Note that I am operating under the convention that N, n, m, i, j are natural numbers unless otherwise specified. I am also operating under the convention $v_a(b) = \{x \in \mathbb{R} : |x - b| < a\}$

Exercise 1: Suppose $f : A \to \mathbb{R}$ and c is a limit point of A. Suppose $f(x) \geq 0$ for all $x \in A$ and that $lim_{x \to c} f(x)$ exists. Show that the limit is non-negative. Provide two proofs, one ϵ - δ style, and the other using the sequential characterization of limits.

Proof. Suppose $f : A \to \mathbb{R}$ and c is a limit point of A. Suppose $f(x) \geq 0$ for all $x \in A$ and that $lim_{x \to c} f(x) = L$ exists.
Suppose $L < 0$. Define $\epsilon = -L/2 > 0$. There must exist a $\delta > 0$ such that for all $x \in A$ where $0 < |x - c| < \delta$, $|f(x) - L| < \epsilon$. Note that c is a limit point of A, thus $V_\delta(c) \cap A - \{c\} \neq \emptyset$. Take one of the elements of this set $a \in V_\delta(c) \cap A - \{c\}$. Note that $a \neq c$. Note that $a \in A$. Note $|a - c| < \delta$. Thus $|f(a) - L| < \epsilon$ and so $L - \epsilon < f(a) < L + \epsilon = L/2 < 0$. A contradiction, we know that $f(a) \geq 0$ and now we have $f(a) < 0$. $\square$