size independent, not dependent on V) If we take the ideal gass limit (a=0 b=0) we get:

$$\beta = \frac{R}{Pv}$$

Noting that Pv=RT we see imeadiately:

$$\beta = \frac{1}{T}$$

Verifing the expected value for an ideal gas.

1.2 b

The coefficient of compression is defined as follows:

$$\kappa \equiv -\frac{1}{V}(\frac{\partial V}{\partial P})\bigg|_T$$

Find κ for a van der Waals gas. Show that this reduces to the ideal gas result, $\kappa_{ideal} = \frac{1}{P}$, when $a = 0$ and $b = 0$.

Lets start by taking a $\frac{\partial}{\partial P}$ holding T as a constant. Noting that $v = V/n$ we begin taking a implicit derivative on boath sides.

$$\frac{\partial}{\partial P}(P + \frac{an^2}{V^2})(V/n - b) = \frac{\partial}{\partial P}RT$$

$$(1 + \frac{-2an^2}{V^3}\frac{\partial V}{\partial P}\bigg|_T)(V/n - b) + (P + \frac{an^2}{V^2})(1/n\frac{\partial V}{\partial P}\bigg|_T) = 0$$

$$(1 + \frac{2a}{v^2}\kappa)(v - b) - (P + \frac{a}{v^2})(v\kappa) = 0$$

$$2a\kappa(v - b) - (Pv^2 + a)(v\kappa) = (b - v)v^2$$

$$\kappa = \frac{(v - b)v^2}{2ab + Pv^3 - av}$$

This term is also size independent. We note that as a and b become 0 the above becomes $\kappa_{ideal} = \frac{1}{P}$.

2 Q2

We have an elastic band. The Youngs modulus Y is defined by the linear relation between a differential change in length, dL, that results from the application of a differential increment of force, dF: $dF = \frac{YA}{L_\circ}dL$, where the length is $L_\circ$ when no force is applied, and the cross-sectional area is A.

2.1 a

Let the tension on the band be increased quasi-statically from T_1 to T_2. Calculate the work done on the band, assuming that the cross-section does not change appreciably.