Exercise 2.24: (a) Recall that the multiplicity for a two state paramagnet is $\Omega(N, N_\uparrow) = \frac{N!}{N_\uparrow!(N-N_\uparrow)!}$. Applying Sterling's approximation of $A! = \sqrt{2\pi}\sqrt{A}(A/e)^A$ we get

$$\Omega(N, N_\uparrow) = \frac{N!}{N_\uparrow!(N-N_\uparrow)!} \approx \sqrt{\frac{N}{2\pi N_\uparrow(N-N_\uparrow)}}\frac{(N/e)^N}{(N_\uparrow/e)^{N_\uparrow}((N-N_\uparrow)/e)^{(N-N_\uparrow)}}$$

$$= \sqrt{\frac{N}{2\pi N_\uparrow(N-N_\uparrow)}}\left(\frac{N}{N-N_\uparrow}\right)^N\left(\frac{N-N_\uparrow}{N_\uparrow}\right)^{N\uparrow}$$

leting $N_\uparrow = N/2$

$$= \sqrt{\frac{N}{2\pi N/2N/2}}\left(\frac{N}{N/2}\right)^N\left(\frac{N/2}{N/2}\right)^{N/2}$$

$$= \sqrt{\frac{2}{\pi N}}2^N$$

(b)

$$\Omega(N, N_\uparrow) \approx \sqrt{\frac{N}{2\pi N_\uparrow(N-N_\uparrow)}}\left(\frac{N}{N-N_\uparrow}\right)^N\left(\frac{N-N_\uparrow}{N_\uparrow}\right)^{N\uparrow}$$

leting $N_\uparrow = N/2 + x$

$$\Omega(N, N_\uparrow) \approx \sqrt{\frac{N}{2\pi(N/2+x)(N/2-x)}}\left(\frac{N}{N/2-x}\right)^N\left(\frac{N/2-x}{N/2+x}\right)^{N/2+x}$$

$$= \sqrt{\frac{N}{2\pi(N/2+x)(N/2-x)}}\left(\frac{N}{N/2-x}\right)^N\left(\frac{\sqrt{N/2-x}}{\sqrt{N/2+x}}\right)^N\left(\frac{N/2-x}{N/2+x}\right)^x$$

$$= \sqrt{\frac{N}{2\pi(N/2+x)(N/2-x)}}\left(\frac{N}{\sqrt{N/2-x}\sqrt{N/2+x}}\right)^N\left(\frac{N/2-x}{N/2+x}\right)^x$$

If $x = 0$ this approximation becomes

$$\sqrt{\frac{N}{2\pi(N/2)(N/2)}}\left(\frac{N}{\sqrt{N/2}\sqrt{N/2}}\right)^N = \sqrt{\frac{2}{\pi N}}2^N$$

(c)

$$\sqrt{\frac{N}{2\pi(N/2+x)(N/2-x)}}\left(\frac{N}{\sqrt{N/2-x}\sqrt{N/2+x}}\right)^N\left(\frac{N/2-x}{N/2+x}\right)^x$$

(d) I would not be surprised to find off by 1000 as the multiplicity of that result is similar to the peak however I would be surprised to find off by 10000 as that multiplicity is several orders of magnitude less than the peak

Exercise 2.29: The highest entropy corresponds to the highest Ω_{total} and we can see from table 2.5 that this occurs when $q_A = 60$ and corresponds to a entropy of $k264$. The lowest entropy would likewise occur at $q_A = 0$ and is $k187$. Over long time scales the time spent far away from the peak are completely negligible thus the average entropy will be $\approx k264$.

Exercise 2.41:

1. Melting ice - The room will cool down and decrease in entropy but the ice will melt and, by coming out of a crystal, have a huge increase in entropy.
2. Heat transfer across a rod - If one end of a rod is heated up the heat will disperse across the rod. The end that cools looses some entropy, but the end that eventually receives the heat will drastically increase in entropy
3. Breaking - Breaking a car will transfer low entropy energy, in the form of velocity to heat on the break pads, a very high entropy energy.