Note that I am operating under the convention that N, n, m, i, j are natural numbers unless otherwise specified. I am also operating under the convention $v_a(b) = \{x \in \mathbb{R} : b - a < x < b + a\}$

Exercise : IVT

Proof. Suppose $f : [a, b] \to \mathbb{R}$ is a continuous function with $f(a) < f(b)$. Choose $v \in \mathbb{R}$ such that $f(a) < v < f(b)$. Define for $Y \subseteq \mathbb{R}$, $f^{-1}(Y) = \{a \in A : f(a) \in Y\}$. Define $A_v = f^{-1}((-\infty, v))$. Note that $f(a) < v$ and so $a \in A_v$. Note that for all $x \in A_v$, $x \in A$ and thus $x <= b$ and so b is a upper bound on A_v. Since A_v is bounded and non-empty it has a suppremum. Define $x = \sup(A_v)$. We have previously proven there is a sequence A_v that converges to x, This can be easily proven since $[\sup(S) - 1/n, \sup(S)] \cap S \neq \emptyset$ for all $n \in \mathbb{N}$, call this sequence $\{a_n\}$. Note that $f(a_n) \in (-\infty, v)$ since $a_n \in A_v$, thus $f(a_n) < v$. $\square$