

¹ Text embedding models yield high-resolution insights
² into conceptual knowledge from short multiple-choice
³ quizzes

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⁵ **Abstract**

We develop a mathematical framework, based on natural language processing models, for tracking and characterizing the acquisition of conceptual knowledge. Our approach embeds each concept in a high-dimensional representation space where nearby coordinates reflect similar or related concepts. We test our approach using behavioral data from participants who answered small sets of multiple-choice quiz questions interleaved between watching two Khan Academy course videos. We apply our framework to the videos' transcripts and the text of the quiz questions to quantify the content of each moment of video and each quiz question. We use these embeddings, along with participants' quiz responses, to track how the learners' knowledge changed after watching each video and predict their success on individual quiz questions. Our findings show how a small set of quiz questions may be used to obtain rich and meaningful high-resolution insights into what each learner knows, and how their knowledge changes over time as they learn.

¹⁸ **Keywords:** education, learning, knowledge, concepts, natural language processing

¹⁹ **Introduction**

²⁰ Suppose that a teacher had access to a complete, tangible “map” of everything a student knows.
²¹ Defining what such a map might even look like, let alone how it might be constructed or filled in, is
²² itself a non-trivial problem. But if a teacher *were* to gain access to such a map, how might it change
²³ their ability to teach that student? Perhaps they might start by checking how well the student
²⁴ knows the to-be-learned information already, or how much they know about related concepts.
²⁵ For some students, they could potentially optimize their teaching efforts to maximize efficiency
²⁶ by focusing primarily on not-yet-known content. For other students (or other content areas), it
²⁷ might be more effective to optimize for direct connections between already known content and
²⁸ new material. Observing how the student’s knowledge changed over time, in response to their
²⁹ teaching, could also help to guide the teacher towards the most effective strategy for that individual
³⁰ student.

³¹ A common approach to assessing a student’s knowledge is to present them with a set of quiz
³² questions, calculate the proportion they answer correctly, and provide them with feedback in the
³³ form of a simple numeric or letter grade. While such a grade can provide *some* indication of whether
³⁴ the student has mastered the to-be-learned material, any univariate measure of performance on a
³⁵ complex task sacrifices certain relevant information, risks conflating underlying factors, and so on.
³⁶ For example, consider the relative utility of the theoretical map described above that characterizes
³⁷ a student’s knowledge in detail, versus a single annotation saying that the student answered 85%
³⁸ of their quiz questions correctly, or that they received a ‘B’. Here we show that the same quiz data
³⁹ required to compute proportion-correct scores or letter grades can instead be used to obtain far
⁴⁰ more detailed insights into what a student knew at the time they took the quiz.

⁴¹ Designing and building procedures and tools for mapping out knowledge touches on deep
⁴² questions about what it means to learn. For example, how do we acquire conceptual knowledge?
⁴³ Memorizing course lectures or textbook chapters by rote can lead to the superficial *appearance*
⁴⁴ of understanding the underlying content, but achieving true conceptual understanding seems to
⁴⁵ require something deeper and richer. Does conceptual understanding entail connecting newly

46 acquired information to the scaffolding of one’s existing knowledge or experience [6, 11, 13, 15, 30,
47 65]? Or weaving a lecture’s atomic elements (e.g., its component words) into a structured network
48 that describes how those individual elements are related [40, 70]? Conceptual understanding
49 could also involve building a mental model that transcends the meanings of those individual
50 atomic elements by reflecting the deeper meaning underlying the gestalt whole [37, 41, 62, 69].

51 The difference between “understanding” and “memorizing,” as framed by researchers in ed-
52 ucation, cognitive psychology, and cognitive neuroscience [e.g., 23, 28, 33, 41, 62], has profound
53 analogs in the fields of natural language processing and natural language understanding. For
54 example, considering the raw contents of a document (e.g., its constituent symbols, letters, and
55 words) might provide some clues as to what the document is about, just as memorizing a passage
56 might provide some ability to answer simple questions about it. However, text embedding mod-
57 els [e.g., 7, 8, 10, 12, 16, 39, 51, 71] also attempt to capture the deeper meaning *underlying* those
58 atomic elements. These models consider not only the co-occurrences of those elements within and
59 across documents, but (in many cases) also patterns in how those elements appear across different
60 scales (e.g., sentences, paragraphs, chapters, etc.), their temporal and grammatical properties, and
61 other high-level characteristics of how they are used [42, 43]. To be clear, this is not to say that text
62 embedding models themselves are capable of “understanding” deep conceptual meaning in any
63 traditional sense. But rather, their ability to capture the underlying *structure* of text documents
64 beyond their surface-level contents provides a computational framework through which those
65 documents’ deeper conceptual meanings may be quantified, explored, and understood. Accord-
66 ing to these models, the deep conceptual meaning of a document may be captured by a feature
67 vector in a high-dimensional representation space, wherein nearby vectors reflect conceptually
68 related documents. A model that succeeds at capturing an analogue of “understanding” is able
69 to assign nearby feature vectors to two conceptually related documents *even when the specific words*
70 *contained in those documents have limited overlap*. In this way, “concepts” are defined implicitly by
71 the model’s geometry [e.g., how the embedding coordinate of a given word or document relates
72 to the coordinates of other text embeddings; 56].

73 Given these insights, what form might a representation of the sum total of a person’s knowledge

74 take? First, we might require a means of systematically describing or representing (at least some
75 subset of) the nearly infinite set of possible things a person could know. Second, we might want to
76 account for potential associations between different concepts. For example, the concepts of “fish”
77 and “water” might be associated in the sense that fish live in water. Third, knowledge may have
78 a critical dependency structure, such that knowing about a particular concept might require first
79 knowing about a set of other concepts. For example, understanding the concept of a fish swimming
80 in water first requires understanding what fish and water *are*. Fourth, as we learn, our “current
81 state of knowledge” should change accordingly. Learning new concepts should both update our
82 characterizations of “what is known” and also unlock any now-satisfied dependencies of those
83 newly learned concepts so that they are “tagged” as available for future learning.

84 Here we develop a framework for modeling how conceptual knowledge is acquired during
85 learning. The central idea behind our framework is to use text embedding models to define the
86 coordinate systems of two maps: a *knowledge map* that describes the extent to which each concept is
87 currently known, and a *learning map* that describes changes in knowledge over time. Each location
88 on these maps represents a single concept, and the maps’ geometries are defined such that related
89 concepts are located nearby in space. We use this framework to analyze and interpret behavioral
90 data collected from an experiment that had participants answer sets of multiple-choice questions
91 about a series of recorded course lectures.

92 Our primary research goal is to advance our understanding of what it means to acquire deep,
93 real-world conceptual knowledge. Traditional laboratory approaches to studying learning and
94 memory (e.g., list-learning studies) often draw little distinction between memorization and under-
95 standing. Instead, these studies typically focus on whether information is effectively encoded or
96 retrieved, rather than whether the information is *understood*. Approaches to studying conceptual
97 learning, such as category learning experiments, can begin to investigate the distinction between
98 memorization and understanding, often by training participants to distinguish arbitrary or random
99 features in otherwise meaningless categorized stimuli [1, 20, 21, 24, 31, 59]. However, the objective
100 of real-world training, or learning from life experiences more generally, is often to develop new
101 knowledge that may be applied in *useful* ways in the future. In this sense, the gap between modern

learning theories and modern pedagogical approaches that inform classroom learning strategies is enormous: most of our theories about *how* people learn are inspired by experimental paradigms and models that have only peripheral relevance to the kinds of learning that students and teachers actually seek [28, 41]. To help bridge this gap, our study uses course materials from real online courses to inform, fit, and test models of real-world conceptual learning. We show that these models recover meaningful relationships between concepts presented during course lectures and tested by assessments, and that these relationships can be leveraged to predict students' success on individual quiz questions. We also provide a demonstration of how our models can be used to construct "maps" of what students know, and how their knowledge changes with training. In addition to helping to visually capture knowledge (and changes in knowledge), we hope that such maps might lead to real-world tools for improving how we educate. Taken together, our work shows that existing course materials and evaluative tools like short multiple-choice quizzes may be leveraged to gain highly detailed insights into what students know and how they learn.

Results

At its core, our main modeling approach is based around a simple assumption that we sought to test empirically: all else being equal, knowledge about a given concept is predictive of knowledge about similar or related concepts. From a geometric perspective, this assumption implies that knowledge is fundamentally "smooth." In other words, as one moves through a space representing an individual's knowledge (where similar concepts occupy nearby coordinates), their "level of knowledge" should change relatively gradually. To begin to test this smoothness assumption, we sought to track participants' knowledge and how it changed over time in response to training. Two overarching goals guide our approach. First, we want to gain detailed insights into what learners know at different points in their training. For example, rather than simply reporting on the proportions of questions participants answer correctly (i.e., their overall performance), we seek estimates of their knowledge about a variety of specific concepts. Second, we want our approach to be potentially scalable to large numbers of diverse concepts, courses, and students. This requires

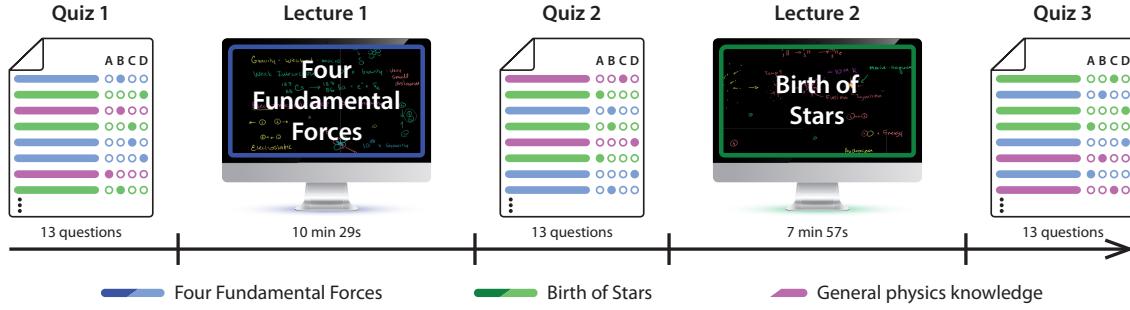


Figure 1: Experimental paradigm. Participants alternate between completing three 13-question multiple-choice quizzes and watching two Khan Academy lectures. Each quiz contains a mix of 5 questions about Lecture 1, 5 questions about Lecture 2, and 3 questions about general physics knowledge. The specific questions appearing on each quiz, and the orders of each quiz’s questions, were randomized across participants.

128 that the conceptual content of interest be discovered *automatically*, rather than relying on manually
 129 produced ratings or labels.

130 We asked participants in our study to complete brief multiple-choice quizzes before, between,
 131 and after watching two lecture videos from the Khan Academy [36] platform (Fig. 1). The first
 132 lecture video, entitled *Four Fundamental Forces*, discussed the four fundamental forces in physics:
 133 gravity, strong and weak interactions, and electromagnetism. The second, entitled *Birth of Stars*,
 134 provided an overview of our current understanding of how stars form. We selected these particular
 135 lectures to satisfy three general criteria. First, we wanted both lectures to be accessible to a broad
 136 audience (i.e., with minimal prerequisite knowledge) so as to limit the impact of prior training
 137 on participants’ abilities to learn from the lectures. To this end, we selected two introductory
 138 videos that were intended to be viewed at the start of students’ training in their respective content
 139 areas. Second, we wanted the two lectures to have some related content so that we could test
 140 our approach’s ability to distinguish similar conceptual content. To this end, we chose two videos
 141 from the same Khan Academy course domain, “Cosmology and Astronomy.” Third, we sought to
 142 minimize dependencies and specific overlap between the videos. For example, we did not want
 143 participants’ abilities to understand one video to (directly) influence their abilities to understand the
 144 other. To satisfy this last criterion, we chose videos from two different lecture series (Lectures 1 and
 145 2 were from the “Scale of the Universe” and “Stars, Black Holes, and Galaxies” series, respectively).



Figure 2: Modeling course content. **A. Building a document pool from sliding windows of text.** We decompose each lecture’s transcript into a series of overlapping sliding windows. The full set of transcript snippets (across all windows) may be treated as a set of “documents” for training a text embedding model. **B. Constructing lecture content trajectories.** After training the model on the sliding windows from both lectures, we transform each lecture into a “trajectory” through text embedding space by joining the embedding coordinates of successive sliding windows parsed from its transcript. **C. Embedding multiple lectures and questions in a shared space.** We apply the same model (trained on the two lectures’ windows) to both lectures, along with the text of each question in our pool (Supp. Tab. 1), to project them into a shared text embedding space. This results in one trajectory per lecture and one coordinate for each question. Here, we have projected the 15-dimensional embeddings onto their first 3 principal components for visualization.

146 We also wrote a set of multiple-choice quiz questions that we hoped would enable us to
 147 evaluate participants’ knowledge about each individual lecture, along with related knowledge
 148 about physics concepts not specifically presented in either video (see Supp. Tab. 1 for the full list
 149 of questions in our stimulus pool). Participants answered questions randomly drawn from each
 150 content area (Lecture 1, Lecture 2, and general physics knowledge) on each of the three quizzes.
 151 Quiz 1 was intended to assess participants’ “baseline” knowledge before training, Quiz 2 assessed
 152 knowledge after watching the *Four Fundamental Forces* video (i.e., Lecture 1), and Quiz 3 assessed
 153 knowledge after watching the *Birth of Stars* video (i.e., Lecture 2).

154 To study in detail how participants’ conceptual knowledge changed over the course of the
 155 experiment, we first sought to model the conceptual content presented to them at each moment
 156 throughout each of the two lectures. We adapted an approach we developed in prior work [29]
 157 to identify the latent themes in the lectures using a topic model [8]. Briefly, topic models take
 158 as input a collection of text documents, and learn a set of “topics” (i.e., latent themes) from their
 159 contents. Once fit, a topic model can be used to transform arbitrary (potentially new) documents
 160 into sets of “topic proportions” describing the weighted blend of learned topics reflected in their

161 texts. We parsed automatically generated transcripts of the two lectures into overlapping sliding
162 windows, where each window contained the text of the lecture transcript from a particular time
163 span. We treated the set of text snippets (across all of these windows) as documents to fit the model
164 (Fig. 2A; see *Constructing text embeddings of multiple lectures and questions*). Transforming the text
165 from every sliding window with the model yielded a number-of-windows by number-of-topics
166 (15) topic-proportions matrix describing the unique mixture of broad themes from both lectures
167 reflected in each window’s text. Each window’s “topic vector” (i.e., column of the topic-proportions
168 matrix) is analogous to a coordinate in a 15-dimensional space whose axes are topics discovered
169 by the model. Within this space, each lecture’s sequence of topic vectors (i.e., corresponding to its
170 transcript’s overlapping text snippets across sliding windows) forms a *trajectory* that captures how
171 its conceptual content unfolds over time (Fig. 2B). We resampled these trajectories to a resolution
172 of one topic vector for each second of video (i.e., 1 Hz).

173 We hypothesized that a topic model trained on transcripts of the two lectures should also
174 capture the conceptual knowledge probed by each quiz question. If indeed the topic model could
175 capture information about the deeper conceptual content of the lectures (i.e., beyond surface-level
176 details such as particular word choices), then we should be able to recover a correspondence
177 between each lecture and questions *about* each lecture. Importantly, such a correspondence could
178 not arise solely from superficial text matching between lecture transcripts and questions, since
179 the lectures and questions often used different words (Supp. Fig. 10) and phrasings. Simply
180 comparing the average topic weights from each lecture and question set (averaging across time
181 and questions, respectively) reveals a striking correspondence (Supp. Fig. 2). Specifically, the
182 average topic weights from Lecture 1 are strongly correlated with the average topic weights from
183 questions about Lecture 1 ($r(13) = 0.809$, $p < 0.001$, 95% confidence interval (CI) = [0.633, 0.962]),
184 and the average topic weights from Lecture 2 are strongly correlated with the average topic weights
185 from questions about Lecture 2 ($r(13) = 0.728$, $p = 0.002$, 95% CI = [0.456, 0.920]). At the same
186 time, the average topic weights from the two lectures are *negatively* correlated with the average
187 topic weights from their non-matching question sets (Lecture 1 video vs. Lecture 2 questions:
188 $r(13) = -0.547$, $p = 0.035$, 95% CI = [-0.812, -0.231]; Lecture 2 video vs. Lecture 1 questions:



Figure 3: Lecture and question topic overlap. A. Topic weight variability. The bar plots display the variance in each topic's weight across lecture timepoints (top row) and questions (bottom row); colors denote topics. The top-weighted words from the most “expressive” (i.e., variable across observations) topic from each lecture are displayed in the upper right (orange: topic 2; yellow-green: topic 4). The top-weighted words from the full set of topics may be found in Supplementary Table 2. **B. Relationships between topic weight variability.** Pairwise correlations between the distributions of topic weight variance for each lecture and question set. Each row and column corresponds to a bar plot in Panel A.

189 $r(13) = -0.612, p = 0.015, 95\% \text{ CI} = [-0.874, -0.281]$), indicating that the topic model also exhibits
190 some degree of specificity. The full set of pairwise comparisons between average topic weights for
191 the lectures and question sets is reported in Supplementary Figure 2.

192 Another, more sensitive, way of summarizing the conceptual content of the lectures and ques-
193 tions is to look at *variability* in how topics are weighted over time and across different questions
194 (Fig. 3). Intuitively, the variability in the expression of a given topic relates to how much “infor-
195 mation” [22] the lecture (or question set) reflects about that topic. For example, suppose a given topic
196 is weighted on heavily throughout a lecture. That topic might be characteristic of some aspect or
197 property of the lecture *overall* (conceptual or otherwise), but unless the topic’s weights change in
198 meaningful ways over time, it would be a poor indicator of any *specific* conceptual content in the
199 lecture. We therefore also compared the variances in topic weights (over time and across questions)
200 between the lectures and questions. The variability in topic expression was similar for the Lecture 1
201 video and questions ($r(13) = 0.824, p < 0.001, 95\% \text{ CI} = [0.696, 0.973]$), and for the Lecture 2 video
202 and questions ($r(13) = 0.801, p < 0.001, 95\% \text{ CI} = [0.539, 0.958]$). Simultaneously, as reported

203 in Figure 3B, the variabilities in topic expression across *different* videos and lecture-specific ques-
204 tions (i.e., Lecture 1 video vs. Lecture 2 questions; Lecture 2 video vs. Lecture 1 questions) were
205 negatively correlated, and neither video’s topic variability was reliably correlated with the topic
206 variability across general physics knowledge questions. Taken together, the analyses reported in
207 Figure 3 and Supplementary Figure 2 indicate that a topic model fit to the videos’ transcripts can
208 also reveal correspondences (at a coarse scale) between the lectures and questions.

209 An individual lecture may be organized around a single broad theme at a coarse scale, but at
210 a finer scale, each moment of a lecture typically covers a narrower range of content. Given the
211 correspondence we found between the variabilities in topic expression across moments of each
212 lecture and questions from its corresponding set (Fig. 3), we wondered whether the text embedding
213 model might additionally capture these conceptual relationships at a finer scale. For example, if a
214 particular question asks about the content from one small part of a lecture, we wondered whether
215 the text embeddings could be used to automatically identify the “matching” moment(s) in the
216 lecture. To explore this, we computed the correlation between each question’s topic weights
217 and the topic weights for each second of its corresponding lecture, and found that each question
218 appeared to be temporally specific (Fig. 4). In particular, most questions’ topic vectors were
219 maximally correlated with a well-defined (and relatively narrow) range of timepoints from their
220 corresponding lectures, outside of which the correlations fell off sharply (Supp. Figs. 3, 4). We also
221 qualitatively examined the best-matching intervals for each question by comparing the questions’
222 text to the transcribed text from the most-correlated parts of the lectures (Supp. Tab. 3). Despite
223 that the questions were excluded from the text embedding model’s training set, in general we
224 found (through manual inspection) a close correspondence between the conceptual content that
225 each question probed and the content covered by the best-matching moments of the lectures. Two
226 representative examples are shown at the bottom of Figure 4.

227 The ability to quantify how much each question is “asking about” the content from each moment
228 of the lectures could enable high-resolution insights into participants’ knowledge. Traditional
229 approaches to estimating how much a student “knows” about the content of a given lecture
230 entail administering some form of assessment (e.g., a quiz) and computing the proportion of

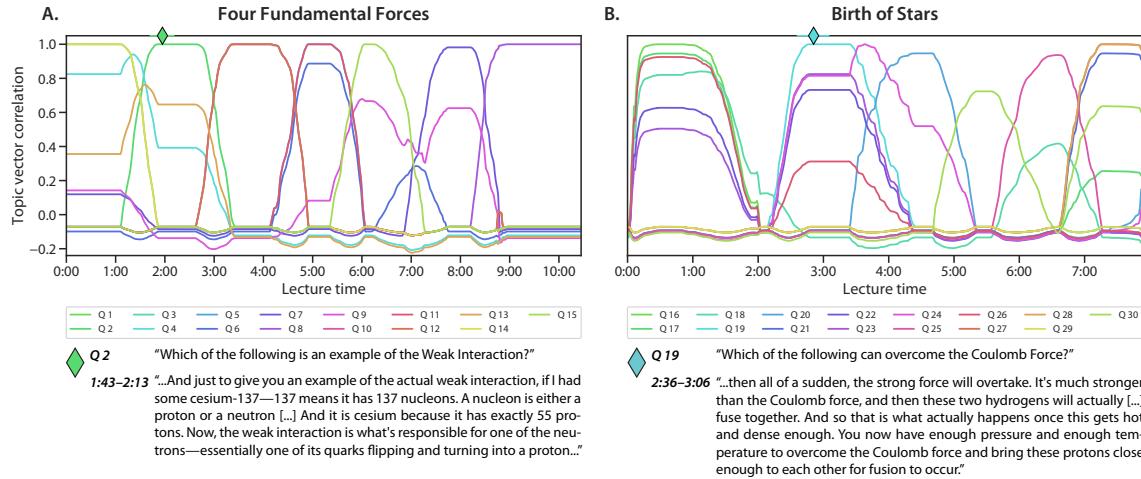


Figure 4: Which parts of each lecture are captured by each question? Each panel displays time series plots showing how each question’s topic vector correlates with each video timepoint’s topic vector (Panel A.: correlations for the *Four Fundamental Forces* lecture and associated questions; Panel B.: correlations for the *Birth of Stars* lecture and associated questions). The colors denote question identities. The diamonds in each panel denote the moment of peak correlation between the indicated question and the lecture trajectory. The associated questions’ text and snippets of the lectures’ transcripts from the surrounding 30 seconds, are displayed at the bottom of the figure.

questions the student answered correctly. But if two students receive identical scores on such an assessment, might our modeling framework help us to gain more nuanced insights into the *specific* content that each student has mastered (or failed to master)? For example, a student who misses three questions that were all about the same concept (e.g., concept *A*) will have gotten the same *proportion* of questions correct as another student who missed three questions about three *different* concepts (e.g., *A*, *B*, and *C*). But if we wanted to help these two students fill in the “gaps” in their understandings, we might do well to focus specifically on concept *A* for the first student, but to also add in materials pertaining to concepts *B* and *C* for the second student. In other words, raw “proportion-correct” measures may capture *how much* a student knows, but not *what* they know. We wondered whether our modeling framework might enable us to (formally and automatically) infer participants’ knowledge at the scale of individual concepts (e.g., as captured by a single moment of a lecture).

We developed a simple formula (Eqn. 1) for using a participant’s responses to a small set

244 of multiple-choice questions to estimate how much that participant “knows” about the concept
245 reflected by any arbitrary coordinate x in text embedding space (e.g., the content reflected by
246 any moment in a lecture they had watched; see *Estimating dynamic knowledge traces*). Essentially,
247 the estimated knowledge at coordinate x is given by the weighted proportion of quiz questions
248 the participant answered correctly, where the weights reflect how much each question is “about”
249 the content at x . When we apply this approach to estimate the participant’s knowledge about
250 the content presented in each moment of each lecture, we can obtain a detailed time course
251 describing how much “knowledge” that participant has about the content presented at any part of
252 the lecture. As shown in Figure 5A and C, we can apply this approach separately for the questions
253 from each quiz participants took throughout the experiment. From just a few questions per quiz
254 (see *Estimating dynamic knowledge traces*), we obtain a high-resolution snapshot (at the time each
255 quiz was taken) of what participants knew about any moment’s content, from either of the two
256 lectures they watched (comprising a total of 1,100 samples across the two lectures).

257 While the time courses in Figure 5A and C provide detailed *estimates* about participants’ knowl-
258 ege, these estimates are of course only *useful* to the extent that they accurately reflect what partic-
259 ipants actually know. As one sanity check, we anticipated that the knowledge estimates should
260 reflect a content-specific “boost” in participants’ knowledge after watching each lecture. In other
261 words, if participants learn about each lecture’s content upon watching it, the knowledge esti-
262 mates should capture that. After watching the *Four Fundamental Forces* lecture, participants should
263 exhibit more knowledge for the content of that lecture than they had before, and that knowledge
264 should persist for the remainder of the experiment. Specifically, knowledge about that lecture’s
265 content should be relatively low when estimated using Quiz 1 responses, but should increase
266 when estimated using Quiz 2 or 3 responses (Fig. 5B). Indeed, we found that participants’ esti-
267 mated knowledge about the content of *Four Fundamental Forces* was substantially higher on Quiz 2
268 versus Quiz 1 ($t(49) = 8.764, p < 0.001$) and on Quiz 3 versus Quiz 1 ($t(49) = 10.519, p < 0.001$).
269 We found no reliable differences in estimated knowledge about that lecture’s content on Quiz 2
270 versus 3 ($t(49) = 0.160, p = 0.874$). Similarly, we hypothesized (and subsequently confirmed) that
271 participants should show greater estimated knowledge about the content of the *Birth of Stars* lec-



Figure 5: Estimating knowledge about the content presented at each moment of each lecture. **A. Knowledge about the time-varying content of *Four Fundamental Forces*.** Each trace displays the weighted proportion of correctly answered questions about the content reflected in each moment of the lecture (see *Estimating dynamic knowledge traces*), using responses from a single quiz (color). The traces are averaged across participants. **B. Average estimated knowledge about *Four Fundamental Forces*.** Each bar displays the across-timepoint average knowledge, estimated using the responses to one quiz's questions. **C. Knowledge about the time-varying content of *Birth of Stars*.** The panel is in the same format as Panel A, but here the knowledge estimates are for the moment-by-moment content of the *Birth of Stars* lecture. **D. Average estimated knowledge about *Birth of Stars*.** The panel is in the same format as Panel B, but here the knowledge estimates are for the content of the *Birth of Stars* lecture. **All panels.** Error ribbons and error bars denote 95% confidence intervals, estimated across participants.

ture after (versus before) watching it (Fig. 5D). Specifically, since participants watched that lecture after taking Quiz 2 (but before Quiz 3), we hypothesized that their knowledge estimates should be relatively low on Quizzes 1 and 2, but should show a “boost” on Quiz 3. Consistent with this prediction, we found no reliable differences in estimated knowledge about the *Birth of Stars* lecture content on Quiz 1 versus 2 ($t(49) = 1.013, p = 0.316$), but estimated knowledge was substantially higher on Quiz 3 versus 2 ($t(49) = 10.561, p < 0.001$) and Quiz 3 versus 1 ($t(49) = 8.969, p < 0.001$).

If we are able to accurately estimate a participant’s knowledge about the content tested by a given question, our estimates of their knowledge should carry some predictive information about whether they are likely to answer that question correctly or incorrectly. We developed a statistical approach to test this claim. For each quiz question a participant answered, in turn, we used Equation 1 to estimate their knowledge at that question’s embedding-space coordinate based on other questions that participant answered on the same quiz. We repeated this for all participants, and for each of the three quizzes. Then, separately for each quiz, we fit a generalized linear mixed model (GLMM) with a logistic link function to explain the probability of correctly answering a question as a function of estimated knowledge at its embedding coordinate, while accounting for varied effects of individual participants and questions (see *Generalized linear mixed models*). To assess the predictive value of the knowledge estimates, we compared each GLMM to an analogous (i.e., nested) “null” model that assumed these estimates carried no predictive information using parametric bootstrap likelihood-ratio tests.

We carried out three different versions of the analyses described above, wherein we considered different sources of information in our estimates of participants’ knowledge for each quiz question. First, we estimated knowledge at each held-out question’s embedding coordinate using *all* other questions answered by the same participant on the same quiz (“All questions”; Fig. 6, top row). This test was intended to assess the overall predictive power of our approach. Second, we estimated knowledge for each question about a given lecture using only the other questions (from the same participant and quiz) about that *same* lecture (“Within-lecture”; Fig. 6, middle rows). This test was intended to assess the *specificity* of our approach by asking whether our predictions could distinguish between questions about different content covered by the same lecture. Third,



Figure 6: Predicting success on held-out questions using estimated knowledge. We used generalized linear mixed models (GLMMs) to model the probability of correctly answering a quiz question as a function of estimated knowledge for its embedding coordinate (see *Generalized linear mixed models*). Separately for each quiz (column), we examined this relationship based on three different sets of knowledge estimates: knowledge for each question based on all other questions the same participant answered on the same quiz (“All questions”; top row), knowledge for each question about one lecture based on all other questions (from the same participant and quiz) about the *same* lecture (“Within-lecture”; middle rows), and knowledge for each question about one lecture based on all questions (from the same participant and quiz) about the *other* lecture (“Across-lecture”; bottom rows). The backgrounds in each panel display kernel density estimates of the relative observed proportions of correctly (blue) versus incorrectly (red) answered questions, for each level of estimated knowledge along the x -axis. The black curves display the (population-level) GLMM-predicted probabilities of correctly answering a question as a function of estimated knowledge. Error ribbons denote 95% confidence intervals.

300 we estimated knowledge for each question about one lecture using only the questions (from the
301 same participant and quiz) about the *other* lecture (“Across-lecture”; Fig. 6, bottom rows). This test
302 was intended to assess the *generalizability* of our approach by asking whether our predictions could
303 extend across the content areas of the two lectures. When estimating participants’ knowledge, we
304 used a rebalancing procedure to ensure that (for a given participant and quiz) their knowledge esti-
305 mates for correctly and incorrectly answered questions were computed from the same underlying
306 proportion of correctly answered questions (see *Generalized linear mixed models*).

307 When we fit a GLMM to estimates of participants’ knowledge for each Quiz 1 question based on
308 all other Quiz 1 questions, we found that higher estimated knowledge for a given question predicted
309 a greater likelihood of answering it correctly (odds ratio (OR) = 8.126, 95% CI = [3.116, 20.123],
310 likelihood-ratio test statistic (λ_{LR}) = 17.002, $p < 0.001$). This relationship held when we repeated
311 this analysis for Quiz 2 (OR = 14.902, 95% CI = [4.976, 39.807], λ_{LR} = 25.408, $p < 0.001$) and again
312 for Quiz 3 (OR = 37.409, 95% CI = [10.425, 107.145], λ_{LR} = 40.948, $p < 0.001$). Taken together,
313 these results suggest that our knowledge estimates can reliably predict participants’ performance
314 on individual questions when they incorporate information from all (other) quiz content.

315 We observed a similar set of results when we restricted our estimates of participants’ knowledge
316 to consider only their performance on other questions about the *same* lecture. Specifically, for
317 Quiz 1, participants’ knowledge of *Four Fundamental Forces*-related questions, estimated from their
318 performance on other *Four Fundamental Forces*-related questions, was predictive of their ability
319 to answer those questions correctly (OR = 15.934, 95% CI = [5.173, 38.005], λ_{LR} = 40.971, $p =$
320 0.001). The same was true of participants’ estimated knowledge for *Birth of Stars*-related questions
321 based on their performance on other *Birth of Stars*-related questions (OR = 9.775, 95% CI =
322 [2.93, 25.08], λ_{LR} = 13.924, $p = 0.001$). These results also held for participants’ Quiz 2 responses
323 (*Four Fundamental Forces*: OR = 35.126, 95% CI = [5.113, 123.868], λ_{LR} = 32.251, $p < 0.001$; *Birth of*
324 *Stars*: OR = 4.717, 95% CI = [2.021, 9.844], λ_{LR} = 16.788, $p < 0.001$) and partially for their Quiz 3
325 responses (*Birth of Stars*: OR = 16.902, 95% CI = [3.353, 53.265], λ_{LR} = 23.233, $p < 0.001$; *Four*
326 *Fundamental Forces*: OR = 2.485, 95% CI = [0.724, 8.366], λ_{LR} = 1.984, $p = 0.170$). Speculatively,
327 the Quiz 3 results suggest that the within-lecture knowledge estimates may be susceptible to ceiling

328 effects in participants' quiz performance. On Quiz 3, after viewing both lectures, no participant
329 answered more than three *Four Fundamental Forces*-related questions incorrectly, and all but five
330 participants (out of 50) answered two or fewer incorrectly. (This was the only subset of questions
331 about either lecture, across all three quizzes, for which this was true.) Consequently, for 90% of
332 participants, our within-lecture estimates of their knowledge for *Four Fundamental Forces*-related
333 questions that they answered incorrectly leveraged information from at most a single other question
334 they were *not* able to correctly answer. This likely hampered our ability to accurately characterize
335 the specific (and by the time they took Quiz 3, relatively few) aspects of the lecture content these
336 participants did *not* know about, and successfully distinguish them from the far more numerous
337 aspects of the lecture content they now *did* know about. Taken together, these within-lecture
338 results suggest that our knowledge estimates can reliably distinguish between questions about
339 different content covered by a single lecture, provided there is sufficient diversity in participants'
340 quiz responses to extract meaningful information about both what they know and what they do
341 not know.

342 Finally, we estimated participants' knowledge for each question about each lecture using only
343 their performance on questions (from the same quiz) about the *other* lecture. This is an especially
344 stringent test of our approach. Our primary assumption in constructing our knowledge estimates is
345 that knowledge about a given concept is similar to knowledge about other concepts that are nearby
346 in the embedding space. However, our analyses in Figure 3 and Supplementary Figure 2 show
347 that the embeddings of content from the two lectures (and of their associated quiz questions) are
348 largely distinct from each other. Therefore, any predictive power of these across-lecture knowledge
349 estimates must overcome large distances in the embedding space. To put this in concrete terms,
350 this test requires predicting participants' performance on individual, highly specific questions
351 about the formation of stars from their responses to just five multiple-choice questions about the
352 fundamental forces of the universe (and vice versa).

353 We found that, before viewing either lecture (i.e., on Quiz 1), participants' abilities to answer
354 *Four Fundamental Forces*-related questions could not be predicted from their responses to *Birth of*
355 *Stars*-related questions ($OR = 1.896$, 95% CI = [0.419, 9.088], $\lambda_{LR} = 0.712$, $p = 0.404$), nor could

356 their abilities to answer *Birth of Stars*-related questions be predicted from their responses to *Four*
357 *Fundamental Forces*-related questions ($OR = 1.522$, 95% CI = [0.332, 6.835], $\lambda_{LR} = 0.286$, $p = 0.611$).
358 Similarly, we found that participants' performance on questions about either lecture could not
359 be predicted given their responses to questions about the other lecture after viewing *Four Funda-*
360 *mental Forces* but before viewing *Birth of Stars* (i.e., on Quiz 2; *Four Fundamental Forces* ques-
361 tions given *Birth of Stars* questions: $OR = 3.49$, 95% CI = [0.739, 12.849], $\lambda_{LR} = 3.266$, $p =$
362 0.083; *Birth of Stars* questions given *Four Fundamental Forces* questions: $OR = 2.199$, 95% CI =
363 [0.711, 5.623], $\lambda_{LR} = 2.304$, $p = 0.141$). Only after viewing *both* lectures (i.e., on Quiz 3) did
364 these across-lecture knowledge estimates reliably predict participants' success on individual quiz
365 questions (*Four Fundamental Forces* questions given *Birth of Stars* questions: $OR = 11.294$, 95% CI =
366 [1.375, 47.744], $\lambda_{LR} = 10.396$, $p < 0.001$; *Birth of Stars* questions given *Four Fundamental Forces* ques-
367 tions: $OR = 7.302$, 95% CI = [1.077, 44.879], $\lambda_{LR} = 4.708$, $p = 0.038$). Taken together, these results
368 suggest that our ability to form estimates solely across different content areas is more limited than
369 our ability to form estimates that incorporate responses to questions from both content areas (as in
370 Fig. 6, "All questions") or within a single content area (as in Fig. 6, "Within-lecture"). However, if
371 participants have recently received some training on both content areas, the knowledge estimates
372 appear to be informative even across content areas.

373 We speculate that these "Across-lecture" results might relate to some of our earlier work on
374 the nature of semantic representations [44]. In that work, we asked whether semantic similarities
375 could be captured through behavioral measures, even if participants' "true" internal representa-
376 tions differed from the embeddings used to *characterize* their behaviors. We found that mismatches
377 between an individual's internal representation of a set of concepts and the representation used to
378 characterize their behaviors can lead to underestimates of how semantically driven those behaviors
379 are. Along similar lines, we suspect that in our current study, participants' conceptual representa-
380 tions may initially differ from the representations learned by our topic model. (Although the topic
381 model's representations are still *related* to participants' initial internal representations; otherwise
382 we would have found that knowledge estimates derived from Quizzes 1 and 2 had no predictive
383 power in the other tests we conducted.) After watching both lectures, however, participants'

384 internal representations may become more aligned with the embeddings used to estimate their
385 knowledge (since those embeddings were trained on the lectures' transcripts). This could help
386 explain why the knowledge estimates derived from Quizzes 1 and 2 (before both lectures had been
387 watched) do not reliably predict performance across content areas, whereas estimates derived from
388 Quiz 3 do.

389 That the knowledge predictions derived from the text embedding space reliably distinguish
390 between correctly and incorrectly answered held-out questions (Fig. 6) suggests that geometric
391 relationships within this space can help explain what participants know. But how far does this
392 explanatory power extend? For example, suppose we know that a participant correctly answered a
393 question at embedding coordinate x . As we move farther away from x in the embedding space, how
394 does the likelihood that the participant knows about the content at a given location "fall off" with
395 distance? Conversely, suppose the participant instead answered that same question *incorrectly*.
396 Again, as we move farther away from x in the embedding space, how does the likelihood that
397 the participant does *not* know about the content at a given coordinate change with distance? We
398 reasoned that, assuming our embedding space is capturing something about how individuals
399 actually organize their knowledge, a participant's ability to answer questions embedded very
400 close to x should tend to be similar to their ability to answer the question embedded *at* x . But once
401 we reach some sufficiently large distance from x , our ability to infer whether or not a participant
402 will correctly answer a question based on their ability to answer the question at x should be no
403 better than guessing based on their *overall* proportion of correctly answered questions. In other
404 words, beyond the maximum distance at which a participant's ability to answer the question at x
405 is informative of their ability to answer a second question at location y , guessing the outcome at
406 y based on the outcome at x should be no more successful than guessing based on a measure that
407 does not consider embedding-space distance.

408 With these ideas in mind, we asked: conditioned on a participant's ability to answer a given
409 question correctly, what proportion of all questions within some radius r of its embedding co-
410 ordinate were they able to answer correctly? We plotted this proportion as a function of r for
411 questions that participants answered correctly, and for questions they answered incorrectly. As

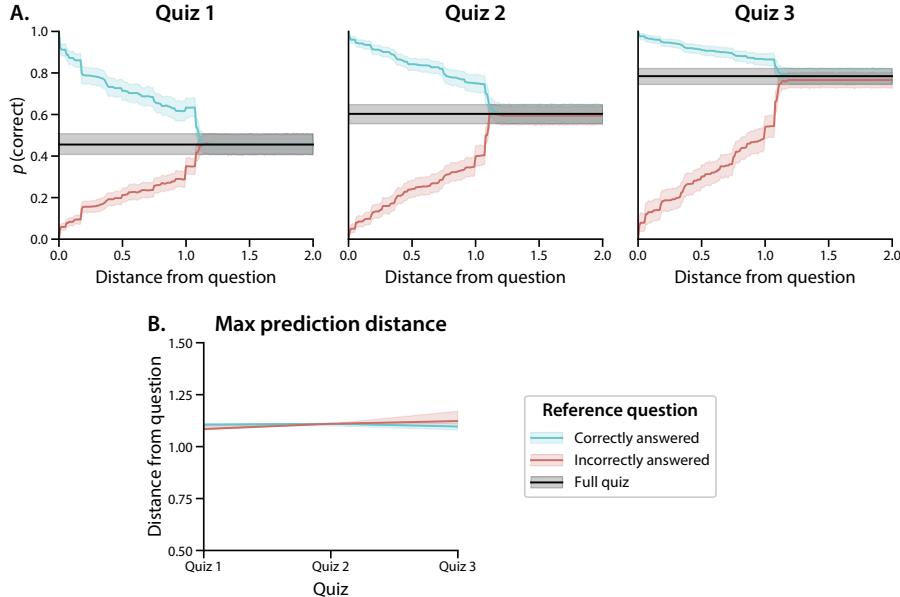


Figure 7: Knowledge falls off gradually in text embedding space. A. Performance versus distance. For each participant, for each correctly answered question (blue) or incorrectly answered question (red), we computed the proportion of correctly answered questions within a given distance of that question’s embedding coordinate. We used these proportions as a proxy for participants’ knowledge about the content within that region of the embedding space. We repeated this analysis for all questions and participants, and separately for each quiz (column). The black lines denote the average proportion correct across *all* questions included in the analysis at the given distance. **B. Maximum distance for which performance is reliably different from the average.** We used a bootstrap procedure (see *Estimating the “smoothness” of knowledge*) to estimate the point at which the blue and red lines in Panel A reliably diverged from the black line. We repeated this analysis separately for correctly and incorrectly answered questions from each quiz. **All panels.** Error ribbons denote bootstrap-estimated 95% confidence intervals.

412 shown in Figure 7, we found that quiz performance falls off smoothly with distance, and the
 413 “rate” at which it falls off does not appear to differ across quizzes, as measured by the distance at
 414 which performance becomes statistically indistinguishable from a simple proportion-correct score
 415 (see *Estimating the “smoothness” of knowledge*). This suggests that, at least within the region of text
 416 embedding space spanned by the questions our study’s participants answered (and as charac-
 417 terized using our topic model), the rate at which knowledge changes with distance is relatively
 418 constant, even as participants’ overall level of knowledge varies across quizzes and regions of the
 419 embedding space.

420 Knowledge estimates need not be limited to the contents of these particular lectures and quizzes.

421 As illustrated in Figure 8, our general approach to estimating knowledge from a small number
422 of quiz questions may be extended to *any* content, given its text embedding coordinate. To
423 visualize how knowledge “spreads” through text embedding space to content beyond the lectures
424 participants watched and the questions they answered, we first fit a new topic model to the lectures’
425 sliding windows with $k = 100$ topics. Conceptually, increasing the number of topics used by the
426 model functions to increase the “resolution” of the embedding space, providing a greater ability
427 to estimate knowledge for content that is highly similar to (but not precisely the same as) that
428 contained in the two lectures used to train the model. Aside from increasing the number of topics
429 from 15 to 100, all other procedures and model parameters were carried over from the preceding
430 analyses. As in our other analyses, we resampled each lecture’s topic trajectory to 1 Hz and
431 projected each question into a shared text embedding space.

432 We projected the resulting 100-dimensional topic vectors (for each second of the lectures and
433 each quiz question) onto a shared 2-dimensional plane (see *Creating knowledge and learning map*
434 *visualizations*). Next, we sampled points from a 100×100 grid of coordinates that evenly tiled a
435 rectangle enclosing the 2D projections of the lectures and questions. We then used Equation 4 to
436 estimate participants’ knowledge at each of these 10,000 sampled locations, and averaged these
437 estimates across participants to obtain an estimated average *knowledge map* (Fig. 8A). Intuitively,
438 the knowledge map constructed from a given quiz’s responses provides a visualization of “how
439 much” participants knew about any content expressible by the fitted text embedding model at
440 the point in time when they completed that quiz. We note that we used these 2D maps solely
441 for visualization; all relevant comparisons, distance computations, and statistical tests we report
442 above were carried out in the original 15-dimensional space, using the 15-topic model.

443 Several features of the resulting knowledge maps are worth noting. The average knowledge
444 map estimated from Quiz 1 responses (Fig. 8A, leftmost map) shows that participants tended to
445 have relatively little knowledge about any parts of the text embedding space (i.e., the shading is
446 relatively dark everywhere). The knowledge map estimated from Quiz 2 responses shows a marked
447 increase in knowledge on the left side of the map (around roughly the same range of coordinates
448 traversed by the *Four Fundamental Forces* lecture, indicated by the dotted blue line). In other words,

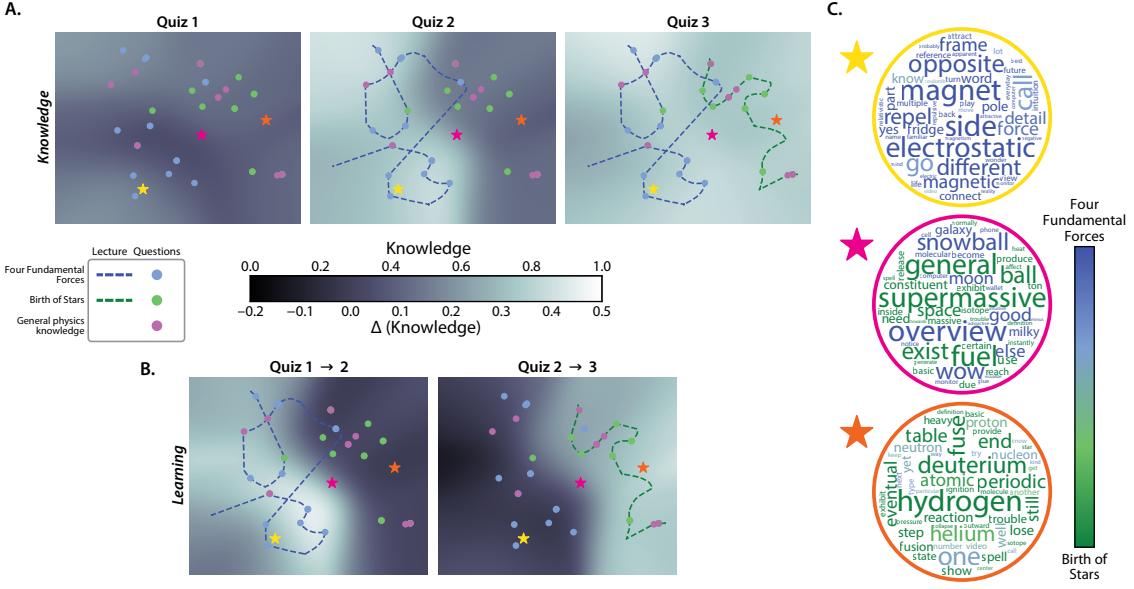


Figure 8: Mapping out the geometry of knowledge and learning. **A. Average “knowledge maps” estimated using each quiz.** Each map displays a 2D projection of participants’ estimated knowledge about the content reflected by *all* regions of topic space (see *Creating knowledge and learning map visualizations*). The topic trajectories of the two lectures are indicated by dotted lines (blue: Lecture 1; green: Lecture 2), and the coordinates of each question are indicated by dots (light blue: Lecture 1-related; light green: Lecture 2-related; purple: general physics knowledge). Each map reflects an average across all participants. For individual participants’ maps, see Supplementary Figures 5, 6, and 7. **B. Average “learning maps” estimated between each successive pair of quizzes.** The learning maps follow the same general format as the knowledge maps in Panel A, but here the shading at each coordinate indicates the *difference* between the corresponding coordinates in the indicated *pair* of knowledge maps—i.e., how much the estimated knowledge “changed” between the two quizzes. Each map reflects an average across all participants. For individual participants’ maps, see Supplementary Figures 8 and 9. **C. Word clouds for sampled points in topic space.** Each word cloud displays the weighted blend of words underlying the topic proportions represented at the corresponding colored star’s location on the maps. In each word cloud, the words’ relative sizes correspond to their relative weights at the starred location, and their colors indicate their relative weights in the *Four Fundamental Forces* (blue) versus *Birth of Stars* (green) lectures, on average, across all timepoints’ topic vectors.

449 participants' estimated increase in knowledge is localized to conceptual content that is nearby (i.e.,
450 related to) the content from the lecture they watched prior to taking Quiz 2. This localization is
451 non-trivial: these knowledge estimates are informed only by the embedded coordinates of the
452 *quiz questions*, not by the embeddings of either lecture (see Eqn. 4). Finally, the knowledge map
453 estimated from Quiz 3 responses shows a second increase in knowledge, localized to the region
454 surrounding the embedding of the *Birth of Stars* lecture participants watched immediately prior to
455 taking Quiz 3.

456 Another way of visualizing these content-specific increases in knowledge after participants
457 viewed each lecture is displayed in Figure 8B. Taking the point-by-point difference between the
458 knowledge maps estimated from responses to a successive pair of quizzes yields a *learning map*
459 that describes the *change* in estimated knowledge from one quiz to the next. These learning maps
460 highlight that the estimated knowledge increases we observed across maps were specific to the
461 regions around the embeddings of each lecture, in turn.

462 Because the 2D projection we used to construct the knowledge and learning maps is invertible,
463 we may gain additional insights into these maps' meanings by reconstructing the original high-
464 dimensional topic vector for any location on the map we are interested in. For example, this could
465 serve as a useful tool for an instructor looking to better understand which content areas a student
466 (or a group of students) knows well (or poorly). As a demonstration, we show the top-weighted
467 words from the blends of topics reconstructed from three example locations on the maps (Fig. 8C):
468 one point near the *Four Fundamental Forces* embedding (yellow), a second point near the *Birth of*
469 *Stars* embedding (orange), and a third point between the two lectures' embeddings (pink). As
470 shown in the word clouds in Panel C, the top-weighted words at the example coordinate near the
471 *Four Fundamental Forces* embedding tended to be weighted more heavily by the topics expressed
472 in that lecture. Similarly, the top-weighted words at the example coordinate near the *Birth of Stars*
473 embedding tended to be weighted more heavily by the topics expressed in *that* lecture. The top-
474 weighted words at the example coordinate between the two lectures' embeddings show a roughly
475 even mix of words most strongly associated with each lecture.

476 **Discussion**

477 We developed a computational framework that uses short multiple-choice quizzes to gain nuanced
478 insights into what learners know and how their knowledge changes with training. First, we show
479 that our approach can automatically match the conceptual knowledge probed by individual quiz
480 questions to the corresponding moments in lecture videos when those concepts were presented
481 (Fig. 4). Next, we demonstrate how we can estimate moment-by-moment “knowledge traces” that
482 reflect the degree of knowledge participants have about each lecture’s time-varying content, and
483 capture temporally specific increases in knowledge after viewing each lecture (Fig. 5). We then
484 show that these knowledge estimates can generalize to held-out questions and predict participants’
485 abilities to answer them correctly (Fig. 6). Finally, we use our framework to construct visual maps
486 that provide snapshot estimates of how much participants know about any concept within the
487 scope of our text embedding model, and how much their knowledge of those concepts changes
488 with training (Fig. 8).

489 Our work makes several contributions to the study of how people acquire conceptual knowl-
490 edge. First, from a methodological standpoint, our modeling framework provides a systematic
491 means of mapping out and characterizing knowledge in maps that have infinite (arbitrarily many)
492 numbers of coordinates, and of “filling out” those maps using relatively small numbers of multiple-
493 choice quiz questions. Our experimental finding that we can use these maps to predict success
494 on held-out questions has several psychological implications as well. For example, concepts that
495 are assigned to nearby coordinates by the text embedding model also appear to be “known to a
496 similar extent” (as reflected by participants’ responses to held-out questions; Fig. 6). This suggests
497 that participants also *conceptualize* similarly the content reflected by nearby embedding coordi-
498 nates. How participants’ knowledge “falls off” with spatial distance is captured by the knowledge
499 maps we infer from their quiz responses (e.g., Figs. 7, 8). In other words, our study shows that
500 knowledge about a given concept implies knowledge about related concepts, and how far this
501 implication extends in text embedding space.

502 In our study, we characterize the “coordinates” of participants’ knowledge using a relatively

503 simple “bag-of-words” text embedding model [LDA; 8]. More sophisticated text embedding
504 models, such as transformer-based models [18, 55, 68, 71], can leverage additional textual infor-
505 mation such as complex grammatical and semantic relationships between words, higher-order
506 syntactic structures, stylistic features, and more. We considered using transformer-based models
507 in our study, but we found that the text embeddings derived from these models were surprisingly
508 uninformative with respect to differentiating or otherwise characterizing the conceptual content
509 of the lectures and questions we used (see *Supplementary results*). We suspect that this reflects a
510 broader challenge in constructing models that are both high-resolution within a given domain (e.g.,
511 the domain of physics lectures and questions) *and* sufficiently broad as to enable them to cover
512 a wide range of domains. Essentially, “larger” language models learn more complex features of
513 language through training on enormous and diverse text corpora. But as a result, their embedding
514 spaces also “span” an enormous and diverse range of conceptual content, sacrificing a degree of
515 specificity in their capacities to distinguish subtle conceptual differences within a more narrow
516 range of content. In comparing our LDA model (trained specifically on the lectures used in our
517 study) to a larger transformer-based model (BERT), we found that our LDA model provides both
518 coverage of the requisite material and specificity at the level of individual questions, while BERT
519 essentially relegates the contents of both lectures and all quiz questions (which are all broadly
520 about “physics”) to a tiny region of its embedding space, thereby blurring out meaningful distinc-
521 tions between different specific concepts covered by the lectures and questions (Supp. Fig. 11). We
522 note that these are not criticisms of BERT, nor of other large language models trained on large and
523 diverse corpora. Rather, our point is that simpler models trained on relatively small but specialized
524 corpora can outperform much more complex models trained on much larger corpora when we are
525 specifically interested in capturing subtle conceptual differences at the level of a single, narrowly
526 focused course lecture or quiz question. On the other hand, if our goal had been to choose a
527 model that generalized to many different content areas simultaneously, we would expect our LDA
528 model to perform comparatively poorly to BERT or other much larger general-purpose models.
529 We suggest that bridging this tradeoff between achieving high resolution within a single content
530 area and the ability to generalize to many diverse content areas will be an important challenge for

531 future work.

532 At the opposite end of the spectrum from large language models, one could also imagine
533 using an even *simpler* “model” than LDA that relates the contents of course lectures and quiz
534 questions through explicit word-overlap metrics (rather than similarities in the latent topics they
535 exhibit). In a supplementary analysis (Supp. Fig. 10), we compared the LDA-based question-lecture
536 matches shown in Figure 4 with analogous matches based on the Jaccard similarity between each
537 question’s text and each sliding window from the corresponding lecture’s transcript. As for
538 the embeddings derived from BERT, we found that this word-matching approach also blurred
539 meaningful distinctions between concepts presented in different parts of each lecture and tested
540 by different quiz questions. But rather than characterizing their contents at too *broad* a semantic
541 scale, the lack of specificity in this approach arises from considering too *narrow* a semantic scale:
542 the sorts of concepts typically conveyed in course lectures and tested by quiz questions are not
543 defined (and meaningful similarities and distinctions between them do not tend to emerge) at the
544 level of individual words.

545 In other words, while the embedding spaces of more complex large language models afford
546 low resolution at the scale of individual course lectures and questions because they “zoom out”
547 too far, simpler word-matching measures afford low resolution because they “zoom *in*” too far. In
548 this way, we view our approach as occupying a sort of “sweet spot” between simpler and more
549 complex alternatives, in that it enables us to characterize the contents of course materials at the
550 appropriate semantic scale where relevant concepts “come into focus.” Our approach enables us to
551 accurately and consistently identify each question’s content in a way that matches it with specific
552 content from the lectures and distinguishes it from other questions about similar content. In turn,
553 this enables us to construct accurate predictions about participants’ knowledge of the conceptual
554 content tested by individual quiz questions (Fig. 6).

555 Another application for large language models that does *not* require explicitly modeling the
556 content of individual lectures or questions is to leverage these models’ abilities to generate text. For
557 example, generative text models like ChatGPT [55] and LLaMa [68] are already being used to build
558 a new generation of interactive tutoring systems [e.g., 45]. Unlike the approach we have taken here,

559 these generative text model-based systems do not explicitly model what learners know, or how
560 their knowledge changes over time with training. One could imagine building a hybrid system
561 that combines the best of both worlds: a large language model that can *generate* text, combined
562 with a smaller model that can *infer* what learners know and how their knowledge changes over
563 time. Such a hybrid system could potentially be used to build the next generation of interactive
564 tutoring systems that are able to adapt to learners' needs in real time, and provide more nuanced
565 feedback about what learners know and what they do not know.

566 One limitation of our approach is that topic models contain no explicit internal representations
567 of more complex aspects of "knowledge," like knowledge graphs, dependencies or associations
568 between concepts, causality, and so on. These representations might (in principle) be added
569 as extensions to our approach to more accurately and precisely capture, characterize, and track
570 learners' knowledge. However, modeling these aspects of knowledge will likely require substantial
571 additional research effort.

572 Within the past several years, a global pandemic forced many educators to suddenly adapt to
573 teaching remotely [35, 52, 64, 72]. This change in world circumstances is happening alongside (and
574 perhaps accelerating) geometric growth in the availability of high-quality online courses from plat-
575 forms such as Khan Academy [36], Coursera [73], EdX [38], and others [60]. Continued expansion
576 of the global internet backbone and improvements in computing hardware have also facilitated
577 improvements in video streaming, enabling videos to be easily shared and viewed by increasingly
578 large segments of the world's population. This exciting time for online course instruction provides
579 an opportunity to re-evaluate how we, as a global community, educate ourselves and each other.
580 For example, we can ask: what defines an effective course or training program? Which aspects of
581 teaching might be optimized and/or augmented by automated tools? How and why do learning
582 needs and goals vary across people? How might we lower barriers to receiving a high-quality
583 education?

584 Alongside these questions, there is a growing desire to extend existing theories beyond the
585 domain of lab testing rooms and into real classrooms [34]. In part, this has led to a recent
586 resurgence of "naturalistic" or "observational" experimental paradigms that attempt to better

587 reflect more ethologically valid phenomena that are more directly relevant to real-world situations
588 and behaviors [53]. In turn, this has brought new challenges in data analysis and interpretation. A
589 key step towards solving these challenges will be to build explicit models of real-world scenarios
590 and how people behave in them (e.g., models of how people learn conceptual content from real-
591 world courses, as in our current study). A second key step will be to understand which sorts
592 of signals derived from behaviors and/or other measurements [e.g., neurophysiological data; 4,
593 19, 50, 54, 57] might help to inform these models. A third major step will be to develop and
594 employ reliable ways of evaluating the complex models and data that are a hallmark of naturalistic
595 paradigms.

596 Beyond specifically predicting what people *know*, the fundamental ideas we develop here also
597 relate to the notion of “theory of mind” of other individuals [26, 32, 49]. Considering others’ unique
598 perspectives, prior experiences, knowledge, goals, etc., can help us to more effectively interact and
599 communicate [58, 63, 67]. One could imagine future extensions of our work (e.g., analogous to
600 the knowledge and learning maps shown in Fig. 8), that attempt to characterize how well-aligned
601 different people’s knowledge bases or backgrounds are. In turn, this might be used to model how
602 knowledge (or other forms of communicable information) flows not just between teachers and
603 students, but between friends having a conversation, individuals on a first date, participants at
604 a business meeting, doctors and patients, experts and non-experts, political allies or adversaries,
605 and more. For example, the extent to which two people’s knowledge maps “match” or “align” in
606 a given region of text embedding space might serve as a predictor of how effectively they will be
607 able to communicate about the corresponding conceptual content.

608 Ultimately, our work suggests a rich new line of questions about the geometric “form” of
609 knowledge, how knowledge changes over time, and how we might map out the full space of
610 what an individual knows. Our finding that detailed estimates about knowledge may be obtained
611 from short quizzes shows one way that traditional approaches to evaluation in education may be
612 extended. We hope that these advances might help pave the way for new approaches to teaching
613 or delivering educational content that are tailored to individual students’ learning needs and goals.

614 **Materials and methods**

615 **Participants**

616 We enrolled a total of 50 Dartmouth undergraduate students in our study. Participants received
617 optional course credit for enrolling. We asked each participant to complete a demographic survey
618 that included questions about their age, gender, native spoken language, ethnicity, race, hearing,
619 color vision, sleep, coffee consumption, level of alertness, and several aspects of their educational
620 background and prior coursework.

621 Participants' ages ranged from 18 to 22 years (mean: 19.52 years; standard deviation: 1.09
622 years). A total of 15 participants reported their gender as male and 35 participants reported their
623 gender as female. A total of 49 participants reported their native language as "English" and 1
624 reported having another native language. A total of 47 participants reported their ethnicity as
625 "Not Hispanic or Latino" and three reported their ethnicity as "Hispanic or Latino." Participants
626 reported their races as White (32 participants), Asian (14 participants), Black or African American
627 (5 participants), American Indian or Alaska Native (1 participant), and Native Hawaiian or Other
628 Pacific Islander (1 participant). (Note that some participants selected multiple racial categories.)

629 A total of 49 participants reporting having normal hearing and 1 participant reported having
630 some hearing impairment. A total of 49 participants reported having normal color vision and 1
631 participant reported being color blind. Participants reported having had, on the night prior to
632 testing, 2–4 hours of sleep (1 participant), 4–6 hours of sleep (9 participants), 6–8 hours of sleep (35
633 participants), or 8+ hours of sleep (5 participants). They reported having consumed, on the same
634 day and leading up to their testing session, 0 cups of coffee (38 participants), 1 cup of coffee (10
635 participants), 3 cups of coffee (1 participant), or 4+ cups of coffee (1 participant).

636 No participants reported that their focus was currently impaired (e.g., by drugs or alcohol).
637 Participants reported their current level of alertness, and we converted their responses to numerical
638 scores as follows: "very sluggish" (-2), "a little sluggish" (-1), "neutral" (0), "fairly alert" (1), and
639 "very alert" (2). Across all participants, a range of alertness levels were reported (range: -2–1;
640 mean: -0.10; standard deviation: 0.84).

Participants reported their undergraduate major(s) as “social sciences” (28 participants), “natural sciences” (16 participants), “professional” (e.g., pre-med or pre-law; 8 participants), “mathematics and engineering” (7 participants), “humanities” (4 participants), or “undecided” (3 participants). Note that some participants selected multiple categories for their undergraduate major(s). We also asked participants about the courses they had taken. In total, 45 participants reported having taken at least one Khan Academy course in the past, and 5 reported not having taken any Khan Academy courses. Of those who reported having watched at least one Khan Academy course, 7 participants reported having watched 1–2 courses, 11 reported having watched 3–5 courses, 8 reported having watched 5–10 courses, and 19 reported having watched 10 or more courses. We also asked participants about the specific courses they had watched, categorized under different subject areas. In the “Mathematics” area, participants reported having watched videos on AP Calculus AB (21 participants), Precalculus (17 participants), Algebra 2 (14 participants), AP Calculus BC (12 participants), Trigonometry (11 participants), Algebra 1 (10 participants), Geometry (8 participants), Pre-algebra (7 participants), Multivariable Calculus (5 participants), Differential Equations (5 participants), Statistics and Probability (4 participants), AP Statistics (2 participants), Linear Algebra (2 participants), Early Math (1 participant), Arithmetic (1 participant), and other videos not listed in our survey (5 participants). In the “Science and engineering” area, participants reported having watched videos on Chemistry, AP Chemistry, or Organic Chemistry (21 participants); Physics, AP Physics I, or AP Physics II (18 participants); Biology, AP Biology, or High school Biology (15 participants); Health and Medicine (1 participant); and other videos not listed in our survey (5 participants). We also asked participants whether they had specifically seen the videos used in our experiment. Of the 45 participants who reported having taken at least one Khan Academy course in the past, 44 participants reported that they had not watched the *Four Fundamental Forces* video and 1 participant reported that they were not sure whether they had watched it. All participants reported that they had not watched the *Birth of Stars* video. When we asked participants about non-Khan Academy online courses, they reported having watched or taken courses on Mathematics (15 participants), Science and engineering (11 participants), Test preparation (9 participants), Economics and finance (3 participants), Arts and humanities (2 participants).

669 ipants), Computing (2 participants), and other categories not listed in our survey (17 participants).
670 Finally, we asked participants about in-person courses they had taken in different subject areas.
671 They reported taking courses in Mathematics (38 participants), Science and engineering (37 partic-
672 ipants), Arts and humanities (34 participants), Test preparation (27 participants), Economics and
673 finance (26 participants), Computing (14 participants), College and careers (7 participants), and
674 other courses not listed in our survey (6 participants).

675 Experiment

676 We hand-selected two course videos from the Khan Academy platform: *Four Fundamental Forces*
677 (an introduction to gravity, electromagnetism, the weak nuclear force, and the strong nuclear force;
678 duration: 10 minutes and 29 seconds) and *Birth of Stars* (an introduction to how stars are formed;
679 duration: 7 minutes and 57 seconds). All participants viewed the videos in the same order (i.e.,
680 *Four Fundamental Forces* followed by *Birth of Stars*).

681 We then hand-created 39 multiple-choice questions: 15 about the conceptual content of *Four*
682 *Fundamental Forces* (i.e., Lecture 1), 15 about the conceptual content of *Birth of Stars* (i.e., Lecture 2),
683 and 9 questions that tested for general conceptual knowledge about basic physics (covering material
684 that was not presented in either video). To help broaden the set of lecture-specific questions, our
685 team worked through each lecture in small segments to identify what each segment was “about”
686 conceptually, and then write a question about that concept. The general physics questions were
687 drawn from our team’s prior coursework and areas of interest, along with internet searches and
688 brainstorming with the project team and other members of J.R.M.’s lab. Although we attempted to
689 design the questions to test “conceptual knowledge,” we note that estimating the specific “amount”
690 of conceptual understanding that each question “requires” to answer is somewhat subjective, and
691 might even come down to the “strategy” a given participant used to answer the question at that
692 particular moment. The full set of questions and answer choices may be found in Supplementary
693 Table 1. The final set of questions (and response options) was reviewed and approved by J.R.M.
694 before we collected or analyzed the text or experimental data.

695 Over the course of the experiment, participants completed three 13-question multiple-choice

696 quizzes: the first before viewing Lecture 1, the second between Lectures 1 and 2, and the third
697 after viewing Lecture 2 (see Fig. 1). The questions appearing on each quiz, for each participant,
698 were randomly chosen from the full set of 39, with the constraints that (a) each quiz contained
699 exactly 5 questions about Lecture 1, 5 questions about Lecture 2, and 3 questions about general
700 physics knowledge, and (b) each question appear exactly once for each participant. The orders of
701 questions on each quiz, and the orders of answer options for each question, were also randomized.
702 We obtained informed consent from all participants, and our experimental protocol was approved
703 by the Committee for the Protection of Human Subjects at Dartmouth College. We used this
704 experiment to develop and test our computational framework for estimating knowledge and
705 learning.

706 **Analysis**

707 **Statistics**

708 All of the statistical tests performed in our study were two-sided. The 95% confidence intervals
709 we report for each correlation were estimated from bootstrap distributions of 10,000 correlation
710 coefficients obtained by sampling (with replacement) from the observed data.

711 **Constructing text embeddings of multiple lectures and questions**

712 We adapted an approach we developed in prior work [29] to embed each moment of the two
713 lectures and each question in our pool in a common representational space. Briefly, our approach
714 uses a topic model [Latent Dirichlet Allocation; 8] trained on a set of documents to discover a set
715 of k “topics” or “themes.” Formally, each topic is defined as a distribution of weights over words in
716 the model’s vocabulary (i.e., the union of all unique words across all documents, excluding “stop
717 words”). Conceptually, each topic is intended to give larger weights to words that are semantically
718 related (as inferred from their tendency to co-occur in the same document). After fitting a topic
719 model, each document in the training set, or any *new* document that contains at least some of
720 the words in the model’s vocabulary, may be represented as a k -dimensional vector describing

721 how much that document (most probably) reflects each topic. To select an appropriate k for our
722 model, as a starting point, we identified the minimum number of topics that yielded at least one
723 “unused” topic (i.e., in which all words in the vocabulary were assigned uniform weights) after
724 training. This indicated that the number of topics was sufficient to capture the set of latent themes
725 present in the two lectures (from which we constructed our document corpus, as described below).
726 We found this value to be $k = 15$ topics. We found that with a limited number of additional
727 adjustments following Boyd-Graber et al. [9], such as removing corpus-specific stop-words, the
728 model yielded (subjectively) sensible and coherent topics. The distribution of weights over words
729 in the vocabulary for each discovered topic is shown in Supplementary Figure 1, and each topic’s
730 top-weighted words may be found in Supplementary Table 2.

731 As illustrated in Figure 2A, we started by building up a corpus of documents using overlapping
732 sliding windows that spanned each lecture’s transcript. Khan Academy provides professionally
733 created, manual transcriptions of all lecture videos for closed captioning. However, such tran-
734 scripts would not be readily available in all contexts to which our framework could potentially be
735 applied. Khan Academy videos are hosted on the YouTube platform, which additionally provides
736 automated captions. We opted to use these automated transcripts [which, in prior work, we have
737 found to be of sufficiently near-human quality to yield reliable data in behavioral studies; 74]
738 when developing our framework in order to make it more directly extensible and adaptable by
739 others in the future.

740 We fetched these automated transcripts using the `youtube-transcript-api` Python pack-
741 age [17]. Each transcript consisted of one timestamped line of text for every few seconds (mean:
742 2.34 s; standard deviation: 0.83 s) of spoken content in the lecture (i.e., corresponding to each in-
743 dividual caption that would appear on-screen if viewing the lecture via YouTube, and when those
744 lines would appear). We defined a sliding window length of (up to) $w = 30$ transcript lines and
745 assigned each window a timestamp corresponding to the midpoint between the timestamps for its
746 first and last lines. This w parameter was chosen to match the same number of words per sliding
747 window (rounded to the nearest whole word, and before preprocessing) as the sliding windows
748 we defined in our prior work [29; i.e., 185 words per sliding window].

749 These sliding windows ramped up and down in length at the beginning and end of each
750 transcript, respectively. In other words, each transcript's first sliding window covered only its first
751 line, the second sliding window covered the first two lines, and so on. This ensured that each line
752 from the transcripts appeared in the same number (w) of sliding windows. We next performed a
753 series of standard text preprocessing steps: normalizing case, lemmatizing, removing punctuation
754 and removing stop-words. We constructed our corpus of stop words by augmenting the Natural
755 Language Toolkit [NLTK; 5] English stop word list with the following additional words, selected
756 using one of the approaches suggested by Boyd-Graber et al. [9]: "actual," "actually," "also," "bit,"
757 "could," "e," "even," "first," "follow," "following," "four," "let," "like," "mc," "really,", "saw,"
758 "see," "seen," "thing," and "two." This yielded sliding windows containing an average of 73.8
759 remaining words, and spanning an average of 62.22 seconds. We treated the text from each sliding
760 window as a single "document" and combined these documents across the two lectures' windows
761 to create a single training corpus for the topic model.

762 After fitting the topic model to the two lectures' transcripts, we could use the trained model to
763 transform arbitrary (potentially new) documents into k -dimensional topic vectors. A convenient
764 property of these topic vectors is that documents that reflect similar blends of topics (i.e., documents
765 that reflect similar themes, according to the model) will yield similar coordinates (in terms of
766 correlation, cosine similarity, Kullback-Leibler divergence, Euclidean distance, or other geometric
767 measures). In general, the similarity between different documents' topic vectors may be used to
768 characterize the similarity in conceptual content between the documents.

769 We transformed each sliding window's text into a topic vector, and then used linear interpolation
770 (independently for each topic dimension) to resample the resulting time series to one vector
771 per second. We also used the fitted model to obtain topic vectors for each quiz question in our pool
772 (see Supp. Tab. 1). Taken together, we obtained a *trajectory* for each lecture video, describing its path
773 through topic space, and a single coordinate for each question (Fig. 2C). Embedding both lectures
774 and all of the questions using a common model enables us to compare the content from different
775 moments of the lectures, compare the content across lectures, and estimate potential associations
776 between specific questions and specific moments of lecture content.

777 **Estimating dynamic knowledge traces**

778 We used the following equation to estimate each participant’s knowledge about timepoint t of a
779 given lecture, $\hat{k}(t)$:

$$\hat{k}(f(t, L)) = \frac{\sum_{i \in \text{correct}} \text{ncorr}(f(t, L), f(i, Q))}{\sum_{j=1}^N \text{ncorr}(f(t, L), f(j, Q))}, \quad (1)$$

780 where

$$\text{ncorr}(x, y) = \frac{\text{corr}(x, y) - \text{mincorr}}{\text{maxcorr} - \text{mincorr}}, \quad (2)$$

781 and where mincorr and maxcorr are the minimum and maximum correlations between the topic
782 vectors for any lecture timepoint and quiz question, taken over all timepoints in the given lecture
783 and all questions *about* that lecture appearing on the given quiz. We also define $f(s, \Omega)$ as the s^{th}
784 topic vector from the set of topic vectors Ω . Here t indexes the time series of lecture topic vectors
785 L , and i and j index the topic vectors of questions Q used to estimate the participant’s knowledge.
786 Note that “correct” denotes the set of indices of the questions the participant answered correctly
787 on the given quiz.

788 Intuitively, $\text{ncorr}(x, y)$ is the correlation between two topic vectors (e.g., the topic vector x
789 for one timepoint in a lecture and the topic vector y for one question on a quiz), normalized
790 by the minimum and maximum correlations (across all timepoints t and questions j) to range
791 between 0 and 1, inclusive. Equation 1 then computes the weighted average proportion of correctly
792 answered questions about the content presented at timepoint t , where the weights are given by the
793 normalized correlations between timepoint t ’s topic vector and the topic vectors for each question.
794 The normalization step (i.e., using ncorr instead of the raw correlations) ensures that every question
795 contributes some non-negative amount to the knowledge estimate.

796 **Generalized linear mixed models**

797 In the set of analyses reported in Figure 6, we assessed whether estimates of participants’ knowl-
798 edge at the embedding coordinates of individual quiz questions could be used to reliably predict

their abilities to correctly answer those questions. In essence, we treated each question a given participant answered on a given quiz as a “lecture” consisting of a single timepoint, and used Equation 1 to estimate the participant’s knowledge for its embedding coordinate based on their performance on all *other* questions they answered on that same quiz (“All questions”; Fig. 6, top row). Additionally, for each lecture-related question (i.e., excluding questions about general physics knowledge), we computed analogous knowledge estimates based on two different subsets of questions the participant answered on the same quiz: (1) all *other* questions about the same lecture as the target question (“Within-lecture”; Fig. 6, middle rows), and (2) all questions about the other of the two lectures (“Across-lecture”; Fig. 6, bottom rows).

In performing these analyses, our null hypothesis is that the knowledge estimates we compute based on the quiz questions’ embedding coordinates do *not* provide useful information about participants’ abilities to correctly answer those questions—in other words, that there is no meaningful difference (on average) between the knowledge estimates we compute for questions participants answered correctly versus incorrectly. Specifically, since we estimate knowledge for a given embedding coordinate as a weighted proportion-correct score (where each question’s weight reflects its embedding-space distance from the target coordinate; see Eqn. 1), if these weights are uninformative (e.g., randomly distributed), then our estimates of participants’ knowledge should be equivalent (on average) to the *unweighted* proportion of correctly answered questions used to compute them. In general, for a given participant and quiz, this expected null value (i.e., that participant’s proportion-correct score on that quiz) is the same for any coordinate in the embedding space (e.g., any lecture timepoint, quiz question, etc.). However, in the “All questions” and “Within-lecture” versions of the analyses shown in Figure 6, we estimate each participant’s knowledge for each target question using all *other* questions (or all *other* questions about the same lecture) they answered on the same quiz. This introduces a systematic dependency between a participant’s success on a target question and their proportion-correct score on the remaining questions available to estimate their knowledge for it. For example, suppose a participant correctly answered n out of q questions on a given quiz. If we hold out a single *correctly* answered question as the target, the proportion of remaining questions answered correctly would be $\frac{n-1}{q-1}$, whereas if we hold out

827 a single *incorrectly* answered question, the proportion of remaining questions answered correctly
828 would be $\frac{n}{q-1}$. Thus, the proportion of correctly answered remaining questions (and therefore the
829 null-hypothesized value of a knowledge estimate computed from them) is always *lower* for target
830 questions a participant answered correctly than for those they answered incorrectly.

831 To correct for this baseline difference under our null hypothesis, we used a rebalancing pro-
832 cedure that ensured our knowledge estimates for questions each participant answered correctly
833 and incorrectly were computed from the *same* proportion of correctly answered questions. For
834 each target question on a given participant’s quiz, we first identified all remaining questions with
835 the opposite “correctness” label (i.e., if the target question was answered correctly, we identified
836 all remaining incorrectly answered questions, and vice versa). We then held out each of these
837 opposite-label questions, in turn, along with the target question, and estimated the participant’s
838 knowledge for the target question using all *other* remaining questions. Since each of these subsets
839 of remaining questions was constructed by holding out one correctly answered question and one
840 incorrectly answered question from the participant’s quiz responses, if the participant correctly
841 answered n out of q questions total, then their proportion-correct score on each subset of questions
842 used to estimate their knowledge would be $\frac{n-1}{q-2}$, regardless of whether they answered the target
843 question correctly or incorrectly. Finally, we averaged over these per-subset knowledge estimates
844 to obtain a rebalanced estimate of the participant’s knowledge for the target question that lever-
845 aged information from all remaining questions’ embedding coordinates, but whose expected value
846 under our null hypothesis was the same as that of each individual subset ($\frac{n-1}{q-2}$). By equalizing the
847 null-hypothesized values of knowledge estimates for correctly and incorrectly answered ques-
848 tions, this procedure ensures that any meaningful relationships we observe between participants’
849 estimated knowledge for individual quiz questions and their abilities to correctly answer them
850 reflect the predictive power of the embedding-space distances we use to weight questions’ con-
851 tributions to the knowledge estimates, rather than an artifact of our testing procedure. Note that
852 if a participant answered all or no questions on a given quiz correctly, their responses contained
853 no opposite-label questions with which to perform this rebalancing, and we therefore excluded
854 their data from our analyses for that quiz. We used this rebalancing procedure when construct-

855 ing knowledge estimates for the “All questions” and “Within-lecture” versions of the analyses
856 shown in Figure 6, but not for the “Across-lecture” analyses as, in this case, the target questions
857 and the questions used to estimate participants’ knowledge for them were drawn from different
858 subsets of quiz questions (those about one lecture, and those about the other), and were therefore
859 independent.

860 In each version of this analysis (i.e., row in Fig. 6), and separately for each of the three quizzes
861 (i.e., column in Fig. 6), we then fit a generalized linear mixed model (GLMM) with a logistic link
862 function to the set of knowledge estimates for all questions (or all questions about a particular
863 lecture) that participants answered on the given quiz. We implemented these models in R using
864 the `lme4` package [3] and fit them following guidance from Bates et al. [2] and Matuschek et al.
865 [46]. Specifically, we initially fit each model with the maximal random effects structure afforded
866 by our design, which we identified as:

$$\text{accuracy} \sim \text{knowledge} + (\text{knowledge} | \text{participant}) + (\text{knowledge} | \text{question})$$

867 where “accuracy” is a binary value indicating whether each target question was answered cor-
868 rectly or incorrectly, “knowledge” is estimated knowledge at each target question’s embedding
869 coordinate, “participant” is a unique identifier assigned to each participant, and “question” is a
870 unique identifier assigned to each quiz question. For models we fit using knowledge estimates for
871 target questions about multiple content areas (i.e., in the “All questions” version of the analysis),
872 we also included an additional random effect term, $(\text{knowledge} | \text{lecture})$, where “lecture” is a
873 categorical value denoting whether the target question was about *Four Fundamental Forces*, *Birth*
874 *of Stars*, or general physics knowledge. Note that with our coding scheme, identifiers for each
875 question are implicitly nested within levels of `lecture` and so do not require explicit nesting in
876 our model formula. We then iteratively removed random effects from the maximal model until it
877 successfully converged with a full-rank random effects variance-covariance matrix. We obtained
878 the odds ratios reported in Figure 6 by exponentiating the estimated coefficient for “knowledge”
879 from each fitted model. Conceptually, these odds ratios represent how many times greater the odds

880 are that a given participant will answer a given question correctly if their estimated knowledge
881 for its embedding coordinate is 1, compared to if it is 0. We estimated 95% confidence intervals
882 for each odds ratio by generating 10,000 random subsamples (of full size, with replacement) from
883 the data used to fit each model, and refitting the models to each subsample to obtain bootstrap
884 distributions of 10,000 odds ratios.

885 To assess the predictive value of our knowledge estimates, we compared each GLMM's ability
886 to explain participants' success on individual quiz questions to that of an analogous model which
887 assumed (as we assume under our null hypothesis) that knowledge estimates for correctly and
888 incorrectly answered questions did *not* systematically differ, on average. Specifically, we used the
889 same sets of observations to which we fit each "full" model to fit a second "null" model with
890 the same random effects structure, but with the coefficient for the fixed effect of "knowledge" con-
891 strained to zero (i.e., we removed this term from the null model). We then compared each full model
892 to its reduced (null) equivalent using a likelihood-ratio test (LRT). Because the standard asymptotic
893 χ_d^2 approximation of the null distribution for the LRT statistic (λ_{LR}) can be anti-conservative for
894 finite sample sizes [25, 61, 66], we computed *p*-values for these tests using a parametric bootstrap
895 procedure [14, 27]. For each of 10,000 bootstraps, we used the fitted null model to simulate a
896 sample of observations of equal size to our original sample. We then re-fit both the null and full
897 models to this simulated sample and compared them via an LRT. This yielded a distribution of λ_{LR}
898 statistics we may expect to observe given data that conforms to our null hypothesis. We computed
899 a corrected *p*-value for our observed λ_{LR} as $\frac{r+1}{n+1}$, where r is the number of simulated model com-
900 parisons that yielded a λ_{LR} greater than our observed value and n is the number of simulations we
901 ran (10,000).

902 Estimating the "smoothness" of knowledge

903 In the analysis reported in Figure 7A, we show how participants' ability to correctly answer
904 quiz questions changes as a function of distance from a given correctly or incorrectly answered
905 reference question. We used a bootstrap-based approach to estimate the maximum distances over
906 which these proportions of correctly answered questions could be reliably distinguished from

907 participants' overall average proportion of correctly answered questions.

908 For each of 10,000 iterations, we drew a random subsample (with replacement) of 50 partic-
909 ipants from our dataset. Within each iteration, we first computed the 95% confidence interval
910 (CI) of the across-subsample-participants mean proportion correct on each of the three quizzes,
911 separately. To compute this interval for each quiz, we repeatedly (1,000 times) subsampled par-
912 ticipants (with replacement, from the outer subsample for the current iteration) and computed
913 the mean proportion correct of each of these inner subsamples. We then identified the 2.5th and
914 97.5th percentiles of the resulting distributions of 1,000 means. These three intervals (one for each
915 quiz) served as our thresholds for confidence that the proportion correct within a given distance
916 from a reference question was reliably different (at the $p < 0.05$ significance level) from the average
917 proportion correct across all questions on the given quiz.

918 Next, for each participant in the current subsample, and for each of the three quizzes they
919 completed (separately), we iteratively treated each of the 15 questions appearing on the given
920 quiz as the "reference" question. We constructed a series of concentric 15-dimensional "spheres"
921 centered on the reference question's embedding-space coordinate, where each successive sphere's
922 radius increased by 0.01 (correlation distance) between 0 and 2, inclusive (i.e., tiling the range
923 of possible correlation distances with 201 spheres in total). We then computed the proportion
924 of questions enclosed within each sphere that the participant answered correctly, and averaged
925 these per-radius proportion-correct scores across reference questions that were answered correctly,
926 and those that were answered incorrectly. This resulted in two number-of-spheres sequences of
927 proportion-correct scores for each subsample participant and quiz: one derived from correctly
928 answered reference questions, and one derived from incorrectly answered reference questions.

929 We computed the across-subsample-participants mean proportion correct for each radius value
930 (i.e., sphere) and "correctness" of reference question. This yielded two sequences of proportion-
931 correct scores for each quiz, analogous to the blue and red lines displayed in Figure 7A, but for
932 the present subsample. For each quiz, we then found the minimum distance from the reference
933 question (i.e., sphere radius) at which each of these two sequences of per-radius proportion-correct
934 scores intersected the 95% confidence interval for the overall proportion correct (i.e., analogous to

935 the black error bands in Fig. 7A).

936 This resulted in two “intersection” distances for each quiz (for correctly answered and incor-
937 rectly answered reference questions). Repeating this full process for each of the 10,000 bootstrap
938 iterations output two distributions of intersection distances for each of the three quizzes. The
939 means and 95% confidence intervals for these distributions are plotted in Figure 7B.

940 **Creating knowledge and learning map visualizations**

941 An important feature of our approach is that, given a trained text embedding model and partic-
942 ipants’ performance on each quiz question, we can estimate their knowledge about *any* content
943 expressible by the embedding model—not solely the content explicitly probed by the quiz ques-
944 tions, or even appearing in the lectures. To visualize these estimates (Fig. 8, Supp. Figs. 5, 6, 7, 8,
945 and 9), we used Uniform Manifold Approximation and Projection [UMAP; 47, 48] to construct a
946 2D projection of the text embedding space. Whereas our main analyses used a 15-topic embedding
947 space, we used a 100-topic embedding space for these visualizations. This change in the number
948 of topics overcame an undesirable behavior in the UMAP embedding procedure, whereby embed-
949 ding coordinates for the 15-topic model tended to be “clumped” into separated clusters, rather
950 than forming a smooth trajectory through the 2D space. When we increased the number of topics
951 to 100, the embedding coordinates in the 2D space formed a smooth trajectory through the space,
952 with substantially less clumping (Fig. 8). Creating a “map” by sampling this 100-dimensional
953 space at high resolution to obtain an adequate set of topic vectors spanning the embedding space
954 would be computationally intractable. However, sampling a 2D grid is trivial.

955 At a high level, the UMAP algorithm obtains low-dimensional embeddings by minimizing
956 the cross-entropy between the pairwise (clustered) distances between the observations in their
957 original (e.g., 100-dimensional) space and the pairwise (clustered) distances in the low-dimensional
958 embedding space (in our approach, the embedding space is 2D). In our implementation, pairwise
959 distances in the original high-dimensional space were defined as 1 minus the correlation between
960 each pair of coordinates, and pairwise distances in the low-dimensional embedding space were
961 defined as the Euclidean distance between each pair of coordinates.

962 In our application, all of the coordinates we embedded were topic vectors, whose elements
 963 are always non-negative and sum to one. Although UMAP is an invertible transformation at
 964 the embedding locations of the original data, other locations in the embedding space will not
 965 necessarily follow the same implicit “rules” as the original high-dimensional data. For example,
 966 inverting an arbitrary coordinate in the embedding space might result in negative-valued vectors,
 967 which are incompatible with the topic modeling framework. To protect against this issue, we
 968 log-transformed the topic vectors prior to embedding them in the 2D space. When we inverted
 969 the embedded vectors (e.g., to estimate topic vectors for word clouds, as in Fig. 8C), we passed
 970 the inverted (log-transformed) values through the exponential function to obtain a vector of non-
 971 negative values, and normalized them to sum to one.

972 After embedding both lectures’ topic trajectories and the topic vectors of every question, we
 973 defined a rectangle enclosing the 2D projections of the lectures’ and quizzes’ embeddings. We then
 974 sampled points from a regular 100×100 grid of coordinates that evenly tiled this enclosing rectangle.
 975 We sought to estimate participants’ knowledge (and learning, i.e., changes in knowledge) at each
 976 of the resulting 10,000 coordinates.

977 To generate our estimates, we placed a set of 39 radial basis functions (RBFs) throughout the
 978 embedding space, centered on the 2D projections for each question (i.e., we included one RBF for
 979 each question). At coordinate x , the value of an RBF centered on a question’s coordinate μ is given
 980 by:

$$\text{RBF}(x, \mu, \lambda) = \exp\left\{-\frac{\|x - \mu\|^2}{\lambda}\right\}. \quad (3)$$

981 The λ term in the RBF equation controls the “smoothness” of the function, where larger values
 982 of λ result in smoother maps. In our implementation we used $\lambda = 50$. Next, we estimated the
 983 “knowledge” at each coordinate, x , using:

$$\hat{k}(x) = \frac{\sum_{i \in \text{correct}} \text{RBF}(x, q_i, \lambda)}{\sum_{j=1}^N \text{RBF}(x, q_j, \lambda)}. \quad (4)$$

984 Equation 4 computes the weighted proportion of correctly answered questions, where the weights
 985 are given by how nearby (in the 2D space) each question is to the x . We also defined *learning maps*

986 as the coordinate-by-coordinate differences between any pair of knowledge maps. Intuitively,
987 learning maps reflect the *change* in knowledge across two maps.

988 **Author contributions**

989 Conceptualization: P.C.F., A.C.H., and J.R.M. Methodology: P.C.F., A.C.H., and J.R.M. Software:
990 P.C.F. Validation: P.C.F. Formal analysis: P.C.F. Resources: P.C.F., A.C.H., and J.R.M. Data curation:
991 P.C.F. Writing (original draft): J.R.M. Writing (review and editing): P.C.F., A.C.H., and J.R.M. Visu-
992 alization: P.C.F. and J.R.M. Supervision: J.R.M. Project administration: P.C.F. Funding acquisition:
993 J.R.M.

994 **Data availability**

995 All of the data analyzed in this manuscript may be found at [https://github.com/ContextLab/effic-
996 ient-learning-khan](https://github.com/ContextLab/efficient-learning-khan).

997 **Code availability**

998 All of the code for running our experiment and carrying out the analyses may be found at
999 <https://github.com/ContextLab/efficient-learning-khan>.

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1008 References

- 1009 [1] Ashby, F. G. and Maddox, W. T. (2005). Human category learning. *Annual Review of Psychology*,
1010 56:149–178.
- 1011 [2] Bates, D., Kliegl, R., Vasishth, S., and Baayen, H. (2015a). Parsimonious mixed models. *arXiv*,
1012 1506.04967.
- 1013 [3] Bates, D., Mächler, M., Bolker, B., and Walker, S. (2015b). Fitting linear mixed-effects models
1014 using lme4. *Journal of Statistical Software*, 67(1):1–48.
- 1015 [4] Bevilacqua, D., Davidesco, I., Wan, L., and Chaloner, K. (2019). Brain-to-brain synchrony and
1016 learning outcomes vary by student-teacher dynamics: evidence from a real-world classroom
1017 electroencephalography study. *Journal of Cognitive Neuroscience*, 31(3):401–411.
- 1018 [5] Bird, S., Klein, E., and Loper, E. (2009). *Nature language processing with Python: analyzing text
1019 with the natural language toolkit*. Reilly Media, Inc.
- 1020 [6] Blaye, A., Bernard-Peyron, V., Paour, J.-L., and Bonthoux, F. (2006). Category flexibility in chil-
1021 dren: distinguishing response flexibility from conceptual flexibility; the protracted development
1022 of taxonomic representations. *European Journal of Developmental Psychology*, 3(2):163–188.
- 1023 [7] Blei, D. M. and Lafferty, J. D. (2006). Dynamic topic models. In *Proceedings of the International
1024 Conference on Machine Learning*, pages 113–120, New York, NY. Association for Computing
1025 Machinery.
- 1026 [8] Blei, D. M., Ng, A. Y., and Jordan, M. I. (2003). Latent dirichlet allocation. *Journal of Machine
1027 Learning Research*, 3:993–1022.

- 1028 [9] Boyd-Graber, J., Mimno, D., and Newman, D. (2014). Care and feeding of topic models:
1029 problems, diagnostics, and improvements. In Airoldi, E. M., Blei, D. M., Erosheva, E. A., and
1030 Fienberg, S. E., editors, *Handbook of Mixed Membership Models and Their Applications*. CRC Press.
- 1031 [10] Brown, T. B., Mann, B., Ryder, N., Subbiah, M., Kaplan, J., Dhariwal, P., Neelakantan, A.,
1032 Shyam, P., Sastry, G., Askell, A., Agarwal, S., Herbert-Voss, A., Krueger, G., Henighan, T., Child,
1033 R., Ramesh, A., Ziegler, D. M., Wu, J., Winter, C., Hesse, C., Chen, M., Sigler, E., Litwin, M.,
1034 Gray, S., Chess, B., Clark, J., Berner, C., McCandlish, S., Radford, A., Sutskever, I., and Amodei,
1035 D. (2020). Language models are few-shot learners. *arXiv*, 2005.14165.
- 1036 [11] Caramazza, A. and Mahon, B. Z. (2003). The organization of conceptual knowledge: the
1037 evidence from category-specific semantic deficits. *Trends in Cognitive Sciences*, 7(8):354–361.
- 1038 [12] Cer, D., Yang, Y., Kong, S. Y., Hua, N., Limtiaco, N., John, R. S., Constant, N., Guajardo-
1039 Cespedes, M., Yuan, S., Tar, C., Sung, Y.-H., Strope, B., and Kurzweil, R. (2018). Universal
1040 sentence encoder. *arXiv*, 1803.11175.
- 1041 [13] Constantinescu, A. O., O'Reilly, J. X., and Behrens, T. E. J. (2016). Organizing conceptual
1042 knowledge in humans with a gridlike code. *Science*, 352(6292):1464–1468.
- 1043 [14] Davison, A. C. and Hinkley, D. V. (1997). *Bootstrap Methods and their Application*. Cambridge
1044 Series in Statistical and Probabilistic Mathematics. Cambridge University Press.
- 1045 [15] Deacon, D., Grose-Fifer, J., Yang, C. M., Stanick, V., Hewitt, S., and Dynowska, A. (2004).
1046 Evidence for a new conceptualization of semantic representation in the left and right cerebral
1047 hemispheres. *Cortex*, 40(3):467–478.
- 1048 [16] Deerwester, S., Dumais, S. T., Furnas, G. W., Landauer, T. K., and Harshman, R. (1990).
1049 Indexing by latent semantic analysis. *Journal of the American Society for Information Science*,
1050 41(6):391–407.
- 1051 [17] Depoix, J. (2018). YouTube transcript API. <https://github.com/jdepoix/youtube-transcript-api>.

- 1053 [18] Devlin, J., Chang, M.-W., Lee, K., and Toutanova, K. (2018). BERT: pre-training of deep
1054 bidirectional transformers for language understanding. *arXiv*, 1810.04805.
- 1055 [19] Dikker, S., Wan, L., Davidesco, I., Kaggen, L., Oostrik, M., McClintock, J., Rowland, J.,
1056 Michalareas, G., van Bavel, J. J., Ding, M., and Poeppel, D. (2017). Brain-to-brain synchrony
1057 tracks real-world dynamic group interactions in the classroom. *Current Biology*, 27(9):1375–1380.
- 1058 [20] Estes, W. K. (1986a). Array models for category learning. *Cognitive Psychology*, 18(4):500–549.
- 1059 [21] Estes, W. K. (1986b). Memory storage and retrieval processes in category learning. *Journal of*
1060 *Experimental Psychology: General*, 115:155–174.
- 1061 [22] Fisher, R. A. (1922). On the mathematical foundations of theoretical statistics. *Philosophical*
1062 *Transactions of the Royal Society A*, 222(602):309–368.
- 1063 [23] Gallagher, J. J. (2000). Teaching for understanding and application of science knowledge.
1064 *School Science and Mathematics*, 100(6):310–318.
- 1065 [24] Gluck, M. A., Shohamy, D., and Myers, C. E. (2002). How do people solve the “weather
1066 prediction” task? individual variability in strategies for probabilistic category learning. *Learning*
1067 *and Memory*, 9:408–418.
- 1068 [25] Goldman, N. and Whelan, S. (2000). Statistical Tests of Gamma-Distributed Rate Heterogeneity
1069 in Models of Sequence Evolution in Phylogenetics. *Molecular Biology and Evolution*, 17(6):975–978.
- 1070 [26] Goldstein, T. R. and Winner, E. (2012). Enhancing empathy and theory of mind. *Journal of*
1071 *Cognition and Development*, 13(1):19–37.
- 1072 [27] Halekoh, U. and Højsgaard, S. (2014). A Kenward-Roger Approximation and Parametric
1073 Bootstrap Methods for Tests in Linear Mixed Models – The R Package pbkrtest. *Journal of*
1074 *Statistical Software*, 59(9):1–32.
- 1075 [28] Hall, R. and Greeno, J. (2008). *21st century education: A reference handbook*, chapter Conceptual
1076 learning, pages 212–221. Sage Publications.

- 1077 [29] Heusser, A. C., Fitzpatrick, P. C., and Manning, J. R. (2021). Geometric models reveal behav-
1078 ioral and neural signatures of transforming experiences into memories. *Nature Human Behaviour*,
1079 5:905–919.
- 1080 [30] Huebner, P. A. and Willits, J. A. (2018). Structured semantic knowledge can emerge au-
1081 tomatically from predicting word sequences in child-directed speech. *Frontiers in Psychology*,
1082 9:doi.org/10.3389/fpsyg.2018.00133.
- 1083 [31] Hulbert, J. C. and Norman, K. A. (2015). Neural differentiation tracks improved recall of com-
1084 peting memories following interleaved study and retrieval practice. *Cerebral Cortex*, 25(10):3994–
1085 4008.
- 1086 [32] Kanske, P., Böckler, A., and Singer, T. (2015). Models, mechanisms and moderators dissociating
1087 empathy and theory of mind. In *Social Behavior From Rodents to Humans*, pages 193–206. Springer.
- 1088 [33] Katona, G. (1940). *Organizing and memorizing: studies in the psychology of learning and teaching*.
1089 Columbia University Press.
- 1090 [34] Kaufman, D. M. (2003). Applying educational theory in practice. *British Medical Journal*,
1091 326(7382):213–216.
- 1092 [35] Kawasaki, H., Yamasaki, S., Masuoka, Y., Iwasa, M., Fukita, S., and Matsuyama, R. (2021).
1093 Remote teaching due to COVID-19: an exploration of its effectiveness and issues. *International
1094 Journal of Environmental Research and Public Health*, 18(5):2672.
- 1095 [36] Khan, S. (2004). *The Khan Academy*. Salman Khan.
- 1096 [37] Kintsch (1970). *Learning, memory, and conceptual processes*. Wiley.
- 1097 [38] Kolowich, S. (2013). How EdX plans to earn, and share, revenue from its free online courses.
1098 *The Chronicle of Higher Education*, 21:1–5.
- 1099 [39] Landauer, T. K. and Dumais, S. T. (1997). A solution to Plato’s problem: the latent semantic
1100 analysis theory of acquisition, induction, and representation of knowledge. *Psychological Review*,
1101 104:211–240.

- 1102 [40] Lee, H. and Chen, J. (2022). Predicting memory from the network structure of naturalistic
1103 events. *Nature Communications*, 13(4235):doi.org/10.1038/s41467-022-31965-2.
- 1104 [41] Macellan, E. (2005). Conceptual learning: the priority for higher education. *British Journal of
1105 Educational Studies*, 53(2):129–147.
- 1106 [42] Manning, J. R. (2021). Episodic memory: mental time travel or a quantum “memory wave”
1107 function? *Psychological Review*, 128(4):711–725.
- 1108 [43] Manning, J. R. (2023). Context reinstatement. In Kahana, M. J. and Wagner, A. D., editors,
1109 *Handbook of Human Memory*. Oxford University Press.
- 1110 [44] Manning, J. R. and Kahana, M. J. (2012). Interpreting semantic clustering effects in free recall.
1111 *Memory*, 20(5):511–517.
- 1112 [45] Manning, J. R., Menjunatha, H., and Kording, K. (2023). Chatify: A Jupyter extension
1113 for adding LLM-driven chatbots to interactive notebooks. [https://github.com/ContextLab/
1114 chatify](https://github.com/ContextLab/chatify).
- 1115 [46] Matuschek, H., Kliegl, R., Vasishth, S., Baayen, H., and Bates, D. (2017). Balancing type i error
1116 and power in linear mixed models. *Journal of Memory and Language*, 94:305–315.
- 1117 [47] McInnes, L., Healy, J., and Melville, J. (2018a). UMAP: Uniform manifold approximation and
1118 projection for dimension reduction. *arXiv*, 1802(03426).
- 1119 [48] McInnes, L., Healy, J., Saul, N., and Großberger, L. (2018b). UMAP: Uniform Manifold
1120 Approximation and Projection. *Journal of Open Source Software*, 3(29):861.
- 1121 [49] Meltzoff, A. N. (2011). Social cognition and the origins of imitation, empathy, and theory of
1122 mind. In *The Wiley-Blackwell Handbook of Childhood Cognitive Development*. Wiley-Blackwell.
- 1123 [50] Meshulam, M., Hasenfratz, L., Hillman, H., Liu, Y. F., Nguyen, M., Norman, K. A., and Hasson,
1124 U. (2020). Neural alignment predicts learning outcomes in students taking an introduction to
1125 computer science course. *Nature Communications*, 12(1922):doi.org/10.1038/s41467-021-22202-3.

- 1126 [51] Mikolov, T., Chen, K., Corrado, G., and Dean, J. (2013). Efficient estimation of word representations in vector space. *arXiv*, 1301.3781.
- 1127
- 1128 [52] Moser, K. M., Wei, T., and Brenner, D. (2021). Remote teaching during COVID-19: implications from a national survey of language educators. *System*, 97:102431.
- 1129
- 1130 [53] Nastase, S. A., Goldstein, A., and Hasson, U. (2020). Keep it real: rethinking the primacy of experimental control in cognitive neuroscience. *NeuroImage*, 15(222):117254–117261.
- 1131
- 1132 [54] Nguyen, M., Chang, A., Micciche, E., Meshulam, M., Nastase, S. A., and Hasson, U. (2022). Teacher-student neural coupling during teaching and learning. *Social Cognitive and Affective Neuroscience*, 17(4):367–376.
- 1133
- 1134
- 1135 [55] OpenAI (2023). ChatGPT. <https://chat.openai.com>.
- 1136 [56] Piantadosi, S. T. and Hill, F. (2022). Meaning without reference in large language models. *arXiv*, 2208.02957.
- 1137
- 1138 [57] Poulsen, A. T., Kamronn, S., Dmochowski, J., Parra, L. C., and Hansen, L. K. (2017). EEG in the classroom: synchronised neural recordings during video presentation. *Scientific Reports*, 7:43916.
- 1139
- 1140
- 1141 [58] Ratka, A. (2018). Empathy and the development of affective skills. *American Journal of Pharmaceutical Education*, 82(10):doi.org/10.5688/ajpe7192.
- 1142
- 1143 [59] Reilly, D. L., Cooper, L. N., and Elbaum, C. (1982). A neural model for category learning. *Biological Cybernetics*, 45(1):35–41.
- 1144
- 1145 [60] Rhoads, R. A., Berdan, J., and Toven-Lindsey, B. (2013). The open courseware movement in higher education: unmasking power and raising questions about the movement’s democratic potential. *Educational Theory*, 63(1):87–110.
- 1146
- 1147
- 1148 [61] Scheipl, F., Greven, S., and Küchenhoff, H. (2008). Size and power of tests for a zero random effect variance or polynomial regression in additive and linear mixed models. *Computational Statistics & Data Analysis*, 52(7):3283–3299.
- 1149
- 1150

- 1151 [62] Scott, P., Asoko, H., and Leach, J. (2007). *Handbook of research on science education*, chapter
1152 Student conceptions and conceptual learning in science. Routledge.
- 1153 [63] Shao, Y. N., Sun, H. M., Huang, J. W., Li, M. L., Huang, R. R., and Li, N. (2018). Simulation-
1154 based empathy training improves the communication skills of neonatal nurses. *Clinical Simula-*
1155 *tion in Nursing*, 22:32–42.
- 1156 [64] Shim, T. E. and Lee, S. Y. (2020). College students' experience of emergency remote teaching
1157 during COVID-19. *Children and Youth Services Review*, 119:105578.
- 1158 [65] Simon, M. A., Tzur, R., Heinz, K., and Kinzel, M. (2004). Explicating a mechanism for
1159 conceptual learning: elaborating the construct of reflective abstraction. *Journal for Research in*
1160 *Mathematics Education*, 35(5):305–329.
- 1161 [66] Snijders, T. A. B. and Bosker, R. (2011). More powerful tests for variance parameters. In
1162 *Multilevel Analysis: An Introduction to Basic and Advanced Multilevel Modeling*, chapter 6, pages
1163 94–108. Sage Publications, 2nd edition.
- 1164 [67] Stepien, K. A. and Baernstein, A. (2006). Education for empathy. *Journal of General Internal*
1165 *Medicine*, 21:524–530.
- 1166 [68] Touvron, H., Lavril, T., Izacard, G., Martinet, X., Lachaux, M.-A., Lacroix, T., Rozière, B.,
1167 Goyal, N., Hambro, E., Azhar, F., Rodriguz, A., Joulin, A., Grave, E., and Lample, G. (2023).
1168 LLaMA: open and efficient foundation language models. *arXiv*, 2302.13971.
- 1169 [69] Tulchinskii, E., Kuznetsov, K., Kushnareva, L., Cherniavskii, D., Barannikov, S., Pio-
1170 ntkovskaya, I., Nikolenko, S., and Burnaev, E. (2023). Intrinsic dimension estimation for robust
1171 detection of AI-generated texts. *arXiv*, 2306.04723.
- 1172 [70] van Paridon, J., Liu, Q., and Lupyan, G. (2021). How do blind people know that blue is cold?
1173 distributional semantics encode color-adjective associations. *Proceedings of the Annual Meeting of*
1174 *the Cognitive Science Society*, 43(43).

- 1175 [71] Viswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A. N., Kaiser, L., and
1176 Polosukhin, I. (2017). Attention is all you need. In *Advances in Neural Information Processing*
1177 *Systems*.
- 1178 [72] Whalen, J. (2020). Should teachers be trained in emergency remote teaching? Lessons learned
1179 from the COVID-19 pandemic. *Journal of Technology and Teacher Education*, 28(2):189–199.
- 1180 [73] Young, J. R. (2012). Inside the Coursera contract: how an upstart company might profit from
1181 free courses. *The Chronicle of Higher Education*, 19(7):1–4.
- 1182 [74] Ziman, K., Heusser, A. C., Fitzpatrick, P. C., Field, C. E., and Manning, J. R. (2018). Is
1183 automatic speech-to-text transcription ready for use in psychological experiments? *Behavior*
1184 *Research Methods*, 50:2597–2605.