Exercise : Use the worksheet to wite down the PLU factorization.

$$P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, L_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, U_1 = \begin{bmatrix} 1/3 & 0 & 0 \\ 1 & 1 & 1 \\ 1/2 & 0 & 1 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, U_2 = \begin{bmatrix} 1 & 1 & 1 \\ 1/3 & 0 & 0 \\ 1/2 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, L_3 = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ 1/2 & 0 & 1 \end{bmatrix}, U_3 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1/3 & -1/3 \\ 0 & -1/2 & 1/2 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, L_4 = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/3 & 0 & 1 \end{bmatrix}, U_4 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1/2 & 1/2 \\ 0 & -1/3 & -1/3 \end{bmatrix}$$

$$P_5 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, L_5 = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/3 & 2/3 & 1 \end{bmatrix}, U_5 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1/2 & 1/2 \\ 0 & 0 & -2/3 \end{bmatrix}$$

Exercise : Let

$$A = \begin{bmatrix} 10^{-16} & 1 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

(a) Solve $Ax = b$ exactly.

$$\begin{bmatrix} 10^{-16} & 1 \\ 1 & 1 \end{bmatrix} x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 10^{-16} & 1 \\ 0 & 1 - 10^{16} \end{bmatrix} x = \begin{bmatrix} 2 \\ 3 - 2 * 10^{16} \end{bmatrix}$$

$$\begin{bmatrix} 10^{-16} & 0 \\ 0 & 1 - 10^{16} \end{bmatrix} x = \begin{bmatrix} 2 - \frac{3-2*10^{16}}{1-10^{16}} \\ 3 - 2 * 10^{16} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} \frac{2}{10^{-16}} - \frac{3-2*10^{16}}{10^{-16}(1-10^{16})} \\ \frac{3-2*10^{16}}{1-10^{16}} \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{2}{10^{-16}} - \frac{3-2*10^{16}}{10^{-16}(1-10^{16})} \\ \frac{3-2*10^{16}}{1-10^{16}} \end{bmatrix} = \begin{bmatrix} \frac{2-2*10^{16}-(3-2*10^{16})}{10^{-16}(1-10^{16})} \\ \frac{3-2*10^{16}}{1-10^{16}} \end{bmatrix} = \begin{bmatrix} \frac{-1}{10^{-16}-1} \\ \frac{3-2*10^{16}}{1-10^{16}} \end{bmatrix} \approx \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

As a check:

$$Ax = \begin{bmatrix} 10^{-16} & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10^{-16} + 2 \\ 3 \end{bmatrix} \approx \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

(b) What is the 2-norm condition number for A? Is A well behaved in the 2-norm?
The condition number is 2.6180. This is well conditioned since errors is output will be on the same order as errors in imput.

(c) Here is my solution to solving a matrix without pivoting

```
function x=matrixSolve(A,b)
#make A upper triangular
for i=1:(length(A)-1)
    for j=i+1:length(A)
        k=A(j,i)/A(i,i);
        #A(j,i)=0; These values are never used again,
        #don't overide just remember they are zero
        A(j,[i+1:length(A)])-=k*A(i,[i+1:length(A)]);
        b(j)-=k*b(i);
    end
end
#our matrix A is "upper diagonal", not really but we have implied zeros.
#now make A diagonal
for i=length(A):-1:2
    for j=1:i-1
        b(j)-=b(i)*A(j,i)/A(i,i);
        #implied zeroing out of A
    end
end
#our matrix A is "diagonal", not really but we have implied zeros.
#now make A the identity matrix
for i=1:length(A)
    b(i)/=A(i,i);
    #A(i,i)/=A(i,i); implied
end
#our matrix A is "ientity", not really but implied.
x=b;

endfunction
```

And the solution I get:

```
>> x=matrixSolve(A,b)
x =

   4.4409
   2.0000

>> A\b
ans =

   1
   2

>> diary off
```

The problem with this method is that adding and subtracting large numbers causes errors:

```
>> 3+10^16
ans =    1.0000e+16
>> ans-10^16
ans =  4
>> diary off
```

Exercise : From the worksheet on implementing partial pivoting, show your code for mylu.m. Then show your answer to problem 10. Also, use your function usolve from the last homework and lsolve from the course web page to solve $Ax = b$ where A is the matrix from problem 10 of the worksheet and $b = \begin{bmatrix} -1 \\ 6 \\ -8 \end{bmatrix}$.

```
function [L,U]=mylu(A)
    n=length(A);
    L=eye(n);
    U=A;
    for i=1:(n-1)
        for j=i+1:n
            k=U(j,i)/U(i,i);
            L(j,i)=k;
            U(j,i)=0;
            U(j,i+1:n)-=k*U(i,i+1:n);
        end
    end
endfunction
```