Proof HW

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1 2.1.13

Theorem. For all $a \neq 0$,$(a^{-1})^{-1} = a$.
Proof.
Consider a non-zero number, lets call it a. Note that by the definition of multiplicative inverse $a^{-1} \cdot a = 1$. Note $a^{-1} \cdot a = 1 \Rightarrow (a^{-1})^{-1} \cdot a^{-1} \cdot a = (a^{-1})^{-1}$. Noting that $(a^{-1})^{-1} \cdot a^{-1} = 1$ by the definition of multiplicative inverse we see that $(a^{-1})^{-1} \cdot a^{-1} \cdot a = (a^{-1})^{-1} \Rightarrow a = (a^{-1})^{-1}$. Since we have concluded that $a = (a^{-1})^{-1}$ for a arbitrary non-zero a we conclude that the theorem holds for all non-zero numbers.
$\square$

2 2.1.15

Note that $1 \cdot 1 = 1$. Also note that $1^{-1} \cdot 1 = 1$ by the definition of multiplicative inverse. We conclude that one is a value for the multiplicative inverse of one, and as shown in class there is only one multiplicative inverse for a number thus $1^{-1} = 1$.

3 2.1.16

Theorem. For all $a \in \mathbb{R}$,$(-1) \cdot a = -a$.
Proof.
Select a arbitrary element of $\mathbb{R}$ lets call it a. First note that by the definition of additive inverse $1 + (-1) = 0$. Note $1 + (-1) = 0 \Rightarrow a(1 + (-1)) = a \cdot 0 \Rightarrow a + (-1) \cdot a = 0 \Rightarrow (-a) + a + (-1) \cdot a = (-a) + 0 \Rightarrow (-1) \cdot a = (-a)$. Since we have shown that the premise holds for an arbitrary element of $\mathbb{R}$ it must hold for all elements of $\mathbb{R}$.
$\square$