1 Introduction

In all structures, nonlinearities which will affect the dynamic behavior of the structure are present to some extend. Depending on excitation level, the structure will exhibit linear or nonlinear dynamics. Nonlinearities have always existed, but are often neglected.

To get a further understanding of the nonlinear effects present, an efficient set of tools for identification, characterization and estimation of nonlinearities in engineering structures from experimental observations would be useful.

The motivation for this thesis is to provide that understanding, hopefully giving the reader a “toolbox” applicable to nonlinear systems. A toolbox is to be understood as a collection of methods, applicable to nonlinear problems. Like modal testing is one method in the linear toolbox.

To exemplify and test this toolbox, a part of the thesis is dedicated to developing, implementing and exemplifying the methods of the toolbox numerically.

By using such a “numerical toolbox”, further understanding of the nonlinear dynamics can be obtained by simulation, then what is gained by pure experiment, and hopefully assist in virtual prototyping.

One requirement for the toolbox is that it works on experimental data, e.g. time series, alone. Methods exists which can do identification and estimation, but they often require either that the Equations of Motions (EOM) are assembled or a detailed Finite Element Model is constructed, and are thus often difficult and time consuming to use.

Another aspect of the toolbox is to quantify the size, or importance, of nonlinearity:

Even if the area of nonlinear identification and modeling have received a great deal more attention within the last ten years, it is still far behind the linear counterpart, both in theory and application. Thus a nonlinear toolbox certainly requires some specialization to use. If the nonlinearity is weak it might suffice with linear analysis, giving access to all the traditional methods, with reduced time spend on the analysis as a consequence.

1.1 Why nonlinear modeling

For nonlinear systems, the superposition and thus invariance of modes and uniqueness of solutions (e.g. the forced steady state response is dependent on the initial transient behavior) does no longer hold, and many of the techniques from linear analysis cannot be used.

Linear system is an exception. If excited hard enough, all system displays some nonlinear behavior. But often the nonlinearity stems from joints (damping), contact (stiffness) or geometrical nonlinearities, which is why most of the literature today treats localized nonlinearity, assuming the location of the nonlinearity is known. Another reason for dealing with localized nonlinearities, is that no robust method for localization exists. In his thesis Kragh (2010) test different methods for localizing nonlinearity and concludes that “it was not possible to obtain consistent localization of the nonlinearities” even for simple structures.

1. Detection: *Is there?*
   Ascertain if nonlinearity exist in the structural behavior, e.g. yes or no.

2. Characterisation: *Where, what and how?*

   - Localize the nonlinearity, e.g. at the joint
   - Determine the type of nonlinearity e.g. Coulomb friction
     More general: is it stiffness or damping nonlinearity or both. In the case of stiffness: is it hardening or softening
   - Select the functional form of the nonlinearity, e.g. $f(x, \dot{x}) = c\,\mathrm{sign}(\dot{y})$

3. Parameter estimation: *How much?*
   Calculate the coefficients of the nonlinearity model, e.g. $c = 5.47$.
   (Ideally the uncertainty should be quantified, e.g. in a probabilistic sense, $c \sim N(5.47, 1)$. But that is a very difficult task and not within the scope of this thesis)

Figure 1.1: *Identification process for nonlinear structural models*

With the introduction of ever lighter structures, exotic materials, high speed machinery, etc., nonlinear tools are needed to fully understand the dynamics. Also to determine if nonlinear analysis is indeed needed, since this kind of analysis requires substantial more effort than linear analysis would do.

For a general introduction to nonlinear dynamics, the textbook Juel Thomsen (2003) can be recommended.

1.2 Nonlinear system identification?

Kerschen et al. (2006) proposed to regard the identification of nonlinear structural models as a progression through three steps: *detection*, *characterization* and *estimation*, as outlined in figure **??**.

The first book on the topic was Worden and Tomlinson (2000), and even though many new methods has been introduced since then, it still gives a good and well written introduction to the subject as well as a overview of the common types of nonlinearity.

A comprehensive review of the development in nonlinear system identification has been given by Kerschen et al. (2006) and just recently in Noël and Kerschen (2016). For comparison of the many techniques in use, the reader is refereed to these reviews. In this thesis, the choice of technique will be motivated but alternative techniques will not necessarily be mentioned.

1.2.1 Detection

Of the three steps, detection is the easiest. During test, the structure should be excited by a sine-sweep and a mere visual inspection of the time series will show if nonlinearity is present. Signs includes skewness of the envelope, discontinuity, jumps and lack of invariance with increasing force level. The excitation level needs to be at an amplitude where the nonlinearity is activated.

Random excitation is in general not useful, as the randomness of the amplitude and phase of the excitation creates "linearized" frequency response functions (FRFs). At least multiple test with different rms levels are required, and still then it might be difficult to excited the nonlinearities, since the total power of the input spectrum is spread over the band-limited frequency range used.

The use of impact excitation, as often used in linear analysis, suffers from the same problems as random excitation. That is, the input is a broad spectrum and the energy associated with each frequency is low.

Formal methods for detection includes

- Homogeneity check
  Comparing the response of two sweeps with different forcing and calculating the cross correlation. It is a test of superposition, by testing if the two FRFs normalized with forcing overlay as they will for linear systems.

- (ordinary) Coherence function
  The coherence function compares power spectral densities (PSDs) and are required to be unity for all accessible frequencies for the system to be linear *and* free of noise. The advantage is that only one test is needed, but the method does not distinguish between cases of noise and nonlinearity. Instead it is recommend do use:

- Hilbert transforms
  This method detects nonlinearity by doing a Hilbert transform of the FRF, which is invariant for a linear FRF. A Hilbert transform also only requires one data set and is more sensitive to nonlinearity than the coherence function, but still reasonable easy to implement. Kragh (2010) shows that the homogeneity check is superior to the Hilbert transforms, having higher sensitivity to nonlinearity.

For all of these methods it is a requirement that the nonlinearities are activated, e.g. the forcing level end frequency interval should be chosen adequately. Also, the methods are better at detecting nonlinear stiffness than nonlinear damping. This is due to the fact that the resonance peak is not shifted as with the stiffness nonlinearity case. Since the FRF is not shifted but only lowered, the cross correlation coefficient will not decrease as much as in the stiffness nonlinearity case.

1.2.2 Characterization

The second step is the most important and also the most difficult, when localization is not considered.

This step seeks to identify the aspects of the motion that drives the nonlinearity, e.g. displacement or velocity and a representative functional form to represent the nonlinearity.

The most used technique is the *restoring force surface* (RFS). The RFS provides information of restoring force in the excited range. To visualize the functional of the restoring force and the dissipative force, two slices in the RFS is made: at zero displacement and zero velocity. The functional form is then found by fitting polynomials to the slices and perform goodnees of fit. The RFS requires an estimate of the inertia of the system. The approach from Dossogne et al. (2015) will be used, which is a multiple degree of freedom (MDOF) application of the RFS. The RFS is described in section 2.2.

Other characterization methods include blackbox modeling, which do characterization without regards to the underlying physics, instead using a rich and flexible mathematical structure to capture all relevant dynamics.

1.2.3 Estimation

The RFS method can be used for estimation as well, fitting the functional form to the surface. But in order to scale the RFS correct, an estimate of the mass (or inertia) is needed or the full EOM has to be assembled. This is often difficult for MDOF systems and violates the ambition of the toolbox: that it works on time series alone.

A new method, introduced in the ph.d. thesis Noël (2014), is the frequency-domain nonlinear subspace identification (FNSI) and used here. This method works on time series alone and is not sensitive to noise. FNSI is descibed in section 4.1.

1.3 Beyond nonlinear system identification

When the identification steps is completed, a structural model can be build from a FEM of the underlying linear structure with the identified nonlinearities incorporated. It shall be thought of as (larger) chunks of linear sections connected through nonlinear elements. To reduce the computational time, the linear model is reduced using the Craig-Bampton reduction technique, Craig and Bampton (1968).

If the predictions from the nonlinear FEM can be verified by the experimental results, the numerical model can be used to *get further understanding of the nonlinear dynamics*. The latter is the whole goal of the identification, as it allows for uncovering new nonlinear regimes of motion and to make design modifications. The concept of using numerical experiments to assist with the design is referred to as *virtual prototyping*.

1.3.1 Internal resonance

Nonlinear resonances are investigated using an extension of linear normal modes (LNMs) to nonlinear theory, the nonlinear normal modes (NNMs), described in section 2. Where a LNM is interpreted as the deformation along the axis of the vibrating structure or the rotation, a NNM does not have such a clear interpretation. An NNM is said to be a periodic oscillation of the underlaying unforced and unforced nonlinear system and depends on the frequency and energy of the system. Additionally, the backbone of NNMs plotted in a frequency-energy plot (FEP), track the lotus of the nonlinear frequency response function (NFRF) for the system. Thus knowing the FEP, the resonant response at an arbitrary forcing level can be found.

The (numerical) calculations of NNMs were introduced in Peeters et al. (2009), based of the ph.d. work of Peeters (2010), using a shooting method and pseudo-archlength continuation technique.

The shooting method requires substantial computational effort for larger FEM structures. A method based on harmonic balance (HB) and continuation was recently introduced in the review Renson et al. (2016), which, among several benefits, reduces the computational burden. The HB method for NNMs originated from Detroux et al. (2016), based on the ph.d. Detroux (2016). A method for calculating NNMs for nonconservative systems, mentioned in the review and originating from Renson (2014), will not be studied in this thesis.

This is because for simple (and weekly) damping mechanism, the damped dynamics can often be interpreted based on the topological structure and bifurcation of the NNMs of the underlying Hamiltonian (conservative) system, Renson et al. (2016, sec. 4)

1.3.2 Bifurcation

Using the nonlinear FEM, forced responses for sine sweeps can be calculated using Newmark time integration. The corresponding NFRF, including stability and detection of bifurcation points, will be calculated using harmonic balance based continuation, as described in Detroux et al. (2015).

1.4 Thesis outline

Section 2 introduces the theoretical methods used: NNMs, RFS and FNSI,

Section 3 introduces the numerical methods: FEM discretization and model reduction, Newmark time integration, harmonic balance and continuation for calculating NNMs and NFRF.

It also briefly discusses methods for integrating and differentiating time signals and filtering techniques.

Section 4 introduce identification and simulation of benchmark data from a nonlinear system, the COST beam, GOLINVAL and LINK (2003), which have a cubic stiffening nonlinear due to geometry and a squared nonlinearity due to clamping.

Section 5 introduces numerical experiments to investigate how the methods performs and their sensitivity to noise.

Finally section 6 contains an discussion and conclusion and suggest further studies and implementations.