Proof HW

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1

Therm.
$x < 0 \Rightarrow x^{-1} < 0$.
Proof.
We will proceed with a proof by contradiction.
Suppose $x < 0$.
Now suppose $x^{-1} > 0$. Note that we proved on the previous homework that $(\forall a \in \mathbb{R})(-1)a = -a$ thus $(-1)x = (-x)$. Note $x \cdot x^{-1} = 1$ by the definition of multiplicative inverse. Since multiplication is well defined we may say $(-1) \cdot x \cdot x^{-1} = (-1) \cdot 1$. With the property stated above and noting that one is the multiplicative identity we see that $(-x) \cdot x^{-1} = (-1)$. By corollary 2.2.3 we can say $x < 0 \Rightarrow (-x) > 0$. Since $(-x) > 0$ and $x^{-1} > 0$ we can say by the trichotomy law (A18) that $(-x) \cdot x^{-1} > 0$ thus $(-1) > 0$. By corollary 2.2.3 we can say $1 < 0$. We showed in class that $1 > 0$. Contradiction, by the law of trichotomy it is impossible for $1 < 0$ and $1 > 0$ thus our initial assumption that $x^{-1} > 0$ must be false.
Next let us suppose $x^{-1} = 0$. Note $x \cdot x^{-1} = 1$ by the definition of multiplicative inverse. Also note that $x \cdot x^{-1} = x \cdot 0 = 0$ by therm 2.1.11. Thus $1 = 0$. Contradiction, (A15) states $1 \neq 0$ so our assumption $x^{-1} = 0$ must be false.
By the law of trichotomy $x^{-1} \not> 0$ and $x^{-1} \neq 0$ must mean $x^{-1} < 0$.
$\square$

2

Therm.
$x \neq 0 \Rightarrow x^2 > 0$ Proof.
Suppose $x \neq 0$.
Note, by the definition of additive inverse, $(-1) + 1 = 0 \Rightarrow (-(-1)) + (-1) + 1 = (-(-1)) \Rightarrow 1 = -(-1)$. also note $(-1) \in \mathbb{R}$ thus $(-1) \cdot (-1) = -(-1)$ so $1 = (-1)(-1)$ (we will need this later). Note $x^2 = x \cdot x$ by the definition of square. Let us consider the two remaining possibilities given by the law of trichotomy $x < 0$ or $x > 0$.
Suppose $x < 0$. Note that $x \cdot x = 1 \cdot x \cdot x = (-1) \cdot (-1) \cdot x \cdot x = (-1) \cdot x \cdot (-1) \cdot x = (-x) \cdot (-x)$. By corollary 2.2.3 note that $x < 0 \Rightarrow (-x) > 0$ thus by the law of trichotomy $(-x) \cdot (-x) > 0$ thus in this case $x^2 > 0$.

Suppose $x > 0$. By the law of trichotomy $x \cdot x > 0$ thus in this case $x^2 > 0$.
Since $x^2 > 0$ in both of the remaining cases we can say that $x^2 > 0$
$\square$

3

Therm.
If $m|j \wedge m|k$ then m divides any integer linear combination of j and k.
proof.
Suppose $m|j$ and $m|k$. Choose a integer linear combination of j and k, lets call it n. Since n is a integer linear combination of j and k we know there exist two integers, let us call them i and l, such that $n = i \cdot j + l \cdot k$. By definition we know that there exists a integer x such that $m \cdot x = j$, and some integer y such that $m \cdot y = k$. Note $n = i \cdot m \cdot x + l \cdot m \cdot y = m(i \cdot x + l \cdot y)$. Note that $i \cdot x + l \cdot y$ is a integer thus by definition $m|n$. Since n was a arbitrary integer linear combination of j and k we can generalize and say that m divides all integer linear combination of j and k.
$\square$

4

Therm.
If $m|n$ for all n where n is a integer linear combination of j and k then $m|j$ and $m|k$.
Proof.
Suppose $m|n$ for all n where n is a integer linear combination of j and k. Note that $0 \cdot j + 1 \cdot k$ is a integer linear combination of j and k thus $m|(0 \cdot j + 1 \cdot k) \Rightarrow m|k$. Note that $1 \cdot j + 0 \cdot k$ is a integer linear combination of j and k thus $m|(1 \cdot j + 0 \cdot k) \Rightarrow m|j$. We have thus shown that $m|j$ and $m|k$.
$\square$