Note that I am operating under the convention that N, n, m, i, j are natural numbers unless otherwise specified. Also note that I am operating under a convention that $\sum = \sum_{n=1}^{\infty}$ and the convention that $\sum_i^j = \sum_{n=i}^j$.

Exercise : Let A and B be nonempty sets that are bounded above. Suppose $\sup A < \sup B$. Prove that there is an element of B that is an upper bound for A.

Suppose A and B are nonempty sets that are bounded above. Furthur suppose $\sup A < \sup B$. Define $a = \sup A$ and $b = \sup B$. Note that a is less than the suppremum of B thus a is not a upper bound on B. Since a is not a upper bound on B there must exist at least one element of B grater than a, take one of these elements lets call it k, $k \in B$, $a < k$.