Proof HW

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1 3.6.13

We are asked to consider the set of all sets, F. We then define the $\equiv$ operator to be, for sets A and B, $A \equiv B \Leftrightarrow A \subset B \vee B \subset A$. Is $\equiv$ a equivalence relation on F?

No, I will demonstrate that it is not with a proof by contradiction.

Suppose that the operator $\equiv$ is a equivalence relation.
Consider the sets $A = \{1\}$, $B = \{1, 2\}$, $C = \{2\}$. Note that $A \equiv B$ since $A \subseteq B$. Also note that $B \equiv C$ since $C \subseteq B$. Since $\equiv$ is a equivalence relation amongst A, B, and C we can use the transitive property to say that $A \equiv B \wedge B \equiv C \rightarrow A \equiv C$. Note that, since $A \nsubseteq C \wedge C \nsubseteq A \Rightarrow \neg(A \subseteq C \vee C \subseteq A)$, DeMorgans law, it must be that $A \not\equiv C$.
We have thus reached a contradiction, it is not possible for $A \equiv B$ and $A \not\equiv B$ so our initial supposition that $\equiv$ is a equivalence operator must be false.

2 3.6.15

We are asked to consider F, the set of all non empty sets. We define $\equiv$ to be, for sets A and B, $A \equiv B \Leftrightarrow A \cap B$ is a non-empty set. Is $\equiv$ a equivalence relation on F?

No, I will demonstrate that it is not with a proof by contradiction.

Suppose that the operator $\equiv$ is a equivalence relation.
Consider the sets $A = \{1\}$, $B = \{1, 2\}$, $C = \{2\}$. Note that $A \equiv B$ since $A \cap B = \{1\}$. Also note that $B \equiv C$ since $C \cap B = \{2\}$. Since $\equiv$ is a equivalence relation amongst A, B, and C we can use the transitive property to say that $A \equiv B \wedge B \equiv C \rightarrow A \equiv C$. Note that, since $A \cap C = \varnothing$ it must be that $A \not\equiv C$.
We have thus reached a contradiction, it is not possible for $A \equiv B$ and $A \not\equiv B$ so our initial supposition that $\equiv$ is a equivalence operator must be false.

2.1 3.6.17

Is $\neq$ an equivalence relation on $\mathbb{R}$?

No, I will demonstrate this with a proof by contradiction.

Suppose $\neq$ is a equivalence relation on $\mathbb{R}$.
Consider $a = 0$. Note that $a \in \mathbb{R}$. Thus since $\neq$ is a equivalence relation and therefore must be reflexive we can say $a \neq a$. However we know $0 = 0$ thus $a = a$.
We have reached a contradiction it is impossible for $a \neq a$ and $a = a$. Thus our initial supposition that $\neq$ is a equivalence relation must be false.