TRANSCENDENTAL EQUATIONS

1. BISECTION METHOD

AIM: TO WRITE A PROGRAM IN PYTHON TO DEMONSTARTE BISECTION METHOD

PROGRAM:

```
import math

# Evaluate the user-defined function safely
def f(x, func_str):
    try:
        return eval(func_str, {"x": x, "math": math, "_builtins_": None})
    except Exception as e:
        print("Error evaluating function:", e)
        return None


# Bisection Method
def bisection(func_str, a, b, tol):
    if f(a, func_str) * f(b, func_str) >= 0:
        print("Invalid interval. f(a) and f(b) must have opposite signs.")
        return


    print("Iter\t a\t b\t Xr\t f(Xr)")
    iter = 1
    while (b - a) / 2 > tol:
        Xr = (a + b) / 2
        fx = f(Xr, func_str)
        print(f"{iter}\t{a:.3f}\t{b:.3f}\t{Xr:.3f}\t{fx:.3f}")
        if abs(fx) < tol:
                break
        if f(a, func_str) * fx < 0:
            b = Xr
        else:
```

```
        a = Xr
      iter += 1

    print(f"\nApproximate root = {Xr:.3f} (correct to 3 decimal places)")

# === Main Program ===
print("=== Bisection Method ===")
func_str = input("Enter the function f(x): ")     # Example: x**3 - 4*x + 1
a = float(input("Enter the starting value a: "))  # Example: 0
b = float(input("Enter the ending value b: "))    # Example: 1
tol = 0.00003   # 3 decimal place accuracy

bisection(func_str, a, b, tol)
```

OUTPUT:

```
=== Bisection Method ===
Enter the function f(x): x*x*x -4*x +1
Enter the starting value a: 1
Enter the ending value b: 2

Iter      a        b        Xr       f(x)
  1     1.000    2.000    1.500    -1.625
  2     1.500    2.000    1.750    -0.641
  3     1.750    2.000    1.875    0.092
  4     1.750    1.875    1.812    -0.296
  5     1.812    1.875    1.844    -0.107
  6     1.844    1.875    1.859    -0.009
  7     1.859    1.875    1.867    0.041
  8     1.859    1.867    1.863    0.016
  9     1.859    1.863    1.861    0.003
 10     1.859    1.861    1.860    -0.003
 11     1.860    1.861    1.861    0.000
 12     1.860    1.861    1.861    -0.001
 13     1.861    1.861    1.861    -0.001
 14     1.861    1.861    1.861    -0.000
 15     1.861    1.861    1.861    0.000

Approximate root = 1.861 (correct to 3 decimal places)
```

CONCLUSION: The above program has been executed successfully.

2. REGULAR FALSI

Q. WRITE A PROGRAM IN PYTHON TO DEMONSTRATE REGULAR FALSI METHOD

AIM: TO WRITE A PROGRAM IN PYTHON TO DEMONSTRATE NEWTON RAPHSON METHOD

PROGRAM:

```
import math
# Evaluate the user-defined function safely
def safe_eval(expr, x):
  try:
    return eval(expr.strip(), {"x": x, "math": math, "m": math, "_builtins_": None})
  except (NameError, TypeError, ZeroDivisionError, SyntaxError) as e:
    print(f"Error evaluating function: {e}")
    return None

def Regula_Falsi(Func_str, a, b, tol):
  Fa = safe_eval(Func_str, a)
  Fb = safe_eval(Func_str, b)

  if Fa is None or Fb is None:
    return None

  if Fa * Fb >= 0:
    print("Invalid interval. F(a) and F(b) must have opposite signs.")
    return None

  print("\nIter.\t a\t\t b\t\t F(a)\t\t F(b)\t\t Xr\t\t F(Xr)")
    Xr_old = a # Initial guess to calculate error if needed
      for i in range(1, 101):
    # Regula Falsi Formula
```

```
        Xr = (a * Fb - b * Fa) / (Fb - Fa)
        FXr = safe_eval(Func_str, Xr)

        print(f"{i:<6}\t {a:.4f}\t {b:.4f}\t {Fa:.4f}\t {Fb:.4f}\t {Xr:.4f}\t {FXr:.4f}")

        if abs(FXr) < tol:
            return Xr

        if Fa * FXr < 0:
            b = Xr
            Fb = FXr
        else:
            a = Xr
            Fa = FXr

    print(f"\nRoot not found within 100 iterations (Current error: {abs(FXr):.6f})")
    return Xr

print("## Regula Falsi Method ##")
# Example: "x*x - 4*x - 4"
# Example: "m.cos(x) - x"
# Example: "x**3 - x - 1"
# Example: "x*x*x - 4*x - 4"
Func_str = input("Enter the function f(x): ")
a = float(input("Enter the starting value a: "))
b = float(input("Enter the starting value b:
 tol = float(input("Enter the tolerance value: "))
root = Regula_Falsi(Func_str, a, b, tol)
if root is not None:
    print(f"\nApproximate root = {root:.3f} (correct to 3 decimal places)")
```

OUTPUT:

```
=== Regula Falsi Method ===
Enter the function f(x): x*x*x -4*x +1
Enter the starting value a: 1
Enter the ending value b: 2

Iter       a        b        f(a)     f(b)      Xr      f(Xr)
  1      1.0000  2.0000  -2.0000 1.0000  1.6667  -1.0370
  2      1.6667  2.0000  -1.0370 1.0000  1.8364  -0.1528
  3      1.8364  2.0000  -0.1528 1.0000  1.8581  -0.0175
  4      1.8581  2.0000  -0.0175 1.0000  1.8605  -0.0020
  5      1.8605  2.0000  -0.0020 1.0000  1.8608  -0.0002
  6      1.8608  2.0000  -0.0002 1.0000  1.8608  -0.0000

Approximate root = 1.8608 (correct to 3 decimal places)
```

CONCLUSION: The above program has been executed successfully.

3. NEWTON'S RAPHSON METHOD

Q. WRITE A PROGRAM IN PYTHON TO DEMONSTRATE NEWTON RAPHSON METHOD

AIM: TO WRITE A PROGRAM IN PYTHON TO DEMONSTRATE NEWTON RAPHSON METHOD

PROGRAM:

```
Import math
# Safely evaluate the user-defined function
def safe_eval(expr, x):
  try:
    return eval(expr.strip(), {"x": x, "math": math, "_builtins_": None})
  except (NameError, TypeError, ZeroDivisionError, SyntaxError) as e:
    print(f"Error evaluating function: {e}")
    return None

# Safely evaluate the derivative of the function
def df(x, deriv_str):
  try:
    return eval(deriv_str.strip(), {"x": x, "math": math, "_builtins_": None})
  except Exception as e:
    print(f"Error evaluating derivative: {e}")
    return None

def Newton_Raphson_Method(func_str, deriv_str, x0, tol, max_iter=100):
  ai = x0

  print("\nIter.\t ai\t\t f(ai)\t\t df(ai)\t\t ai+1")
  for i in range(1, max_iter + 1):
    fai = safe_eval(func_str, ai)
    dfai = df(ai, deriv_str)
```

```
        if dfai == 0:
            print("Derivative is zero. Method fails.")
            return None

        # Newton-Raphson Formula
        ai_p1 = ai - fai / dfai

        print(f"{i:<6}\t {ai:.4f}\t {fai:.4f}\t {dfai:.4f}\t {ai_p1:.4f}")

        if abs(ai_p1 - ai) < tol:
            print(f"\nApproximate root = {ai_p1:.3f} (correct to 3 decimal places)")
            return ai_p1

        ai = ai_p1

    print(f"\nMaximum iterations reached without convergence.")
    return ai_p1

import math
print("## Newton-Raphson Method ##")

# Example 1: "x*x - 4*x - 4"
# Example 2: "m.cos(x) - x"
# Example 3: "x**3 - x - 1"
# Example of derivative: "3*x*x - 1" for f(x)=x**3 - x - 1
 func_str = input("Enter the function f(x): ")
deriv_str = input("Enter the derivative df(x): ")
x0 = float(input("Enter the initial guess x0: "))
tol = float(input("Enter the tolerance for X decimal place accuracy: "))
newton_raphson(func_str, deriv_str, x0, tol)
```

OUTPUT:

```
=== Newton-Raphson Method ===
Enter the function f(x): x*x*x -2*x -5
Enter the derivative f'(x): 3*x*x -2
Enter the initial guess x0: 2

================ Newton-Raphson Iteration Table ================
Iter  x0          f(x0)       f(x0)       x1
---------------------------------------------------------------
1     2.000000    -1.000000   10.000000   2.100000
2     2.100000    0.061000    11.230000   2.094568
3     2.094568    0.000186    11.161647   2.094551
===============================================================

Approximate root = 2.0946 (correct to 3 decimal places)
```

CONCLUSION:

The above program has been executed successfully.

INTERPOLATION

Q. WRITE A PROGRAM IN PYTHON TO DEMONSTRATE NEWTON FORWARD INTERPOLATION.

AIM: TO WRITE A PROGRAM IN PYTHON TO DEMONSTRATE NEWTON FORWARD INTERPOLATION.

PROGRAM:

```
def forward_difference_table(x, y):
  n = len(y)
  diff_table = [y.copy()] # First row is just y values

  # Generate the forward difference table
  for i in range(1, n):
    row = []
    for j in range(n - i):
      # Calculate the i-th difference: diff(j) = diff(j+1) - diff(j)
      value = diff_table[i-1][j+1] - diff_table[i-1][j]
      row.append(value)
    diff_table.append(row)

  return diff_table

def display_table(x, diff_table):
  n = len(x)
  print("\nForward Difference Table:")
  header = "i\t x\t\t y" + "\t\t Δy" * (n - 1)
  print(header)
  print("-" * len(header) * 2) # For visual separation

  for i in range(n):

row = [str(i), f"{x[i]:.2f}", f"{diff_table[0][i]:.2f}"]
    # Add the differences
    for j in range(1, n - i):
```

```
            row.append(f"{diff_table[j][i]:.2f}")
        print("\t".join(row))


def main():
    n = int(input("Enter the number of data points: ")
    x = []
    y = []
    print("Enter x values (equally spaced):")
    for i in range(n):
        x.append(float(input(f"x[{i}] = ")))
    print("Enter corresponding y values:")
    for i in range(n):
        y.append(float(input(f"y[{i}] = "))
    # Check equal spacing
    h_values = []
    for i in range(n - 1):
        h_values.append(x[i+1] - x[i])
    # Check if all h values are approximately equal
    if not all(abs(h_values[i] - h_values[0]) < 1e-5 for i in range(n - 1)):
        print("\nError: X values are not equally spaced.")
        return
    diff_table = forward_difference_table(x, y)
    display_table(x, diff_table
if _name_ == "_main_":
    main()
```

OUTPUT:

Forward Difference Table:

x	Δ^0y	Δ^1y	Δ^2y	Δ^3y	Δ^4y	Δ^5y	Δ^6y	Δ^7y	Δ^8y
-1.00	-13.00	6.00	0.00	6.00	0.00	0.00	0.00	0.00	0.00
0.00	-7.00	6.00	6.00	6.00	0.00	0.00	0.00	0.00	
1.00	-1.00	12.00	12.00	6.00	0.00	0.00	0.00		
2.00	11.00	24.00	18.00	6.00	0.00	0.00			
3.00	35.00	42.00	24.00	6.00	0.00				
4.00	77.00	66.00	30.00	6.00					
5.00	143.00	96.00	36.00						
6.00	239.00	132.00							
7.00	371.00								

CONCLUSION:

THE ABOVE PROGRAM HAS BEEN EXECUTED SUCCESSFULLY

Q. WRITE A PROGRAM IN PYTHON TO DEMONSTRATE NEWTON BACKWARD INTERPOLATION.

AIM: TO WRITE A PROGRAM IN PYTHON TO DEMONSTRATE NEWTON BACKWARD INTERPOLATION.

PROGRAM:

```
import math


x = [0, 30, 60, 90]

y = [1, 0.85, 0.5, 0]

xp = 70


h = x[1] - x[0]

p = (xp - x[-1]) / h


dy1_3 = y[3] - y[2]

dy1_2 = y[2] - y[1]

dy1_1 = y[1] - y[0]


d2y2 = dy1_3 - dy1_2

d2y1 = dy1_2 - dy1_1


d3y1 = d2y2 - d2y1


print("x\t y\t ∇y\t ∇²y\t ∇³y")

print(f"{x[0]}\t {y[0]}")

print(f"{x[1]}\t {y[1]}\t {dy1_1:.4f}")

print(f"{x[2]}\t {y[2]}\t {dy1_2:.4f}\t {d2y1:.4f}")

print(f"{x[3]}\t {y[3]}\t {dy1_3:.4f}\t {d2y2:.4f}\t {d3y1:.4f}")
```

```
yp = (y[-1]
     + p * dy1_3
     + (p * (p + 1) / math.factorial(2)) * d2y2
     + (p * (p + 1) * (p + 2) / math.factorial(3)) * d3y1)

print(f"\nEstimated cos(70°) using Backward formula = {yp:.5f}")
```

OUTPUT:

```
============================================================
x         y        ∇y       ∇2y      ∇3y
0         1
30        0.85     -0.1500
60        0.5      -0.3500 -0.2000
90        0        -0.5000 -0.1500 0.0500

Estimated cos(70°) using Backward formula = 0.34753
```


CONCLUSION:

THE ABOVE PROGRAM HAS BEEN EXECUTED SUCCESSFULLY.

CURVE FITTING

1. STRAIGHT LINE

PROGRAM:

```
# Curve_fit_no_numpy.py
# Least-squares straight-line fit (no numpy, no pandas)

from typing import List, Optional, Tuple

def fit_line(x_values: List[float], y_values: List[float]) -> Tuple[float, float]:
    """Return (a0, a1) for best fit line y = a0 + a1*x using least squares."""
    if len(x_values) != len(y_values) or len(x_values) == 0:
        raise ValueError("x_values and y_values must have same non-zero length.")
    n = len(x_values)
    sum_x = sum(x_values)
    sum_y = sum(y_values)
    sum_x2 = sum(x * x for x in x_values)
    sum_xy = sum(x * y for x, y in zip(x_values, y_values))

    denom = n * sum_x2 - sum_x * sum_x
    if abs(denom) < 1e-12:
        raise ValueError("Denominator nearly zero: can't compute unique fit (collinear x?).")

    a1 = (n * sum_xy - sum_x * sum_y) / denom
    a0 = (sum_y - a1 * sum_x) / n
    return a0, a1
def print_table(x_values: List[float], y_values: List[float]) -> None:
    """Print table of x, y, x^2, x*y and the sums."""
    n = len(x_values)
    rows = []
    for x, y in zip(x_values, y_values):
        rows.append((x, y, x*x, x*y))

    # Column widths
    w = [8, 8, 10, 10]
    header = f"{'i':<{w[0]}} {'y':<{w[1]}} {'x^2':<{w[2]}} {'x*y':<{w[3]}}"
    print(header)
    print("*" * (sum(w) + 3))

    for r in rows:
        print(f"{r[0]:<{w[0]}.4g} {r[1]:<{w[1]}.4g} {r[2]:<{w[2]}.4g} {r[3]:<{w[3]}.4g}")
```

```
    sum_x = sum(r[0] for r in rows)
    sum_y = sum(r[1] for r in rows)
    sum_x2 = sum(r[2] for r in rows)
    sum_xy = sum(r[3] for r in rows)

    # print the sums
    print("-" * (sum(w) + 3))
    print(f"{'SUM':<{w[0]}} {sum_y:>{w[1]}.4g} {sum_x2:>{w[2]}.4g} {sum_xy:>{w[3]}.4g}")
    print()

    # The image shows extra print statements for the sums:
    print(f"{'SUM_X':<{w[0]}} {sum_y:>{w[1]}.4g} {sum_x2:>{w[2]}.4g} {sum_xy:>{w[3]}.4g}")
    print() # extra line break from image 1000040410.jpg

    print(f"Σx = {sum_x:.4g}, Σy = {sum_y:.4g}, Σx^2 = {sum_x2:.4g}, Σxy = {sum_xy:.4g}")
    print()


def predict(a0: float, a1: float, x: float) -> float:
    return a0 + a1 * x
def interactive():
    print("Curve fitting (straight line) - enter data points.")
    n = int(input("How many points? "))
    x_values = []
    y_values = []

    for i in range(n):
        raw = input(f"Point {i+1} as 'x y' (e.g. 2 5): ").strip().split()
        if len(raw) < 2:
            print("Invalid input, try again.")
            return
        x_values.append(float(raw[0]))
        y_values.append(float(raw[1]))

    print()
    print_table(x_values, y_values)
    a0, a1 = fit_line(x_values, y_values)

    print(f"Best fit line: y = ({a0:.6f}) + ({a1:.6f}) x")

    choice = input("Predict y for some x? (y/n): ").strip().lower()
    if choice and choice[0] == 'y':
```

```
    xv = float(input("Enter x: "))
   print(f"Predicted y = {predict(a0, a1, xv):.6f}")
 if _name_ == "_main_":
    # Example usage (change values directly if you prefer):
    x_values = [0, 2, 5, 7]
    y_values = [-1, 5, 12, 20]

    # Print table and compute
    print_table(x_values, y_values)
    a0, a1 = fit_line(x_values, y_values)

    print(f"Best fit line: y = ({a0:.6f}) + ({a1:.6f}) x")
    print(f"For x=0, predicted y = {predict(a0, a1, 0):.6f}")
```

OUTPUT:

```
        x          y          x^2         x*y
----------------------------------------------
        0         -1            0           0
        2          5            4          10
        5         12           25          60
        7         20           49         140
----------------------------------------------
        Σ         36           78         210

Σx = 14, Σy = 36, Σx^2 = 78, Σxy = 210

Best fit line: y = -1.137931 + 2.896552 x
For x=8, predicted y = 22.034483
```


CONCLUSION: THE ABOVE PROGRAM HAS BEEN EXECUTED SUCCESSFULLY.

2. DEGREE POLYNOMIAL

PROGRAM:

```
# quad_fit_no_numpy.py
# Fit quadratic y = a0 + a1*x + a2*x^2 using normal equations (no numpy, no pandas)

from typing import List, Tuple

def build_sums(x_values: List[float], y_values: List[float]) -> dict:
    """Calculates the necessary sums for the normal equations."""
    s = {
        'n': 0.0,
        'sx': 0.0,
        'sx2': 0.0,
        'sx3': 0.0,
        'sx4': 0.0,
        'sy': 0.0,
        'sxy': 0.0,
        'sx2y': 0.0
    }

    for x, y in zip(x_values, y_values):
        s['n'] += 1
        s['sx'] += x
        s['sx2'] += x**2
        s['sx3'] += x**3
        s['sx4'] += x**4
        s['sy'] += y
        s['sxy'] += x * y
```

```
s['sx2y'] += (x**2) * y
return s
def print_table_and_sums(x_values: List[float], y_values: List[float]) -> None:
    """Prints the data points and the calculated sums in a formatted table."""

    # Header
    print(f"{'x':>8}{'y':>10}{'x^2':>12}{'x^3':>12}{'x^4':>12}{'x*y':>12}{'x^2*y':>12}")
    print("-" * 78) # Separator

    # Data rows
    for x, y in zip(x_values, y_values):
        print(f"{x:8.4g}{y:10.4g}{x*2:12.4g}{x3:12.4g}{x4:12.4g}{x*y:12.4g}{(x*2)*y:12.4g}")

    # Sums
    s = build_sums(x_values, y_values)
    print("-" * 78) # Separator
    print(f"n = {s['n']:3.4g}, sx = {s['sx']:12.4g}, sx2 = {s['sx2']:12.4g}, sx3 = {s['sx3']:12.4g}, sx4 = {s['sx4']:12.4g}")
    print(f"sy = {s['sy']:12.4g}, sxy = {s['sxy']:12.4g}, sx2y = {s['sx2y']:12.4g}")
    print()

def solve_3x3(A: List[List[float]], b: List[float]) -> List[float]:
    """
    Simple Gaussian elimination (in-place) to solve Ax = b for a 3x3 A.
    Returns the solution vector x.
    """
    # Make copies
```

```
M = [row[:] for row in A]
 rhs = b[:]
  n = 3

  # Forward elimination
  for k in range(n):
    # find pivot
    pivot = M[k][k]

    # Check for singularity/pivot too small (1e-14 is a common threshold)
    if abs(pivot) < 1e-14:
      # try to swap with a lower row
      for i in range(k + 1, n):
        if abs(M[i][k]) > 1e-14:
          M[k], M[i] = M[i], M[k]
          rhs[k], rhs[i] = rhs[i], rhs[k]
          pivot = M[k][k]
          break

    if abs(pivot) < 1e-14:
      raise ValueError("Singular matrix in solve_3x3")

    # normalize row k
    for j in range(k, n):
      M[k][j] /= pivot
    rhs[k] /= pivot

    # eliminate
    for i in range(k + 1, n):
```

```
            factor = M[i][k]
        for j in range(k, n):
                M[i][j] -= factor * M[k][j]
            rhs[i] -= factor * rhs[k]

    # Back substitution
    x = [0.0] * n
    for i in range(n - 1, -1, -1):
        val = rhs[i]
        for j in range(i + 1, n):
            val -= M[i][j] * x[j]

        # The diagonal element M[i][i] should be 1.0 from normalization,
        # but we check for singularity one last time just in case.
        x[i] = val / M[i][i] if abs(M[i][i]) > 1e-14 else val

    return x

def fit_quadratic(x_values: List[float], y_values: List[float]) -> Tuple[float, float, float]:
    """Calculates the coefficients (a0, a1, a2) for the least-squares quadratic fit."""

    if len(x_values) != len(y_values) or len(x_values) == 0:
        raise ValueError("X-values and Y-values must have same non-zero length.")

    s = build_sums(x_values, y_values)

    # Normal equations matrix for [a0, a1, a2]
    # [ n    Σx   Σx^2 ] [a0] = [ Σy   ]
    # [ Σx   Σx^2 Σx^3 ] [a1] = [ Σxy  ]
```

```
    # [ Σx  ^2 Σx^3 Σx^4 ] [a2] = [ Σxy  ]
 A = [
       [s['n'],   s['sx'],  s['sx2']],
       [s['sx'],  s['sx2'], s['sx3']],
       [s['sx2'], s['sx3'], s['sx4']]
    ]

    b = [s['sy'], s['sxy'], s['sx2y']]

    # Solve for a0, a1, a2
    a0, a1, a2 = solve_3x3(A, b)

    return a0, a1, a2

def predict(a0: float, a1: float, a2: float, x: float) -> float:
    """Calculates the predicted y value for a given x using the fitted quadratic."""
    return a0 + a1*x + a2*(x**2)

if _name_ == "_main_":
    # Example points from your notebook: (0, 1), (1, 6), (2, 17)
    x_values = [0.0, 1.0, 2.0]
    y_values = [1.0, 6.0, 17.0]

    print("### Input Data and Sums ###")
    print_table_and_sums(x_values, y_values)

    # Fit quadratic
    a0, a1, a2 = fit_quadratic(x_values, y_values)
```

```
    print("### Fitting Results ###")
print(f"Fitted quadratic: y = {a0:.6f} + {a1:.6f} x + {a2:.6f} x^2")


    # Predictions requested in the notebook
    print("\n### Predictions ###")


    # Prediction for x=1.6
    print(f"y(1.6) = {predict(a0, a1, a2, 1.6):.6f}")


    # Prediction for x=3.0
    print(f"y(3)   = {predict(a0, a1, a2, 3.0):.6f}")
```

OUTPUT:

```
       x         y         x^2         x^3         x^4         x*y       x^2*y
--------------------------------------------------------------------------------
       0         1           0           0           0           0           0
       1         6           1           1           1           6           6
       2        17           4           8          16          34          68
--------------------------------------------------------------------------------
       Σ        24           5           9          17          40          74

Σx = 3.0, Σy = 24.0, Σx^2 = 5.0, Σx^3 = 9.0, Σx^4 = 17.0
Σ(xy) = 40.0, Σ(x^2 y) = 74.0

Fitted quadratic: y = 1.000000 + 2.000000 x + 3.000000 x^2
y(1.6) = 11.880000
y(3)   = 34.000000
```


CONCLUSION:

THE ABOVE PROGRAM HAS BEEN EXECUTED SUCCESSFULLY.

SOLUTION OF SIMULTANEOUS ALGEBRAIC EQUATIONS

GUASSIAN ELIMINATION METHOD

AIM: TO WRITE A PROGRAM IN PYTHON TO DEMONSTRATE GUASSIAN ELIMINATION METHOD

PROGRAM:

```
#Gaussian elimination with partial pivoting row operations
# Solve the system:
# x1 + 10 x2 - x3 = 3
# 2x1 + 3 x2 + 20 x3 = 7
# 10x1 - x2 + 2 x3 = 4
# Augmented matrix (each row: [a11, a12, a13, b])
A = [
   [1.0, 10.0, -1.0, 3.0],
   [2.0, 3.0, 20.0, 7.0],
   [10.0, -1.0, 2.0, 4.0]
]
n = len(A) # Number of equations/variables (n=3)

def print_matrix(M: List[List[float]], msg=None) -> None:
   """Prints the augmented matrix with 10.6f formatting."""
   if msg:
      print(msg)
   for r in M:
      # Join values with spaces, formatting each to 10 characters with 6 decimal places
      print("[" + " ".join(f"{val:10.6f}" for val in r) + "]")
   print()
```

```
def swap_rows(M: List[List[float]], i: int, j: int) -> None:
"""Swaps row i and row j in matrix M and prints the operation."""
   M[i], M[j] = M[j], M[i]
   # Print R(i+1) <-> R(j+1) to use 1-based indexing for output
   print(f"R({i+1}) <-> R({j+1})")
   print_matrix(M)


def scale_and_add(M: List[List[float]], col: int, factor: float, row: int) -> None:
   """
   Performs R_dest = R_dest - k * R_src.
   In the context of elimination, row is dest (row to eliminate in), col is src.
   """
   n_cols = len(M[0])

   # Print R(dest+1) <- R(dest+1) - (k) * R(src+1) to use 1-based indexing
   print(f"R({row+1}) <- R({row+1}) - ({factor:.6f})*R({col+1})")

   # Perform the operation: M[row] = M[row] - factor * M[col]
   for c in range(n_cols):
      M[row][c] = M[row][c] - factor * M[col][c]

   print_matrix(M)

# --- Main Solution Logic ---

# Work on a copy of the augmented matrix
M = deepcopy(A)
print_matrix(M, "Initial augmented matrix [A | b]:")
```

```
for col in range(n):
# Partial pivot: find row with max abs value in column 'col' from rows col..n-1
 # max() returns the row index 'r'
   pivot_row = max(range(col, n), key=lambda r: abs(M[r][col]))

   if pivot_row != col:
      swap_rows(M, pivot_row, col)

   pivot = M[col][col]

   if abs(pivot) < 1e-12: # Check for near-zero pivot (singularity)
      raise ValueError("Zero pivot encountered")
   # Eliminate below
   for row in range(col + 1, n):
      factor = M[row][col] / pivot
      # scale_and_add(Matrix, source_row, factor, destination_row)
      scale_and_add(M, col, factor, row)
print("Upper-triangular matrix after forward elimination:")
print_matrix(M)

# Back substitution
x = [0.0] * n # Solution vector [x1, x2, x3]

# Loop backward from the last row (n-1) to the first row (0)
for i in range(n - 1, -1, -1):

 # s is the RHS (augmented column), which is M[i][n]
   s = M[i][n]
# Subtract known x[j]'s multiplied by their coefficients M[i][j]
   for j in range(i + 1, n)
      s -= M[i][j] * x[j]
```

```
  # Solve for x[i]
   x[i] = s / M[i][i]
# Print Solution vector
print("Solution vector:")
# Enumerate x starting from 1 for x1, x2, x3 display
for i, xi in enumerate(x, 1):
   print(f"x{i} = {xi:.8f}")
```

OUTPUT:

```
Initial augmented matrix [A | b]:
  [  1.000000   10.000000   -1.000000    3.000000]
  [  2.000000    3.000000   20.000000    7.000000]
  [ 10.000000   -1.000000    2.000000    4.000000]

R3 <-> R1
  [ 10.000000   -1.000000    2.000000    4.000000]
  [  2.000000    3.000000   20.000000    7.000000]
  [  1.000000   10.000000   -1.000000    3.000000]

R2 = R2 - (0.200000)*R1
  [ 10.000000   -1.000000    2.000000    4.000000]
  [  0.000000    3.200000   19.600000    6.200000]
  [  1.000000   10.000000   -1.000000    3.000000]

R3 = R3 - (0.100000)*R1
  [ 10.000000   -1.000000    2.000000    4.000000]
  [  0.000000    3.200000   19.600000    6.200000]
  [  0.000000   10.100000   -1.200000    2.600000]

R3 <-> R2
  [ 10.000000   -1.000000    2.000000    4.000000]
  [  0.000000   10.100000   -1.200000    2.600000]
  [  0.000000    3.200000   19.600000    6.200000]

R3 = R3 - (0.316832)*R2
  [ 10.000000   -1.000000    2.000000    4.000000]
  [  0.000000   10.100000   -1.200000    2.600000]
  [  0.000000    0.000000   19.980198    5.376238]

Upper-triangular matrix after forward elimination:
  [ 10.000000   -1.000000    2.000000    4.000000]
  [  0.000000   10.100000   -1.200000    2.600000]
  [  0.000000    0.000000   19.980198    5.376238]

Solution vector:
 x1 = 0.37512389
 x2 = 0.28939544
 x3 = 0.26907830
```


CONCLUSION:

THE ABOVE PROGRAM HAS BEEN EXECUTED SUCCESSFULLY.

NUMERIACAL SOLUTIONS OF FIRST AND SECOND ORDER DIFFERENTIAL EQUATIONS

1. TAYLOR SERIES

AIM: TO WRITE A PROGRAM IN PYTHONTO DEMONSTRATE TAYLOR SERIES

PROGRAM:

```
# Taylor_ode_no_sympy.py
# Compute y(n0 + h) using Taylor series for ODE dy/dn = n - y^2, y(n0)=y0
# No external libraries beyond 'math'

from math import factorial

def compute_derivatives_at(n0: float, y0: float) -> List[float]:
    """
    Compute derivatives y', y'', y''', y^(4), y^(5) at (n0, y0)
    using formulas obtained by differentiating dy/dn = n - y^2.
    Returns list [y0, y1, y2, y3, y4, y5], where yk is the kth derivative.
    """

    # y^(0) = y0
    y_0 = y0

    # y' = n - y^2
    y_1 = n0 - (y_0 ** 2)

    # Second derivative: y'' = d/dn(n - y^2) = 1 - 2*y*(dy/dn) = 1 - 2*y*y'
    y_2 = 1.0 - 2.0 * y_0 * y_1
    # Third derivative: y''' = d/dn(1 - 2*y*y') = 0 - 2 * [ y'*y' + y*y'' ]
    # y''' = -2*y'^2 - 2*y*y''
    y_3 = -2.0 * (y_1 ** 2) - 2.0 * y_0 * y_2
        # Fourth derivative: y^(4) = d/dn(-2*y'^2 - 2*y*y'')
```

```
# y^(4) = -2*(2*y'y'') - 2[ y'*y'' + y*y''' ]
    # y^(4) = -4*y'*y'' - 2*y'*y'' - 2*y*y''' = -6*y'*y'' - 2*y*y'''
    y_4 = -2.0 * y_0 * y_3 - 6.0 * y_1 * y_2

    # Fifth derivative: y^(5) = d/dn(-6*y'*y'' - 2*y*y''')
    # y^(5) = -6*[ y''y'' + y'*y''' ] - 2[ y'*y''' + y*y^(4) ]
    # y^(5) = -6*y''^2 - 6*y'*y''' - 2*y'*y''' - 2*y*y^(4)
    # y^(5) = -2*y*y^(4) - 8*y'*y''' - 6*y''^2
    y_5 = -2.0 * y_0 * y_4 - 8.0 * y_1 * y_3 - 6.0 * (y_2 ** 2)

    return [y_0, y_1, y_2, y_3, y_4, y_5]

def taylor_at(n0: float, y0: float, h: float, order: int = 5) -> Tuple[float, List[float]]:
    """
    Evaluate Taylor polynomial of given order (<=5) for y at n0+h.
    Returns (approx_value, derivatives_list).
    """

    if order > 5:
        raise ValueError("This implementation supports up to 5th derivative (order<=5).")

    derivs = compute_derivatives_at(n0, y0)

    # Build Taylor sum: y(n0+h) approx y(n0) + h*y'(n0)/1! + h^2*y''(n0)/2! + ...

    taylor_sum = 0.0
    for k in range(order + 1):
# Term = y^(k) * h^k / k!
  taylor_sum += derivs[k] * (h ** k) / factorial(k)
 return taylor_sum, derivs
```

```
if _name_ == "_main_":
    # Initial point and step
    n0 = 0.0
    y0 = 1.0
    h = 0.1
    order = 5 # use terms up to y^(5)/5!
    # Compute the approximation and the derivatives
    approx, derivs = taylor_at(n0, y0, h, order)
    print("Derivatives at n0 = {:.4g}, y0 = {:.4g}:".format(n0, y0))
    print(f"y'(0)    = {derivs[1]:.6g}")
    print(f"y''(0)   = {derivs[2]:.6g}")
    print(f"y'''(0)  = {derivs[3]:.6g}")
    print(f"y^(4)(0) = {derivs[4]:.6g}")
    print(f"y^(5)(0) = {derivs[5]:.6g}")
    print()
    print("Taylor approximation up to order {}:".format(order))
    print(f"y({n0 + h:.4g}) ≈ {approx:.10f}")
    print(f"Rounded to 4 decimal places: {approx:.4f}")
```


OUTPUT:

```
======================================================================
Derivatives at n0 = 0, y0 = 1:
 y(0)     = 1
 y'(0)    = -1
 y''(0)   = 3
 y'''(0)  = -8
 y^(4)(0)= 34
 y^(5)(0)= -186

Taylor approximation up to order 5:
 y(0.1) ≈ 0.9137928333
 Rounded to 4 decimal places: 0.9138
```

CONCLUSION: THE ABOVE PROGRAM HAS BEEN EXECUTED SUCCESSFULLY.

2. EULER'S METHOD

AIM: TO WRITE A PROGRAM IN PYTHON TO DEMONSTARTE EULER'S METHOD.

PROGRAM:

```
# Euler_method.py

# Solve dy/dx = -y with y(0)=1 using Euler's method


def f(x, y):
    """The ODE: dy/dx = -y"""
    return -y


def euler(x0, y0, h, x_target):
    """Euler's method to approximate y(x_target)"""
    steps = int((x_target - x0) / h)
    x = x0
    y = y0

    print("Step |   x    |    y   ")
    print("-----|--------|---------")
    print(f"  0  | {x:.2f} | {y:.6f}")

    for i in range(1, steps + 1):
        # Euler formula: y(i+1) = y(i) + h * f(x(i), y(i))
        y = y + h * f(x, y)
        x = x + h
print(f"{i:3d}  | {x:.2f} | {y:.6f}"
    return y
```

```
if _name_ == "_main_":
    # initial values
    x0 = 0.0
    y0 = 1.0
    h = 0.01
    x_target = 0.04

    result = euler(x0, y0, h, x_target)

    print(f"\nApproximate value at x={x_target:.2f}:", round(result, 6))
```


OUTPUT:

```
==================================================
Step |   x    |     y
------------------------
  0  | 0.00 | 1.000000
  1  | 0.01 | 0.990000
  2  | 0.02 | 0.980100
  3  | 0.03 | 0.970299
  4  | 0.04 | 0.960596

Approximate value at x=0.04: 0.960596
```

CONCLUSION: THE ABOVE PRAOGRAM HAS BEEN EXECUTED SIUCCESSFULLY.

3.MODIFIED EULER'S METHOD

AIM: TO WRITE A PROGRAM IN PYTHON TO DEMONSTRATE MODIFIED EULER'S METHOD

PROGRAM:

```
# Modified_Euler's_Method.py
# Equation: dy/dx = x^2 + y, y(0) = 1
# Find y(0.2) with step size h = 0.02

def f(x, y):
   """The ODE: dy/dx = x^2 + y"""
   return x**2 + y

# Initial conditions
x0 = 0
y0 = 1
h = 0.02
x_end = 0.2
n = int((x_end - x0) / h) # Number of steps: (0.2 - 0) / 0.02 = 10

# Table header
print("-------------------------------------------------------------------------------------------")
print("i |    x(i)    |    y(i)    |   f(x(i),y(i)) (f1)   |   y'(Pred) |   f(x+h, y') (f2) |   y(i+1)")
print("-------------------------------------------------------------------------------------------")

# Print initial condition (Step 0)
print(f"{0:<2d} | {x0:10.6f} | {y0:10.6f} | {'':20} | {'':10} | {'':20} | {'':10}")
# Iterative Modified Euler Calculation
     # 1. Predictor (Standard Euler): y* = y_i + h * f(x_i, y_i)
   f1 = f(x0, y0)
  y_pred = y0 + h * f1
```

```
    # 2. Corrector (Heun's Formula): y_i+1 = y_i + (h / 2) * [ f(x_i, y_i) + f(x_i+1, y*) ]
    x_next = x0 + h
    f2 = f(x_next, y_pred)
    y_next = y0 + (h / 2) * (f1 + f2)

    # Print intermediate results for the current step (i+1)
    print(f"{i+1:<2d} | {x_next:10.6f} | {y0:10.6f} | {f1:20.6f} | {y_pred:10.6f} | {f2:20.6f} | {y_next:10.6f}")

    # Update for next iteration
    x0 = x_next
    y0 = y_next

print("-----------------------------------------------------------------------------------------")
print(f"h = {h:.2f}, Number of steps = {n}")
print(f"Formula used:")
print(f"y*(i+1) = y_i + h * f(x_i, y_i) (Euler Predictor)")
print(f"y(i+1)  = y_i + (h/2) * [ f(x_i, y_i) + f(x_i + h, y*(i+1)) ] (Heun Corrector)")
print(f"Approximate value of y({x_end:.1f}) = {y0:.6f}")
print("-----------------------------------------------------------------------------------------")
```


OUTPUT:

```
-------------------------------------------------------------------
  i     x(i)          y(i)          f(x(i),y(i))     y* (Pred)      f(x+h, y*)      y(i+1)
-------------------------------------------------------------------
  0      0.0000        1.000000        1.000000        1.020000        1.020400        1.020204
  1      0.0200        1.020204        1.020604        1.040616        1.042216        1.040832
  2      0.0400        1.040832        1.042432        1.061681        1.065281        1.061909
  3      0.0600        1.061909        1.065509        1.083220        1.089620        1.083461
  4      0.0800        1.083461        1.089861        1.105258        1.115258        1.105512
  5      0.1000        1.105512        1.115512        1.127822        1.142222        1.128089
  6      0.1200        1.128089        1.142489        1.150939        1.170539        1.151219
  7      0.1400        1.151219        1.170819        1.174636        1.200236        1.174930
  8      0.1600        1.174930        1.200530        1.198941        1.231341        1.199249
  9      0.1800        1.199249        1.231649        1.223882        1.263882        1.224204
-------------------------------------------------------------------
h = 0.02, Number of steps = 10
Formula used:
y_(i+1) = y_i + (h/2) * [f(x_i, y_i) + f(x_i + h, y*)]
-------------------------------------------------------------------
Approximate value of y(0.2) = 1.224204
-------------------------------------------------------------------
```

CONCLUSION: THE ABOVE PROGRAM HAS BEEN EXECUTED SUCCESSFULLY.

4. RUNGE-KUTTA 4th ORDER METHOD

AIM: AIM: TO WRITE A PROGRAM IN PYTHON TO DEMONSTRATE RUNGE-KUTTA 4th ORDER METHOD

PROGRAM:

```
RK4 step-by-step for y' = x + y, y(0)=1

import math


def f(x, y):
    """The ODE: y' = x + y"""
    return x + y


def exact_solution(x):
    """The exact solution of y' - y = x with y(0)=1 is y = 2*e^x - x - 1"""
    return 2 * math.exp(x) - x - 1


# Initial values
x = 0.0
y = 1.0
h = 0.1
steps = int(0.2 / h) # Compute up to x = 0.2 (steps = 2)


print("Runge-Kutta 4th order (RK4) step-by-step")
print(f"Equation: y' = x + y, y(0)={y}")
print(f"Step | x_n | y_n (before) |   k1   |   k2   |   k3   |   k4   | y_{steps * h:.1f}")
print("-" * 100)


for n in range(steps):


    # RK4 coefficients
    k1 = f(x, y) * h
k2 = f(x + h/2.0, y + (k1/2.0)) * h
```

```
    k3 = f(x + h/2.0, y + (k2/2.0)) * h
    k4 = f(x + h, y + k3) * h

    # RK4 Update Formula
    increment = (h/6.0) * (k1 + 2.0*k2 + 2.0*k3 + k4)
    y_next = y + increment

    # Print step details
    # Using 'n' for the step count (0 and 1) and 'n+1' for the y_next index (1 and 2)
    print(f"{n+1:3d}  | {x:6.3f} | {y:16.10f} | "
          f"{k1:7.6f} | {k2:7.6f} | {k3:7.6f} | {k4:7.6f} | {y_next:10.9f}")

    # Update
    x += h
    x = round(x, 10) # avoid floating accumulation
    y = y_next

print("-" * 100)
# Final output
exact_y = exact_solution(x)
absolute_error = abs(y - exact_y)

print(f"Final RK4 approximation: y({x:.3f}) = {y:.9f}")
print(f"Exact value          : y({x:.3f}) = {exact_y:.9f}")
print(f"Absolute error       : |abs({absolute_error:.12e})")
```

OUTPUT:

```
Runge-Kutta 4th order (RK4) step-by-step
Equation: y' = x + y , y(0)=1

 Step |   x_n   |    y_n (before)   |   k1        k2        k3        k4      |   y_(n+1)
--------------------------------------------------------------------------------------------------
   1  |  0.000  |     1.0000000000 | 1.000000  1.100000  1.105000  1.210500 | 1.110341667
   2  |  0.100  |     1.1103416667 | 1.210342  1.320859  1.326385  1.442980 | 1.242805142
--------------------------------------------------------------------------------------------------
Final RK4 approximation: y(0.200) = 1.242805142
Exact value            : y(0.200) = 1.242805516
Absolute error         : 3.746189507492e-07
```

CONCLUSION: THE ABOVE PROGRAM HAS BEEN EXECUTED SUCCESSFULLY,

NUMERICAL INTEGRATION

1. TRAPEZOIDAL RULE

AIM: TO WRITE A PROGRAM IN PYTHON TO DEMONSTRATE TRAPEZOIDAL RULE.

PROGRAM:

```
# Trapezoidal Rule for I = integral(f(x) dx from 0 to 1) with 2 subintervals


def f(x):
   """The integrand: 1 / (1 + x^2)"""
   return 1 / (1 + x**2)


# Given limits
a = 0
b = 1
n = 2 # number of subintervals


# Step size
h = (b - a) / n


# Compute x values
x = [a + i * h for i in range(n + 1)] # [0.0, 0.5, 1.0]


# Compute f(x) values
f_values = [f(xi) for xi in x] # [1.0, 0.8, 0.5]


# Display table header
print("-----------------------------------")
 print("i | x(i)   | f(x(i)) = 1/(1+x^2)")
print("-----------------------------------")
```

```
# Display table values
for i in range(n + 1):
    print(f"{i:<3} | {x[i]:<8.4f} | {f_values[i]:<8.4f}")
print("----------------------------------")

# Apply Trapezoidal Rule
# The formula in Python: I = (h / 2) * (f[0] + 2 * sum(f[1:-1]) + f[-1])
I = (h / 2) * (f_values[0] + 2 * sum(f_values[1:-1]) + f_values[-1])

# Step-by-step explanation
print(f"h = (b - a) / n = ({b} - {a}) / {n} = {h}")
print("\nUsing Trapezoidal Rule:")
print(f"I = (h / 2) * [f(x0) + 2*f(x1) + f(x2)]")
print(f"I = ({h/2}) * [{f_values[0]:.4f} + 2*{f_values[1]:.4f} + {f_values[2]:.4f}]")

# Final result
print("\n----------------------------------")
print(f"Approximate value of the integral I = {I:.4f}")
print("----------------------------------")
```

OUTPUT:

```
-----------------------------------------------
  i     x(i)       f(x(i)) = 1/(1+x^2)
-----------------------------------------------
  0     0.0000        1.0000
  1     0.5000        0.8000
  2     1.0000        0.5000
-----------------------------------------------

h = (b - a) / n = (1 - 0) / 2 = 0.5

Using Trapezoidal Rule:
I = (h/2) * [f(x0) + 2*f(x1) + f(x2)]
I = (0.5/2) * [1.0000 + 2*0.8000 + 0.5000]

--------------------------------------------
Approximate value of the integral I = 0.7750
--------------------------------------------
```

CONCLUSION: THE ABOVE PROGRAM HAS BEEN EXECUTED SUCCESSFULLY.

2. SIMPSON'S 1/3 RULE

AIM: AIM: TO WRITE A PROGRAM IN PYTHON TO DEMONSTRATE SIMPSON'S 1/3 RULE.

PROGRAM:

```
# Simpson's 1/3 Rule for I = ∫(0 to 1) e^(-x^2) dx with n = 4
import math

# Define the function
def f(x):
   return math.exp(-x**2)

# Given values
a = 0  # lower limit
b = 1  # upper limit
n = 4  # number of subintervals (must be even)

# Step size
h = (b - a) / n

# Generate x and f(x) values
x = [a + i * h for i in range(n + 1)]
f_values = [f(xi) for xi in x]

# Display table
print("--------------------------------------------------")
print(" i   | x(i)   | f(x(i)) = e^(-x^2)")
print("--------------------------------------------------")
for i in range(n + 1):
   print(f"{i:<3} | {x[i]:<8.4f} | {f_values[i]:<10.6f}")
print("--------------------------------------------------")

# Simpson's 1/3 rule computation
sum_odd = sum(f_values[i] for i in range(1, n, 2))
```

```
sum_even = sum(f_values[i] for i in range(2, n, 2))
I = (h / 3) * (f_values[0] + 4 * sum_odd + 2 * sum_even + f_values[-1])


# Show steps
print(f"\nh = (b - a) / n = ({b} - {a}) / {n} = {h}")
print("\nUsing Simpson's 1/3 Rule:")
print(f"I = (h/3) * [f(x0) + 4*(f(x1) + f(x3) + ...) + 2*(f(x2) + f(x4) + ...) + f(xn)]")
print(f"I = ({h}/3) * [{f_values[0]:.6f} + 4*{{{sum_odd:.6f}}} + 2*{{{sum_even:.6f}}} + {f_values[-1]:.6f}]")


# Display final result
print("\n--------------------------------------------------")
print(f"Approximate value of the integral I = {I:.6f}")
print("--------------------------------------------------")
```

OUTPUT:

```
------------------------------------------------
  i     x(i)        f(x(i)) = e^(-x^2)
------------------------------------------------
  0     0.0000       1.000000
  1     0.2500       0.939413
  2     0.5000       0.778801
  3     0.7500       0.569783
  4     1.0000       0.367879
------------------------------------------------

h = (b - a) / n = (1 - 0) / 4 = 0.25

Using Simpson's 1/3 Rule:
I = (h/3) * [f(x0) + 4*(f(x1) + f(x3) + ...) + 2*(f(x2) + f(x4) + ...) + f(xn)]
I = (0.25/3) * [1.000000 + 4*(1.509196) + 2*(0.778801) + 0.367879]

---------------------------------------------
Approximate value of the integral I = 0.746855
---------------------------------------------
```


CONCLUSION: THE ABOVE PROGRAM HAS BEEN EXECUTED SUCCESSFULLY.

2. SIMPSON'S 3/8 RULE

AIM: AIM: TO WRITE A PROGRAM IN PYTHON TO DEMONSTRATE SIMPSON'S 3/8 RULE.

PROGRAM:

```
# Simpson's 3/8 Rule for I = ∫(0 to 1) e^(-x^2) dx with n = 3
import math

# Define the function
def f(x):
   return math.exp(-x**2)

# Given values
a = 0  # lower limit
b = 1  # upper limit
n = 3  # must be a multiple of 3 for Simpson's 3/8 rule

# Step size
h = (b - a) / n

# Generate x and f(x)
x = [a + i * h for i in range(n + 1)]
f_values = [f(xi) for xi in x]

# Display table
print("--------------------------------------------------")
print(" i   | x(i)   | f(x(i)) = e^(-x^2)")
print("--------------------------------------------------")
          for i in range(n + 1):
   print(f"{i:<3} | {x[i]:<8.6f} | {f_values[i]:<10.6f}")
print("--------------------------------------------------")
```

```
# Apply Simpson's 3/8 rule formula (for n=3)
I = (3 * h / 8) * (f_values[0] + 3*f_values[1] + 3*f_values[2] + f_values[3])

# Step-by-step output
print(f"\nh = (b - a) / n = ({b}) / ({n}) = {h:.6f}")
print("\nUsing Simpson's 3/8 Rule:")
print(f"I = (3h/8) * [f(x0) + 3f(x1) + 3f(x2) + f(x3)]")
print(f"I = (3*{h:.6f}/8) * [{f_values[0]:.6f} + 3*{f_values[1]:.6f} + 3*{f_values[2]:.6f} + {f_values[3]:.6f}]")

# Final result
print("\n--------------------------------------------------")
print(f"Approximate value of the integral I = {I:.6f}")
print("--------------------------------------------------")
```

OUTPUT:


```
  i     x(i)         f(x(i)) = e^(-x^2)
------------------------------------------------
  0     0.000000     1.000000
  1     0.333333     0.894839
  2     0.666667     0.641180
  3     1.000000     0.367879
------------------------------------------------

h = (b - a)/n = (1 - 0)/3 = 0.333333

Using Simpson's 3/8 Rule:
I = (3h/8) * [f(x0) + 3f(x1) + 3f(x2) + f(x3)]
I = (3*0.333333/8) * [1.000000 + 3*0.894839 + 3*0.641180 + 0.367879]

---------------------------------------------
Approximate value of the integral I = 0.746992
---------------------------------------------
```

CONCLUSION: THE ABOVE PROGRAM HAS BEEN EXECUTED SUCCESSFULLY.

TRANSPORTATION PROBLEM

TRANSPORTATION PROBLEM USING NORTHWEST METHOD

AIM: TO WRITE A PROGRAM IN PYTHON TO DEMONSTRATE TRANSPORTATION PROBLEM USING NORTHWEST METHOD.

PROGRAM:

```
# Northwest Corner Method - step-by-step
# Problem data (from your sheet)
# Costs matrix: rows = origins O1,O2 ; cols = destinations D1,D2,D3
costs = [
    [8, 6, 10],  # O1
    [10, 4, 9]   # O2
]

supply = [2000, 2500]  # supplies for O1, O2
demand = [1500, 2000, 1000] # demands for D1, D2, D3

# Make copies so we don't destroy originals if we want to reuse them
sup = supply.copy()
dem = demand.copy()

# Prepare an allocation matrix initialized to zeros
alloc = [[0 for _ in range(len(demand))] for _ in range(len(supply))]

print("Northwest Corner Method - step by step\n")
print("Initial supply:", supply)
print("Initial demand:", demand)
print()
i = 0  # origin index (row)
j = 0  # destination index (col)
step = 0
```

```
# Loop until all supplies and demands are satisfied
while i < len(sup) and j < len(dem):
    step += 1
    qty = min(sup[i], dem[j])
    alloc[i][j] = qty
    sup[i] -= qty
    dem[j] -= qty

    # Print step details
    print(f"Step {step}: Allocate {qty} units to cell O{i+1}, D{j+1}")
    print(f"    cost per unit = {costs[i][j]}")
    print(f"    Remaining supply for O{i+1} = {sup[i]}")
    print(f"    Remaining demand for D{j+1} = {dem[j]}\n")

    # Move to next row or column (if supply exhausted move down, if demand exhausted move right)
    # If both become zero, move one and then the other: standard choice is to advance column (j) after row
    if sup[i] == 0 and dem[j] == 0:
        # If both exhausted, advance (commonly advance row or column) - advance column then row to avoid
skipping
        # But we must ensure not to go out of bounds: handle carefully:

        # Advance column if possible, otherwise advance row.
        if j + 1 < len(dem):
   j += 1
        elif i + 1 < len(sup):
            i += 1
        else:
  break  # Finished
    elif sup[i] == 0:
        i += 1
    elif dem[j] == 0:
        j += 1
```

```
        # This else normally won't happen because qty = min(sup[i], dem[j]) forces one to zero
        else:
          pass

# Display final allocation matrix
    print("Final allocation matrix (rows = O1,O2 ; cols = D1,D2,D3):\n")
    header = ["  |"] + [f" D{c+1}" for c in range(len(demand))] + ["| Supply"]
    print("".join(header))
    for r in range(len(alloc)):
      row_str = [f"O{r+1}|"] + [f"{alloc[r][c]:6d}" for c in range(len(alloc[r]))] + [f"| {supply[r]:6d}"]
      print("".join(row_str))
    print()

    # Compute total cost
    total_cost = 0
    for r in range(len(alloc)):

     for c in range(len(alloc[0])):
        total_cost += alloc[r][c] * costs[r][c]
    # Print non-zero allocations
    print("Allocations (non-zero):")
    for r in range(len(alloc)):
      for c in range(len(alloc[0])):
        if alloc[r][c] != 0:
          print(f"O{r+1}, D{c+1} -> {alloc[r][c]} units at cost {costs[r][c]} => contribution = {alloc[r][c] *
    costs[r][c]}")

    print(f"\nTotal transportation cost (initial NW-corner solution) = {total_cost}")
```

OUTPUT:

```
Northwest Corner Method - step by step

Initial supply: [2000, 2500]
Initial demand: [1500, 2000, 1000]

Step 1: Allocate 1500 units to cell (O1, D1)
        cost per unit = 8
        Remaining supply for O1 = 500
        Remaining demand for D1 = 0

Step 2: Allocate 500 units to cell (O1, D2)
        cost per unit = 6
        Remaining supply for O1 = 0
        Remaining demand for D2 = 1500

Step 3: Allocate 1500 units to cell (O2, D2)
        cost per unit = 4
        Remaining supply for O2 = 1000
        Remaining demand for D2 = 0

Step 4: Allocate 1000 units to cell (O2, D3)
        cost per unit = 9
        Remaining supply for O2 = 0
        Remaining demand for D3 = 0

Final allocation matrix (rows = O1,O2 ; cols = D1,D2,D3):

     D1  D2  D3  | Supply
O1    1500     500       0  |   2000
O2       0    1500    1000  |   2500

Allocations (non-zero):
  (O1, D1) -> 1500 units  at cost 8  =>  contribution = 12000
  (O1, D2) -> 500 units  at cost 6  =>  contribution = 3000
  (O2, D2) -> 1500 units  at cost 4  =>  contribution = 6000
  (O2, D3) -> 1000 units  at cost 9  =>  contribution = 9000

Total transportation cost (initial NW-corner solution) = 30000
```


CONCLUSION: THE ABOVE PROGRAM HAS BEEN EXECUTED SUCCESSFULLY.