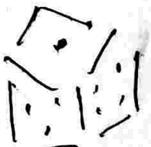
Probability:

equally likely possibilities:

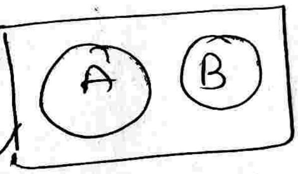
$$P(H) = \frac{\#\text{ of possibilities that meet by}}{\#\text{ of equally likely possibilites}}$$

$$= \frac{1}{2} = 50\%$$



(#) $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$ → Addition Rule

(#) Mutually Exclusive ← No overlap ←



$P(A \cap B) = 0$ ←

(#) $P(HH) = 1/4$ ← Independent Event

Head Head
↓ ↓
2 2 → 4.

HH
HT
TH
TT

$= P(H_1) \cdot P(H_2)$

Head on 1st flip, on 2nd flip

(#) $P(THT) \Rightarrow P(T_1) \cdot P(H_2) \cdot P(T_3)$ ← Independent events.

$\Rightarrow \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 1/8.$

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

(#) $P(\text{atleast } 1\ H) = \frac{7}{8}$ ← $P(H_1) P(H_2) P(H_3)$
if flipping a coin 3 times

⇕

$P(\text{Not getting all tails in 3 flips}) = 1 - P(TTT)$

P(Atleast 1 head in 10 flips) = ?

$\Updownarrow$

P(Not all tails in 10 flips) = 1 − P(10 tails in a row)

$$= 1 - \left(\frac{1}{2}\right)^{10}$$

$$= 1 - \frac{1}{1024} = \frac{1024}{1024} - \frac{1}{1024}$$

$$= \frac{1023}{1024} \quad \underline{99.9\%}$$

$P(A) = \dfrac{\text{\# of events satisfy } A}{\text{\# of equally likely events}}$

Unfair Coins frequency

---

fair coin: Flip 4 times

P(exactly 1 "Heads") is

$\_ \ \_ \ \_ \ \_$

$2 \times 2 \times 2 \times 2$ ← Possibility of H/T

$= 16$

= Probability of

P(HTTT) + ~~P(…)~~ P(THTT) + P(TTHT) + P(TTTH)

$$\Rightarrow \underbrace{\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}}_{\text{mutually exclusive}} = \frac{4}{16}$$

Probability :

P(exactly 2 Head) :- _ _ _ _

2 × 2 × 2 × 2 → 16 total poss

= P(HHTT) + P(THHT) + P(TTHH) + P(HTHT) + P(HTTH) + P(THTH)

$= \frac{6}{16}$

How to calculate for 10 flips or 1000 flips ?

Lets take the same example :

① H_A H_B $(2)^4 = 16$

$\underbrace{\_\ \_\ \_\ \_}_{4\ flip}$

② ~~If H_A is at position 3~~ At how many diff. spots H_A can show up :-

_ _ H_A _ 4 places ✓

③ Now how many diff spots H_B can show up :- 3 places.

= 4 places · 3 places = 12 diff. scenario.

but H_A & H_B are differen

but here are not.

(4) Number of ways to swaps:

2 Heads → 2 ways to swaps.

(5) $\frac{\text{12 different scenario}}{2}$ = 6 different scenario

(6) ⇒ $\frac{6}{16}$ ← equally likely scenario.

(#) fair coin: 5 flip.

(a) How many equally likely possibility

_ _ _ _ _

$2 \times 2 \times 2 \times 2 \times 2 = 32$

p(exactly 3 Head's).

(b) How many possibilities involves 3 Heads in 32 ?

HA HB HC

~~Head opp~~

_ _ _ _ _

# of position available of HA . # of pos. available of HB ...

⇒ 5 places * 4 places * 3 places

⇒ 60 places

(c) ∵ HA = HB = HC : ~~we can swap them~~

3 _ _ _

3 * 2 * 1

How many ways we can arrange them in 3 places

$60/6 = 10$ places

(d) = $\frac{10}{32}$

P(K-heads in 'n' flips of the fair coin):-

(a) — — — - - - — — = $\underline{2^n}$ possibilities.

(b) • $n\cdot(n-1)\,(n-2)\,\cdots\cdots\,(n-(k-1))$ $\rightarrow$ $\boxed{\dfrac{n!}{(n-k)!}}$

K heads

1st head, HB, HC, HK

$\hookrightarrow$ $\dfrac{5\cdot4\cdot3\cdot2!}{2!}$ $k=3$

(c) How many ways, we can arrange them — K-heads K-places?

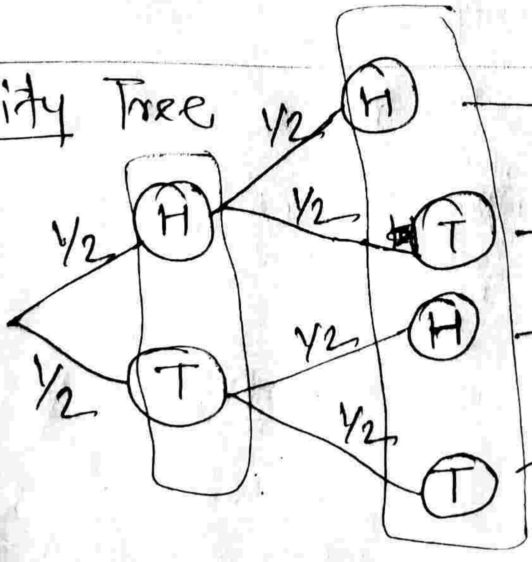
$k\cdot(k-1)\cdot(k-2)\;\cdots\cdots\;1 \Rightarrow \underline{k!}$

HA, HB, HK

(d) $\dfrac{n!}{2^n\,k!\,(n-k)!}$; $\boxed{\binom{n}{k} = {}_nC_k = \dfrac{n!}{k!\,(n-k)!}}$

Probability Tree

P(H·H).



$\rightarrow P(H\cdot H) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

$\rightarrow P(HT) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$,

$\rightarrow P(TH)$

$\rightarrow P(TT)$

⇒ $P(1 \text{ head}, 1 \text{ tail}) = P(TH) + P(HT)$

$= P(TH \underset{\text{or}}{\cup} HT) = \frac{2}{4}$

⇒ Note: for mutually exclusive events you can multiple the probability.

$P(5 \text{ Heads in row})$:- $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

from Probability tree.

$P(\text{Not getting any heads}$ out of 7 times$)$:

$\Rightarrow \underline{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}} \cdot \frac{1}{2} \cdot \frac{1}{2} = \underline{\frac{1}{2^7}}$

↳ p(7 tails row)

P(all heads & last time tail)

$P(\underline{HHHHH}T) = \underline{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}} \cdot \underline{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}}$

P(exactly 1 heads) out of 5 flip $= \frac{5}{32}$

TTTTH — $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \rightarrow \frac{1}{32}$

TTTHT — $(\frac{1}{2})^5 \longrightarrow \frac{1}{32}$

TTHTT — $(\frac{1}{2})^5 \longrightarrow \frac{1}{32}$

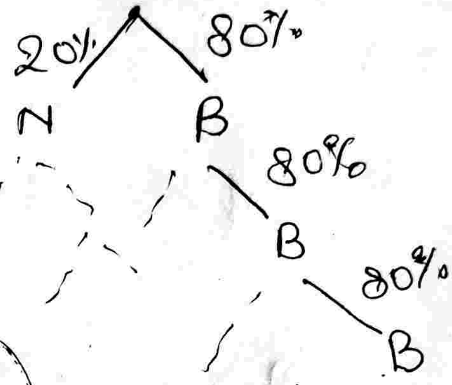
THTTT — $(\frac{1}{2})^5 \longrightarrow \frac{1}{32}$

HTTTT — $(\frac{1}{2})^5 \longrightarrow \frac{1}{32}$

P(not getting exactly 1 head)

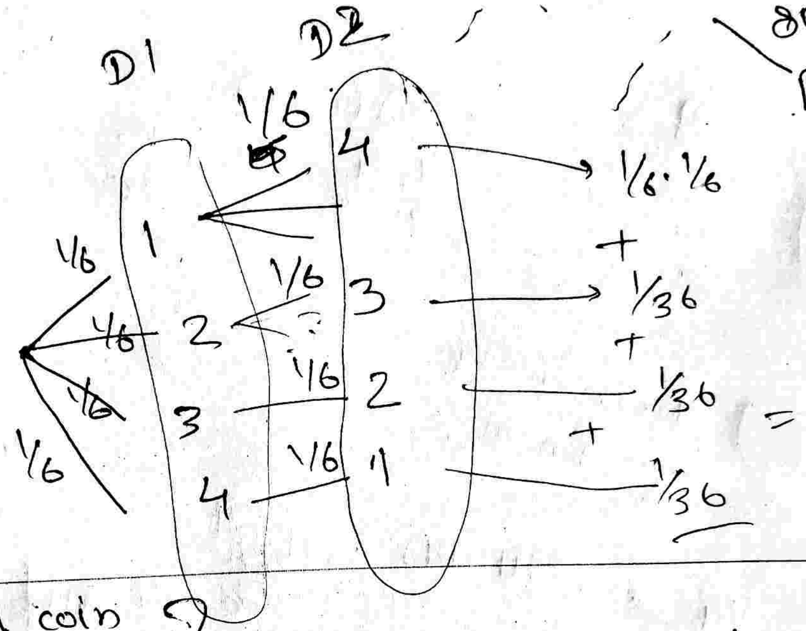
$\Rightarrow 1 - P(\text{exactly 1 heads}) :- 1 - \frac{5}{32}$

Probability tree ✓



P(5) in two dice
sum

D1	D2
1	4
2	3
3	2
4	1



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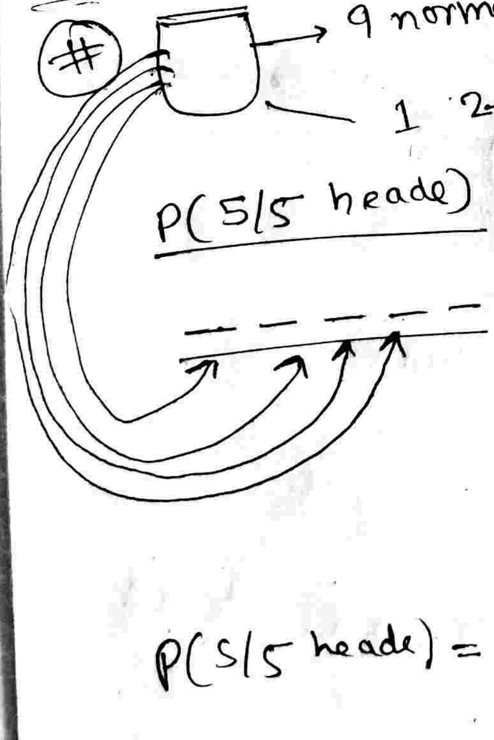


→ 9 normal coin

1 2-sided head

→ probability to take out normal coin = 9/10

$P(n) = 9/10$

$P(2s) = 1/10$

P(5/5 heads) :

9/10 Norm $P(5\text{ heads} \mid \text{Normal coin}) = (1/2)^5 = 1/32$

1/10 2s $P(5\text{ heads} \mid 2\text{ sided coin}) = (1/1)^5 = 1$

$$P(5/5\text{ heads}) = \frac{9}{10} \cdot \frac{1}{32} + \frac{1}{10} \cdot 1 =$$

$$= P(5/5H \mid \text{Normal}) \cdot P(\text{Norm}) + P(5/5H \mid 2s)\, P(2s)$$

#

8 coin total
3 unfair coin

$5/8$ → $P(HH)$: $\frac{1}{2} \cdot \frac{1}{2} = \rightarrow \frac{1}{4}$

$3/8$ → $(0.60)(0.60)$

$\frac{4}{5} \cdot \frac{4}{8} = \frac{16}{25}$

$\frac{60}{100} = \frac{4}{5}$

$\Rightarrow \frac{5}{8} \cdot \frac{1}{4} + \frac{3}{8} \cdot \frac{16}{25}$ ✓

$\Rightarrow$

#

4 coin in bag
3 - unfair

$\frac{1}{4} \cdot (\frac{1}{2})^4 + (\frac{3}{4}) \cdot (1-0.45)^4$

$1/4$ → $P(HHHH) \longrightarrow (\frac{1}{2})^4 \leftarrow P(5H \mid \text{Normal coin}) \longrightarrow P(N \cap 5/5H)$

$3/4$ → $(1-0.45)^4$ ~~(1-0.45)~~ — → $P(2s \cap 5/5H)$

→ Probability $(5/5H \mid 2s)$

total probability:

$\Rightarrow P(5/5H \mid \text{Normal Coin}) \cdot P(\text{Normal Coin})$

$+ P(5/5H \mid 2s) \cdot P(2s)$ // ✓

$\Rightarrow P(\text{Normal Coin} \cap 5/5H) + P(2s \text{ coin} \cap 5/5H)$

so; $P(a \cap b) = P(a \mid b) \cdot P(b)$

b → Normal coin / 2s coin
a → 5/5 H.

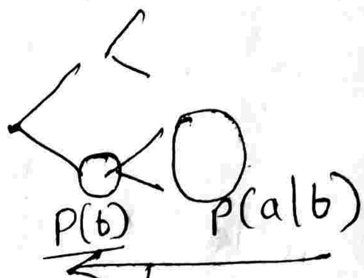
#

$P(5/5H \mid 2s) = 1$

$P(2s \mid 5/5H) = ?$

b, a

$P(a \cap b) = P(b) \cdot P(a|b)$



or

$P(a \cap b) = P(a|b) \cdot P(b)$ — ①

$P(a \cap b) = P(b \cap a) = P(b|a) \cdot P(a)$ — ②

$P(a|b) \cdot P(b) = P(b|a) \cdot P(a)$

$$P(b|a) = \frac{P(a|b) \cdot P(b)}{P(a)}$$ → Baye's Theorem

$$P(2s \mid 5/5H) = \frac{P(5/5H \mid 2s) \cdot P(2s)}{P(5/5)}$$ ⇒ ✓

Permutations

ABCDEFG → 7 people

chair 1 2 3

→ 7 6 5 ⇒ $7*6*5 = 210$

7 possibity, 6 possibily, 5 possibility

$\Rightarrow P^7_3 \rightarrow$ How to put 7 things in 3 places $\rightarrow P^7_3$

7 things $\rightarrow n$, 3 places $\rightarrow K$

$\Rightarrow P^3_2 \rightarrow$ 3 things, 2 places.

3 things $\rightarrow n$, 2 places $\rightarrow K$

$\Rightarrow nP_K \rightarrow P^n_K \rightarrow$

$\hookrightarrow \underset{1}{\_}\ \_\ \_\ \_\ \ldots\ \_\ \_\ \underset{K}{\_}$

$\hookrightarrow \cancel{K(K-1)(K-2)\ldots 1 = K!}$

n different people at 1st pos. $\rightarrow$ $n.(n-1).(n-2)\ldots\ldots(n-(K-1))$

$$\boxed{P^n_K \Rightarrow \frac{n!}{(n-K)!}}\ \checkmark$$

ex: ABC
ACB
BAC....

$\hookrightarrow$ we recount here.

$\_\ \_\ \_\ \cancel{\_}$

$5 \cdot 4 \cdot 3 \cdot \cancel{2\cdot1}$ $\leftarrow K=3$

$\frac{n!}{(n-K)}$ $n=5$ $= \frac{5\cdot4\cdot3\cdot\cancel{2\cdot1}}{\cancel{2\cdot1}}$

$5-3$

Combinations :

Permutation ~~with order~~ :- without ~~order~~ caring about order.

how many groups?

$_5C_3 \rightarrow$ how many groups of 3 can be seated on 3 chair when total number of people are 5.

$$\boxed{_5C_3 = \frac{_5P_3}{3!}}$$

$3! \rightarrow$ (swap)

$\hookrightarrow$ groups of 3 (number of spot, they ~~are~~ can rearrange themself).

$$\boxed{nC_r = \frac{P^n_r}{r!}} = \frac{\frac{n!}{(n-K)!}}{r!} = \frac{n!}{r!(n-K)!}$$

$K=r$

$P(\frac{3}{8}H)$

3 Heads exactly.

K → heads

${}_8P_3 \Rightarrow \frac{n!}{(n-k)!}$

$= \frac{8!}{(5)!}$

$\Rightarrow 8\times7\times6$

$\Rightarrow$

combination

① possible ~~→~~ total equally probable ~~events~~ outcomes

$\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_$

$2\times2 \cdots \times 2 = 2^8$

(ii) $\_\ \_\ \_\ \_\ \_$ $H_A H_B H_C$.

at how many diff. spots H_A can show up.

8

(iii) $H_B \to 7$

$H_C \to 6$

total - $8\cdot7\cdot6 =$

(iii) common (swap) $(\because H_A = H_B = H_C)$

3 places $3! = 6$ ✓

(iv) $\frac{8\cdot7\cdot\cancel{6}}{6} = 56$

(v) $\frac{56}{2^8}$

Steps:

1. possible equally probable outcomes. (total)
2. Combination $\binom{8}{3}$
3. $\Rightarrow \frac{\text{Combination}}{\text{total ~~pos~~ outcomes}} \Rightarrow \boxed{\frac{n!}{k!(n-k)!} / 2^n}$

~~01629565029~~

$2\times2\times2\times2\times2$

\# 5 fair coin

10 unfair coin → 80% H, 20% T

$P(\text{fair} \mid 4/6 \text{ heads})$

$\frac{5}{15} = \frac{1}{3}$ fair → (4/6 Heads) → $15/2^6 = \frac{15}{64}$

total outcomes $= \left(\frac{1}{2}\right)^6$

$6\cdot5\cdot4\cdot3$

$4! \to$ swap → 1·2·3·4

$\frac{6\cdot5\cdot4\cdot3}{4\cdot3\cdot2\cdot1} = \frac{30}{2} = 15$

$\frac{10}{15} = \frac{2}{3}$ Unfair → ?

total outcome $(0.8)^4(0.2)^2$

$15(0.8)^4(0.2)^2$

$P(4/6 H \mid F)$

$P(B \mid A)$

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$P(\text{fair} \cap 4/6 \text{ Head})$

$= P(\text{fair}) \cdot P(4/6 \text{ Head} \mid \text{fair})$

$P(\overset{A}{4/6 \text{ Head}} \cap \overset{B}{\text{fair}})$

$= P(4/6 \text{ Head}) \cdot P(\text{fair} \mid 4/6 \text{ Head})$

$$\boxed{\begin{aligned} P(A \cap B) &= P(A) \cdot P(B \mid A) \\ &= P(B) \cdot P(A \mid B) \end{aligned}}$$

Conditional probability

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$

A = Picked fair

B = 4/6 Head.

$$= \frac{(15/64)\cdot(1/3)}{(2/3)}$$

$=$

30 people

P(atleast 2 people have the same b-day)

$\Rightarrow$ P(someone shares with atleast someone less)

Sol: P(No one share the birthday)

$\Rightarrow$ 1 − P(Noone shares the birthday)

P(distinct birthday)

$365 \cdot 364 \cdot 363 \cdots (365-30)$

$$1 - \left(\frac{365!}{(365-30)!} \right) * \frac{1}{(365)^{30}}$$

↑ total probable outcomes

$\frac{365!}{(365-30)!}$

Random variables

Discrete

Continuous $\longrightarrow$ x = Exact amount of inches tomorrow

1", 1.1" 2.1" ....

Roll a coin → 2 discrete cases

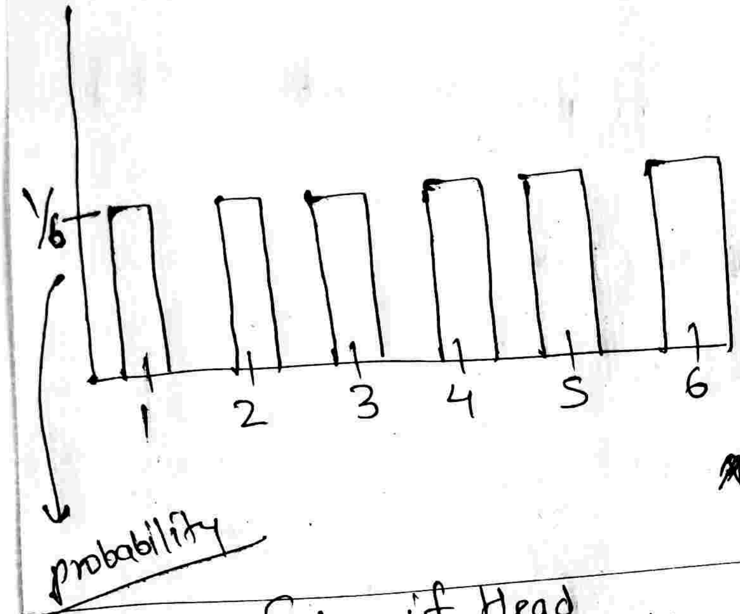
Roll a die → 1/6 outcomes

Rain or Not → 1/2 outcomes (Yes/No)

finite number of variable.

probability distribution

$X =$ # facing up on a fair dice.

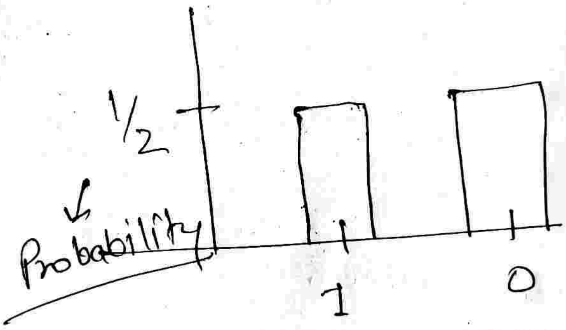


Probability Distribution of Discrete variable

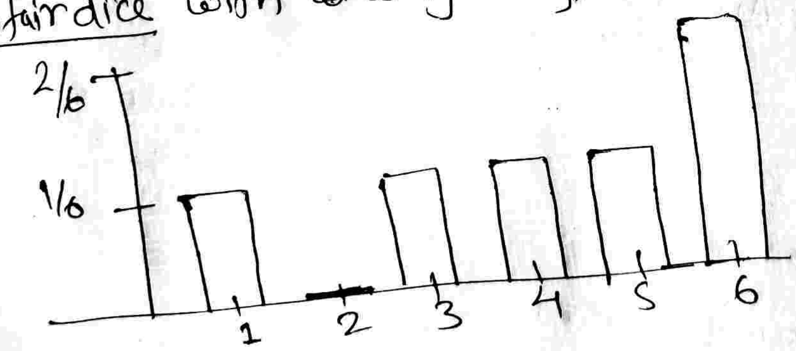
← outcomes

Normal distribution / gaussian
uniform distribution / binomial

$$X = \begin{cases} 1 & \text{if Head} \\ 0 & \text{if tail} \end{cases}$$



Unfair dice with weighting.



probability distribution f^n

$X = \{$ # on unfair dice

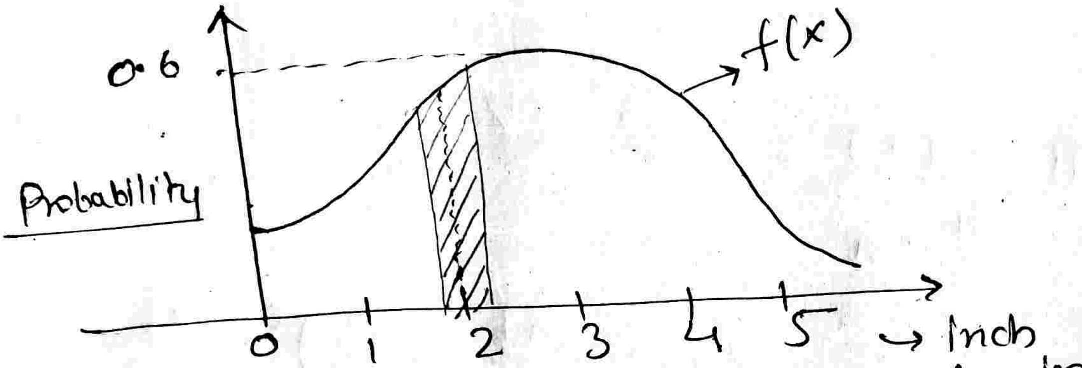
$P(X=6) = 2/6$; $P(X \geq 0) = 1/6 + 2/6 = 3/6 = 1/2$

Rom

Continuous Random variable:

Y = exact amount of rain tomorrow

Probability Density function (Probability distribution)



What is the probability that tomorrow, we will have exactly 2" of rain? ~~2.01~~, ~~1.99~~, ~~1.99995~~, ~~2.000001~~

$\Rightarrow P(|Y-2| < .1)$

↳ Probability $1.9 < Y < 2.1$ (shaded area)

$$\rightarrow \boxed{P(|Y-2| < .01) = \int_{1.9}^{2.1} f(x)\, dx}$$ ✓

Note: $\boxed{\int_{0}^{\infty} f(x)\, dx = 1}$ → all probability should add to 1.

fair coin : flip → 5 times X = # of heads after 5 flips

- $P(X=0)$

$P(TTTTT) = (1/2)^5 = (1/32)$

- $P(X=1)$

$P(HTTTT) = (1/2)^5 = (1/32)$

H H H H

- $P(X=2)$

$P(HHTTT) = \left(\frac{1}{2}\right)^5 \cdot \frac{n!}{k!(n-k)!} \Rightarrow (1/2)^5 \cdot \frac{5!}{2!\cdot 3!}$ — $5*4$ — $5*2^2$

as we don't care about order

$= \frac{1}{2^4} \cdot 5 = \left(\frac{5}{16}\right)$ ✓

- $P(X=3) \Rightarrow \left(\frac{1}{2}\right)^5 \cdot \frac{5!}{3!\cdot 2!}$

HHHTT

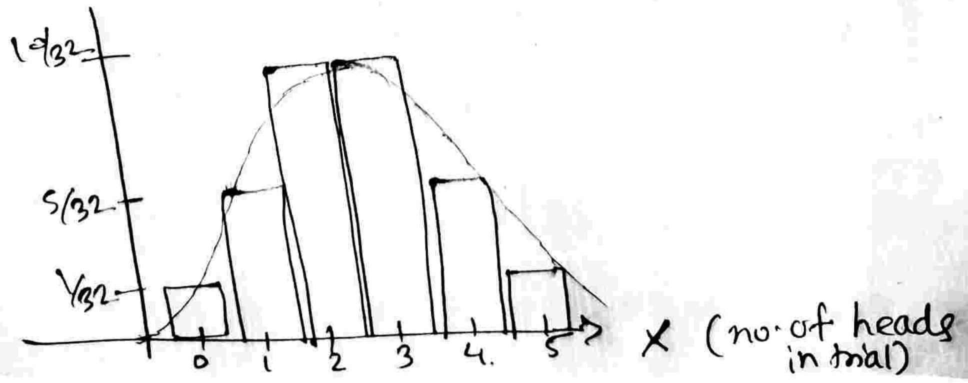
$\Rightarrow 5/16$

- $P(X=4) \Rightarrow 5/32$; = Probability of getting 1 ~~heads~~ ~~tail~~ Head (tail) ✓

Probability of getting 4 heads

- $P(X=5) = \frac{1}{32}$; ; = Prob. of No tail

all heads



$$\boxed{P(X=n) = \frac{5!}{n!\,(5-2)!}} = \binom{5}{n} \checkmark$$

$\binom{5}{n}$ → 5 flips choosing 'n'

Binomial coefficient

$$\frac{n!}{K!\,(n-K)!} = ($$

$n = 6$; Basketball shots, $X = \{$ # of shots I make.

$P = 30\% \rightarrow$ (success)

$$P(X=0) = \overset{1^{st}}{(0.70)}\,\overset{2^{nd}}{(0.70)}\,\overset{3^{rd}}{(0.70)}\,\overset{4^{th}}{(0.7)}\,\overset{5^{th}}{(0.7)}\,\overset{6^{th}}{(0.7)}$$

$$= (0.7)^6 \Rightarrow \binom{6}{0} \Rightarrow \frac{6!}{0!\,(6-0)!} =$$

$$P(X=1) = 6\,(0.30)^1 (0.7)^5 = \binom{6}{1} \Rightarrow \frac{6!}{1!\,5!} \quad \boxed{6} \checkmark$$

$$P(X=2) = \underline{15} \cdot \underbrace{(0.30)^2 (0.7)^4}_{\text{outcomes}}$$

$$\frac{6!}{2!\,4!} = \frac{\overset{3}{\cancel{6}} * 5}{\cancel{2}}$$

$$P(X=3) = \underline{20} \cdot (0.30)^3 (0.7)^3$$

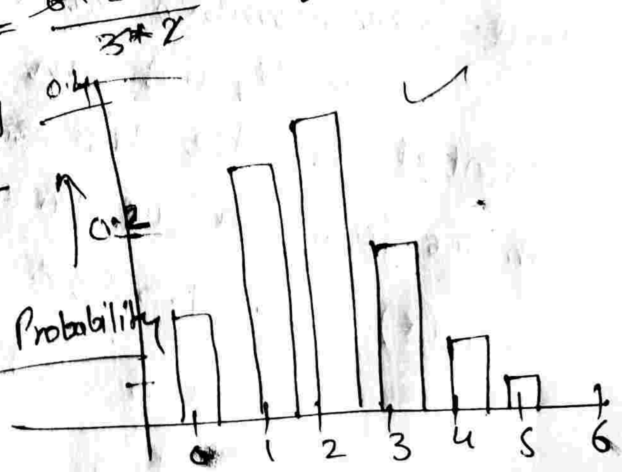
$$\frac{6!}{3!\,3!} = \frac{\cancel{6} * 5 * \overset{2}{\cancel{4}}\ 2}{\cancel{3 * 2}}$$

binomial distribution

$$P(X=4) \Rightarrow \binom{6}{4} (0.7)^2 (0.3)^4$$

$$P(X=5) \Rightarrow \binom{6}{5} (0.7)^1 (0.3)^5$$

$$P(X=6) \Rightarrow \binom{6}{5} (0.3)^6$$



Expected value of random variables

is exact same thing as population mean; sometimes it's called population mean, but in the expected value of random variable, you have a infinite population.

↳ So you can't just add up all the numbers & divide by the number of numbers you have.

But what you can do is if you said wow, I know the frequency of the numbers.

$60\% \cdot 3 + 20\% \cdot 4 + 20\% \cdot 5$.

Then even if you have an infinite number of numbers, you can actually still calculate a mean. And that's how you do it for an expected value of a random variable.

So, how do you figure out the frequencies that the number shows up?

Well, we can look at the probability distribution, the discreat probability distribution.

examples from previous probability distribution.

$$E(x) = \underset{(\text{Heads})}{0} \cdot 1.563\% + \underset{(\text{Heads})}{1} \cdot 9.375\% + 2 \cdot 23.438\% + 3 \cdot 31.25\% + 4 \cdot 23.438\% + 5 \cdot 9.375\% + 6 \cdot 1.563\%$$

$= 3.0\%$

Expected value

example 1 X = # of success with probability P after n-trials.

$$E(X) = n \cdot p$$ (① n times (probability))

ex: 2 X = # of baskets I make after 10 shots

$p = 40\%$ of making shot

$$E(X) = 0.4 \times 10 \Rightarrow 4 \text{ shots}.$$

$$P(X=K) = \underbrace{\binom{n}{k} \underbrace{p^k (1-p)^{n-k}}_{\text{total possible probable outcomes}}}_{\text{probability of having } K}$$

so:

$$\boxed{E(X) = \sum_{K=0}^{n} K \cdot \underbrace{\binom{n}{k} p^k (1-p)^{n-k}}_{\text{probability of } K \text{ shots}}}$$

(same as, $\underset{\text{(Heads)}}{0} \cdot \underbrace{(1.563\%)}_{\text{probability of 0 heads}} + 1 \cdot \underline{\times} + 2 \cdot \ldots$

$$\boxed{E(X) = \sum_{K=1}^{n} K \cdot \binom{n}{k} p^k (1-p)^{n-k}}$$ → because $K=0$ terms will be zero.

$$E(X) = \sum_{K=1}^{n} K \cdot \frac{n!}{K!(n-K)!} \cdot p^k (1-p)^{n-K}$$

$$\boxed{E(X) = \sum_{K=1}^{n} \frac{n!}{(K-1)!(n-K)!} \cdot p^K \cdot (1-p)^{n-K}}$$

$$E(x) = n\cdot p\cdot \sum_{k=1}^{n} \frac{(n-1)!}{(k-1)!\,(n-k)!}\cdot p^{k-1}\cdot (1-p)^{n-k}$$

$a = k-1, \quad b = n-1,$

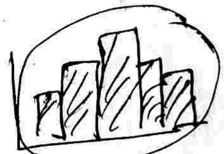
$a+1 = k; \quad b+1 = n$

$n-k = \cancel{a-b}\ b-a$

if $k=n$

$$E(x) = np \boxed{\sum_{a=0}^{b} \frac{b!}{a!\,(b-a)!}\cdot p^{a}\cdot (1-p)^{b-a}}$$

$$= np\cdot \underbrace{\sum_{a=0}^{b} \binom{b}{a}\cdot p^{a}\cdot (1-p)^{b-a}}_{=1} = 1$$



$\boxed{E(x) = np}$ → p = probability of success

n = # of trials

only true; for binomial distribution.

#

$\underline{\underline{X}}$ = # of cars pass in an hour.
↳ probability distribution ?

$$E(X) = \overset{\text{success in hour}}{\lambda} = n \cdot p \quad \text{—— (0)}$$

$$\lambda \frac{\text{cars}}{\text{hours}} = 60 \text{ min/hour} \rightarrow \frac{\lambda}{60} \text{ cars/min}$$

$$P(X=K) = \binom{60}{K}\left(\frac{\lambda}{60}\right)^K\left(1-\frac{\lambda}{60}\right)^{60-K}.$$

↳ problem ?

What if more than 1 car passes in an hour.

more granular

$$\hookrightarrow P(X=K) = \binom{3600}{K}\left(\frac{\lambda}{3600}\right)^K\left(1-\frac{\lambda}{3600}\right)^{3600-K} \quad \text{—— (1)}$$

↳ so, 2 car may come in 1 sec;
so more granular & granular

↳ Poisson distribution

$$\because \lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x = e^a$$

$1/n = a/x$; $x = na$

$$\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{na} = \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^a\right.$$

$$= \left(\lim_{n\to\infty}\left(1+1/n\right)^n\right)^a = e^a \quad \text{—— (2)}$$

$$\because \frac{x!}{(x-K)!} = (x)(x-1)(x-2)\ldots(x-K+1)! \quad \text{--- (3)}$$

Updated (1):

$$P(X=K) = \lim_{n\to\infty} \binom{n}{K} \underbrace{\left(\frac{\lambda}{n}\right)^K}_{\text{from eq-(0)} \Rightarrow p=\lambda/n} \left(1-\frac{\lambda}{n}\right)^{n-K}$$

in ∞ time

$$\lim_{n\to\infty} \underbrace{\frac{n!}{(n-K)!\,K!}}_{\text{from eq-(3)}} \left(\frac{\lambda}{n}\right)^K \left(1-\frac{\lambda}{n}\right)^{n-K}$$

$$\Rightarrow \lim_{n\to\infty} \frac{n(n-1)(n-2)\ldots(n-K+1)}{K!} \frac{\lambda^K}{n^K} \cdot \left(1-\frac{\lambda}{n}\right)^n \left(1-\frac{\lambda}{n}\right)^{-K}$$

$$\because \Rightarrow \lim_{x\to a} f(x)\,g(x) = \lim_{x\to a} f(x) \lim_{x\to 0} g(a)$$

$$\Rightarrow \lim_{n\to\infty} \underbrace{\frac{n^K + \ldots\ldots}{n^K}}_{=1} \cdot \left(\frac{\lambda^K}{K!}\right) \underbrace{\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n}_{e^{\lambda}\ \text{eq-(2)}} \underbrace{\left(1-\frac{\lambda}{n}\right)^{-K}}_{1}$$

$\leftarrow$ height degree

$$\Rightarrow 1 \cdot \frac{\lambda^K}{K!} \cdot e^{-\lambda}$$

$$P(X=K) = \lim_{n\to\infty} \binom{n}{K} \left(\frac{\lambda}{n}\right)^K \left(1-\frac{\lambda}{n}\right)^{n-K}$$

$$\boxed{P(X=K) = \frac{\lambda^K}{K!} e^{-\lambda}} \quad //$$

Poisson distribution //

example: $X = 9$ car pass per hour

$$P(X=2) = \frac{\lambda^k}{k!} \cdot e^{-\lambda} = \frac{9^2}{2!} e^{-9}.$$

Law of Large Numbers

$$\overline{X_n} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

tells that my sample mean will approach $E(X)$.

$$\overline{X_n} \to E(X) \quad \text{for } n \to \infty$$

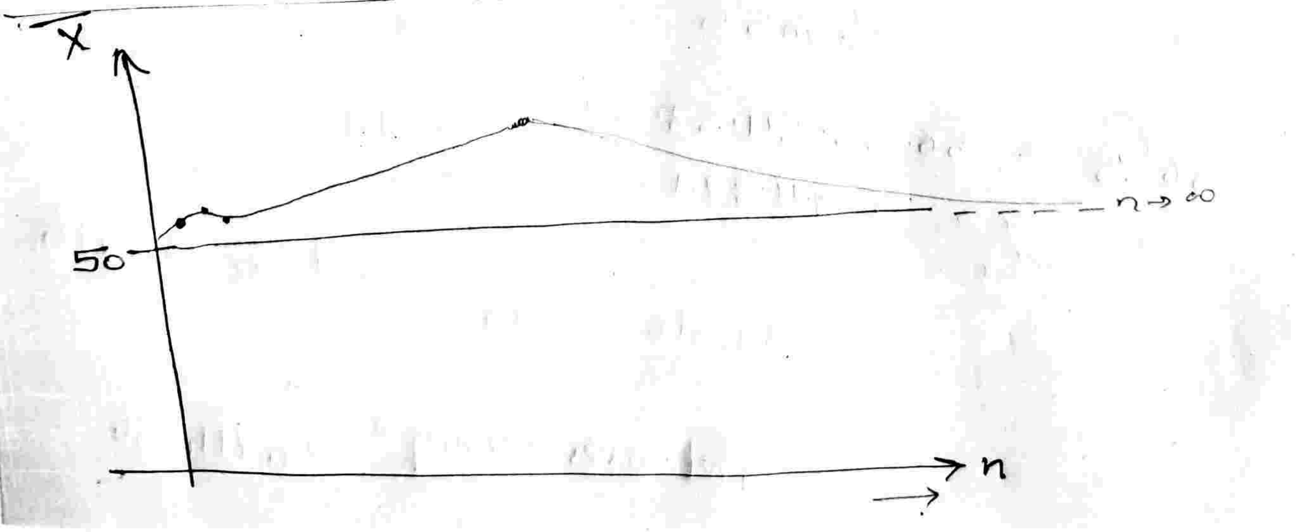
$$\overline{X_n} \to \mu$$

ex: X = # of heads after 100 tosses of fair coin

$$E(X) = 100 \cdot (0.5) = \underline{50}$$

$$\overline{X_n} = \frac{55 + 65 + 45 + \cdots + n}{n}$$

converge to 50 as $n \to \infty$



example 6-

$1 million policy

$500/year premium

* term - 20 years

P(Sal's death in 20 years) = ?

Sol:

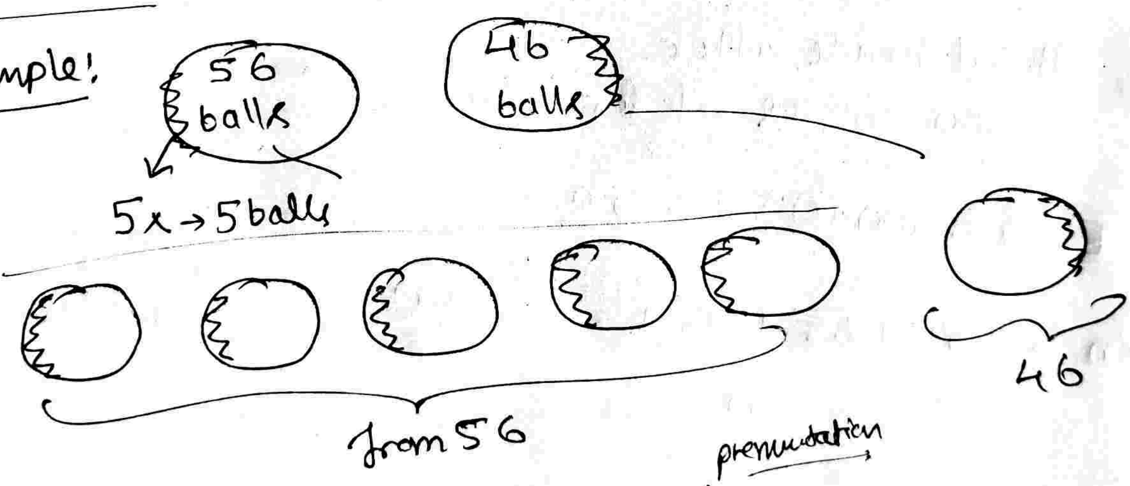
$$500 \times 20 = \frac{\$10,000}{\$1,000,000} = \frac{\$1 \text{ - premium}}{\$100 \text{ - insurance}}$$

↓ probability

$\underbrace{\text{100 people}}_{\text{\$100 in premium}}$

Break even: only 1 people dies.

example!



$$_{56}C_5 = \frac{56 \cdot 55 \cdot 54 \cdot 53 \cdot 52}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \times 46$$

premutation

for any order

↓

$3,819,816 \times 46 = 175,711,536.$

Prob. over winning = $\frac{1}{175,711,536}$