Exercise : Write down the 4th order Taylor polynomial of $\sqrt{x}$ centered at $x = 1$. Let $P(x)$ denote this polynomial. If $1 \leq x \leq 2$, what can you say about the size of $|\sqrt{x} - P(x)|$? Hint: Use the remainder term!

Lets start by geting the derivitives of $f(x) = \sqrt{x}$.

$$f(x) = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

$$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$$

$$f'''(x) = \frac{3}{8}x^{-\frac{5}{2}}$$

$$f^{(4)}(x) = -\frac{15}{16}x^{-\frac{7}{2}}$$

$$f^{(5)}(x) = \frac{105}{32}x^{-\frac{9}{2}}$$

Now we can construct $P(x)$ as

$$P(x) = 1^{\frac{1}{2}} + \frac{1}{2}1^{-\frac{1}{2}}(x-1) + -\frac{1}{2}\frac{1}{4}1^{-\frac{3}{2}}(x-1)^2 + \frac{1}{6}\frac{3}{8}1^{-\frac{5}{2}}(x-1)^3 + -\frac{1}{24}\frac{15}{16}1^{-\frac{7}{2}}(x-1)^4$$

$$P(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \frac{5}{128}(x-1)^4$$

Note that the remander term would be

$$R(x) = \frac{1}{5!}\frac{105}{32}\xi^{-\frac{9}{2}}(x-1)^5 = \frac{7}{256}\xi^{-\frac{9}{2}}(x-1)^5$$

Where $f(x) = P(x) + R(x)$. We are now asked to compute the error, E, in the case that $1 \leq x \leq 2$. We see that $E = |f(x) - P(x)| = |R(x)| = |\frac{7}{256}\xi^{-\frac{9}{2}}(x-1)^5|$ and noting that $\xi, x \in [1,2]$ we get $E = \frac{7}{256}\xi^{-\frac{9}{2}}(x-1)^5$. The maximum value E could have would be $x = 2$ and $\xi = 1$ so $E_{max} = \frac{7}{256}$ and of course the smallest value is at $x = 1$ where by our construction $f(x) = p(x)$ so $0 \leq E \leq \frac{7}{256}$.

Exercise : Chapter 4: 2 (b)

```
function [guess x]=newton(guess,f,df,tol_y)
step=@(x) x-f(x)/df(x);
err_y=@(x) abs(f(x));
%abselute max number of steps, sometimes this method does not converge
N=60;
x=[guess];
disp("        x            f(x)        error");
%    "   4.9663e+00  -1.6564e-01   1.6564e-01"
```

```
disp([guess, f(guess), err_y(guess)]);
while(N>0 && err_y(guess)>tol_y)
    N=N-1;
    guess=step(guess);
    x=[x guess];
    disp([guess, f(guess), err_y(guess)]);
end
endfunction
```