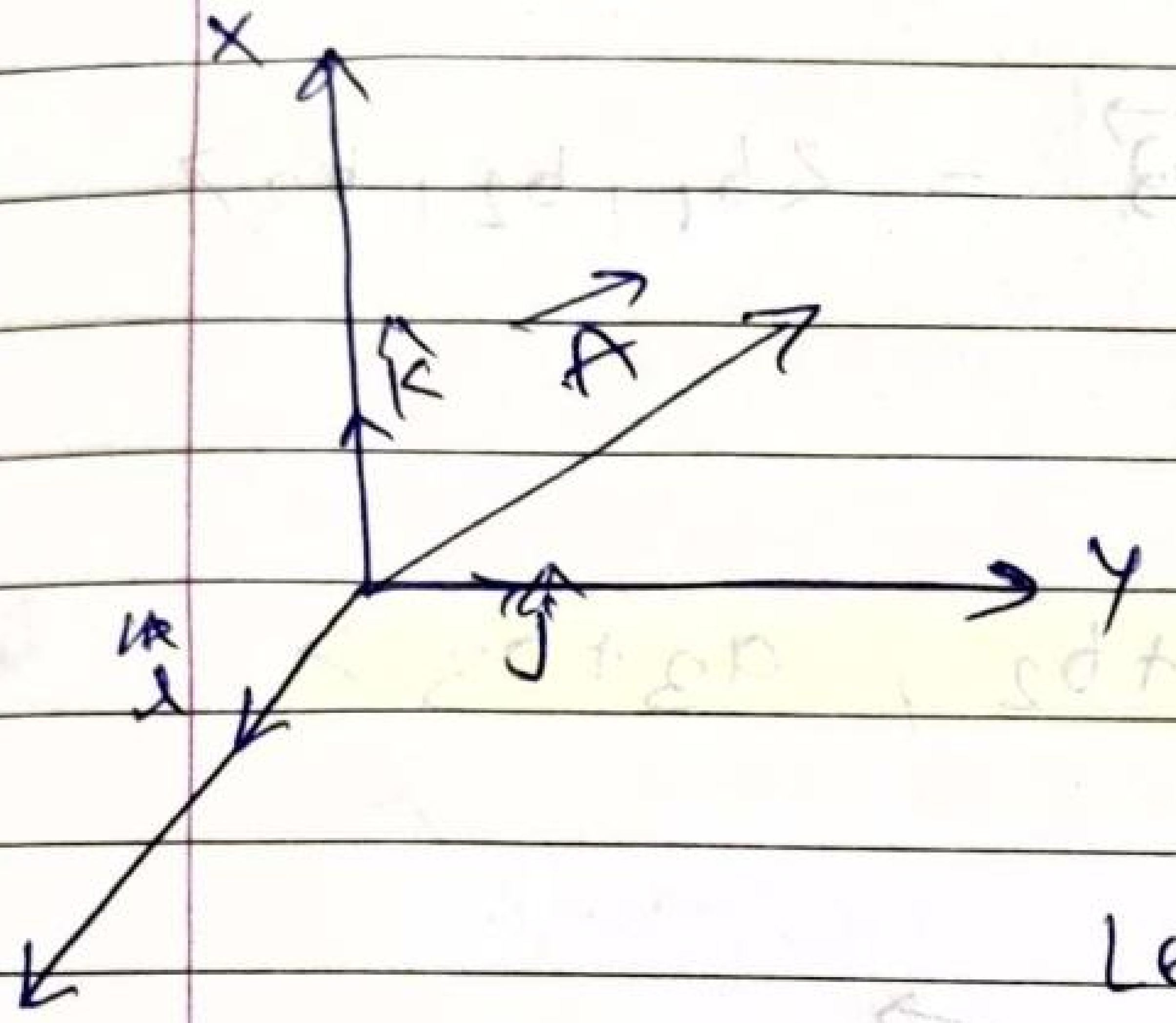
MVC

Lectuor: 1

Dot Product

Vectors



- direction
- magnitude / length

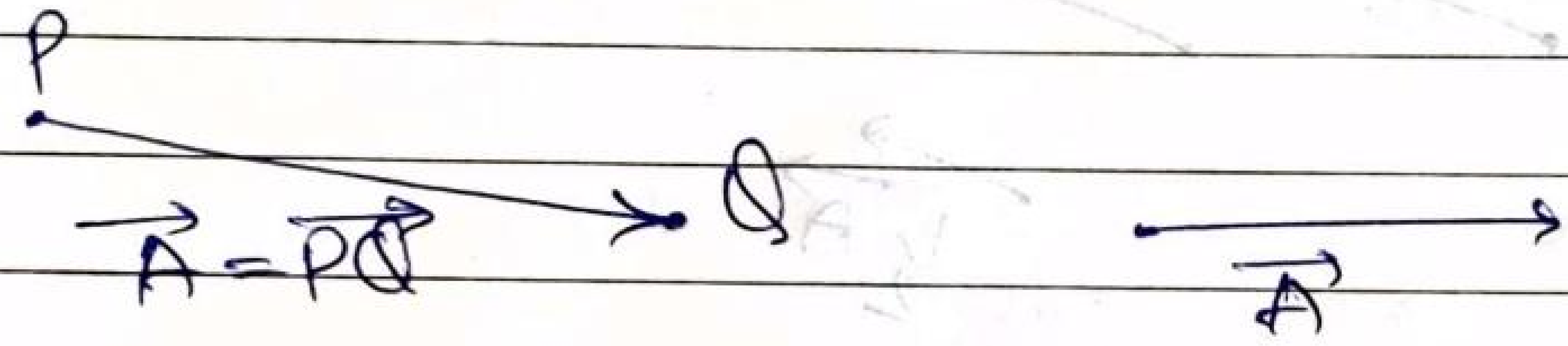
$\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$= \langle a_1, a_2, a_3 \rangle$

Length $|\vec{A}|$ (a scalar),

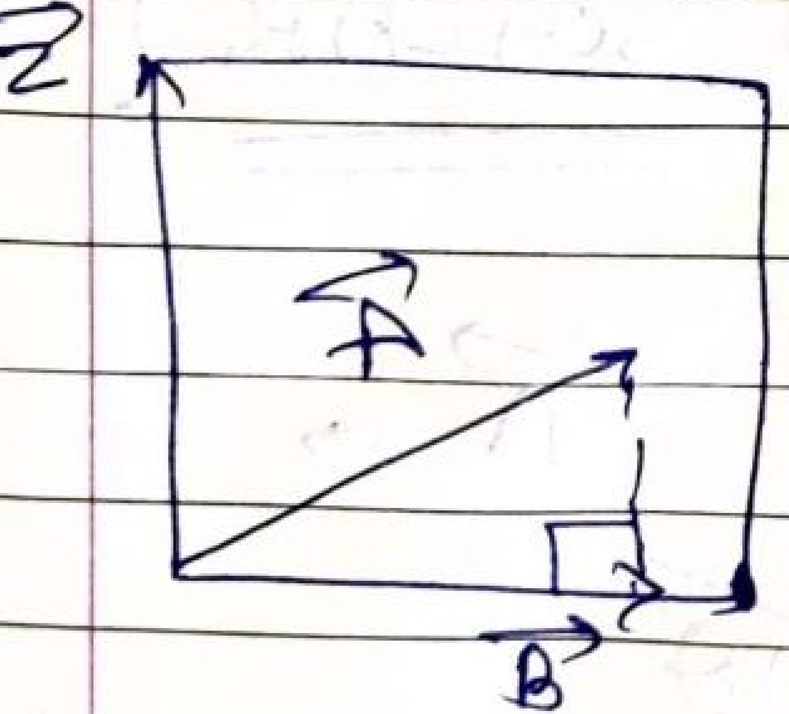
direction :- $dir(\vec{A})$

#

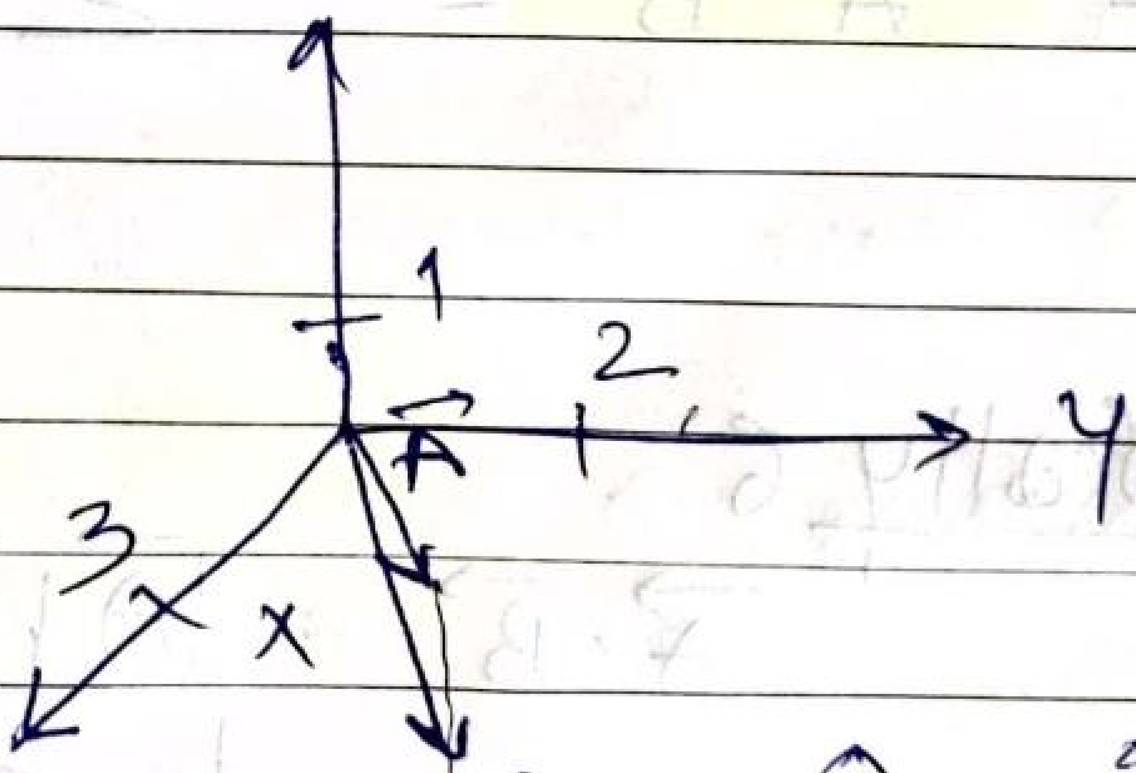


$\vec{A} = 3\hat{i} + 2\hat{j} + \hat{k} = \langle 3, 2, 1 \rangle$

Length of $\vec{A}$



$\xleftarrow{\text{in plane}}$



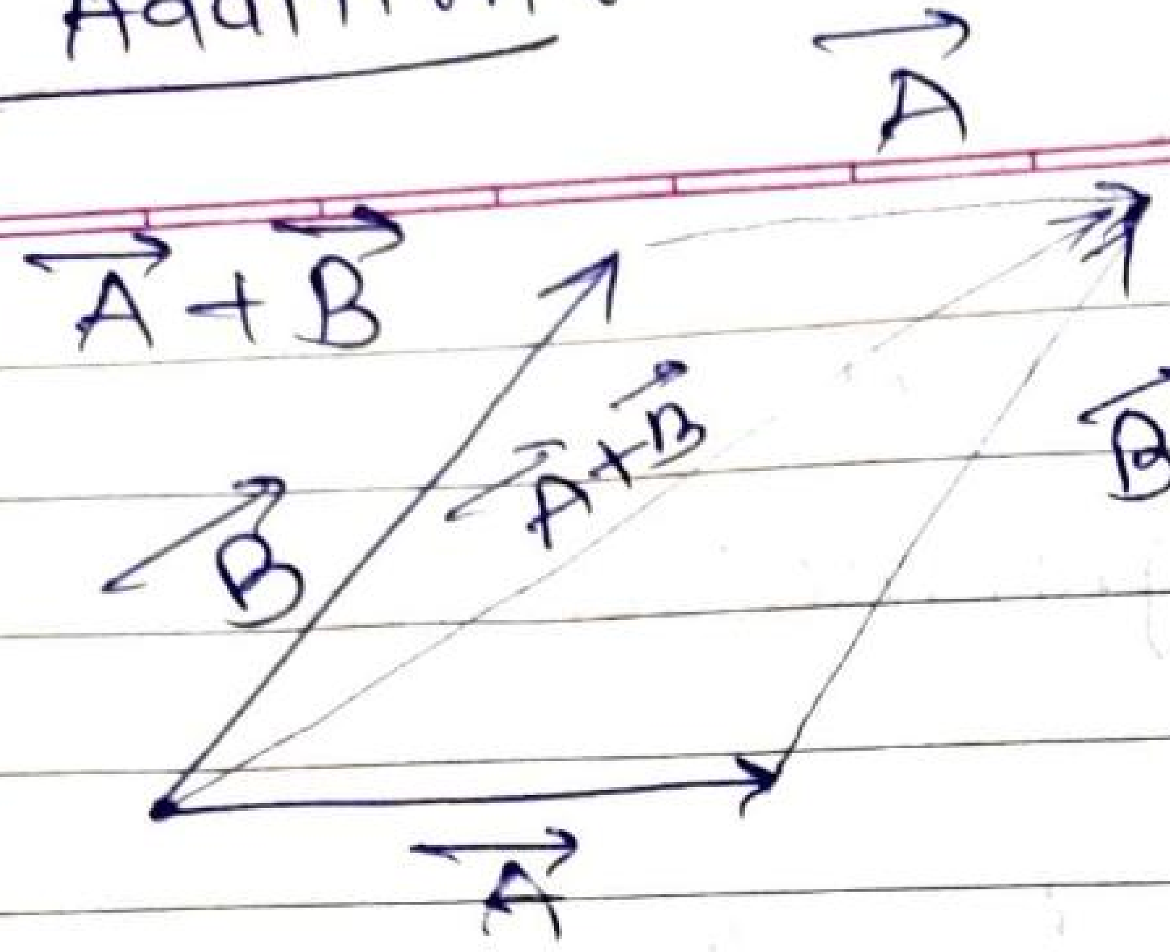
$\vec{B} = 3\hat{i} + 2\hat{j}$

$|\vec{B}| = \sqrt{3^2 + 2^2} = \sqrt{13}$

$|\vec{A}| = \sqrt{|\vec{B}|^2 + 1} = \sqrt{13+1} = \sqrt{14}$

In general, $\vec{A} = \langle a_1 a_2 a_3 \rangle$; $\boxed{|\vec{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2}}$

Addition :

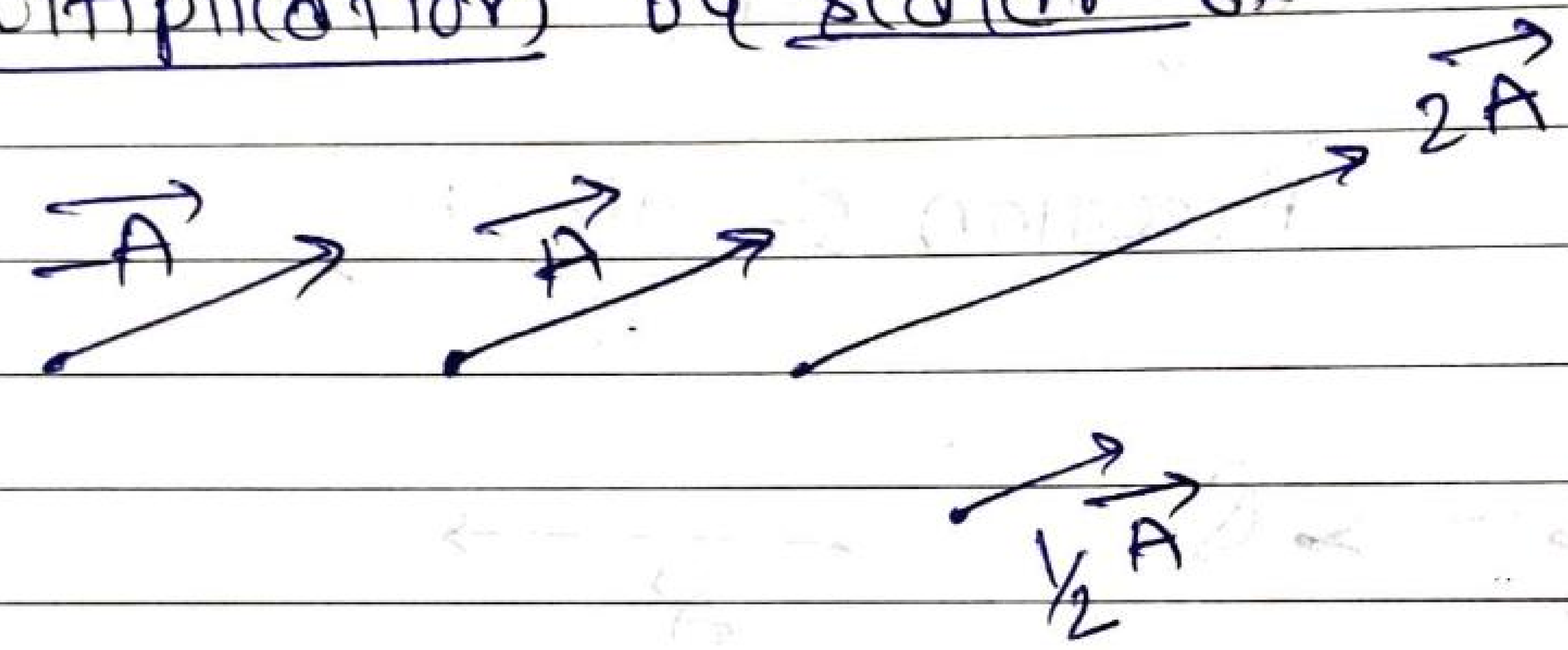


$\vec{A} = \langle a_1, a_2, a_3 \rangle$

$\vec{B} = \langle b_1, b_2, b_3 \rangle$

$$\vec{A}+\vec{B} = \langle a_1+b_1,\ a_2+b_2,\ a_3+b_3 \rangle$$

Multiplication by scalar :



DOT PRODUCT :

Def :- $\vec{A}\cdot\vec{B} = \sum a_i b_i = a_1b_1 + a_2b_2 + a_3b_3$

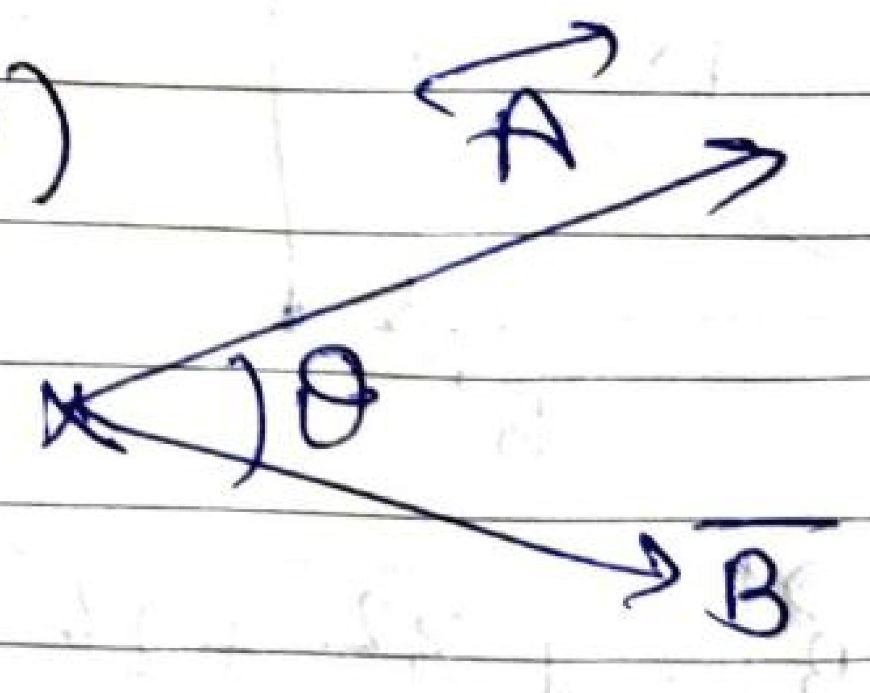
is a SCALAR !

(I)

Geometrically :

$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}| \cos(\theta)$

(II)

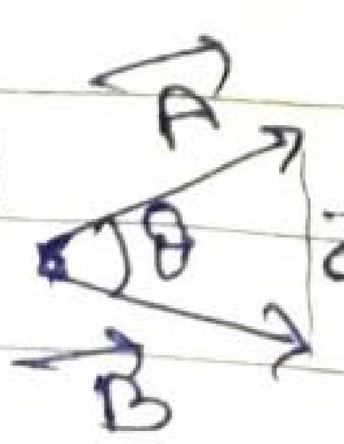


prove (II) to (I)

What does geom. def. mean?

(1) $\vec{A}\cdot\vec{A} = |\vec{A}|^2 \cos(0) = |\vec{A}|^2$
$= a_1^2 + a_2^2 + a_3^2$

(2)



: Law of cosines :

$\vec{C} = \vec{A} - \vec{B}$

$|\vec{C}|^2 = |\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}||\vec{B}|\cos\theta$

$$|\vec{C}|^2 = \vec{C}\cdot\vec{C} = (\vec{A}-\vec{B})\cdot(\vec{A}-\vec{B})$$

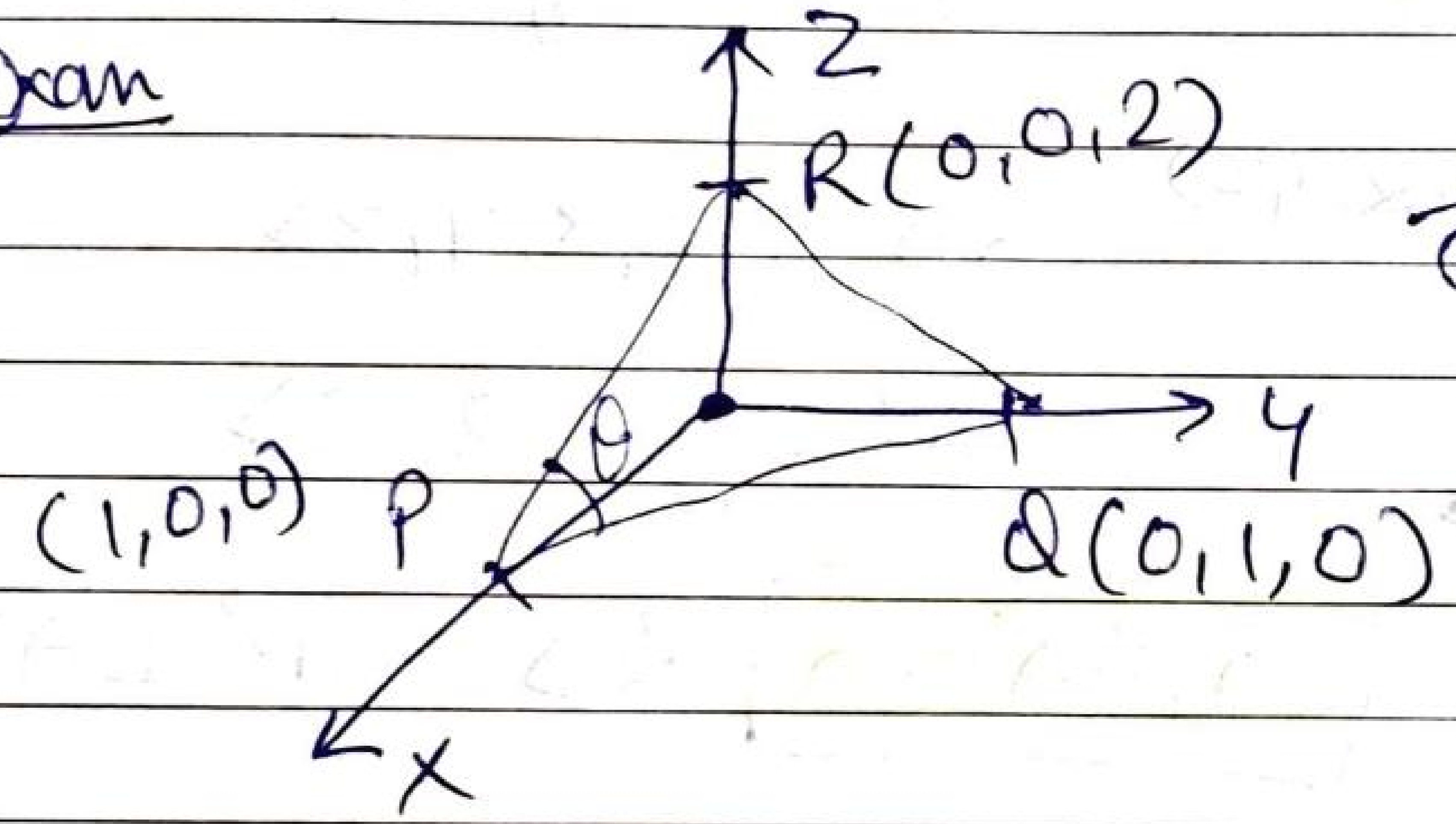
$$= \vec{A}\cdot\vec{A} - \vec{A}\cdot\vec{B} - \bar{B}\cdot\bar{A} + \bar{B}\cdot\bar{B}$$

$$= |\bar{A}|^2 + |\bar{B}|^2 - 2\,\bar{A}\cdot\bar{B}$$

(#) Application of dot product:

1) computing lengths & angles.

Exam



find θ ?

Sol:

$$\overline{PQ}\cdot\overline{PR} = |\overline{PQ}|\,.\,|\overline{PR}|\cos\theta$$

$$\boxed{\cos\theta = \frac{\overline{PQ}\cdot\overline{PR}}{|\overline{PQ}|\,|\overline{PR}|}}$$

To go from P to Q, I need to move $\langle -1, 1, 0\rangle$
from P to R $= \langle -1, 0, 2\rangle$

$$= \frac{\langle -1, 1, 0\rangle \cdot \langle -1, 0, 2\rangle}{\sqrt{(-1)^2+1^2+0^2}\,\sqrt{(-1)^2+0^2+2^2}}$$

$$\cos\theta = \frac{(-1)\cdot(-1)+(1)(0)+(0)(2)}{\sqrt{2}\cdot\sqrt{5}} = \frac{1}{\sqrt{10}}$$

$$\theta = \cos^{-1}(1/\sqrt{10}) \simeq 71.5^\circ$$

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Page No.
Date: / /

Sign of $\vec{A}\cdot\vec{B}$:

> 0 if $\theta < 90°$

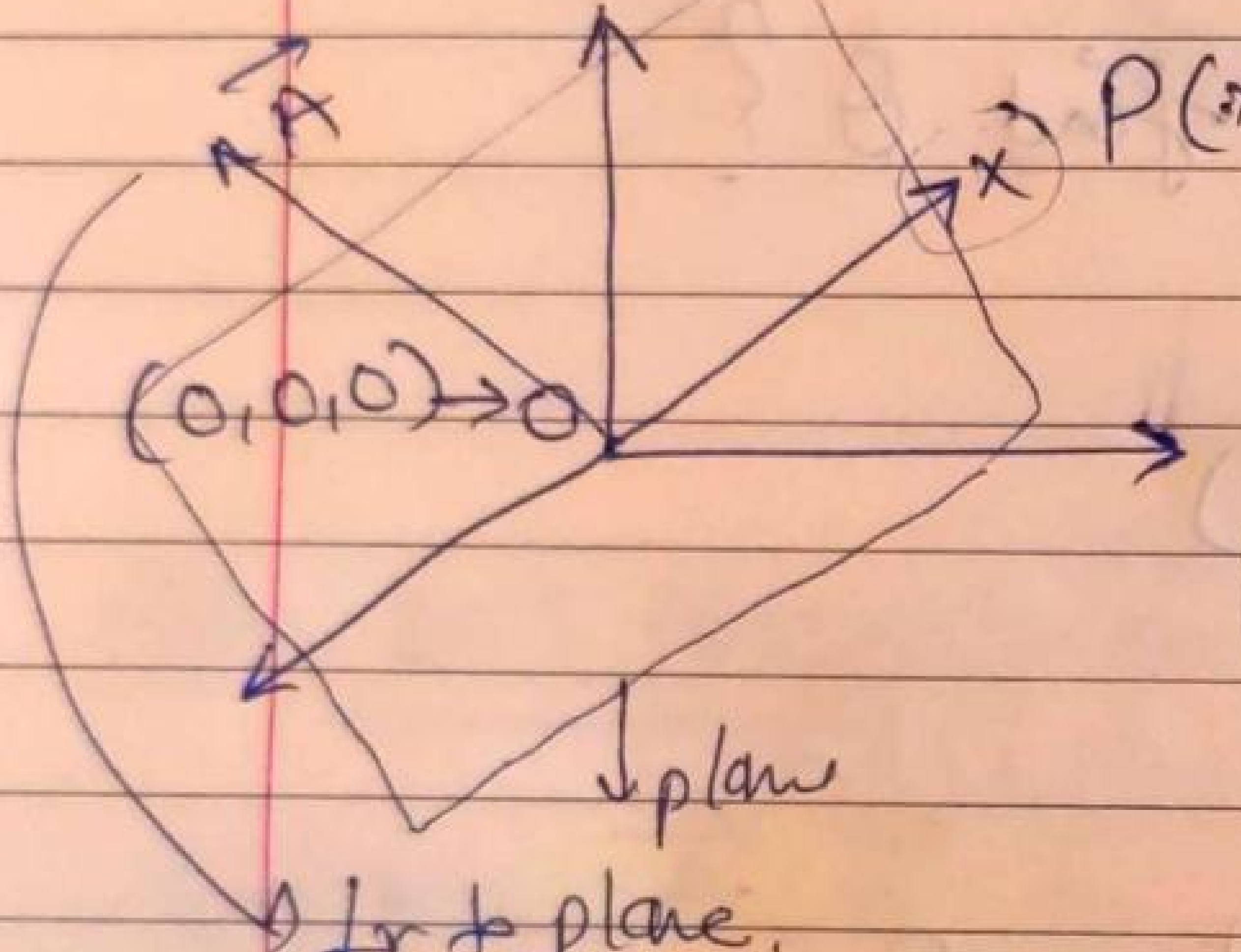
$= 0$ if $\theta = 90°$

< 0 if $\theta > 90$

Applications

② Detect Orthogonality :-

ex:- $x + 2y + 3z = 0$ is eq^n of a plane.



$\vec{OP} = \langle x, y, z \rangle$

$\vec{A} = \langle 1, 2, 3 \rangle$

$\boxed{\vec{OP}\cdot\vec{A} = 0} \iff \vec{OP} \perp \vec{A}$

Here plan through O, ⊥r to $\vec{A}$

Remeber $\vec{A}\cdot\vec{B} = 0 \iff \cos\theta = 0$

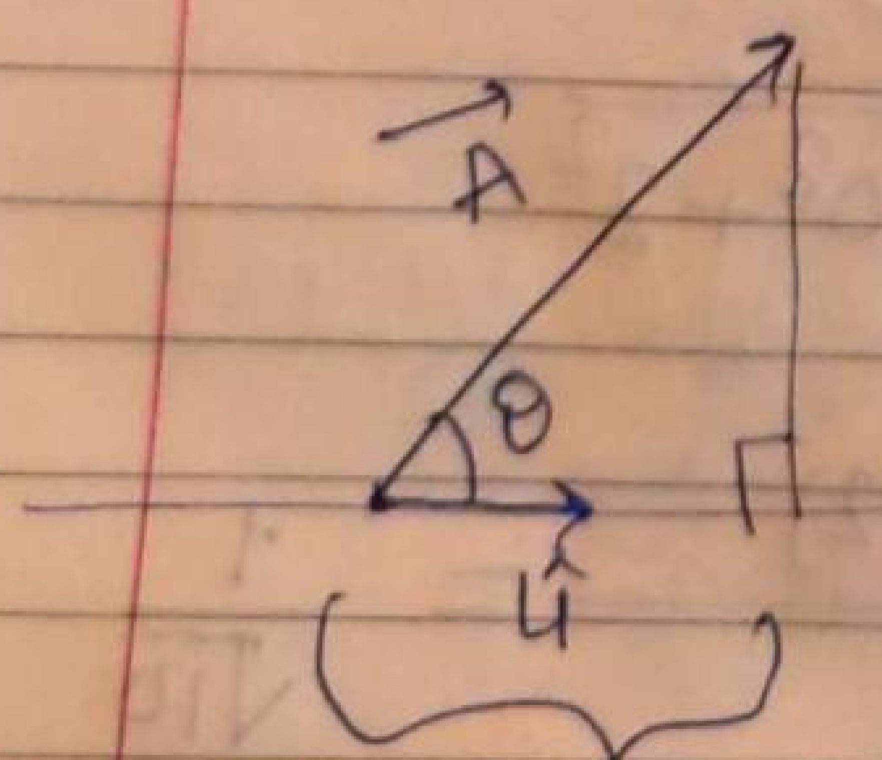
$\iff \theta = 90°$

$\iff \vec{A} \perp \vec{B}$

③ Components of $\vec{A}$ along direction $\hat{u}$ (unit vector

what is the component along $\hat{u}$?

↳ means what is the length of A projection to the given direction



$= \boxed{|\vec{A}|\cos\theta} = \boxed{\vec{A}\cdot\hat{u}}$

$= |\vec{A}|\,|\hat{u}|\cos\theta$