PHYS 472L #13#18

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1.1 Newtonian

In a Newtonian system we see that momentum must be conserved so if in the instantaneous rest frame the rocket ejects a mass dm with a momentum udm it must adopt the momentum dp=-udm. in other words mdv=-udm or

$$v = \int_{m_0}^{m} -u/mdm = u * ln(m) \mid_{m_0}^{m} = u * ln(m_0/m)$$

1.2 relativistic

(take all interactions to be along the $\hat{x}$ axis)

In relativistic mechanics in the instantaneous rest frame of the rocket the rocket ejects the fuel of mass δm with 4-velocity $(\gamma', \gamma' u)$ where u is the velocity of the fuel and 4-momentum $\delta m(\gamma', \gamma' u)$ Consider that the rocket must have started at rest in this frame and then has some velocity. We see by conservation of relativistic momentum $(M, 0) = M'(\gamma, \gamma dv) + \delta m(\gamma', \gamma' u)$. Let's calculate dv, $M'\gamma dv = -\delta m \gamma' u \Rightarrow \delta m = -M' \frac{\gamma dv}{\gamma' u}$. Now $M = M'\gamma + \delta m \gamma'$ becomes $M = M'\gamma - M' \frac{\gamma dv}{\gamma' u}\gamma' \Rightarrow M = M'\gamma(1 - \frac{dv}{u}) \Rightarrow M\frac{\sqrt{1-dv^2}}{1-dv/u} = M'$ doing a Taylor expansion and eliminating higher order terms in dv we end up at $dv = \frac{u}{M}dM$.

In our lab frame this dv causes a change in the rockets velocity $v' + dv' = \frac{v'+dv}{1+v'dv} = (v' + dv) * (1 - v'dv + O(dv^2))$ witch linearisation yields: $dv' = (1 - v'^2)\frac{u}{M}dM$. Now we need only integrate over our change in velocity

$$\int_{M_0}^{M} \frac{u}{M} dM = \int_0^{v_f} \frac{1}{(1 - v'^2)} dv'$$

After calculus

$$v_f = \tanh(-u \ln(M_0/M))$$

1.3 relativistic mass conservation

Consider a frame that the rocket is at rest in that watches the rocket move in the $\hat{x}$ for a bit and sees all of the ejected fuel moving in the $-\hat{x}$ direction. We immediately note that there is more kinetic energy then there was initially so by conservation of the first terms of the four momentums -the terms that are equivalent to M+T we see that $\sum M = \sum M' + \sum T'$ since some T' are not 0 the sum of masses must have changed. Mass can not be conserved.

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In all frames $P \bullet P = (P^1)^2 + p^2$ in this lab the energy of the particle P^1 is given by the first quantity of the particles 4-momentum, witch can be obtained in a frame independent way by taking $u \bullet P = E = P^1_{lab}$ so we can now put this in for the energy we get $P \bullet P = (u \bullet P)^2 + p^2$ so $p = \sqrt{P \bullet P - (u \bullet P)^2}$.