

<sup>1</sup> Text embedding models yield high-resolution insights  
<sup>2</sup> into conceptual knowledge from short multiple-choice  
<sup>3</sup> quizzes

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<sup>5</sup> **Abstract**

<sup>6</sup> We develop a mathematical framework, based on natural language processing models, for track-  
<sup>7</sup> ing and characterizing the acquisition of conceptual knowledge. Our approach embeds each  
<sup>8</sup> concept in a high-dimensional representation space, where nearby coordinates reflect similar or  
<sup>9</sup> related concepts. We test our approach using behavioral data from participants who answered  
<sup>10</sup> small sets of multiple-choice quiz questions interleaved between watching two course videos  
<sup>11</sup> from the Khan Academy platform. We apply our framework to the videos' transcripts and  
<sup>12</sup> the text of the quiz questions to quantify the content of each moment of video and each quiz  
<sup>13</sup> question. We use these embeddings, along with participants' quiz responses, to track how the  
<sup>14</sup> learners' knowledge changed after watching each video. Our findings show how a small set of  
<sup>15</sup> quiz questions may be used to obtain rich and meaningful high-resolution insights into what  
<sup>16</sup> each learner knows, and how their knowledge changes over time as they learn.

<sup>17</sup> **Keywords:** education, learning, knowledge, concepts, natural language processing

<sup>18</sup> **Introduction**

<sup>19</sup> Suppose that a teacher had access to a complete, tangible “map” of everything a student knows.  
<sup>20</sup> Defining what such a map might even look like, let alone how it might be constructed or filled in, is  
<sup>21</sup> itself a non-trivial problem. But if a teacher *were* to gain access to such a map, how might it change  
<sup>22</sup> their ability to teach that student? Perhaps they might start by checking how well the student  
<sup>23</sup> knows the to-be-learned information already, or how much they know about related concepts.  
<sup>24</sup> For some students, they could potentially optimize their teaching efforts to maximize efficiency  
<sup>25</sup> by focusing primarily on not-yet-known content. For other students (or other content areas), it  
<sup>26</sup> might be more effective to optimize for direct connections between already known content and  
<sup>27</sup> new material. Observing how the student’s knowledge changed over time, in response to their  
<sup>28</sup> teaching, could also help to guide the teacher towards the most effective strategy for that individual  
<sup>29</sup> student.

<sup>30</sup> A common approach to assessing a student’s knowledge is to present them with a set of quiz  
<sup>31</sup> questions, calculate the proportion they answer correctly, and provide them with feedback in the  
<sup>32</sup> form of a simple numeric or letter grade. While such a grade can provide *some* indication of whether  
<sup>33</sup> the student has mastered the to-be-learned material, any univariate measure of performance on a  
<sup>34</sup> complex task sacrifices certain relevant information, risks conflating underlying factors, and so on.  
<sup>35</sup> For example, consider the relative utility of the theoretical map described above that characterizes  
<sup>36</sup> a student’s knowledge in detail, versus a single annotation saying that the student answered 85%  
<sup>37</sup> of their quiz questions correctly, or that they received a ‘B’. Here, we show that the same quiz data  
<sup>38</sup> required to compute proportion-correct scores or letter grades can instead be used to obtain far  
<sup>39</sup> more detailed insights into what a student knew at the time they took the quiz.

<sup>40</sup> Designing and building procedures and tools for mapping out knowledge touches on deep  
<sup>41</sup> questions about what it means to learn. For example, how do we acquire conceptual knowledge?  
<sup>42</sup> Memorizing course lectures or textbook chapters by rote can lead to the superficial *appearance*  
<sup>43</sup> of understanding the underlying content, but achieving true conceptual understanding seems to  
<sup>44</sup> require something deeper and richer. Does conceptual understanding entail connecting newly

45 acquired information to the scaffolding of one’s existing knowledge or experience [6, 11, 13, 15, 30,  
46 64]? Or weaving a lecture’s atomic elements (e.g., its component words) into a structured network  
47 that describes how those individual elements are related [40, 69]? Conceptual understanding  
48 could also involve building a mental model that transcends the meanings of those individual  
49 atomic elements by reflecting the deeper meaning underlying the gestalt whole [37, 41, 61, 68].

50 The difference between “understanding” and “memorizing,” as framed by researchers in ed-  
51 ucation, cognitive psychology, and cognitive neuroscience [e.g., 23, 28, 33, 41, 61], has profound  
52 analogs in the fields of natural language processing and natural language understanding. For  
53 example, considering the raw contents of a document (e.g., its constituent symbols, letters, and  
54 words) might provide some clues as to what the document is about, just as memorizing a passage  
55 might provide some ability to answer simple questions about it. However, text embedding mod-  
56 els [e.g., 7, 8, 10, 12, 16, 39, 50, 70] also attempt to capture the deeper meaning *underlying* those  
57 atomic elements. These models consider not only the co-occurrences of those elements within and  
58 across documents, but (in many cases) also patterns in how those elements appear across different  
59 scales (e.g., sentences, paragraphs, chapters, etc.), the temporal and grammatical properties of the  
60 elements, and other high-level characteristics of how they are used [42, 43]. To be clear, this is not  
61 to say that text embedding models themselves are capable of “understanding” deep conceptual  
62 meaning in any traditional sense. But rather, their ability to capture the underlying *structure* of  
63 text documents beyond their surface-level contents provides a computational framework through  
64 which those documents’ deeper conceptual meanings may be quantified, explored, and under-  
65 stood. According to these models, the deep conceptual meaning of a document may be captured  
66 by a feature vector in a high-dimensional representation space, wherein nearby vectors reflect con-  
67 ceptually related documents. A model that succeeds at capturing an analogue of “understanding”  
68 is able to assign nearby feature vectors to two conceptually related documents, *even when the specific*  
69 *words contained in those documents have limited overlap*. In this way, “concepts” are defined implicitly  
70 by the model’s geometry [e.g., how the embedding coordinate of a given word or document relates  
71 to the coordinates of other text embeddings; 55].

72 Given these insights, what form might a representation of the sum total of a person’s knowledge

73 take? First, we might require a means of systematically describing or representing (at least some  
74 subset of) the nearly infinite set of possible things a person could know. Second, we might want to  
75 account for potential associations between different concepts. For example, the concepts of “fish”  
76 and “water” might be associated in the sense that fish live in water. Third, knowledge may have  
77 a critical dependency structure, such that knowing about a particular concept might require first  
78 knowing about a set of other concepts. For example, understanding the concept of a fish swimming  
79 in water first requires understanding what fish and water *are*. Fourth, as we learn, our “current  
80 state of knowledge” should change accordingly. Learning new concepts should both update our  
81 characterizations of “what is known” and also unlock any now-satisfied dependencies of those  
82 newly learned concepts so that they are “tagged” as available for future learning.

83 Here we develop a framework for modeling how conceptual knowledge is acquired during  
84 learning. The central idea behind our framework is to use text embedding models to define the  
85 coordinate systems of two maps: a *knowledge map* that describes the extent to which each concept is  
86 currently known, and a *learning map* that describes changes in knowledge over time. Each location  
87 on these maps represents a single concept, and the maps’ geometries are defined such that related  
88 concepts are located nearby in space. We use this framework to analyze and interpret behavioral  
89 data collected from an experiment that had participants answer sets of multiple-choice questions  
90 about a series of recorded course lectures.

91 Our primary research goal is to advance our understanding of what it means to acquire deep,  
92 real-world conceptual knowledge. Traditional laboratory approaches to studying learning and  
93 memory (e.g., list-learning studies) often draw little distinction between memorization and under-  
94 standing. Instead, these studies typically focus on whether information is effectively encoded or  
95 retrieved, rather than whether the information is *understood*. Approaches to studying conceptual  
96 learning, such as category learning experiments, can begin to investigate the distinction between  
97 memorization and understanding, often by training participants to distinguish arbitrary or random  
98 features in otherwise meaningless categorized stimuli [1, 20, 21, 24, 31, 58]. However the objective  
99 of real-world training, or learning from life experiences more generally, is often to develop new  
100 knowledge that may be applied in *useful* ways in the future. In this sense, the gap between modern

learning theories and modern pedagogical approaches that inform classroom learning strategies is enormous: most of our theories about *how* people learn are inspired by experimental paradigms and models that have only peripheral relevance to the kinds of learning that students and teachers actually seek [28, 41]. To help bridge this gap, our study uses course materials from real online courses to inform, fit, and test models of real-world conceptual learning. We also provide a demonstration of how our models can be used to construct “maps” of what students know, and how their knowledge changes with training. In addition to helping to visually capture knowledge (and changes in knowledge), we hope that such maps might lead to real-world tools for improving how we educate. Taken together, our work shows that existing course materials and evaluative tools like short multiple-choice quizzes may be leveraged to gain highly detailed insights into what students know and how they learn.

## Results

At its core, our main modeling approach is based around a simple assumption that we sought to test empirically: all else being equal, knowledge about a given concept is predictive of knowledge about similar or related concepts. From a geometric perspective, this assumption implies that knowledge is fundamentally “smooth.” In other words, as one moves through a space representing an individual’s knowledge (where similar concepts occupy nearby coordinates), their “level of knowledge” should change relatively gradually. To begin to test this smoothness assumption, we sought to track participants’ knowledge and how it changed over time in response to training. Two overarching goals guide our approach. First, we want to gain detailed insights into what learners know at different points in their training. For example, rather than simply reporting on the proportions of questions participants answer correctly (i.e., their overall performance), we seek estimates of their knowledge about a variety of specific concepts. Second, we want our approach to be potentially scalable to large numbers of diverse concepts, courses, and students. This requires that the conceptual content of interest be discovered *automatically*, rather than relying on manually produced ratings or labels.



**Figure 1: Experimental paradigm.** Participants alternate between completing three 13-question multiple-choice quizzes and watching two Khan Academy lectures. Each quiz contains a mix of 5 questions about Lecture 1, 5 questions about Lecture 2, and 3 questions about general physics knowledge. The specific questions reflected on each quiz, and the orders of each quiz's questions, were randomized across participants.

127 We asked participants in our study to complete brief multiple-choice quizzes before, between,  
 128 and after watching two lecture videos from the Khan Academy [36] platform (Fig. 1). The first  
 129 lecture video, entitled *Four Fundamental Forces*, discussed the four fundamental forces in physics:  
 130 gravity, strong and weak interactions, and electromagnetism. The second, entitled *Birth of Stars*,  
 131 provided an overview of our current understanding of how stars form. We selected these particular  
 132 lectures to satisfy three general criteria. First, we wanted both lectures to be accessible to a broad  
 133 audience (i.e., with minimal prerequisite knowledge) so as to limit the impact of prior training  
 134 on participants' abilities to learn from the lectures. To this end, we selected two introductory  
 135 videos that were intended to be viewed at the start of students' training in their respective content  
 136 areas. Second, we wanted the two lectures to have some related content, so that we could test  
 137 our approach's ability to distinguish similar conceptual content. To this end, we chose two videos  
 138 from the same Khan Academy course domain, "Cosmology and Astronomy." Third, we sought to  
 139 minimize dependencies and specific overlap between the videos. For example, we did not want  
 140 participants' abilities to understand one video to (directly) influence their abilities to understand the  
 141 other. To satisfy this last criterion, we chose videos from two different lecture series (Lectures 1 and  
 142 2 were from the "Scale of the Universe" and "Stars, Black Holes, and Galaxies" series, respectively).

143 We also wrote a set of multiple-choice quiz questions that we hoped would enable us to  
 144 evaluate participants' knowledge about each individual lecture, along with related knowledge



**Figure 2: Modeling course content.** **A. Building a document pool from sliding windows of text.** We decompose each lecture’s transcript into a series of overlapping sliding windows. The full set of transcript snippets (across all windows) may be treated as a set of “documents” for training a text embedding model. **B. Constructing lecture content trajectories.** After training the model on the sliding windows from both lectures, we transform each lecture into a “trajectory” through text embedding space by joining the embedding coordinates of successive sliding windows parsed from its transcript. **C. Embedding multiple lectures and questions in a shared space.** We apply the same model (trained on the two lectures’ windows) to both lectures, along with the text of each question in our pool (Supp. Tab. 1), to project them into a shared text embedding space. This results in one trajectory per lecture and one coordinate for each question. Here, we have projected the 15-dimensional embeddings onto their first 3 principal components for visualization.

145 about physics concepts not specifically presented in either video (see Supp. Tab. 1 for the full list  
 146 of questions in our stimulus pool). Participants answered questions randomly drawn from each  
 147 content area (Lecture 1, Lecture 2, and general physics knowledge) on each of the three quizzes.  
 148 Quiz 1 was intended to assess participants’ “baseline” knowledge before training, Quiz 2 assessed  
 149 knowledge after watching the *Four Fundamental Forces* video (i.e., Lecture 1), and Quiz 3 assessed  
 150 knowledge after watching the *Birth of Stars* video (i.e., Lecture 2).

151 To study in detail how participants’ conceptual knowledge changed over the course of the  
 152 experiment, we first sought to model the conceptual content presented to them at each moment  
 153 throughout each of the two lectures. We adapted an approach we developed in prior work [29]  
 154 to identify the latent themes in the lectures using a topic model [8]. Briefly, topic models take  
 155 as input a collection of text documents, and learn a set of “topics” (i.e., latent themes) from their  
 156 contents. Once fit, a topic model can be used to transform arbitrary (potentially new) documents  
 157 into sets of “topic proportions,” describing the weighted blend of learned topics reflected in their  
 158 texts. We parsed automatically generated transcripts of the two lectures into overlapping sliding  
 159 windows, where each window contained the text of the lecture transcript from a particular time

span. We treated the set of text snippets (across all of these windows) as documents to fit the model (Fig. 2A; see *Constructing text embeddings of multiple lectures and questions*). Transforming the text from every sliding window with the model yielded a number-of-windows by number-of-topics (15) topic-proportions matrix describing the unique mixture of broad themes from both lectures reflected in each window’s text. Each window’s “topic vector” (i.e., column of the topic-proportions matrix) is analogous to a coordinate in a 15-dimensional space whose axes are topics discovered by the model. Within this space, each lecture’s sequence of topic vectors (i.e., corresponding to its transcript’s overlapping text snippets across sliding windows) forms a *trajectory* that captures how its conceptual content unfolds over time (Fig. 2B). We resampled these trajectories to a resolution of one topic vector for each second of video (i.e., 1 Hz).

We hypothesized that a topic model trained on transcripts of the two lectures should also capture the conceptual knowledge probed by each quiz question. If indeed the topic model could capture information about the deeper conceptual content of the lectures (i.e., beyond surface-level details such as particular word choices), then we should be able to recover a correspondence between each lecture and questions *about* each lecture. Importantly, such a correspondence could not solely arise from superficial text matching between lecture transcripts and questions, since the lectures and questions often used different words (Supp. Fig. 5) and phrasings. Simply comparing the average topic weights from each lecture and question set (averaging across time and questions, respectively) reveals a striking correspondence (Supp. Fig. 2). Specifically, the average topic weights from Lecture 1 are strongly correlated with the average topic weights from Lecture 1 questions ( $r(13) = 0.809$ ,  $p < 0.001$ , 95% confidence interval (CI) = [0.633, 0.962]), and the average topic weights from Lecture 2 are strongly correlated with the average topic weights from Lecture 2 questions ( $r(13) = 0.728$ ,  $p = 0.002$ , 95% CI = [0.456, 0.920]). At the same time, the average topic weights from the two lectures are *negatively* correlated with the average topic weights from their non-matching question sets (Lecture 1 video vs. Lecture 2 questions:  $r(13) = -0.547$ ,  $p = 0.035$ , 95% CI = [-0.812, -0.231]; Lecture 2 video vs. Lecture 1 questions:  $r(13) = -0.612$ ,  $p = 0.015$ , 95% CI = [-0.874, -0.281]), indicating that the topic model also exhibits some degree of specificity. The full set of pairwise comparisons between average topic weights for the lectures and question sets



**Figure 3: Lecture and question topic overlap. A. Topic weight variability.** The bar plots display the variance of each topic's weight across lecture timepoints (top row) and questions (bottom row); colors denote topics. The top-weighted words from the most “expressive” (i.e., variable across observations) topic from each lecture are displayed in the upper right (orange: topic 2; yellow-green: topic 4). The top-weighted words from the full set of topics may be found in Supplementary Table 2. **B. Relationships between topic weight variability.** Pairwise correlations between the distributions of topic weight variance for each lecture and question set. Each row and column corresponds to a bar plot in Panel A.

188 is reported in Supplementary Figure 2.

189 Another, more sensitive, way of summarizing the conceptual content of the lectures and questions is to look at *variability* in how topics are weighted over time and across different questions 190 (Fig. 3). Intuitively, the variability in the expression of a given topic relates to how much “information” [22] the lecture (or question set) reflects about that topic. For example, suppose a given 191 topic is weighted on heavily throughout a lecture. That topic might be characteristic of some 192 aspect or property of the lecture *overall* (conceptual or otherwise), but unless the topic’s weights 193 changed in meaningful ways over time, the topic would be a poor indicator of any *specific* conceptual 194 content in the lecture. We therefore also compared the variances in topic weights (across time 195 or questions) between the lectures and questions. The variability in topic expression (over time 196 or questions) was similar for the Lecture 1 video and questions ( $r(13) = 0.824, p < 0.001$ , 197 95% CI = [0.696, 0.973]) and the Lecture 2 video and questions ( $r(13) = 0.801, p < 0.001$ , 95% 198 CI = [0.539, 0.958]). Simultaneously, as reported in Figure 3B, the variabilities in topic expression 199 across *different* videos and lecture-specific questions (i.e., Lecture 1 video vs. Lecture 2 questions; 200 201

202 Lecture 2 video vs. Lecture 1 questions) were negatively correlated, and neither video’s topic  
203 variability was reliably correlated with the topic variability across general physics knowledge  
204 questions. Taken together, the analyses reported in Figure 3 and Supplementary Figure 2 indicate  
205 that a topic model fit to the videos’ transcripts can also reveal correspondences (at a coarse scale)  
206 between the lectures and questions.

207 While an individual lecture may be organized around a single broad theme at a coarse scale,  
208 at a finer scale, each moment of a lecture typically covers a narrower range of content. Given the  
209 correspondence we found between the variabilities in topic expression across moments of each  
210 lecture and questions from its corresponding set (Fig. 3), we wondered whether the text embedding  
211 model might additionally capture these conceptual relationships at a finer scale. For example, if a  
212 particular question asks about the content from one small part of a lecture, we wondered whether  
213 the text embeddings could be used to automatically identify the “matching” moment(s) in the  
214 lecture. To explore this, we computed the correlation between each question’s topic weights  
215 and the topic weights for each second of its corresponding lecture, and found that each question  
216 appeared to be temporally specific (Fig. 4). In particular, most questions’ topic vectors were  
217 maximally correlated with a well-defined (and relatively narrow) range of timepoints from their  
218 corresponding lectures, and the correlations fell off sharply outside of that range (Supp. Figs. 3, 4).  
219 We also qualitatively examined the best-matching intervals for each question by comparing the  
220 question’s text to the transcribed text from the most-correlated parts of the lectures (Supp. Tab. 3).  
221 Despite that the questions were excluded from the text embedding model’s training set, in general  
222 we found (through manual inspection) a close correspondence between the conceptual content  
223 that each question probed and the content covered by the best-matching moments of the lectures.  
224 Two representative examples are shown at the bottom of Figure 4.

225 The ability to quantify how much each question is “asking about” the content from each moment  
226 of the lectures could enable high-resolution insights into participants’ knowledge. Traditional  
227 approaches to estimating how much a student “knows” about the content of a given lecture entail  
228 administering some form of assessment (e.g., a quiz) and computing the proportion of correctly  
229 answered questions. But if two students receive identical scores on such an exam, might our



**Figure 4: Which parts of each lecture are captured by each question?** Each panel displays time series plots showing how each question’s topic vector correlates with each video timepoint’s topic vector (Panel A.: correlations for the *Four Fundamental Forces* lecture and associated questions; Panel B.: correlations for the *Birth of Stars* lecture and associated questions). The colors denote question identities. The diamonds in each panel denote the moment of peak correlation between the indicated question and the lecture trajectory. The associated questions’ text and snippets of the lectures’ transcripts from the surrounding 30 seconds, are displayed at the bottom of the figure.

modeling framework help us to gain more nuanced insights into the *specific* content that each student has mastered (or failed to master)? For example, a student who misses three questions that were all about the same concept (e.g., concept *A*) will have gotten the same *proportion* of questions correct as another student who missed three questions about three *different* concepts (e.g., *A*, *B*, and *C*). But if we wanted to help these two students fill in the “gaps” in their understandings, we might do well to focus specifically on concept *A* for the first student, but to also add in materials pertaining to concepts *B* and *C* for the second student. In other words, raw “proportion-correct” measures may capture *how much* a student knows, but not *what* they know. We wondered whether our modeling framework might enable us to (formally and automatically) infer participants’ knowledge at the scale of individual concepts (e.g., as captured by a single moment of a lecture).

We developed a simple formula (Eqn. 1) for using a participant’s responses to a small set of multiple-choice questions to estimate how much the participant “knows” about the concept reflected by any arbitrary coordinate  $x$  in text embedding space (e.g., the content reflected by

any moment in a lecture they had watched; see *Estimating dynamic knowledge traces*). Essentially, the estimated knowledge at coordinate  $x$  is given by the weighted proportion of quiz questions the participant answered correctly, where the weights reflect how much each question is “about” the content at  $x$ . When we apply this approach to estimate the participant’s knowledge about the content presented in each moment of each lecture, we can obtain a detailed time course describing how much “knowledge” that participant has about the content presented at any part of the lecture. As shown in Figure 5A and C, we can apply this approach separately for the questions from each quiz participants took throughout the experiment. From just a few questions per quiz (see *Estimating dynamic knowledge traces*), we obtain a high-resolution snapshot (at the time each quiz was taken) of what the participants knew about any moment’s content, from either of the two lectures they watched (comprising a total of 1,100 samples across the two lectures).

While the time courses in Figure 5A and C provide detailed *estimates* about participants’ knowledge, these estimates are of course only *useful* to the extent that they accurately reflect what participants actually know. As one sanity check, we anticipated that the knowledge estimates should reflect a content-specific “boost” in participants’ knowledge after watching each lecture. In other words, if participants learn about each lecture’s content upon watching it, the knowledge estimates should capture that. After watching the *Four Fundamental Forces* lecture, participants should exhibit more knowledge for the content of that lecture than they had before, and that knowledge should persist for the remainder of the experiment. Specifically, knowledge about that lecture’s content should be relatively low when estimated using Quiz 1 responses, but should increase when estimated using Quiz 2 or 3 responses (Fig. 5B). Indeed, we found that participants’ estimated knowledge about the content of *Four Fundamental Forces* was substantially higher on Quiz 2 versus Quiz 1 ( $t(49) = 8.764, p < 0.001$ ) and on Quiz 3 versus Quiz 1 ( $t(49) = 10.519, p < 0.001$ ). We found no reliable differences in estimated knowledge about that lecture’s content on Quiz 2 versus 3 ( $t(49) = 0.160, p = 0.874$ ). Similarly, we hypothesized (and subsequently confirmed) that participants should show greater estimated knowledge about the content of the *Birth of Stars* lecture after (versus before) watching it (Fig. 5D). Specifically, since participants watched that lecture after taking Quiz 2 (but before Quiz 3), we hypothesized that their knowledge estimates



**Figure 5: Estimating knowledge about the content presented at each moment of each lecture.** **A. Knowledge about the time-varying content of *Four Fundamental Forces*.** Each trace displays the weighted proportion of correctly answered questions about the content reflected in each moment of the lecture (see *Estimating dynamic knowledge traces*), using responses from a single quiz (color). The traces are averaged across participants. **B. Average estimated knowledge about *Four Fundamental Forces*.** Each bar displays the across-timepoint average knowledge, estimated using the responses to one quiz's questions. **C. Knowledge about the time-varying content of *Birth of Stars*.** The panel is in the same format as Panel A, but here the knowledge estimates are for the moment-by-moment content of the *Birth of Stars* lecture. **D. Average estimated knowledge about *Birth of Stars*.** The panel is in the same format as Panel B, but here the knowledge estimates are for the content of the *Birth of Stars* lecture. All panels: error ribbons and error bars denote 95% confidence intervals, estimated across participants.

271 should be relatively low on Quizzes 1 and 2, but should show a “boost” on Quiz 3. Consistent  
272 with this prediction, we found no reliable differences in estimated knowledge about the *Birth of*  
273 *Stars* lecture content on Quizzes 1 versus 2 ( $t(49) = 1.013, p = 0.316$ ), but the estimated knowl-  
274 edge was substantially higher on Quiz 3 versus 2 ( $t(49) = 10.561, p < 0.001$ ) and Quiz 3 versus 1  
275 ( $t(49) = 8.969, p < 0.001$ ).

276 If we are able to accurately estimate a participant’s knowledge about the content tested by a  
277 given question, our estimates of their knowledge should carry some predictive information about  
278 whether they are likely to answer that question correctly or incorrectly. We developed a statistical  
279 approach to test this claim. For each quiz question a participant answered, in turn, we used  
280 Equation 1 to estimate their knowledge at the given question’s embedding space coordinate based  
281 on other questions that participant answered on the same quiz. We repeated this for all participants,  
282 and for each of the three quizzes. Then, separately for each quiz, we fit a generalized linear mixed  
283 model (GLMM) with a logistic link function to explain the probability of correctly answering a  
284 question as a function of estimated knowledge for its embedding coordinate, while accounting  
285 for random variation among participants and questions (see *Generalized linear mixed models*). To  
286 assess the predictive value of the knowledge estimates, we compared each GLMM to an analogous  
287 (i.e., nested) “null” model that did not consider estimated knowledge using parametric bootstrap  
288 likelihood-ratio tests.

289 We carried out three different versions of the analyses described above, wherein we considered  
290 different sources of information in our estimates of participants’ knowledge for each quiz question.  
291 First, we estimated knowledge at each question’s embedding coordinate using *all* other questions  
292 answered by the same participant on the same quiz (“All questions”; Fig. 6, top row). This test was  
293 intended to assess the overall predictive power of our approach. Second, we estimated knowledge  
294 for each question about a given lecture using only the other questions (from the same participant  
295 and quiz) about that *same* lecture (“Within-lecture”; Fig. 6, middle rows). This test was intended to  
296 assess the *specificity* of our approach by asking whether our predictions could distinguish between  
297 questions about different content covered by the same lecture. Third, we estimated knowledge  
298 for each question about one lecture using only the questions (from the same participant and quiz)



**Figure 6: Predicting success on held-out questions using estimated knowledge.** We used generalized linear mixed models (GLMMs) to model the likelihood of correctly answering a quiz question as a function of estimated knowledge for its embedding coordinate (see *Generalized linear mixed models*). Separately for each quiz (column), we examined this relationship based on three different sets of knowledge estimates: knowledge for each question based on all other questions the same participant answered on the same quiz (“All questions”; top row), knowledge for each question about one lecture based on all other questions (from the same participant and quiz) about the *same* lecture (“Within-lecture”; middle rows), and knowledge for each question about one lecture based on all questions (from the same participant and quiz) about the *other* lecture (“Across-lecture”; bottom rows). The backgrounds in each panel display kernel density estimates of the relative observed proportions of correctly (blue) versus incorrectly (red) answered questions, for each level of estimated knowledge along the  $x$ -axis. The black curves display the (population-level) GLMM-predicted probabilities of correctly answering a question as a function of estimated knowledge. Error ribbons denote 95% confidence intervals.

299 about the *other* lecture (“Across-lecture”; Fig. 6, bottom rows). This test was intended to assess the  
300 *generalizability* of our approach by asking whether our predictions could extend across the content  
301 areas of the two lectures. When computing these knowledge estimates, we used a rebalancing  
302 procedure to ensure that (for a given participant and quiz) the knowledge estimates for correctly  
303 and incorrectly answered questions were based on the same proportion of correctly answered  
304 questions (see *Generalized linear mixed models*).

305 When we fit a GLMM to estimates of participants’ knowledge for each Quiz 1 question based on  
306 all other Quiz 1 questions, we found that higher estimated knowledge for a given question predicted  
307 a greater likelihood of answering it correctly (odds ratio ( $OR$ ) = 8.126, 95% CI = [3.116, 20.123],  
308 likelihood-ratio test statistic ( $\lambda_{LR}$ ) = 17.002,  $p < 0.001$ ). This relationship held when we repeated  
309 this analysis for Quiz 2 ( $OR$  = 14.902, 95% CI = [4.976, 39.807],  $\lambda_{LR}$  = 25.408,  $p < 0.001$ ) and again  
310 for Quiz 3 ( $OR$  = 37.409, 95% CI = [10.425, 107.145],  $\lambda_{LR}$  = 40.948,  $p < 0.001$ ). Taken together,  
311 these results suggest that our knowledge estimates can reliably predict participants’ performance  
312 on individual questions when aggregated across all quiz content.

313 We observed a similar set of results when we restricted our estimates of participants’ knowl-  
314 edge for questions about each lecture to consider only their performance on other questions  
315 about the *same* lecture. Specifically, for Quiz 1, participants’ knowledge of *Four Fundamental Forces*-  
316 related questions, estimated from their performance on other *Four Fundamental Forces*-related ques-  
317 tions, was predictive of their ability to answer those questions correctly ( $OR$  = 15.934, 95% CI =  
318 [5.173, 38.005],  $\lambda_{LR}$  = 40.971,  $p = 0.001$ ). The same was true of participants’ estimated knowledge  
319 for *Birth of Stars*-related questions based on their performance on other *Birth of Stars*-related ques-  
320 tions ( $OR$  = 9.775, 95% CI = [2.93, 25.08],  $\lambda_{LR}$  = 13.924,  $p = 0.001$ ). These within-lecture knowl-  
321 edge estimates also predicted success on questions about both lectures when we computed them  
322 analogously for Quiz 2 (*Four Fundamental Forces*:  $OR$  = 35.126, 95% CI = [5.113, 123.868],  $\lambda_{LR}$  =  
323 32.251,  $p < 0.001$ ; *Birth of Stars*:  $OR$  = 4.717, 95% CI = [2.021, 9.844],  $\lambda_{LR}$  = 16.788,  $p < 0.001$ ).  
324 For Quiz 3, we found that within-lecture knowledge estimates predicted participants’ success on  
325 *Birth of Stars*-related questions ( $OR$  = 16.902, 95% CI = [3.353, 53.265],  $\lambda_{LR}$  = 23.233,  $p < 0.001$ )  
326 but not on *Four Fundamental Forces*-related questions ( $OR$  = 2.485, 95% CI = [0.724, 8.366],  $\lambda_{LR}$  =

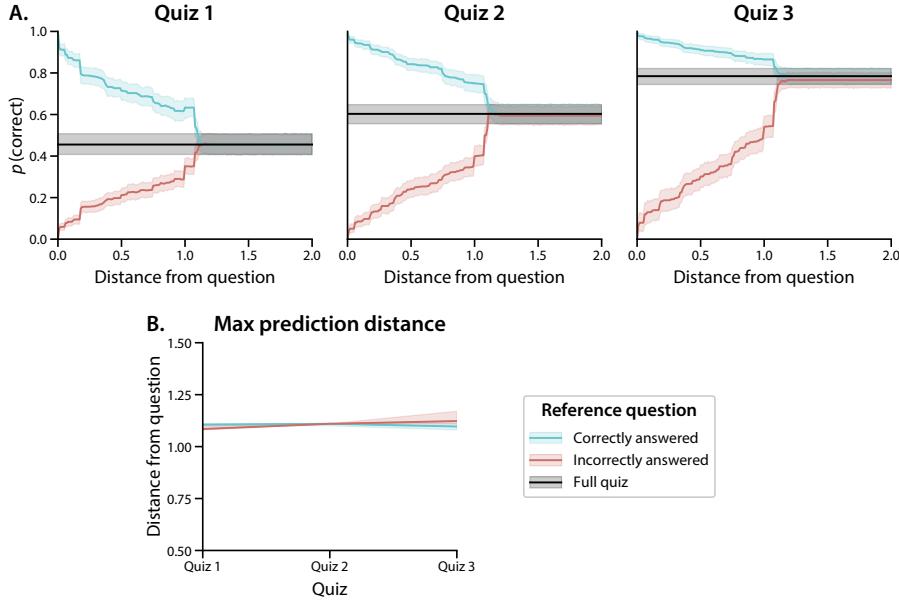
327 1.984,  $p = 0.170$ ). This may indicate that the within-lecture knowledge estimates are susceptible  
328 to ceiling effects in participants' quiz performance. On Quiz 3, after viewing both lectures, no  
329 participant answered more than three *Four Fundamental Forces*-related questions incorrectly, and  
330 all but five participants (out of 50) answered two or fewer questions incorrectly. (This was the  
331 only subset of questions about either lecture, across all three quizzes, for which this was true.)  
332 Because of this, when we held out one incorrectly answered *Four Fundamental Forces* question from  
333 a given participant's Quiz 3 responses and estimated their knowledge at its embedding coordinate  
334 using the remaining *Four Fundamental Forces* questions they answered, for 90% of participants,  
335 that estimate leveraged information about at most a single other question they were *not* able to  
336 correctly answer. This broad homogeneity in participants' success on questions used to estimate  
337 their knowledge may have hurt our ability to accurately characterize the specific (and by Quiz 3,  
338 relatively few) aspects of the lecture content they did *not* know about. Taken together, these results  
339 suggest that our knowledge estimates can reliably distinguish between questions about different  
340 content covered by a single lecture, provided there is sufficient diversity in participants' quiz  
341 responses to extract meaningful information about both what they know and what they do not  
342 know.

343 Finally, when we estimated participants' knowledge for each question about one lecture us-  
344 ing their performance on questions (from the same quiz) about the *other* lecture, we observed  
345 a somewhat different pattern of results. Here we found that before viewing either lecture (i.e.,  
346 on Quiz 1), participants' abilities to answer *Four Fundamental Forces*-related questions could not  
347 be predicted from their responses to *Birth of Stars*-related questions ( $OR = 1.896$ , 95% CI =  
348  $[0.419, 9.088]$ ,  $\lambda_{LR} = 0.712$ ,  $p = 0.404$ ), nor could their abilities to answer *Birth of Stars*-related  
349 questions be predicted from their responses to *Four Fundamental Forces*-related questions ( $OR =$   
350  $1.522$ , 95% CI =  $[0.332, 6.835]$ ,  $\lambda_{LR} = 0.286$ ,  $p = 0.611$ ). We similarly found that participants'  
351 success on questions about either lecture could not be predicted given their responses to questions  
352 about the other lecture after viewing *Four Fundamental Forces* but before viewing *Birth of Stars* (i.e.,  
353 on Quiz 2; *Four Fundamental Forces* questions given *Birth of Stars* questions:  $OR = 3.49$ , 95% CI =  
354  $[0.739, 12.849]$ ,  $\lambda_{LR} = 3.266$ ,  $p = 0.083$ ; *Birth of Stars* questions given *Four Fundamental Forces*

355 questions:  $OR = 2.199$ , 95% CI = [0.711, 5.623],  $\lambda_{LR} = 2.304$ ,  $p = 0.141$ ). Only after viewing *both*  
356 lectures (i.e., on Quiz 3) did these across-lecture knowledge estimates reliably predict participants'  
357 success on individual quiz questions (*Four Fundamental Forces* questions given *Birth of Stars* ques-  
358 tions:  $OR = 11.294$ , 95% CI = [1.375, 47.744],  $\lambda_{LR} = 10.396$ ,  $p < 0.001$ ; *Birth of Stars* questions given  
359 *Four Fundamental Forces* questions:  $OR = 7.302$ , 95% CI = [1.077, 44.879],  $\lambda_{LR} = 4.708$ ,  $p = 0.038$ ).  
360 Taken together, these results suggest that our knowledge estimates can be used to predict partic-  
361 ipants' success across content areas once they have received some training on both the content  
362 about which their knowledge is estimated, and the content used to construct these estimates.

363 That the knowledge predictions derived from the text embedding space reliably distinguish  
364 between held-out correctly versus incorrectly answered questions (Fig. 6) suggests that spatial  
365 relationships within this space can help explain what participants know. But how far does this  
366 explanatory power extend? For example, suppose we know that a participant correctly answered a  
367 question at embedding coordinate  $x$ . As we move farther away from  $x$  in the embedding space, how  
368 does the likelihood that the participant knows about the content at a given location "fall off" with  
369 distance? Conversely, suppose the participant instead answered that same question *incorrectly*.  
370 Again, as we move farther away from  $x$  in the embedding space, how does the likelihood that the  
371 participant does *not* know about a coordinate's content change with distance? We reasoned that,  
372 assuming our embedding space is capturing something about how individuals actually organize  
373 their knowledge, a participant's ability to answer questions embedded very close to  $x$  should  
374 tend to be similar to their ability to answer the question embedded *at*  $x$ . Whereas at another  
375 extreme, once we reach some sufficiently large distance from  $x$ , our ability to infer whether or  
376 not a participant will correctly answer a question based on their ability to answer the question  
377 at  $x$  should be no better than guessing based on their *overall* proportion of correctly answered  
378 questions. In other words, beyond the maximum distance at which the participant's ability to  
379 answer the question at  $x$  is informative of their ability to answer a second question at location  $y$ ,  
380 then guessing the outcome at  $y$  based on  $x$  should be no more successful than guessing based on a  
381 measure that does not consider embedding space distance.

382 With these ideas in mind, we asked: conditioned on answering a question correctly, what



**Figure 7: Knowledge falls off gradually in text embedding space. A. Performance versus distance.** For each participant, for each correctly answered question (blue) or incorrectly answered question (red), we computed the proportion of correctly answered questions within a given distance of that question’s embedding coordinate. We used these proportions as a proxy for participants’ knowledge about the content within that region of the embedding space. We repeated this analysis for all questions and participants, and separately for each quiz (column). The black lines denote the average proportion correct across *all* questions included in the analysis at the given distance. **B. Maximum distance for which performance is reliably different from the average.** We used a bootstrap procedure (see *Estimating the “smoothness” of knowledge*) to estimate the point at which the blue and red lines in Panel A reliably diverged from the black line. We repeated this analysis separately for correctly and incorrectly answered questions from each quiz. **All panels.** Error ribbons denote bootstrap-estimated 95% confidence intervals.

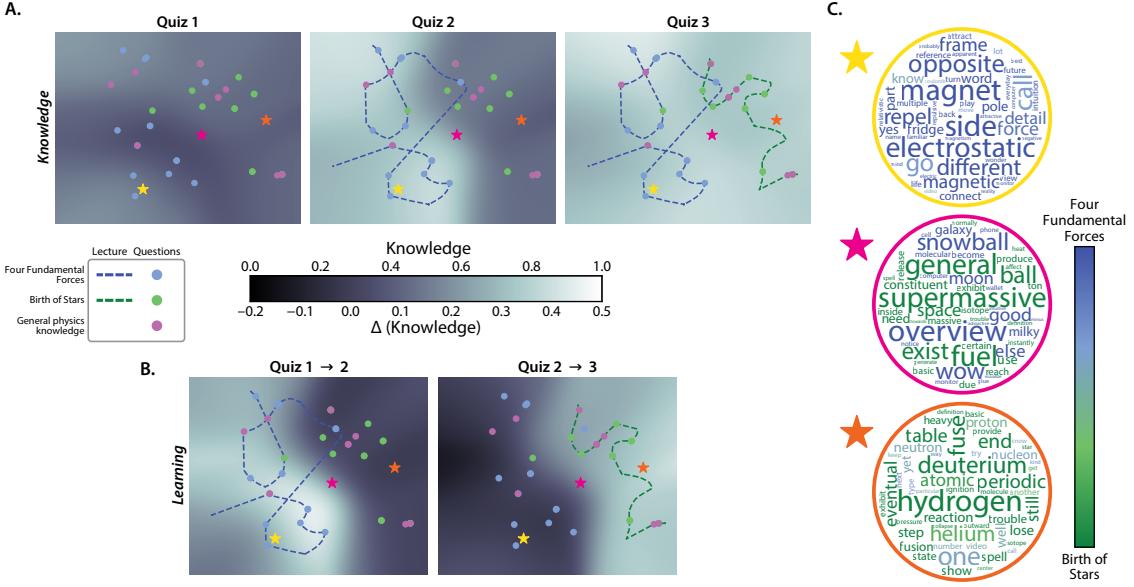
383 proportion of all questions (within some radius,  $r$ , of that question’s embedding coordinate)  
 384 were answered correctly? We plotted this proportion as a function of  $r$ . Similarly, we could  
 385 ask, conditioned on answering a question incorrectly, how the proportion of correct responses  
 386 changed with  $r$ . As shown in Figure 7, we found that quiz performance falls off smoothly with  
 387 distance, and the “rate” of the falloff does not appear to change across the different quizzes, as  
 388 measured by the distance at which performance becomes statistically indistinguishable from a  
 389 simple proportion correct score (see *Estimating the “smoothness” of knowledge*). This suggests that,  
 390 at least within the region of text embedding space covered by the questions our participants  
 391 answered (and as characterized using our topic model), the rate at which knowledge changes

392 with distance is relatively constant, even as participants' overall level of knowledge varies across  
393 quizzes or regions of the embedding space.

394 Knowledge estimates need not be limited to the content of the lectures. As illustrated in  
395 Figure 8, our general approach to estimating knowledge from a small number of quiz questions  
396 may be extended to *any* content, given its text embedding coordinate. To visualize how knowledge  
397 "spreads" through text embedding space to content beyond the lectures participants watched, we  
398 first fit a new topic model to the lectures' sliding windows with  $k = 100$  topics. Conceptually,  
399 increasing the number of topics used by the model functions to increase the "resolution" of the  
400 embedding space, providing a greater ability to estimate knowledge for content that is highly  
401 similar to (but not precisely the same as) that contained in the two lectures. We note that we  
402 used these 2D maps solely for visualization; all relevant comparisons, distance computations, and  
403 statistical tests we report above were carried out in the original 15-dimensional space, using the  
404 15-topic model. Aside from increasing the number of topics from 15 to 100, all other procedures  
405 and model parameters were carried over from the preceding analyses. As in our other analyses,  
406 we resampled each lecture's topic trajectory to 1 Hz and projected each question into a shared text  
407 embedding space.

408 We projected the resulting 100-dimensional topic vectors (for each second of video and each quiz  
409 question) onto a shared 2-dimensional plane (see *Creating knowledge and learning map visualizations*).  
410 Next, we sampled points from a  $100 \times 100$  grid of coordinates that evenly tiled a rectangle enclos-  
411 ing the 2D projections of the videos and questions. We used Equation 4 to estimate participants'  
412 knowledge at each of these 10,000 sampled locations, and averaged these estimates across par-  
413 ticipants to obtain an estimated average *knowledge map* (Fig. 8A). Intuitively, the knowledge map  
414 constructed from a given quiz's responses provides a visualization of "how much" participants  
415 knew about any content expressible by the fitted text embedding model at the point in time when  
416 they completed that quiz.

417 Several features of the resulting knowledge maps are worth noting. The average knowledge  
418 map estimated from Quiz 1 responses (Fig. 8A, leftmost map) shows that participants tended to  
419 have relatively little knowledge about any parts of the text embedding space (i.e., the shading is



**Figure 8: Mapping out the geometry of knowledge and learning.** **A.** Average “knowledge maps” estimated using each quiz. Each map displays a 2D projection of the estimated knowledge about the content reflected by *all* regions of topic space (see *Creating knowledge and learning map visualizations*). The topic trajectories of the two lectures are indicated by dotted lines (blue: Lecture 1; green: Lecture 2), and the coordinates of each question are indicated by dots (light blue: Lecture 1-related; light green: Lecture 2-related; purple: general physics knowledge). Each map reflects an average across all participants. For individual participants’ maps, see Supplementary Figures 7, 8, and 9. **B.** Average “learning maps” estimated between each successive pair of quizzes. The learning maps follow the same general format as the knowledge maps in Panel A, but here the shading at each coordinate indicates the *difference* between the corresponding coordinates in the indicated pair of knowledge maps—i.e., how much the estimated knowledge “changed” between the two quizzes. Each map reflects an average across all participants. For individual participants’ maps, see Supplementary Figures 10 and 11. **C.** Word clouds for sampled points in topic space. Each word cloud displays the weighted blend of words underlying the topic proportions represented at the corresponding colored star’s location on the maps. In each word cloud, the words’ relative sizes correspond to their relative weights at the starred location, and their colors indicate their relative weights in *Four Fundamental Forces* (blue) versus *Birth of Stars* (green) lectures, on average, across all timepoints’ topic vectors.

420 relatively dark everywhere). The knowledge map estimated from Quiz 2 responses shows a marked  
421 increase in knowledge on the left side of the map (around roughly the same range of coordinates  
422 traversed by the *Four Fundamental Forces* lecture, indicated by the dotted blue line). In other words,  
423 participants' estimated increase in knowledge is localized to conceptual content that is nearby (i.e.,  
424 related to) the content from the lecture they watched prior to taking Quiz 2. This localization is  
425 non-trivial: these knowledge estimates are informed only by the embedded coordinates of the  
426 *quiz questions*, not by the embeddings of either lecture (see Eqn. 4). Finally, the knowledge map  
427 estimated from Quiz 3 responses shows a second increase in knowledge, localized to the region  
428 surrounding the embedding of the *Birth of Stars* lecture participants watched immediately prior to  
429 taking Quiz 3.

430 Another way of visualizing these content-specific increases in knowledge after participants  
431 viewed each lecture is displayed in Figure 8B. Taking the point-by-point difference between the  
432 knowledge maps estimated from responses to a successive pair of quizzes yields a *learning map*  
433 that describes the *change* in knowledge estimates from one quiz to the next. These learning maps  
434 highlight that the estimated knowledge increases we observed across maps were specific to the  
435 regions around the embeddings of each lecture, in turn.

436 Because the 2D projection we used to construct the knowledge and learning maps is invertible,  
437 we may gain additional insights into these maps' meanings by reconstructing the original high-  
438 dimensional topic vector for any location on the map we are interested in. For example, this could  
439 serve as a useful tool for an instructor looking to better understand which content areas a student  
440 (or a group of students) knows well (or poorly). As a demonstration, we show the top-weighted  
441 words from the blends of topics reconstructed from three example locations on the maps (Fig. 8C):  
442 one point near the *Four Fundamental Forces* embedding (yellow), a second point near the *Birth of*  
443 *Stars* embedding (orange), and a third point between the two lectures' embeddings (pink). As  
444 shown in the word clouds in the panel, the top-weighted words at the example coordinate near the  
445 *Four Fundamental Forces* embedding tended to be weighted more heavily by the topics expressed  
446 in that lecture. Similarly, the top-weighted words at the example coordinate near the *Birth of Stars*  
447 embedding tended to be weighted more heavily by the topics expressed in *that* lecture. And the

448 top-weighted words at the example coordinate between the two lectures' embeddings show a  
449 roughly even mix of words most strongly associated with each lecture.

## 450 Discussion

451 We developed a computational framework that uses short multiple-choice quizzes to gain nuanced  
452 insights into what learners know and how their knowledge changes with training. First, we show  
453 that our approach can automatically match the conceptual knowledge probed by individual quiz  
454 questions to the corresponding moments in lecture videos when those concepts were presented  
455 (Fig. 4). Next, we demonstrate how we can estimate moment-by-moment "knowledge traces"  
456 that reflect the degree of knowledge participants have about each video's time-varying content,  
457 and capture temporally specific increases in knowledge after viewing each lecture (Fig. 5). We  
458 also show that these knowledge estimates can generalize to held-out questions (Fig. 6). Finally,  
459 we use our framework to construct visual maps that provide snapshot estimates of how much  
460 participants know about any concept within the scope of our text embedding model, and how  
461 much their knowledge of those concepts changes with training (Fig. 8).

462 We view our work as making several contributions to the study of how people acquire con-  
463 ceptual knowledge. First, from a methodological standpoint, our modeling framework provides  
464 a systematic means of mapping out and characterizing knowledge in maps that have infinite (ar-  
465 bitrarily many) numbers of coordinates, and of "filling out" those maps using relatively small  
466 numbers of multiple choice quiz questions. Our experimental finding that we can use these maps  
467 to predict responses to held-out questions has several psychological implications as well. For ex-  
468 ample, concepts that are assigned to nearby coordinates by the text embedding model also appear  
469 to be "known to a similar extent" (as reflected by participants' responses to held-out questions;  
470 Fig. 6). This suggests that participants also *conceptualize* similarly the content reflected by nearby  
471 embedding coordinates. How participants' knowledge falls off with spatial distance is captured  
472 by the knowledge maps we infer from their quiz responses (e.g., Figs. 7, 8). In other words, our  
473 study shows that knowledge about a given concept implies knowledge about related concepts,

474 and we also show how estimated knowledge falls off with distance in text embedding space.

475 In our study, we characterize the “coordinates” of participants’ knowledge using a relatively  
476 simple “bag of words” text embedding model [LDA; 8]. More sophisticated text embedding mod-  
477 els, such as transformer-based models [18, 54, 67, 70] can learn complex grammatical and semantic  
478 relationships between words, higher-order syntactic structures, stylistic features, and more. We  
479 considered using transformer-based models in our study, but we found that the text embeddings  
480 derived from these models were surprisingly uninformative with respect to differentiating or oth-  
481 erwise characterizing the conceptual content of the lectures and questions we used. We suspect  
482 that this reflects a broader challenge in constructing models that are high-resolution within a given  
483 domain (e.g., the domain of physics lectures and questions) *and* sufficiently broad so as to enable  
484 them to cover a wide range of domains. For example, we found that the embeddings derived even  
485 from much larger and more modern models like BERT [18], GPT [70], LLaMa [67], and others that  
486 are trained on enormous text corpora, end up yielding poor resolution within the content space  
487 spanned by individual course videos (Supp. Fig. 6). Whereas the LDA embeddings of the lectures  
488 and questions are “near” each other (i.e., the convex hull enclosing the two lectures’ trajectories is  
489 highly overlapping with the convex hull enclosing the questions’ embeddings), the BERT embed-  
490 dings of the lectures and questions are instead largely distinct (top row of Supp. Fig. 6). The LDA  
491 embeddings of the questions for each lecture and the corresponding lecture’s trajectory are also  
492 similar. For example, as shown in Fig. 2C, the LDA embeddings for *Four Fundamental Forces* ques-  
493 tions (blue dots) appear closer to the *Four Fundamental Forces* lecture trajectory (blue line), whereas  
494 the LDA embeddings for *Birth of Stars* questions (green dots) appear closer to the *Birth of Stars*  
495 lecture trajectory (green line). The BERT embeddings of the lectures and questions do not show  
496 this property (Supp. Fig. 6). We also examined per-question “content matches” between individual  
497 questions and individual moments of each lecture (Figs. 4, 6). The time series plot of individual  
498 questions’ correlations are different from each other when computed using LDA (e.g., the traces  
499 can be clearly visually separated), whereas the correlations computed from BERT embeddings of  
500 different questions all look very similar. This tells us that LDA is capturing some differences in  
content between the questions, whereas BERT is not. The time series plots of individual ques-

tions’ correlations have clear “peaks” when computed using LDA, but not when computed using BERT. This tells us that LDA is capturing a “match” between the content of each question and a relatively well-defined time window of the corresponding lectures. The BERT embeddings appear to blur together the content of the questions versus specific moments of each lecture. Finally, we also compared the pairwise correlations between embeddings of questions within versus across content areas (i.e., content covered by the individual lectures, lecture-specific questions, and by the “general physics knowledge” questions). The LDA embeddings show a strong contrast between same-content embeddings versus across-content embeddings. In other words, the embeddings of questions about the *Four Fundamental Forces* material are highly correlated with the embeddings of the *Four Fundamental Forces* lecture, but not with the embeddings of *Birth of Stars*, questions about *Birth of Stars*, or general physics knowledge questions. We see a similar pattern with the LDA embeddings of the *Birth of Stars* questions (Fig. 3, Supp. Fig. 2). In contrast, the BERT embeddings are all highly correlated with each other (Supp. Fig. 6). Taken together, these comparisons illustrate how LDA (trained on the specific content in question) provides both coverage of the requisite material and specificity at the level of the content covered by individual questions. BERT, on the other hand, essentially assigns both lectures and all of the questions (which are all broadly about “physics”) into a tiny region of its embedding space, thereby blurring out meaningful distinctions between different specific concepts covered by the lectures and questions. We note that these are not criticisms of BERT (or other large language models trained on large and diverse corpora). Rather, our point is that simple fine-tuned models trained on a relatively small but specialized corpus can outperform much more complicated models trained on much larger corpora, when we are specifically interested in capturing subtle conceptual differences at the level of a single course lecture or question. Of course if our goal had been to find a model that generalized to many different content areas, we would expect our approach to perform comparatively poorly relative to BERT or other much larger models. We suggest that bridging the tradeoff between high resolution within each content area versus the ability to generalize to many different content areas will be an important challenge for future work in this domain.

Another application for large language models that does *not* require explicitly modeling the

530 content of individual lectures or questions is to leverage the models' abilities to generate text. For  
531 example, generative text models like ChatGPT [54] and LLaMa [67] are already being used to build  
532 a new generation of interactive tutoring systems [e.g., 44]. Unlike the approach we have taken here,  
533 these generative text model-based systems do not explicitly model what learners know, or how  
534 their knowledge changes over time with training. One could imagine building a hybrid system  
535 that combines the best of both worlds: a large language model that can *generate* text, combined  
536 with a smaller model that can *infer* what learners know and how their knowledge changes over  
537 time. Such a hybrid system could potentially be used to build the next generation of interactive  
538 tutoring systems that are able to adapt to learners' needs in real time, and that are able to provide  
539 more nuanced feedback about what learners know and what they do not know.

540 At the opposite end of the spectrum from large language models, one could also imagine  
541 *simplifying* some aspects of our LDA-based approach by computing simple word overlap metrics.  
542 For example, the Jaccard similarity between text  $A$  and  $B$  is computed as the number of unique  
543 words in the intersection of words from  $A$  and  $B$  divided by the number of unique words in the  
544 union of words from  $A$  and  $B$ . In a supplementary analysis (Supp. Fig. 5), we compared the  
545 LDA-based question-lecture matches we reported in Figure 4 with the Jaccard similarities between  
546 each question and each sliding window of text from the corresponding lecture. As shown in  
547 Supplementary Figure 5, this simple word-matching approach does not appear to capture the same  
548 level of specificity as the LDA-based approach. Whereas the LDA-based approach often yields a  
549 clear peak in the time series of correlations between each question and the corresponding lecture,  
550 the Jaccard similarity-based approach does not. Furthermore, these LDA-based matches appear  
551 to capture conceptual overlaps between the questions and lectures (Supp. Tab. 3), whereas simple  
552 word matching does not. For example, one of the example questions examined in Supplementary  
553 Figure 5 asks "Which of the following occurs as a cloud of atoms gets more dense?" The LDA-based  
554 matches identify lecture timepoints where the relevant *topics* are discussed (e.g., when words like  
555 "cloud," "atom," "dense," etc., are mentioned *together*). The Jaccard similarity-based matches,  
556 on the other hand, are strong when *any* of these words are mentioned, even if they do not occur  
557 together.

558 We view our approach as occupying a sort of “sweet spot,” between much larger language  
559 models and simple word matching-based approaches, that enables us to capture the relevant  
560 conceptual content of course materials at an appropriate semantic scale. Our approach enables us  
561 to accurately and consistently identify each question’s content in a way that also matches up with  
562 what is presented in the lectures. In turn, this enables us to construct accurate predictions about  
563 participants’ knowledge of the conceptual content tested by held-out questions (Fig. 6).

564 One limitation of our approach is that topic models contain no explicit internal representations  
565 of more complex aspects of “knowledge,” like knowledge graphs, dependencies or associations  
566 between concepts, causality, and so on. These representations might (in principle) be added  
567 as extensions to our approach to more accurately and precisely capture, characterize, and track  
568 learners’ knowledge. However, modeling these aspects of knowledge will likely require substantial  
569 additional research effort.

570 Within the past several years, the global pandemic forced many educators to suddenly adapt to  
571 teaching remotely [35, 51, 63, 71]. This change in world circumstances is happening alongside (and  
572 perhaps accelerating) geometric growth in the availability of high-quality online courses from plat-  
573 forms such as Khan Academy [36], Coursera [72], EdX [38], and others [59]. Continued expansion  
574 of the global internet backbone and improvements in computing hardware have also facilitated  
575 improvements in video streaming, enabling videos to be easily shared and viewed by increasingly  
576 large segments of the world’s population. This exciting time for online course instruction provides  
577 an opportunity to re-evaluate how we, as a global community, educate ourselves and each other.  
578 For example, we can ask: what defines an effective course or training program? Which aspects of  
579 teaching might be optimized and/or augmented by automated tools? How and why do learning  
580 needs and goals vary across people? How might we lower barriers to receiving a high-quality  
581 education?

582 Alongside these questions, there is a growing desire to extend existing theories beyond the  
583 domain of lab testing rooms and into real classrooms [34]. In part, this has led to a recent  
584 resurgence of “naturalistic” or “observational” experimental paradigms that attempt to better  
585 reflect more ethologically valid phenomena that are more directly relevant to real-world situations

586 and behaviors [52]. In turn, this has brought new challenges in data analysis and interpretation. A  
587 key step towards solving these challenges will be to build explicit models of real-world scenarios  
588 and how people behave in them (e.g., models of how people learn conceptual content from real-  
589 world courses, as in our current study). A second key step will be to understand which sorts  
590 of signals derived from behaviors and/or other measurements [e.g., neurophysiological data; 4,  
591 19, 49, 53, 56] might help to inform these models. A third major step will be to develop and  
592 employ reliable ways of evaluating the complex models and data that are a hallmark of naturalistic  
593 paradigms.

594 Beyond specifically predicting what people *know*, the fundamental ideas we develop here also  
595 relate to the notion of “theory of mind” of other individuals [26, 32, 48]. Considering others’ unique  
596 perspectives, prior experiences, knowledge, goals, etc., can help us to more effectively interact and  
597 communicate [57, 62, 66]. One could imagine future extensions of our work (e.g., analogous to  
598 the knowledge and learning maps shown in Fig. 8), that attempt to characterize how well-aligned  
599 different people’s knowledge bases or backgrounds are. In turn, this might be used to model how  
600 knowledge (or other forms of communicable information) flows not just between teachers and  
601 students, but between friends having a conversation, individuals on a first date, participants at  
602 a business meeting, doctors and patients, experts and non-experts, political allies or adversaries,  
603 and more. For example, the extent to which two people’s knowledge maps “match” or “align” in  
604 a given region of text embedding space might serve as a predictor of how effectively they will be  
605 able to communicate about the corresponding conceptual content.

606 Ultimately, our work suggests a rich new line of questions about the geometric “form” of  
607 knowledge, how knowledge changes over time, and how we might map out the full space of  
608 what an individual knows. Our finding that detailed estimates about knowledge may be obtained  
609 from short quizzes shows one way that traditional approaches to evaluation in education may be  
610 extended. We hope that these advances might help pave the way for new approaches to teaching  
611 or delivering educational content that are tailored to individual students’ learning needs and goals.

612 **Materials and methods**

613 **Participants**

614 We enrolled a total of 50 Dartmouth undergraduate students in our study. Participants received  
615 optional course credit for enrolling. We asked each participant to complete a demographic survey  
616 that included questions about their age, gender, native spoken language, ethnicity, race, hearing,  
617 color vision, sleep, coffee consumption, level of alertness, and several aspects of their educational  
618 background and prior coursework.

619 Participants' ages ranged from 18 to 22 years (mean: 19.52 years; standard deviation: 1.09  
620 years). A total of 15 participants reported their gender as male and 35 participants reported their  
621 gender as female. A total of 49 participants reported their native language as "English" and 1  
622 reported having another native language. A total of 47 participants reported their ethnicity as  
623 "Not Hispanic or Latino" and three reported their ethnicity as "Hispanic or Latino." Participants  
624 reported their races as White (32 participants), Asian (14 participants), Black or African American  
625 (5 participants), American Indian or Alaska Native (1 participant), and Native Hawaiian or Other  
626 Pacific Islander (1 participant). (Note that some participants selected multiple racial categories.)

627 A total of 49 participants reporting having normal hearing and 1 participant reported having  
628 some hearing impairment. A total of 49 participants reported having normal color vision and 1  
629 participant reported being color blind. Participants reported having had, on the night prior to  
630 testing, 2–4 hours of sleep (1 participant), 4–6 hours of sleep (9 participants), 6–8 hours of sleep (35  
631 participants), or 8+ hours of sleep (5 participants). They reported having consumed, on the same  
632 day and leading up to their testing session, 0 cups of coffee (38 participants), 1 cup of coffee (10  
633 participants), 3 cups of coffee (1 participant), or 4+ cups of coffee (1 participant).

634 No participants reported that their focus was currently impaired (e.g., by drugs or alcohol).  
635 Participants reported their current level of alertness, and we converted their responses to numerical  
636 scores as follows: "very sluggish" (-2), "a little sluggish" (-1), "neutral" (0), "fairly alert" (1), and  
637 "very alert" (2). Across all participants, a range of alertness levels were reported (range: -2–1;  
638 mean: -0.10; standard deviation: 0.84).

Participants reported their undergraduate major(s) as “social sciences” (28 participants), “natural sciences” (16 participants), “professional” (e.g., pre-med or pre-law; 8 participants), “mathematics and engineering” (7 participants), “humanities” (4 participants), or “undecided” (3 participants). Note that some participants selected multiple categories for their undergraduate major(s). We also asked participants about the courses they had taken. In total, 45 participants reported having taken at least one Khan Academy course in the past, and 5 reported not having taken any Khan Academy courses. Of those who reported having watched at least one Khan Academy course, 7 participants reported having watched 1–2 courses, 11 reported having watched 3–5 courses, 8 reported having watched 5–10 courses, and 19 reported having watched 10 or more courses. We also asked participants about the specific courses they had watched, categorized under different subject areas. In the “Mathematics” area, participants reported having watched videos on AP Calculus AB (21 participants), Precalculus (17 participants), Algebra 2 (14 participants), AP Calculus BC (12 participants), Trigonometry (11 participants), Algebra 1 (10 participants), Geometry (8 participants), Pre-algebra (7 participants), Multivariable Calculus (5 participants), Differential Equations (5 participants), Statistics and Probability (4 participants), AP Statistics (2 participants), Linear Algebra (2 participants), Early Math (1 participant), Arithmetic (1 participant), and other videos not listed in our survey (5 participants). In the “Science and engineering” area, participants reported having watched videos on Chemistry, AP Chemistry, or Organic Chemistry (21 participants); Physics, AP Physics I, or AP Physics II (18 participants); Biology, AP Biology; or High school Biology (15 participants); Health and Medicine (1 participant); or other videos not listed in our survey (5 participants). We also asked participants whether they had specifically seen the videos used in our experiment. Of the 45 participants who reported having having taken at least one Khan Academy course in the past, 44 participants reported that they had not watched the *Four Fundamental Forces* video, and 1 participant reported that they were not sure whether they had watched it. All participants reported that they had not watched the *Birth of Stars* video. When we asked participants about non-Khan Academy online courses, they reported having watched or taken courses on Mathematics (15 participants), Science and engineering (11 participants), Test preparation (9 participants), Economics and finance (3 participants), Arts and humanities (2 participants).

667 ipants), Computing (2 participants), and other categories not listed in our survey (17 participants).  
668 Finally, we asked participants about in-person courses they had taken in different subject areas.  
669 They reported taking courses in Mathematics (38 participants), Science and engineering (37 par-  
670 ticipants), Arts and humanities (34 participants), Test preparation (27 participants), Economics  
671 and finance (26 participants), Computing (14 participants), College and careers (7 participants), or  
672 other courses not listed in our survey (6 participants).

## 673 Experiment

674 We hand-selected two course videos from the Khan Academy platform: *Four Fundamental Forces*  
675 (an introduction to gravity, electromagnetism, the weak nuclear force, and the strong nuclear force;  
676 duration: 10 minutes and 29 seconds) and *Birth of Stars* (an introduction to how stars are formed;  
677 duration: 7 minutes and 57 seconds). All participants viewed the videos in the same order (i.e.,  
678 *Four Fundamental Forces* followed by *Birth of Stars*).

679 We then hand-created 39 multiple-choice questions: 15 about the conceptual content of *Four*  
680 *Fundamental Forces* (i.e., Lecture 1), 15 about the conceptual content of *Birth of Stars* (i.e., Lecture 2),  
681 and 9 questions that tested for general conceptual knowledge about basic physics (covering material  
682 that was not presented in either video). To help broaden the set of lecture-specific questions,  
683 our team worked through each lecture in small segments to identify what each segment was  
684 “about” conceptually, and then write a question about that concept. The general physics questions  
685 were drawn our team’s prior coursework and areas of interest, along with internet searches and  
686 brainstorming with the project team and other members of J.R.M.’s lab. Although we attempted to  
687 design the questions to test “conceptual knowledge,” we note that estimating the specific “amount”  
688 of conceptual understanding that each question “requires” to answer is somewhat subjective, and  
689 might even come down to the “strategy” a given participant uses to answer the question at that  
690 particular moment. The full set of questions and answer choices may be found in Supplementary  
691 Table 1. The final set of questions (and response options) was reviewed and approved by J.R.M.  
692 before we collected or analyzed the text or experimental data.

693 Over the course of the experiment, participants completed three 13-question multiple-choice

694 quizzes: the first before viewing Lecture 1, the second between Lectures 1 and 2, and the third  
695 after viewing Lecture 2 (see Fig. 1). The questions appearing on each quiz, for each participant,  
696 were randomly chosen from the full set of 39, with the constraints that (a) each quiz contained  
697 exactly 5 questions about Lecture 1, 5 questions about Lecture 2, and 3 questions about general  
698 physics knowledge, and (b) each question appear exactly once for each participant. The orders of  
699 questions on each quiz, and the orders of answer options for each question, were also randomized.  
700 We obtained informed consent from all participants, and our experimental protocol was approved  
701 by the Committee for the Protection of Human Subjects at Dartmouth College. We used this  
702 experiment to develop and test our computational framework for estimating knowledge and  
703 learning.

## 704 **Analysis**

### 705 **Statistics**

706 All of the statistical tests performed in our study were two-sided. The 95% confidence intervals  
707 we reported for each correlation were estimated by generating 10,000 bootstrap distributions of  
708 correlation coefficients by sampling (with replacement) from the observed data.

### 709 **Constructing text embeddings of multiple lectures and questions**

710 We adapted an approach we developed in prior work [29] to embed each moment of the two  
711 lectures and each question in our pool in a common representational space. Briefly, our approach  
712 uses a topic model [Latent Dirichlet Allocation; 8] trained on a set of documents, to discover a set  
713 of  $k$  “topics” or “themes.” Formally, each topic is defined as a distribution of weights over words  
714 in the model’s vocabulary (i.e., the union of all unique words, across all documents, excluding  
715 “stop words.”). Conceptually, each topic is intended to give larger weights to words that are  
716 semantically related (as inferred from their tendency to co-occur in the same document). After  
717 fitting a topic model, each document in the training set, or any *new* document that contains at  
718 least some of the words in the model’s vocabulary, may be represented as a  $k$ -dimensional vector

719 describing how much the document (most probably) reflects each topic. To select an appropriate  $k$   
720 for our model, as a starting point, we identified the minimum number of topics that yielded at least  
721 one “unused” topic (i.e., in which all words in the vocabulary were assigned uniform weights)  
722 after training. This indicated that the number of topics was sufficient to capture the set of latent  
723 themes present in the two lectures (from which we constructed our document corpus, as described  
724 below). We found this value to be  $k = 15$  topics. We found that with a limited number of additional  
725 adjustments following Boyd-Graber et al. [9], such as removing corpus-specific stop-words, the  
726 model yielded (subjectively) sensible and coherent topics. The distribution of weights over words  
727 in the vocabulary for each discovered topic is shown in Supplementary Figure 1, and each topic’s  
728 top-weighted words may be found in Supplementary Table 2.

729 As illustrated in Figure 2A, we start by building up a corpus of documents using overlapping  
730 sliding windows that span each video’s transcript. Khan Academy provides professionally created,  
731 manual transcriptions of all videos for closed captioning. However, such transcripts would not  
732 be readily available in all contexts to which our framework could potentially be applied. Khan  
733 Academy videos are hosted on the YouTube platform, which additionally provides automated  
734 captions. We opted to use these automated transcripts [which, in prior work, we have found to be  
735 of sufficiently near-human quality to yield reliable data in behavioral studies; 73] when developing  
736 our framework in order to make it more directly extensible and adaptable by others in the future.

737 We fetched these automated transcripts using the `youtube-transcript-api` Python pack-  
738 age [17]. The transcripts consisted of one timestamped line of text for every few seconds (mean:  
739 2.34 s; standard deviation: 0.83 s) of spoken content in the video (i.e., corresponding to each indi-  
740 vidual caption that would appear on-screen if viewing the lecture via YouTube, and when those  
741 lines would appear). We defined a sliding window length of (up to)  $w = 30$  transcript lines, and  
742 assigned each window a timestamp corresponding to the midpoint between the timestamps for its  
743 first and last lines. This  $w$  parameter was chosen to match the same number of words per sliding  
744 window (rounded to the nearest whole word, and before preprocessing) as the sliding windows  
745 we defined in our prior work [29; i.e., 185 words per sliding window].

746 These sliding windows ramped up and down in length at the beginning and end of each

747 transcript, respectively. In other words, each transcript’s first sliding window covered only its first  
748 line, the second sliding window covered the first two lines, and so on. This ensured that each line  
749 from the transcripts appeared in the same number ( $w$ ) of sliding windows. We next performed a  
750 series of standard text preprocessing steps: normalizing case, lemmatizing, removing punctuation  
751 and removing stop-words. We constructed our corpus of stop words by augmenting the Natural  
752 Language Toolkit [NLTK; 5] English stop word list with the following additional words, selected  
753 using one of the approaches suggested by Boyd-Graber et al. [9]: “actual,” “actually,” “also,” “bit,”  
754 “could,” “e,” “even,” “first,” “follow,” “following,” “four,” “let,” “like,” “mc,” “really,”, “saw,”  
755 “see,” “seen,” “thing,” and “two.” This yielded sliding windows with an average of 73.8 remaining  
756 words, and lasting for an average of 62.22 seconds. We treated the text from each sliding window  
757 as a single “document,” and combined these documents across the two videos’ windows to create  
758 a single training corpus for the topic model.

759 After fitting a topic model to the two videos’ transcripts, we could use the trained model to  
760 transform arbitrary (potentially new) documents into  $k$ -dimensional topic vectors. A convenient  
761 property of these topic vectors is that documents that reflect similar blends of topics (i.e., documents  
762 that reflect similar themes, according to the model) will yield similar coordinates (in terms of  
763 correlation, cosine similarity, Kullback-Leibler divergence, Euclidean distance, or other geometric  
764 measures). In general, the similarity between different documents’ topic vectors may be used to  
765 characterize the similarity in conceptual content between the documents.

766 We transformed each sliding window’s text into a topic vector, and then used linear interpolation  
767 (independently for each topic dimension) to resample the resulting time series to one vector  
768 per second. We also used the fitted model to obtain topic vectors for each question in our pool (see  
769 Supp. Tab. 1). Taken together, we obtained a *trajectory* for each video, describing its path through  
770 topic space, and a single coordinate for each question (Fig. 2C). Embedding both videos and all of  
771 the questions using a common model enables us to compare the content from different moments  
772 of videos, compare the content across videos, and estimate potential associations between specific  
773 questions and specific moments of video.

774 **Estimating dynamic knowledge traces**

775 We used the following equation to estimate each participant’s knowledge about timepoint  $t$  of a  
776 given lecture,  $\hat{k}(t)$ :

$$\hat{k}(f(t, L)) = \frac{\sum_{i \in \text{correct}} \text{ncorr}(f(t, L), f(i, Q))}{\sum_{j=1}^N \text{ncorr}(f(t, L), f(j, Q))}, \quad (1)$$

777 where

$$\text{ncorr}(x, y) = \frac{\text{corr}(x, y) - \text{mincorr}}{\text{maxcorr} - \text{mincorr}}, \quad (2)$$

778 and where mincorr and maxcorr are the minimum and maximum correlations between any lecture  
779 timepoint and question, taken over all timepoints in the given lecture, and all five questions *about*  
780 that lecture appearing on the given quiz. We also define  $f(s, \Omega)$  as the  $s^{\text{th}}$  topic vector from the set  
781 of topic vectors  $\Omega$ . Here  $t$  indexes the set of lecture topic vectors,  $L$ , and  $i$  and  $j$  index the topic  
782 vectors of questions used to estimate the knowledge trace,  $Q$ . Note that “correct” denotes the set  
783 of indices of the questions the participant answered correctly on the given quiz.

784 Intuitively,  $\text{ncorr}(x, y)$  is the correlation between two topic vectors (e.g., the topic vector from one  
785 timepoint in a lecture,  $x$ , and the topic vector for one question,  $y$ ), normalized by the minimum and  
786 maximum correlations (across all timepoints  $t$  and questions  $Q$ ) to range between 0 and 1, inclusive.  
787 Equation 1 then computes the weighted average proportion of correctly answered questions about  
788 the content presented at timepoint  $t$ , where the weights are given by the normalized correlations  
789 between timepoint  $t$ ’s topic vector and the topic vectors for each question. The normalization step  
790 (i.e., using ncorr instead of the raw correlations) ensures that every question contributes some  
791 non-negative amount to the knowledge estimate.

792 **Generalized linear mixed models**

793 In the set of analyses reported in Figure 6, we assessed whether estimates of participants’ knowledge  
794 at the embedding coordinates of individual quiz questions could be used to reliably predict their  
795 ability to correctly answer those questions. In essence, we treated each question a given participant

796 answered on a given quiz as a “lecture” consisting of a single timepoint, and used Equation 1 to  
797 estimate the participant’s knowledge for its embedding coordinate based on their performance on  
798 all *other* questions they answered on that same quiz (“All questions”; Fig. 6, top row). Additionally,  
799 for each lecture-related question (i.e., excluding questions about general physics knowledge), we  
800 computed analogous knowledge estimates based on all other questions the participant answered  
801 on the same quiz about (1) the same lecture as the target question (“Within-lecture”; Fig. 6, middle  
802 rows), and (2) the other of the two lectures (“Across-lecture”; Fig. 6, bottom rows).

803 In each version of this analysis (i.e., row in Fig. 6), and separately for each of the three quizzes  
804 (i.e., column in Fig. 6), we then fit a generalized linear mixed model (GLMM) with a logistic link  
805 function to the set of knowledge estimates for all questions that participants answered on the  
806 given quiz. We implemented these models in R using the `lme4` package [3] and fit them following  
807 guidance from Bates et al. [2] and Matuschek et al. [45]. Specifically, we initially fit each model  
808 with the maximal random effects structure afforded by our design, which we identified as:

$$\text{accuracy} \sim \text{knowledge} + (\text{knowledge} | \text{participant}) + (\text{knowledge} | \text{question})$$

809 where “accuracy” is a binary value indicating whether each target question was answered cor-  
810 rectly or incorrectly, “knowledge” is estimated knowledge at each target question’s embedding  
811 coordinate, “participant” is a unique identifier assigned to each participant, and “question” is a  
812 unique identifier assigned to each quiz question. For models we fit using knowledge estimates for  
813 target questions about multiple content areas (i.e., in the “All questions” version of the analysis),  
814 we also included an additional random effect term,  $(\text{knowledge} | \text{lecture})$ , where “lecture” is a  
815 categorical value denoting whether the target question was about *Four Fundamental Forces*, *Birth*  
816 *of Stars*, or general physics knowledge. Note that with our coding scheme, identifiers for each  
817 question are implicitly nested within levels of lecture and do not require explicit nesting in  
818 our model formula. We then iteratively removed random effects from the maximal model until  
819 it successfully converged with a full rank (i.e., non-singular) random effects variance-covariance  
820 matrix.

821 To assess the predictive value of our knowledge estimates, we compared each GLMM's ability  
822 to discriminate between correctly and incorrectly answered questions to that of an analogous model  
823 that did *not* consider estimated knowledge. Specifically, we used the same sets of observations  
824 with which we fit each "full" model to fit a second "null" model, with the formula:

$$\text{accuracy} \sim (1 | \text{participant}) + (1 | \text{question})$$

825 where "accuracy", "participant", and "question" are as defined above. As with our full models,  
826 the null models we fit for the "All questions" version of the analysis for each quiz contained an  
827 additional term,  $(1 | \text{lecture})$ , where "lecture" is as defined above. We then compared each  
828 full model to its reduced (null) equivalent using a likelihood-ratio test (LRT). Because the typical  
829 asymptotic  $\chi_d^2$  approximation of the null distribution for the LRT statistic ( $\lambda_{LR}$ ) is anti-conservative  
830 for models that differ in their random slope terms [25, 60, 65], we computed  $p$ -values for these  
831 tests using a parametric bootstrap procedure [14, 27]. For each of 1,000 bootstraps, we used the  
832 fitted null model to simulate a sample of observations of equal size to our original sample. We  
833 then re-fit both the null and full models to this simulated sample and compared them via an LRT.  
834 This yielded a distribution of  $\lambda_{LR}$  statistics we may expect to observe under our null hypothesis.  
835 We computed a corrected  $p$ -value for our observed  $\lambda_{LR}$  as  $\frac{r+1}{n+1}$ , where  $r$  is the number of simulated  
836 model comparisons that yielded a  $\lambda_{LR}$  greater than our observed value and  $n$  is the number of  
837 simulations we ran (1,000).

838 **Estimating the "smoothness" of knowledge**

839 In the analysis reported in Figure 7A, we show how participants' ability to correctly answer  
840 quiz questions changes as a function of distance from a given correctly or incorrectly answered  
841 reference question. We used a bootstrap-based approach to estimate the maximum distances over  
842 which these proportions of correctly answered questions could be reliably distinguished from  
843 participants' overall average proportion of correctly answered questions.

844 For each of 10,000 iterations, we drew a random subsample (with replacement) of 50 partic-

845 ipants from our dataset. Within each iteration, we first computed the 95% confidence interval  
846 (CI) of the across-subsample-participants mean proportion correct on each of the three quizzes,  
847 separately. To compute this interval for each quiz, we repeatedly (1,000 times) subsampled par-  
848 ticipants (with replacement, from the outer subsample for the current iteration) and computed  
849 the mean proportion correct of each of these inner subsamples. We then identified the 2.5<sup>th</sup> and  
850 97.5<sup>th</sup> percentiles of the resulting distributions of 1,000 means. These three intervals (one for each  
851 quiz) served as our thresholds for confidence that the proportion correct within a given distance  
852 from a reference question was reliably different (at the  $p < 0.05$  significance level) from the average  
853 proportion correct across all questions on the given quiz.

854 Next, for each participant in the current subsample, and for each of the three quizzes they  
855 completed (separately), we iteratively treated each of the 15 questions appearing on the given  
856 quiz as the “reference” question. We constructed a series of concentric 15-dimensional “spheres”  
857 centered on the reference question’s embedding space coordinate, where each successive sphere’s  
858 radius increased by 0.01 (correlation distance) between 0 and 2, inclusive (i.e., tiling the range  
859 of possible correlation distances with 201 spheres in total). We then computed the proportion  
860 of questions enclosed within each sphere that the participant answered correctly, and averaged  
861 these per-radius proportion correct scores across reference questions that were answered correctly,  
862 and those that were answered incorrectly. This resulted in two number-of-spheres sequences of  
863 proportion-correct scores for each subsample participant and quiz: one derived from correctly  
864 answered reference questions, and one derived from incorrectly answered reference questions.

865 We computed the across-subsample-participants mean proportion correct for each radius value  
866 (i.e., sphere) and “correctness” of reference question. This yielded two sequences of proportion-  
867 correct scores for each quiz, analogous to the blue and red lines displayed in Figure 7A, but for  
868 the present subsample. For each quiz, we then found the minimum distance from the reference  
869 question (i.e., sphere radius) at which each of these two sequences of per-radius proportion correct  
870 scores intersected the 95% confidence interval for the overall proportion correct (i.e., analogous to  
871 the black error bands in Fig. 7A).

872 This resulted in two “intersection” distances for each quiz (for correctly answered and incor-

873 rectly answered reference questions). Repeating this full process for each of the 10,000 bootstrap  
874 iterations output two distributions of intersection distances for each of the three quizzes. The  
875 means and 95% confidence intervals for these distributions are plotted in Figure 7B.

876 **Creating knowledge and learning map visualizations**

877 An important feature of our approach is that, given a trained text embedding model and partic-  
878 ipants' quiz performance on each question, we can estimate their knowledge about *any* content  
879 expressible by the embedding model—not solely the content explicitly probed by the quiz ques-  
880 tions, or even appearing in the lectures. To visualize these estimates (Fig. 8, Supp. Figs. 7, 8, 9, 10,  
881 and 11), we used Uniform Manifold Approximation and Projection [UMAP; 46, 47] to construct a  
882 2D projection of the text embedding space. Whereas our main analyses used a 15-topic embedding  
883 space, we used a 100-topic embedding space for these visualizations. This change in the number  
884 of topics overcame an undesirable behavior in the UMAP embedding procedure, whereby embed-  
885 ding coordinates for the 15-topic model tended to be “clumped” into separated clusters, rather  
886 than forming a smooth trajectory through the 2D space. When we increased the number of topics  
887 to 100, the embedding coordinates in the 2D space formed a smooth trajectory through the space,  
888 with substantially less clumping (Fig. 8). Creating a “map” by sampling this 100-dimensional  
889 space at high resolution to obtain an adequate set of topic vectors spanning the embedding space  
890 would be computationally intractable. However, sampling a 2D grid is trivial.

891 At a high level, the UMAP algorithm obtains low-dimensional embeddings by minimizing  
892 the cross-entropy between the pairwise (clustered) distances between the observations in their  
893 original (e.g., 100-dimensional) space and the pairwise (clustered) distances in the low-dimensional  
894 embedding space (in our approach, the embedding space is 2D). In our implementation, pairwise  
895 distances in the original high-dimensional space were defined as 1 minus the correlation between  
896 each pair of coordinates, and pairwise distances in the low-dimensional embedding space were  
897 defined as the Euclidean distance between each pair of coordinates.

898 In our application, all of the coordinates we embedded were topic vectors, whose elements  
899 are always non-negative and sum to one. Although UMAP is an invertible transformation at

900 the embedding locations of the original data, other locations in the embedding space will not  
901 necessarily follow the same implicit “rules” as the original high-dimensional data. For example,  
902 inverting an arbitrary coordinate in the embedding space might result in negative-valued vectors,  
903 which are incompatible with the topic modeling framework. To protect against this issue, we  
904 log-transformed the topic vectors prior to embedding them in the 2D space. When we inverted  
905 the embedded vectors (e.g., to estimate topic vectors or word clouds, as in Fig. 8C), we passed  
906 the inverted (log-transformed) values through the exponential function to obtain a vector of non-  
907 negative values, and normalized them to sum to one.

908 After embedding both lectures’ topic trajectories and the topic vectors of every question, we  
909 defined a rectangle enclosing the 2D projections of the lectures’ and quizzes’ embeddings. We then  
910 sampled points from a regular  $100 \times 100$  grid of coordinates that evenly tiled this enclosing rectangle.  
911 We sought to estimate participants’ knowledge (and learning, i.e., changes in knowledge) at each  
912 of the resulting 10,000 coordinates.

913 To generate our estimates, we placed a set of 39 radial basis functions (RBFs) throughout the  
914 embedding space, centered on the 2D projections for each question (i.e., we included one RBF for  
915 each question). At coordinate  $x$ , the value of an RBF centered on a question’s coordinate  $\mu$ , is given  
916 by:

$$\text{RBF}(x, \mu, \lambda) = \exp \left\{ -\frac{\|x - \mu\|^2}{\lambda} \right\}. \quad (3)$$

917 The  $\lambda$  term in the RBF equation controls the “smoothness” of the function, where larger values  
918 of  $\lambda$  result in smoother maps. In our implementation we used  $\lambda = 50$ . Next, we estimated the  
919 “knowledge” at each coordinate,  $x$ , using:

$$\hat{k}(x) = \frac{\sum_{i \in \text{correct}} \text{RBF}(x, q_i, \lambda)}{\sum_{j=1}^N \text{RBF}(x, q_j, \lambda)}. \quad (4)$$

920 Intuitively, Equation 4 computes the weighted proportion of correctly answered questions, where  
921 the weights are given by how nearby (in the 2D space) each question is to the  $x$ . We also defined  
922 *learning maps* as the coordinate-by-coordinate differences between any pair of knowledge maps.  
923 Intuitively, learning maps reflect the *change* in knowledge across two maps.

924 **Author contributions**

925 Conceptualization: P.C.F., A.C.H., and J.R.M. Methodology: P.C.F., A.C.H., and J.R.M. Software:  
926 P.C.F. Validation: P.C.F. Formal analysis: P.C.F. Resources: P.C.F., A.C.H., and J.R.M. Data curation:  
927 P.C.F. Writing (original draft): J.R.M. Writing (review and editing): P.C.F., A.C.H., and J.R.M. Visu-  
928 alization: P.C.F. and J.R.M. Supervision: J.R.M. Project administration: P.C.F. Funding acquisition:  
929 J.R.M.

930 **Data availability**

931 All of the data analyzed in this manuscript may be found at <https://github.com/ContextLab/efficient-learning-khan>.  
932

933 **Code availability**

934 All of the code for running our experiment and carrying out the analyses may be found at  
935 <https://github.com/ContextLab/efficient-learning-khan>.

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## 944 References

- 945 [1] Ashby, F. G. and Maddox, W. T. (2005). Human category learning. *Annual Review of Psychology*,  
946 56:149–178.
- 947 [2] Bates, D., Kliegl, R., Vasishth, S., and Baayen, H. (2015a). Parsimonious mixed models. *arXiv*,  
948 1506.04967.
- 949 [3] Bates, D., Mächler, M., Bolker, B., and Walker, S. (2015b). Fitting linear mixed-effects models  
950 using lme4. *Journal of Statistical Software*, 67(1):1–48.
- 951 [4] Bevilacqua, D., Davidesco, I., Wan, L., and Chaloner, K. (2019). Brain-to-brain synchrony and  
952 learning outcomes vary by student-teacher dynamics: evidence from a real-world classroom  
953 electroencephalography study. *Journal of Cognitive Neuroscience*, 31(3):401–411.
- 954 [5] Bird, S., Klein, E., and Loper, E. (2009). *Nature language processing with Python: analyzing text  
955 with the natural language toolkit*. Reilly Media, Inc.
- 956 [6] Blaye, A., Bernard-Peyron, V., Paour, J.-L., and Bonthoux, F. (2006). Category flexibility in chil-  
957 dren: distinguishing response flexibility from conceptual flexibility; the protracted development  
958 of taxonomic representations. *European Journal of Developmental Psychology*, 3(2):163–188.
- 959 [7] Blei, D. M. and Lafferty, J. D. (2006). Dynamic topic models. In *Proceedings of the International  
960 Conference on Machine Learning*, pages 113–120, New York, NY. Association for Computing  
961 Machinery.
- 962 [8] Blei, D. M., Ng, A. Y., and Jordan, M. I. (2003). Latent dirichlet allocation. *Journal of Machine  
963 Learning Research*, 3:993–1022.
- 964 [9] Boyd-Graber, J., Mimno, D., and Newman, D. (2014). Care and feeding of topic models:  
965 problems, diagnostics, and improvements. In Airolidi, E. M., Blei, D. M., Erosheva, E. A., and  
966 Fienberg, S. E., editors, *Handbook of Mixed Membership Models and Their Applications*. CRC Press.

- 967 [10] Brown, T. B., Mann, B., Ryder, N., Subbiah, M., Kaplan, J., Dhariwal, P., Neelakantan, A.,  
968 Shyam, P., Sastry, G., Askell, A., Agarwal, S., Herbert-Voss, A., Krueger, G., Henighan, T., Child,  
969 R., Ramesh, A., Ziegler, D. M., Wu, J., Winter, C., Hesse, C., Chen, M., Sigler, E., Litwin, M.,  
970 Gray, S., Chess, B., Clark, J., Berner, C., McCandlish, S., Radford, A., Sutskever, I., and Amodei,  
971 D. (2020). Language models are few-shot learners. *arXiv*, 2005.14165.
- 972 [11] Caramazza, A. and Mahon, B. Z. (2003). The organization of conceptual knowledge: the  
973 evidence from category-specific semantic deficits. *Trends in Cognitive Sciences*, 7(8):354–361.
- 974 [12] Cer, D., Yang, Y., Kong, S. Y., Hua, N., Limtiaco, N., John, R. S., Constant, N., Guajardo-  
975 Cespedes, M., Yuan, S., Tar, C., Sung, Y.-H., Strope, B., and Kurzweil, R. (2018). Universal  
976 sentence encoder. *arXiv*, 1803.11175.
- 977 [13] Constantinescu, A. O., O'Reilly, J. X., and Behrens, T. E. J. (2016). Organizing conceptual  
978 knowledge in humans with a gridlike code. *Science*, 352(6292):1464–1468.
- 979 [14] Davison, A. C. and Hinkley, D. V. (1997). *Bootstrap Methods and their Application*. Cambridge  
980 Series in Statistical and Probabilistic Mathematics. Cambridge University Press.
- 981 [15] Deacon, D., Grose-Fifer, J., Yang, C. M., Stanick, V., Hewitt, S., and Dynowska, A. (2004).  
982 Evidence for a new conceptualization of semantic representation in the left and right cerebral  
983 hemispheres. *Cortex*, 40(3):467–478.
- 984 [16] Deerwester, S., Dumais, S. T., Furnas, G. W., Landauer, T. K., and Harshman, R. (1990).  
985 Indexing by latent semantic analysis. *Journal of the American Society for Information Science*,  
986 41(6):391–407.
- 987 [17] Depoix, J. (2018). YouTube transcript API. <https://github.com/jdepoix/youtube-transcript-api>.
- 988 [18] Devlin, J., Chang, M.-W., Lee, K., and Toutanova, K. (2018). BERT: pre-training of deep  
989 bidirectional transformers for language understanding. *arXiv*, 1810.04805.

- 991 [19] Dikker, S., Wan, L., Davidesco, I., Kaggen, L., Oostrik, M., McClintock, J., Rowland, J.,  
992 Michalareas, G., van Bavel, J. J., Ding, M., and Poeppel, D. (2017). Brain-to-brain synchrony  
993 tracks real-world dynamic group interactions in the classroom. *Current Biology*, 27(9):1375–1380.
- 994 [20] Estes, W. K. (1986a). Array models for category learning. *Cognitive Psychology*, 18(4):500–549.
- 995 [21] Estes, W. K. (1986b). Memory storage and retrieval processes in category learning. *Journal of*  
996 *Experimental Psychology: General*, 115:155–174.
- 997 [22] Fisher, R. A. (1922). On the mathematical foundations of theoretical statistics. *Philosophical*  
998 *Transactions of the Royal Society A*, 222(602):309–368.
- 999 [23] Gallagher, J. J. (2000). Teaching for understanding and application of science knowledge.  
1000 *School Science and Mathematics*, 100(6):310–318.
- 1001 [24] Gluck, M. A., Shohamy, D., and Myers, C. E. (2002). How do people solve the “weather  
1002 prediction” task? individual variability in strategies for probabilistic category learning. *Learning*  
1003 *and Memory*, 9:408–418.
- 1004 [25] Goldman, N. and Whelan, S. (2000). Statistical Tests of Gamma-Distributed Rate Heterogeneity  
1005 in Models of Sequence Evolution in Phylogenetics. *Molecular Biology and Evolution*, 17(6):975–978.
- 1006 [26] Goldstein, T. R. and Winner, E. (2012). Enhancing empathy and theory of mind. *Journal of*  
1007 *Cognition and Development*, 13(1):19–37.
- 1008 [27] Halekoh, U. and Højsgaard, S. (2014). A Kenward-Roger Approximation and Parametric  
1009 Bootstrap Methods for Tests in Linear Mixed Models – The R Package pbkrtest. *Journal of*  
1010 *Statistical Software*, 59(9):1–32.
- 1011 [28] Hall, R. and Greeno, J. (2008). *21st century education: A reference handbook*, chapter Conceptual  
1012 learning, pages 212–221. Sage Publications.
- 1013 [29] Heusser, A. C., Fitzpatrick, P. C., and Manning, J. R. (2021). Geometric models reveal behav-  
1014 ioral and neural signatures of transforming experiences into memories. *Nature Human Behaviour*,  
1015 5:905–919.

- 1016 [30] Huebner, P. A. and Willits, J. A. (2018). Structured semantic knowledge can emerge au-  
1017 tomatically from predicting word sequences in child-directed speech. *Frontiers in Psychology*,  
1018 9:doi.org/10.3389/fpsyg.2018.00133.
- 1019 [31] Hulbert, J. C. and Norman, K. A. (2015). Neural differentiation tracks improved recall of com-  
1020 peting memories following interleaved study and retrieval practice. *Cerebral Cortex*, 25(10):3994–  
1021 4008.
- 1022 [32] Kanske, P., Böckler, A., and Singer, T. (2015). Models, mechanisms and moderators dissociating  
1023 empathy and theory of mind. In *Social Behavior From Rodents to Humans*, pages 193–206. Springer.
- 1024 [33] Katona, G. (1940). *Organizing and memorizing: studies in the psychology of learning and teaching*.  
1025 Columbia University Press.
- 1026 [34] Kaufman, D. M. (2003). Applying educational theory in practice. *British Medical Journal*,  
1027 326(7382):213–216.
- 1028 [35] Kawasaki, H., Yamasaki, S., Masuoka, Y., Iwasa, M., Fukita, S., and Matsuyama, R. (2021).  
1029 Remote teaching due to COVID-19: an exploration of its effectiveness and issues. *International  
1030 Journal of Environmental Research and Public Health*, 18(5):2672.
- 1031 [36] Khan, S. (2004). *The Khan Academy*. Salman Khan.
- 1032 [37] Kintsch (1970). *Learning, memory, and conceptual processes*. Wiley.
- 1033 [38] Kolowich, S. (2013). How EdX plans to earn, and share, revenue from its free online courses.  
1034 *The Chronicle of Higher Education*, 21:1–5.
- 1035 [39] Landauer, T. K. and Dumais, S. T. (1997). A solution to Plato’s problem: the latent semantic  
1036 analysis theory of acquisition, induction, and representation of knowledge. *Psychological Review*,  
1037 104:211–240.
- 1038 [40] Lee, H. and Chen, J. (2022). Predicting memory from the network structure of naturalistic  
1039 events. *Nature Communications*, 13(4235):doi.org/10.1038/s41467-022-31965-2.

- 1040 [41] Maclellan, E. (2005). Conceptual learning: the priority for higher education. *British Journal of*  
1041 *Educational Studies*, 53(2):129–147.
- 1042 [42] Manning, J. R. (2021). Episodic memory: mental time travel or a quantum “memory wave”  
1043 function? *Psychological Review*, 128(4):711–725.
- 1044 [43] Manning, J. R. (2023). Context reinstatement. In Kahana, M. J. and Wagner, A. D., editors,  
1045 *Handbook of Human Memory*. Oxford University Press.
- 1046 [44] Manning, J. R., Menjunatha, H., and Kording, K. (2023). Chatify: A Jupyter extension  
1047 for adding LLM-driven chatbots to interactive notebooks. [https://github.com/ContextLab/  
1048 chatify](https://github.com/ContextLab/chatify).
- 1049 [45] Matuschek, H., Kliegl, R., Vasishth, S., Baayen, H., and Bates, D. (2017). Balancing type i error  
1050 and power in linear mixed models. *Journal of Memory and Language*, 94:305–315.
- 1051 [46] McInnes, L., Healy, J., and Melville, J. (2018a). UMAP: Uniform manifold approximation and  
1052 projection for dimension reduction. *arXiv*, 1802(03426).
- 1053 [47] McInnes, L., Healy, J., Saul, N., and Großberger, L. (2018b). UMAP: Uniform Manifold  
1054 Approximation and Projection. *Journal of Open Source Software*, 3(29):861.
- 1055 [48] Meltzoff, A. N. (2011). Social cognition and the origins of imitation, empathy, and theory of  
1056 mind. In *The Wiley-Blackwell Handbook of Childhood Cognitive Development*. Wiley-Blackwell.
- 1057 [49] Meshulam, M., Hasenfratz, L., Hillman, H., Liu, Y. F., Nguyen, M., Norman, K. A., and Hasson,  
1058 U. (2020). Neural alignment predicts learning outcomes in students taking an introduction to  
1059 computer science course. *Nature Communications*, 12(1922):doi.org/10.1038/s41467-021-22202-3.
- 1060 [50] Mikolov, T., Chen, K., Corrado, G., and Dean, J. (2013). Efficient estimation of word represen-  
1061 tations in vector space. *arXiv*, 1301.3781.
- 1062 [51] Moser, K. M., Wei, T., and Brenner, D. (2021). Remote teaching during COVID-19: implications  
1063 from a national survey of language educators. *System*, 97:102431.

- 1064 [52] Nastase, S. A., Goldstein, A., and Hasson, U. (2020). Keep it real: rethinking the primacy of  
1065 experimental control in cognitive neuroscience. *NeuroImage*, 15(222):117254–117261.
- 1066 [53] Nguyen, M., Chang, A., Micciche, E., Meshulam, M., Nastase, S. A., and Hasson, U. (2022).  
1067 Teacher-student neural coupling during teaching and learning. *Social Cognitive and Affective*  
1068 *Neuroscience*, 17(4):367–376.
- 1069 [54] OpenAI (2023). ChatGPT. <https://chat.openai.com>.
- 1070 [55] Piantadosi, S. T. and Hill, F. (2022). Meaning without reference in large language models.  
1071 *arXiv*, 2208.02957.
- 1072 [56] Poulsen, A. T., Kamronn, S., Dmochowski, J., Parra, L. C., and Hansen, L. K. (2017). EEG  
1073 in the classroom: synchronised neural recordings during video presentation. *Scientific Reports*,  
1074 7:43916.
- 1075 [57] Ratka, A. (2018). Empathy and the development of affective skills. *American Journal of*  
1076 *Pharmaceutical Education*, 82(10):doi.org/10.5688/ajpe7192.
- 1077 [58] Reilly, D. L., Cooper, L. N., and Elbaum, C. (1982). A neural model for category learning.  
1078 *Biological Cybernetics*, 45(1):35–41.
- 1079 [59] Rhoads, R. A., Berdan, J., and Toven-Lindsey, B. (2013). The open courseware movement in  
1080 higher education: unmasking power and raising questions about the movement’s democratic  
1081 potential. *Educational Theory*, 63(1):87–110.
- 1082 [60] Scheipl, F., Greven, S., and Küchenhoff, H. (2008). Size and power of tests for a zero random  
1083 effect variance or polynomial regression in additive and linear mixed models. *Computational*  
1084 *Statistics & Data Analysis*, 52(7):3283–3299.
- 1085 [61] Scott, P., Asoko, H., and Leach, J. (2007). *Handbook of research on science education*, chapter  
1086 Student conceptions and conceptual learning in science. Routledge.

- 1087 [62] Shao, Y. N., Sun, H. M., Huang, J. W., Li, M. L., Huang, R. R., and Li, N. (2018). Simulation-  
1088 based empathy training improves the communication skills of neonatal nurses. *Clinical Simula-*  
1089 *tion in Nursing*, 22:32–42.
- 1090 [63] Shim, T. E. and Lee, S. Y. (2020). College students' experience of emergency remote teaching  
1091 during COVID-19. *Children and Youth Services Review*, 119:105578.
- 1092 [64] Simon, M. A., Tzur, R., Heinz, K., and Kinzel, M. (2004). Explicating a mechanism for  
1093 conceptual learning: elaborating the construct of reflective abstraction. *Journal for Research in*  
1094 *Mathematics Education*, 35(5):305–329.
- 1095 [65] Snijders, T. A. B. and Bosker, R. (2011). More powerful tests for variance parameters. In  
1096 *Multilevel Analysis: An Introduction to Basic and Advanced Multilevel Modeling*, chapter 6, pages  
1097 94–108. Sage Publications, 2nd edition.
- 1098 [66] Stepien, K. A. and Baernstein, A. (2006). Education for empathy. *Journal of General Internal*  
1099 *Medicine*, 21:524–530.
- 1100 [67] Touvron, H., Lavril, T., Izacard, G., Martinet, X., Lachaux, M.-A., Lacroix, T., Rozière, B.,  
1101 Goyal, N., Hambro, E., Azhar, F., Rodriguez, A., Joulin, A., Grave, E., and Lample, G. (2023).  
1102 LLaMA: open and efficient foundation language models. *arXiv*, 2302.13971.
- 1103 [68] Tulchinskii, E., Kuznetsov, K., Kushnareva, L., Cherniavskii, D., Barannikov, S., Pio-  
1104 ntkovskaya, I., Nikolenko, S., and Burnaev, E. (2023). Intrinsic dimension estimation for robust  
1105 detection of AI-generated texts. *arXiv*, 2306.04723.
- 1106 [69] van Paridon, J., Liu, Q., and Lupyan, G. (2021). How do blind people know that blue is cold?  
1107 distributional semantics encode color-adjective associations. *Proceedings of the Annual Meeting of*  
1108 *the Cognitive Science Society*, 43(43).
- 1109 [70] Viswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A. N., Kaiser, L., and  
1110 Polosukhin, I. (2017). Attention is all you need. In *Advances in Neural Information Processing*  
1111 *Systems*.

1112 [71] Whalen, J. (2020). Should teachers be trained in emergency remote teaching? Lessons learned  
1113 from the COVID-19 pandemic. *Journal of Technology and Teacher Education*, 28(2):189–199.

1114 [72] Young, J. R. (2012). Inside the Coursera contract: how an upstart company might profit from  
1115 free courses. *The Chronicle of Higher Education*, 19(7):1–4.

1116 [73] Ziman, K., Heusser, A. C., Fitzpatrick, P. C., Field, C. E., and Manning, J. R. (2018). Is  
1117 automatic speech-to-text transcription ready for use in psychological experiments? *Behavior*  
1118 *Research Methods*, 50:2597–2605.