

- 1 Text embedding models yield high-resolution insights
- 2 into conceptual knowledge from short multiple-choice

3 quizzes

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Abstract

We develop a mathematical framework, based on natural language processing models, for tracking and characterizing the acquisition of conceptual knowledge. Our approach embeds each concept in a high-dimensional representation space, where nearby coordinates reflect similar or related concepts. We test our approach using behavioral data from participants who answered small sets of multiple-choice quiz questions interleaved between watching two course videos from the Khan Academy platform. We apply our framework to the videos' transcripts and the text of the quiz questions to quantify the content of each moment of video and each quiz question. We use these embeddings, along with participants' quiz responses, to track how the learners' knowledge changed after watching each video and predict their success on individual quiz questions. Our findings show how a small set of quiz questions may be used to obtain rich and meaningful high-resolution insights into what each learner knows, and how their knowledge changes over time as they learn.

Keywords: education, learning, knowledge, concepts, natural language processing

¹⁹ **Introduction**

²⁰ Suppose that a teacher had access to a complete, tangible “map” of everything a student knows.
²¹ Defining what such a map might even look like, let alone how it might be constructed or filled in, is
²² itself a non-trivial problem. But if a teacher *were* to gain access to such a map, how might it change
²³ their ability to teach that student? Perhaps they might start by checking how well the student
²⁴ knows the to-be-learned information already, or how much they know about related concepts.
²⁵ For some students, they could potentially optimize their teaching efforts to maximize efficiency
²⁶ by focusing primarily on not-yet-known content. For other students (or other content areas), it
²⁷ might be more effective to optimize for direct connections between already known content and
²⁸ new material. Observing how the student’s knowledge changed over time, in response to their
²⁹ teaching, could also help to guide the teacher towards the most effective strategy for that individual
³⁰ student.

³¹ A common approach to assessing a student’s knowledge is to present them with a set of quiz
³² questions, calculate the proportion they answer correctly, and provide them with feedback in the
³³ form of a simple numeric or letter grade. While such a grade can provide *some* indication of whether
³⁴ the student has mastered the to-be-learned material, any univariate measure of performance on a
³⁵ complex task sacrifices certain relevant information, risks conflating underlying factors, and so on.
³⁶ For example, consider the relative utility of the theoretical map described above that characterizes
³⁷ a student’s knowledge in detail, versus a single annotation saying that the student answered 85%
³⁸ of their quiz questions correctly, or that they received a ‘B’. Here, we show that the same quiz data
³⁹ required to compute proportion-correct scores or letter grades can instead be used to obtain far
⁴⁰ more detailed insights into what a student knew at the time they took the quiz.

⁴¹ Designing and building procedures and tools for mapping out knowledge touches on deep
⁴² questions about what it means to learn. For example, how do we acquire conceptual knowledge?
⁴³ Memorizing course lectures or textbook chapters by rote can lead to the superficial *appearance*
⁴⁴ of understanding the underlying content, but achieving true conceptual understanding seems to
⁴⁵ require something deeper and richer. Does conceptual understanding entail connecting newly

46 acquired information to the scaffolding of one’s existing knowledge or experience [6, 11, 13, 15, 30,
47 65]? Or weaving a lecture’s atomic elements (e.g., its component words) into a structured network
48 that describes how those individual elements are related [40, 70]? Conceptual understanding
49 could also involve building a mental model that transcends the meanings of those individual
50 atomic elements by reflecting the deeper meaning underlying the gestalt whole [37, 41, 62, 69].

51 The difference between “understanding” and “memorizing,” as framed by researchers in ed-
52 ucation, cognitive psychology, and cognitive neuroscience [e.g., 23, 28, 33, 41, 62], has profound
53 analogs in the fields of natural language processing and natural language understanding. For
54 example, considering the raw contents of a document (e.g., its constituent symbols, letters, and
55 words) might provide some clues as to what the document is about, just as memorizing a passage
56 might provide some ability to answer simple questions about it. However, text embedding mod-
57 els [e.g., 7, 8, 10, 12, 16, 39, 51, 71] also attempt to capture the deeper meaning *underlying* those
58 atomic elements. These models consider not only the co-occurrences of those elements within and
59 across documents, but (in many cases) also patterns in how those elements appear across different
60 scales (e.g., sentences, paragraphs, chapters, etc.), the temporal and grammatical properties of the
61 elements, and other high-level characteristics of how they are used [42, 43]. To be clear, this is not
62 to say that text embedding models themselves are capable of “understanding” deep conceptual
63 meaning in any traditional sense. But rather, their ability to capture the underlying *structure* of
64 text documents beyond their surface-level contents provides a computational framework through
65 which those documents’ deeper conceptual meanings may be quantified, explored, and under-
66 stood. According to these models, the deep conceptual meaning of a document may be captured
67 by a feature vector in a high-dimensional representation space, wherein nearby vectors reflect con-
68 ceptually related documents. A model that succeeds at capturing an analogue of “understanding”
69 is able to assign nearby feature vectors to two conceptually related documents, *even when the specific*
70 *words contained in those documents have limited overlap*. In this way, “concepts” are defined implicitly
71 by the model’s geometry [e.g., how the embedding coordinate of a given word or document relates
72 to the coordinates of other text embeddings; 56].

73 Given these insights, what form might a representation of the sum total of a person’s knowledge

74 take? First, we might require a means of systematically describing or representing (at least some
75 subset of) the nearly infinite set of possible things a person could know. Second, we might want to
76 account for potential associations between different concepts. For example, the concepts of “fish”
77 and “water” might be associated in the sense that fish live in water. Third, knowledge may have
78 a critical dependency structure, such that knowing about a particular concept might require first
79 knowing about a set of other concepts. For example, understanding the concept of a fish swimming
80 in water first requires understanding what fish and water *are*. Fourth, as we learn, our “current
81 state of knowledge” should change accordingly. Learning new concepts should both update our
82 characterizations of “what is known” and also unlock any now-satisfied dependencies of those
83 newly learned concepts so that they are “tagged” as available for future learning.

84 Here we develop a framework for modeling how conceptual knowledge is acquired during
85 learning. The central idea behind our framework is to use text embedding models to define the
86 coordinate systems of two maps: a *knowledge map* that describes the extent to which each concept is
87 currently known, and a *learning map* that describes changes in knowledge over time. Each location
88 on these maps represents a single concept, and the maps’ geometries are defined such that related
89 concepts are located nearby in space. We use this framework to analyze and interpret behavioral
90 data collected from an experiment that had participants answer sets of multiple-choice questions
91 about a series of recorded course lectures.

92 Our primary research goal is to advance our understanding of what it means to acquire deep,
93 real-world conceptual knowledge. Traditional laboratory approaches to studying learning and
94 memory (e.g., list-learning studies) often draw little distinction between memorization and under-
95 standing. Instead, these studies typically focus on whether information is effectively encoded or
96 retrieved, rather than whether the information is *understood*. Approaches to studying conceptual
97 learning, such as category learning experiments, can begin to investigate the distinction between
98 memorization and understanding, often by training participants to distinguish arbitrary or random
99 features in otherwise meaningless categorized stimuli [1, 20, 21, 24, 31, 59]. However, the objective
100 of real-world training, or learning from life experiences more generally, is often to develop new
101 knowledge that may be applied in *useful* ways in the future. In this sense, the gap between modern

learning theories and modern pedagogical approaches that inform classroom learning strategies is enormous: most of our theories about *how* people learn are inspired by experimental paradigms and models that have only peripheral relevance to the kinds of learning that students and teachers actually seek [28, 41]. To help bridge this gap, our study uses course materials from real online courses to inform, fit, and test models of real-world conceptual learning. We show that these models recover meaningful relationships between concepts presented during course lectures and tested by assessments, and that these relationships can be leveraged to predict students' success on individual quiz questions. We also provide a demonstration of how our models can be used to construct "maps" of what students know, and how their knowledge changes with training. In addition to helping to visually capture knowledge (and changes in knowledge), we hope that such maps might lead to real-world tools for improving how we educate. Taken together, our work shows that existing course materials and evaluative tools like short multiple-choice quizzes may be leveraged to gain highly detailed insights into what students know and how they learn.

Results

At its core, our main modeling approach is based around a simple assumption that we sought to test empirically: all else being equal, knowledge about a given concept is predictive of knowledge about similar or related concepts. From a geometric perspective, this assumption implies that knowledge is fundamentally "smooth." In other words, as one moves through a space representing an individual's knowledge (where similar concepts occupy nearby coordinates), their "level of knowledge" should change relatively gradually. To begin to test this smoothness assumption, we sought to track participants' knowledge and how it changed over time in response to training. Two overarching goals guide our approach. First, we want to gain detailed insights into what learners know at different points in their training. For example, rather than simply reporting on the proportions of questions participants answer correctly (i.e., their overall performance), we seek estimates of their knowledge about a variety of specific concepts. Second, we want our approach to be potentially scalable to large numbers of diverse concepts, courses, and students. This requires



Figure 1: Experimental paradigm. Participants alternate between completing three 13-question multiple-choice quizzes and watching two Khan Academy lectures. Each quiz contains a mix of 5 questions about Lecture 1, 5 questions about Lecture 2, and 3 questions about general physics knowledge. The specific questions reflected on each quiz, and the orders of each quiz's questions, were randomized across participants.

128 that the conceptual content of interest be discovered *automatically*, rather than relying on manually
 129 produced ratings or labels.

130 We asked participants in our study to complete brief multiple-choice quizzes before, between,
 131 and after watching two lecture videos from the Khan Academy [36] platform (Fig. 1). The first
 132 lecture video, entitled *Four Fundamental Forces*, discussed the four fundamental forces in physics:
 133 gravity, strong and weak interactions, and electromagnetism. The second, entitled *Birth of Stars*,
 134 provided an overview of our current understanding of how stars form. We selected these particular
 135 lectures to satisfy three general criteria. First, we wanted both lectures to be accessible to a broad
 136 audience (i.e., with minimal prerequisite knowledge) so as to limit the impact of prior training
 137 on participants' abilities to learn from the lectures. To this end, we selected two introductory
 138 videos that were intended to be viewed at the start of students' training in their respective content
 139 areas. Second, we wanted the two lectures to have some related content, so that we could test
 140 our approach's ability to distinguish similar conceptual content. To this end, we chose two videos
 141 from the same Khan Academy course domain, "Cosmology and Astronomy." Third, we sought to
 142 minimize dependencies and specific overlap between the videos. For example, we did not want
 143 participants' abilities to understand one video to (directly) influence their abilities to understand the
 144 other. To satisfy this last criterion, we chose videos from two different lecture series (Lectures 1 and
 145 2 were from the "Scale of the Universe" and "Stars, Black Holes, and Galaxies" series, respectively).



Figure 2: Modeling course content. **A. Building a document pool from sliding windows of text.** We decompose each lecture’s transcript into a series of overlapping sliding windows. The full set of transcript snippets (across all windows) may be treated as a set of “documents” for training a text embedding model. **B. Constructing lecture content trajectories.** After training the model on the sliding windows from both lectures, we transform each lecture into a “trajectory” through text embedding space by joining the embedding coordinates of successive sliding windows parsed from its transcript. **C. Embedding multiple lectures and questions in a shared space.** We apply the same model (trained on the two lectures’ windows) to both lectures, along with the text of each question in our pool (Supp. Tab. 1), to project them into a shared text embedding space. This results in one trajectory per lecture and one coordinate for each question. Here, we have projected the 15-dimensional embeddings onto their first 3 principal components for visualization.

146 We also wrote a set of multiple-choice quiz questions that we hoped would enable us to
 147 evaluate participants’ knowledge about each individual lecture, along with related knowledge
 148 about physics concepts not specifically presented in either video (see Supp. Tab. 1 for the full list
 149 of questions in our stimulus pool). Participants answered questions randomly drawn from each
 150 content area (Lecture 1, Lecture 2, and general physics knowledge) on each of the three quizzes.
 151 Quiz 1 was intended to assess participants’ “baseline” knowledge before training, Quiz 2 assessed
 152 knowledge after watching the *Four Fundamental Forces* video (i.e., Lecture 1), and Quiz 3 assessed
 153 knowledge after watching the *Birth of Stars* video (i.e., Lecture 2).

154 To study in detail how participants’ conceptual knowledge changed over the course of the
 155 experiment, we first sought to model the conceptual content presented to them at each moment
 156 throughout each of the two lectures. We adapted an approach we developed in prior work [29]
 157 to identify the latent themes in the lectures using a topic model [8]. Briefly, topic models take
 158 as input a collection of text documents, and learn a set of “topics” (i.e., latent themes) from their
 159 contents. Once fit, a topic model can be used to transform arbitrary (potentially new) documents
 160 into sets of “topic proportions,” describing the weighted blend of learned topics reflected in their

¹⁶¹ texts. We parsed automatically generated transcripts of the two lectures into overlapping sliding
¹⁶² windows, where each window contained the text of the lecture transcript from a particular time
¹⁶³ span. We treated the set of text snippets (across all of these windows) as documents to fit the
¹⁶⁴ model (Fig. 2A; see *Constructing text embeddings of multiple lectures and questions*). Transforming the
¹⁶⁵ text from every sliding window with the model yielded a number-of-windows by number-of-topics
¹⁶⁶ (15) topic-proportions matrix describing the unique mixture of broad themes from both lectures
¹⁶⁷ reflected in each window’s text. Each window’s “topic vector” (i.e., column of the topic-proportions
¹⁶⁸ matrix) is analogous to a coordinate in a 15-dimensional space whose axes are topics discovered
¹⁶⁹ by the model. Within this space, each lecture’s sequence of topic vectors (i.e., corresponding to its
¹⁷⁰ transcript’s overlapping text snippets across sliding windows) forms a *trajectory* that captures how
¹⁷¹ its conceptual content unfolds over time (Fig. 2B). We resampled these trajectories to a resolution
¹⁷² of one topic vector for each second of video (i.e., 1 Hz).

¹⁷³ We hypothesized that a topic model trained on transcripts of the two lectures should also
¹⁷⁴ capture the conceptual knowledge probed by each quiz question. If indeed the topic model could
¹⁷⁵ capture information about the deeper conceptual content of the lectures (i.e., beyond surface-level
¹⁷⁶ details such as particular word choices), then we should be able to recover a correspondence
¹⁷⁷ between each lecture and questions *about* each lecture. Importantly, such a correspondence could
¹⁷⁸ not solely arise from superficial text matching between lecture transcripts and questions, since the
¹⁷⁹ lectures and questions often used different words (Supp. Fig. 5) and phrasings. Simply comparing
¹⁸⁰ the average topic weights from each lecture and question set (averaging across time and questions,
¹⁸¹ respectively) reveals a striking correspondence (Supp. Fig. 2). Specifically, the average topic
¹⁸² weights from Lecture 1 are strongly correlated with the average topic weights from Lecture 1
¹⁸³ questions ($r(13) = 0.809$, $p < 0.001$, 95% confidence interval (CI) = [0.633, 0.962]), and the average
¹⁸⁴ topic weights from Lecture 2 are strongly correlated with the average topic weights from Lecture 2
¹⁸⁵ questions ($r(13) = 0.728$, $p = 0.002$, 95% CI = [0.456, 0.920]). At the same time, the average topic
¹⁸⁶ weights from the two lectures are *negatively* correlated with the average topic weights from their
¹⁸⁷ non-matching question sets (Lecture 1 video vs. Lecture 2 questions: $r(13) = -0.547$, $p = 0.035$,
¹⁸⁸ 95% CI = [-0.812, -0.231]; Lecture 2 video vs. Lecture 1 questions: $r(13) = -0.612$, $p = 0.015$, 95%



Figure 3: Lecture and question topic overlap. A. Topic weight variability. The bar plots display the variance of each topic's weight across lecture timepoints (top row) and questions (bottom row); colors denote topics. The top-weighted words from the most “expressive” (i.e., variable across observations) topic from each lecture are displayed in the upper right (orange: topic 2; yellow-green: topic 4). The top-weighted words from the full set of topics may be found in Supplementary Table 2. **B. Relationships between topic weight variability.** Pairwise correlations between the distributions of topic weight variance for each lecture and question set. Each row and column corresponds to a bar plot in Panel A.

189 CI = [-0.874, -0.281]), indicating that the topic model also exhibits some degree of specificity. The
190 full set of pairwise comparisons between average topic weights for the lectures and question sets
191 is reported in Supplementary Figure 2.

192 Another, more sensitive, way of summarizing the conceptual content of the lectures and ques-
193 tions is to look at *variability* in how topics are weighted over time and across different questions
194 (Fig. 3). Intuitively, the variability in the expression of a given topic relates to how much “infor-
195 mation” [22] the lecture (or question set) reflects about that topic. For example, suppose a given
196 topic is weighted on heavily throughout a lecture. That topic might be characteristic of some
197 aspect or property of the lecture *overall* (conceptual or otherwise), but unless the topic’s weights
198 changed in meaningful ways over time, the topic would be a poor indicator of any *specific* concep-
199 tual content in the lecture. We therefore also compared the variances in topic weights (across time
200 or questions) between the lectures and questions. The variability in topic expression (over time
201 and across questions) was similar for the Lecture 1 video and questions ($r(13) = 0.824$, $p < 0.001$,
202 95% CI = [0.696, 0.973]) and the Lecture 2 video and questions ($r(13) = 0.801$, $p < 0.001$, 95%

203 CI = [0.539, 0.958]). Simultaneously, as reported in Figure 3B, the variabilities in topic expression
204 across *different* videos and lecture-specific questions (i.e., Lecture 1 video vs. Lecture 2 questions;
205 Lecture 2 video vs. Lecture 1 questions) were negatively correlated, and neither video’s topic
206 variability was reliably correlated with the topic variability across general physics knowledge
207 questions. Taken together, the analyses reported in Figure 3 and Supplementary Figure 2 indicate
208 that a topic model fit to the videos’ transcripts can also reveal correspondences (at a coarse scale)
209 between the lectures and questions.

210 While an individual lecture may be organized around a single broad theme at a coarse scale,
211 at a finer scale, each moment of a lecture typically covers a narrower range of content. Given the
212 correspondence we found between the variabilities in topic expression across moments of each
213 lecture and questions from its corresponding set (Fig. 3), we wondered whether the text embedding
214 model might additionally capture these conceptual relationships at a finer scale. For example, if a
215 particular question asks about the content from one small part of a lecture, we wondered whether
216 the text embeddings could be used to automatically identify the “matching” moment(s) in the
217 lecture. To explore this, we computed the correlation between each question’s topic weights
218 and the topic weights for each second of its corresponding lecture, and found that each question
219 appeared to be temporally specific (Fig. 4). In particular, most questions’ topic vectors were
220 maximally correlated with a well-defined (and relatively narrow) range of timepoints from their
221 corresponding lectures, and the correlations fell off sharply outside of that range (Supp. Figs. 3, 4).
222 We also qualitatively examined the best-matching intervals for each question by comparing the
223 question’s text to the transcribed text from the most-correlated parts of the lectures (Supp. Tab. 3).
224 Despite that the questions were excluded from the text embedding model’s training set, in general
225 we found (through manual inspection) a close correspondence between the conceptual content
226 that each question probed and the content covered by the best-matching moments of the lectures.
227 Two representative examples are shown at the bottom of Figure 4.

228 The ability to quantify how much each question is “asking about” the content from each moment
229 of the lectures could enable high-resolution insights into participants’ knowledge. Traditional
230 approaches to estimating how much a student “knows” about the content of a given lecture entail



Figure 4: Which parts of each lecture are captured by each question? Each panel displays time series plots showing how each question’s topic vector correlates with each video timepoint’s topic vector (Panel A.: correlations for the *Four Fundamental Forces* lecture and associated questions; Panel B.: correlations for the *Birth of Stars* lecture and associated questions). The colors denote question identities. The diamonds in each panel denote the moment of peak correlation between the indicated question and the lecture trajectory. The associated questions’ text and snippets of the lectures’ transcripts from the surrounding 30 seconds, are displayed at the bottom of the figure.

231 administering some form of assessment (e.g., a quiz) and computing the proportion of correctly
 232 answered questions. But if two students receive identical scores on such an exam, might our
 233 modeling framework help us to gain more nuanced insights into the *specific* content that each
 234 student has mastered (or failed to master)? For example, a student who misses three questions that
 235 were all about the same concept (e.g., concept *A*) will have gotten the same *proportion* of questions
 236 correct as another student who missed three questions about three *different* concepts (e.g., *A, B*, and
 237 *C*). But if we wanted to help these two students fill in the “gaps” in their understandings, we might
 238 do well to focus specifically on concept *A* for the first student, but to also add in materials pertaining
 239 to concepts *B* and *C* for the second student. In other words, raw “proportion-correct” measures may
 240 capture *how much* a student knows, but not *what* they know. We wondered whether our modeling
 241 framework might enable us to (formally and automatically) infer participants’ knowledge at the
 242 scale of individual concepts (e.g., as captured by a single moment of a lecture).

243 We developed a simple formula (Eqn. 1) for using a participant’s responses to a small set

244 of multiple-choice questions to estimate how much the participant “knows” about the concept
245 reflected by any arbitrary coordinate x in text embedding space (e.g., the content reflected by
246 any moment in a lecture they had watched; see *Estimating dynamic knowledge traces*). Essentially,
247 the estimated knowledge at coordinate x is given by the weighted proportion of quiz questions
248 the participant answered correctly, where the weights reflect how much each question is “about”
249 the content at x . When we apply this approach to estimate the participant’s knowledge about
250 the content presented in each moment of each lecture, we can obtain a detailed time course
251 describing how much “knowledge” that participant has about the content presented at any part of
252 the lecture. As shown in Figure 5A and C, we can apply this approach separately for the questions
253 from each quiz participants took throughout the experiment. From just a few questions per quiz
254 (see *Estimating dynamic knowledge traces*), we obtain a high-resolution snapshot (at the time each
255 quiz was taken) of what the participants knew about any moment’s content, from either of the two
256 lectures they watched (comprising a total of 1,100 samples across the two lectures).

257 While the time courses in Figure 5A and C provide detailed *estimates* about participants’ knowl-
258 ege, these estimates are of course only *useful* to the extent that they accurately reflect what partic-
259 ipants actually know. As one sanity check, we anticipated that the knowledge estimates should
260 reflect a content-specific “boost” in participants’ knowledge after watching each lecture. In other
261 words, if participants learn about each lecture’s content upon watching it, the knowledge esti-
262 mated should capture that. After watching the *Four Fundamental Forces* lecture, participants should
263 exhibit more knowledge for the content of that lecture than they had before, and that knowledge
264 should persist for the remainder of the experiment. Specifically, knowledge about that lecture’s
265 content should be relatively low when estimated using Quiz 1 responses, but should increase
266 when estimated using Quiz 2 or 3 responses (Fig. 5B). Indeed, we found that participants’ esti-
267 mated knowledge about the content of *Four Fundamental Forces* was substantially higher on Quiz 2
268 versus Quiz 1 ($t(49) = 8.764, p < 0.001$) and on Quiz 3 versus Quiz 1 ($t(49) = 10.519, p < 0.001$).
269 We found no reliable differences in estimated knowledge about that lecture’s content on Quiz 2
270 versus 3 ($t(49) = 0.160, p = 0.874$). Similarly, we hypothesized (and subsequently confirmed)
271 that participants should show greater estimated knowledge about the content of the *Birth of Stars*



Figure 5: Estimating knowledge about the content presented at each moment of each lecture. **A. Knowledge about the time-varying content of *Four Fundamental Forces*.** Each trace displays the weighted proportion of correctly answered questions about the content reflected in each moment of the lecture (see *Estimating dynamic knowledge traces*), using responses from a single quiz (color). The traces are averaged across participants. **B. Average estimated knowledge about *Four Fundamental Forces*.** Each bar displays the across-timepoint average knowledge, estimated using the responses to one quiz's questions. **C. Knowledge about the time-varying content of *Birth of Stars*.** The panel is in the same format as Panel A, but here the knowledge estimates are for the moment-by-moment content of the *Birth of Stars* lecture. **D. Average estimated knowledge about *Birth of Stars*.** The panel is in the same format as Panel B, but here the knowledge estimates are for the content of the *Birth of Stars* lecture. All panels: error ribbons and error bars denote 95% confidence intervals, estimated across participants.

lecture after (versus before) watching it (Fig. 5D). Specifically, since participants watched that lecture after taking Quiz 2 (but before Quiz 3), we hypothesized that their knowledge estimates should be relatively low on Quizzes 1 and 2, but should show a “boost” on Quiz 3. Consistent with this prediction, we found no reliable differences in estimated knowledge about the *Birth of Stars* lecture content on Quizzes 1 versus 2 ($t(49) = 1.013, p = 0.316$), but the estimated knowledge was substantially higher on Quiz 3 versus 2 ($t(49) = 10.561, p < 0.001$) and Quiz 3 versus 1 ($t(49) = 8.969, p < 0.001$).

If we are able to accurately estimate a participant’s knowledge about the content tested by a given question, our estimates of their knowledge should carry some predictive information about whether they are likely to answer that question correctly or incorrectly. We developed a statistical approach to test this claim. For each quiz question a participant answered, in turn, we used Equation 1 to estimate their knowledge at the given question’s embedding-space coordinate based on other questions that participant answered on the same quiz. We repeated this for all participants, and for each of the three quizzes. Then, separately for each quiz, we fit a generalized linear mixed model (GLMM) with a logistic link function to explain the probability of correctly answering a question as a function of estimated knowledge for its embedding coordinate, while accounting for varied effects of individual participants and questions (see *Generalized linear mixed models*). To assess the predictive value of the knowledge estimates, we compared each GLMM to an analogous (i.e., nested) “null” model that assumed these estimates carried no predictive information using parametric bootstrap likelihood-ratio tests.

We carried out three different versions of the analyses described above, wherein we considered different sources of information in our estimates of participants’ knowledge for each quiz question. First, we estimated knowledge at each question’s embedding coordinate using *all* other questions answered by the same participant on the same quiz (“All questions”; Fig. 6, top row). This test was intended to assess the overall predictive power of our approach. Second, we estimated knowledge for each question about a given lecture using only the other questions (from the same participant and quiz) about that *same* lecture (“Within-lecture”; Fig. 6, middle rows). This test was intended to assess the *specificity* of our approach by asking whether our predictions could distinguish between



Figure 6: Predicting success on held-out questions using estimated knowledge. We used generalized linear mixed models (GLMMs) to model the probability of correctly answering a quiz question as a function of estimated knowledge for its embedding coordinate (see *Generalized linear mixed models*). Separately for each quiz (column), we examined this relationship based on three different sets of knowledge estimates: knowledge for each question based on all other questions the same participant answered on the same quiz (“All questions”; top row), knowledge for each question about one lecture based on all other questions (from the same participant and quiz) about the *same* lecture (“Within-lecture”; middle rows), and knowledge for each question about one lecture based on all questions (from the same participant and quiz) about the *other* lecture (“Across-lecture”; bottom rows). The backgrounds in each panel display kernel density estimates of the relative observed proportions of correctly (blue) versus incorrectly (red) answered questions, for each level of estimated knowledge along the x -axis. The black curves display the (population-level) GLMM-predicted probabilities of correctly answering a question as a function of estimated knowledge. Error ribbons denote 95% confidence intervals.

300 questions about different content covered by the same lecture. Third, we estimated knowledge
301 for each question about one lecture using only the questions (from the same participant and quiz)
302 about the *other* lecture (“Across-lecture”; Fig. 6, bottom rows). This test was intended to assess the
303 *generalizability* of our approach by asking whether our predictions could extend across the content
304 areas of the two lectures. When estimating participants’ knowledge, we used a rebalancing
305 procedure to ensure that (for a given participant and quiz) their knowledge estimates for correctly
306 and incorrectly answered questions were computed from the same underlying proportion of
307 correctly answered questions (see *Generalized linear mixed models*).

308 When we fit a GLMM to estimates of participants’ knowledge for each Quiz 1 question based on
309 all other Quiz 1 questions, we found that higher estimated knowledge for a given question predicted
310 a greater likelihood of answering it correctly (odds ratio (OR) = 8.126, 95% CI = [3.116, 20.123],
311 likelihood-ratio test statistic (λ_{LR}) = 17.002, $p < 0.001$). This relationship held when we repeated
312 this analysis for Quiz 2 (OR = 14.902, 95% CI = [4.976, 39.807], λ_{LR} = 25.408, $p < 0.001$) and again
313 for Quiz 3 (OR = 37.409, 95% CI = [10.425, 107.145], λ_{LR} = 40.948, $p < 0.001$). Taken together,
314 these results suggest that our knowledge estimates can reliably predict participants’ performance
315 on individual questions when they incorporate information from all (other) quiz content.

316 We observed a similar set of results when we restricted our estimates of participants’ knowl-
317 edge for questions about each lecture to consider only their performance on other questions
318 about the *same* lecture. Specifically, for Quiz 1, participants’ knowledge of *Four Fundamental Forces*-
319 related questions, estimated from their performance on other *Four Fundamental Forces*-related ques-
320 tions, was predictive of their ability to answer those questions correctly (OR = 15.934, 95% CI =
321 [5.173, 38.005], λ_{LR} = 40.971, $p = 0.001$). The same was true of participants’ estimated knowledge
322 for *Birth of Stars*-related questions based on their performance on other *Birth of Stars*-related ques-
323 tions (OR = 9.775, 95% CI = [2.93, 25.08], λ_{LR} = 13.924, $p = 0.001$). These within-lecture knowl-
324 edge estimates also predicted success on questions about both lectures when we computed them
325 analogously for Quiz 2 (*Four Fundamental Forces*: OR = 35.126, 95% CI = [5.113, 123.868], λ_{LR} =
326 32.251, $p < 0.001$; *Birth of Stars*: OR = 4.717, 95% CI = [2.021, 9.844], λ_{LR} = 16.788, $p < 0.001$).
327 For Quiz 3, we found that within-lecture knowledge estimates predicted participants’ success on

328 *Birth of Stars*-related questions ($OR = 16.902$, 95% CI = [3.353, 53.265], $\lambda_{LR} = 23.233$, $p < 0.001$)
329 but not on *Four Fundamental Forces*-related questions ($OR = 2.485$, 95% CI = [0.724, 8.366], $\lambda_{LR} =$
330 1.984, $p = 0.170$). This may indicate that the within-lecture knowledge estimates are susceptible
331 to ceiling effects in participants' quiz performance. On Quiz 3, after viewing both lectures, no
332 participant answered more than three *Four Fundamental Forces*-related questions incorrectly, and
333 all but five participants (out of 50) answered two or fewer incorrectly. (This was the only subset
334 of questions about either lecture, across all three quizzes, for which this was true.) Because of
335 this, for 90% of participants, our within-lecture estimates of their knowledge for *Four Fundamen-*
336 *tal Forces*-related questions that they answered incorrectly leveraged information from at most a
337 single other question they were *not* able to correctly answer. This likely hampered our ability to
338 accurately characterize the specific (and by the time they took Quiz 3, relatively few) aspects of the
339 lecture content these participants did *not* know about, and successfully distinguish them from the
340 far more numerous aspects of the lecture content they now *did* know about. Taken together, these
341 results suggest that our knowledge estimates can reliably distinguish between questions about
342 different content covered by a single lecture, provided there is sufficient diversity in participants'
343 quiz responses to extract meaningful information about both what they know and what they do
344 not know.

345 Finally, we estimated participants' knowledge for each question about one lecture using their
346 performance on questions (from the same quiz) about the *other* lecture. This is an especially
347 stringent test of our approach. Our primary assumption in building our knowledge estimates is
348 that knowledge about a given concept is similar to knowledge about other concepts that are nearby
349 in the embedding space. However, our analyses in Figure 3 and Supplementary Figure 2 show
350 that the embeddings of content from the two lectures (and of their associated quiz questions) are
351 largely distinct from each other. Therefore, any predictive power of these across-lecture knowledge
352 estimates must overcome large distances in the embedding space. To put this in concrete terms,
353 this test requires predicting participants' performance on individual, highly specific questions
354 about the formation of stars from their responses to just five multiple-choice questions about the
355 fundamental forces of the universe (and vice versa).

356 We found that, before viewing either lecture (i.e., on Quiz 1), participants' abilities to answer
357 *Four Fundamental Forces*-related questions could not be predicted from their responses to *Birth of*
358 *Stars*-related questions ($OR = 1.896$, 95% CI = [0.419, 9.088], $\lambda_{LR} = 0.712$, $p = 0.404$), nor could
359 their abilities to answer *Birth of Stars*-related questions be predicted from their responses to *Four*
360 *Fundamental Forces*-related questions ($OR = 1.522$, 95% CI = [0.332, 6.835], $\lambda_{LR} = 0.286$, $p = 0.611$).
361 Similarly, we found that participants' performance on questions about either lecture could not
362 be predicted given their responses to questions about the other lecture after viewing *Four Funda-*
363 *mental Forces* but before viewing *Birth of Stars* (i.e., on Quiz 2; *Four Fundamental Forces* ques-
364 tions given *Birth of Stars* questions: $OR = 3.49$, 95% CI = [0.739, 12.849], $\lambda_{LR} = 3.266$, $p =$
365 0.083; *Birth of Stars* questions given *Four Fundamental Forces* questions: $OR = 2.199$, 95% CI =
366 [0.711, 5.623], $\lambda_{LR} = 2.304$, $p = 0.141$). Only after viewing *both* lectures (i.e., on Quiz 3) did
367 these across-lecture knowledge estimates reliably predict participants' success on individual quiz
368 questions (*Four Fundamental Forces* questions given *Birth of Stars* questions: $OR = 11.294$, 95% CI =
369 [1.375, 47.744], $\lambda_{LR} = 10.396$, $p < 0.001$; *Birth of Stars* questions given *Four Fundamental Forces* ques-
370 tions: $OR = 7.302$, 95% CI = [1.077, 44.879], $\lambda_{LR} = 4.708$, $p = 0.038$). Taken together, these results
371 suggest that our ability to form estimates solely across different content areas is more limited than
372 our ability to form estimates that incorporate responses to questions from both content areas (as in
373 Fig. 6, "All questions") or within a single content area (as in Fig. 6, "Within-lecture"). However, if
374 participants have recently received some training on both content areas, the knowledge estimates
375 appear to be informative even across content areas.

376 We speculate that these "Across-lecture" results might relate to some of our earlier work on
377 the nature of semantic representations [44]. In that work, we asked whether semantic similarities
378 could be captured through behavioral measures, even if participants' "true" internal representa-
379 tions differed from the embeddings used to characterize their behaviors. We found that mismatches
380 between an individual's internal representation of a set of concepts and the representation used to
381 characterize their behaviors can lead to underestimates of how semantically driven those behaviors
382 are. Along similar lines, we suspect that in our current study, participants' conceptual representa-
383 tions may initially differ from the representations learned by our topic model. (Although the topic

384 model’s representations are still *related* to participants’ initial internal representations; otherwise
385 we would have found that knowledge estimates derived from Quizzes 1 and 2 had no predictive
386 power in the other tests we conducted.) After watching both lectures, however, participants’
387 internal representations may become more aligned with the embeddings used to estimate their
388 knowledge (since those embeddings were trained on the lectures’ transcripts). This could help
389 explain why the knowledge estimates derived from Quizzes 1 and 2 (before both lectures had been
390 watched) do not reliably predict performance across content areas, whereas estimates derived from
391 Quiz 3 do.

392 That the knowledge predictions derived from the text embedding space reliably distinguish
393 between held-out correctly versus incorrectly answered questions (Fig. 6) suggests that spatial
394 relationships within this space can help explain what participants know. But how far does this
395 explanatory power extend? For example, suppose we know that a participant correctly answered a
396 question at embedding coordinate x . As we move farther away from x in the embedding space, how
397 does the likelihood that the participant knows about the content at a given location “fall off” with
398 distance? Conversely, suppose the participant instead answered that same question *incorrectly*.
399 Again, as we move farther away from x in the embedding space, how does the likelihood that the
400 participant does *not* know about a coordinate’s content change with distance? We reasoned that,
401 assuming our embedding space is capturing something about how individuals actually organize
402 their knowledge, a participant’s ability to answer questions embedded very close to x should
403 tend to be similar to their ability to answer the question embedded *at* x . Whereas at another
404 extreme, once we reach some sufficiently large distance from x , our ability to infer whether or
405 not a participant will correctly answer a question based on their ability to answer the question
406 at x should be no better than guessing based on their *overall* proportion of correctly answered
407 questions. In other words, beyond the maximum distance at which the participant’s ability to
408 answer the question at x is informative of their ability to answer a second question at location y ,
409 then guessing the outcome at y based on x should be no more successful than guessing based on a
410 measure that does not consider embedding-space distance.

411 With these ideas in mind, we asked: conditioned on answering a question correctly, what

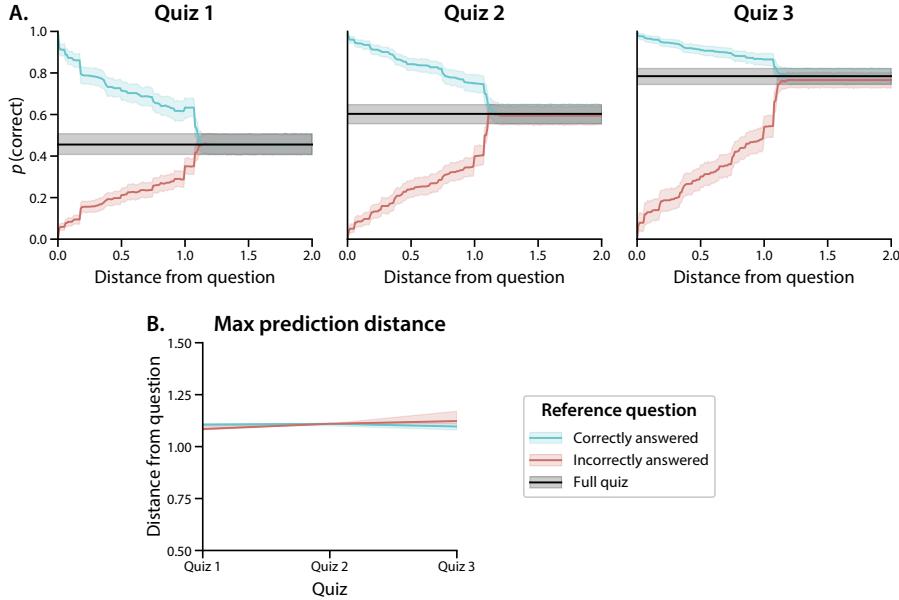


Figure 7: Knowledge falls off gradually in text embedding space. A. Performance versus distance. For each participant, for each correctly answered question (blue) or incorrectly answered question (red), we computed the proportion of correctly answered questions within a given distance of that question’s embedding coordinate. We used these proportions as a proxy for participants’ knowledge about the content within that region of the embedding space. We repeated this analysis for all questions and participants, and separately for each quiz (column). The black lines denote the average proportion correct across *all* questions included in the analysis at the given distance. **B. Maximum distance for which performance is reliably different from the average.** We used a bootstrap procedure (see *Estimating the “smoothness” of knowledge*) to estimate the point at which the blue and red lines in Panel A reliably diverged from the black line. We repeated this analysis separately for correctly and incorrectly answered questions from each quiz. **All panels.** Error ribbons denote bootstrap-estimated 95% confidence intervals.

412 proportion of all questions (within some radius, r , of that question’s embedding coordinate)
 413 were answered correctly? We plotted this proportion as a function of r . Similarly, we could
 414 ask, conditioned on answering a question incorrectly, how the proportion of correct responses
 415 changed with r . As shown in Figure 7, we found that quiz performance falls off smoothly with
 416 distance, and the “rate” of the falloff does not appear to change across the different quizzes, as
 417 measured by the distance at which performance becomes statistically indistinguishable from a
 418 simple proportion-correct score (see *Estimating the “smoothness” of knowledge*). This suggests that,
 419 at least within the region of text embedding space covered by the questions our participants
 420 answered (and as characterized using our topic model), the rate at which knowledge changes

421 with distance is relatively constant, even as participants' overall level of knowledge varies across
422 quizzes or regions of the embedding space.

423 Knowledge estimates need not be limited to the contents of these particular lectures and quizzes.
424 As illustrated in Figure 8, our general approach to estimating knowledge from a small number
425 of quiz questions may be extended to *any* content, given its text embedding coordinate. To
426 visualize how knowledge "spreads" through text embedding space to content beyond the lectures
427 participants watched and the questions they answered, we first fit a new topic model to the lectures'
428 sliding windows with $k = 100$ topics. Conceptually, increasing the number of topics used by the
429 model functions to increase the "resolution" of the embedding space, providing a greater ability
430 to estimate knowledge for content that is highly similar to (but not precisely the same as) that
431 contained in the two lectures used to train the model. We note that we used these 2D maps solely
432 for visualization; all relevant comparisons, distance computations, and statistical tests we report
433 above were carried out in the original 15-dimensional space, using the 15-topic model. Aside from
434 increasing the number of topics from 15 to 100, all other procedures and model parameters were
435 carried over from the preceding analyses. As in our other analyses, we resampled each lecture's
436 topic trajectory to 1 Hz and projected each question into a shared text embedding space.

437 We projected the resulting 100-dimensional topic vectors (for each second of video and each quiz
438 question) onto a shared 2-dimensional plane (see *Creating knowledge and learning map visualizations*).
439 Next, we sampled points from a 100×100 grid of coordinates that evenly tiled a rectangle enclos-
440 ing the 2D projections of the videos and questions. We used Equation 4 to estimate participants'
441 knowledge at each of these 10,000 sampled locations, and averaged these estimates across par-
442 ticipants to obtain an estimated average *knowledge map* (Fig. 8A). Intuitively, the knowledge map
443 constructed from a given quiz's responses provides a visualization of "how much" participants
444 knew about any content expressible by the fitted text embedding model at the point in time when
445 they completed that quiz.

446 Several features of the resulting knowledge maps are worth noting. The average knowledge
447 map estimated from Quiz 1 responses (Fig. 8A, leftmost map) shows that participants tended to
448 have relatively little knowledge about any parts of the text embedding space (i.e., the shading is

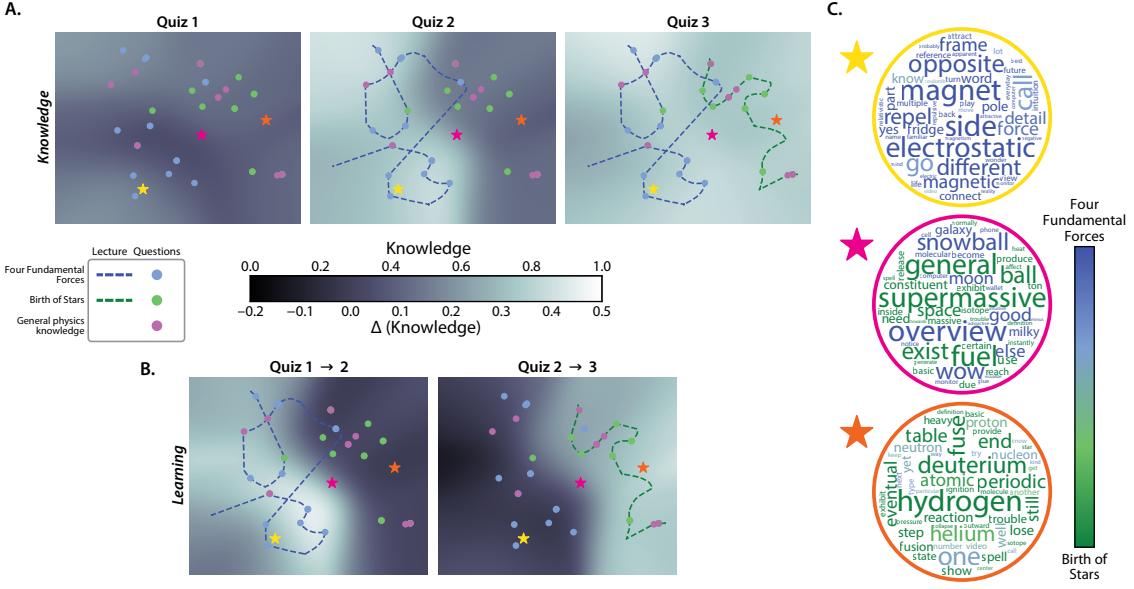


Figure 8: Mapping out the geometry of knowledge and learning. **A.** Average “knowledge maps” estimated using each quiz. Each map displays a 2D projection of the estimated knowledge about the content reflected by *all* regions of topic space (see *Creating knowledge and learning map visualizations*). The topic trajectories of the two lectures are indicated by dotted lines (blue: Lecture 1; green: Lecture 2), and the coordinates of each question are indicated by dots (light blue: Lecture 1-related; light green: Lecture 2-related; purple: general physics knowledge). Each map reflects an average across all participants. For individual participants’ maps, see Supplementary Figures 7, 8, and 9. **B.** Average “learning maps” estimated between each successive pair of quizzes. The learning maps follow the same general format as the knowledge maps in Panel A, but here the shading at each coordinate indicates the *difference* between the corresponding coordinates in the indicated pair of knowledge maps—i.e., how much the estimated knowledge “changed” between the two quizzes. Each map reflects an average across all participants. For individual participants’ maps, see Supplementary Figures 10 and 11. **C.** Word clouds for sampled points in topic space. Each word cloud displays the weighted blend of words underlying the topic proportions represented at the corresponding colored star’s location on the maps. In each word cloud, the words’ relative sizes correspond to their relative weights at the starred location, and their colors indicate their relative weights in *Four Fundamental Forces* (blue) versus *Birth of Stars* (green) lectures, on average, across all timepoints’ topic vectors.

449 relatively dark everywhere). The knowledge map estimated from Quiz 2 responses shows a marked
450 increase in knowledge on the left side of the map (around roughly the same range of coordinates
451 traversed by the *Four Fundamental Forces* lecture, indicated by the dotted blue line). In other words,
452 participants' estimated increase in knowledge is localized to conceptual content that is nearby (i.e.,
453 related to) the content from the lecture they watched prior to taking Quiz 2. This localization is
454 non-trivial: these knowledge estimates are informed only by the embedded coordinates of the
455 *quiz questions*, not by the embeddings of either lecture (see Eqn. 4). Finally, the knowledge map
456 estimated from Quiz 3 responses shows a second increase in knowledge, localized to the region
457 surrounding the embedding of the *Birth of Stars* lecture participants watched immediately prior to
458 taking Quiz 3.

459 Another way of visualizing these content-specific increases in knowledge after participants
460 viewed each lecture is displayed in Figure 8B. Taking the point-by-point difference between the
461 knowledge maps estimated from responses to a successive pair of quizzes yields a *learning map*
462 that describes the *change* in knowledge estimates from one quiz to the next. These learning maps
463 highlight that the estimated knowledge increases we observed across maps were specific to the
464 regions around the embeddings of each lecture, in turn.

465 Because the 2D projection we used to construct the knowledge and learning maps is invertible,
466 we may gain additional insights into these maps' meanings by reconstructing the original high-
467 dimensional topic vector for any location on the map we are interested in. For example, this could
468 serve as a useful tool for an instructor looking to better understand which content areas a student
469 (or a group of students) knows well (or poorly). As a demonstration, we show the top-weighted
470 words from the blends of topics reconstructed from three example locations on the maps (Fig. 8C):
471 one point near the *Four Fundamental Forces* embedding (yellow), a second point near the *Birth of*
472 *Stars* embedding (orange), and a third point between the two lectures' embeddings (pink). As
473 shown in the word clouds in the panel, the top-weighted words at the example coordinate near the
474 *Four Fundamental Forces* embedding tended to be weighted more heavily by the topics expressed
475 in that lecture. Similarly, the top-weighted words at the example coordinate near the *Birth of Stars*
476 embedding tended to be weighted more heavily by the topics expressed in *that* lecture. And the

477 top-weighted words at the example coordinate between the two lectures' embeddings show a
478 roughly even mix of words most strongly associated with each lecture.

479 Discussion

480 We developed a computational framework that uses short multiple-choice quizzes to gain nuanced
481 insights into what learners know and how their knowledge changes with training. First, we show
482 that our approach can automatically match the conceptual knowledge probed by individual quiz
483 questions to the corresponding moments in lecture videos when those concepts were presented
484 (Fig. 4). Next, we demonstrate how we can estimate moment-by-moment "knowledge traces"
485 that reflect the degree of knowledge participants have about each video's time-varying content,
486 and capture temporally specific increases in knowledge after viewing each lecture (Fig. 5). We also
487 show that these knowledge estimates can generalize to held-out questions and predict participants'
488 abilities to answer them correctly (Fig. 6). Finally, we use our framework to construct visual maps
489 that provide snapshot estimates of how much participants know about any concept within the
490 scope of our text embedding model, and how much their knowledge of those concepts changes
491 with training (Fig. 8).

492 Our work makes several contributions to the study of how people acquire conceptual knowl-
493 edge. First, from a methodological standpoint, our modeling framework provides a systematic
494 means of mapping out and characterizing knowledge in maps that have infinite (arbitrarily many)
495 numbers of coordinates, and of "filling out" those maps using relatively small numbers of multiple-
496 choice quiz questions. Our experimental finding that we can use these maps to predict responses
497 to held-out questions has several psychological implications as well. For example, concepts that
498 are assigned to nearby coordinates by the text embedding model also appear to be "known to a
499 similar extent" (as reflected by participants' responses to held-out questions; Fig. 6). This suggests
500 that participants also *conceptualize* similarly the content reflected by nearby embedding coordi-
501 nates. How participants' knowledge "falls off" with spatial distance is captured by the knowledge
502 maps we infer from their quiz responses (e.g., Figs. 7, 8). In other words, our study shows that

503 knowledge about a given concept implies knowledge about related concepts, and we also show
504 how estimated knowledge falls off with distance in text embedding space.

505 In our study, we characterize the “coordinates” of participants’ knowledge using a relatively
506 simple “bag of words” text embedding model [LDA; 8]. More sophisticated text embedding mod-
507 els, such as transformer-based models [18, 55, 68, 71], can leverage additional textual information
508 such as complex grammatical and semantic relationships between words, higher-order syntactic
509 structures, stylistic features, and more. We considered using transformer-based models in our
510 study, but we found that the text embeddings derived from these models were surprisingly unin-
511 formative with respect to differentiating or otherwise characterizing the conceptual content of the
512 lectures and questions we used (see *Supplementary results*). We suspect that this reflects a broader
513 challenge in constructing models that are both high-resolution within a given domain (e.g., the
514 domain of physics lectures and questions) *and* sufficiently broad as to enable them to cover a wide
515 range of domains. Essentially, these “larger” language models learn these more complex features
516 of language through pre-training on enormous and diverse text corpora. But as a result, their
517 embedding spaces also “span” an enormous and diverse range of conceptual content, sacrificing a
518 degree of specificity in their capacities to distinguish subtle conceptual differences within a more
519 narrow range of content. In comparing our LDA model (trained specifically on the lectures used
520 in our study) to a larger transformer-based model (BERT), we found that LDA provides both
521 coverage of the requisite material and specificity at the level of individual questions, while BERT
522 essentially relegates the contents of both lectures and all quiz questions (which are all broadly
523 about “physics”) to a tiny region of its embedding space, thereby blurring out meaningful distinc-
524 tions between different specific concepts covered by the lectures and questions (Supp. Fig. 6). We
525 note that these are not criticisms of BERT, nor of other large language models trained on large and
526 diverse corpora. Rather, our point is that simpler models trained on relatively small but specialized
527 corpora can outperform much more complex models trained on much larger corpora when we
528 are specifically interested in capturing subtle conceptual differences at the level of a single course
529 lecture or question. On the other hand, if our goal had been to choose a model that generalized
530 to many different content domains, we would expect our LDA model to perform comparatively

531 poorly to BERT or other much larger models. We suggest that bridging this tradeoff will be an
532 important challenge for future work.

533 At the opposite end of the spectrum from large language models, one could also imagine
534 using an even *simpler* “model” than LDA that relates the contents of course lectures and quiz
535 questions through explicit word-overlap metrics (rather than similarities in the latent topics they
536 exhibit). In a supplementary analysis (Supp. Fig. 5), we compared the LDA-based question-lecture
537 matches shown in Figure 4 with analogous matches based on the Jaccard similarity between each
538 question’s text and each sliding window from the corresponding lecture’s transcript. Similarly
539 to the embeddings derived from BERT, we found that this approach also blurred meaningful
540 distinctions between concepts presented in different parts of each lecture and tested by different
541 quiz questions. In this case, this lack of specificity reflects that meaningful distinctions between
542 concepts do not tend to emerge at the level of individual words.

543 In this way, we view our approach as occupying a sort of “sweet spot” between much larger
544 language models and simpler word-matching-based approaches, in that it enables us to capture the
545 relevant conceptual content of course materials at the appropriate semantic scale. Our approach
546 enables us to accurately and consistently identify each question’s content in a way that matches
547 it with specific content from the lectures and distinguishes it from other questions about similar
548 content. In turn, this enables us to construct accurate predictions about participants’ knowledge
549 of the conceptual content tested by individual quiz questions (Fig. 6).

550 Another application for large language models that does *not* require explicitly modeling the
551 content of individual lectures or questions is to leverage the models’ abilities to generate text. For
552 example, generative text models like ChatGPT [55] and LLaMa [68] are already being used to build
553 a new generation of interactive tutoring systems [e.g., 45]. Unlike the approach we have taken here,
554 these generative text model-based systems do not explicitly model what learners know, or how
555 their knowledge changes over time with training. One could imagine building a hybrid system
556 that combines the best of both worlds: a large language model that can *generate* text, combined
557 with a smaller model that can *infer* what learners know and how their knowledge changes over
558 time. Such a hybrid system could potentially be used to build the next generation of interactive

559 tutoring systems that are able to adapt to learners' needs in real time, and that are able to provide
560 more nuanced feedback about what learners know and what they do not know.

561 One limitation of our approach is that topic models contain no explicit internal representations
562 of more complex aspects of "knowledge," like knowledge graphs, dependencies or associations
563 between concepts, causality, and so on. These representations might (in principle) be added
564 as extensions to our approach to more accurately and precisely capture, characterize, and track
565 learners' knowledge. However, modeling these aspects of knowledge will likely require substantial
566 additional research effort.

567 Within the past several years, the global pandemic forced many educators to suddenly adapt to
568 teaching remotely [35, 52, 64, 72]. This change in world circumstances is happening alongside (and
569 perhaps accelerating) geometric growth in the availability of high-quality online courses from plat-
570 forms such as Khan Academy [36], Coursera [73], EdX [38], and others [60]. Continued expansion
571 of the global internet backbone and improvements in computing hardware have also facilitated
572 improvements in video streaming, enabling videos to be easily shared and viewed by increasingly
573 large segments of the world's population. This exciting time for online course instruction provides
574 an opportunity to re-evaluate how we, as a global community, educate ourselves and each other.
575 For example, we can ask: what defines an effective course or training program? Which aspects of
576 teaching might be optimized and/or augmented by automated tools? How and why do learning
577 needs and goals vary across people? How might we lower barriers to receiving a high-quality
578 education?

579 Alongside these questions, there is a growing desire to extend existing theories beyond the
580 domain of lab testing rooms and into real classrooms [34]. In part, this has led to a recent
581 resurgence of "naturalistic" or "observational" experimental paradigms that attempt to better
582 reflect more ethologically valid phenomena that are more directly relevant to real-world situations
583 and behaviors [53]. In turn, this has brought new challenges in data analysis and interpretation. A
584 key step towards solving these challenges will be to build explicit models of real-world scenarios
585 and how people behave in them (e.g., models of how people learn conceptual content from real-
586 world courses, as in our current study). A second key step will be to understand which sorts

587 of signals derived from behaviors and/or other measurements [e.g., neurophysiological data; 4,
588 19, 50, 54, 57] might help to inform these models. A third major step will be to develop and
589 employ reliable ways of evaluating the complex models and data that are a hallmark of naturalistic
590 paradigms.

591 Beyond specifically predicting what people *know*, the fundamental ideas we develop here also
592 relate to the notion of “theory of mind” of other individuals [26, 32, 49]. Considering others’ unique
593 perspectives, prior experiences, knowledge, goals, etc., can help us to more effectively interact and
594 communicate [58, 63, 67]. One could imagine future extensions of our work (e.g., analogous to
595 the knowledge and learning maps shown in Fig. 8), that attempt to characterize how well-aligned
596 different people’s knowledge bases or backgrounds are. In turn, this might be used to model how
597 knowledge (or other forms of communicable information) flows not just between teachers and
598 students, but between friends having a conversation, individuals on a first date, participants at
599 a business meeting, doctors and patients, experts and non-experts, political allies or adversaries,
600 and more. For example, the extent to which two people’s knowledge maps “match” or “align” in
601 a given region of text embedding space might serve as a predictor of how effectively they will be
602 able to communicate about the corresponding conceptual content.

603 Ultimately, our work suggests a rich new line of questions about the geometric “form” of
604 knowledge, how knowledge changes over time, and how we might map out the full space of
605 what an individual knows. Our finding that detailed estimates about knowledge may be obtained
606 from short quizzes shows one way that traditional approaches to evaluation in education may be
607 extended. We hope that these advances might help pave the way for new approaches to teaching
608 or delivering educational content that are tailored to individual students’ learning needs and goals.

609 **Materials and methods**

610 **Participants**

611 We enrolled a total of 50 Dartmouth undergraduate students in our study. Participants received
612 optional course credit for enrolling. We asked each participant to complete a demographic survey
613 that included questions about their age, gender, native spoken language, ethnicity, race, hearing,
614 color vision, sleep, coffee consumption, level of alertness, and several aspects of their educational
615 background and prior coursework.

616 Participants' ages ranged from 18 to 22 years (mean: 19.52 years; standard deviation: 1.09
617 years). A total of 15 participants reported their gender as male and 35 participants reported their
618 gender as female. A total of 49 participants reported their native language as "English" and 1
619 reported having another native language. A total of 47 participants reported their ethnicity as
620 "Not Hispanic or Latino" and three reported their ethnicity as "Hispanic or Latino." Participants
621 reported their races as White (32 participants), Asian (14 participants), Black or African American
622 (5 participants), American Indian or Alaska Native (1 participant), and Native Hawaiian or Other
623 Pacific Islander (1 participant). (Note that some participants selected multiple racial categories.)

624 A total of 49 participants reporting having normal hearing and 1 participant reported having
625 some hearing impairment. A total of 49 participants reported having normal color vision and 1
626 participant reported being color blind. Participants reported having had, on the night prior to
627 testing, 2–4 hours of sleep (1 participant), 4–6 hours of sleep (9 participants), 6–8 hours of sleep (35
628 participants), or 8+ hours of sleep (5 participants). They reported having consumed, on the same
629 day and leading up to their testing session, 0 cups of coffee (38 participants), 1 cup of coffee (10
630 participants), 3 cups of coffee (1 participant), or 4+ cups of coffee (1 participant).

631 No participants reported that their focus was currently impaired (e.g., by drugs or alcohol).
632 Participants reported their current level of alertness, and we converted their responses to numerical
633 scores as follows: "very sluggish" (-2), "a little sluggish" (-1), "neutral" (0), "fairly alert" (1), and
634 "very alert" (2). Across all participants, a range of alertness levels were reported (range: -2–1;
635 mean: -0.10; standard deviation: 0.84).

Participants reported their undergraduate major(s) as “social sciences” (28 participants), “natural sciences” (16 participants), “professional” (e.g., pre-med or pre-law; 8 participants), “mathematics and engineering” (7 participants), “humanities” (4 participants), or “undecided” (3 participants). Note that some participants selected multiple categories for their undergraduate major(s).

We also asked participants about the courses they had taken. In total, 45 participants reported having taken at least one Khan Academy course in the past, and 5 reported not having taken any Khan Academy courses. Of those who reported having watched at least one Khan Academy course, 7 participants reported having watched 1–2 courses, 11 reported having watched 3–5 courses, 8 reported having watched 5–10 courses, and 19 reported having watched 10 or more courses. We also asked participants about the specific courses they had watched, categorized under different subject areas. In the “Mathematics” area, participants reported having watched videos on AP Calculus AB (21 participants), Precalculus (17 participants), Algebra 2 (14 participants), AP Calculus BC (12 participants), Trigonometry (11 participants), Algebra 1 (10 participants), Geometry (8 participants), Pre-algebra (7 participants), Multivariable Calculus (5 participants), Differential Equations (5 participants), Statistics and Probability (4 participants), AP Statistics (2 participants), Linear Algebra (2 participants), Early Math (1 participant), Arithmetic (1 participant), and other videos not listed in our survey (5 participants). In the “Science and engineering” area, participants reported having watched videos on Chemistry, AP Chemistry, or Organic Chemistry (21 participants); Physics, AP Physics I, or AP Physics II (18 participants); Biology, AP Biology; or High school Biology (15 participants); Health and Medicine (1 participant); or other videos not listed in our survey (5 participants). We also asked participants whether they had specifically seen the videos used in our experiment. Of the 45 participants who reported having having taken at least one Khan Academy course in the past, 44 participants reported that they had not watched the *Four Fundamental Forces* video, and 1 participant reported that they were not sure whether they had watched it. All participants reported that they had not watched the *Birth of Stars* video. When we asked participants about non-Khan Academy online courses, they reported having watched or taken courses on Mathematics (15 participants), Science and engineering (11 participants), Test preparation (9 participants), Economics and finance (3 participants), Arts and humanities (2 participants).

664 ipants), Computing (2 participants), and other categories not listed in our survey (17 participants).
665 Finally, we asked participants about in-person courses they had taken in different subject areas.
666 They reported taking courses in Mathematics (38 participants), Science and engineering (37 par-
667 ticipants), Arts and humanities (34 participants), Test preparation (27 participants), Economics
668 and finance (26 participants), Computing (14 participants), College and careers (7 participants), or
669 other courses not listed in our survey (6 participants).

670 **Experiment**

671 We hand-selected two course videos from the Khan Academy platform: *Four Fundamental Forces*
672 (an introduction to gravity, electromagnetism, the weak nuclear force, and the strong nuclear force;
673 duration: 10 minutes and 29 seconds) and *Birth of Stars* (an introduction to how stars are formed;
674 duration: 7 minutes and 57 seconds). All participants viewed the videos in the same order (i.e.,
675 *Four Fundamental Forces* followed by *Birth of Stars*).

676 We then hand-created 39 multiple-choice questions: 15 about the conceptual content of *Four*
677 *Fundamental Forces* (i.e., Lecture 1), 15 about the conceptual content of *Birth of Stars* (i.e., Lecture 2),
678 and 9 questions that tested for general conceptual knowledge about basic physics (covering material
679 that was not presented in either video). To help broaden the set of lecture-specific questions,
680 our team worked through each lecture in small segments to identify what each segment was
681 “about” conceptually, and then write a question about that concept. The general physics questions
682 were drawn our team’s prior coursework and areas of interest, along with internet searches and
683 brainstorming with the project team and other members of J.R.M.’s lab. Although we attempted to
684 design the questions to test “conceptual knowledge,” we note that estimating the specific “amount”
685 of conceptual understanding that each question “requires” to answer is somewhat subjective, and
686 might even come down to the “strategy” a given participant uses to answer the question at that
687 particular moment. The full set of questions and answer choices may be found in Supplementary
688 Table 1. The final set of questions (and response options) was reviewed and approved by J.R.M.
689 before we collected or analyzed the text or experimental data.

690 Over the course of the experiment, participants completed three 13-question multiple-choice

691 quizzes: the first before viewing Lecture 1, the second between Lectures 1 and 2, and the third
692 after viewing Lecture 2 (see Fig. 1). The questions appearing on each quiz, for each participant,
693 were randomly chosen from the full set of 39, with the constraints that (a) each quiz contained
694 exactly 5 questions about Lecture 1, 5 questions about Lecture 2, and 3 questions about general
695 physics knowledge, and (b) each question appear exactly once for each participant. The orders of
696 questions on each quiz, and the orders of answer options for each question, were also randomized.
697 We obtained informed consent from all participants, and our experimental protocol was approved
698 by the Committee for the Protection of Human Subjects at Dartmouth College. We used this
699 experiment to develop and test our computational framework for estimating knowledge and
700 learning.

701 **Analysis**

702 **Statistics**

703 All of the statistical tests performed in our study were two-sided. The 95% confidence intervals
704 we reported for each correlation were estimated from bootstrap distributions of 10,000 correlation
705 coefficients obtained by sampling (with replacement) from the observed data.

706 **Constructing text embeddings of multiple lectures and questions**

707 We adapted an approach we developed in prior work [29] to embed each moment of the two
708 lectures and each question in our pool in a common representational space. Briefly, our approach
709 uses a topic model [Latent Dirichlet Allocation; 8] trained on a set of documents, to discover a set
710 of k “topics” or “themes.” Formally, each topic is defined as a distribution of weights over words in
711 the model’s vocabulary (i.e., the union of all unique words, across all documents, excluding “stop
712 words”). Conceptually, each topic is intended to give larger weights to words that are semantically
713 related (as inferred from their tendency to co-occur in the same document). After fitting a topic
714 model, each document in the training set, or any *new* document that contains at least some of
715 the words in the model’s vocabulary, may be represented as a k -dimensional vector describing

716 how much the document (most probably) reflects each topic. To select an appropriate k for our
717 model, as a starting point, we identified the minimum number of topics that yielded at least one
718 “unused” topic (i.e., in which all words in the vocabulary were assigned uniform weights) after
719 training. This indicated that the number of topics was sufficient to capture the set of latent themes
720 present in the two lectures (from which we constructed our document corpus, as described below).
721 We found this value to be $k = 15$ topics. We found that with a limited number of additional
722 adjustments following Boyd-Graber et al. [9], such as removing corpus-specific stop-words, the
723 model yielded (subjectively) sensible and coherent topics. The distribution of weights over words
724 in the vocabulary for each discovered topic is shown in Supplementary Figure 1, and each topic’s
725 top-weighted words may be found in Supplementary Table 2.

726 As illustrated in Figure 2A, we start by building up a corpus of documents using overlapping
727 sliding windows that span each video’s transcript. Khan Academy provides professionally created,
728 manual transcriptions of all videos for closed captioning. However, such transcripts would not
729 be readily available in all contexts to which our framework could potentially be applied. Khan
730 Academy videos are hosted on the YouTube platform, which additionally provides automated
731 captions. We opted to use these automated transcripts [which, in prior work, we have found to be
732 of sufficiently near-human quality to yield reliable data in behavioral studies; 74] when developing
733 our framework in order to make it more directly extensible and adaptable by others in the future.

734 We fetched these automated transcripts using the `youtube-transcript-api` Python pack-
735 age [17]. The transcripts consisted of one timestamped line of text for every few seconds (mean:
736 2.34 s; standard deviation: 0.83 s) of spoken content in the video (i.e., corresponding to each indi-
737 vidual caption that would appear on-screen if viewing the lecture via YouTube, and when those
738 lines would appear). We defined a sliding window length of (up to) $w = 30$ transcript lines, and
739 assigned each window a timestamp corresponding to the midpoint between the timestamps for its
740 first and last lines. This w parameter was chosen to match the same number of words per sliding
741 window (rounded to the nearest whole word, and before preprocessing) as the sliding windows
742 we defined in our prior work [29; i.e., 185 words per sliding window].

743 These sliding windows ramped up and down in length at the beginning and end of each

744 transcript, respectively. In other words, each transcript’s first sliding window covered only its first
745 line, the second sliding window covered the first two lines, and so on. This ensured that each line
746 from the transcripts appeared in the same number (w) of sliding windows. We next performed a
747 series of standard text preprocessing steps: normalizing case, lemmatizing, removing punctuation
748 and removing stop-words. We constructed our corpus of stop words by augmenting the Natural
749 Language Toolkit [NLTK; 5] English stop word list with the following additional words, selected
750 using one of the approaches suggested by Boyd-Graber et al. [9]: “actual,” “actually,” “also,” “bit,”
751 “could,” “e,” “even,” “first,” “follow,” “following,” “four,” “let,” “like,” “mc,” “really,”, “saw,”
752 “see,” “seen,” “thing,” and “two.” This yielded sliding windows with an average of 73.8 remaining
753 words, and lasting for an average of 62.22 seconds. We treated the text from each sliding window
754 as a single “document,” and combined these documents across the two videos’ windows to create
755 a single training corpus for the topic model.

756 After fitting a topic model to the two videos’ transcripts, we could use the trained model to
757 transform arbitrary (potentially new) documents into k -dimensional topic vectors. A convenient
758 property of these topic vectors is that documents that reflect similar blends of topics (i.e., documents
759 that reflect similar themes, according to the model) will yield similar coordinates (in terms of
760 correlation, cosine similarity, Kullback-Leibler divergence, Euclidean distance, or other geometric
761 measures). In general, the similarity between different documents’ topic vectors may be used to
762 characterize the similarity in conceptual content between the documents.

763 We transformed each sliding window’s text into a topic vector, and then used linear interpolation
764 (independently for each topic dimension) to resample the resulting time series to one vector
765 per second. We also used the fitted model to obtain topic vectors for each question in our pool (see
766 Supp. Tab. 1). Taken together, we obtained a *trajectory* for each video, describing its path through
767 topic space, and a single coordinate for each question (Fig. 2C). Embedding both videos and all of
768 the questions using a common model enables us to compare the content from different moments
769 of videos, compare the content across videos, and estimate potential associations between specific
770 questions and specific moments of video.

771 **Estimating dynamic knowledge traces**

772 We used the following equation to estimate each participant’s knowledge about timepoint t of a
773 given lecture, $\hat{k}(t)$:

$$\hat{k}(f(t, L)) = \frac{\sum_{i \in \text{correct}} \text{ncorr}(f(t, L), f(i, Q))}{\sum_{j=1}^N \text{ncorr}(f(t, L), f(j, Q))}, \quad (1)$$

774 where

$$\text{ncorr}(x, y) = \frac{\text{corr}(x, y) - \text{mincorr}}{\text{maxcorr} - \text{mincorr}}, \quad (2)$$

775 and where mincorr and maxcorr are the minimum and maximum correlations between any lecture
776 timepoint and question, taken over all timepoints in the given lecture and all questions *about* that
777 lecture appearing on the given quiz. We also define $f(s, \Omega)$ as the s^{th} topic vector from the set of
778 topic vectors Ω . Here t indexes the set of lecture topic vectors L , and i and j index the topic vectors
779 of questions Q used to estimate the knowledge trace. Note that “correct” denotes the set of indices
780 of the questions the participant answered correctly on the given quiz.

781 Intuitively, $\text{ncorr}(x, y)$ is the correlation between two topic vectors (e.g., the topic vector x
782 for one timepoint in a lecture and the topic vector y for one question on a quiz), normalized
783 by the minimum and maximum correlations (across all timepoints t and questions j) to range
784 between 0 and 1, inclusive. Equation 1 then computes the weighted average proportion of correctly
785 answered questions about the content presented at timepoint t , where the weights are given by the
786 normalized correlations between timepoint t ’s topic vector and the topic vectors for each question.
787 The normalization step (i.e., using ncorr instead of the raw correlations) ensures that every question
788 contributes some non-negative amount to the knowledge estimate.

789 **Generalized linear mixed models**

790 In the set of analyses reported in Figure 6, we assessed whether estimates of participants’ knowl-
791 edge at the embedding coordinates of individual quiz questions could be used to reliably predict
792 their abilities to correctly answer those questions. In essence, we treated each question a given

793 participant answered on a given quiz as a “lecture” consisting of a single timepoint, and used
794 Equation 1 to estimate the participant’s knowledge for its embedding coordinate based on their
795 performance on all *other* questions they answered on that same quiz (“All questions”; Fig. 6,
796 top row). Additionally, for each lecture-related question (i.e., excluding questions about general
797 physics knowledge), we computed analogous knowledge estimates based on two different subsets
798 of questions the participant answered on the same quiz: (1) all *other* questions about the same
799 lecture as the target question (“Within-lecture”; Fig. 6, middle rows), and (2) all questions about
800 the other of the two lectures (“Across-lecture”; Fig. 6, bottom rows).

801 In performing these analyses, our null hypothesis is that the knowledge estimates we compute
802 based on the quiz questions’ embedding coordinates do *not* provide useful information about
803 participants’ abilities to answer those questions—in other words, that there is no meaningful
804 difference (on average) between the knowledge estimates we compute for questions participants
805 answered correctly and those they answered incorrectly. Specifically, since we estimate knowledge
806 for a given embedding coordinate as a weighted proportion-correct score (where each question’s
807 weight reflects its embedding-space distance from the target coordinate; see Eqn. 1), if these weights
808 are uninformative (e.g., randomly distributed), then our estimates of participants’ knowledge
809 should be equivalent (on average) to the *unweighted* proportion of correctly answered questions
810 used to compute them. In general, for a given participant and quiz, this expected value (i.e.,
811 that participant’s proportion-correct score on that quiz) is the same for any coordinate in the
812 embedding space (e.g., any lecture timepoint, quiz question, etc.). However, in the “All questions”
813 and “Within-lecture” versions of the analyses shown in Figure 6, we estimate each participant’s
814 knowledge for each target question using all *other* questions (or all *other* questions about the same
815 lecture) they answered on the same quiz. This introduces a systematic dependency between
816 a participant’s success on a target question and their proportion-correct score on the remaining
817 questions available to estimate their knowledge for it. For example, suppose a participant correctly
818 answered n out of q questions on a given quiz. If we hold out a single *correctly* answered question as
819 the target, the proportion of remaining questions answered correctly would be $\frac{n-1}{q-1}$. Whereas if we
820 hold out a single *incorrectly* answered question, the proportion of remaining questions answered

821 correctly would be $\frac{n}{q-1}$. Thus, the proportion of correctly answered remaining questions (and
822 therefore the null-hypothesized value of a knowledge estimate computed from them) is always
823 *lower* for target questions a participant answered correctly than for those they answered incorrectly.

824 To correct for this baseline inverse relationship between a participant's success on a target
825 question and their estimated knowledge for it, we used a rebalancing procedure that ensured
826 our knowledge estimates for questions each participant answered correctly and incorrectly were
827 computed from the *same* proportion of correctly answered questions. For each target question on
828 a given participant's quiz, we identified all remaining questions with the opposite "correctness"
829 label (i.e., if the target question was answered correctly, we identified all remaining incorrectly
830 answered questions, and vice versa). We then held out each of these opposite-label questions,
831 in turn, along with the target question, and estimated the participant's knowledge for the target
832 question using all *other* remaining questions. Since each of these subsets of remaining questions
833 was constructed by holding out one correctly answered question and one incorrectly answered
834 question from the participant's quiz, if the participant correctly answered n out of q questions total,
835 then their proportion-correct score on each subset of questions used to estimate their knowledge
836 for the target question is $\frac{n-1}{q-2}$, regardless of whether they answered the target question correctly
837 or incorrectly. Finally, averaging over these per-subset knowledge estimates yielded a rebalanced
838 estimate of the participant's knowledge for the target question that leveraged information from all
839 remaining questions' embedding coordinates, but whose expected value under our null hypothesis
840 was the same as that of each individual subset ($\frac{n-1}{q-2}$). By equalizing the null-hypothesized values of
841 knowledge estimates for correctly and incorrectly answered questions, this procedure ensures that
842 any meaningful relationships we observe between participants' estimated knowledge for individ-
843 ual quiz questions and their abilities to correctly answer them are attributable to the predictive
844 power of the embedding-space distances used to weight questions' contributions to the knowledge
845 estimates, rather than an artifact of our estimation procedure. Note that if a participant answered
846 all or no questions on a given quiz correctly, their responses contained no opposite-label questions
847 with which to perform this rebalancing, and we therefore excluded their data from our analyses
848 for that quiz. We used this rebalancing procedure when constructing knowledge estimates for the

849 “All questions” and “Within-lecture” versions of the analyses shown in Figure 6, but not for the
850 “Across-lecture” analyses as, in this case, the target questions and the questions used to estimate
851 participants’ knowledge for them were drawn from different subsets of quiz questions (those about
852 one lecture, and those about the other), and were therefore independent.

853 In each version of this analysis (i.e., row in Fig. 6), and separately for each of the three quizzes
854 (i.e., column in Fig. 6), we then fit a generalized linear mixed model (GLMM) with a logistic link
855 function to the set of knowledge estimates for all questions (or all questions about a particular
856 lecture) that participants answered on the given quiz. We implemented these models in R using
857 the `lme4` package [3] and fit them following guidance from Bates et al. [2] and Matuschek et al.
858 [46]. Specifically, we initially fit each model with the maximal random effects structure afforded
859 by our design, which we identified as:

$$\text{accuracy} \sim \text{knowledge} + (\text{knowledge} | \text{participant}) + (\text{knowledge} | \text{question})$$

860 where “accuracy” is a binary value indicating whether each target question was answered cor-
861 rectly or incorrectly, “knowledge” is estimated knowledge at each target question’s embedding
862 coordinate, “participant” is a unique identifier assigned to each participant, and “question” is a
863 unique identifier assigned to each quiz question. For models we fit using knowledge estimates for
864 target questions about multiple content areas (i.e., in the “All questions” version of the analysis),
865 we also included an additional random effect term, $(\text{knowledge} | \text{lecture})$, where “lecture” is a
866 categorical value denoting whether the target question was about *Four Fundamental Forces*, *Birth*
867 of *Stars*, or general physics knowledge. Note that with our coding scheme, identifiers for each
868 question are implicitly nested within levels of lecture and so do not require explicit nesting in
869 our model formula. We then iteratively removed random effects from the maximal model until it
870 successfully converged with a full-rank random effects variance-covariance matrix. We obtained
871 the odds ratios reported in Figure 6 by exponentiating the estimated coefficient for “knowledge”
872 from each fitted model. Conceptually, these odds ratios represent how many times greater the odds
873 are that a given participant will answer a given question correctly if their estimated knowledge

874 for its embedding coordinate is 1, compared to if it is 0. We estimated 95% confidence intervals
875 for each odds ratio by generating 10,000 random subsamples (of full size, with replacement) from
876 the data used to fit each model, and refitting the models to each subsample to obtain bootstrap
877 distributions of 10,000 odds ratios.

878 To assess the predictive value of our knowledge estimates, we compared each GLMM's ability
879 to explain participants' success on individual quiz questions to that of an analogous model which
880 assumed (as we assume under our null hypothesis) that knowledge estimates for correctly and
881 incorrectly answered questions did *not* systematically differ, on average. Specifically, we used the
882 same sets of observations with which we fit each "full" model to fit a second "null" model that had
883 the same random effects structure, but in which the coefficient for the fixed effect of "knowledge"
884 was fixed at 0 (i.e., we removed this term from the null model). We then compared each full model
885 to its reduced (null) equivalent using a likelihood-ratio test (LRT). Because the standard asymptotic
886 χ_d^2 approximation of the null distribution for the LRT statistic (λ_{LR}) can be anti-conservative for
887 finite sample sizes [25, 61, 66], we computed *p*-values for these tests using a parametric bootstrap
888 procedure [14, 27]. For each of 10,000 bootstraps, we used the fitted null model to simulate a
889 sample of observations of equal size to our original sample. We then re-fit both the null and
890 full models to this simulated sample and compared them via an LRT. This yielded a distribution
891 of λ_{LR} statistics we may expect to observe given data that conforms to our null hypothesis. We
892 computed a corrected *p*-value for our observed λ_{LR} as $\frac{r+1}{n+1}$, where r is the number of simulated
893 model comparisons that yielded a λ_{LR} greater than our observed value and n is the number of
894 simulations we ran (10,000).

895 **Estimating the "smoothness" of knowledge**

896 In the analysis reported in Figure 7A, we show how participants' ability to correctly answer
897 quiz questions changes as a function of distance from a given correctly or incorrectly answered
898 reference question. We used a bootstrap-based approach to estimate the maximum distances over
899 which these proportions of correctly answered questions could be reliably distinguished from
900 participants' overall average proportion of correctly answered questions.

901 For each of 10,000 iterations, we drew a random subsample (with replacement) of 50 partic-
902 ipants from our dataset. Within each iteration, we first computed the 95% confidence interval
903 (CI) of the across-subsample-participants mean proportion correct on each of the three quizzes,
904 separately. To compute this interval for each quiz, we repeatedly (1,000 times) subsampled par-
905 ticipants (with replacement, from the outer subsample for the current iteration) and computed
906 the mean proportion correct of each of these inner subsamples. We then identified the 2.5th and
907 97.5th percentiles of the resulting distributions of 1,000 means. These three intervals (one for each
908 quiz) served as our thresholds for confidence that the proportion correct within a given distance
909 from a reference question was reliably different (at the $p < 0.05$ significance level) from the average
910 proportion correct across all questions on the given quiz.

911 Next, for each participant in the current subsample, and for each of the three quizzes they
912 completed (separately), we iteratively treated each of the 15 questions appearing on the given
913 quiz as the “reference” question. We constructed a series of concentric 15-dimensional “spheres”
914 centered on the reference question’s embedding-space coordinate, where each successive sphere’s
915 radius increased by 0.01 (correlation distance) between 0 and 2, inclusive (i.e., tiling the range
916 of possible correlation distances with 201 spheres in total). We then computed the proportion
917 of questions enclosed within each sphere that the participant answered correctly, and averaged
918 these per-radius proportion-correct scores across reference questions that were answered correctly,
919 and those that were answered incorrectly. This resulted in two number-of-spheres sequences of
920 proportion-correct scores for each subsample participant and quiz: one derived from correctly
921 answered reference questions, and one derived from incorrectly answered reference questions.

922 We computed the across-subsample-participants mean proportion correct for each radius value
923 (i.e., sphere) and “correctness” of reference question. This yielded two sequences of proportion-
924 correct scores for each quiz, analogous to the blue and red lines displayed in Figure 7A, but for
925 the present subsample. For each quiz, we then found the minimum distance from the reference
926 question (i.e., sphere radius) at which each of these two sequences of per-radius proportion-correct
927 scores intersected the 95% confidence interval for the overall proportion correct (i.e., analogous to
928 the black error bands in Fig. 7A).

929 This resulted in two “intersection” distances for each quiz (for correctly answered and incor-
930 rectly answered reference questions). Repeating this full process for each of the 10,000 bootstrap
931 iterations output two distributions of intersection distances for each of the three quizzes. The
932 means and 95% confidence intervals for these distributions are plotted in Figure 7B.

933 **Creating knowledge and learning map visualizations**

934 An important feature of our approach is that, given a trained text embedding model and partic-
935 ipants’ quiz performance on each question, we can estimate their knowledge about *any* content
936 expressible by the embedding model—not solely the content explicitly probed by the quiz ques-
937 tions, or even appearing in the lectures. To visualize these estimates (Fig. 8, Supp. Figs. 7, 8, 9, 10,
938 and 11), we used Uniform Manifold Approximation and Projection [UMAP; 47, 48] to construct a
939 2D projection of the text embedding space. Whereas our main analyses used a 15-topic embedding
940 space, we used a 100-topic embedding space for these visualizations. This change in the number
941 of topics overcame an undesirable behavior in the UMAP embedding procedure, whereby embed-
942 ding coordinates for the 15-topic model tended to be “clumped” into separated clusters, rather
943 than forming a smooth trajectory through the 2D space. When we increased the number of topics
944 to 100, the embedding coordinates in the 2D space formed a smooth trajectory through the space,
945 with substantially less clumping (Fig. 8). Creating a “map” by sampling this 100-dimensional
946 space at high resolution to obtain an adequate set of topic vectors spanning the embedding space
947 would be computationally intractable. However, sampling a 2D grid is trivial.

948 At a high level, the UMAP algorithm obtains low-dimensional embeddings by minimizing
949 the cross-entropy between the pairwise (clustered) distances between the observations in their
950 original (e.g., 100-dimensional) space and the pairwise (clustered) distances in the low-dimensional
951 embedding space (in our approach, the embedding space is 2D). In our implementation, pairwise
952 distances in the original high-dimensional space were defined as 1 minus the correlation between
953 each pair of coordinates, and pairwise distances in the low-dimensional embedding space were
954 defined as the Euclidean distance between each pair of coordinates.

955 In our application, all of the coordinates we embedded were topic vectors, whose elements

956 are always non-negative and sum to one. Although UMAP is an invertible transformation at
 957 the embedding locations of the original data, other locations in the embedding space will not
 958 necessarily follow the same implicit “rules” as the original high-dimensional data. For example,
 959 inverting an arbitrary coordinate in the embedding space might result in negative-valued vectors,
 960 which are incompatible with the topic modeling framework. To protect against this issue, we
 961 log-transformed the topic vectors prior to embedding them in the 2D space. When we inverted
 962 the embedded vectors (e.g., to estimate topic vectors for word clouds, as in Fig. 8C), we passed
 963 the inverted (log-transformed) values through the exponential function to obtain a vector of non-
 964 negative values, and normalized them to sum to one.

965 After embedding both lectures’ topic trajectories and the topic vectors of every question, we
 966 defined a rectangle enclosing the 2D projections of the lectures’ and quizzes’ embeddings. We then
 967 sampled points from a regular 100×100 grid of coordinates that evenly tiled this enclosing rectangle.
 968 We sought to estimate participants’ knowledge (and learning, i.e., changes in knowledge) at each
 969 of the resulting 10,000 coordinates.

970 To generate our estimates, we placed a set of 39 radial basis functions (RBFs) throughout the
 971 embedding space, centered on the 2D projections for each question (i.e., we included one RBF for
 972 each question). At coordinate x , the value of an RBF centered on a question’s coordinate μ , is given
 973 by:

$$\text{RBF}(x, \mu, \lambda) = \exp\left\{-\frac{\|x - \mu\|^2}{\lambda}\right\}. \quad (3)$$

974 The λ term in the RBF equation controls the “smoothness” of the function, where larger values
 975 of λ result in smoother maps. In our implementation we used $\lambda = 50$. Next, we estimated the
 976 “knowledge” at each coordinate, x , using:

$$\hat{k}(x) = \frac{\sum_{i \in \text{correct}} \text{RBF}(x, q_i, \lambda)}{\sum_{j=1}^N \text{RBF}(x, q_j, \lambda)}. \quad (4)$$

977 Equation 4 computes the weighted proportion of correctly answered questions, where the weights
 978 are given by how nearby (in the 2D space) each question is to the x . We also defined *learning maps*
 979 as the coordinate-by-coordinate differences between any pair of knowledge maps. Intuitively,

980 learning maps reflect the *change* in knowledge across two maps.

981 **Author contributions**

982 Conceptualization: P.C.F., A.C.H., and J.R.M. Methodology: P.C.F., A.C.H., and J.R.M. Software:
983 P.C.F. Validation: P.C.F. Formal analysis: P.C.F. Resources: P.C.F., A.C.H., and J.R.M. Data curation:
984 P.C.F. Writing (original draft): J.R.M. Writing (review and editing): P.C.F., A.C.H., and J.R.M. Visu-
985 alization: P.C.F. and J.R.M. Supervision: J.R.M. Project administration: P.C.F. Funding acquisition:
986 J.R.M.

987 **Data availability**

988 All of the data analyzed in this manuscript may be found at <https://github.com/ContextLab/efficient-learning-khan>.
989

990 **Code availability**

991 All of the code for running our experiment and carrying out the analyses may be found at
992 <https://github.com/ContextLab/efficient-learning-khan>.

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