# PHYS 472L #6#17

Parker Whaley

February 6, 2016

## 1 #6

Using $\vec{\beta} = \vec{x}$ by re-scaling our velocities to units of c and with $\gamma = \frac{1}{\sqrt{1-\vec{\beta}^2}}$ we can construct our four velocity as $u^\mu = (\gamma, \gamma\vec{\beta})$. Now that we have our 4-vector velocity we can take the $\frac{d}{d\tau}$ witch we recall $d\tau^2 = dt^2 - \vec{dx}^2 = dt^2(1-\vec{\beta}^2) \iff \frac{d}{d\tau} = \gamma\frac{d}{dt}$. Now we can calculate $\alpha^\mu = \frac{d}{d\tau}u^\mu = \gamma\frac{d}{dt}[\gamma(1,\vec{\beta})] = \gamma\frac{d\gamma}{dt}(1,\vec{\beta}) + \gamma^2\frac{d}{dt}[(1,\vec{\beta})]$ so we need to understand what $\frac{d\gamma}{dt}$ is. This is simply a chain rule problem and we get $\frac{d\gamma}{dt} = -\frac{1}{2(1-\vec{\beta}^2)^{\frac{3}{2}}}(-2)(\vec{\beta}\bullet\vec{a}) = \gamma^3(\vec{\beta}\bullet\vec{a})$. So $\alpha^\mu = \gamma^2(0,\vec{a}) + (1,\vec{\beta})\gamma^4(\vec{\beta}\bullet\vec{a})$. Now that we have $\alpha^\mu$ let's verify that $\alpha^\mu u_\mu = 0$ we see that $\alpha^\mu u_\mu = \gamma^3(0,\vec{a})\bullet(1,\vec{\beta}) + \gamma^5(1,\vec{\beta})\bullet(1,\vec{\beta})(\vec{\beta}\bullet\vec{a}) = \gamma^3(-\vec{a}\bullet\vec{\beta} + \gamma^2(1-\vec{\beta}^2)(\vec{\beta}\bullet\vec{a}))$ (in Tr=-2) noting that $1-\vec{\beta}^2 = \frac{1}{\gamma^2}$ we immediately see $\alpha^\mu u_\mu = \gamma^3(-\vec{a}\bullet\vec{\beta} + \vec{a}\bullet\vec{\beta}) = 0$ witch verifies that the projection of acceleration onto velocity is 0.

## 2 #17

In the S' frame the velocity of a particle is $\beta = \tanh(\theta)$ witch would mean $\gamma = \cosh(\theta)$ and $\gamma\beta = \sinh(\theta)$ let us also define the direction of travel of the particle as $\hat{x}$. We are told that there is another frame S and in frame S frame S' is travelling in the same direction as the particle with velocity $\beta'$ by inspection in frame S' frame S is moving with velocity $-\beta'\hat{x}$. Using standard rapidity as above we can define a $\psi$ such that $\beta' = \tanh(\psi)$ and so $\gamma' = \cosh(\psi)$ and $\beta'\gamma' = \sinh(\psi)$. Let's now construct the 4-velocity of the particle in frame S' $u^\mu_{S'} = (\gamma, \gamma\beta, 0, 0) = (\cosh(\theta), \sinh(\theta), 0, 0)$. We also know how to boost a 4-velocity so we can calculate the 4-velocity in frame S $u^\mu_S = (\gamma' u^0_{S'} - (-\beta'\gamma')u^1_{S'}, \gamma' u^1_{S'} - (-\beta'\gamma')u^0_{S'}, 0, 0) = (\cosh(\psi)\cosh(\theta) + \sinh(\psi)\sinh(\theta), \cosh(\psi)\sinh(\theta) + \sinh(\psi)\cosh(\theta), 0, 0) = (\cosh(\psi+\theta), \sinh(\psi+\theta), 0, 0)$ it is immediately apparent that the rapidity are additive, since to find the velocity of our particle in frame S we need only calculate $\frac{u^1_S}{u^0_S} = \tanh(\psi+\theta)$.