Proof HW

Parker Whaley

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1

1.1

Theorem.
If * is closed on sets A and B and $A \cap B \neq \emptyset$ then * is closed on $A \cap B$.
Proof.
Suppose * is closed on sets A and B and $A \cap B \neq \emptyset$. Chose two arbitrary elements of $A \cap B$ lets call these c and d. By definition c and d are elements of A and elements of B. By simplification c and d are elements of A. Since A is closed under * note that $c * d \in A$. By simplification c and d are elements of B. Since B is closed under * note that $c * d \in B$. Note that $c * d \in A \wedge c * d \in B \rightarrow c * d \in A \cap B$. Since c and d were chosen arbitrarily from $A \cap B$ and $c * d$ was shown to be in $A \cap B$ we can say that $A \cap B$ is closed under *.

1.2

This is false, counterexample.
Suppose * is a operator on the $\mathbb{R}$ that returns the left side if both numbers are the same otherwise it returns the multiple.
Consider sets A=2 B=3. Note that since $2 * 2 = 2$ and $3 * 3 = 3$ that * is closed on both A and B. We also note that $2 * 3 = 6$ witch is not a element of $A \cap B$ thus * is not closed in the set $A \cap B$.

2

Show that $\oplus$ is communicative.
Consider two arbitrary / ratios n/m and j/k. Note that:

$$n/m \oplus j/k = (nk + mj)/(mk) = (jm + kn)/(km) = n/m \oplus j/k$$

So we can say that for arbitrary ratios $\oplus$ commutes.

3

Show that $a/b = a/b \otimes c/c$ given that $c \neq 0 \neq b$.
Note that $a/b \otimes c/c = (ac)/(bc)$. Also note that since $abc = bac$ we can say $a/b = (ac)/(bc)$.
Note $a/b = (ac)/(bc) = a/b \otimes c/c$ thus $a/b = a/b \otimes c/c$.