

- 1 Text embedding models yield high-resolution insights
- 2 into conceptual knowledge from short multiple-choice

3 quizzes

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Abstract

We develop a mathematical framework, based on natural language processing models, for tracking and characterizing the acquisition of conceptual knowledge. Our approach embeds each concept in a high-dimensional representation space, where nearby coordinates reflect similar or related concepts. We test our approach using behavioral data from participants who answered small sets of multiple-choice quiz questions interleaved between watching two course videos from the Khan Academy platform. We apply our framework to the videos' transcripts and the text of the quiz questions to quantify the content of each moment of video and each quiz question. We use these embeddings, along with participants' quiz responses, to track how the learners' knowledge changed after watching each video and predict their success on individual quiz questions. Our findings show how a small set of quiz questions may be used to obtain rich and meaningful high-resolution insights into what each learner knows, and how their knowledge changes over time as they learn.

Keywords: education, learning, knowledge, concepts, natural language processing

¹⁹ **Introduction**

²⁰ Suppose that a teacher had access to a complete, tangible “map” of everything a student knows.
²¹ Defining what such a map might even look like, let alone how it might be constructed or filled in, is
²² itself a non-trivial problem. But if a teacher *were* to gain access to such a map, how might it change
²³ their ability to teach that student? Perhaps they might start by checking how well the student
²⁴ knows the to-be-learned information already, or how much they know about related concepts.
²⁵ For some students, they could potentially optimize their teaching efforts to maximize efficiency
²⁶ by focusing primarily on not-yet-known content. For other students (or other content areas), it
²⁷ might be more effective to optimize for direct connections between already known content and
²⁸ new material. Observing how the student’s knowledge changed over time, in response to their
²⁹ teaching, could also help to guide the teacher towards the most effective strategy for that individual
³⁰ student.

³¹ A common approach to assessing a student’s knowledge is to present them with a set of quiz
³² questions, calculate the proportion they answer correctly, and provide them with feedback in the
³³ form of a simple numeric or letter grade. While such a grade can provide *some* indication of whether
³⁴ the student has mastered the to-be-learned material, any univariate measure of performance on a
³⁵ complex task sacrifices certain relevant information, risks conflating underlying factors, and so on.
³⁶ For example, consider the relative utility of the theoretical map described above that characterizes
³⁷ a student’s knowledge in detail, versus a single annotation saying that the student answered 85%
³⁸ of their quiz questions correctly, or that they received a ‘B’. Here, we show that the same quiz data
³⁹ required to compute proportion-correct scores or letter grades can instead be used to obtain far
⁴⁰ more detailed insights into what a student knew at the time they took the quiz.

⁴¹ Designing and building procedures and tools for mapping out knowledge touches on deep
⁴² questions about what it means to learn. For example, how do we acquire conceptual knowledge?
⁴³ Memorizing course lectures or textbook chapters by rote can lead to the superficial *appearance*
⁴⁴ of understanding the underlying content, but achieving true conceptual understanding seems to
⁴⁵ require something deeper and richer. Does conceptual understanding entail connecting newly

46 acquired information to the scaffolding of one’s existing knowledge or experience [6, 11, 13, 15, 30,
47 64]? Or weaving a lecture’s atomic elements (e.g., its component words) into a structured network
48 that describes how those individual elements are related [40, 69]? Conceptual understanding
49 could also involve building a mental model that transcends the meanings of those individual
50 atomic elements by reflecting the deeper meaning underlying the gestalt whole [37, 41, 61, 68].

51 The difference between “understanding” and “memorizing,” as framed by researchers in ed-
52 ucation, cognitive psychology, and cognitive neuroscience [e.g., 23, 28, 33, 41, 61], has profound
53 analogs in the fields of natural language processing and natural language understanding. For
54 example, considering the raw contents of a document (e.g., its constituent symbols, letters, and
55 words) might provide some clues as to what the document is about, just as memorizing a passage
56 might provide some ability to answer simple questions about it. However, text embedding mod-
57 els [e.g., 7, 8, 10, 12, 16, 39, 50, 70] also attempt to capture the deeper meaning *underlying* those
58 atomic elements. These models consider not only the co-occurrences of those elements within and
59 across documents, but (in many cases) also patterns in how those elements appear across different
60 scales (e.g., sentences, paragraphs, chapters, etc.), the temporal and grammatical properties of the
61 elements, and other high-level characteristics of how they are used [42, 43]. To be clear, this is not
62 to say that text embedding models themselves are capable of “understanding” deep conceptual
63 meaning in any traditional sense. But rather, their ability to capture the underlying *structure* of
64 text documents beyond their surface-level contents provides a computational framework through
65 which those documents’ deeper conceptual meanings may be quantified, explored, and under-
66 stood. According to these models, the deep conceptual meaning of a document may be captured
67 by a feature vector in a high-dimensional representation space, wherein nearby vectors reflect con-
68 ceptually related documents. A model that succeeds at capturing an analogue of “understanding”
69 is able to assign nearby feature vectors to two conceptually related documents, *even when the specific*
70 *words contained in those documents have limited overlap*. In this way, “concepts” are defined implicitly
71 by the model’s geometry [e.g., how the embedding coordinate of a given word or document relates
72 to the coordinates of other text embeddings; 55].

73 Given these insights, what form might a representation of the sum total of a person’s knowledge

74 take? First, we might require a means of systematically describing or representing (at least some
75 subset of) the nearly infinite set of possible things a person could know. Second, we might want to
76 account for potential associations between different concepts. For example, the concepts of “fish”
77 and “water” might be associated in the sense that fish live in water. Third, knowledge may have
78 a critical dependency structure, such that knowing about a particular concept might require first
79 knowing about a set of other concepts. For example, understanding the concept of a fish swimming
80 in water first requires understanding what fish and water *are*. Fourth, as we learn, our “current
81 state of knowledge” should change accordingly. Learning new concepts should both update our
82 characterizations of “what is known” and also unlock any now-satisfied dependencies of those
83 newly learned concepts so that they are “tagged” as available for future learning.

84 Here we develop a framework for modeling how conceptual knowledge is acquired during
85 learning. The central idea behind our framework is to use text embedding models to define the
86 coordinate systems of two maps: a *knowledge map* that describes the extent to which each concept is
87 currently known, and a *learning map* that describes changes in knowledge over time. Each location
88 on these maps represents a single concept, and the maps’ geometries are defined such that related
89 concepts are located nearby in space. We use this framework to analyze and interpret behavioral
90 data collected from an experiment that had participants answer sets of multiple-choice questions
91 about a series of recorded course lectures.

92 Our primary research goal is to advance our understanding of what it means to acquire deep,
93 real-world conceptual knowledge. Traditional laboratory approaches to studying learning and
94 memory (e.g., list-learning studies) often draw little distinction between memorization and under-
95 standing. Instead, these studies typically focus on whether information is effectively encoded or
96 retrieved, rather than whether the information is *understood*. Approaches to studying conceptual
97 learning, such as category learning experiments, can begin to investigate the distinction between
98 memorization and understanding, often by training participants to distinguish arbitrary or random
99 features in otherwise meaningless categorized stimuli [1, 20, 21, 24, 31, 58]. However, the objective
100 of real-world training, or learning from life experiences more generally, is often to develop new
101 knowledge that may be applied in *useful* ways in the future. In this sense, the gap between modern

learning theories and modern pedagogical approaches that inform classroom learning strategies is enormous: most of our theories about *how* people learn are inspired by experimental paradigms and models that have only peripheral relevance to the kinds of learning that students and teachers actually seek [28, 41]. To help bridge this gap, our study uses course materials from real online courses to inform, fit, and test models of real-world conceptual learning. We show that these models recover meaningful relationships between concepts presented during course lectures and tested by assessments, and that these relationships can be leveraged to predict students' success on individual quiz questions. We also provide a demonstration of how our models can be used to construct "maps" of what students know, and how their knowledge changes with training. In addition to helping to visually capture knowledge (and changes in knowledge), we hope that such maps might lead to real-world tools for improving how we educate. Taken together, our work shows that existing course materials and evaluative tools like short multiple-choice quizzes may be leveraged to gain highly detailed insights into what students know and how they learn.

Results

At its core, our main modeling approach is based around a simple assumption that we sought to test empirically: all else being equal, knowledge about a given concept is predictive of knowledge about similar or related concepts. From a geometric perspective, this assumption implies that knowledge is fundamentally "smooth." In other words, as one moves through a space representing an individual's knowledge (where similar concepts occupy nearby coordinates), their "level of knowledge" should change relatively gradually. To begin to test this smoothness assumption, we sought to track participants' knowledge and how it changed over time in response to training. Two overarching goals guide our approach. First, we want to gain detailed insights into what learners know at different points in their training. For example, rather than simply reporting on the proportions of questions participants answer correctly (i.e., their overall performance), we seek estimates of their knowledge about a variety of specific concepts. Second, we want our approach to be potentially scalable to large numbers of diverse concepts, courses, and students. This requires



Figure 1: Experimental paradigm. Participants alternate between completing three 13-question multiple-choice quizzes and watching two Khan Academy lectures. Each quiz contains a mix of 5 questions about Lecture 1, 5 questions about Lecture 2, and 3 questions about general physics knowledge. The specific questions reflected on each quiz, and the orders of each quiz's questions, were randomized across participants.

128 that the conceptual content of interest be discovered *automatically*, rather than relying on manually
 129 produced ratings or labels.

130 We asked participants in our study to complete brief multiple-choice quizzes before, between,
 131 and after watching two lecture videos from the Khan Academy [36] platform (Fig. 1). The first
 132 lecture video, entitled *Four Fundamental Forces*, discussed the four fundamental forces in physics:
 133 gravity, strong and weak interactions, and electromagnetism. The second, entitled *Birth of Stars*,
 134 provided an overview of our current understanding of how stars form. We selected these particular
 135 lectures to satisfy three general criteria. First, we wanted both lectures to be accessible to a broad
 136 audience (i.e., with minimal prerequisite knowledge) so as to limit the impact of prior training
 137 on participants' abilities to learn from the lectures. To this end, we selected two introductory
 138 videos that were intended to be viewed at the start of students' training in their respective content
 139 areas. Second, we wanted the two lectures to have some related content, so that we could test
 140 our approach's ability to distinguish similar conceptual content. To this end, we chose two videos
 141 from the same Khan Academy course domain, "Cosmology and Astronomy." Third, we sought to
 142 minimize dependencies and specific overlap between the videos. For example, we did not want
 143 participants' abilities to understand one video to (directly) influence their abilities to understand the
 144 other. To satisfy this last criterion, we chose videos from two different lecture series (Lectures 1 and
 145 2 were from the "Scale of the Universe" and "Stars, Black Holes, and Galaxies" series, respectively).



Figure 2: Modeling course content. **A. Building a document pool from sliding windows of text.** We decompose each lecture’s transcript into a series of overlapping sliding windows. The full set of transcript snippets (across all windows) may be treated as a set of “documents” for training a text embedding model. **B. Constructing lecture content trajectories.** After training the model on the sliding windows from both lectures, we transform each lecture into a “trajectory” through text embedding space by joining the embedding coordinates of successive sliding windows parsed from its transcript. **C. Embedding multiple lectures and questions in a shared space.** We apply the same model (trained on the two lectures’ windows) to both lectures, along with the text of each question in our pool (Supp. Tab. 1), to project them into a shared text embedding space. This results in one trajectory per lecture and one coordinate for each question. Here, we have projected the 15-dimensional embeddings onto their first 3 principal components for visualization.

146 We also wrote a set of multiple-choice quiz questions that we hoped would enable us to
 147 evaluate participants’ knowledge about each individual lecture, along with related knowledge
 148 about physics concepts not specifically presented in either video (see Supp. Tab. 1 for the full list
 149 of questions in our stimulus pool). Participants answered questions randomly drawn from each
 150 content area (Lecture 1, Lecture 2, and general physics knowledge) on each of the three quizzes.
 151 Quiz 1 was intended to assess participants’ “baseline” knowledge before training, Quiz 2 assessed
 152 knowledge after watching the *Four Fundamental Forces* video (i.e., Lecture 1), and Quiz 3 assessed
 153 knowledge after watching the *Birth of Stars* video (i.e., Lecture 2).

154 To study in detail how participants’ conceptual knowledge changed over the course of the
 155 experiment, we first sought to model the conceptual content presented to them at each moment
 156 throughout each of the two lectures. We adapted an approach we developed in prior work [29]
 157 to identify the latent themes in the lectures using a topic model [8]. Briefly, topic models take
 158 as input a collection of text documents, and learn a set of “topics” (i.e., latent themes) from their
 159 contents. Once fit, a topic model can be used to transform arbitrary (potentially new) documents
 160 into sets of “topic proportions,” describing the weighted blend of learned topics reflected in their

¹⁶¹ texts. We parsed automatically generated transcripts of the two lectures into overlapping sliding
¹⁶² windows, where each window contained the text of the lecture transcript from a particular time
¹⁶³ span. We treated the set of text snippets (across all of these windows) as documents to fit the
¹⁶⁴ model (Fig. 2A; see *Constructing text embeddings of multiple lectures and questions*). Transforming the
¹⁶⁵ text from every sliding window with the model yielded a number-of-windows by number-of-topics
¹⁶⁶ (15) topic-proportions matrix describing the unique mixture of broad themes from both lectures
¹⁶⁷ reflected in each window’s text. Each window’s “topic vector” (i.e., column of the topic-proportions
¹⁶⁸ matrix) is analogous to a coordinate in a 15-dimensional space whose axes are topics discovered
¹⁶⁹ by the model. Within this space, each lecture’s sequence of topic vectors (i.e., corresponding to its
¹⁷⁰ transcript’s overlapping text snippets across sliding windows) forms a *trajectory* that captures how
¹⁷¹ its conceptual content unfolds over time (Fig. 2B). We resampled these trajectories to a resolution
¹⁷² of one topic vector for each second of video (i.e., 1 Hz).

¹⁷³ We hypothesized that a topic model trained on transcripts of the two lectures should also
¹⁷⁴ capture the conceptual knowledge probed by each quiz question. If indeed the topic model could
¹⁷⁵ capture information about the deeper conceptual content of the lectures (i.e., beyond surface-level
¹⁷⁶ details such as particular word choices), then we should be able to recover a correspondence
¹⁷⁷ between each lecture and questions *about* each lecture. Importantly, such a correspondence could
¹⁷⁸ not solely arise from superficial text matching between lecture transcripts and questions, since the
¹⁷⁹ lectures and questions often used different words (Supp. Fig. 5) and phrasings. Simply comparing
¹⁸⁰ the average topic weights from each lecture and question set (averaging across time and questions,
¹⁸¹ respectively) reveals a striking correspondence (Supp. Fig. 2). Specifically, the average topic
¹⁸² weights from Lecture 1 are strongly correlated with the average topic weights from Lecture 1
¹⁸³ questions ($r(13) = 0.809$, $p < 0.001$, 95% confidence interval (CI) = [0.633, 0.962]), and the average
¹⁸⁴ topic weights from Lecture 2 are strongly correlated with the average topic weights from Lecture 2
¹⁸⁵ questions ($r(13) = 0.728$, $p = 0.002$, 95% CI = [0.456, 0.920]). At the same time, the average topic
¹⁸⁶ weights from the two lectures are *negatively* correlated with the average topic weights from their
¹⁸⁷ non-matching question sets (Lecture 1 video vs. Lecture 2 questions: $r(13) = -0.547$, $p = 0.035$,
¹⁸⁸ 95% CI = [-0.812, -0.231]; Lecture 2 video vs. Lecture 1 questions: $r(13) = -0.612$, $p = 0.015$, 95%



Figure 3: Lecture and question topic overlap. A. Topic weight variability. The bar plots display the variance of each topic's weight across lecture timepoints (top row) and questions (bottom row); colors denote topics. The top-weighted words from the most “expressive” (i.e., variable across observations) topic from each lecture are displayed in the upper right (orange: topic 2; yellow-green: topic 4). The top-weighted words from the full set of topics may be found in Supplementary Table 2. **B. Relationships between topic weight variability.** Pairwise correlations between the distributions of topic weight variance for each lecture and question set. Each row and column corresponds to a bar plot in Panel A.

189 CI = [-0.874, -0.281]), indicating that the topic model also exhibits some degree of specificity. The
190 full set of pairwise comparisons between average topic weights for the lectures and question sets
191 is reported in Supplementary Figure 2.

192 Another, more sensitive, way of summarizing the conceptual content of the lectures and ques-
193 tions is to look at *variability* in how topics are weighted over time and across different questions
194 (Fig. 3). Intuitively, the variability in the expression of a given topic relates to how much “infor-
195 mation” [22] the lecture (or question set) reflects about that topic. For example, suppose a given
196 topic is weighted on heavily throughout a lecture. That topic might be characteristic of some
197 aspect or property of the lecture *overall* (conceptual or otherwise), but unless the topic’s weights
198 changed in meaningful ways over time, the topic would be a poor indicator of any *specific* concep-
199 tual content in the lecture. We therefore also compared the variances in topic weights (across time
200 or questions) between the lectures and questions. The variability in topic expression (over time
201 and across questions) was similar for the Lecture 1 video and questions ($r(13) = 0.824$, $p < 0.001$,
202 95% CI = [0.696, 0.973]) and the Lecture 2 video and questions ($r(13) = 0.801$, $p < 0.001$, 95%

203 CI = [0.539, 0.958]). Simultaneously, as reported in Figure 3B, the variabilities in topic expression
204 across *different* videos and lecture-specific questions (i.e., Lecture 1 video vs. Lecture 2 questions;
205 Lecture 2 video vs. Lecture 1 questions) were negatively correlated, and neither video’s topic
206 variability was reliably correlated with the topic variability across general physics knowledge
207 questions. Taken together, the analyses reported in Figure 3 and Supplementary Figure 2 indicate
208 that a topic model fit to the videos’ transcripts can also reveal correspondences (at a coarse scale)
209 between the lectures and questions.

210 While an individual lecture may be organized around a single broad theme at a coarse scale,
211 at a finer scale, each moment of a lecture typically covers a narrower range of content. Given the
212 correspondence we found between the variabilities in topic expression across moments of each
213 lecture and questions from its corresponding set (Fig. 3), we wondered whether the text embedding
214 model might additionally capture these conceptual relationships at a finer scale. For example, if a
215 particular question asks about the content from one small part of a lecture, we wondered whether
216 the text embeddings could be used to automatically identify the “matching” moment(s) in the
217 lecture. To explore this, we computed the correlation between each question’s topic weights
218 and the topic weights for each second of its corresponding lecture, and found that each question
219 appeared to be temporally specific (Fig. 4). In particular, most questions’ topic vectors were
220 maximally correlated with a well-defined (and relatively narrow) range of timepoints from their
221 corresponding lectures, and the correlations fell off sharply outside of that range (Supp. Figs. 3, 4).
222 We also qualitatively examined the best-matching intervals for each question by comparing the
223 question’s text to the transcribed text from the most-correlated parts of the lectures (Supp. Tab. 3).
224 Despite that the questions were excluded from the text embedding model’s training set, in general
225 we found (through manual inspection) a close correspondence between the conceptual content
226 that each question probed and the content covered by the best-matching moments of the lectures.
227 Two representative examples are shown at the bottom of Figure 4.

228 The ability to quantify how much each question is “asking about” the content from each moment
229 of the lectures could enable high-resolution insights into participants’ knowledge. Traditional
230 approaches to estimating how much a student “knows” about the content of a given lecture entail



Figure 4: Which parts of each lecture are captured by each question? Each panel displays time series plots showing how each question’s topic vector correlates with each video timepoint’s topic vector (Panel A.: correlations for the *Four Fundamental Forces* lecture and associated questions; Panel B.: correlations for the *Birth of Stars* lecture and associated questions). The colors denote question identities. The diamonds in each panel denote the moment of peak correlation between the indicated question and the lecture trajectory. The associated questions’ text and snippets of the lectures’ transcripts from the surrounding 30 seconds, are displayed at the bottom of the figure.

231 administering some form of assessment (e.g., a quiz) and computing the proportion of correctly
 232 answered questions. But if two students receive identical scores on such an exam, might our
 233 modeling framework help us to gain more nuanced insights into the *specific* content that each
 234 student has mastered (or failed to master)? For example, a student who misses three questions that
 235 were all about the same concept (e.g., concept *A*) will have gotten the same *proportion* of questions
 236 correct as another student who missed three questions about three *different* concepts (e.g., *A, B*, and
 237 *C*). But if we wanted to help these two students fill in the “gaps” in their understandings, we might
 238 do well to focus specifically on concept *A* for the first student, but to also add in materials pertaining
 239 to concepts *B* and *C* for the second student. In other words, raw “proportion-correct” measures may
 240 capture *how much* a student knows, but not *what* they know. We wondered whether our modeling
 241 framework might enable us to (formally and automatically) infer participants’ knowledge at the
 242 scale of individual concepts (e.g., as captured by a single moment of a lecture).

243 We developed a simple formula (Eqn. 1) for using a participant’s responses to a small set

244 of multiple-choice questions to estimate how much the participant “knows” about the concept
245 reflected by any arbitrary coordinate x in text embedding space (e.g., the content reflected by
246 any moment in a lecture they had watched; see *Estimating dynamic knowledge traces*). Essentially,
247 the estimated knowledge at coordinate x is given by the weighted proportion of quiz questions
248 the participant answered correctly, where the weights reflect how much each question is “about”
249 the content at x . When we apply this approach to estimate the participant’s knowledge about
250 the content presented in each moment of each lecture, we can obtain a detailed time course
251 describing how much “knowledge” that participant has about the content presented at any part of
252 the lecture. As shown in Figure 5A and C, we can apply this approach separately for the questions
253 from each quiz participants took throughout the experiment. From just a few questions per quiz
254 (see *Estimating dynamic knowledge traces*), we obtain a high-resolution snapshot (at the time each
255 quiz was taken) of what the participants knew about any moment’s content, from either of the two
256 lectures they watched (comprising a total of 1,100 samples across the two lectures).

257 While the time courses in Figure 5A and C provide detailed *estimates* about participants’ knowl-
258 ege, these estimates are of course only *useful* to the extent that they accurately reflect what partic-
259 ipants actually know. As one sanity check, we anticipated that the knowledge estimates should
260 reflect a content-specific “boost” in participants’ knowledge after watching each lecture. In other
261 words, if participants learn about each lecture’s content upon watching it, the knowledge esti-
262 mated should capture that. After watching the *Four Fundamental Forces* lecture, participants should
263 exhibit more knowledge for the content of that lecture than they had before, and that knowledge
264 should persist for the remainder of the experiment. Specifically, knowledge about that lecture’s
265 content should be relatively low when estimated using Quiz 1 responses, but should increase
266 when estimated using Quiz 2 or 3 responses (Fig. 5B). Indeed, we found that participants’ esti-
267 mated knowledge about the content of *Four Fundamental Forces* was substantially higher on Quiz 2
268 versus Quiz 1 ($t(49) = 8.764, p < 0.001$) and on Quiz 3 versus Quiz 1 ($t(49) = 10.519, p < 0.001$).
269 We found no reliable differences in estimated knowledge about that lecture’s content on Quiz 2
270 versus 3 ($t(49) = 0.160, p = 0.874$). Similarly, we hypothesized (and subsequently confirmed)
271 that participants should show greater estimated knowledge about the content of the *Birth of Stars*



Figure 5: Estimating knowledge about the content presented at each moment of each lecture. **A. Knowledge about the time-varying content of *Four Fundamental Forces*.** Each trace displays the weighted proportion of correctly answered questions about the content reflected in each moment of the lecture (see *Estimating dynamic knowledge traces*), using responses from a single quiz (color). The traces are averaged across participants. **B. Average estimated knowledge about *Four Fundamental Forces*.** Each bar displays the across-timepoint average knowledge, estimated using the responses to one quiz's questions. **C. Knowledge about the time-varying content of *Birth of Stars*.** The panel is in the same format as Panel A, but here the knowledge estimates are for the moment-by-moment content of the *Birth of Stars* lecture. **D. Average estimated knowledge about *Birth of Stars*.** The panel is in the same format as Panel B, but here the knowledge estimates are for the content of the *Birth of Stars* lecture. All panels: error ribbons and error bars denote 95% confidence intervals, estimated across participants.

lecture after (versus before) watching it (Fig. 5D). Specifically, since participants watched that lecture after taking Quiz 2 (but before Quiz 3), we hypothesized that their knowledge estimates should be relatively low on Quizzes 1 and 2, but should show a “boost” on Quiz 3. Consistent with this prediction, we found no reliable differences in estimated knowledge about the *Birth of Stars* lecture content on Quizzes 1 versus 2 ($t(49) = 1.013, p = 0.316$), but the estimated knowledge was substantially higher on Quiz 3 versus 2 ($t(49) = 10.561, p < 0.001$) and Quiz 3 versus 1 ($t(49) = 8.969, p < 0.001$).

If we are able to accurately estimate a participant’s knowledge about the content tested by a given question, our estimates of their knowledge should carry some predictive information about whether they are likely to answer that question correctly or incorrectly. We developed a statistical approach to test this claim. For each quiz question a participant answered, in turn, we used Equation 1 to estimate their knowledge at the given question’s embedding-space coordinate based on other questions that participant answered on the same quiz. We repeated this for all participants, and for each of the three quizzes. Then, separately for each quiz, we fit a generalized linear mixed model (GLMM) with a logistic link function to explain the probability of correctly answering a question as a function of estimated knowledge for its embedding coordinate, while accounting for varied effects of individual participants and questions (see *Generalized linear mixed models*). To assess the predictive value of the knowledge estimates, we compared each GLMM to an analogous (i.e., nested) “null” model that assumed these estimates carried no predictive information using parametric bootstrap likelihood-ratio tests.

We carried out three different versions of the analyses described above, wherein we considered different sources of information in our estimates of participants’ knowledge for each quiz question. First, we estimated knowledge at each question’s embedding coordinate using *all* other questions answered by the same participant on the same quiz (“All questions”; Fig. 6, top row). This test was intended to assess the overall predictive power of our approach. Second, we estimated knowledge for each question about a given lecture using only the other questions (from the same participant and quiz) about that *same* lecture (“Within-lecture”; Fig. 6, middle rows). This test was intended to assess the *specificity* of our approach by asking whether our predictions could distinguish between



Figure 6: Predicting success on held-out questions using estimated knowledge. We used generalized linear mixed models (GLMMs) to model the probability of correctly answering a quiz question as a function of estimated knowledge for its embedding coordinate (see *Generalized linear mixed models*). Separately for each quiz (column), we examined this relationship based on three different sets of knowledge estimates: knowledge for each question based on all other questions the same participant answered on the same quiz (“All questions”; top row), knowledge for each question about one lecture based on all other questions (from the same participant and quiz) about the *same* lecture (“Within-lecture”; middle rows), and knowledge for each question about one lecture based on all questions (from the same participant and quiz) about the *other* lecture (“Across-lecture”; bottom rows). The backgrounds in each panel display kernel density estimates of the relative observed proportions of correctly (blue) versus incorrectly (red) answered questions, for each level of estimated knowledge along the x -axis. The black curves display the (population-level) GLMM-predicted probabilities of correctly answering a question as a function of estimated knowledge. Error ribbons denote 95% confidence intervals.

300 questions about different content covered by the same lecture. Third, we estimated knowledge
301 for each question about one lecture using only the questions (from the same participant and quiz)
302 about the *other* lecture (“Across-lecture”; Fig. 6, bottom rows). This test was intended to assess the
303 *generalizability* of our approach by asking whether our predictions could extend across the content
304 areas of the two lectures. When estimating participants’ knowledge, we used a rebalancing
305 procedure to ensure that (for a given participant and quiz) their knowledge estimates for correctly
306 and incorrectly answered questions were computed from the same underlying proportion of
307 correctly answered questions (see *Generalized linear mixed models*).

308 When we fit a GLMM to estimates of participants’ knowledge for each Quiz 1 question based on
309 all other Quiz 1 questions, we found that higher estimated knowledge for a given question predicted
310 a greater likelihood of answering it correctly (odds ratio (OR) = 8.126, 95% CI = [3.116, 20.123],
311 likelihood-ratio test statistic (λ_{LR}) = 17.002, $p < 0.001$). This relationship held when we repeated
312 this analysis for Quiz 2 (OR = 14.902, 95% CI = [4.976, 39.807], λ_{LR} = 25.408, $p < 0.001$) and again
313 for Quiz 3 (OR = 37.409, 95% CI = [10.425, 107.145], λ_{LR} = 40.948, $p < 0.001$). Taken together,
314 these results suggest that our knowledge estimates can reliably predict participants’ performance
315 on individual questions when they incorporate information from all (other) quiz content.

316 We observed a similar set of results when we restricted our estimates of participants’ knowl-
317 edge for questions about each lecture to consider only their performance on other questions
318 about the *same* lecture. Specifically, for Quiz 1, participants’ knowledge of *Four Fundamental Forces*-
319 related questions, estimated from their performance on other *Four Fundamental Forces*-related ques-
320 tions, was predictive of their ability to answer those questions correctly (OR = 15.934, 95% CI =
321 [5.173, 38.005], λ_{LR} = 40.971, $p = 0.001$). The same was true of participants’ estimated knowledge
322 for *Birth of Stars*-related questions based on their performance on other *Birth of Stars*-related ques-
323 tions (OR = 9.775, 95% CI = [2.93, 25.08], λ_{LR} = 13.924, $p = 0.001$). These within-lecture knowl-
324 edge estimates also predicted success on questions about both lectures when we computed them
325 analogously for Quiz 2 (*Four Fundamental Forces*: OR = 35.126, 95% CI = [5.113, 123.868], λ_{LR} =
326 32.251, $p < 0.001$; *Birth of Stars*: OR = 4.717, 95% CI = [2.021, 9.844], λ_{LR} = 16.788, $p < 0.001$).
327 For Quiz 3, we found that within-lecture knowledge estimates predicted participants’ success on

328 *Birth of Stars*-related questions ($OR = 16.902$, 95% CI = [3.353, 53.265], $\lambda_{LR} = 23.233$, $p < 0.001$)
329 but not on *Four Fundamental Forces*-related questions ($OR = 2.485$, 95% CI = [0.724, 8.366], $\lambda_{LR} =$
330 1.984, $p = 0.170$). This may indicate that the within-lecture knowledge estimates are susceptible to
331 ceiling effects in participants' quiz performance. On Quiz 3, after viewing both lectures, no partic-
332 ipant answered more than three *Four Fundamental Forces*-related questions incorrectly, and all but
333 five participants (out of 50) answered two or fewer questions incorrectly. (This was the only subset
334 of questions about either lecture, across all three quizzes, for which this was true.) Because of this,
335 when we held out one incorrectly answered *Four Fundamental Forces*-related question from a given
336 participant's Quiz 3 responses and estimated their knowledge at its embedding coordinate using
337 the remaining *Four Fundamental Forces*-related questions they answered, for 90% of participants,
338 that estimate leveraged information about at most a single other question they were *not* able to
339 correctly answer. This broad homogeneity in participants' success on questions used to estimate
340 their knowledge may have hurt our ability to accurately characterize the specific (and by Quiz 3,
341 relatively few) aspects of the lecture content they did *not* know about. Taken together, these results
342 suggest that our knowledge estimates can reliably distinguish between questions about different
343 content covered by a single lecture, provided there is sufficient diversity in participants' quiz
344 responses to extract meaningful information about both what they know and what they do not
345 know.

346 Finally, when we estimated participants' knowledge for each question about one lecture us-
347 ing their performance on questions (from the same quiz) about the *other* lecture, we observed
348 a somewhat different pattern of results. Here we found that before viewing either lecture (i.e.,
349 on Quiz 1), participants' abilities to answer *Four Fundamental Forces*-related questions could not
350 be predicted from their responses to *Birth of Stars*-related questions ($OR = 1.896$, 95% CI =
351 [0.419, 9.088], $\lambda_{LR} = 0.712$, $p = 0.404$), nor could their abilities to answer *Birth of Stars*-related
352 questions be predicted from their responses to *Four Fundamental Forces*-related questions ($OR =$
353 1.522, 95% CI = [0.332, 6.835], $\lambda_{LR} = 0.286$, $p = 0.611$). We similarly found that participants'
354 success on questions about either lecture could not be predicted given their responses to questions
355 about the other lecture after viewing *Four Fundamental Forces* but before viewing *Birth of Stars* (i.e.,

356 on Quiz 2; *Four Fundamental Forces* questions given *Birth of Stars* questions: $OR = 3.49$, 95% CI =
357 [0.739, 12.849], $\lambda_{LR} = 3.266$, $p = 0.083$; *Birth of Stars* questions given *Four Fundamental Forces*
358 questions: $OR = 2.199$, 95% CI = [0.711, 5.623], $\lambda_{LR} = 2.304$, $p = 0.141$). Only after viewing *both*
359 lectures (i.e., on Quiz 3) did these across-lecture knowledge estimates reliably predict participants'
360 success on individual quiz questions (*Four Fundamental Forces* questions given *Birth of Stars* ques-
361 tions: $OR = 11.294$, 95% CI = [1.375, 47.744], $\lambda_{LR} = 10.396$, $p < 0.001$; *Birth of Stars* questions given
362 *Four Fundamental Forces* questions: $OR = 7.302$, 95% CI = [1.077, 44.879], $\lambda_{LR} = 4.708$, $p = 0.038$).
363 Taken together, these results suggest that our knowledge estimates can be used to predict partic-
364 ipants' success across content areas once they have received some training on both the content
365 about which their knowledge is estimated and the content used to construct these estimates.

366 That the knowledge predictions derived from the text embedding space reliably distinguish
367 between held-out correctly versus incorrectly answered questions (Fig. 6) suggests that spatial
368 relationships within this space can help explain what participants know. But how far does this
369 explanatory power extend? For example, suppose we know that a participant correctly answered a
370 question at embedding coordinate x . As we move farther away from x in the embedding space, how
371 does the likelihood that the participant knows about the content at a given location "fall off" with
372 distance? Conversely, suppose the participant instead answered that same question *incorrectly*.
373 Again, as we move farther away from x in the embedding space, how does the likelihood that the
374 participant does *not* know about a coordinate's content change with distance? We reasoned that,
375 assuming our embedding space is capturing something about how individuals actually organize
376 their knowledge, a participant's ability to answer questions embedded very close to x should
377 tend to be similar to their ability to answer the question embedded *at* x . Whereas at another
378 extreme, once we reach some sufficiently large distance from x , our ability to infer whether or
379 not a participant will correctly answer a question based on their ability to answer the question
380 at x should be no better than guessing based on their *overall* proportion of correctly answered
381 questions. In other words, beyond the maximum distance at which the participant's ability to
382 answer the question at x is informative of their ability to answer a second question at location y ,
383 then guessing the outcome at y based on x should be no more successful than guessing based on a

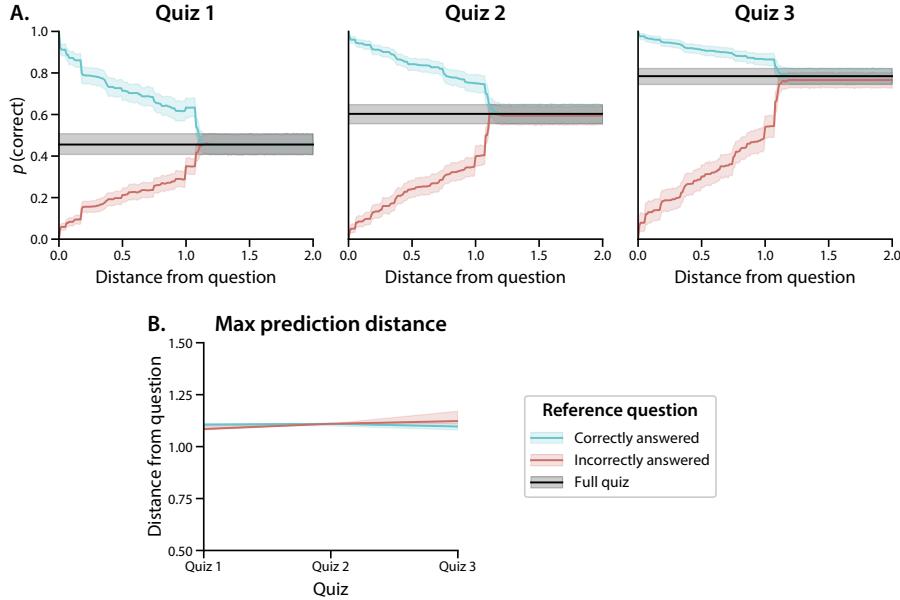


Figure 7: Knowledge falls off gradually in text embedding space. **A. Performance versus distance.** For each participant, for each correctly answered question (blue) or incorrectly answered question (red), we computed the proportion of correctly answered questions within a given distance of that question’s embedding coordinate. We used these proportions as a proxy for participants’ knowledge about the content within that region of the embedding space. We repeated this analysis for all questions and participants, and separately for each quiz (column). The black lines denote the average proportion correct across *all* questions included in the analysis at the given distance. **B. Maximum distance for which performance is reliably different from the average.** We used a bootstrap procedure (see *Estimating the “smoothness” of knowledge*) to estimate the point at which the blue and red lines in Panel A reliably diverged from the black line. We repeated this analysis separately for correctly and incorrectly answered questions from each quiz. **All panels.** Error ribbons denote bootstrap-estimated 95% confidence intervals.

384 measure that does not consider embedding-space distance.

385 With these ideas in mind, we asked: conditioned on answering a question correctly, what
 386 proportion of all questions (within some radius, r , of that question’s embedding coordinate)
 387 were answered correctly? We plotted this proportion as a function of r . Similarly, we could
 388 ask, conditioned on answering a question incorrectly, how the proportion of correct responses
 389 changed with r . As shown in Figure 7, we found that quiz performance falls off smoothly with
 390 distance, and the “rate” of the falloff does not appear to change across the different quizzes, as
 391 measured by the distance at which performance becomes statistically indistinguishable from a
 392 simple proportion-correct score (see *Estimating the “smoothness” of knowledge*). This suggests that,

393 at least within the region of text embedding space covered by the questions our participants
394 answered (and as characterized using our topic model), the rate at which knowledge changes
395 with distance is relatively constant, even as participants' overall level of knowledge varies across
396 quizzes or regions of the embedding space.

397 Knowledge estimates need not be limited to the contents of these particular lectures and quizzes.
398 As illustrated in Figure 8, our general approach to estimating knowledge from a small number
399 of quiz questions may be extended to *any* content, given its text embedding coordinate. To
400 visualize how knowledge "spreads" through text embedding space to content beyond the lectures
401 participants watched and the questions they answered, we first fit a new topic model to the lectures'
402 sliding windows with $k = 100$ topics. Conceptually, increasing the number of topics used by the
403 model functions to increase the "resolution" of the embedding space, providing a greater ability
404 to estimate knowledge for content that is highly similar to (but not precisely the same as) that
405 contained in the two lectures used to train the model. We note that we used these 2D maps solely
406 for visualization; all relevant comparisons, distance computations, and statistical tests we report
407 above were carried out in the original 15-dimensional space, using the 15-topic model. Aside from
408 increasing the number of topics from 15 to 100, all other procedures and model parameters were
409 carried over from the preceding analyses. As in our other analyses, we resampled each lecture's
410 topic trajectory to 1 Hz and projected each question into a shared text embedding space.

411 We projected the resulting 100-dimensional topic vectors (for each second of video and each quiz
412 question) onto a shared 2-dimensional plane (see *Creating knowledge and learning map visualizations*).
413 Next, we sampled points from a 100×100 grid of coordinates that evenly tiled a rectangle enclosing
414 the 2D projections of the videos and questions. We used Equation 4 to estimate participants'
415 knowledge at each of these 10,000 sampled locations, and averaged these estimates across participants
416 to obtain an estimated average *knowledge map* (Fig. 8A). Intuitively, the knowledge map
417 constructed from a given quiz's responses provides a visualization of "how much" participants
418 knew about any content expressible by the fitted text embedding model at the point in time when
419 they completed that quiz.

420 Several features of the resulting knowledge maps are worth noting. The average knowledge

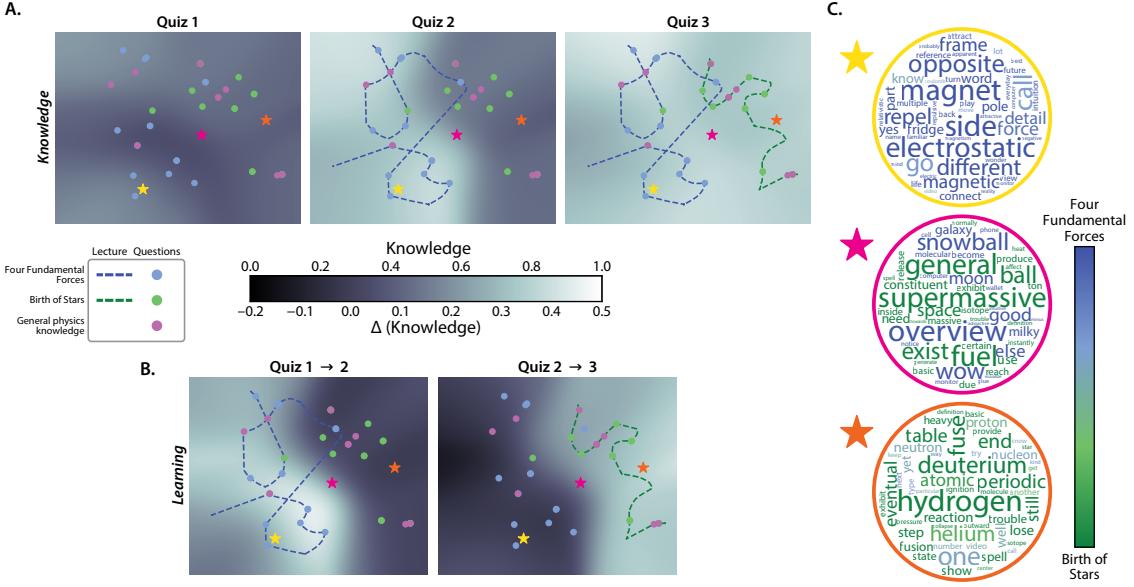


Figure 8: Mapping out the geometry of knowledge and learning. **A.** Average “knowledge maps” estimated using each quiz. Each map displays a 2D projection of the estimated knowledge about the content reflected by *all* regions of topic space (see *Creating knowledge and learning map visualizations*). The topic trajectories of the two lectures are indicated by dotted lines (blue: Lecture 1; green: Lecture 2), and the coordinates of each question are indicated by dots (light blue: Lecture 1-related; light green: Lecture 2-related; purple: general physics knowledge). Each map reflects an average across all participants. For individual participants’ maps, see Supplementary Figures 7, 8, and 9. **B.** Average “learning maps” estimated between each successive pair of quizzes. The learning maps follow the same general format as the knowledge maps in Panel A, but here the shading at each coordinate indicates the *difference* between the corresponding coordinates in the indicated pair of knowledge maps—i.e., how much the estimated knowledge “changed” between the two quizzes. Each map reflects an average across all participants. For individual participants’ maps, see Supplementary Figures 10 and 11. **C.** Word clouds for sampled points in topic space. Each word cloud displays the weighted blend of words underlying the topic proportions represented at the corresponding colored star’s location on the maps. In each word cloud, the words’ relative sizes correspond to their relative weights at the starred location, and their colors indicate their relative weights in *Four Fundamental Forces* (blue) versus *Birth of Stars* (green) lectures, on average, across all timepoints’ topic vectors.

421 map estimated from Quiz 1 responses (Fig. 8A, leftmost map) shows that participants tended to
422 have relatively little knowledge about any parts of the text embedding space (i.e., the shading is
423 relatively dark everywhere). The knowledge map estimated from Quiz 2 responses shows a marked
424 increase in knowledge on the left side of the map (around roughly the same range of coordinates
425 traversed by the *Four Fundamental Forces* lecture, indicated by the dotted blue line). In other words,
426 participants' estimated increase in knowledge is localized to conceptual content that is nearby (i.e.,
427 related to) the content from the lecture they watched prior to taking Quiz 2. This localization is
428 non-trivial: these knowledge estimates are informed only by the embedded coordinates of the
429 *quiz questions*, not by the embeddings of either lecture (see Eqn. 4). Finally, the knowledge map
430 estimated from Quiz 3 responses shows a second increase in knowledge, localized to the region
431 surrounding the embedding of the *Birth of Stars* lecture participants watched immediately prior to
432 taking Quiz 3.

433 Another way of visualizing these content-specific increases in knowledge after participants
434 viewed each lecture is displayed in Figure 8B. Taking the point-by-point difference between the
435 knowledge maps estimated from responses to a successive pair of quizzes yields a *learning map*
436 that describes the *change* in knowledge estimates from one quiz to the next. These learning maps
437 highlight that the estimated knowledge increases we observed across maps were specific to the
438 regions around the embeddings of each lecture, in turn.

439 Because the 2D projection we used to construct the knowledge and learning maps is invertible,
440 we may gain additional insights into these maps' meanings by reconstructing the original high-
441 dimensional topic vector for any location on the map we are interested in. For example, this could
442 serve as a useful tool for an instructor looking to better understand which content areas a student
443 (or a group of students) knows well (or poorly). As a demonstration, we show the top-weighted
444 words from the blends of topics reconstructed from three example locations on the maps (Fig. 8C):
445 one point near the *Four Fundamental Forces* embedding (yellow), a second point near the *Birth of*
446 *Stars* embedding (orange), and a third point between the two lectures' embeddings (pink). As
447 shown in the word clouds in the panel, the top-weighted words at the example coordinate near the
448 *Four Fundamental Forces* embedding tended to be weighted more heavily by the topics expressed

449 in that lecture. Similarly, the top-weighted words at the example coordinate near the *Birth of Stars*
450 embedding tended to be weighted more heavily by the topics expressed in *that* lecture. And the
451 top-weighted words at the example coordinate between the two lectures' embeddings show a
452 roughly even mix of words most strongly associated with each lecture.

453 Discussion

454 We developed a computational framework that uses short multiple-choice quizzes to gain nuanced
455 insights into what learners know and how their knowledge changes with training. First, we show
456 that our approach can automatically match the conceptual knowledge probed by individual quiz
457 questions to the corresponding moments in lecture videos when those concepts were presented
458 (Fig. 4). Next, we demonstrate how we can estimate moment-by-moment “knowledge traces”
459 that reflect the degree of knowledge participants have about each video’s time-varying content,
460 and capture temporally specific increases in knowledge after viewing each lecture (Fig. 5). We also
461 show that these knowledge estimates can generalize to held-out questions and predict participants’
462 abilities to answer them correctly (Fig. 6). Finally, we use our framework to construct visual maps
463 that provide snapshot estimates of how much participants know about any concept within the
464 scope of our text embedding model, and how much their knowledge of those concepts changes
465 with training (Fig. 8).

466 We view our work as making several contributions to the study of how people acquire con-
467 ceptual knowledge. First, from a methodological standpoint, our modeling framework provides
468 a systematic means of mapping out and characterizing knowledge in maps that have infinite (ar-
469 bitrarily many) numbers of coordinates, and of “filling out” those maps using relatively small
470 numbers of multiple choice quiz questions. Our experimental finding that we can use these maps
471 to predict responses to held-out questions has several psychological implications as well. For ex-
472 ample, concepts that are assigned to nearby coordinates by the text embedding model also appear
473 to be “known to a similar extent” (as reflected by participants’ responses to held-out questions;
474 Fig. 6). This suggests that participants also *conceptualize* similarly the content reflected by nearby

embedding coordinates. How participants' knowledge falls off with spatial distance is captured by the knowledge maps we infer from their quiz responses (e.g., Figs. 7, 8). In other words, our study shows that knowledge about a given concept implies knowledge about related concepts, and we also show how estimated knowledge falls off with distance in text embedding space.

In our study, we characterize the "coordinates" of participants' knowledge using a relatively simple "bag of words" text embedding model [LDA; 8]. More sophisticated text embedding models, such as transformer-based models [18, 54, 67, 70] can learn complex grammatical and semantic relationships between words, higher-order syntactic structures, stylistic features, and more. We considered using transformer-based models in our study, but we found that the text embeddings derived from these models were surprisingly uninformative with respect to differentiating or otherwise characterizing the conceptual content of the lectures and questions we used. We suspect that this reflects a broader challenge in constructing models that are high-resolution within a given domain (e.g., the domain of physics lectures and questions) *and* sufficiently broad so as to enable them to cover a wide range of domains. For example, we found that the embeddings derived even from much larger and more modern models like BERT [18], GPT [70], LLaMa [67], and others that are trained on enormous text corpora, end up yielding poor resolution within the content space spanned by individual course videos (Supp. Fig. 6). Whereas the LDA embeddings of the lectures and questions are "near" each other (i.e., the convex hull enclosing the two lectures' trajectories is highly overlapping with the convex hull enclosing the questions' embeddings), the BERT embeddings of the lectures and questions are instead largely distinct (top row of Supp. Fig. 6). The LDA embeddings of the questions for each lecture and the corresponding lecture's trajectory are also similar. For example, as shown in Fig. 2C, the LDA embeddings for *Four Fundamental Forces* questions (blue dots) appear closer to the *Four Fundamental Forces* lecture trajectory (blue line), whereas the LDA embeddings for *Birth of Stars* questions (green dots) appear closer to the *Birth of Stars* lecture trajectory (green line). The BERT embeddings of the lectures and questions do not show this property (Supp. Fig. 6). We also examined per-question "content matches" between individual questions and individual moments of each lecture (Fig. 4, Supp. Fig. 6). The time series plot of individual questions' correlations are different from each other when computed using LDA

503 (e.g., the traces can be clearly visually separated), whereas the correlations computed from BERT
504 embeddings of different questions all look very similar. This tells us that LDA is capturing some
505 differences in content between the questions, whereas BERT is not. The time series plots of individ-
506 ual questions' correlations have clear "peaks" when computed using LDA, but not when computed
507 using BERT. This tells us that LDA is capturing a "match" between the content of each question and
508 a relatively well-defined time window of the corresponding lectures. The BERT embeddings ap-
509 pear to blur together the content of the questions versus specific moments of each lecture. Finally,
510 we also compared the pairwise correlations between embeddings of questions within versus across
511 content areas (i.e., content covered by the individual lectures, lecture-specific questions, and by the
512 "general physics knowledge" questions). The LDA embeddings show a strong contrast between
513 same-content embeddings versus across-content embeddings. In other words, the embeddings of
514 questions about the *Four Fundamental Forces* material are highly correlated with the embeddings of
515 the *Four Fundamental Forces* lecture, but not with the embeddings of *Birth of Stars*, questions about
516 *Birth of Stars*, or general physics knowledge questions. We see a similar pattern with the LDA
517 embeddings of the *Birth of Stars* questions (Fig. 3, Supp. Fig. 2). In contrast, the BERT embeddings
518 are all highly correlated with each other (Supp. Fig. 6). Taken together, these comparisons illus-
519 trate how LDA (trained on the specific content in question) provides both coverage of the requisite
520 material and specificity at the level of the content covered by individual questions. BERT, on the
521 other hand, essentially assigns both lectures and all of the questions (which are all broadly about
522 "physics") into a tiny region of its embedding space, thereby blurring out meaningful distinctions
523 between different specific concepts covered by the lectures and questions. We note that these are
524 not criticisms of BERT (or other large language models trained on large and diverse corpora).
525 Rather, our point is that simple fine-tuned models trained on a relatively small but specialized
526 corpus can outperform much more complicated models trained on much larger corpora, when we
527 are specifically interested in capturing subtle conceptual differences at the level of a single course
528 lecture or question. Of course if our goal had been to find a model that generalized to many
529 different content areas, we would expect our approach to perform comparatively poorly relative to
530 BERT or other much larger models. We suggest that bridging the tradeoff between high resolution

531 within each content area versus the ability to generalize to many different content areas will be an
532 important challenge for future work in this domain.

533 Another application for large language models that does *not* require explicitly modeling the
534 content of individual lectures or questions is to leverage the models' abilities to generate text. For
535 example, generative text models like ChatGPT [54] and LLaMa [67] are already being used to build
536 a new generation of interactive tutoring systems [e.g., 44]. Unlike the approach we have taken here,
537 these generative text model-based systems do not explicitly model what learners know, or how
538 their knowledge changes over time with training. One could imagine building a hybrid system
539 that combines the best of both worlds: a large language model that can *generate* text, combined
540 with a smaller model that can *infer* what learners know and how their knowledge changes over
541 time. Such a hybrid system could potentially be used to build the next generation of interactive
542 tutoring systems that are able to adapt to learners' needs in real time, and that are able to provide
543 more nuanced feedback about what learners know and what they do not know.

544 At the opposite end of the spectrum from large language models, one could also imagine
545 *simplifying* some aspects of our LDA-based approach by computing simple word overlap metrics.
546 For example, the Jaccard similarity between text A and B is computed as the number of unique
547 words in the intersection of words from A and B divided by the number of unique words in the
548 union of words from A and B . In a supplementary analysis (Supp. Fig. 5), we compared the
549 LDA-based question-lecture matches we reported in Figure 4 with the Jaccard similarities between
550 each question and each sliding window of text from the corresponding lecture. As shown in
551 Supplementary Figure 5, this simple word-matching approach does not appear to capture the same
552 level of specificity as the LDA-based approach. Whereas the LDA-based approach often yields a
553 clear peak in the time series of correlations between each question and the corresponding lecture,
554 the Jaccard similarity-based approach does not. Furthermore, these LDA-based matches appear
555 to capture conceptual overlaps between the questions and lectures (Supp. Tab. 3), whereas simple
556 word matching does not. For example, one of the example questions examined in Supplementary
557 Figure 5 asks "Which of the following occurs as a cloud of atoms gets more dense?" The LDA-based
558 matches identify lecture timepoints where the relevant *topics* are discussed (e.g., when words like

559 “cloud,” “atom,” “dense,” etc., are mentioned *together*). The Jaccard similarity-based matches,
560 on the other hand, are strong when *any* of these words are mentioned, even if they do not occur
561 together.

562 We view our approach as occupying a sort of “sweet spot,” between much larger language
563 models and simple word matching-based approaches, that enables us to capture the relevant
564 conceptual content of course materials at an appropriate semantic scale. Our approach enables us
565 to accurately and consistently identify each question’s content in a way that also matches up with
566 what is presented in the lectures. In turn, this enables us to construct accurate predictions about
567 participants’ knowledge of the conceptual content tested by held-out questions (Fig. 6).

568 One limitation of our approach is that topic models contain no explicit internal representations
569 of more complex aspects of “knowledge,” like knowledge graphs, dependencies or associations
570 between concepts, causality, and so on. These representations might (in principle) be added
571 as extensions to our approach to more accurately and precisely capture, characterize, and track
572 learners’ knowledge. However, modeling these aspects of knowledge will likely require substantial
573 additional research effort.

574 Within the past several years, the global pandemic forced many educators to suddenly adapt to
575 teaching remotely [35, 51, 63, 71]. This change in world circumstances is happening alongside (and
576 perhaps accelerating) geometric growth in the availability of high-quality online courses from plat-
577 forms such as Khan Academy [36], Coursera [72], EdX [38], and others [59]. Continued expansion
578 of the global internet backbone and improvements in computing hardware have also facilitated
579 improvements in video streaming, enabling videos to be easily shared and viewed by increasingly
580 large segments of the world’s population. This exciting time for online course instruction provides
581 an opportunity to re-evaluate how we, as a global community, educate ourselves and each other.
582 For example, we can ask: what defines an effective course or training program? Which aspects of
583 teaching might be optimized and/or augmented by automated tools? How and why do learning
584 needs and goals vary across people? How might we lower barriers to receiving a high-quality
585 education?

586 Alongside these questions, there is a growing desire to extend existing theories beyond the

587 domain of lab testing rooms and into real classrooms [34]. In part, this has led to a recent
588 resurgence of “naturalistic” or “observational” experimental paradigms that attempt to better
589 reflect more ethologically valid phenomena that are more directly relevant to real-world situations
590 and behaviors [52]. In turn, this has brought new challenges in data analysis and interpretation. A
591 key step towards solving these challenges will be to build explicit models of real-world scenarios
592 and how people behave in them (e.g., models of how people learn conceptual content from real-
593 world courses, as in our current study). A second key step will be to understand which sorts
594 of signals derived from behaviors and/or other measurements [e.g., neurophysiological data; 4,
595 19, 49, 53, 56] might help to inform these models. A third major step will be to develop and
596 employ reliable ways of evaluating the complex models and data that are a hallmark of naturalistic
597 paradigms.

598 Beyond specifically predicting what people *know*, the fundamental ideas we develop here also
599 relate to the notion of “theory of mind” of other individuals [26, 32, 48]. Considering others’ unique
600 perspectives, prior experiences, knowledge, goals, etc., can help us to more effectively interact and
601 communicate [57, 62, 66]. One could imagine future extensions of our work (e.g., analogous to
602 the knowledge and learning maps shown in Fig. 8), that attempt to characterize how well-aligned
603 different people’s knowledge bases or backgrounds are. In turn, this might be used to model how
604 knowledge (or other forms of communicable information) flows not just between teachers and
605 students, but between friends having a conversation, individuals on a first date, participants at
606 a business meeting, doctors and patients, experts and non-experts, political allies or adversaries,
607 and more. For example, the extent to which two people’s knowledge maps “match” or “align” in
608 a given region of text embedding space might serve as a predictor of how effectively they will be
609 able to communicate about the corresponding conceptual content.

610 Ultimately, our work suggests a rich new line of questions about the geometric “form” of
611 knowledge, how knowledge changes over time, and how we might map out the full space of
612 what an individual knows. Our finding that detailed estimates about knowledge may be obtained
613 from short quizzes shows one way that traditional approaches to evaluation in education may be
614 extended. We hope that these advances might help pave the way for new approaches to teaching

615 or delivering educational content that are tailored to individual students' learning needs and goals.

616 **Materials and methods**

617 **Participants**

618 We enrolled a total of 50 Dartmouth undergraduate students in our study. Participants received
619 optional course credit for enrolling. We asked each participant to complete a demographic survey
620 that included questions about their age, gender, native spoken language, ethnicity, race, hearing,
621 color vision, sleep, coffee consumption, level of alertness, and several aspects of their educational
622 background and prior coursework.

623 Participants' ages ranged from 18 to 22 years (mean: 19.52 years; standard deviation: 1.09
624 years). A total of 15 participants reported their gender as male and 35 participants reported their
625 gender as female. A total of 49 participants reported their native language as "English" and 1
626 reported having another native language. A total of 47 participants reported their ethnicity as
627 "Not Hispanic or Latino" and three reported their ethnicity as "Hispanic or Latino." Participants
628 reported their races as White (32 participants), Asian (14 participants), Black or African American
629 (5 participants), American Indian or Alaska Native (1 participant), and Native Hawaiian or Other
630 Pacific Islander (1 participant). (Note that some participants selected multiple racial categories.)

631 A total of 49 participants reporting having normal hearing and 1 participant reported having
632 some hearing impairment. A total of 49 participants reported having normal color vision and 1
633 participant reported being color blind. Participants reported having had, on the night prior to
634 testing, 2–4 hours of sleep (1 participant), 4–6 hours of sleep (9 participants), 6–8 hours of sleep (35
635 participants), or 8+ hours of sleep (5 participants). They reported having consumed, on the same
636 day and leading up to their testing session, 0 cups of coffee (38 participants), 1 cup of coffee (10
637 participants), 3 cups of coffee (1 participant), or 4+ cups of coffee (1 participant).

638 No participants reported that their focus was currently impaired (e.g., by drugs or alcohol).
639 Participants reported their current level of alertness, and we converted their responses to numerical

640 scores as follows: “very sluggish” (-2), “a little sluggish” (-1), “neutral” (0), “fairly alert” (1), and
641 “very alert” (2). Across all participants, a range of alertness levels were reported (range: -2–1;
642 mean: -0.10; standard deviation: 0.84).

643 Participants reported their undergraduate major(s) as “social sciences” (28 participants), “nat-
644 ural sciences” (16 participants), “professional” (e.g., pre-med or pre-law; 8 participants), “mathe-
645 matics and engineering” (7 participants), “humanities” (4 participants), or “undecided” (3 partici-
646 pants). Note that some participants selected multiple categories for their undergraduate major(s).
647 We also asked participants about the courses they had taken. In total, 45 participants reported hav-
648 ing taken at least one Khan Academy course in the past, and 5 reported not having taken any Khan
649 Academy courses. Of those who reported having watched at least one Khan Academy course,
650 7 participants reported having watched 1–2 courses, 11 reported having watched 3–5 courses, 8
651 reported having watched 5–10 courses, and 19 reported having watched 10 or more courses. We
652 also asked participants about the specific courses they had watched, categorized under different
653 subject areas. In the “Mathematics” area, participants reported having watched videos on AP
654 Calculus AB (21 participants), Precalculus (17 participants), Algebra 2 (14 participants), AP Cal-
655 culus BC (12 participants), Trigonometry (11 participants), Algebra 1 (10 participants), Geometry
656 (8 participants), Pre-algebra (7 participants), Multivariable Calculus (5 participants), Differential
657 Equations (5 participants), Statistics and Probability (4 participants), AP Statistics (2 participants),
658 Linear Algebra (2 participants), Early Math (1 participant), Arithmetic (1 participant), and other
659 videos not listed in our survey (5 participants). In the “Science and engineering” area, participants
660 reported having watched videos on Chemistry, AP Chemistry, or Organic Chemistry (21 partic-
661 ipants); Physics, AP Physics I, or AP Physics II (18 participants); Biology, AP Biology; or High
662 school Biology (15 participants); Health and Medicine (1 participant); or other videos not listed
663 in our survey (5 participants). We also asked participants whether they had specifically seen the
664 videos used in our experiment. Of the 45 participants who reported having having taken at least
665 one Khan Academy course in the past, 44 participants reported that they had not watched the *Four*
666 *Fundamental Forces* video, and 1 participant reported that they were not sure whether they had
667 watched it. All participants reported that they had not watched the *Birth of Stars* video. When

668 we asked participants about non-Khan Academy online courses, they reported having watched
669 or taken courses on Mathematics (15 participants), Science and engineering (11 participants), Test
670 preparation (9 participants), Economics and finance (3 participants), Arts and humanities (2 partic-
671 ipants), Computing (2 participants), and other categories not listed in our survey (17 participants).
672 Finally, we asked participants about in-person courses they had taken in different subject areas.
673 They reported taking courses in Mathematics (38 participants), Science and engineering (37 par-
674 ticipants), Arts and humanities (34 participants), Test preparation (27 participants), Economics
675 and finance (26 participants), Computing (14 participants), College and careers (7 participants), or
676 other courses not listed in our survey (6 participants).

677 **Experiment**

678 We hand-selected two course videos from the Khan Academy platform: *Four Fundamental Forces*
679 (an introduction to gravity, electromagnetism, the weak nuclear force, and the strong nuclear force;
680 duration: 10 minutes and 29 seconds) and *Birth of Stars* (an introduction to how stars are formed;
681 duration: 7 minutes and 57 seconds). All participants viewed the videos in the same order (i.e.,
682 *Four Fundamental Forces* followed by *Birth of Stars*).

683 We then hand-created 39 multiple-choice questions: 15 about the conceptual content of *Four*
684 *Fundamental Forces* (i.e., Lecture 1), 15 about the conceptual content of *Birth of Stars* (i.e., Lecture 2),
685 and 9 questions that tested for general conceptual knowledge about basic physics (covering material
686 that was not presented in either video). To help broaden the set of lecture-specific questions,
687 our team worked through each lecture in small segments to identify what each segment was
688 “about” conceptually, and then write a question about that concept. The general physics questions
689 were drawn our team’s prior coursework and areas of interest, along with internet searches and
690 brainstorming with the project team and other members of J.R.M.’s lab. Although we attempted to
691 design the questions to test “conceptual knowledge,” we note that estimating the specific “amount”
692 of conceptual understanding that each question “requires” to answer is somewhat subjective, and
693 might even come down to the “strategy” a given participant uses to answer the question at that
694 particular moment. The full set of questions and answer choices may be found in Supplementary

695 Table 1. The final set of questions (and response options) was reviewed and approved by J.R.M.
696 before we collected or analyzed the text or experimental data.

697 Over the course of the experiment, participants completed three 13-question multiple-choice
698 quizzes: the first before viewing Lecture 1, the second between Lectures 1 and 2, and the third
699 after viewing Lecture 2 (see Fig. 1). The questions appearing on each quiz, for each participant,
700 were randomly chosen from the full set of 39, with the constraints that (a) each quiz contained
701 exactly 5 questions about Lecture 1, 5 questions about Lecture 2, and 3 questions about general
702 physics knowledge, and (b) each question appear exactly once for each participant. The orders of
703 questions on each quiz, and the orders of answer options for each question, were also randomized.
704 We obtained informed consent from all participants, and our experimental protocol was approved
705 by the Committee for the Protection of Human Subjects at Dartmouth College. We used this
706 experiment to develop and test our computational framework for estimating knowledge and
707 learning.

708 **Analysis**

709 **Statistics**

710 All of the statistical tests performed in our study were two-sided. The 95% confidence intervals
711 we reported for each correlation were estimated from bootstrap distributions of 10,000 correlation
712 coefficients obtained by sampling (with replacement) from the observed data.

713 **Constructing text embeddings of multiple lectures and questions**

714 We adapted an approach we developed in prior work [29] to embed each moment of the two
715 lectures and each question in our pool in a common representational space. Briefly, our approach
716 uses a topic model [Latent Dirichlet Allocation; 8] trained on a set of documents, to discover a set
717 of k “topics” or “themes.” Formally, each topic is defined as a distribution of weights over words in
718 the model’s vocabulary (i.e., the union of all unique words, across all documents, excluding “stop
719 words”). Conceptually, each topic is intended to give larger weights to words that are semantically

720 related (as inferred from their tendency to co-occur in the same document). After fitting a topic
721 model, each document in the training set, or any *new* document that contains at least some of
722 the words in the model’s vocabulary, may be represented as a k -dimensional vector describing
723 how much the document (most probably) reflects each topic. To select an appropriate k for our
724 model, as a starting point, we identified the minimum number of topics that yielded at least one
725 “unused” topic (i.e., in which all words in the vocabulary were assigned uniform weights) after
726 training. This indicated that the number of topics was sufficient to capture the set of latent themes
727 present in the two lectures (from which we constructed our document corpus, as described below).
728 We found this value to be $k = 15$ topics. We found that with a limited number of additional
729 adjustments following Boyd-Graber et al. [9], such as removing corpus-specific stop-words, the
730 model yielded (subjectively) sensible and coherent topics. The distribution of weights over words
731 in the vocabulary for each discovered topic is shown in Supplementary Figure 1, and each topic’s
732 top-weighted words may be found in Supplementary Table 2.

733 As illustrated in Figure 2A, we start by building up a corpus of documents using overlapping
734 sliding windows that span each video’s transcript. Khan Academy provides professionally created,
735 manual transcriptions of all videos for closed captioning. However, such transcripts would not
736 be readily available in all contexts to which our framework could potentially be applied. Khan
737 Academy videos are hosted on the YouTube platform, which additionally provides automated
738 captions. We opted to use these automated transcripts [which, in prior work, we have found to be
739 of sufficiently near-human quality to yield reliable data in behavioral studies; 73] when developing
740 our framework in order to make it more directly extensible and adaptable by others in the future.

741 We fetched these automated transcripts using the `youtube-transcript-api` Python pack-
742 age [17]. The transcripts consisted of one timestamped line of text for every few seconds (mean:
743 2.34 s; standard deviation: 0.83 s) of spoken content in the video (i.e., corresponding to each indi-
744 vidual caption that would appear on-screen if viewing the lecture via YouTube, and when those
745 lines would appear). We defined a sliding window length of (up to) $w = 30$ transcript lines, and
746 assigned each window a timestamp corresponding to the midpoint between the timestamps for its
747 first and last lines. This w parameter was chosen to match the same number of words per sliding

748 window (rounded to the nearest whole word, and before preprocessing) as the sliding windows
749 we defined in our prior work [29; i.e., 185 words per sliding window].

750 These sliding windows ramped up and down in length at the beginning and end of each
751 transcript, respectively. In other words, each transcript’s first sliding window covered only its first
752 line, the second sliding window covered the first two lines, and so on. This ensured that each line
753 from the transcripts appeared in the same number (w) of sliding windows. We next performed a
754 series of standard text preprocessing steps: normalizing case, lemmatizing, removing punctuation
755 and removing stop-words. We constructed our corpus of stop words by augmenting the Natural
756 Language Toolkit [NLTK; 5] English stop word list with the following additional words, selected
757 using one of the approaches suggested by Boyd-Graber et al. [9]: “actual,” “actually,” “also,” “bit,”
758 “could,” “e,” “even,” “first,” “follow,” “following,” “four,” “let,” “like,” “mc,” “really,”, “saw,”
759 “see,” “seen,” “thing,” and “two.” This yielded sliding windows with an average of 73.8 remaining
760 words, and lasting for an average of 62.22 seconds. We treated the text from each sliding window
761 as a single “document,” and combined these documents across the two videos’ windows to create
762 a single training corpus for the topic model.

763 After fitting a topic model to the two videos’ transcripts, we could use the trained model to
764 transform arbitrary (potentially new) documents into k -dimensional topic vectors. A convenient
765 property of these topic vectors is that documents that reflect similar blends of topics (i.e., documents
766 that reflect similar themes, according to the model) will yield similar coordinates (in terms of
767 correlation, cosine similarity, Kullback-Leibler divergence, Euclidean distance, or other geometric
768 measures). In general, the similarity between different documents’ topic vectors may be used to
769 characterize the similarity in conceptual content between the documents.

770 We transformed each sliding window’s text into a topic vector, and then used linear interpolation
771 (independently for each topic dimension) to resample the resulting time series to one vector
772 per second. We also used the fitted model to obtain topic vectors for each question in our pool (see
773 Supp. Tab. 1). Taken together, we obtained a *trajectory* for each video, describing its path through
774 topic space, and a single coordinate for each question (Fig. 2C). Embedding both videos and all of
775 the questions using a common model enables us to compare the content from different moments

776 of videos, compare the content across videos, and estimate potential associations between specific
777 questions and specific moments of video.

778 **Estimating dynamic knowledge traces**

779 We used the following equation to estimate each participant’s knowledge about timepoint t of a
780 given lecture, $\hat{k}(t)$:

$$\hat{k}(f(t, L)) = \frac{\sum_{i \in \text{correct}} \text{ncorr}(f(t, L), f(i, Q))}{\sum_{j=1}^N \text{ncorr}(f(t, L), f(j, Q))}, \quad (1)$$

781 where

$$\text{ncorr}(x, y) = \frac{\text{corr}(x, y) - \text{mincorr}}{\text{maxcorr} - \text{mincorr}}, \quad (2)$$

782 and where mincorr and maxcorr are the minimum and maximum correlations between any lecture
783 timepoint and question, taken over all timepoints in the given lecture and all questions *about* that
784 lecture appearing on the given quiz. We also define $f(s, \Omega)$ as the s^{th} topic vector from the set of
785 topic vectors Ω . Here t indexes the set of lecture topic vectors L , and i and j index the topic vectors
786 of questions Q used to estimate the knowledge trace. Note that “correct” denotes the set of indices
787 of the questions the participant answered correctly on the given quiz.

788 Intuitively, $\text{ncorr}(x, y)$ is the correlation between two topic vectors (e.g., the topic vector x
789 for one timepoint in a lecture and the topic vector y for one question on a quiz), normalized
790 by the minimum and maximum correlations (across all timepoints t and questions j) to range
791 between 0 and 1, inclusive. Equation 1 then computes the weighted average proportion of correctly
792 answered questions about the content presented at timepoint t , where the weights are given by the
793 normalized correlations between timepoint t ’s topic vector and the topic vectors for each question.
794 The normalization step (i.e., using ncorr instead of the raw correlations) ensures that every question
795 contributes some non-negative amount to the knowledge estimate.

796 **Generalized linear mixed models**

797 In the set of analyses reported in Figure 6, we assessed whether estimates of participants' knowl-
798 edge at the embedding coordinates of individual quiz questions could be used to reliably predict
799 their abilities to correctly answer those questions. In essence, we treated each question a given
800 participant answered on a given quiz as a "lecture" consisting of a single timepoint, and used
801 Equation 1 to estimate the participant's knowledge for its embedding coordinate based on their
802 performance on all *other* questions they answered on that same quiz ("All questions"; Fig. 6,
803 top row). Additionally, for each lecture-related question (i.e., excluding questions about general
804 physics knowledge), we computed analogous knowledge estimates based on two different subsets
805 of questions the participant answered on the same quiz: (1) all *other* questions about the same
806 lecture as the target question ("Within-lecture"; Fig. 6, middle rows), and (2) all questions about
807 the other of the two lectures ("Across-lecture"; Fig. 6, bottom rows).

808 In performing these analyses, our null hypothesis is that the knowledge estimates we compute
809 based on the quiz questions' embedding coordinates do *not* provide useful information about
810 participants' abilities to answer those questions—in other words, that there is no meaningful
811 difference (on average) between the knowledge estimates we compute for questions participants
812 answered correctly and those they answered incorrectly. Specifically, since we estimate knowledge
813 for a given embedding coordinate as a weighted proportion-correct score (where each question's
814 weight reflects its embedding-space distance from the target coordinate; see Eqn. 1), if these weights
815 are uninformative (e.g., randomly distributed), then our estimates of participants' knowledge
816 should be equivalent (on average) to the *unweighted* proportion of correctly answered questions
817 used to compute them. In general, for a given participant and quiz, this expected value (i.e.,
818 that participant's proportion-correct score on that quiz) is the same for any coordinate in the
819 embedding space (e.g., any lecture timepoint, quiz question, etc.). However, in the "All questions"
820 and "Within-lecture" versions of the analyses shown in Figure 6, we estimate each participant's
821 knowledge for each target question using all *other* questions (or all *other* questions about the same
822 lecture) they answered on the same quiz. This introduces a systematic dependency between

823 a participant’s success on a target question and their proportion-correct score on the remaining
824 questions available to estimate their knowledge for it. For example, suppose a participant correctly
825 answered n out of q questions on a given quiz. If we hold out a single *correctly* answered question as
826 the target, the proportion of remaining questions answered correctly would be $\frac{n-1}{q-1}$. Whereas if we
827 hold out a single *incorrectly* answered question, the proportion of remaining questions answered
828 correctly would be $\frac{n}{q-1}$. Thus, the proportion of correctly answered remaining questions (and
829 therefore the null-hypothesized value of a knowledge estimate computed from them) is always
830 *lower* for target questions a participant answered correctly than for those they answered incorrectly.

831 To correct for this baseline inverse relationship between a participant’s success on a target
832 question and their estimated knowledge for it, we used a rebalancing procedure that ensured
833 our knowledge estimates for questions each participant answered correctly and incorrectly were
834 computed from the *same* proportion of correctly answered questions. For each target question on
835 a given participant’s quiz, we identified all remaining questions with the opposite “correctness”
836 label (i.e., if the target question was answered correctly, we identified all remaining incorrectly
837 answered questions, and vice versa). We then held out each of these opposite-label questions,
838 in turn, along with the target question, and estimated the participant’s knowledge for the target
839 question using all *other* remaining questions. Since each of these subsets of remaining questions
840 was constructed by holding out one correctly answered question and one incorrectly answered
841 question from the participant’s quiz, if the participant correctly answered n out of q questions total,
842 then their proportion-correct score on each subset of questions used to estimate their knowledge
843 for the target question is $\frac{n-1}{q-2}$, regardless of whether they answered the target question correctly
844 or incorrectly. Finally, averaging over these per-subset knowledge estimates yielded a rebalanced
845 estimate of the participant’s knowledge for the target question that leveraged information from all
846 remaining questions’ embedding coordinates, but whose expected value under our null hypothesis
847 was the same as that of each individual subset ($\frac{n-1}{q-2}$). By equalizing the null-hypothesized values of
848 knowledge estimates for correctly and incorrectly answered questions, this procedure ensures that
849 any meaningful relationships we observe between participants’ estimated knowledge for individ-
850 ual quiz questions and their abilities to correctly answer them are attributable to the predictive

power of the embedding-space distances used to weight questions' contributions to the knowledge estimates, rather than an artifact of our estimation procedure. Note that if a participant answered all or no questions on a given quiz correctly, their responses contained no opposite-label questions with which to perform this rebalancing, and we therefore excluded their data from our analyses for that quiz. We used this rebalancing procedure when constructing knowledge estimates for the "All questions" and "Within-lecture" versions of the analyses shown in Figure 6, but not for the "Across-lecture" analyses as, in this case, the target questions and the questions used to estimate participants' knowledge for them were drawn from different subsets of quiz questions (those about one lecture, and those about the other), and were therefore independent.

In each version of this analysis (i.e., row in Fig. 6), and separately for each of the three quizzes (i.e., column in Fig. 6), we then fit a generalized linear mixed model (GLMM) with a logistic link function to the set of knowledge estimates for all questions (or all questions about a particular lecture) that participants answered on the given quiz. We implemented these models in R using the `lme4` package [3] and fit them following guidance from Bates et al. [2] and Matuschek et al. [45]. Specifically, we initially fit each model with the maximal random effects structure afforded by our design, which we identified as:

$$\text{accuracy} \sim \text{knowledge} + (\text{knowledge} | \text{participant}) + (\text{knowledge} | \text{question})$$

where "accuracy" is a binary value indicating whether each target question was answered correctly or incorrectly, "knowledge" is estimated knowledge at each target question's embedding coordinate, "participant" is a unique identifier assigned to each participant, and "question" is a unique identifier assigned to each quiz question. For models we fit using knowledge estimates for target questions about multiple content areas (i.e., in the "All questions" version of the analysis), we also included an additional random effect term, $(\text{knowledge} | \text{lecture})$, where "lecture" is a categorical value denoting whether the target question was about *Four Fundamental Forces*, *Birth of Stars*, or general physics knowledge. Note that with our coding scheme, identifiers for each question are implicitly nested within levels of lecture and so do not require explicit nesting in

876 our model formula. We then iteratively removed random effects from the maximal model until it
877 successfully converged with a full-rank random effects variance-covariance matrix. We obtained
878 the odds ratios reported in Figure 6 by exponentiating the estimated coefficient for “knowledge”
879 from each fitted model. Conceptually, these odds ratios represent how many times greater the odds
880 are that a given participant will answer a given question correctly if their estimated knowledge
881 for its embedding coordinate is 1, compared to if it is 0. We estimated 95% confidence intervals
882 for each odds ratio by generating 10,000 random subsamples (of full size, with replacement) from
883 the data used to fit each model, and refitting the models to each subsample to obtain bootstrap
884 distributions of 10,000 odds ratios.

885 To assess the predictive value of our knowledge estimates, we compared each GLMM’s ability
886 to explain participants’ success on individual quiz questions to that of an analogous model which
887 assumed (as we assume under our null hypothesis) that knowledge estimates for correctly and
888 incorrectly answered questions did *not* systematically differ, on average. Specifically, we used the
889 same sets of observations with which we fit each “full” model to fit a second “null” model that had
890 the same random effects structure, but in which the coefficient for the fixed effect of “knowledge”
891 was fixed at 0 (i.e., we removed this term from the null model). We then compared each full model
892 to its reduced (null) equivalent using a likelihood-ratio test (LRT). Because the standard asymptotic
893 χ_d^2 approximation of the null distribution for the LRT statistic (λ_{LR}) can be anti-conservative for
894 finite sample sizes [25, 60, 65], we computed p -values for these tests using a parametric bootstrap
895 procedure [14, 27]. For each of 10,000 bootstraps, we used the fitted null model to simulate a
896 sample of observations of equal size to our original sample. We then re-fit both the null and
897 full models to this simulated sample and compared them via an LRT. This yielded a distribution
898 of λ_{LR} statistics we may expect to observe given data that conforms to our null hypothesis. We
899 computed a corrected p -value for our observed λ_{LR} as $\frac{r+1}{n+1}$, where r is the number of simulated
900 model comparisons that yielded a λ_{LR} greater than our observed value and n is the number of
901 simulations we ran (10,000).

902 **Estimating the “smoothness” of knowledge**

903 In the analysis reported in Figure 7A, we show how participants’ ability to correctly answer
904 quiz questions changes as a function of distance from a given correctly or incorrectly answered
905 reference question. We used a bootstrap-based approach to estimate the maximum distances over
906 which these proportions of correctly answered questions could be reliably distinguished from
907 participants’ overall average proportion of correctly answered questions.

908 For each of 10,000 iterations, we drew a random subsample (with replacement) of 50 partic-
909 ipants from our dataset. Within each iteration, we first computed the 95% confidence interval
910 (CI) of the across-subsample-participants mean proportion correct on each of the three quizzes,
911 separately. To compute this interval for each quiz, we repeatedly (1,000 times) subsampled par-
912 ticipants (with replacement, from the outer subsample for the current iteration) and computed
913 the mean proportion correct of each of these inner subsamples. We then identified the 2.5th and
914 97.5th percentiles of the resulting distributions of 1,000 means. These three intervals (one for each
915 quiz) served as our thresholds for confidence that the proportion correct within a given distance
916 from a reference question was reliably different (at the $p < 0.05$ significance level) from the average
917 proportion correct across all questions on the given quiz.

918 Next, for each participant in the current subsample, and for each of the three quizzes they
919 completed (separately), we iteratively treated each of the 15 questions appearing on the given
920 quiz as the “reference” question. We constructed a series of concentric 15-dimensional “spheres”
921 centered on the reference question’s embedding-space coordinate, where each successive sphere’s
922 radius increased by 0.01 (correlation distance) between 0 and 2, inclusive (i.e., tiling the range
923 of possible correlation distances with 201 spheres in total). We then computed the proportion
924 of questions enclosed within each sphere that the participant answered correctly, and averaged
925 these per-radius proportion-correct scores across reference questions that were answered correctly,
926 and those that were answered incorrectly. This resulted in two number-of-spheres sequences of
927 proportion-correct scores for each subsample participant and quiz: one derived from correctly
928 answered reference questions, and one derived from incorrectly answered reference questions.

929 We computed the across-subsample-participants mean proportion correct for each radius value
930 (i.e., sphere) and “correctness” of reference question. This yielded two sequences of proportion-
931 correct scores for each quiz, analogous to the blue and red lines displayed in Figure 7A, but for
932 the present subsample. For each quiz, we then found the minimum distance from the reference
933 question (i.e., sphere radius) at which each of these two sequences of per-radius proportion-correct
934 scores intersected the 95% confidence interval for the overall proportion correct (i.e., analogous to
935 the black error bands in Fig. 7A).

936 This resulted in two “intersection” distances for each quiz (for correctly answered and incor-
937 rectly answered reference questions). Repeating this full process for each of the 10,000 bootstrap
938 iterations output two distributions of intersection distances for each of the three quizzes. The
939 means and 95% confidence intervals for these distributions are plotted in Figure 7B.

940 **Creating knowledge and learning map visualizations**

941 An important feature of our approach is that, given a trained text embedding model and partic-
942 ipants’ quiz performance on each question, we can estimate their knowledge about *any* content
943 expressible by the embedding model—not solely the content explicitly probed by the quiz ques-
944 tions, or even appearing in the lectures. To visualize these estimates (Fig. 8, Supp. Figs. 7, 8, 9, 10,
945 and 11), we used Uniform Manifold Approximation and Projection [UMAP; 46, 47] to construct a
946 2D projection of the text embedding space. Whereas our main analyses used a 15-topic embedding
947 space, we used a 100-topic embedding space for these visualizations. This change in the number
948 of topics overcame an undesirable behavior in the UMAP embedding procedure, whereby embed-
949 ding coordinates for the 15-topic model tended to be “clumped” into separated clusters, rather
950 than forming a smooth trajectory through the 2D space. When we increased the number of topics
951 to 100, the embedding coordinates in the 2D space formed a smooth trajectory through the space,
952 with substantially less clumping (Fig. 8). Creating a “map” by sampling this 100-dimensional
953 space at high resolution to obtain an adequate set of topic vectors spanning the embedding space
954 would be computationally intractable. However, sampling a 2D grid is trivial.

955 At a high level, the UMAP algorithm obtains low-dimensional embeddings by minimizing

956 the cross-entropy between the pairwise (clustered) distances between the observations in their
957 original (e.g., 100-dimensional) space and the pairwise (clustered) distances in the low-dimensional
958 embedding space (in our approach, the embedding space is 2D). In our implementation, pairwise
959 distances in the original high-dimensional space were defined as 1 minus the correlation between
960 each pair of coordinates, and pairwise distances in the low-dimensional embedding space were
961 defined as the Euclidean distance between each pair of coordinates.

962 In our application, all of the coordinates we embedded were topic vectors, whose elements
963 are always non-negative and sum to one. Although UMAP is an invertible transformation at
964 the embedding locations of the original data, other locations in the embedding space will not
965 necessarily follow the same implicit “rules” as the original high-dimensional data. For example,
966 inverting an arbitrary coordinate in the embedding space might result in negative-valued vectors,
967 which are incompatible with the topic modeling framework. To protect against this issue, we
968 log-transformed the topic vectors prior to embedding them in the 2D space. When we inverted
969 the embedded vectors (e.g., to estimate topic vectors for word clouds, as in Fig. 8C), we passed
970 the inverted (log-transformed) values through the exponential function to obtain a vector of non-
971 negative values, and normalized them to sum to one.

972 After embedding both lectures’ topic trajectories and the topic vectors of every question, we
973 defined a rectangle enclosing the 2D projections of the lectures’ and quizzes’ embeddings. We then
974 sampled points from a regular 100×100 grid of coordinates that evenly tiled this enclosing rectangle.
975 We sought to estimate participants’ knowledge (and learning, i.e., changes in knowledge) at each
976 of the resulting 10,000 coordinates.

977 To generate our estimates, we placed a set of 39 radial basis functions (RBFs) throughout the
978 embedding space, centered on the 2D projections for each question (i.e., we included one RBF for
979 each question). At coordinate x , the value of an RBF centered on a question’s coordinate μ , is given
980 by:

$$\text{RBF}(x, \mu, \lambda) = \exp \left\{ -\frac{\|x - \mu\|^2}{\lambda} \right\}. \quad (3)$$

981 The λ term in the RBF equation controls the “smoothness” of the function, where larger values

982 of λ result in smoother maps. In our implementation we used $\lambda = 50$. Next, we estimated the
983 “knowledge” at each coordinate, x , using:

$$\hat{k}(x) = \frac{\sum_{i \in \text{correct}} \text{RBF}(x, q_i, \lambda)}{\sum_{j=1}^N \text{RBF}(x, q_j, \lambda)}. \quad (4)$$

984 Intuitively, Equation 4 computes the weighted proportion of correctly answered questions, where
985 the weights are given by how nearby (in the 2D space) each question is to the x . We also defined
986 *learning maps* as the coordinate-by-coordinate differences between any pair of knowledge maps.
987 Intuitively, learning maps reflect the *change* in knowledge across two maps.

988 Author contributions

989 Conceptualization: P.C.F., A.C.H., and J.R.M. Methodology: P.C.F., A.C.H., and J.R.M. Software:
990 P.C.F. Validation: P.C.F. Formal analysis: P.C.F. Resources: P.C.F., A.C.H., and J.R.M. Data curation:
991 P.C.F. Writing (original draft): J.R.M. Writing (review and editing): P.C.F., A.C.H., and J.R.M. Visu-
992 alization: P.C.F. and J.R.M. Supervision: J.R.M. Project administration: P.C.F. Funding acquisition:
993 J.R.M.

994 Data availability

995 All of the data analyzed in this manuscript may be found at <https://github.com/ContextLab/efficient-learning-khan>.

997 Code availability

998 All of the code for running our experiment and carrying out the analyses may be found at
999 <https://github.com/ContextLab/efficient-learning-khan>.

1000 **Acknowledgements**

1001 We acknowledge useful discussions, assistance in setting up an earlier (unpublished) version of
1002 this study, and assistance with data collection efforts from Will Baxley, Max Bluestone, Daniel
1003 Carstensen, Kunal Jha, Caroline Lee, Lucy Owen, Xinming Xu, and Kirsten Ziman. Our work was
1004 supported in part by NSF CAREER Award Number 2145172 to J.R.M. The content is solely the
1005 responsibility of the authors and does not necessarily represent the official views of our supporting
1006 organizations. The funders had no role in study design, data collection and analysis, decision to
1007 publish, or preparation of the manuscript.

1008 **References**

- 1009 [1] Ashby, F. G. and Maddox, W. T. (2005). Human category learning. *Annual Review of Psychology*,
1010 56:149–178.
- 1011 [2] Bates, D., Kliegl, R., Vasishth, S., and Baayen, H. (2015a). Parsimonious mixed models. *arXiv*,
1012 1506.04967.
- 1013 [3] Bates, D., Mächler, M., Bolker, B., and Walker, S. (2015b). Fitting linear mixed-effects models
1014 using lme4. *Journal of Statistical Software*, 67(1):1–48.
- 1015 [4] Bevilacqua, D., Davidesco, I., Wan, L., and Chaloner, K. (2019). Brain-to-brain synchrony and
1016 learning outcomes vary by student-teacher dynamics: evidence from a real-world classroom
1017 electroencephalography study. *Journal of Cognitive Neuroscience*, 31(3):401–411.
- 1018 [5] Bird, S., Klein, E., and Loper, E. (2009). *Nature language processing with Python: analyzing text
1019 with the natural language toolkit*. Reilly Media, Inc.
- 1020 [6] Blaye, A., Bernard-Peyron, V., Paour, J.-L., and Bonthoux, F. (2006). Category flexibility in chil-
1021 dren: distinguishing response flexibility from conceptual flexibility; the protracted development
1022 of taxonomic representations. *European Journal of Developmental Psychology*, 3(2):163–188.

- 1023 [7] Blei, D. M. and Lafferty, J. D. (2006). Dynamic topic models. In *Proceedings of the International*
1024 *Conference on Machine Learning*, pages 113–120, New York, NY. Association for Computing
1025 Machinery.
- 1026 [8] Blei, D. M., Ng, A. Y., and Jordan, M. I. (2003). Latent dirichlet allocation. *Journal of Machine*
1027 *Learning Research*, 3:993–1022.
- 1028 [9] Boyd-Graber, J., Mimno, D., and Newman, D. (2014). Care and feeding of topic models:
1029 problems, diagnostics, and improvements. In Airolidi, E. M., Blei, D. M., Erosheva, E. A., and
1030 Fienberg, S. E., editors, *Handbook of Mixed Membership Models and Their Applications*. CRC Press.
- 1031 [10] Brown, T. B., Mann, B., Ryder, N., Subbiah, M., Kaplan, J., Dhariwal, P., Neelakantan, A.,
1032 Shyam, P., Sastry, G., Askell, A., Agarwal, S., Herbert-Voss, A., Krueger, G., Henighan, T., Child,
1033 R., Ramesh, A., Ziegler, D. M., Wu, J., Winter, C., Hesse, C., Chen, M., Sigler, E., Litwin, M.,
1034 Gray, S., Chess, B., Clark, J., Berner, C., McCandlish, S., Radford, A., Sutskever, I., and Amodei,
1035 D. (2020). Language models are few-shot learners. *arXiv*, 2005.14165.
- 1036 [11] Caramazza, A. and Mahon, B. Z. (2003). The organization of conceptual knowledge: the
1037 evidence from category-specific semantic deficits. *Trends in Cognitive Sciences*, 7(8):354–361.
- 1038 [12] Cer, D., Yang, Y., Kong, S. Y., Hua, N., Limtiaco, N., John, R. S., Constant, N., Guajardo-
1039 Cespedes, M., Yuan, S., Tar, C., Sung, Y.-H., Strope, B., and Kurzweil, R. (2018). Universal
1040 sentence encoder. *arXiv*, 1803.11175.
- 1041 [13] Constantinescu, A. O., O'Reilly, J. X., and Behrens, T. E. J. (2016). Organizing conceptual
1042 knowledge in humans with a gridlike code. *Science*, 352(6292):1464–1468.
- 1043 [14] Davison, A. C. and Hinkley, D. V. (1997). *Bootstrap Methods and their Application*. Cambridge
1044 Series in Statistical and Probabilistic Mathematics. Cambridge University Press.
- 1045 [15] Deacon, D., Grose-Fifer, J., Yang, C. M., Stanick, V., Hewitt, S., and Dynowska, A. (2004).
1046 Evidence for a new conceptualization of semantic representation in the left and right cerebral
1047 hemispheres. *Cortex*, 40(3):467–478.

- 1048 [16] Deerwester, S., Dumais, S. T., Furnas, G. W., Landauer, T. K., and Harshman, R. (1990).
1049 Indexing by latent semantic analysis. *Journal of the American Society for Information Science*,
1050 41(6):391–407.
- 1051 [17] Depoix, J. (2018). YouTube transcript API. <https://github.com/jdepoix/youtube-transcript-api>.
- 1053 [18] Devlin, J., Chang, M.-W., Lee, K., and Toutanova, K. (2018). BERT: pre-training of deep
1054 bidirectional transformers for language understanding. *arXiv*, 1810.04805.
- 1055 [19] Dikker, S., Wan, L., Davidesco, I., Kaggen, L., Oostrik, M., McClintock, J., Rowland, J.,
1056 Michalareas, G., van Bavel, J. J., Ding, M., and Poeppel, D. (2017). Brain-to-brain synchrony
1057 tracks real-world dynamic group interactions in the classroom. *Current Biology*, 27(9):1375–1380.
- 1058 [20] Estes, W. K. (1986a). Array models for category learning. *Cognitive Psychology*, 18(4):500–549.
- 1059 [21] Estes, W. K. (1986b). Memory storage and retrieval processes in category learning. *Journal of*
1060 *Experimental Psychology: General*, 115:155–174.
- 1061 [22] Fisher, R. A. (1922). On the mathematical foundations of theoretical statistics. *Philosophical*
1062 *Transactions of the Royal Society A*, 222(602):309–368.
- 1063 [23] Gallagher, J. J. (2000). Teaching for understanding and application of science knowledge.
1064 *School Science and Mathematics*, 100(6):310–318.
- 1065 [24] Gluck, M. A., Shohamy, D., and Myers, C. E. (2002). How do people solve the “weather
1066 prediction” task? individual variability in strategies for probabilistic category learning. *Learning*
1067 *and Memory*, 9:408–418.
- 1068 [25] Goldman, N. and Whelan, S. (2000). Statistical Tests of Gamma-Distributed Rate Heterogeneity
1069 in Models of Sequence Evolution in Phylogenetics. *Molecular Biology and Evolution*, 17(6):975–978.
- 1070 [26] Goldstein, T. R. and Winner, E. (2012). Enhancing empathy and theory of mind. *Journal of*
1071 *Cognition and Development*, 13(1):19–37.

- 1072 [27] Halekoh, U. and Højsgaard, S. (2014). A Kenward-Roger Approximation and Parametric
1073 Bootstrap Methods for Tests in Linear Mixed Models – The R Package pbkrtest. *Journal of*
1074 *Statistical Software*, 59(9):1–32.
- 1075 [28] Hall, R. and Greeno, J. (2008). *21st century education: A reference handbook*, chapter Conceptual
1076 learning, pages 212–221. Sage Publications.
- 1077 [29] Heusser, A. C., Fitzpatrick, P. C., and Manning, J. R. (2021). Geometric models reveal behav-
1078 ioral and neural signatures of transforming experiences into memories. *Nature Human Behaviour*,
1079 5:905–919.
- 1080 [30] Huebner, P. A. and Willits, J. A. (2018). Structured semantic knowledge can emerge au-
1081 tomatically from predicting word sequences in child-directed speech. *Frontiers in Psychology*,
1082 9:doi.org/10.3389/fpsyg.2018.00133.
- 1083 [31] Hulbert, J. C. and Norman, K. A. (2015). Neural differentiation tracks improved recall of com-
1084 peting memories following interleaved study and retrieval practice. *Cerebral Cortex*, 25(10):3994–
1085 4008.
- 1086 [32] Kanske, P., Böckler, A., and Singer, T. (2015). Models, mechanisms and moderators dissociating
1087 empathy and theory of mind. In *Social Behavior From Rodents to Humans*, pages 193–206. Springer.
- 1088 [33] Katona, G. (1940). *Organizing and memorizing: studies in the psychology of learning and teaching*.
1089 Columbia University Press.
- 1090 [34] Kaufman, D. M. (2003). Applying educational theory in practice. *British Medical Journal*,
1091 326(7382):213–216.
- 1092 [35] Kawasaki, H., Yamasaki, S., Masuoka, Y., Iwasa, M., Fukita, S., and Matsuyama, R. (2021).
1093 Remote teaching due to COVID-19: an exploration of its effectiveness and issues. *International*
1094 *Journal of Environmental Research and Public Health*, 18(5):2672.
- 1095 [36] Khan, S. (2004). *The Khan Academy*. Salman Khan.

- 1096 [37] Kintsch (1970). *Learning, memory, and conceptual processes*. Wiley.
- 1097 [38] Kolowich, S. (2013). How EdX plans to earn, and share, revenue from its free online courses.
- 1098 *The Chronicle of Higher Education*, 21:1–5.
- 1099 [39] Landauer, T. K. and Dumais, S. T. (1997). A solution to Plato’s problem: the latent semantic
1100 analysis theory of acquisition, induction, and representation of knowledge. *Psychological Review*,
1101 104:211–240.
- 1102 [40] Lee, H. and Chen, J. (2022). Predicting memory from the network structure of naturalistic
1103 events. *Nature Communications*, 13(4235):doi.org/10.1038/s41467-022-31965-2.
- 1104 [41] Maclellan, E. (2005). Conceptual learning: the priority for higher education. *British Journal of
1105 Educational Studies*, 53(2):129–147.
- 1106 [42] Manning, J. R. (2021). Episodic memory: mental time travel or a quantum “memory wave”
1107 function? *Psychological Review*, 128(4):711–725.
- 1108 [43] Manning, J. R. (2023). Context reinstatement. In Kahana, M. J. and Wagner, A. D., editors,
1109 *Handbook of Human Memory*. Oxford University Press.
- 1110 [44] Manning, J. R., Menjunatha, H., and Kording, K. (2023). Chatify: A Jupyter extension
1111 for adding LLM-driven chatbots to interactive notebooks. [https://github.com/ContextLab/
1112 chatify](https://github.com/ContextLab/chatify).
- 1113 [45] Matuschek, H., Kliegl, R., Vasishth, S., Baayen, H., and Bates, D. (2017). Balancing type i error
1114 and power in linear mixed models. *Journal of Memory and Language*, 94:305–315.
- 1115 [46] McInnes, L., Healy, J., and Melville, J. (2018a). UMAP: Uniform manifold approximation and
1116 projection for dimension reduction. *arXiv*, 1802(03426).
- 1117 [47] McInnes, L., Healy, J., Saul, N., and Großberger, L. (2018b). UMAP: Uniform Manifold
1118 Approximation and Projection. *Journal of Open Source Software*, 3(29):861.

- 1119 [48] Meltzoff, A. N. (2011). Social cognition and the origins of imitation, empathy, and theory of
1120 mind. In *The Wiley-Blackwell Handbook of Childhood Cognitive Development*. Wiley-Blackwell.
- 1121 [49] Meshulam, M., Hasenfratz, L., Hillman, H., Liu, Y. F., Nguyen, M., Norman, K. A., and Hasson,
1122 U. (2020). Neural alignment predicts learning outcomes in students taking an introduction to
1123 computer science course. *Nature Communications*, 12(1922):doi.org/10.1038/s41467-021-22202-3.
- 1124 [50] Mikolov, T., Chen, K., Corrado, G., and Dean, J. (2013). Efficient estimation of word represen-
1125 tations in vector space. *arXiv*, 1301.3781.
- 1126 [51] Moser, K. M., Wei, T., and Brenner, D. (2021). Remote teaching during COVID-19: implications
1127 from a national survey of language educators. *System*, 97:102431.
- 1128 [52] Nastase, S. A., Goldstein, A., and Hasson, U. (2020). Keep it real: rethinking the primacy of
1129 experimental control in cognitive neuroscience. *NeuroImage*, 15(222):117254–117261.
- 1130 [53] Nguyen, M., Chang, A., Micciche, E., Meshulam, M., Nastase, S. A., and Hasson, U. (2022).
1131 Teacher-student neural coupling during teaching and learning. *Social Cognitive and Affective*
1132 *Neuroscience*, 17(4):367–376.
- 1133 [54] OpenAI (2023). ChatGPT. <https://chat.openai.com>.
- 1134 [55] Piantadosi, S. T. and Hill, F. (2022). Meaning without reference in large language models.
1135 *arXiv*, 2208.02957.
- 1136 [56] Poulsen, A. T., Kamronn, S., Dmochowski, J., Parra, L. C., and Hansen, L. K. (2017). EEG
1137 in the classroom: synchronised neural recordings during video presentation. *Scientific Reports*,
1138 7:43916.
- 1139 [57] Ratka, A. (2018). Empathy and the development of affective skills. *American Journal of*
1140 *Pharmaceutical Education*, 82(10):doi.org/10.5688/ajpe7192.
- 1141 [58] Reilly, D. L., Cooper, L. N., and Elbaum, C. (1982). A neural model for category learning.
1142 *Biological Cybernetics*, 45(1):35–41.

- 1143 [59] Rhoads, R. A., Berdan, J., and Toven-Lindsey, B. (2013). The open courseware movement in
1144 higher education: unmasking power and raising questions about the movement's democratic
1145 potential. *Educational Theory*, 63(1):87–110.
- 1146 [60] Scheipl, F., Greven, S., and Küchenhoff, H. (2008). Size and power of tests for a zero random
1147 effect variance or polynomial regression in additive and linear mixed models. *Computational
1148 Statistics & Data Analysis*, 52(7):3283–3299.
- 1149 [61] Scott, P., Asoko, H., and Leach, J. (2007). *Handbook of research on science education*, chapter
1150 Student conceptions and conceptual learning in science. Routledge.
- 1151 [62] Shao, Y. N., Sun, H. M., Huang, J. W., Li, M. L., Huang, R. R., and Li, N. (2018). Simulation-
1152 based empathy training improves the communication skills of neonatal nurses. *Clinical Simula-
1153 tion in Nursing*, 22:32–42.
- 1154 [63] Shim, T. E. and Lee, S. Y. (2020). College students' experience of emergency remote teaching
1155 during COVID-19. *Children and Youth Services Review*, 119:105578.
- 1156 [64] Simon, M. A., Tzur, R., Heinz, K., and Kinzel, M. (2004). Explicating a mechanism for
1157 conceptual learning: elaborating the construct of reflective abstraction. *Journal for Research in
1158 Mathematics Education*, 35(5):305–329.
- 1159 [65] Snijders, T. A. B. and Bosker, R. (2011). More powerful tests for variance parameters. In
1160 *Multilevel Analysis: An Introduction to Basic and Advanced Multilevel Modeling*, chapter 6, pages
1161 94–108. Sage Publications, 2nd edition.
- 1162 [66] Stepien, K. A. and Baernstein, A. (2006). Education for empathy. *Journal of General Internal
1163 Medicine*, 21:524–530.
- 1164 [67] Touvron, H., Lavril, T., Izacard, G., Martinet, X., Lachaux, M.-A., Lacroix, T., Rozière, B.,
1165 Goyal, N., Hambro, E., Azhar, F., Rodriguez, A., Joulin, A., Grave, E., and Lample, G. (2023).
1166 LLaMA: open and efficient foundation language models. *arXiv*, 2302.13971.

- 1167 [68] Tulchinskii, E., Kuznetsov, K., Kushnareva, L., Cherniavskii, D., Barannikov, S., Pio-
1168 ntkovskaya, I., Nikolenko, S., and Burnaev, E. (2023). Intrinsic dimension estimation for robust
1169 detection of AI-generated texts. *arXiv*, 2306.04723.
- 1170 [69] van Paridon, J., Liu, Q., and Lupyan, G. (2021). How do blind people know that blue is cold?
1171 distributional semantics encode color-adjective associations. *Proceedings of the Annual Meeting of*
1172 *the Cognitive Science Society*, 43(43).
- 1173 [70] Viswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A. N., Kaiser, L., and
1174 Polosukhin, I. (2017). Attention is all you need. In *Advances in Neural Information Processing*
1175 *Systems*.
- 1176 [71] Whalen, J. (2020). Should teachers be trained in emergency remote teaching? Lessons learned
1177 from the COVID-19 pandemic. *Journal of Technology and Teacher Education*, 28(2):189–199.
- 1178 [72] Young, J. R. (2012). Inside the Coursera contract: how an upstart company might profit from
1179 free courses. *The Chronicle of Higher Education*, 19(7):1–4.
- 1180 [73] Ziman, K., Heusser, A. C., Fitzpatrick, P. C., Field, C. E., and Manning, J. R. (2018). Is
1181 automatic speech-to-text transcription ready for use in psychological experiments? *Behavior*
1182 *Research Methods*, 50:2597–2605.