CARNESS analysis package

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Alessandro Filisetti

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CHAPTER

ONE

CHEMISTRY GRAPH ANALYSIS

This python tool evaluates a particular chemistry findind RAF, SCC and saving the multigraph bipartite network and the catalyst-product network

NETWORKX formats :: http://networkx.lanl.gov/reference/readwrite.html

graph_chemistry_analysis.**beta**(*a*, *b*, *size=None*)

The Beta distribution over `[0, 1]`.

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

graph_chemistry_analysis.**binomial**(*n*, *p*, *size=None*)

Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

graph_chemistry_analysis.**chisquare**(*df*, *size=None*)

Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\texttt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

NIST/SEMATECH e-Handbook of Statistical Methods

```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

graph_chemistry_analysis.**exponential**(*scale=1.0*, *size=None*)

Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

graph_chemistry_analysis.**f**(*dfnum*, *dfden*, *size=None*)

Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

graph_chemistry_analysis.**gamma**(*shape*, *scale=1.0*, *size=None*)

Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

graph_chemistry_analysis.**geometric**(*p*, *size=None*)

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where p is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

graph_chemistry_analysis.**get_state**()

Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

graph_chemistry_analysis.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)

Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

graph_chemistry_analysis.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

`graph_chemistry_analysis.`**`laplace`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...      np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

graph_chemistry_analysis.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

graph_chemistry_analysis.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)

Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

graph_chemistry_analysis.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

graph_chemistry_analysis.**multinomial**(*n*, *pvals*, *size=None*)

Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

graph_chemistry_analysis.**multivariate_normal**(*mean*, *cov*[, *size*])

Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

•Spherical covariance (*cov* is a multiple of the identity matrix)

•Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

graph_chemistry_analysis.**negative_binomial**(*n*, *p*, *size=None*)

Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N; n, p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...     probability = sum(s<i) / 100000.
...     print i, "wells drilled, probability of one success =", probability
```

graph_chemistry_analysis.**noncentral_chisquare**(*df*, *nonc*, *size=None*)

Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

graph_chemistry_analysis.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)

Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

graph_chemistry_analysis.**normal**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

graph_chemistry_analysis.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has

also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

graph_chemistry_analysis.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

graph_chemistry_analysis.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k; \lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

graph_chemistry_analysis.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')
```

```
>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

graph_chemistry_analysis.**rand**(*d0*, *d1*, ..., *dn*)

Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

graph_chemistry_analysis.**randint**(*low*, *high=None*, *size=None*)

Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

graph_chemistry_analysis.**randn**(*d0*, *d1*, ..., *dn*)

Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

graph_chemistry_analysis.**random**()

random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

graph_chemistry_analysis.**random_integers**(*low*, *high=None*, *size=None*)

Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

graph_chemistry_analysis.**random_sample**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

graph_chemistry_analysis.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

graph_chemistry_analysis.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

graph_chemistry_analysis.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

graph_chemistry_analysis.**seed**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

graph_chemistry_analysis.**set_state**(*state*)
Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

graph_chemistry_analysis.**shuffle**(*x*)
Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

graph_chemistry_analysis.**standard_cauchy**(*size=None*)
Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

graph_chemistry_analysis.**standard_exponential**(*size=None*)

Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

graph_chemistry_analysis.**standard_gamma**(*shape*, *size=None*)

Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

graph_chemistry_analysis.**standard_normal**(*size=None*)

Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

graph_chemistry_analysis.**standard_t**(*df*, *size=None*)

Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1+\frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

graph_chemistry_analysis.**triangular**(*left*, *mode*, *right*, *size=None*)

Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

graph_chemistry_analysis.**uniform**(*low=0.0*, *high=1.0*, *size=1*)

Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

graph_chemistry_analysis.**vonmises**(*mu*, *kappa*, *size=None*)

Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode (“center”) of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

graph_chemistry_analysis.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

graph_chemistry_analysis.**weibull**(*a*, *size=None*)

Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1}e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

graph_chemistry_analysis.**zipf**(*a*, *size=None*)

Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

CHAPTER

TWO

ACSATTRACTORANALYSIS MODULE

Created on 4 February 2014

Author: Alessandro Filisetti <alessandro.filisetti@gmail.com>

Function to analyze the different final dynamical states from different simulations (final states). The algorithm compares all the final states (in terms of concentrations) of the simulations contained in StrPath. If there are both several generations and simulations the script will process everything.

acsAttractorAnalysis.**beta**(*a*, *b*, *size=None*)

The Beta distribution over `[0, 1]`.

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

acsAttractorAnalysis.**binomial**(*n*, *p*, *size=None*)

Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

acsAttractorAnalysis.**chisquare**(*df*, *size=None*)

Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\texttt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

NIST/SEMATECH e-Handbook of Statistical Methods

```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

acsAttractorAnalysis.**exponential**(*scale=1.0*, *size=None*)

Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

acsAttractorAnalysis.**f**(*dfnum*, *dfden*, *size=None*)

Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

acsAttractorAnalysis.**gamma**(*shape*, *scale=1.0*, *size=None*)

Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsAttractorAnalysis.**geometric**(*p*, *size=None*)

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where p is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

acsAttractorAnalysis.**get_state**()

Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

acsAttractorAnalysis.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)

Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

acsAttractorAnalysis.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

`acsAttractorAnalysis.`**`laplace`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...      np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

acsAttractorAnalysis.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

acsAttractorAnalysis.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)

Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

acsAttractorAnalysis.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

acsAttractorAnalysis.**multinomial**(*n*, *pvals*, *size=None*)

Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

acsAttractorAnalysis.**multivariate_normal**(*mean*, *cov*[, *size*])

Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

- Spherical covariance (*cov* is a multiple of the identity matrix)
- Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

acsAttractorAnalysis.**negative_binomial**(*n*, *p*, *size=None*)

Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N;n,p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...     probability = sum(s<i) / 100000.
...     print i, "wells drilled, probability of one success =", probability
```

acsAttractorAnalysis.**noncentral_chisquare**(*df*, *nonc*, *size=None*)

Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

acsAttractorAnalysis.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)

Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

`acsAttractorAnalysis.`**`normal`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

acsAttractorAnalysis.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has

also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

acsAttractorAnalysis.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

acsAttractorAnalysis.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k; \lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

acsAttractorAnalysis.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')
```

```
>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

acsAttractorAnalysis.**rand**(*d0*, *d1*, ..., *dn*)

Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

acsAttractorAnalysis.**randint**(*low*, *high=None*, *size=None*)

Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

acsAttractorAnalysis.**randn**(*d0*, *d1*, ..., *dn*)

Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

acsAttractorAnalysis.**random**()

random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsAttractorAnalysis.**random_integers**(*low*, *high=None*, *size=None*)

Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

acsAttractorAnalysis.**random_sample**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsAttractorAnalysis.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsAttractorAnalysis.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

acsAttractorAnalysis.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsAttractorAnalysis.**seed**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

acsAttractorAnalysis.**set_state**(*state*)

Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

acsAttractorAnalysis.**shuffle**(*x*)

Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

acsAttractorAnalysis.**standard_cauchy**(*size=None*)

Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

acsAttractorAnalysis.**standard_exponential**(*size=None*)

Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

acsAttractorAnalysis.**standard_gamma**(*shape*, *size=None*)

Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsAttractorAnalysis.**standard_normal**(*size=None*)

Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

acsAttractorAnalysis.**standard_t**(*df*, *size=None*)

Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1+\frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

acsAttractorAnalysis.**triangular**(*left*, *mode*, *right*, *size=None*)

Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

acsAttractorAnalysis.**uniform**(*low=0.0*, *high=1.0*, *size=1*)

Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

acsAttractorAnalysis.**vonmises**(*mu*, *kappa*, *size=None*)

Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode (“center”) of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

acsAttractorAnalysis.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

acsAttractorAnalysis.**weibull**(*a*, *size=None*)

Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1}e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

acsAttractorAnalysis.**zeroBeforeStrNum**(*tmpl*, *tmpL*)

Function to create string zero string vector before graph filename. According to the total number of reactions N zeros will be add before the instant reaction number (e.g. reaction 130 of 10000 the string became '00130') :param tmpl: length (e.g. 1 = 1, 10 = 2, 100 = 3, ...) of the current folder :param tmpL: total length last folder :returns: A zero string numbers to complete the length of tmpL

acsAttractorAnalysis.**zipf**(*a*, *size=None*)

Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

CHAPTER

THREE

ACSATTRACTORANALYSISINTIME MODULE

Function to analyse the different attractors emerging from different simulations in time. python ~/Dropbox/python/GIT/ACS_analysis/initializator.py -t2 -k3 -K-1 -f2 -s6 -m6 -p5 -I ./acsm2s.conf -H1 -v3 -c0.5 -F PROTO_ac3_f2_s6_m6_p5_RAF -N5 -x0 -i 100

acsAttractorAnalysisInTime.**beta**(*a*, *b*, *size=None*)

The Beta distribution over `[0, 1]`.

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

acsAttractorAnalysisInTime.**binomial**(*n*, *p*, *size=None*)

Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

acsAttractorAnalysisInTime.**chisquare**(*df*, *size=None*)

Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\texttt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

NIST/SEMATECH e-Handbook of Statistical Methods

```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

acsAttractorAnalysisInTime.**exponential**(*scale=1.0*, *size=None*)

Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

acsAttractorAnalysisInTime.**f**(*dfnum*, *dfden*, *size=None*)

Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

acsAttractorAnalysisInTime.**gamma**(*shape*, *scale=1.0*, *size=None*)

Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsAttractorAnalysisInTime.**geometric**(*p*, *size=None*)

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where p is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

acsAttractorAnalysisInTime.**get_state**()

Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

acsAttractorAnalysisInTime.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)

Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

acsAttractorAnalysisInTime.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

`acsAttractorAnalysisInTime.`**`laplace`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...      np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

acsAttractorAnalysisInTime.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

acsAttractorAnalysisInTime.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)

Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

acsAttractorAnalysisInTime.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

acsAttractorAnalysisInTime.**multinomial**(*n*, *pvals*, *size=None*)

Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

acsAttractorAnalysisInTime.**multivariate_normal**(*mean*, *cov*[, *size*])

Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

- Spherical covariance (*cov* is a multiple of the identity matrix)
- Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

acsAttractorAnalysisInTime.**negative_binomial**(*n*, *p*, *size=None*)

Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N;n,p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...    probability = sum(s<i) / 100000.
...    print i, "wells drilled, probability of one success =", probability
```

acsAttractorAnalysisInTime.**noncentral_chisquare**(*df*, *nonc*, *size=None*)

Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

acsAttractorAnalysisInTime.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)

Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

acsAttractorAnalysisInTime.**normal**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

acsAttractorAnalysisInTime.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has

also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

acsAttractorAnalysisInTime.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

acsAttractorAnalysisInTime.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k; \lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

acsAttractorAnalysisInTime.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')
```

```
>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

acsAttractorAnalysisInTime.**rand**(*d0*, *d1*, ..., *dn*)

Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

acsAttractorAnalysisInTime.**randint**(*low*, *high=None*, *size=None*)

Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

acsAttractorAnalysisInTime.**randn**(*d0*, *d1*, ..., *dn*)

Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

acsAttractorAnalysisInTime.**random**()

random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsAttractorAnalysisInTime.**random_integers**(*low*, *high=None*, *size=None*)

Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

acsAttractorAnalysisInTime.**random_sample**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsAttractorAnalysisInTime.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsAttractorAnalysisInTime.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

acsAttractorAnalysisInTime.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsAttractorAnalysisInTime.**seed**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

acsAttractorAnalysisInTime.**set_state**(*state*)
Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

acsAttractorAnalysisInTime.**shuffle**(*x*)
Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

acsAttractorAnalysisInTime.**standard_cauchy**(*size=None*)
Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

acsAttractorAnalysisInTime.**standard_exponential**(*size=None*)

Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

acsAttractorAnalysisInTime.**standard_gamma**(*shape*, *size=None*)

Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsAttractorAnalysisInTime.**standard_normal**(*size=None*)

Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

acsAttractorAnalysisInTime.**standard_t**(*df*, *size=None*)

Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1+\frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

acsAttractorAnalysisInTime.**triangular**(*left*, *mode*, *right*, *size=None*)

Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

acsAttractorAnalysisInTime.**uniform**(*low=0.0*, *high=1.0*, *size=1*)

Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

acsAttractorAnalysisInTime.**vonmises**(*mu*, *kappa*, *size=None*)

Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode (“center”) of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

acsAttractorAnalysisInTime.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

acsAttractorAnalysisInTime.**weibull**(*a*, *size=None*)

Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1}e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

acsAttractorAnalysisInTime.**zeroBeforeStrNum**(*tmpl*, *tmpL*)

acsAttractorAnalysisInTime.**zipf**(*a*, *size=None*)

Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

CHAPTER

FOUR

ACSBUFFEREDFLUXES MODULE

Function to evaluate the activity of each species during the simulation, catalyst substrate product or nothing

acsBufferedFluxes.**zeroBeforeStrNum**(*tmpl*, *tmpL*)

CHAPTER

FIVE

ACSDYNSTATINTIME MODULE

Script to order the analysis of the divergences in time.

acsDynStatInTime.**beta**(*a*, *b*, *size=None*)

The Beta distribution over `[0, 1]`.

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

acsDynStatInTime.**binomial**(*n*, *p*, *size=None*)

Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

acsDynStatInTime.**chisquare**(*df*, *size=None*)

Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\texttt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

NIST/SEMATECH e-Handbook of Statistical Methods

```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

acsDynStatInTime.**exponential**(*scale=1.0*, *size=None*)

Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

acsDynStatInTime.**f**(*dfnum*, *dfden*, *size=None*)

Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

acsDynStatInTime.**gamma**(*shape*, *scale=1.0*, *size=None*)

Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsDynStatInTime.**geometric**(*p*, *size=None*)

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where p is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

acsDynStatInTime.**get_state**()

Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

acsDynStatInTime.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)

Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

acsDynStatInTime.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

`acsDynStatInTime.`**`laplace`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...      np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

acsDynStatInTime.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

acsDynStatInTime.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)

Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

acsDynStatInTime.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

acsDynStatInTime.**multinomial**(*n*, *pvals*, *size=None*)

Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

acsDynStatInTime.**multivariate_normal**(*mean*, *cov*[, *size*])

Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

•Spherical covariance (*cov* is a multiple of the identity matrix)

•Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

acsDynStatInTime.**negative_binomial**(*n*, *p*, *size=None*)

Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N; n, p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...     probability = sum(s<i) / 100000.
...     print i, "wells drilled, probability of one success =", probability
```

acsDynStatInTime.**noncentral_chisquare**(*df*, *nonc*, *size=None*)

Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

acsDynStatInTime.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)

Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

acsDynStatInTime.**normal**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

acsDynStatInTime.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has

also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

acsDynStatInTime.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

acsDynStatInTime.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k;\lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

acsDynStatInTime.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')
```

```
>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

acsDynStatInTime.**rand**(*d0*, *d1*, ..., *dn*)

Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

acsDynStatInTime.**randint**(*low*, *high=None*, *size=None*)

Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

acsDynStatInTime.**randn**(*d0*, *d1*, ..., *dn*)

Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

acsDynStatInTime.**random**()

random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsDynStatInTime.**random_integers**(*low*, *high=None*, *size=None*)

Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

acsDynStatInTime.**random_sample**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsDynStatInTime.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsDynStatInTime.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

acsDynStatInTime.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsDynStatInTime.**seed**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

acsDynStatInTime.**set_state**(*state*)
Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

acsDynStatInTime.**shuffle**(*x*)
Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

acsDynStatInTime.**standard_cauchy**(*size=None*)
Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

acsDynStatInTime.**standard_exponential**(*size=None*)

Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

acsDynStatInTime.**standard_gamma**(*shape*, *size=None*)

Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsDynStatInTime.**standard_normal**(*size=None*)

Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

acsDynStatInTime.**standard_t**(*df*, *size=None*)

Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1 + \frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

acsDynStatInTime.**triangular**(*left*, *mode*, *right*, *size=None*)

Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \le x \le m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \le x \le r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

acsDynStatInTime.**uniform**(*low=0.0*, *high=1.0*, *size=1*)

Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

acsDynStatInTime.**vonmises**(*mu*, *kappa*, *size=None*)

Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode (“center”) of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

acsDynStatInTime.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

acsDynStatInTime.**weibull**(*a*, *size=None*)

Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1}e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

acsDynStatInTime.**zipf**(*a*, *size=None*)

Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

CHAPTER

SIX

ACSFROMWIM2CARNESS MODULE

File to convert Wim files in my files.

acsFromWim2Carness.**beta**(*a*, *b*, *size=None*)

The Beta distribution over `[0, 1]`.

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

acsFromWim2Carness.**binomial**(*n*, *p*, *size=None*)

Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

acsFromWim2Carness.**chisquare**(*df*, *size=None*)

Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\texttt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

NIST/SEMATECH e-Handbook of Statistical Methods

```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

acsFromWim2Carness.**exponential**(*scale=1.0*, *size=None*)

Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

acsFromWim2Carness.**f**(*dfnum*, *dfden*, *size=None*)

Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

acsFromWim2Carness.**gamma**(*shape*, *scale=1.0*, *size=None*)

Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsFromWim2Carness.**geometric**(*p*, *size=None*)

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where p is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

acsFromWim2Carness.**get_state**()

Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

acsFromWim2Carness.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)

Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

acsFromWim2Carness.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

`acsFromWim2Carness.`**`laplace`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...      np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

acsFromWim2Carness.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

acsFromWim2Carness.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)

Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

acsFromWim2Carness.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

acsFromWim2Carness.**multinomial**(*n*, *pvals*, *size=None*)

Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

acsFromWim2Carness.**multivariate_normal**(*mean*, *cov*[, *size*])

Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

•Spherical covariance (*cov* is a multiple of the identity matrix)

•Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

acsFromWim2Carness.**negative_binomial**(*n*, *p*, *size=None*)

Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N;n,p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...    probability = sum(s<i) / 100000.
...    print i, "wells drilled, probability of one success =", probability
```

acsFromWim2Carness.**noncentral_chisquare**(*df*, *nonc*, *size=None*)

Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

acsFromWim2Carness.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)

Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

acsFromWim2Carness.**normal**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

acsFromWim2Carness.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has

also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

acsFromWim2Carness.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

acsFromWim2Carness.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k;\lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

acsFromWim2Carness.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')
```

```
>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

acsFromWim2Carness.**rand**(*d0*, *d1*, ..., *dn*)

Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

acsFromWim2Carness.**randint**(*low*, *high=None*, *size=None*)

Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

acsFromWim2Carness.**randn**(*d0*, *d1*, ..., *dn*)

Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

acsFromWim2Carness.**random**()

random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsFromWim2Carness.**random_integers**(*low*, *high=None*, *size=None*)

Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

acsFromWim2Carness.**random_sample**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsFromWim2Carness.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsFromWim2Carness.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

acsFromWim2Carness.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsFromWim2Carness.**seed**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

acsFromWim2Carness.**set_state**(*state*)

Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

acsFromWim2Carness.**shuffle**(*x*)

Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

acsFromWim2Carness.**standard_cauchy**(*size=None*)

Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

acsFromWim2Carness.**standard_exponential**(*size=None*)

Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

acsFromWim2Carness.**standard_gamma**(*shape*, *size=None*)

Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsFromWim2Carness.**standard_normal**(*size=None*)

Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

acsFromWim2Carness.**standard_t**(*df*, *size=None*)

Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1+\frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

acsFromWim2Carness.**triangular**(*left*, *mode*, *right*, *size=None*)

Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

acsFromWim2Carness.**uniform**(*low=0.0*, *high=1.0*, *size=1*)

Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

acsFromWim2Carness.**vonmises**(*mu*, *kappa*, *size=None*)

Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode (“center”) of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

acsFromWim2Carness.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

acsFromWim2Carness.**weibull**(*a*, *size=None*)

Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1}e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

acsFromWim2Carness.**zipf**(*a*, *size=None*)

Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

CHAPTER

SEVEN

ACSSCCANALYSIS MODULE

script to analyze the emerging strongly connected components.

acsSCCanalysis.**beta**(*a*, *b*, *size=None*)

The Beta distribution over `[0, 1]`.

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

acsSCCanalysis.**binomial**(*n*, *p*, *size=None*)

Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

acsSCCanalysis.**chisquare**(*df*, *size=None*)

Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\texttt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

NIST/SEMATECH e-Handbook of Statistical Methods

```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

acsSCCanalysis.**exponential**(*scale=1.0*, *size=None*)

Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

acsSCCanalysis.**f**(*dfnum*, *dfden*, *size=None*)

Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

acsSCCanalysis.**gamma**(*shape*, *scale=1.0*, *size=None*)

Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsSCCanalysis.**geometric**(*p*, *size=None*)

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where p is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

acsSCCanalysis.**get_state**()

Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

acsSCCanalysis.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)

Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

acsSCCanalysis.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

acsSCCanalysis.**laplace**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...       np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

acsSCCanalysis.**loadReactionGraph**()

acsSCCanalysis.**loadSpecificReactionGraph**()

acsSCCanalysis.**loadSpecificReactionSubGraph**()

acsSCCanalysis.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

acsSCCanalysis.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)

Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

acsSCCanalysis.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

acsSCCanalysis.**multinomial**(*n*, *pvals*, *size=None*)

Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

acsSCCanalysis.**multivariate_normal**(*mean*, *cov*[, *size*])

Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

•Spherical covariance (*cov* is a multiple of the identity matrix)

•Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

acsSCCanalysis.**negative_binomial**(*n*, *p*, *size=None*)

Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N; n, p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...     probability = sum(s<i) / 100000.
...     print i, "wells drilled, probability of one success =", probability
```

acsSCCanalysis.**noncentral_chisquare**(*df*, *nonc*, *size=None*)

Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

acsSCCanalysis.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)

Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

acsSCCanalysis.**normal**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

acsSCCanalysis.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has

also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

acsSCCanalysis.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

acsSCCanalysis.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k;\lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

acsSCCanalysis.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')
```

```
>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

acsSCCanalysis.**rand**(*d0*, *d1*, ..., *dn*)

Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

acsSCCanalysis.**randint**(*low*, *high=None*, *size=None*)

Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

acsSCCanalysis.**randn**(*d0*, *d1*, ..., *dn*)

Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

acsSCCanalysis.**random**()

random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsSCCanalysis.**random_integers**(*low*, *high=None*, *size=None*)

Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

acsSCCanalysis.**random_sample**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsSCCanalysis.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsSCCanalysis.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

acsSCCanalysis.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsSCCanalysis.**saveGillToFile**()

acsSCCanalysis.**saveGraphSUBToFile**()

acsSCCanalysis.**saveGraphToFile**()

acsSCCanalysis.**saveNrgToFile**()

acsSCCanalysis.**seed**(*seed=None*)

Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

acsSCCanalysis.**set_state**(*state*)

Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

acsSCCanalysis.**shuffle**(*x*)

Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

acsSCCanalysis.**standard_cauchy**(*size=None*)
Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

acsSCCanalysis.**standard_exponential**(*size=None*)
Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

acsSCCanalysis.**standard_gamma**(*shape*, *size=None*)
Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsSCCanalysis.**standard_normal**(*size=None*)

Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

acsSCCanalysis.**standard_t**(*df*, *size=None*)

Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1+\frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

acsSCCanalysis.**triangular**(*left*, *mode*, *right*, *size=None*)

Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

acsSCCanalysis.**uniform**(*low=0.0*, *high=1.0*, *size=1*)

Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

acsSCCanalysis.**vonmises**(*mu*, *kappa*, *size=None*)

Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode ("center") of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

acsSCCanalysis.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

acsSCCanalysis.**weibull**(*a*, *size=None*)

Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1} e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

acsSCCanalysis.**zeroBeforeStrNum**(*tmpl*, *tmpL*)

acsSCCanalysis.**zipf**(*a*, *size=None*)

Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

CHAPTER

EIGHT

ACSSPECIESACTIVITIES MODULE

Function to evaluate the activity of each species during the simulation, catalyst substrate product or nothing. Moreover the script recognize all those molecules functioning as hub

`acsSpeciesActivities.`**`beta`**(*a*, *b*, *size=None*)

The Beta distribution over `[0, 1]`.

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

`acsSpeciesActivities.`**`binomial`**(*n*, *p*, *size=None*)

Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

acsSpeciesActivities.**chisquare**(*df*, *size=None*)

Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\mathtt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

NIST/SEMATECH e-Handbook of Statistical Methods

```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

acsSpeciesActivities.**exponential**(*scale=1.0*, *size=None*)

Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

acsSpeciesActivities.**f**(*dfnum*, *dfden*, *size=None*)

Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

acsSpeciesActivities.**gamma**(*shape*, *scale=1.0*, *size=None*)

Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsSpeciesActivities.**geometric**(*p*, *size=None*)

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where p is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

acsSpeciesActivities.**get_state**()

Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

acsSpeciesActivities.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)

Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

acsSpeciesActivities.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

`acsSpeciesActivities.`**`laplace`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...      np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

acsSpeciesActivities.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

acsSpeciesActivities.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)

Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

acsSpeciesActivities.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

acsSpeciesActivities.**multinomial**(*n*, *pvals*, *size=None*)

Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

acsSpeciesActivities.**multivariate_normal**(*mean*, *cov*[, *size*])

Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

•Spherical covariance (*cov* is a multiple of the identity matrix)

•Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

acsSpeciesActivities.**negative_binomial**(*n*, *p*, *size=None*)

Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N;n,p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...    probability = sum(s<i) / 100000.
...    print i, "wells drilled, probability of one success =", probability
```

acsSpeciesActivities.**noncentral_chisquare**(*df*, *nonc*, *size=None*)

Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

acsSpeciesActivities.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)

Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

acsSpeciesActivities.**normal**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

acsSpeciesActivities.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has

also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

acsSpeciesActivities.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

acsSpeciesActivities.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k; \lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

acsSpeciesActivities.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')
```

```
>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

acsSpeciesActivities.**rand**(*d0*, *d1*, ..., *dn*)

Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

acsSpeciesActivities.**randint**(*low*, *high=None*, *size=None*)

Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

acsSpeciesActivities.**randn**(*d0*, *d1*, ..., *dn*)

Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

acsSpeciesActivities.**random**()

random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsSpeciesActivities.**random_integers**(*low*, *high=None*, *size=None*)

Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

acsSpeciesActivities.**random_sample**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsSpeciesActivities.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsSpeciesActivities.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

acsSpeciesActivities.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsSpeciesActivities.**seed**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

acsSpeciesActivities.**set_state**(*state*)

Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

acsSpeciesActivities.**shuffle**(*x*)

Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

acsSpeciesActivities.**standard_cauchy**(*size=None*)

Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

acsSpeciesActivities.**standard_exponential**(*size=None*)

Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

acsSpeciesActivities.**standard_gamma**(*shape*, *size=None*)

Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsSpeciesActivities.**standard_normal**(*size=None*)

Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

acsSpeciesActivities.**standard_t**(*df*, *size=None*)

Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1+\frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

acsSpeciesActivities.**triangular**(*left*, *mode*, *right*, *size=None*)

Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

acsSpeciesActivities.**uniform**(*low=0.0*, *high=1.0*, *size=1*)

Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

acsSpeciesActivities.**vonmises**(*mu*, *kappa*, *size=None*)

Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode (“center”) of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

acsSpeciesActivities.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

acsSpeciesActivities.**weibull**(*a*, *size=None*)

Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1}e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

acsSpeciesActivities.**zeroBeforeStrNum**(*tmpl*, *tmpL*)

acsSpeciesActivities.**zipf**(*a*, *size=None*)

Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

CHAPTER

NINE

ACSSTATESANALYSIS MODULE

Script to compute the distance between different state of the same simulation. comparison between t0 and t, t-1 and t adopting three different distance misure: angle, euclidian distance and hamming distance. Moreover the script make an analysis of all the aggregative variables. https://help.github.com/articles/fork-a-repo

acsStatesAnalysis.**beta**(*a*, *b*, *size=None*)

The Beta distribution over `[0, 1]`.

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

acsStatesAnalysis.**binomial**(*n*, *p*, *size=None*)

Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

acsStatesAnalysis.**chisquare**(*df*, *size=None*)

Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\texttt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

NIST/SEMATECH e-Handbook of Statistical Methods

```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

acsStatesAnalysis.**distanceMisures**(*tmpSeqX*, *tmpConcX*, *tmpSeqY*, *tmpConcY*, *tmpIDs*)
Function to compute the angle between two multidimensional vectors

acsStatesAnalysis.**exponential**(*scale=1.0*, *size=None*)
Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

acsStatesAnalysis.**f**(*dfnum*, *dfden*, *size=None*)
Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

acsStatesAnalysis.**gamma**(*shape*, *scale=1.0*, *size=None*)

Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsStatesAnalysis.**geometric**(*p*, *size=None*)

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where p is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

acsStatesAnalysis.**get_state**()

Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

acsStatesAnalysis.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)

Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

acsStatesAnalysis.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

`acsStatesAnalysis.`**`laplace`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...      np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

acsStatesAnalysis.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

acsStatesAnalysis.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)

Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

acsStatesAnalysis.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

acsStatesAnalysis.**multinomial**(*n*, *pvals*, *size=None*)

Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

acsStatesAnalysis.**multivariate_normal**(*mean*, *cov*[, *size*])

Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

- Spherical covariance (*cov* is a multiple of the identity matrix)
- Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

acsStatesAnalysis.**negative_binomial**(*n*, *p*, *size=None*)

Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N; n, p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...    probability = sum(s<i) / 100000.
...    print i, "wells drilled, probability of one success =", probability
```

acsStatesAnalysis.**noncentral_chisquare**(*df*, *nonc*, *size=None*)

Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

acsStatesAnalysis.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)

Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

`acsStatesAnalysis.`**`normal`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

acsStatesAnalysis.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has

also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

acsStatesAnalysis.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

acsStatesAnalysis.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k;\lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

acsStatesAnalysis.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')
```

```
>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

acsStatesAnalysis.**rand**(*d0*, *d1*, ..., *dn*)

Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

acsStatesAnalysis.**randint**(*low*, *high=None*, *size=None*)

Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

acsStatesAnalysis.**randn**(*d0*, *d1*, ..., *dn*)

Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

acsStatesAnalysis.**random**()

random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsStatesAnalysis.**random_integers**(*low*, *high=None*, *size=None*)

Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

acsStatesAnalysis.**random_sample**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsStatesAnalysis.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsStatesAnalysis.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

acsStatesAnalysis.**returnZeroSpeciesList**(*tmpLastSpeciesFile*)
Function to create a zero vector for each species (NO COMPLEXES)

acsStatesAnalysis.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

acsStatesAnalysis.**seed**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

acsStatesAnalysis.**set_state**(*state*)
Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

acsStatesAnalysis.**shuffle**(*x*)
Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

acsStatesAnalysis.**standard_cauchy**(*size=None*)
Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

acsStatesAnalysis.**standard_exponential**(*size=None*)

Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

acsStatesAnalysis.**standard_gamma**(*shape*, *size=None*)

Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

acsStatesAnalysis.**standard_normal**(*size=None*)

Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

acsStatesAnalysis.**standard_t**(*df*, *size=None*)

Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1+\frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

acsStatesAnalysis.**triangular**(*left*, *mode*, *right*, *size=None*)

Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

acsStatesAnalysis.**uniform**(*low=0.0*, *high=1.0*, *size=1*)

Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

acsStatesAnalysis.**vonmises**(*mu*, *kappa*, *size=None*)

Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode (“center”) of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

acsStatesAnalysis.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

acsStatesAnalysis.**weibull**(*a*, *size=None*)

Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1}e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

acsStatesAnalysis.**zeroBeforeStrNum**(*tmpl*, *tmpL*)

Function to create string zero string vector before graph filename. According to the total number of reactions N zeros will be add before the instant reaction number (e.g. reaction 130 of 10000 the string became '00130')

acsStatesAnalysis.**zipf**(*a*, *size=None*)

Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

CHAPTER

TEN

INITIALIZATOR MODULE

Script to initialize random catalytic nets python <path>/GIT/ACS_analysis/initializator.py -t2 -a0 -o ~/Documents/lavoro/protocell/init/

> -k3 -d2 -K10 -f2 -n2 -s6 -m6 -p7 -I ~/Documents/lavoro/protocell/init/acsm2s.conf -N600 -B600 -x1 -O0 -H10 -v2.5 -c0.5 -F TEST -i 1 -S2 -u -P2 -S2 -A0.1

2015 - experiments on synchronization, init command python <path>/initializator.py t2 -H20 -K20 -u -v1.0 -P2 -S2 -F <folder_name> -A0.01 and python <path>/initializator.py t2 -H20 -K20 -u -v2.5 -P2 -S2 -F <folder_name> -A0.01

CHAPTER

ELEVEN

LIB PACKAGE

11.1 Subpackages

11.1.1 IO Package

IO Package

readfiles Module

lib.IO.readfiles.**beta**(*a*, *b*, *size=None*)
The Beta distribution over [0, 1].

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

lib.IO.readfiles.**binomial**(*n*, *p*, *size=None*)
Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., (m, n, k), then m * n * k samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

lib.IO.readfiles.**chisquare**(*df*, *size=None*)

Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\texttt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

NIST/SEMATECH e-Handbook of Statistical Methods

```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

lib.IO.readfiles.**exponential**(*scale=1.0*, *size=None*)
Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

lib.IO.readfiles.**f**(*dfnum*, *dfden*, *size=None*)
Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

lib.IO.readfiles.**gamma**(*shape*, *scale=1.0*, *size=None*)
Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

lib.IO.readfiles.**geometric**(*p*, *size=None*)
Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where *p* is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

lib.IO.readfiles.**get_state**()
Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

lib.IO.readfiles.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)
Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

lib.IO.readfiles.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)
Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

lib.IO.readfiles.**laplace**(*loc=0.0*, *scale=1.0*, *size=None*)
Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x-\mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...       np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

lib.IO.readfiles.**loadAllData**(*tmpPath*, *tmpFname*)

lib.IO.readfiles.**loadRandomSeed**(*tmpRndPath*)
Function to load a previously saved random seed

lib.IO.readfiles.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)
Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

lib.IO.readfiles.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)

Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))
```

```
>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

lib.IO.readfiles.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

lib.IO.readfiles.**multinomial**(*n*, *pvals*, *size=None*)

Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

lib.IO.readfiles.**multivariate_normal**(*mean*, *cov*[, *size*])

Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

•Spherical covariance (*cov* is a multiple of the identity matrix)

•Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

lib.IO.readfiles.**negative_binomial**(*n*, *p*, *size=None*)
Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N;n,p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...    probability = sum(s<i) / 100000.
...    print i, "wells drilled, probability of one success =", probability
```

lib.IO.readfiles.**noncentral_chisquare**(*df*, *nonc*, *size=None*)

Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

lib.IO.readfiles.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)
Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

lib.IO.readfiles.**normal**(*loc=0.0*, *scale=1.0*, *size=None*)
Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

lib.IO.readfiles.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

lib.IO.readfiles.**permutation**(*x*)
Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

lib.IO.readfiles.**poisson**(*lam=1.0*, *size=None*)
Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k;\lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

lib.IO.readfiles.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)
```

```
>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

lib.IO.readfiles.**rand**(*d0*, *d1*, ..., *dn*)
Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

lib.IO.readfiles.**randint**(*low*, *high=None*, *size=None*)
Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

lib.IO.readfiles.**randn**(*d0*, *d1*, ..., *dn*)

Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

lib.IO.readfiles.**random**()

random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.IO.readfiles.**random_integers**(*low*, *high=None*, *size=None*)

Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

lib.IO.readfiles.**random_sample**(*size=None*)
Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.IO.readfiles.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.IO.readfiles.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

lib.IO.readfiles.**readBufferedID**(*tmpPath*)

lib.IO.readfiles.**readCSTRflux**(*tmpPath*)

lib.IO.readfiles.**readConfFile**(*tmpPath*)

lib.IO.readfiles.**readInitConfFile**(*tmpPath*)

lib.IO.readfiles.**read_sims_conf_file**(*paramFile='acsm2s.conf'*)

lib.IO.readfiles.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.IO.readfiles.**seed**(*seed=None*)

Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

lib.IO.readfiles.**set_state**(*state*)

Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

lib.IO.readfiles.**shuffle**(*x*)

Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

lib.IO.readfiles.**splitRctParsLine**(*tmpLine*)

lib.IO.readfiles.**standard_cauchy**(*size=None*)
Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

lib.IO.readfiles.**standard_exponential**(*size=None*)
Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

lib.IO.readfiles.**standard_gamma**(*shape*, *size=None*)
Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

lib.IO.readfiles.**standard_normal**(*size=None*)
Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

lib.IO.readfiles.**standard_t**(*df*, *size=None*)
Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1 + \frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

lib.IO.readfiles.**triangular**(*left*, *mode*, *right*, *size=None*)
Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

lib.IO.readfiles.**uniform**(*low=0.0*, *high=1.0*, *size=1*)

Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b-a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

lib.IO.readfiles.**vonmises**(*mu*, *kappa*, *size=None*)

Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode ("center") of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

lib.IO.readfiles.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

lib.IO.readfiles.**weibull**(*a*, *size=None*)
Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1} e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

lib.IO.readfiles.**zeroBeforeStrNum**(*tmpl*, *tmpL*)

lib.IO.readfiles.**zipf**(*a*, *size=None*)
Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
```

```
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

writefiles Module

lib.IO.writefiles.**beta**(*a*, *b*, *size=None*)
The Beta distribution over [0, 1].

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

lib.IO.writefiles.**binomial**(*n*, *p*, *size=None*)
Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., (m, n, k), then m * n * k samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

lib.IO.writefiles.**chisquare**(*df*, *size=None*)
Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\mathtt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

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```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

lib.IO.writefiles.**exponential**(*scale=1.0*, *size=None*)
Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

lib.IO.writefiles.**f**(*dfnum*, *dfden*, *size=None*)
Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

lib.IO.writefiles.**gamma**(*shape*, *scale=1.0*, *size=None*)
Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

lib.IO.writefiles.**geometric**(*p*, *size=None*)

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where p is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

lib.IO.writefiles.**get_state**()
Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

lib.IO.writefiles.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)
Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

lib.IO.writefiles.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

lib.IO.writefiles.**laplace**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...       np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

lib.IO.writefiles.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)
Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

lib.IO.writefiles.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)
Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))
```

```
>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

lib.IO.writefiles.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

lib.IO.writefiles.**multinomial**(*n*, *pvals*, *size=None*)

Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

lib.IO.writefiles.**multivariate_normal**(*mean*, *cov*[, *size*])
Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

- Spherical covariance (*cov* is a multiple of the identity matrix)
- Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis
```

```
>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

lib.IO.writefiles.**negative_binomial**(*n*, *p*, *size=None*)
Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N;n,p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...     probability = sum(s<i) / 100000.
...     print i, "wells drilled, probability of one success =", probability
```

lib.IO.writefiles.**noncentral_chisquare**(*df*, *nonc*, *size=None*)
Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

lib.IO.writefiles.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)
Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

lib.IO.writefiles.**normal**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True
```

```
>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

lib.IO.writefiles.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

lib.IO.writefiles.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

lib.IO.writefiles.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k; \lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

lib.IO.writefiles.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

`lib.IO.writefiles.`**`rand`**(*d0*, *d1*, ..., *dn*)
Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

`lib.IO.writefiles.`**`randint`**(*low*, *high=None*, *size=None*)
Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

`lib.IO.writefiles.`**`randn`**(*d0*, *d1*, ..., *dn*)
Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

lib.IO.writefiles.**random**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.IO.writefiles.**random_integers**(*low*, *high=None*, *size=None*)
Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

lib.IO.writefiles.**random_sample**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.IO.writefiles.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.IO.writefiles.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

lib.IO.writefiles.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.IO.writefiles.**saveRandomSeed**(*tmpPath*)
Function to save the random seed

lib.IO.writefiles.**seed**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

lib.IO.writefiles.**set_state**(*state*)
Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.

2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

`lib.IO.writefiles.`**`shuffle`**(*x*)
Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

`lib.IO.writefiles.`**`standard_cauchy`**(*size=None*)
Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + \left(\frac{x-x_0}{\gamma}\right)^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

lib.IO.writefiles.**standard_exponential**(*size=None*)
Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

lib.IO.writefiles.**standard_gamma**(*shape*, *size=None*)
Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

lib.IO.writefiles.**standard_normal**(*size=None*)
Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                                #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

lib.IO.writefiles.**standard_t**(*df*, *size=None*)
Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1+\frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

lib.IO.writefiles.**triangular**(*left*, *mode*, *right*, *size=None*)
Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

lib.IO.writefiles.**uniform**(*low=0.0*, *high=1.0*, *size=1*)
Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

lib.IO.writefiles.**vonmises**(*mu*, *kappa*, *size=None*)
Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode ("center") of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

lib.IO.writefiles.**wald**(*mean*, *scale*, *size=None*)
Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

lib.IO.writefiles.**weibull**(*a*, *size=None*)
Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1}e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

lib.IO.writefiles.**writeAllFilesAndCreateResFolder**(*pathFile*, *resFolderName*, *cats*, *rcts*, *food*, *spontRatio=None*, *kspontass=None*, *kspontdiss=None*, *conf=False*)

lib.IO.writefiles.**write_acsCatalysis_file**(*path_file*, *catStr*)

lib.IO.writefiles.**write_acsReactions_file**(*path_file*, *rctStr*, *spontRatio=None*, *kspontass=None*, *kspontdiss=None*)

lib.IO.writefiles.**write_acsms_file**(*path_file*, *nGen=10*, *nSim=1*, *nSec=1000*, *nRct=200000000*, *nH=0*, *nA=0*, *rs=0*, *dl=0*, *tssi=10*, *ftsi=0*, *nspmt=1*, *lfds=13*, *oc=0.0001*, *ecc=0*, *alf='AB'*, *v=1e-18*, *vg=0*, *sd=0*, *nrg=0*, *rse=0*, *ncml=2*, *P=0.00103306*, *cp=0.5*, *mrevrct=0*, *rr=0*, *rrr=0*, *sr=0*, *K_ass=50*, *K_diss=25*, *K_cpx=50*, *K_cpxDiss=1*, *K_nrg=0*, *K_nrg_decay=0*, *K_spont_ass=0*, *K_spont_diss=0*, *moleculeDecay_KineticConstant=0.02*, *diffusion_contribute=0*, *solubility_threshold=0*, *influx_rate=0*, *maxLOut=3*, *fileAmountSaveInterval=10*, *saveRtcInfo=1*, *randInitSpeciesConc=0*, *tmpTheta=0*)

lib.IO.writefiles.**write_and_createInfluxFile**(*path_file*, *tmpFood*)

lib.IO.writefiles.**write_and_create_std_nrgFile**(*path_file*)

lib.IO.writefiles.**write_init_raf_all**(*fid*, *rafinfo*, *folder*, *rcts*, *cats*)

lib.IO.writefiles.**write_init_raf_list**(*fid*, *rafinfo*, *folder*)

lib.IO.writefiles.**zipf**(*a*, *size=None*)

Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
```

```
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

11.1.2 dyn Package

dynamics Module

lib.dyn.dynamics.**beta**(*a*, *b*, *size=None*)

The Beta distribution over `[0, 1]`.

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

lib.dyn.dynamics.**binomial**(*n*, *p*, *size=None*)

Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number

of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

lib.dyn.dynamics.**chisquare**(*df*, *size=None*)

Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\mathtt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

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```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

lib.dyn.dynamics.**exponential**(*scale=1.0*, *size=None*)
Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

lib.dyn.dynamics.**f**(*dfnum*, *dfden*, *size=None*)
Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

lib.dyn.dynamics.**fluxAnalysis**(*tmpDir*, *resDirPath*, *strZeros*, *ngen*)

lib.dyn.dynamics.**gamma**(*shape*, *scale=1.0*, *size=None*)
Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

lib.dyn.dynamics.**generateFluxList**(*tmpPath*, *tmpSysType*, *tmpLastID=None*)

lib.dyn.dynamics.**geometric**(*p*, *size=None*)
Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where *p* is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

lib.dyn.dynamics.**get_state**()
Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

lib.dyn.dynamics.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)
Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

lib.dyn.dynamics.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)
Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

`lib.dyn.dynamics.`**`laplace`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...      np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

lib.dyn.dynamics.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)
Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

lib.dyn.dynamics.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)

Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

lib.dyn.dynamics.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

lib.dyn.dynamics.**multinomial**(*n*, *pvals*, *size=None*)
Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

lib.dyn.dynamics.**multivariate_normal**(*mean*, *cov*[, *size*])
Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

•Spherical covariance (*cov* is a multiple of the identity matrix)

•Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

lib.dyn.dynamics.**negative_binomial**(*n*, *p*, *size=None*)

Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N; n, p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...    probability = sum(s<i) / 100000.
...    print i, "wells drilled, probability of one success =", probability
```

lib.dyn.dynamics.**noncentral_chisquare**(*df*, *nonc*, *size=None*)
Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

lib.dyn.dynamics.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)
Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

`lib.dyn.dynamics.`**`normal`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

lib.dyn.dynamics.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has

also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

lib.dyn.dynamics.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

lib.dyn.dynamics.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k;\lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

lib.dyn.dynamics.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')
```

```
>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

lib.dyn.dynamics.**rand**(*d0*, *d1*, ..., *dn*)
Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

lib.dyn.dynamics.**randint**(*low*, *high=None*, *size=None*)
Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

lib.dyn.dynamics.**randn**(*d0*, *d1*, ..., *dn*)

Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

lib.dyn.dynamics.**random**()

random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.dyn.dynamics.**random_integers**(*low*, *high=None*, *size=None*)

Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

lib.dyn.dynamics.**random_sample**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.dyn.dynamics.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.dyn.dynamics.**rangeFloat**(*start*, *step*, *stop*)

lib.dyn.dynamics.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

lib.dyn.dynamics.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.dyn.dynamics.**seed**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

lib.dyn.dynamics.**set_state**(*state*)
Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

lib.dyn.dynamics.**shuffle**(*x*)
Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

lib.dyn.dynamics.**standard_cauchy**(*size=None*)
Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

lib.dyn.dynamics.**standard_exponential**(*size=None*)
Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

lib.dyn.dynamics.**standard_gamma**(*shape*, *size=None*)
Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

lib.dyn.dynamics.**standard_normal**(*size=None*)

Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

lib.dyn.dynamics.**standard_t**(*df*, *size=None*)

Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1+\frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

`lib.dyn.dynamics.`**`triangular`**(*left*, *mode*, *right*, *size=None*)
Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

lib.dyn.dynamics.**uniform**(*low=0.0*, *high=1.0*, *size=1*)
Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

lib.dyn.dynamics.**vonmises**(*mu*, *kappa*, *size=None*)
Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode ("center") of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

lib.dyn.dynamics.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

lib.dyn.dynamics.**weibull**(*a*, *size=None*)
Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1} e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

lib.dyn.dynamics.**zipf**(*a*, *size=None*)
Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

11.1.3 graph Package

network Module

lib.graph.network.**beta**(*a*, *b*, *size=None*)
The Beta distribution over `[0, 1]`.

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

lib.graph.network.**binomial**(*n*, *p*, *size=None*)
Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

lib.graph.network.**chisquare**(*df*, *size=None*)
Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\mathtt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

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```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

lib.graph.network.**create_chemistry**(*args*, *originalSpeciesList*, *parameters*, *rctToCat*, *totCleavage*, *totCond*, *tmpac*, *autocat=True*)

lib.graph.network.**exponential**(*scale=1.0*, *size=None*)
Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

lib.graph.network.**f**(*dfnum*, *dfden*, *size=None*)
Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

lib.graph.network.**fixCondensationReaction**(*m1*, *m2*, *m3*, *rcts*)

lib.graph.network.**gamma**(*shape*, *scale=1.0*, *size=None*)
Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

lib.graph.network.**geometric**(*p*, *size=None*)
Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where *p* is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

lib.graph.network.**get_state**()
Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

`lib.graph.network.`**`gumbel`**(*loc=0.0*, *scale=1.0*, *size=None*)
Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

lib.graph.network.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

lib.graph.network.**laplace**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...      np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

lib.graph.network.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

lib.graph.network.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)

Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x \sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

lib.graph.network.**logseries**(*p*, *size=None*)
Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

lib.graph.network.**multinomial**(*n*, *pvals*, *size=None*)
Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

lib.graph.network.**multivariate_normal**(*mean*, *cov*[, *size*])

Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

- Spherical covariance (*cov* is a multiple of the identity matrix)
- Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

lib.graph.network.**negative_binomial**(*n*, *p*, *size=None*)

Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N;n,p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...     probability = sum(s<i) / 100000.
...     print i, "wells drilled, probability of one success =", probability
```

lib.graph.network.**net_analysis_of_dynamic_graphs**(*fid_dynRafRes*, *tmpTime*, *rcts*, *cats*, *foodList*, *growth=False*, *rctsALL=None*, *catsALL=None*, *completeRCTS=None*, *debug=False*)

lib.graph.network.**net_analysis_of_static_graphs**(*fid_initRafRes*, *fid_initRafResALL*, *fid_initRafResLIST*, *tmpDir*, *rctProb*, *avgCon*, *rcts*, *cats*, *foodList*, *maxDim*, *debug=False*)

lib.graph.network.**noncentral_chisquare**(*df*, *nonc*, *size=None*)
Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

lib.graph.network.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)
Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

lib.graph.network.**normal**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

lib.graph.network.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

lib.graph.network.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

lib.graph.network.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k;\lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

lib.graph.network.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

lib.graph.network.**rand**(*d0*, *d1*, ..., *dn*)
Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

lib.graph.network.**randint**(*low*, *high=None*, *size=None*)
Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

lib.graph.network.**randn**(*d0*, *d1*, ..., *dn*)
Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

lib.graph.network.**random**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.graph.network.**random_integers**(*low*, *high=None*, *size=None*)
Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

`lib.graph.network.`**`random_sample`**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.graph.network.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.graph.network.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

lib.graph.network.**removeRareRcts**(*graph*, *dt*, *life*, *nrg*, *deltat*)

lib.graph.network.**return_scc_in_raf**(*tmpRAF*, *tmpClosure*, *tmpCats*)

lib.graph.network.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.graph.network.**seed**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

lib.graph.network.**set_state**(*state*)
Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

lib.graph.network.**shuffle**(*x*)
Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

lib.graph.network.**standard_cauchy**(*size=None*)
Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

lib.graph.network.**standard_exponential**(*size=None*)
Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

lib.graph.network.**standard_gamma**(*shape*, *size=None*)
Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                        (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

lib.graph.network.**standard_normal**(*size=None*)

Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

lib.graph.network.**standard_t**(*df*, *size=None*)

Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1 + \frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

lib.graph.network.**triangular**(*left*, *mode*, *right*, *size=None*)
Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

lib.graph.network.**uniform**(*low=0.0*, *high=1.0*, *size=1*)
Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

`lib.graph.network.`**`vonmises`**(*mu*, *kappa*, *size=None*)
Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode ("center") of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

lib.graph.network.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

lib.graph.network.**weibull**(*a*, *size=None*)
Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1}e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

lib.graph.network.**zipf**(*a*, *size=None*)
Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

raf Module

lib.graph.raf.**Fcondition**(*tmpCL*, *tmpRA*, *rcts*, *debug=False*)

lib.graph.raf.**RAcondition**(*tmpCL*, *rcts*, *cats*, *debug=False*)

lib.graph.raf.**beta**(*a*, *b*, *size=None*)
The Beta distribution over [0, 1].

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

lib.graph.raf.**binomial**(*n*, *p*, *size=None*)
Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

lib.graph.raf.**chisquare**(*df*, *size=None*)
Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\texttt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

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```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

lib.graph.raf.**exponential**(*scale=1.0*, *size=None*)
Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

lib.graph.raf.**f**(*dfnum*, *dfden*, *size=None*)
Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

`lib.graph.raf.`**`findCatforRAF`**(*tmpCat*, *tmpRAF*, *tmpClosure*, *debug=False*)

`lib.graph.raf.`**`findRAFrcts`**(*RAF*, *rcts*, *actrcts*)

`lib.graph.raf.`**`gamma`**(*shape*, *scale=1.0*, *size=None*)
Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

lib.graph.raf.**generateClosure**(*tmpF*, *rcts*, *debug=False*)

lib.graph.raf.**geometric**(*p*, *size=None*)

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where p is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

lib.graph.raf.**get_state**()

Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

lib.graph.raf.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)
Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

lib.graph.raf.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

lib.graph.raf.**laplace**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...      np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

lib.graph.raf.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

lib.graph.raf.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)
Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

lib.graph.raf.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

`lib.graph.raf.`**`multinomial`**(*n*, *pvals*, *size=None*)
Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

lib.graph.raf.**multivariate_normal**(*mean*, *cov*[, *size*])
Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

- Spherical covariance (*cov* is a multiple of the identity matrix)
- Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

lib.graph.raf.**negative_binomial**(*n*, *p*, *size=None*)
Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N;n,p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a “1” appears, then the probability distribution of the number of non-“1”s that appear before the third “1” is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...     probability = sum(s<i) / 100000.
...     print i, "wells drilled, probability of one success =", probability
```

lib.graph.raf.**noncentral_chisquare**(*df*, *nonc*, *size=None*)
Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

lib.graph.raf.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)
Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

lib.graph.raf.**normal**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

lib.graph.raf.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

lib.graph.raf.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

lib.graph.raf.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k; \lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

lib.graph.raf.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x;a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

lib.graph.raf.**rafComputation**(*fid_initRafRes*, *fid_initRafResALL*, *fid_initRafResLIST*, *tmpDir*, *rctProb*, *avgCon*, *rcts*, *cats*, *foodList*, *maxDim*, *debug=False*)

lib.graph.raf.**rafDynamicComputation**(*fid_dynRafRes*, *tmpTime*, *rcts*, *cats*, *foodList*, *growth=False*, *rctsALL=None*, *catsALL=None*, *completeRCTS=None*, *debug=False*)

lib.graph.raf.**rafsearch**(*rcts*, *cats*, *closure*, *debug=False*)

lib.graph.raf.**rand**(*d0*, *d1*, ..., *dn*)
Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

`lib.graph.raf.`**`randint`**(*low*, *high=None*, *size=None*)

Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

`lib.graph.raf.`**`randn`**(*d0*, *d1*, ..., *dn*)

Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

lib.graph.raf.**random**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.graph.raf.**random_integers**(*low*, *high=None*, *size=None*)
Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

`lib.graph.raf.`**`random_sample`**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.graph.raf.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.graph.raf.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

lib.graph.raf.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.graph.raf.**seed**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

lib.graph.raf.**set_state**(*state*)
Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

`lib.graph.raf.`**`shuffle`**(*x*)
Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

`lib.graph.raf.`**`standard_cauchy`**(*size=None*)
Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + \left(\frac{x-x_0}{\gamma}\right)^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

lib.graph.raf.**standard_exponential**(*size=None*)
Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

lib.graph.raf.**standard_gamma**(*shape*, *size=None*)
Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
```

```
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

lib.graph.raf.**standard_normal**(*size=None*)

Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

lib.graph.raf.**standard_t**(*df*, *size=None*)

Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1 + \frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

lib.graph.raf.**triangular**(*left*, *mode*, *right*, *size=None*)
Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

lib.graph.raf.**uniform**(*low=0.0*, *high=1.0*, *size=1*)
Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

lib.graph.raf.**vonmises**(*mu*, *kappa*, *size=None*)
Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode ("center") of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

lib.graph.raf.**wald**(*mean*, *scale*, *size=None*)
Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

lib.graph.raf.**weibull**(*a*, *size=None*)
Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1}e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

lib.graph.raf.**zipf**(*a*, *size=None*)
Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

scc Module

lib.graph.scc.**beta**(*a*, *b*, *size=None*)
The Beta distribution over `[0, 1]`.

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

lib.graph.scc.**binomial**(*n*, *p*, *size=None*)
Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

lib.graph.scc.**checkMinimalSCCdimension**(*tmpDig*, *tmpMinDim*)

lib.graph.scc.**chisquare**(*df*, *size=None*)
Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\texttt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi^2_k.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

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```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

lib.graph.scc.**createNetXGraph**(*tmpCstr*, *tmpCats*)
|- Cat -> Prod graph creation...

lib.graph.scc.**createNetXGraphForRAF**(*tmpCstr*, *tmpClosure*, *tmpCats*)
|- Cat -> Prod graph creation...

lib.graph.scc.**createSimpleGraph**(*tmpCstr*, *tmpCats*)
|- Cat -> Prod graph creation...

lib.graph.scc.**diGraph_netX_stats**(*tmpDig*)

lib.graph.scc.**exponential**(*scale=1.0*, *size=None*)
Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

lib.graph.scc.**f**(*dfnum*, *dfden*, *size=None*)
Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

lib.graph.scc.**gamma**(*shape*, *scale=1.0*, *size=None*)
Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

lib.graph.scc.**geometric**(*p*, *size=None*)

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where *p* is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

lib.graph.scc.**get_state**()

Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

lib.graph.scc.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)
Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

lib.graph.scc.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

lib.graph.scc.**laplace**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...       np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

lib.graph.scc.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

lib.graph.scc.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)
Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...    a = 10. + np.random.random(100)
...    b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

lib.graph.scc.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

lib.graph.scc.**multinomial**(*n*, *pvals*, *size=None*)
Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

lib.graph.scc.**multivariate_normal**(*mean*, *cov*[, *size*])
Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

•Spherical covariance (*cov* is a multiple of the identity matrix)

•Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

lib.graph.scc.**negative_binomial**(*n*, *p*, *size=None*)
Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N;n,p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...     probability = sum(s<i) / 100000.
...     print i, "wells drilled, probability of one success =", probability
```

lib.graph.scc.**noncentral_chisquare**(*df*, *nonc*, *size=None*)
Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

lib.graph.scc.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)

Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
```

```
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

lib.graph.scc.**normal**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

lib.graph.scc.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of

the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

lib.graph.scc.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])
```

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

lib.graph.scc.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k;\lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

lib.graph.scc.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x;a) = ax^{a-1}, 0 \le x \le 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

lib.graph.scc.**printSCConFile**(*tmpSCCL*, *tmpfolderName*, *filePrefix*)

lib.graph.scc.**rand**(*d0*, *d1*, ..., *dn*)

Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

lib.graph.scc.**randint**(*low*, *high=None*, *size=None*)
Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

lib.graph.scc.**randn**(*d0*, *d1*, ..., *dn*)
Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...) + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

lib.graph.scc.**random**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.graph.scc.**random_integers**(*low*, *high=None*, *size=None*)
Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

lib.graph.scc.**random_sample**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.graph.scc.**ranf**()

random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

lib.graph.scc.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

lib.graph.scc.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

`lib.graph.scc.`**`seed`**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

`lib.graph.scc.`**`set_state`**(*state*)
Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

lib.graph.scc.**shuffle**(*x*)
Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

lib.graph.scc.**standard_cauchy**(*size=None*)
Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

lib.graph.scc.**standard_exponential**(*size=None*)
Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

lib.graph.scc.**standard_gamma**(*shape*, *size=None*)
Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

lib.graph.scc.**standard_normal**(*size=None*)
Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
```

```
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

lib.graph.scc.**standard_t**(*df*, *size=None*)
Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\Gamma(\frac{df}{2})}\left(1 + \frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

lib.graph.scc.**triangular**(*left*, *mode*, *right*, *size=None*)
Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

lib.graph.scc.**uniform**(*low=0.0*, *high=1.0*, *size=1*)

Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b-a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

lib.graph.scc.**vonmises**(*mu*, *kappa*, *size=None*)

Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode ("center") of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

lib.graph.scc.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

lib.graph.scc.**weibull**(*a*, *size=None*)

Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1}e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

lib.graph.scc.**zipf**(*a*, *size=None*)
Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

11.1.4 model Package

reactions Module

lib.model.reactions.**createRandomCleavage**(*tmpSpecies*, *alphabet*, *tmpInitLMax*)

lib.model.reactions.**createRandomCleavageForCompleteFiringDisk**(*tmpSpecies*, *alphabet*, *tmpInitLMax*)

lib.model.reactions.**createRandomCondensation**(*tmpSpecies*, *tmpInitLMax*)

lib.model.reactions.**getNumOfCleavages**(*tmpSpecies*)

lib.model.reactions.**getNumOfCondensations**(*N*)

species Module

lib.model.species.**createCompleteSpeciesPopulation**(*M*, *alphabet*)

lib.model.species.**createFileSpecies**(*tmpFolder*, *args*, *pars*, *tmpScale=1*, *specieslist=None*, *tmpCatInRAF=None*, *tmpRafCatContribute2C=1*)

lib.model.species.**getTotNumberOfSpeciesFromCompletePop**(*M*)

lib.model.species.**weightedChoice**(*weights*, *objects*)
Return a random item from objects, with the weighting defined by weights (which must sum to 1).

CHAPTER

TWELVE

MAIN MODULE

MAIN analysis script package for CaRNeSS simulations Main file analysis

`main.`**`beta`**(*a*, *b*, *size=None*)

The Beta distribution over `[0, 1]`.

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

`main.`**`binomial`**(*n*, *p*, *size=None*)

Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

main.**chisquare**(*df*, *size=None*)

Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\texttt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

NIST/SEMATECH e-Handbook of Statistical Methods

```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

main.**exponential**(*scale=1.0*, *size=None*)

Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

main.**f**(*dfnum*, *dfden*, *size=None*)

Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

main.**gamma**(*shape*, *scale=1.0*, *size=None*)

Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

main.**geometric**(*p*, *size=None*)

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where p is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

main.**get_state**()

Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

main.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)

Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

main.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

`main.`**`laplace`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...      np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

main.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

main.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)

Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

main.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

main.**multinomial**(*n*, *pvals*, *size=None*)

Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

main.**multivariate_normal**(*mean*, *cov*[, *size*])

Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

•Spherical covariance (*cov* is a multiple of the identity matrix)

•Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

main.**negative_binomial**(*n*, *p*, *size=None*)

Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N; n, p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...    probability = sum(s<i) / 100000.
...    print i, "wells drilled, probability of one success =", probability
```

main.**noncentral_chisquare**(*df*, *nonc*, *size=None*)

Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

main.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)

Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

main.**normal**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

main.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has

also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

main.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

main.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k; \lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

main.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., (m, n, k), then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')
```

```
>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

main.**rand**(*d0*, *d1*, ..., *dn*)

Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

main.**randint**(*low*, *high=None*, *size=None*)

Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

main.**randn**(*d0*, *d1*, ..., *dn*)

Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

main.**random**()

random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

main.**random_integers**(*low*, *high=None*, *size=None*)

Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

main.**random_sample**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

main.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

main.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

main.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

main.**seed**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

main.**set_state**(*state*)

Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos).`

main.**shuffle**(*x*)

Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

main.**standard_cauchy**(*size=None*)

Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

main.**standard_exponential**(*size=None*)

Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

main.**standard_gamma**(*shape*, *size=None*)

Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

main.**standard_normal**(*size=None*)

Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

main.**standard_t**(*df*, *size=None*)

Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1 + \frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

`main.`**`triangular`**(*left*, *mode*, *right*, *size=None*)

Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

`main.`**`uniform`**(*low=0.0*, *high=1.0*, *size=1*)

Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

main.**vonmises**(*mu*, *kappa*, *size=None*)

Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode (“center”) of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

main.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

main.**weibull**(*a*, *size=None*)

Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1}e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

main.**zipf**(*a*, *size=None*)

Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

CHAPTER

THIRTEEN

PREPARENEWSIM MODULE

CHAPTER

FOURTEEN

TOPOLOGY_ANALYSIS MODULE

This python program assess different network structures in term of topological (NON DYNAMICAL) RAF and SCC presence according to different structural parameters.

topology_analysis.**beta**(*a*, *b*, *size=None*)

The Beta distribution over `[0, 1]`.

The Beta distribution is a special case of the Dirichlet distribution, and is related to the Gamma distribution. It has the probability distribution function

$$f(x; a, b) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

where the normalisation, B, is the beta function,

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt.$$

It is often seen in Bayesian inference and order statistics.

a [float] Alpha, non-negative.

b [float] Beta, non-negative.

size [tuple of ints, optional] The number of samples to draw. The output is packed according to the size given.

out [ndarray] Array of the given shape, containing values drawn from a Beta distribution.

topology_analysis.**binomial**(*n*, *p*, *size=None*)

Draw samples from a binomial distribution.

Samples are drawn from a Binomial distribution with specified parameters, n trials and p probability of success where n an integer >= 0 and p is in the interval [0,1]. (n may be input as a float, but it is truncated to an integer in use)

n [float (but truncated to an integer)] parameter, >= 0.

p [float] parameter, >= 0 and <=1.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.binom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Binomial distribution is

$$P(N) = \binom{n}{N} p^N (1-p)^{n-N},$$

where n is the number of trials, p is the probability of success, and N is the number of successes.

When estimating the standard error of a proportion in a population by using a random sample, the normal distribution works well unless the product p*n <=5, where p = population proportion estimate, and n = number of samples, in which case the binomial distribution is used instead. For example, a sample of 15 people shows 4 who are left handed, and 11 who are right handed. Then p = 4/15 = 27%. 0.27*15 = 4, so the binomial distribution should be used in this case.

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9,0.1,20000)==0)/20000.
answer = 0.38885, or 38%.
```

topology_analysis.**chisquare**(*df*, *size=None*)

Draw samples from a chi-square distribution.

When *df* independent random variables, each with standard normal distributions (mean 0, variance 1), are squared and summed, the resulting distribution is chi-square (see Notes). This distribution is often used in hypothesis testing.

df [int] Number of degrees of freedom.

size [tuple of ints, int, optional] Size of the returned array. By default, a scalar is returned.

output [ndarray] Samples drawn from the distribution, packed in a *size*-shaped array.

ValueError When *df* <= 0 or when an inappropriate *size* (e.g. `size=-1`) is given.

The variable obtained by summing the squares of *df* independent, standard normally distributed random variables:

$$Q = \sum_{i=0}^{\mathtt{df}} X_i^2$$

is chi-square distributed, denoted

$$Q \sim \chi_k^2.$$

The probability density function of the chi-squared distribution is

$$p(x) = \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2},$$

where Γ is the gamma function,

$$\Gamma(x) = \int_0^{-\infty} t^{x-1} e^{-t} dt.$$

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```
>>> np.random.chisquare(2,4)
array([ 1.89920014,  9.00867716,  3.13710533,  5.62318272])
```

topology_analysis.**exponential**(*scale=1.0*, *size=None*)

Exponential distribution.

Its probability density function is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}),$$

for `x > 0` and 0 elsewhere. β is the scale parameter, which is the inverse of the rate parameter $\lambda = 1/\beta$. The rate parameter is an alternative, widely used parameterization of the exponential distribution [3]_.

The exponential distribution is a continuous analogue of the geometric distribution. It describes many common situations, such as the size of raindrops measured over many rainstorms [1]_, or the time between page requests to Wikipedia [2]_.

scale [float] The scale parameter, $\beta = 1/\lambda$.

size [tuple of ints] Number of samples to draw. The output is shaped according to *size*.

topology_analysis.**f**(*dfnum*, *dfden*, *size=None*)

Draw samples from a F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters should be greater than zero.

The random variate of the F distribution (also known as the Fisher distribution) is a continuous probability distribution that arises in ANOVA tests, and is the ratio of two chi-square variates.

dfnum [float] Degrees of freedom in numerator. Should be greater than zero.

dfden [float] Degrees of freedom in denominator. Should be greater than zero.

size [{tuple, int}, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn. By default only one sample is returned.

samples [{ndarray, scalar}] Samples from the Fisher distribution.

scipy.stats.distributions.f [probability density function,] distribution or cumulative density function, etc.

The F statistic is used to compare in-group variances to between-group variances. Calculating the distribution depends on the sampling, and so it is a function of the respective degrees of freedom in the problem. The variable *dfnum* is the number of samples minus one, the between-groups degrees of freedom, while *dfden* is the within-groups degrees of freedom, the sum of the number of samples in each group minus the number of groups.

An example from Glantz[1], pp 47-40. Two groups, children of diabetics (25 people) and children from people without diabetes (25 controls). Fasting blood glucose was measured, case group had a mean value of 86.1, controls had a mean value of 82.2. Standard deviations were 2.09 and 2.49 respectively. Are these data consistent with the null hypothesis that the parents diabetic status does not affect their children's blood glucose levels? Calculating the F statistic from the data gives a value of 36.01.

Draw samples from the distribution:

```
>>> dfnum = 1. # between group degrees of freedom
>>> dfden = 48. # within groups degrees of freedom
>>> s = np.random.f(dfnum, dfden, 1000)
```

The lower bound for the top 1% of the samples is :

```
>>> sort(s)[-10]
7.61988120985
```

So there is about a 1% chance that the F statistic will exceed 7.62, the measured value is 36, so the null hypothesis is rejected at the 1% level.

topology_analysis.**gamma**(*shape*, *scale=1.0*, *size=None*)

Draw samples from a Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, *shape* (sometimes designated "k") and *scale* (sometimes designated "theta"), where both parameters are > 0.

shape [scalar > 0] The shape of the gamma distribution.

scale [scalar > 0, optional] The scale of the gamma distribution. Default is equal to 1.

size [shape_tuple, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray, float] Returns one sample unless *size* parameter is specified.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 2. # mean and dispersion
>>> s = np.random.gamma(shape, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1)*(np.exp(-bins/scale) /
...                      (sps.gamma(shape)*scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

topology_analysis.**geometric**(*p*, *size=None*)

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1-p)^{k-1}p$$

where p is the probability of success of an individual trial.

p [float] The probability of success of an individual trial.

size [tuple of ints] Number of values to draw from the distribution. The output is shaped according to *size*.

out [ndarray] Samples from the geometric distribution, shaped according to *size*.

Draw ten thousand values from the geometric distribution, with the probability of an individual success equal to 0.35:

```
>>> z = np.random.geometric(p=0.35, size=10000)
```

How many trials succeeded after a single run?

```
>>> (z == 1).sum() / 10000.
0.34889999999999999 #random
```

topology_analysis.**get_state**()

Return a tuple representing the internal state of the generator.

For more details, see *set_state*.

out [tuple(str, ndarray of 624 uints, int, int, float)] The returned tuple has the following items:

1. the string 'MT19937'.
2. a 1-D array of 624 unsigned integer keys.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

set_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

topology_analysis.**gumbel**(*loc=0.0*, *scale=1.0*, *size=None*)

Gumbel distribution.

Draw samples from a Gumbel distribution with specified location and scale. For more information on the Gumbel distribution, see Notes and References below.

loc [float] The location of the mode of the distribution.

scale [float] The scale parameter of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

out [ndarray] The samples

scipy.stats.gumbel_l scipy.stats.gumbel_r scipy.stats.genextreme

probability density function, distribution, or cumulative density function, etc. for each of the above

weibull

The Gumbel (or Smallest Extreme Value (SEV) or the Smallest Extreme Value Type I) distribution is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. The Gumbel is a special case of the Extreme Value Type I distribution for maximums from distributions with "exponential-like" tails.

The probability density for the Gumbel distribution is

$$p(x) = \frac{e^{-(x-\mu)/\beta}}{\beta} e^{-e^{-(x-\mu)/\beta}},$$

where μ is the mode, a location parameter, and β is the scale parameter.

The Gumbel (named for German mathematician Emil Julius Gumbel) was used very early in the hydrology literature, for modeling the occurrence of flood events. It is also used for modeling maximum wind speed and rainfall rates. It is a "fat-tailed" distribution - the probability of an event in the tail of the distribution is larger than if one used a Gaussian, hence the surprisingly frequent occurrence of 100-year floods. Floods were initially modeled as a Gaussian process, which underestimated the frequency of extreme events.

It is one of a class of extreme value distributions, the Generalized Extreme Value (GEV) distributions, which also includes the Weibull and Frechet.

The function has a mean of $\mu + 0.57721\beta$ and a variance of $\frac{\pi^2}{6}\beta^2$.

Gumbel, E. J., *Statistics of Extremes*, New York: Columbia University Press, 1958.

Reiss, R.-D. and Thomas, M., *Statistical Analysis of Extreme Values from Insurance, Finance, Hydrology and Other Fields*, Basel: Birkhauser Verlag, 2001.

Draw samples from the distribution:

```
>>> mu, beta = 0, 0.1 # location and scale
>>> s = np.random.gumbel(mu, beta, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp( -np.exp( -(bins - mu) /beta) ),
...          linewidth=2, color='r')
>>> plt.show()
```

Show how an extreme value distribution can arise from a Gaussian process and compare to a Gaussian:

```
>>> means = []
>>> maxima = []
>>> for i in range(0,1000) :
...    a = np.random.normal(mu, beta, 1000)
...    means.append(a.mean())
...    maxima.append(a.max())
>>> count, bins, ignored = plt.hist(maxima, 30, normed=True)
>>> beta = np.std(maxima)*np.pi/np.sqrt(6)
>>> mu = np.mean(maxima) - 0.57721*beta
>>> plt.plot(bins, (1/beta)*np.exp(-(bins - mu)/beta)
...          * np.exp(-np.exp(-(bins - mu)/beta)),
...          linewidth=2, color='r')
>>> plt.plot(bins, 1/(beta * np.sqrt(2 * np.pi))
...          * np.exp(-(bins - mu)**2 / (2 * beta**2)),
...          linewidth=2, color='g')
>>> plt.show()
```

topology_analysis.**hypergeometric**(*ngood*, *nbad*, *nsample*, *size=None*)

Draw samples from a Hypergeometric distribution.

Samples are drawn from a Hypergeometric distribution with specified parameters, ngood (ways to make a good selection), nbad (ways to make a bad selection), and nsample = number of items sampled, which is less than or equal to the sum ngood + nbad.

ngood [int or array_like] Number of ways to make a good selection. Must be nonnegative.

nbad [int or array_like] Number of ways to make a bad selection. Must be nonnegative.

nsample [int or array_like] Number of items sampled. Must be at least 1 and at most `ngood + nbad`.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The values are all integers in [0, n].

scipy.stats.distributions.hypergeom [probability density function,] distribution or cumulative density function, etc.

The probability density for the Hypergeometric distribution is

$$P(x) = \frac{\binom{m}{n}\binom{N-m}{n-x}}{\binom{N}{n}},$$

where $0 \leq x \leq m$ and $n + m - N \leq x \leq n$

for P(x) the probability of x successes, n = ngood, m = nbad, and N = number of samples.

Consider an urn with black and white marbles in it, ngood of them black and nbad are white. If you draw nsample balls without replacement, then the Hypergeometric distribution describes the distribution of black balls in the drawn sample.

Note that this distribution is very similar to the Binomial distribution, except that in this case, samples are drawn without replacement, whereas in the Binomial case samples are drawn with replacement (or the sample space is infinite). As the sample space becomes large, this distribution approaches the Binomial.

Draw samples from the distribution:

```
>>> ngood, nbad, nsamp = 100, 2, 10
# number of good, number of bad, and number of samples
>>> s = np.random.hypergeometric(ngood, nbad, nsamp, 1000)
>>> hist(s)
#   note that it is very unlikely to grab both bad items
```

Suppose you have an urn with 15 white and 15 black marbles. If you pull 15 marbles at random, how likely is it that 12 or more of them are one color?

```
>>> s = np.random.hypergeometric(15, 15, 15, 100000)
>>> sum(s>=12)/100000. + sum(s<=3)/100000.
#   answer = 0.003 ... pretty unlikely!
```

`topology_analysis.`**`laplace`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from the Laplace or double exponential distribution with specified location (or mean) and scale (decay).

The Laplace distribution is similar to the Gaussian/normal distribution, but is sharper at the peak and has fatter tails. It represents the difference between two independent, identically distributed exponential random variables.

loc [float] The position, μ, of the distribution peak.

scale [float] λ, the exponential decay.

It has the probability density function

$$f(x; \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|x - \mu|}{\lambda}\right).$$

The first law of Laplace, from 1774, states that the frequency of an error can be expressed as an exponential function of the absolute magnitude of the error, which leads to the Laplace distribution. For many problems in Economics and Health sciences, this distribution seems to model the data better than the standard Gaussian distribution

Draw samples from the distribution

```
>>> loc, scale = 0., 1.
>>> s = np.random.laplace(loc, scale, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> x = np.arange(-8., 8., .01)
>>> pdf = np.exp(-abs(x-loc/scale))/(2.*scale)
>>> plt.plot(x, pdf)
```

Plot Gaussian for comparison:

```
>>> g = (1/(scale * np.sqrt(2 * np.pi)) *
...      np.exp( - (x - loc)**2 / (2 * scale**2) ))
>>> plt.plot(x,g)
```

topology_analysis.**logistic**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw samples from a Logistic distribution.

Samples are drawn from a Logistic distribution with specified parameters, loc (location or mean, also median), and scale (>0).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logistic [probability density function,] distribution or cumulative density function, etc.

The probability density for the Logistic distribution is

$$P(x) = P(x) = \frac{e^{-(x-\mu)/s}}{s(1+e^{-(x-\mu)/s})^2},$$

where μ = location and s = scale.

The Logistic distribution is used in Extreme Value problems where it can act as a mixture of Gumbel distributions, in Epidemiology, and by the World Chess Federation (FIDE) where it is used in the Elo ranking system, assuming the performance of each player is a logistically distributed random variable.

Draw samples from the distribution:

```
>>> loc, scale = 10, 1
>>> s = np.random.logistic(loc, scale, 10000)
>>> count, bins, ignored = plt.hist(s, bins=50)
```

plot against distribution

```
>>> def logist(x, loc, scale):
...     return exp((loc-x)/scale)/(scale*(1+exp((loc-x)/scale))**2)
>>> plt.plot(bins, logist(bins, loc, scale)*count.max()/\
... logist(bins, loc, scale).max())
>>> plt.show()
```

topology_analysis.**lognormal**(*mean=0.0*, *sigma=1.0*, *size=None*)

Return samples drawn from a log-normal distribution.

Draw samples from a log-normal distribution with specified mean, standard deviation, and array shape. Note that the mean and standard deviation are not the values for the distribution itself, but of the underlying normal distribution it is derived from.

mean [float] Mean value of the underlying normal distribution

sigma [float, > 0.] Standard deviation of the underlying normal distribution

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or float] The desired samples. An array of the same shape as *size* if given, if *size* is None a float is returned.

scipy.stats.lognorm [probability density function, distribution,] cumulative density function, etc.

A variable *x* has a log-normal distribution if *log(x)* is normally distributed. The probability density function for the log-normal distribution is:

$$p(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{(-\frac{(ln(x)-\mu)^2}{2\sigma^2})}$$

where μ is the mean and σ is the standard deviation of the normally distributed logarithm of the variable. A log-normal distribution results if a random variable is the *product* of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the *sum* of a large number of independent, identically-distributed variables.

Limpert, E., Stahel, W. A., and Abbt, M., "Log-normal Distributions across the Sciences: Keys and Clues," *BioScience*, Vol. 51, No. 5, May, 2001. http://stat.ethz.ch/~stahel/lognormal/bioscience.pdf

Reiss, R.D. and Thomas, M., *Statistical Analysis of Extreme Values*, Basel: Birkhauser Verlag, 2001, pp. 31-32.

Draw samples from the distribution:

```
>>> mu, sigma = 3., 1. # mean and standard deviation
>>> s = np.random.lognormal(mu, sigma, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='mid')

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, linewidth=2, color='r')
>>> plt.axis('tight')
>>> plt.show()
```

Demonstrate that taking the products of random samples from a uniform distribution can be fit well by a log-normal probability density function.

```
>>> # Generate a thousand samples: each is the product of 100 random
>>> # values, drawn from a normal distribution.
>>> b = []
>>> for i in range(1000):
...     a = 10. + np.random.random(100)
...     b.append(np.product(a))

>>> b = np.array(b) / np.min(b) # scale values to be positive
>>> count, bins, ignored = plt.hist(b, 100, normed=True, align='center')
>>> sigma = np.std(np.log(b))
>>> mu = np.mean(np.log(b))

>>> x = np.linspace(min(bins), max(bins), 10000)
>>> pdf = (np.exp(-(np.log(x) - mu)**2 / (2 * sigma**2))
...        / (x * sigma * np.sqrt(2 * np.pi)))

>>> plt.plot(x, pdf, color='r', linewidth=2)
>>> plt.show()
```

topology_analysis.**logseries**(*p*, *size=None*)

Draw samples from a Logarithmic Series distribution.

Samples are drawn from a Log Series distribution with specified parameter, p (probability, 0 < p < 1).

loc : float

scale : float > 0.

size [{tuple, int}] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] where the values are all integers in [0, n].

scipy.stats.distributions.logser [probability density function,] distribution or cumulative density function, etc.

The probability density for the Log Series distribution is

$$P(k) = \frac{-p^k}{k \ln(1-p)},$$

where p = probability.

The Log Series distribution is frequently used to represent species richness and occurrence, first proposed by Fisher, Corbet, and Williams in 1943 [2]. It may also be used to model the numbers of occupants seen in cars [3].

Draw samples from the distribution:

```
>>> a = .6
>>> s = np.random.logseries(a, 10000)
>>> count, bins, ignored = plt.hist(s)
```

plot against distribution

```
>>> def logseries(k, p):
...     return -p**k/(k*log(1-p))
>>> plt.plot(bins, logseries(bins, a)*count.max()/
             logseries(bins, a).max(), 'r')
>>> plt.show()
```

topology_analysis.**multinomial**(*n*, *pvals*, *size=None*)

Draw samples from a multinomial distribution.

The multinomial distribution is a multivariate generalisation of the binomial distribution. Take an experiment with one of `p` possible outcomes. An example of such an experiment is throwing a dice, where the outcome can be 1 through 6. Each sample drawn from the distribution represents *n* such experiments. Its values, `X_i = [X_0, X_1, ..., X_p]`, represent the number of times the outcome was `i`.

n [int] Number of experiments.

pvals [sequence of floats, length p] Probabilities of each of the `p` different outcomes. These should sum to 1 (however, the last element is always assumed to account for the remaining probability, as long as `sum(pvals[:-1]) <= 1)`.

size [tuple of ints] Given a *size* of `(M, N, K)`, then `M*N*K` samples are drawn, and the output shape becomes `(M, N, K, p)`, since each sample has shape `(p,)`.

Throw a dice 20 times:

```
>>> np.random.multinomial(20, [1/6.]*6, size=1)
array([[4, 1, 7, 5, 2, 1]])
```

It landed 4 times on 1, once on 2, etc.

Now, throw the dice 20 times, and 20 times again:

```
>>> np.random.multinomial(20, [1/6.]*6, size=2)
array([[3, 4, 3, 3, 4, 3],
       [2, 4, 3, 4, 0, 7]])
```

For the first run, we threw 3 times 1, 4 times 2, etc. For the second, we threw 2 times 1, 4 times 2, etc.

A loaded dice is more likely to land on number 6:

```
>>> np.random.multinomial(100, [1/7.]*5)
array([13, 16, 13, 16, 42])
```

topology_analysis.**multivariate_normal**(*mean*, *cov*[, *size*])

Draw random samples from a multivariate normal distribution.

The multivariate normal, multinormal or Gaussian distribution is a generalization of the one-dimensional normal distribution to higher dimensions. Such a distribution is specified by its mean and covariance matrix. These parameters are analogous to the mean (average or "center") and variance (standard deviation, or "width," squared) of the one-dimensional normal distribution.

mean [1-D array_like, of length N] Mean of the N-dimensional distribution.

cov [2-D array_like, of shape (N, N)] Covariance matrix of the distribution. Must be symmetric and positive semi-definite for "physically meaningful" results.

size [int or tuple of ints, optional] Given a shape of, for example, `(m,n,k)`, `m*n*k` samples are generated, and packed in an *m*-by-*n*-by-*k* arrangement. Because each sample is *N*-dimensional, the output shape is `(m,n,k,N)`. If no shape is specified, a single (*N*-D) sample is returned.

out [ndarray] The drawn samples, of shape *size*, if that was provided. If not, the shape is `(N,)`.

In other words, each entry `out[i,j,...,:]` is an N-dimensional value drawn from the distribution.

The mean is a coordinate in N-dimensional space, which represents the location where samples are most likely to be generated. This is analogous to the peak of the bell curve for the one-dimensional or univariate normal distribution.

Covariance indicates the level to which two variables vary together. From the multivariate normal distribution, we draw N-dimensional samples, $X = [x_1, x_2, ...x_N]$. The covariance matrix element C_{ij} is the covariance of x_i and x_j. The element C_{ii} is the variance of x_i (i.e. its "spread").

Instead of specifying the full covariance matrix, popular approximations include:

•Spherical covariance (*cov* is a multiple of the identity matrix)

•Diagonal covariance (*cov* has non-negative elements, and only on the diagonal)

This geometrical property can be seen in two dimensions by plotting generated data-points:

```
>>> mean = [0,0]
>>> cov = [[1,0],[0,100]] # diagonal covariance, points lie on x or y-axis

>>> import matplotlib.pyplot as plt
>>> x,y = np.random.multivariate_normal(mean,cov,5000).T
>>> plt.plot(x,y,'x'); plt.axis('equal'); plt.show()
```

Note that the covariance matrix must be non-negative definite.

Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., New York: McGraw-Hill, 1991.

Duda, R. O., Hart, P. E., and Stork, D. G., *Pattern Classification*, 2nd ed., New York: Wiley, 2001.

```
>>> mean = (1,2)
>>> cov = [[1,0],[1,0]]
>>> x = np.random.multivariate_normal(mean,cov,(3,3))
>>> x.shape
(3, 3, 2)
```

The following is probably true, given that 0.6 is roughly twice the standard deviation:

```
>>> print list( (x[0,0,:] - mean) < 0.6 )
[True, True]
```

topology_analysis.**negative_binomial**(*n*, *p*, *size=None*)

Draw samples from a negative_binomial distribution.

Samples are drawn from a negative_Binomial distribution with specified parameters, *n* trials and *p* probability of success where *n* is an integer > 0 and *p* is in the interval [0, 1].

n [int] Parameter, > 0.

p [float] Parameter, >= 0 and <=1.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [int or ndarray of ints] Drawn samples.

The probability density for the Negative Binomial distribution is

$$P(N;n,p) = \binom{N+n-1}{n-1} p^n (1-p)^N,$$

where $n-1$ is the number of successes, p is the probability of success, and $N+n-1$ is the number of trials.

The negative binomial distribution gives the probability of n-1 successes and N failures in N+n-1 trials, and success on the (N+n)th trial.

If one throws a die repeatedly until the third time a "1" appears, then the probability distribution of the number of non-"1"s that appear before the third "1" is a negative binomial distribution.

Draw samples from the distribution:

A real world example. A company drills wild-cat oil exploration wells, each with an estimated probability of success of 0.1. What is the probability of having one success for each successive well, that is what is the probability of a single success after drilling 5 wells, after 6 wells, etc.?

```
>>> s = np.random.negative_binomial(1, 0.1, 100000)
>>> for i in range(1, 11):
...     probability = sum(s<i) / 100000.
...     print i, "wells drilled, probability of one success =", probability
```

topology_analysis.**noncentral_chisquare**(*df*, *nonc*, *size=None*)

Draw samples from a noncentral chi-square distribution.

The noncentral χ^2 distribution is a generalisation of the χ^2 distribution.

df [int] Degrees of freedom, should be >= 1.

nonc [float] Non-centrality, should be > 0.

size [int or tuple of ints] Shape of the output.

The probability density function for the noncentral Chi-square distribution is

$$P(x;df,nonc) = \sum_{i=0}^{\infty} \frac{e^{-nonc/2}(nonc/2)^i}{i!} P_{Y_{df+2i}}(x),$$

where Y_q is the Chi-square with q degrees of freedom.

In Delhi (2007), it is noted that the noncentral chi-square is useful in bombing and coverage problems, the probability of killing the point target given by the noncentral chi-squared distribution.

Draw values from the distribution and plot the histogram

```
>>> import matplotlib.pyplot as plt
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

Draw values from a noncentral chisquare with very small noncentrality, and compare to a chisquare.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, .0000001, 100000),
...                   bins=np.arange(0., 25, .1), normed=True)
>>> values2 = plt.hist(np.random.chisquare(3, 100000),
...                    bins=np.arange(0., 25, .1), normed=True)
>>> plt.plot(values[1][0:-1], values[0]-values2[0], 'ob')
>>> plt.show()
```

Demonstrate how large values of non-centrality lead to a more symmetric distribution.

```
>>> plt.figure()
>>> values = plt.hist(np.random.noncentral_chisquare(3, 20, 100000),
...                   bins=200, normed=True)
>>> plt.show()
```

topology_analysis.**noncentral_f**(*dfnum*, *dfden*, *nonc*, *size=None*)

Draw samples from the noncentral F distribution.

Samples are drawn from an F distribution with specified parameters, *dfnum* (degrees of freedom in numerator) and *dfden* (degrees of freedom in denominator), where both parameters > 1. *nonc* is the non-centrality parameter.

dfnum [int] Parameter, should be > 1.

dfden [int] Parameter, should be > 1.

nonc [float] Parameter, should be >= 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] Drawn samples.

When calculating the power of an experiment (power = probability of rejecting the null hypothesis when a specific alternative is true) the non-central F statistic becomes important. When the null hypothesis is true, the F statistic follows a central F distribution. When the null hypothesis is not true, then it follows a non-central F statistic.

Weisstein, Eric W. "Noncentral F-Distribution." From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/NoncentralF-Distribution.html

Wikipedia, "Noncentral F distribution", http://en.wikipedia.org/wiki/Noncentral_F-distribution

In a study, testing for a specific alternative to the null hypothesis requires use of the Noncentral F distribution. We need to calculate the area in the tail of the distribution that exceeds the value of the F distribution for the null hypothesis. We'll plot the two probability distributions for comparison.

```
>>> dfnum = 3 # between group deg of freedom
>>> dfden = 20 # within groups degrees of freedom
>>> nonc = 3.0
>>> nc_vals = np.random.noncentral_f(dfnum, dfden, nonc, 1000000)
>>> NF = np.histogram(nc_vals, bins=50, normed=True)
>>> c_vals = np.random.f(dfnum, dfden, 1000000)
>>> F = np.histogram(c_vals, bins=50, normed=True)
>>> plt.plot(F[1][1:], F[0])
>>> plt.plot(NF[1][1:], NF[0])
>>> plt.show()
```

`topology_analysis.`**`normal`**(*loc=0.0*, *scale=1.0*, *size=None*)

Draw random samples from a normal (Gaussian) distribution.

The probability density function of the normal distribution, first derived by De Moivre and 200 years later by both Gauss and Laplace independently [2]_, is often called the bell curve because of its characteristic shape (see the example below).

The normal distributions occurs often in nature. For example, it describes the commonly occurring distribution of samples influenced by a large number of tiny, random disturbances, each with its own unique distribution [2]_.

loc [float] Mean ("centre") of the distribution.

scale [float] Standard deviation (spread or "width") of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.norm [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gaussian distribution is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and σ the standard deviation. The square of the standard deviation, σ^2, is called the variance.

The function has its peak at the mean, and its "spread" increases with the standard deviation (the function reaches 0.607 times its maximum at $x + \sigma$ and $x - \sigma$ [2]_). This implies that *numpy.random.normal* is more likely to return samples lying close to the mean, rather than those far away.

Draw samples from the distribution:

```
>>> mu, sigma = 0, 0.1 # mean and standard deviation
>>> s = np.random.normal(mu, sigma, 1000)
```

Verify the mean and the variance:

```
>>> abs(mu - np.mean(s)) < 0.01
True

>>> abs(sigma - np.std(s, ddof=1)) < 0.01
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 30, normed=True)
>>> plt.plot(bins, 1/(sigma * np.sqrt(2 * np.pi)) *
...                np.exp( - (bins - mu)**2 / (2 * sigma**2) ),
...          linewidth=2, color='r')
>>> plt.show()
```

topology_analysis.**pareto**(*a*, *size=None*)

Draw samples from a Pareto II or Lomax distribution with specified shape.

The Lomax or Pareto II distribution is a shifted Pareto distribution. The classical Pareto distribution can be obtained from the Lomax distribution by adding the location parameter m, see below. The smallest value of the Lomax distribution is zero while for the classical Pareto distribution it is m, where the standard Pareto distribution has location m=1. Lomax can also be considered as a simplified version of the Generalized Pareto distribution (available in SciPy), with the scale set to one and the location set to zero.

The Pareto distribution must be greater than zero, and is unbounded above. It is also known as the "80-20 rule". In this distribution, 80 percent of the weights are in the lowest 20 percent of the range, while the other 20 percent fill the remaining 80 percent of the range.

shape [float, > 0.] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.lomax.pdf [probability density function,] distribution or cumulative density function, etc.

scipy.stats.distributions.genpareto.pdf [probability density function,] distribution or cumulative density function, etc.

The probability density for the Pareto distribution is

$$p(x) = \frac{am^a}{x^{a+1}}$$

where a is the shape and m the location

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution useful in many real world problems. Outside the field of economics it is generally referred to as the Bradford distribution. Pareto developed the distribution to describe the distribution of wealth in an economy. It has

also found use in insurance, web page access statistics, oil field sizes, and many other problems, including the download frequency for projects in Sourceforge [1]. It is one of the so-called "fat-tailed" distributions.

Draw samples from the distribution:

```
>>> a, m = 3., 1. # shape and mode
>>> s = np.random.pareto(a, 1000) + m
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 100, normed=True, align='center')
>>> fit = a*m**a/bins**(a+1)
>>> plt.plot(bins, max(count)*fit/max(fit),linewidth=2, color='r')
>>> plt.show()
```

topology_analysis.**permutation**(*x*)

Randomly permute a sequence, or return a permuted range.

If *x* is a multi-dimensional array, it is only shuffled along its first index.

x [int or array_like] If *x* is an integer, randomly permute `np.arange(x)`. If *x* is an array, make a copy and shuffle the elements randomly.

out [ndarray] Permuted sequence or array range.

```
>>> np.random.permutation(10)
array([1, 7, 4, 3, 0, 9, 2, 5, 8, 6])

>>> np.random.permutation([1, 4, 9, 12, 15])
array([15,  1,  9,  4, 12])

>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.permutation(arr)
array([[6, 7, 8],
       [0, 1, 2],
       [3, 4, 5]])
```

topology_analysis.**poisson**(*lam=1.0*, *size=None*)

Draw samples from a Poisson distribution.

The Poisson distribution is the limit of the Binomial distribution for large N.

lam [float] Expectation of interval, should be >= 0.

size [int or tuple of ints, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

The Poisson distribution

$$f(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For events with an expected separation λ the Poisson distribution $f(k; \lambda)$ describes the probability of k events occurring within the observed interval λ.

Because the output is limited to the range of the C long type, a ValueError is raised when *lam* is within 10 sigma of the maximum representable value.

Draw samples from the distribution:

```
>>> import numpy as np
>>> s = np.random.poisson(5, 10000)
```

Display histogram of the sample:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 14, normed=True)
>>> plt.show()
```

topology_analysis.**power**(*a*, *size=None*)

Draws samples in [0, 1] from a power distribution with positive exponent a - 1.

Also known as the power function distribution.

a [float] parameter, > 0

size [tuple of ints]

Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [{ndarray, scalar}] The returned samples lie in [0, 1].

ValueError If a<1.

The probability density function is

$$P(x; a) = ax^{a-1}, 0 \leq x \leq 1, a > 0.$$

The power function distribution is just the inverse of the Pareto distribution. It may also be seen as a special case of the Beta distribution.

It is used, for example, in modeling the over-reporting of insurance claims.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> samples = 1000
>>> s = np.random.power(a, samples)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, bins=30)
>>> x = np.linspace(0, 1, 100)
>>> y = a*x**(a-1.)
>>> normed_y = samples*np.diff(bins)[0]*y
>>> plt.plot(x, normed_y)
>>> plt.show()
```

Compare the power function distribution to the inverse of the Pareto.

```
>>> from scipy import stats
>>> rvs = np.random.power(5, 1000000)
>>> rvsp = np.random.pareto(5, 1000000)
>>> xx = np.linspace(0,1,100)
>>> powpdf = stats.powerlaw.pdf(xx,5)

>>> plt.figure()
>>> plt.hist(rvs, bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('np.random.power(5)')
```

```
>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of 1 + np.random.pareto(5)')

>>> plt.figure()
>>> plt.hist(1./(1.+rvsp), bins=50, normed=True)
>>> plt.plot(xx,powpdf,'r-')
>>> plt.title('inverse of stats.pareto(5)')
```

topology_analysis.**rand**(*d0*, *d1*, ..., *dn*)

Random values in a given shape.

Create an array of the given shape and propagate it with random samples from a uniform distribution over `[0, 1)`.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should all be positive. If no argument is given a single Python float is returned.

out [ndarray, shape `(d0, d1, ..., dn)`] Random values.

random

This is a convenience function. If you want an interface that takes a shape-tuple as the first argument, refer to np.random.random_sample .

```
>>> np.random.rand(3,2)
array([[ 0.14022471,  0.96360618],  #random
       [ 0.37601032,  0.25528411],  #random
       [ 0.49313049,  0.94909878]]) #random
```

topology_analysis.**randint**(*low*, *high=None*, *size=None*)

Return random integers from *low* (inclusive) to *high* (exclusive).

Return random integers from the "discrete uniform" distribution in the "half-open" interval [*low*, *high*). If *high* is None (the default), then results are from [0, *low*).

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, one above the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.random_integers [similar to *randint*, only for the closed] interval [*low*, *high*], and 1 is the lowest value if *high* is omitted. In particular, this other one is the one to use to generate uniformly distributed discrete non-integers.

```
>>> np.random.randint(2, size=10)
array([1, 0, 0, 0, 1, 1, 0, 0, 1, 0])
>>> np.random.randint(1, size=10)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

Generate a 2 x 4 array of ints between 0 and 4, inclusive:

```
>>> np.random.randint(5, size=(2, 4))
array([[4, 0, 2, 1],
       [3, 2, 2, 0]])
```

topology_analysis.**randn**(*d0*, *d1*, ..., *dn*)

Return a sample (or samples) from the "standard normal" distribution.

If positive, int_like or int-convertible arguments are provided, *randn* generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate "normal" (Gaussian) distribution of mean 0 and variance 1 (if any of the d_i are floats, they are first converted to integers by truncation). A single float randomly sampled from the distribution is returned if no argument is provided.

This is a convenience function. If you want an interface that takes a tuple as the first argument, use *numpy.random.standard_normal* instead.

d0, d1, ..., dn [int, optional] The dimensions of the returned array, should be all positive. If no argument is given a single Python float is returned.

Z [ndarray or float] A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

random.standard_normal : Similar, but takes a tuple as its argument.

For random samples from $N(\mu, \sigma^2)$, use:

```
sigma * np.random.randn(...)  + mu
```

```
>>> np.random.randn()
2.1923875335537315 #random
```

Two-by-four array of samples from N(3, 6.25):

```
>>> 2.5 * np.random.randn(2, 4) + 3
array([[-4.49401501,  4.00950034, -1.81814867,  7.29718677],  #random
       [ 0.39924804,  4.68456316,  4.99394529,  4.84057254]]) #random
```

topology_analysis.**random**()

random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

topology_analysis.**random_integers**(*low*, *high=None*, *size=None*)

Return random integers between *low* and *high*, inclusive.

Return random integers from the "discrete uniform" distribution in the closed interval [*low*, *high*]. If *high* is None (the default), then results are from [1, *low*].

low [int] Lowest (signed) integer to be drawn from the distribution (unless `high=None`, in which case this parameter is the *highest* such integer).

high [int, optional] If provided, the largest (signed) integer to be drawn from the distribution (see above for behavior if `high=None`).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single int is returned.

out [int or ndarray of ints] *size*-shaped array of random integers from the appropriate distribution, or a single such random int if *size* not provided.

random.randint [Similar to *random_integers*, only for the half-open] interval [*low*, *high*), and 0 is the lowest value if *high* is omitted.

To sample from N evenly spaced floating-point numbers between a and b, use:

```
a + (b - a) * (np.random.random_integers(N) - 1) / (N - 1.)
```

```
>>> np.random.random_integers(5)
4
>>> type(np.random.random_integers(5))
<type 'int'>
>>> np.random.random_integers(5, size=(3.,2.))
array([[5, 4],
       [3, 3],
       [4, 5]])
```

Choose five random numbers from the set of five evenly-spaced numbers between 0 and 2.5, inclusive (*i.e.*, from the set $0, 5/8, 10/8, 15/8, 20/8$):

```
>>> 2.5 * (np.random.random_integers(5, size=(5,)) - 1) / 4.
array([ 0.625,  1.25 ,  0.625,  0.625,  2.5  ])
```

Roll two six sided dice 1000 times and sum the results:

```
>>> d1 = np.random.random_integers(1, 6, 1000)
>>> d2 = np.random.random_integers(1, 6, 1000)
>>> dsums = d1 + d2
```

Display results as a histogram:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(dsums, 11, normed=True)
>>> plt.show()
```

topology_analysis.**random_sample**(*size=None*)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

topology_analysis.**ranf**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

topology_analysis.**rayleigh**(*scale=1.0*, *size=None*)
Draw samples from a Rayleigh distribution.

The χ and Weibull distributions are generalizations of the Rayleigh.

scale [scalar] Scale, also equals the mode. Should be >= 0.

size [int or tuple of ints, optional] Shape of the output. Default is None, in which case a single value is returned.

The probability density function for the Rayleigh distribution is

$$P(x; scale) = \frac{x}{scale^2} e^{\frac{-x^2}{2 \cdot scale^2}}$$

The Rayleigh distribution arises if the wind speed and wind direction are both gaussian variables, then the vector wind velocity forms a Rayleigh distribution. The Rayleigh distribution is used to model the expected output from wind turbines.

Draw values from the distribution and plot the histogram

```
>>> values = hist(np.random.rayleigh(3, 100000), bins=200, normed=True)
```

Wave heights tend to follow a Rayleigh distribution. If the mean wave height is 1 meter, what fraction of waves are likely to be larger than 3 meters?

```
>>> meanvalue = 1
>>> modevalue = np.sqrt(2 / np.pi) * meanvalue
>>> s = np.random.rayleigh(modevalue, 1000000)
```

The percentage of waves larger than 3 meters is:

```
>>> 100.*sum(s>3)/1000000.
0.087300000000000003
```

topology_analysis.**sample**()
random_sample(size=None)

Return random floats in the half-open interval [0.0, 1.0).

Results are from the "continuous uniform" distribution over the stated interval. To sample $Unif[a, b), b > a$ multiply the output of *random_sample* by *(b-a)* and add *a*:

```
(b - a) * random_sample() + a
```

size [int or tuple of ints, optional] Defines the shape of the returned array of random floats. If None (the default), returns a single float.

out [float or ndarray of floats] Array of random floats of shape *size* (unless `size=None`, in which case a single float is returned).

```
>>> np.random.random_sample()
0.47108547995356098
>>> type(np.random.random_sample())
<type 'float'>
>>> np.random.random_sample((5,))
array([ 0.30220482,  0.86820401,  0.1654503 ,  0.11659149,  0.54323428])
```

Three-by-two array of random numbers from [-5, 0):

```
>>> 5 * np.random.random_sample((3, 2)) - 5
array([[-3.99149989, -0.52338984],
       [-2.99091858, -0.79479508],
       [-1.23204345, -1.75224494]])
```

topology_analysis.**seed**(*seed=None*)
Seed the generator.

This method is called when *RandomState* is initialized. It can be called again to re-seed the generator. For details, see *RandomState*.

seed [int or array_like, optional] Seed for *RandomState*.

RandomState

topology_analysis.**set_state**(*state*)

Set the internal state of the generator from a tuple.

For use if one has reason to manually (re-)set the internal state of the "Mersenne Twister"[1]_ pseudo-random number generating algorithm.

state [tuple(str, ndarray of 624 uints, int, int, float)] The *state* tuple has the following items:

1. the string 'MT19937', specifying the Mersenne Twister algorithm.
2. a 1-D array of 624 unsigned integers `keys`.
3. an integer `pos`.
4. an integer `has_gauss`.
5. a float `cached_gaussian`.

out [None] Returns 'None' on success.

get_state

set_state and *get_state* are not needed to work with any of the random distributions in NumPy. If the internal state is manually altered, the user should know exactly what he/she is doing.

For backwards compatibility, the form (str, array of 624 uints, int) is also accepted although it is missing some information about the cached Gaussian value: `state = ('MT19937', keys, pos)`.

topology_analysis.**shuffle**(*x*)

Modify a sequence in-place by shuffling its contents.

x [array_like] The array or list to be shuffled.

None

```
>>> arr = np.arange(10)
>>> np.random.shuffle(arr)
>>> arr
[1 7 5 2 9 4 3 6 0 8]
```

This function only shuffles the array along the first index of a multi-dimensional array:

```
>>> arr = np.arange(9).reshape((3, 3))
>>> np.random.shuffle(arr)
>>> arr
array([[3, 4, 5],
       [6, 7, 8],
       [0, 1, 2]])
```

topology_analysis.**standard_cauchy**(*size=None*)

Standard Cauchy distribution with mode = 0.

Also known as the Lorentz distribution.

size [int or tuple of ints] Shape of the output.

samples [ndarray or scalar] The drawn samples.

The probability density function for the full Cauchy distribution is

$$P(x; x_0, \gamma) = \frac{1}{\pi\gamma\left[1 + (\frac{x-x_0}{\gamma})^2\right]}$$

and the Standard Cauchy distribution just sets $x_0 = 0$ and $\gamma = 1$

The Cauchy distribution arises in the solution to the driven harmonic oscillator problem, and also describes spectral line broadening. It also describes the distribution of values at which a line tilted at a random angle will cut the x axis.

When studying hypothesis tests that assume normality, seeing how the tests perform on data from a Cauchy distribution is a good indicator of their sensitivity to a heavy-tailed distribution, since the Cauchy looks very much like a Gaussian distribution, but with heavier tails.

Draw samples and plot the distribution:

```
>>> s = np.random.standard_cauchy(1000000)
>>> s = s[(s>-25) & (s<25)]  # truncate distribution so it plots well
>>> plt.hist(s, bins=100)
>>> plt.show()
```

topology_analysis.**standard_exponential**(*size=None*)

Draw samples from the standard exponential distribution.

standard_exponential is identical to the exponential distribution with a scale parameter of 1.

size [int or tuple of ints] Shape of the output.

out [float or ndarray] Drawn samples.

Output a 3x8000 array:

```
>>> n = np.random.standard_exponential((3, 8000))
```

topology_analysis.**standard_gamma**(*shape*, *size=None*)

Draw samples from a Standard Gamma distribution.

Samples are drawn from a Gamma distribution with specified parameters, shape (sometimes designated "k") and scale=1.

shape [float] Parameter, should be > 0.

size [int or tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [ndarray or scalar] The drawn samples.

scipy.stats.distributions.gamma [probability density function,] distribution or cumulative density function, etc.

The probability density for the Gamma distribution is

$$p(x) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k\Gamma(k)},$$

where k is the shape and θ the scale, and Γ is the Gamma function.

The Gamma distribution is often used to model the times to failure of electronic components, and arises naturally in processes for which the waiting times between Poisson distributed events are relevant.

Draw samples from the distribution:

```
>>> shape, scale = 2., 1. # mean and width
>>> s = np.random.standard_gamma(shape, 1000000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> y = bins**(shape-1) * ((np.exp(-bins/scale))/ \
...                       (sps.gamma(shape) * scale**shape))
>>> plt.plot(bins, y, linewidth=2, color='r')
>>> plt.show()
```

topology_analysis.**standard_normal**(*size=None*)

Returns samples from a Standard Normal distribution (mean=0, stdev=1).

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

out [float or ndarray] Drawn samples.

```
>>> s = np.random.standard_normal(8000)
>>> s
array([ 0.6888893 ,  0.78096262, -0.89086505, ...,  0.49876311, #random
       -0.38672696, -0.4685006 ])                               #random
>>> s.shape
(8000,)
>>> s = np.random.standard_normal(size=(3, 4, 2))
>>> s.shape
(3, 4, 2)
```

topology_analysis.**standard_t**(*df*, *size=None*)

Standard Student's t distribution with df degrees of freedom.

A special case of the hyperbolic distribution. As *df* gets large, the result resembles that of the standard normal distribution (*standard_normal*).

df [int] Degrees of freedom, should be > 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn samples.

The probability density function for the t distribution is

$$P(x, df) = \frac{\Gamma(\frac{df+1}{2})}{\sqrt{\pi df}\,\Gamma(\frac{df}{2})}\left(1+\frac{x^2}{df}\right)^{-(df+1)/2}$$

The t test is based on an assumption that the data come from a Normal distribution. The t test provides a way to test whether the sample mean (that is the mean calculated from the data) is a good estimate of the true mean.

The derivation of the t-distribution was forst published in 1908 by William Gisset while working for the Guinness Brewery in Dublin. Due to proprietary issues, he had to publish under a pseudonym, and so he used the name Student.

From Dalgaard page 83 [1]_, suppose the daily energy intake for 11 women in Kj is:

```
>>> intake = np.array([5260., 5470, 5640, 6180, 6390, 6515, 6805, 7515, \
...                    7515, 8230, 8770])
```

Does their energy intake deviate systematically from the recommended value of 7725 kJ?

We have 10 degrees of freedom, so is the sample mean within 95% of the recommended value?

```
>>> s = np.random.standard_t(10, size=100000)
>>> np.mean(intake)
6753.636363636364
>>> intake.std(ddof=1)
1142.1232221373727
```

Calculate the t statistic, setting the ddof parameter to the unbiased value so the divisor in the standard deviation will be degrees of freedom, N-1.

```
>>> t = (np.mean(intake)-7725)/(intake.std(ddof=1)/np.sqrt(len(intake)))
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(s, bins=100, normed=True)
```

For a one-sided t-test, how far out in the distribution does the t statistic appear?

```
>>> >>> np.sum(s<t) / float(len(s))
0.0090699999999999999  #random
```

So the p-value is about 0.009, which says the null hypothesis has a probability of about 99% of being true.

topology_analysis.**triangular**(*left*, *mode*, *right*, *size=None*)

Draw samples from the triangular distribution.

The triangular distribution is a continuous probability distribution with lower limit left, peak at mode, and upper limit right. Unlike the other distributions, these parameters directly define the shape of the pdf.

left [scalar] Lower limit.

mode [scalar] The value where the peak of the distribution occurs. The value should fulfill the condition `left <= mode <= right`.

right [scalar] Upper limit, should be larger than *left*.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] The returned samples all lie in the interval [left, right].

The probability density function for the Triangular distribution is

$$P(x;l,m,r) = \begin{cases} \frac{2(x-l)}{(r-l)(m-l)} & \text{for } l \leq x \leq m, \\ \frac{2(m-x)}{(r-l)(r-m)} & \text{for } m \leq x \leq r, \\ 0 & \text{otherwise.} \end{cases}$$

The triangular distribution is often used in ill-defined problems where the underlying distribution is not known, but some knowledge of the limits and mode exists. Often it is used in simulations.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.triangular(-3, 0, 8, 100000), bins=200,
...              normed=True)
>>> plt.show()
```

topology_analysis.**uniform**(*low=0.0*, *high=1.0*, *size=1*)

Draw samples from a uniform distribution.

Samples are uniformly distributed over the half-open interval `[low, high)` (includes low, but excludes high). In other words, any value within the given interval is equally likely to be drawn by *uniform*.

low [float, optional] Lower boundary of the output interval. All values generated will be greater than or equal to low. The default value is 0.

high [float] Upper boundary of the output interval. All values generated will be less than high. The default value is 1.0.

size [int or tuple of ints, optional] Shape of output. If the given size is, for example, (m,n,k), m*n*k samples are generated. If no shape is specified, a single sample is returned.

out [ndarray] Drawn samples, with shape *size*.

randint : Discrete uniform distribution, yielding integers. random_integers : Discrete uniform distribution over the closed

interval `[low, high]`.

random_sample : Floats uniformly distributed over `[0, 1)`. random : Alias for *random_sample*. rand : Convenience function that accepts dimensions as input, e.g.,

`rand(2,2)` would generate a 2-by-2 array of floats, uniformly distributed over `[0, 1)`.

The probability density function of the uniform distribution is

$$p(x) = \frac{1}{b - a}$$

anywhere within the interval `[a, b)`, and zero elsewhere.

Draw samples from the distribution:

```
>>> s = np.random.uniform(-1,0,1000)
```

All values are within the given interval:

```
>>> np.all(s >= -1)
True
>>> np.all(s < 0)
True
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> count, bins, ignored = plt.hist(s, 15, normed=True)
>>> plt.plot(bins, np.ones_like(bins), linewidth=2, color='r')
>>> plt.show()
```

topology_analysis.**vonmises**(*mu*, *kappa*, *size=None*)

Draw samples from a von Mises distribution.

Samples are drawn from a von Mises distribution with specified mode (mu) and dispersion (kappa), on the interval [-pi, pi].

The von Mises distribution (also known as the circular normal distribution) is a continuous probability distribution on the unit circle. It may be thought of as the circular analogue of the normal distribution.

mu [float] Mode (“center”) of the distribution.

kappa [float] Dispersion of the distribution, has to be >=0.

size [int or tuple of int] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

samples [scalar or ndarray] The returned samples, which are in the interval [-pi, pi].

scipy.stats.distributions.vonmises [probability density function,] distribution, or cumulative density function, etc.

The probability density for the von Mises distribution is

$$p(x) = \frac{e^{\kappa cos(x-\mu)}}{2\pi I_0(\kappa)},$$

where μ is the mode and κ the dispersion, and $I_0(\kappa)$ is the modified Bessel function of order 0.

The von Mises is named for Richard Edler von Mises, who was born in Austria-Hungary, in what is now the Ukraine. He fled to the United States in 1939 and became a professor at Harvard. He worked in probability theory, aerodynamics, fluid mechanics, and philosophy of science.

Abramowitz, M. and Stegun, I. A. (ed.), *Handbook of Mathematical Functions*, New York: Dover, 1965.

von Mises, R., *Mathematical Theory of Probability and Statistics*, New York: Academic Press, 1964.

Draw samples from the distribution:

```
>>> mu, kappa = 0.0, 4.0 # mean and dispersion
>>> s = np.random.vonmises(mu, kappa, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
>>> count, bins, ignored = plt.hist(s, 50, normed=True)
>>> x = np.arange(-np.pi, np.pi, 2*np.pi/50.)
>>> y = -np.exp(kappa*np.cos(x-mu))/(2*np.pi*sps.jn(0,kappa))
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

topology_analysis.**wald**(*mean*, *scale*, *size=None*)

Draw samples from a Wald, or Inverse Gaussian, distribution.

As the scale approaches infinity, the distribution becomes more like a Gaussian.

Some references claim that the Wald is an Inverse Gaussian with mean=1, but this is by no means universal.

The Inverse Gaussian distribution was first studied in relationship to Brownian motion. In 1956 M.C.K. Tweedie used the name Inverse Gaussian because there is an inverse relationship between the time to cover a unit distance and distance covered in unit time.

mean [scalar] Distribution mean, should be > 0.

scale [scalar] Scale parameter, should be >= 0.

size [int or tuple of ints, optional] Output shape. Default is None, in which case a single value is returned.

samples [ndarray or scalar] Drawn sample, all greater than zero.

The probability density function for the Wald distribution is

$$P(x; mean, scale) = \sqrt{\frac{scale}{2\pi x^3}} e^{\frac{-scale(x-mean)^2}{2 \cdot mean^2 x}}$$

As noted above the Inverse Gaussian distribution first arise from attempts to model Brownian Motion. It is also a competitor to the Weibull for use in reliability modeling and modeling stock returns and interest rate processes.

Draw values from the distribution and plot the histogram:

```
>>> import matplotlib.pyplot as plt
>>> h = plt.hist(np.random.wald(3, 2, 100000), bins=200, normed=True)
>>> plt.show()
```

topology_analysis.**weibull**(*a*, *size=None*)

Weibull distribution.

Draw samples from a 1-parameter Weibull distribution with the given shape parameter a.

$$X = (-ln(U))^{1/a}$$

Here, U is drawn from the uniform distribution over (0,1].

The more common 2-parameter Weibull, including a scale parameter λ is just $X = \lambda(-ln(U))^{1/a}$.

a [float] Shape of the distribution.

size [tuple of ints] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn.

scipy.stats.distributions.weibull_max scipy.stats.distributions.weibull_min scipy.stats.distributions.genextreme gumbel

The Weibull (or Type III asymptotic extreme value distribution for smallest values, SEV Type III, or Rosin-Rammler distribution) is one of a class of Generalized Extreme Value (GEV) distributions used in modeling extreme value problems. This class includes the Gumbel and Frechet distributions.

The probability density for the Weibull distribution is

$$p(x) = \frac{a}{\lambda}(\frac{x}{\lambda})^{a-1}e^{-(x/\lambda)^a},$$

where a is the shape and λ the scale.

The function has its peak (the mode) at $\lambda(\frac{a-1}{a})^{1/a}$.

When `a = 1`, the Weibull distribution reduces to the exponential distribution.

Draw samples from the distribution:

```
>>> a = 5. # shape
>>> s = np.random.weibull(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> x = np.arange(1,100.)/50.
>>> def weib(x,n,a):
...     return (a / n) * (x / n)**(a - 1) * np.exp(-(x / n)**a)

>>> count, bins, ignored = plt.hist(np.random.weibull(5.,1000))
>>> x = np.arange(1,100.)/50.
>>> scale = count.max()/weib(x, 1., 5.).max()
>>> plt.plot(x, weib(x, 1., 5.)*scale)
>>> plt.show()
```

topology_analysis.**zipf**(*a*, *size=None*)

Draw samples from a Zipf distribution.

Samples are drawn from a Zipf distribution with specified parameter $a > 1$.

The Zipf distribution (also known as the zeta distribution) is a continuous probability distribution that satisfies Zipf's law: the frequency of an item is inversely proportional to its rank in a frequency table.

a [float > 1] Distribution parameter.

size [int or tuple of int, optional] Output shape. If the given shape is, e.g., `(m, n, k)`, then `m * n * k` samples are drawn; a single integer is equivalent in its result to providing a mono-tuple, i.e., a 1-D array of length *size* is returned. The default is None, in which case a single scalar is returned.

samples [scalar or ndarray] The returned samples are greater than or equal to one.

scipy.stats.distributions.zipf [probability density function,] distribution, or cumulative density function, etc.

The probability density for the Zipf distribution is

$$p(x) = \frac{x^{-a}}{\zeta(a)},$$

where ζ is the Riemann Zeta function.

It is named for the American linguist George Kingsley Zipf, who noted that the frequency of any word in a sample of a language is inversely proportional to its rank in the frequency table.

Zipf, G. K., *Selected Studies of the Principle of Relative Frequency in Language*, Cambridge, MA: Harvard Univ. Press, 1932.

Draw samples from the distribution:

```
>>> a = 2. # parameter
>>> s = np.random.zipf(a, 1000)
```

Display the histogram of the samples, along with the probability density function:

```
>>> import matplotlib.pyplot as plt
>>> import scipy.special as sps
Truncate s values at 50 so plot is interesting
>>> count, bins, ignored = plt.hist(s[s<50], 50, normed=True)
>>> x = np.arange(1., 50.)
>>> y = x**(-a)/sps.zetac(a)
>>> plt.plot(x, y/max(y), linewidth=2, color='r')
>>> plt.show()
```

CHAPTER

FIFTEEN

INDICES AND TABLES

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