PHYS 472L #19#20

Parker Whaley

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1 #19

1.1 a

Let's consider the Lorentz invariant (ESC) $F^{\mu\nu}F_{\nu\nu} = F^{\mu\nu}F^{\pi\tau}g_{\mu\pi}g_{\nu\tau}$. Breaking ESC we note that g has only diagonal terms and so we get a free $\delta_{\mu\pi}$ and $\delta_{\nu\tau}$ so $\sum F^{\mu\nu}F^{\pi\tau}g_{\mu\pi}g_{\nu\tau}\delta_{\mu\pi}\delta_{\nu\tau} = \sum(F^{\mu\nu})^2 g_{\mu\mu}g_{\nu\nu}$. Using $Tr = -2$ convention g is only 1 for the time components and -1 for the space components so we can quickly evaluate the above statement to $2\vec{E}^2 - 2\vec{B}^2$. Decide by 2 and we conclude $\vec{E}^2 - \vec{B}^2$ is a invariant.

If there were a frame where $\vec{E}$ vanished we would know $\vec{E}^2 - \vec{B}^2 < 0 \Rightarrow \vec{E}^2 < \vec{B}^2$ in all frames.

1.2 b

For this work in analogy to the previous question examining the self projection of $F^{\mu\nu}G_{\nu\nu} = F^{\mu\nu}G^{\pi\tau}g_{\mu\pi}g_{\nu\tau}$. (copy and paste the above arguments) We see immediately that we have the invariant $\sum F^{\mu\nu}G^{\mu\nu}g_{\mu\mu}g_{\nu\nu}$. Witch then gives us $4\vec{E} \bullet \vec{B}$ (same argument about the sign effect of g then notice we have 4 copies of $E_d * B_d$). Now we have our Lorenz invariant $\vec{E} \bullet \vec{B}$.

If there were a frame where $\vec{E}$ or $\vec{B}$ vanished then in all frames $\vec{E} \bullet \vec{B} = 0$ witch would mean $\vec{E}$ and $\vec{B}$ are perpendicular.

1.3 c

Well we demonstrated in class that G's self projection does not yield another invariant. So I can't see how we would arrive at any more invariants.

2 #20

2.1 a

The EM tensor (Faraday) in Tr=-2 is:

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}$$

In our boosted frame $F^{\mu'\nu'} = \bigwedge_{\mu}^{\mu'} \bigwedge_{\nu}^{\nu'} F^{\mu\nu}$ and $\bigwedge_{\nu}^{\nu'} F^{\mu\nu} = F^{\mu\nu'} =$

$$\begin{bmatrix} \gamma\beta E_x & \gamma E_x & \gamma E_y - \gamma\beta B_z & \gamma E_z + \gamma\beta B_y \\ -\gamma E_x & -\gamma\beta E_x & -\gamma\beta E_y - \gamma B_z & -\gamma\beta E_z - \gamma B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}$$

After applying $\bigwedge_{\mu}^{\mu'} F^{\mu\nu'} = F^{\mu'\nu'} =$

$$\begin{bmatrix} 0 & \gamma^2 E_x(1-\beta^2) & \gamma E_y - \gamma\beta B_z & \gamma E_z + \gamma\beta B_y \\ -\gamma^2 E_x(1-\beta^2) & 0 & -\gamma\beta E_y - \gamma B_z & -\gamma\beta E_z - \gamma B_y \\ -\gamma E_y + \gamma\beta B_z & \gamma\beta E_y + \gamma B_z & 0 & B_x \\ -\gamma E_z - \gamma\beta B_y & \gamma\beta E_z + \gamma B_y & -B_x & 0 \end{bmatrix}$$

Noting that $1 - \beta^2 = 1/\gamma^2$ we see that the statements in the assignment hold.

2.2 b

Now let us take the direction of boost as the normal to the plane containing the E field and the B field. Find the boost that paralyses the E and B fields, in other words $E' \times B' = 0$. This means

$$E_{y'} B_{z'} = E_{z'} B_{y'}$$

$$(E_y - \beta B_z)(B_z - \beta E_y) = (E_z + \beta B_y)(B_y + \beta E_z)$$

$$E_y B_z - \beta(E_y^2 + B_z^2) + \beta^2 B_z E_y = E_z B_y + \beta(E_z^2 + B_y^2) + \beta^2 B_y E_z$$

$$\Rightarrow (E \times B) \bullet \hat{x} = E_y B_z - E_z B_y \Leftarrow$$

$$(1 + \beta^2)(E \times B) \bullet \hat{x} = \beta(E \bullet E + B \bullet B)$$

$$C = -\frac{(E \bullet E + B \bullet B)}{(E \times B) \bullet \hat{x}}$$

$$\beta^2 + C\beta + 1 = 0$$

$$\beta = \frac{-C \pm \sqrt{C^2 - 4}}{2}$$