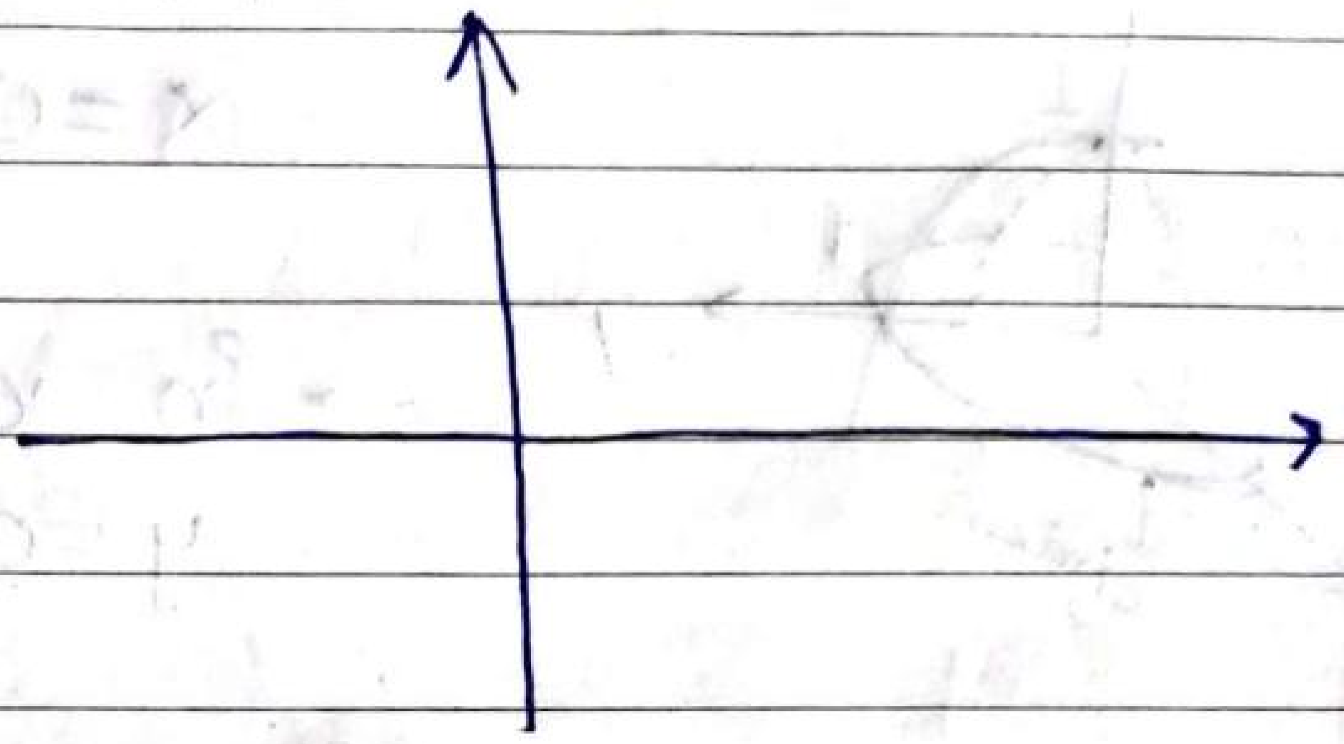
Lecture :- 7

(Review) :

Lecture :- 8 (Level curves, partial derivative, tangent plane approximation)

functions

function of 1 variables



$f(x) = \sin(x)$

function of 2 variables

given (x,y) ⇝ get a number $f(x,y)$.

example: • $f(x,y) = x^2+y^2$

• $f(x,y) = \sqrt{y}$ ... only defined if $y \geq 0$

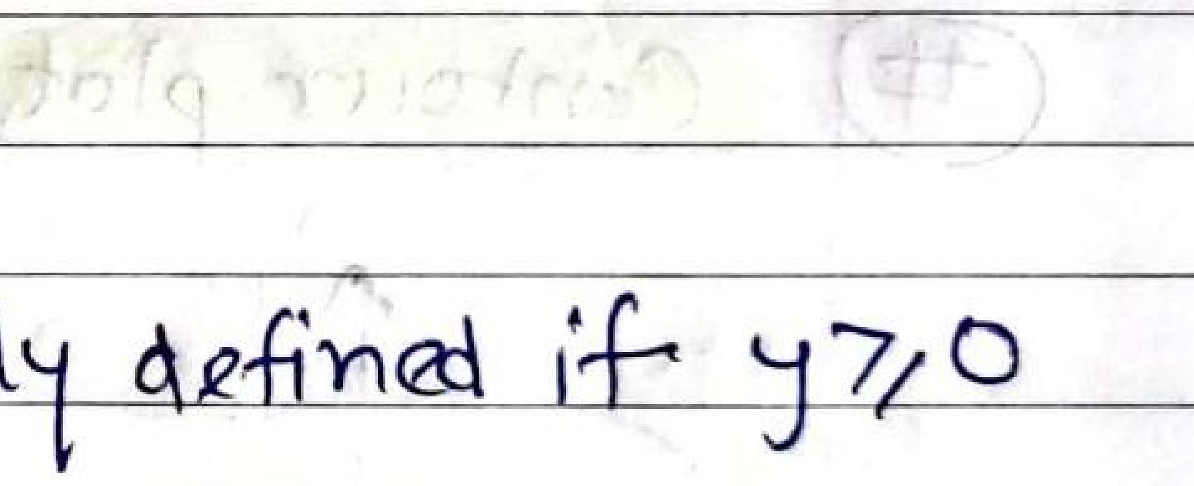
• $f(x,y) = 1/(x+y)$ only if $x+y \neq 0$

example: $f(x,y)$ = temperature at point (x,y). or 3 or more parameters!

For simplicity, focus mostly on 2 (or 3) variable.

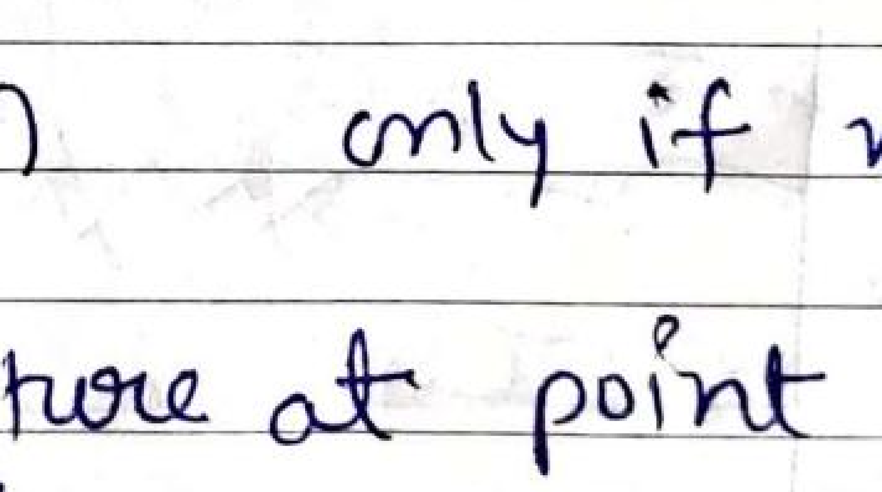
How to visulize f^n of 2 variable ?

→ graph $z = f(x,y)$



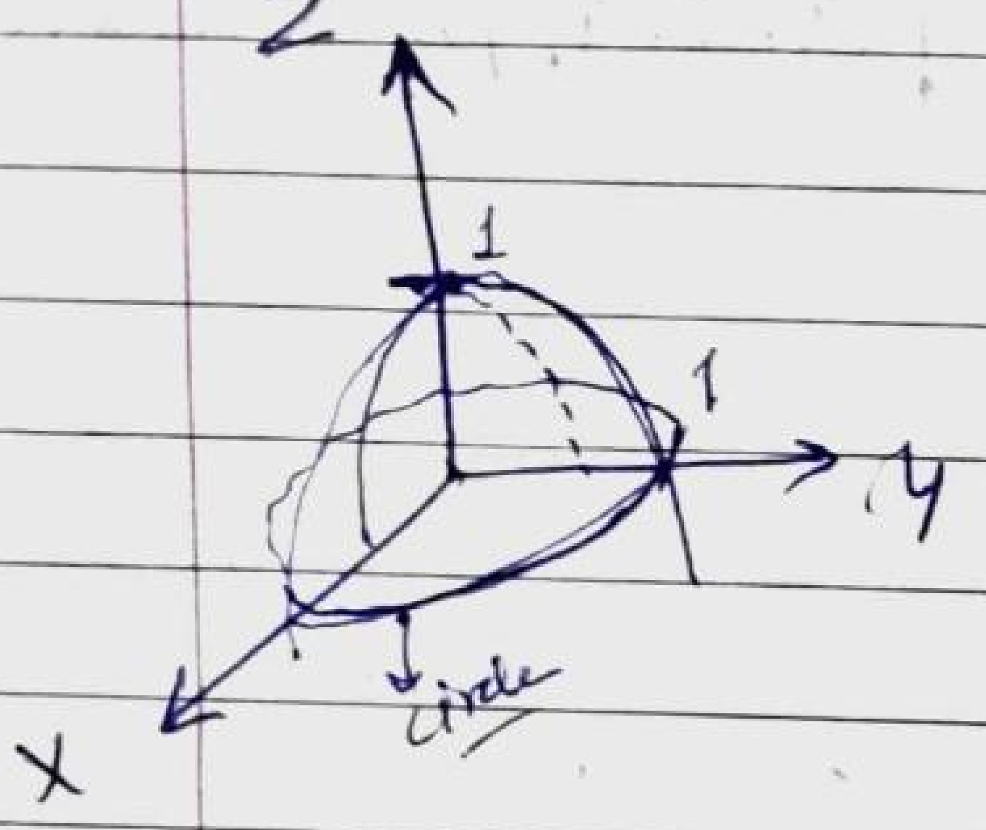
example: $f(x,y) = -y$

graph: $z = -y$



example

$$f(x,y) = 1 - x^2 - y^2$$



→ in yz-plane

$x=0;\quad z = 1-y^2$ → parabola

→ in xz plane;

$y=0;\quad z = 1-x^2$

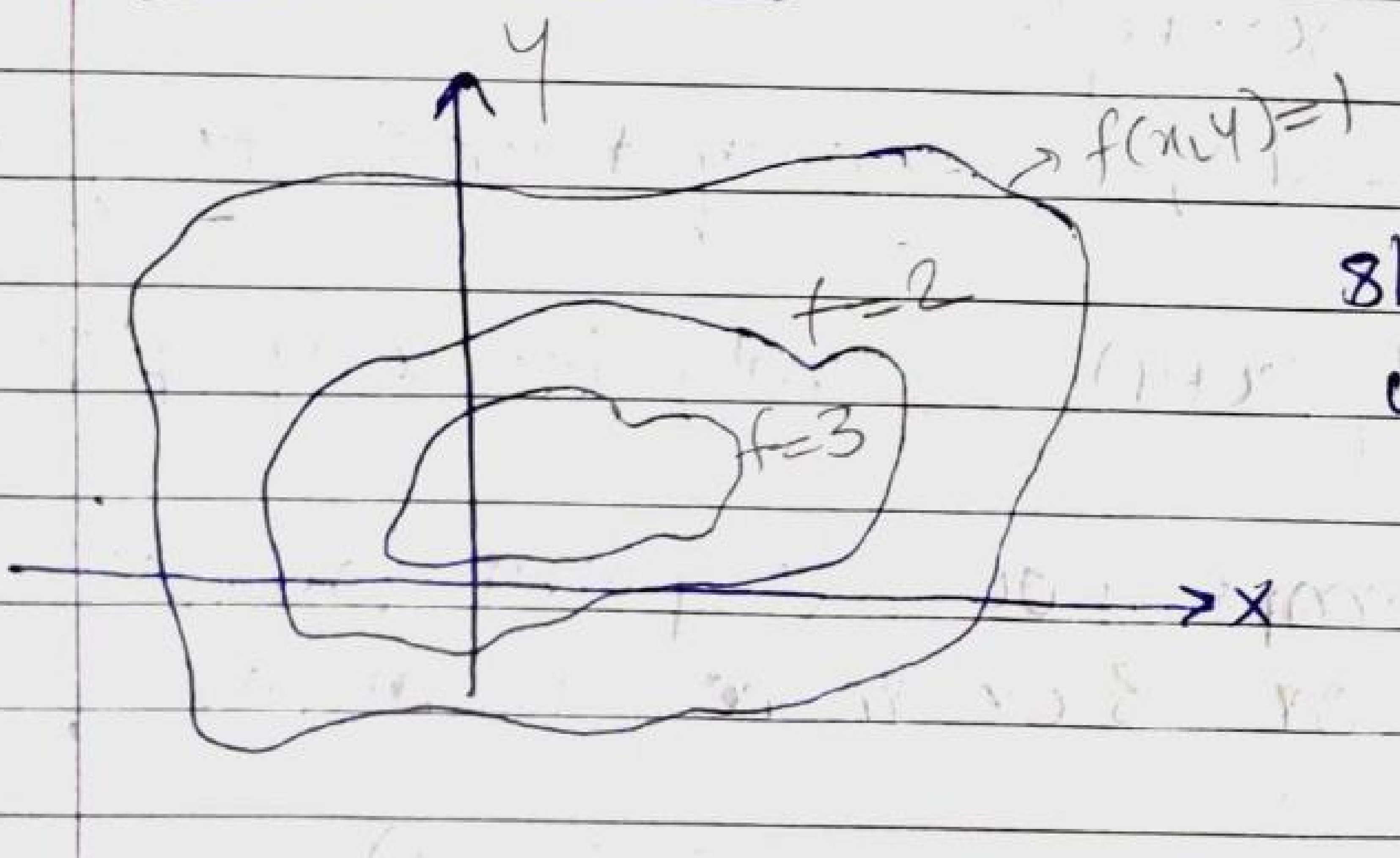
→ hit xy-plane

$z=0;\quad 1-x^2-y^2=0$

$x^2+y^2=1$

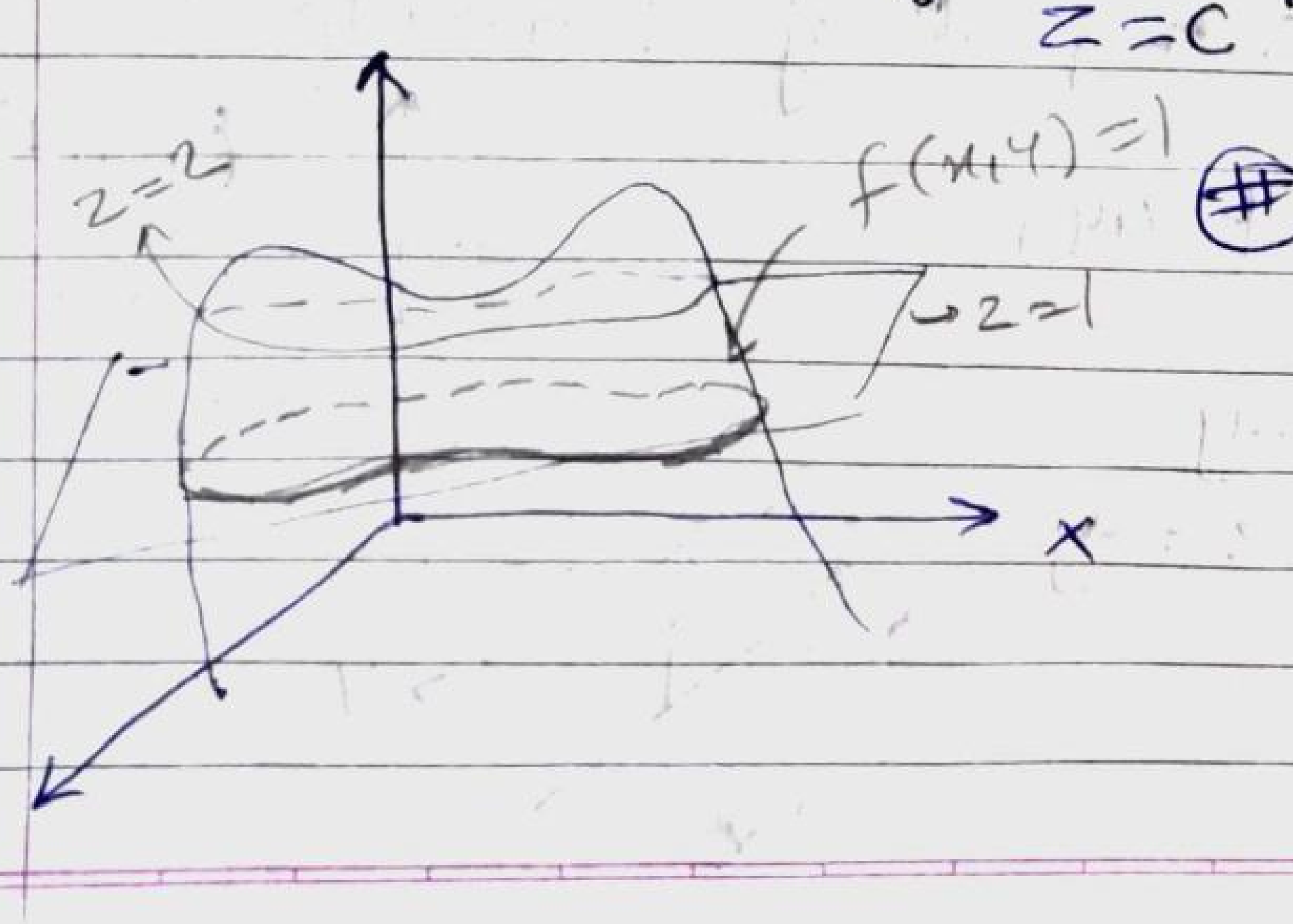
unit circle.

Contour plots



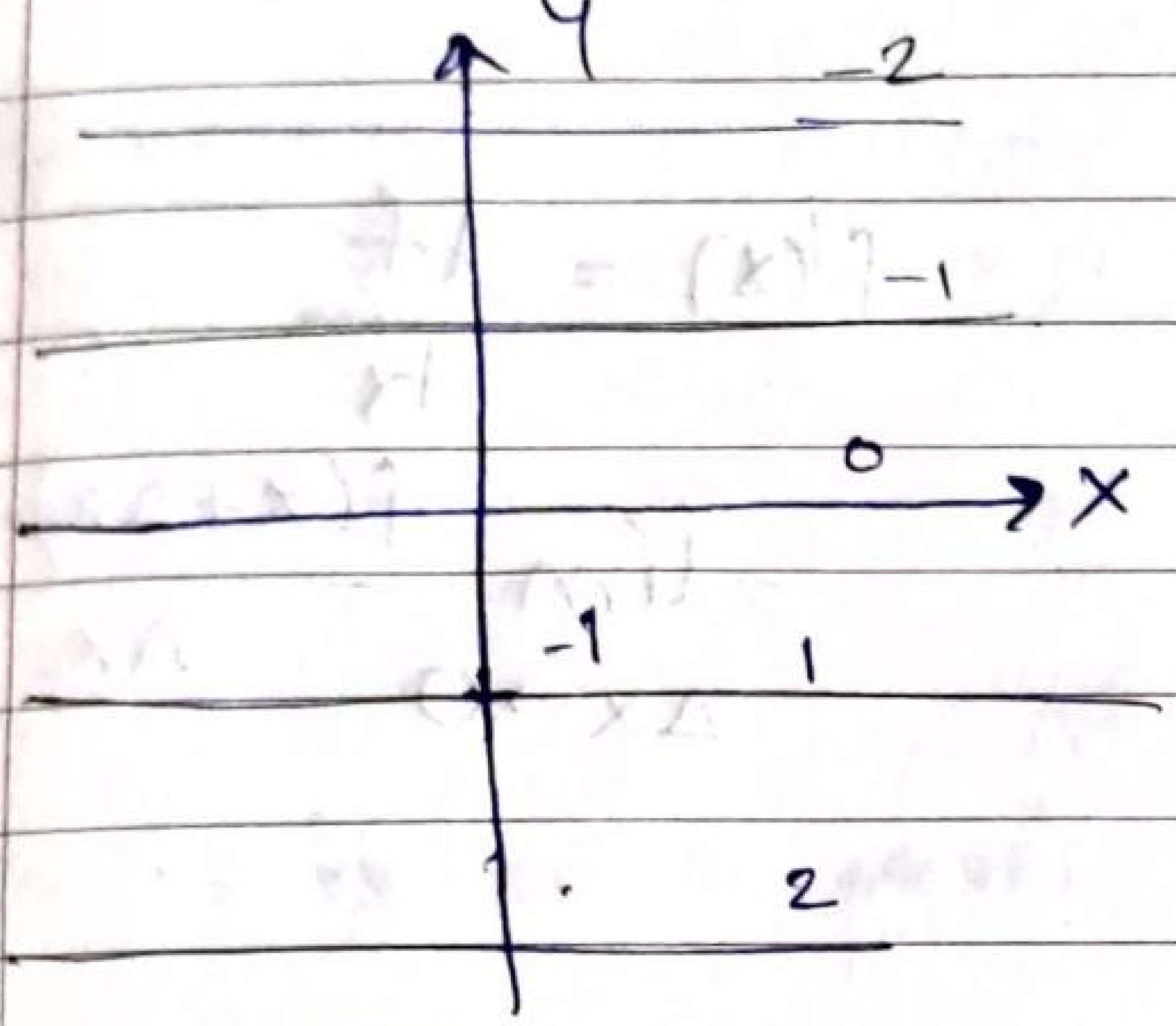
shows all the points where $f(x,y) =$ some fixed constants. (choose at regular interval)

⟺ we slice the graph by horizontal plane: $z=c$

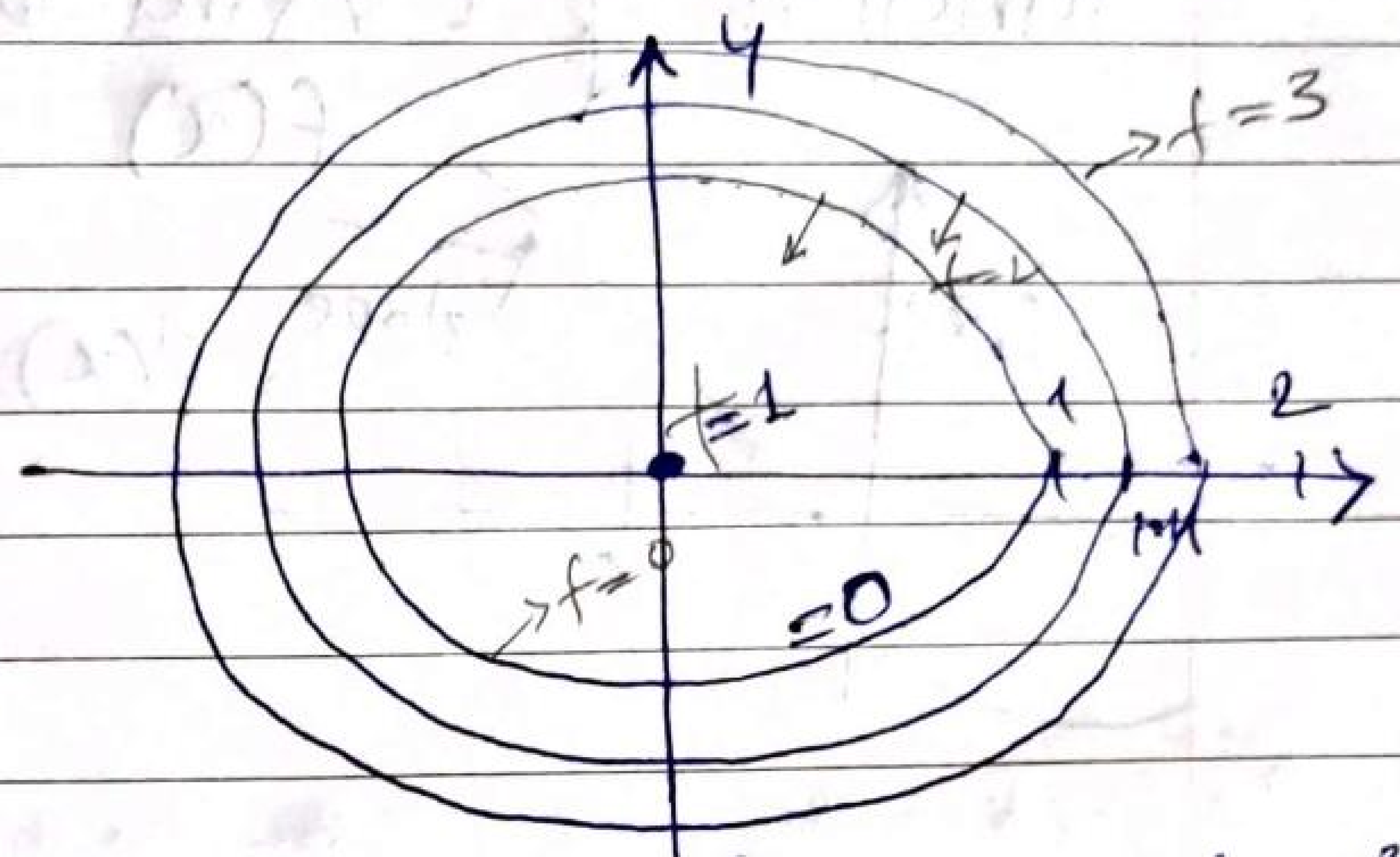


Level curve of f.

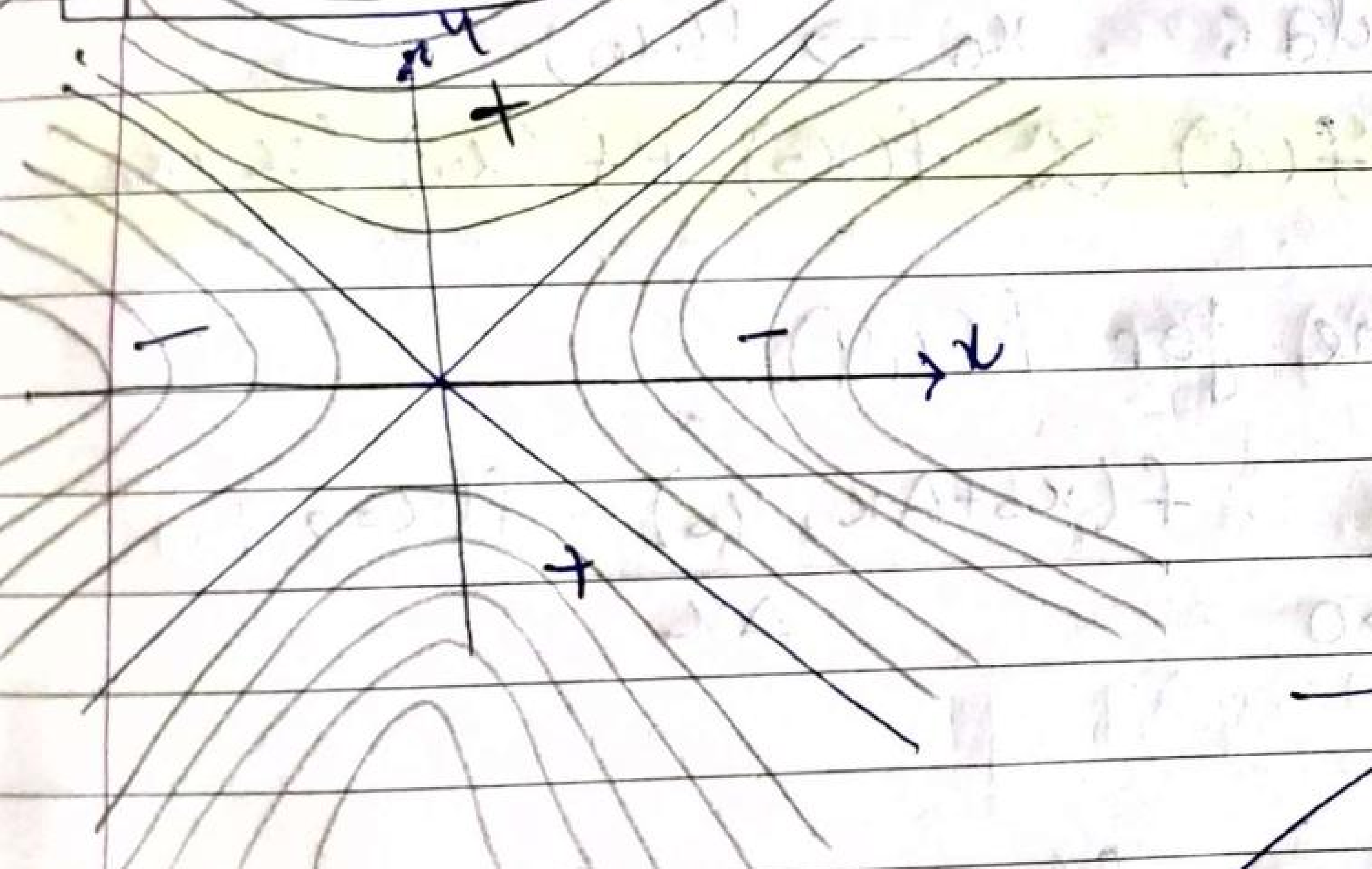
example: $f(x,y) = -y$



$f(x,y) = 1-(x^2+y^2)$

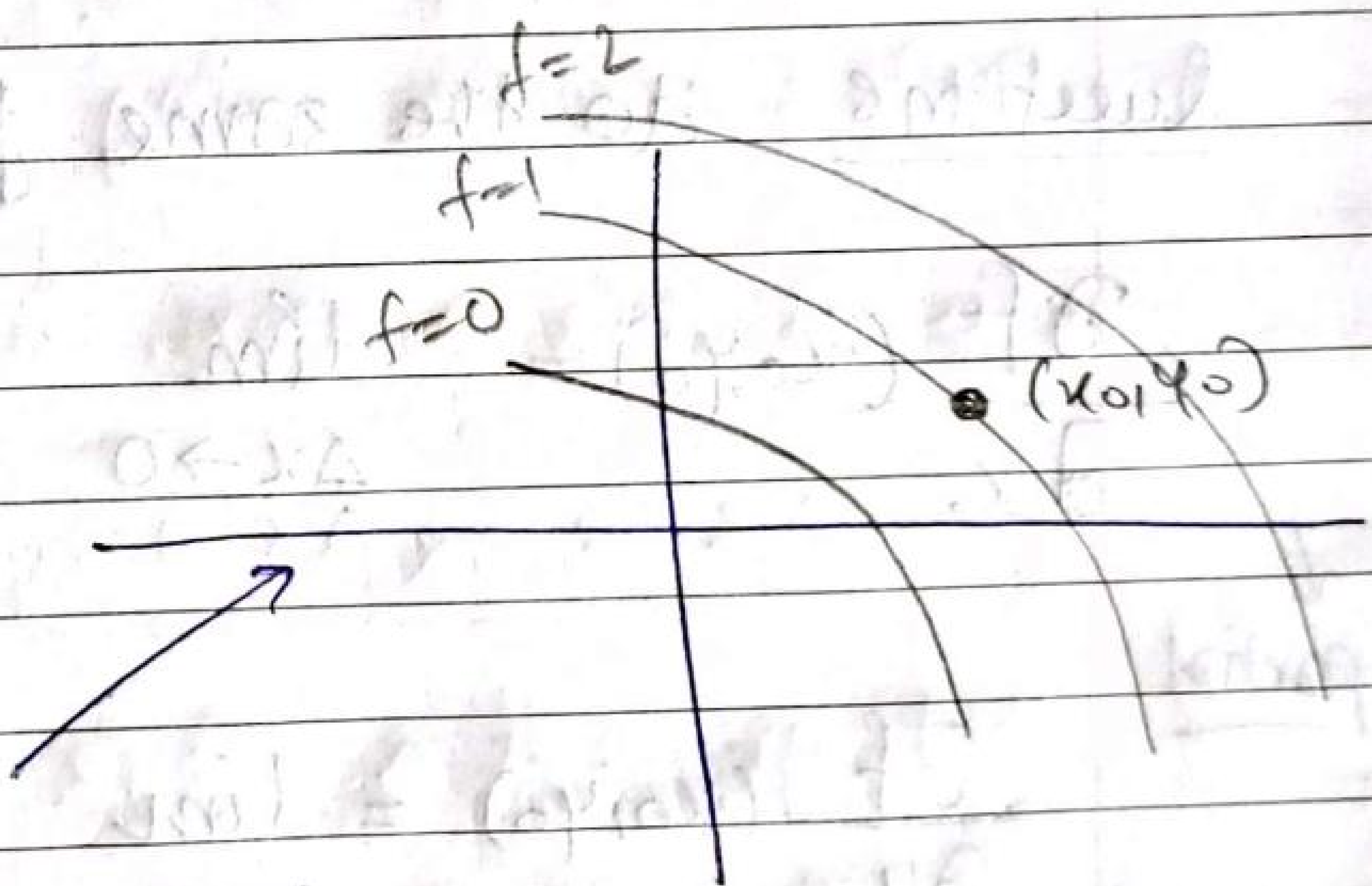


$\boxed{y^2 - x^2 = z}$



$f=0 \Leftrightarrow x^2+y^2=1$

$f=1 \Leftrightarrow x^2+y^2=0$



example:

Contour plot tells me that:

→ If x increases and keep 'y' constant then f(x,y) increases
↓ ↓

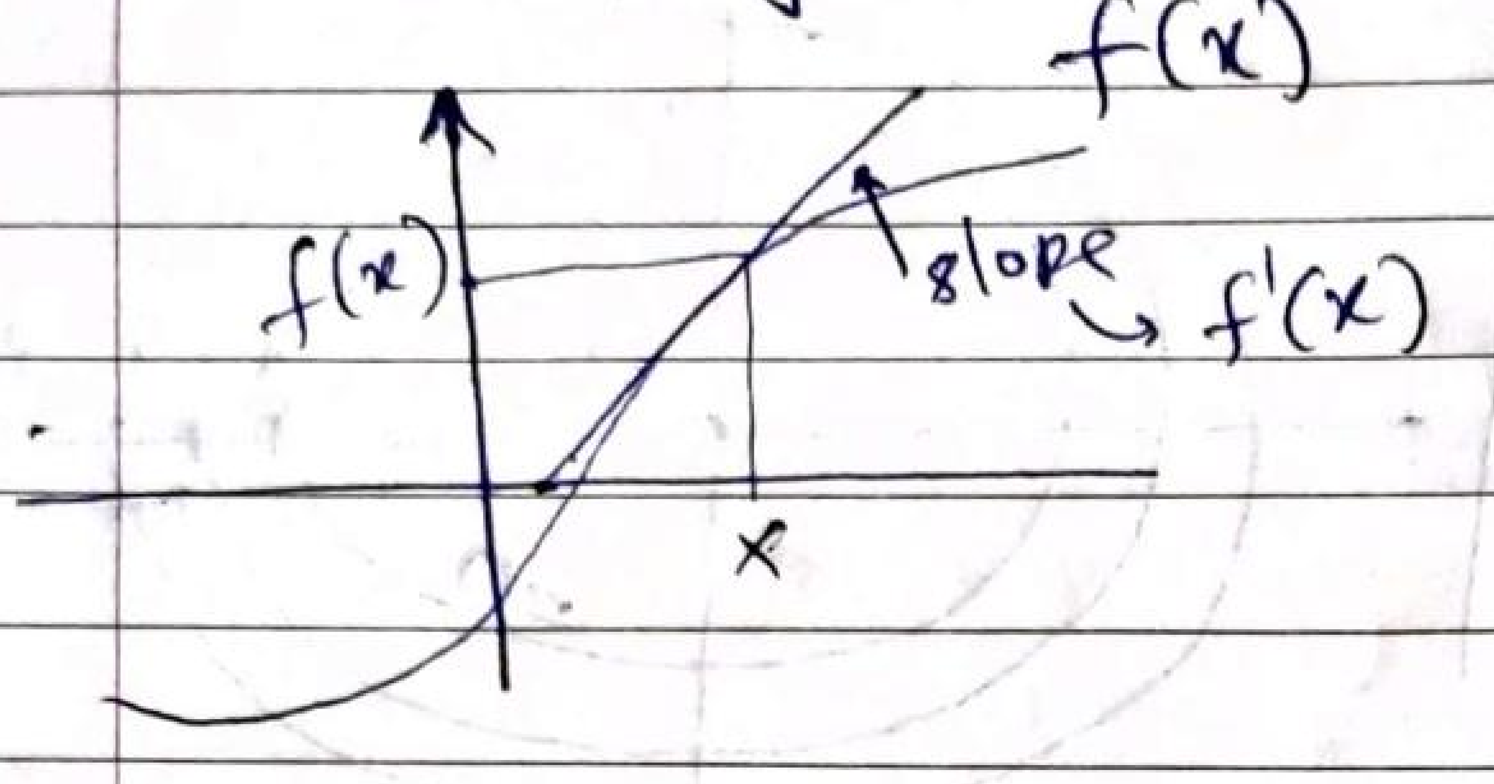
→ y increases then f(x,y) decreases
↓ ↓

How fast f changes if I change x & y?

To ~~rate~~ find rate of change, that's exactly where we use derivatives.

→ Partial derivatives:

function of 1 variable : $f'(x) = \frac{df}{dx}$



$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Approximation formula: $x_0 \to f(x_0)$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

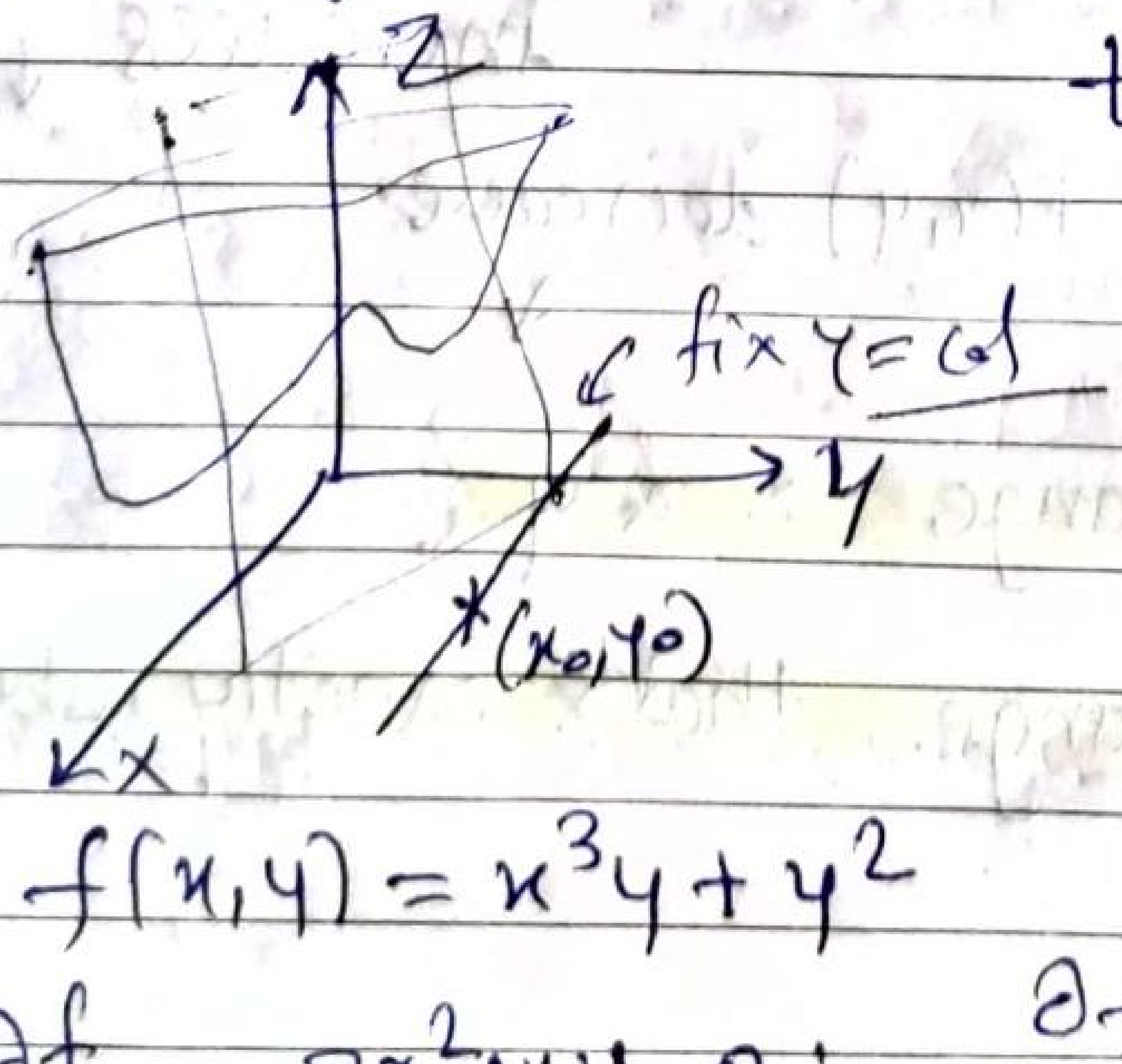
Question: do the same for $f(x,y)$?

$$\frac{\partial f}{\partial x}(x_0,y_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

partial

$$\frac{\partial f}{\partial y}(x_0,y_0) = \lim_{\Delta y \to 0} \frac{f(x_0, y_0+\Delta y) - f(x_0, y_0)}{\Delta y}$$

Geometrically:



How to compute?

to find $\frac{\partial f}{\partial x} = f_x$

treat y as constant
x as variable
viceversa.

example: $f(x,y) = x^3y + y^2$

: $\frac{\partial f}{\partial x} = 3x^2y + 0$; $\frac{\partial f}{\partial y} = x^3 + 2y$.