

**Exercise : 7.8**

One norm = sum of absolute terms = 15. Two norm = root of the sum of squares =  $\sqrt{77}$ . Infinity norm = largest absolute term = 6.

**Exercise : 7.9**

one norm = largest of the sum of absolute terms in a column = 14. infinity norm = largest of the sum of absolute terms in a row = 15.

Using octave  $A^{-1} = \begin{bmatrix} -4 & 3 \\ 3.5 & -2.5 \end{bmatrix}$ . Which has a one norm of 7.5 and a infinity norm of 7. So its one norm condition number is 105 and its infinity condition number is 105.

**Exercise : 7.10**

Let  $v$  be a  $n$ -vector. WLOG let  $v_1$  be the maximum absolute entry in  $v$ , and thus  $|v_1| = \|v\|_\infty$ .

(a) Note that for all  $i$ ,  $v_i^2 \leq v_1^2$ . Thus  $v_1^2 \leq v_1^2 + \sum_{i=2}^n v_i^2 \leq n * v_1^2$  or  $\sqrt{v_1^2} \leq \sqrt{v_1^2 + \sum_{i=2}^n v_i^2} \leq \sqrt{n * v_1^2}$  or  $\|v\|_\infty \leq \|v\|_2 \leq \sqrt{n} \|v\|_\infty$ .

(b) Note that  $(|a| + |b|)^2 = a^2 + b^2 + 2|ab| \geq a^2 + b^2$  by trivial induction we can conclude  $\sum |v_i| \geq \sqrt{\sum v_i^2}$  or  $\|v\|_1 \geq \|v\|_2$ .

(c) Note that  $v_i \leq v_1$ . Thus  $\sum v_i \leq n v_1$  or  $\|v\|_1 \leq n \|v\|_\infty$ .

(a) There will never be equality for any non zero vector  $v$  living in  $n > 1$  space. We can see this by simply considering the proposed equality,  $\|v\|_\infty = \|v\|_2 = \sqrt{n} \|v\|_\infty$ , or in other words  $\|v\|_\infty = \sqrt{n} \|v\|_\infty$  so  $\|v\|_\infty = 0$ , which only occurs for  $v = \vec{0}$ .

(b) The vector  $v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  has the property that  $\|v\|_1 = \|v\|_2 = 1$ .

(c) The vector  $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  has the property that  $\|v\|_1 = 2 \|v\|_\infty = 2$ .

**Exercise : 4**

```

1 >> A=[9 3 2 0 7;7 6 9 6 4;2 7 7 8 2;0 9 7 2 2;7 3 6 4 3];
2 >> b=[35, 58, 53, 37, 39]';
3 >> x=PPsolve(A,b)
4 x =
5
6     -5.6251e-15
7     1.0000e+00

```

```

8      2.0000e+00
9      3.0000e+00
10     4.0000e+00
11
12 >> A*x
13 ans =
14
15      35
16      58
17      53
18      37
19      39
20
21 >> diary off

```

```

1 function x=PPsolve(A,x)
2 [P,L,U]=myplu(A);
3 x=usolve(U,lsolve(L,P*x));
4 end

```

```

1 function [P,L,U]=myplu(A)
2     n=length(A);
3     p=[1:n];
4     L=eye(n);
5     U=A;
6     for i=1:(n-1)
7         [x,xi]=max(abs(U(i:n,i)));
8         xi=xi+i-1;
9
10        temp=p(i);
11        p(i)=p(xi);
12        p(xi)=temp;
13
14        temp=U(i,i:n);
15        U(i,i:n)=U(xi,i:n);
16        U(xi,i:n)=temp;
17        if i~=1
18            temp=L(i,1:(i-1));
19            L(i,1:(i-1))=L(xi,1:(i-1));
20            L(xi,1:(i-1))=temp;
21        end
22        #we have now pivoted
23        for j=(i+1):n
24            k=U(j,i)/U(i,i);
25            L(j,i)=k;
26            U(j,i)=0;
27            U(j,i+1:n) -= k*U(i,i+1:n);
28        end
29    end
30    I=eye(n);
31    P=I(p,:);

```

```
32 endfunction
```

```
1 function x=lsolve(L,y)
2 U=L(length(L):-1:1,length(L):-1:1);
3 y=y(length(y):-1:1);
4
5 x=usolve(U,y);
6
7 x=x(length(x):-1:1);
8 endfunction
```

```
1 function x=usolve(U,y)
2     n=length(y);
3     x=zeros(n,1);
4     for(i=n:-1:1)
5         #calc x
6         x(i)=y(i)/U(i,i);
7         #eliminate entries
8         #note that we will not actually change the values of U only
9         #remember that all terms further out than i are zero's
10        y(1:(i-1))=y(1:(i-1))-x(i)*U(1:(i-1),i);
11    end
12 endfunction
```

### Exercise : 5

TBA???