**Exercise 1.4.1:** Recall that $\mathbb{I}$ stands for the set of irrational numbers.

(a) Show that if $a, b \in \mathbb{Q}$ then $ab$ and $a + b \in \mathbb{Q}$ as well.

(b) Show that if $a \in \mathbb{Q}$ and $t \in \mathbb{I}$ then $a + t \in \mathbb{I}$ and if $a \neq 0$ then $at \in \mathbb{I}$ as well.

(c) Part (a) says that the rational numbers are closed under multiplication and addition. What can be said about $st$ and $s + t$ when $s, t \in \mathbb{I}$?

(a) *Proof.* Select two arbitrary elements from the rational numbers, since they are rational we can represent them as $i/j$ and $m/n$ where $j \neq 0$ and $n \neq 0$. Note that $i/j * m/n = \frac{im}{jn}$ from the definition of multiplication of rational numbers. Since the multiple of any two non-zero numbers is non-zero and since the multiple of any two integers is a integer $\frac{im}{jn} \in \mathbb{Q}$. □

**Exercise 1.4.2:** Let $A \subseteq \mathbb{R}$ be nonempty and bounded above. Let $s \in \mathbb{R}$ have the property that for all $n \in \mathbb{N}$, $s + (1/n)$ is an upper bound for $A$ but $s - (1/n)$ is not an upper bound for $A$. Show that $s = \sup A$.

*Proof.* □

**Exercise 1.4.3:** Show that $\cap_{n=1}^{\infty}(0, 1/n) = \emptyset$.

*Proof.* □

**Exercise 1.4.4:** **(W) (Hand this one in to David.)**
Let $a < b$ be real numbers and let $T = [a, b] \cap \mathbb{Q}$. Show that $\sup T = a$.

*Proof.* □

**Exercise 1.4.5:** Use Exercise 1.4.1 to provide a proof of Corollary 1.4.4 by considering real numbers $a - \sqrt{2}$ and $b - \sqrt{2}$.

*Proof.* □

**Exercise Supplemental 1:** Show that the sets $[0, 1)$ and $(0, 1)$ have the same cardinality.