(1) Start with an easy case:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

In this case, all we need to do is clear a_{21}. How many foating point operations are needed? Be eficient! When you have an answer, please discuss it with me!

Since we are storing the values neededd to reduce $A \to U$ in L there are no additional floating point operations needed to construct L. To construct L we need to eliminate a_{21}, to do this we subtract $k = a_{21}/a_{11}$. We need only compute the value going in the 22 spot since the value in the 21 spot will be 0. We will then subteact $k * a_{12}$ from a_{22} to find the value at that spot. There are 3 operations, one to compute k and two to subtract $k * a_{12}$.

(2) Next easiest case:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

We first need to eliminate the first row terms, calculate $k_2 = a_{21}/a_{11}$ and $k_3 = a_{31}/a_{11}$, two floating point ops. Now set a_{21} and a_{31} to zero. Subtract k_2 copies of row 1 from row two, except for the fist columb, four floating point operations, and the same with k_3 and the third row another four operations. Now we have a two by two to reduce, wich we know takes 3 operations, for a total of 13 operations.

(3) What about a 4×4 matrix?

In this case we will compute 3 k's in a similar fassion to the 3×3, 3 floating point operations, then we will set the first columb except the top entry to zero's. now subtract the first row times each multiplyer k from the row beneath, ignoring the first row, we take $2*3*3$ operations. Now we need to reduce a 3×3 wich will take 13 operations, bringing our total to 34.

(4) What about a $n \times n$ matrix?

We will need $n - 1$ operations to compute the k's, then $2(n - 1)^2$ operations to eliminate the first entries. After that we are left with a $n - 1 \times n - 1$. So assuming $f(n)$ is a function giving the number of operations to reduce a $n \times n$, we know that $f(n) = f(n - 1) + n - 1 + 2(n - 1)^2$ and $f(1) = 0$ since a 1×1 is reduced.

$$f(n) = \sum_{i=1}^{n-1} i + 2i^2 = (n - 1)(n)/2 + (n - 1)n(2n - 1)/3$$

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