2025-2026

MATHS

FORMULA SHEET

CLASS 12

Umesh Bhaiya

www.ncertkaksha.com
@NCERTKAKSHA
Contact us: contactncertkaksha@gmail.com


WARNING

Class XII (2025 - 2026)

INDEX

UNIT-1 (RELATIONS AND FUNCTIONS)

CHAPTER-1

Relations And Functions

Relation :- If A and B are two non-empty sets, then any subset R of $A \times B$ is called Relation from set A to B.

i.e. $\boxed{R : A \to B \Leftrightarrow R \subseteq A \times B}$

If $(x, y) \in R$ then we write xRy (read as x is R related to y) and
If $(x, y) \notin R$ then we write $x\not R y$ (read as x is not R related to y)

Domain And Range Of a Relation :-

If R is any relation from set A to set B then,

Domain

Domain of R is the set of all first coordinates of elements of R and is denoted by Dom (R).

Range

- Range of R is the set of all second coordinates of R and it is denoted by Range (R). A relation R on set A means, the relation from A to A i.e. $\boxed{R \subseteq A \times A}$

Types Of Relation :-

- **Empty Relation** :- A relation R in a set A is called empty relation, if no element of A is related to any element of A, i.e. $\boxed{R = \phi \subset A \times A}$

- **Universal Relation** :-

Universal

A relation R in a set A is called universal relation each of A is related to every element of A, i.e. $\boxed{R = A \times A}$

- **Identity Relation** :-

$\boxed{R = \{(x, y) : x \in A, y \in A, x = y\}}$

OR

$\boxed{R = \{(x, x) ; x \in A\}}$

- **Reflexive Relation** :-

If $(a, a) \in R$, for every $a \in A$

- **Symmetric Relation** :- If $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$ for all $a_1, a_2 \in A$.

* **Transitive Relation** :- If $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies $(a_1, a_3) \in R$ for all $a_1, a_2, a_3 \in A$.

* **Equivalence Relation** :- A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

* **Antisymmetric Relation** :- A relation R in a set A is antisymmetric if $\boxed{(a,b) \in R \quad (b,a) \in R \Rightarrow a = b \ \forall \ a, b \in R}$

* **Inverse Relation** :- If A and B are two non-empty sets and R be a relation from A to B, such that $R = \{(a,b) : a \in A, b \in B\}$, then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by $\boxed{R^{-1} = \{(b,a) : (a,b) \in R\}}$

➡ **Equivalence Class** :- let R be an equivalence relation on a non-empty set A for all $a \in A$, the equivalence class of 'a' is defined as the set of all such elements of A which are related to 'a' under R. It is denoted by $[a]$

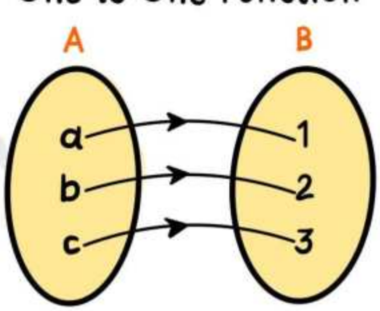
i.e. $\boxed{[a] = \text{equivalence class of 'a'} = \{x \in A : (x, a) \in R\}}$

➡ **Function** :- let X and Y be two non-empty sets. Then a rule f which associates to each element $x \in X$, a unique element, denoted by $f(x)$ of Y, is called a function from X to Y and written as $f : X \rightarrow Y$

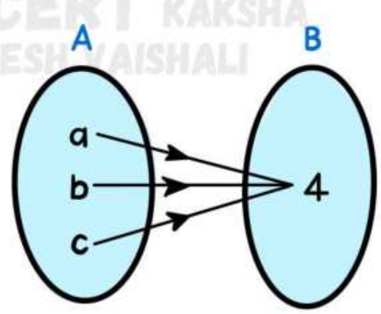
where, $f(x)$ is called image of x and x is called the pre-image of $f(x)$ and set Y is called the co-domain of f and $f(x) = \{f(x) : x \in X\}$ is called the range of f.

Types Of Functions ~

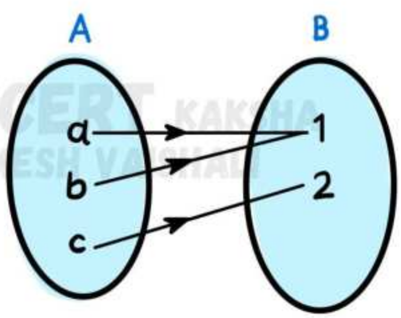
One to One Function



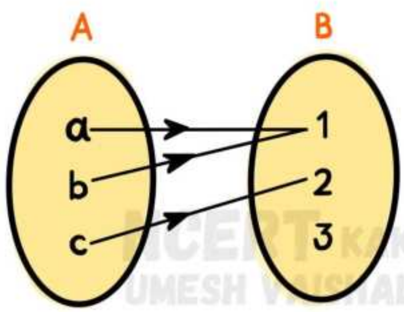
Many to One Function



Onto Function



Into Function



▶ **One - One OR Injective Function** :- A function $f: X \to Y$ is defined to be one-one if the images of distinct element of X under f are distinct;

i.e. $\boxed{x_1, x_2 \in X : f(x_1) = f(x_2) \Rightarrow x_1 = x_2}$ otherwise f is called many-one.

note:- • Set always denoted by capital letters (A, B, C, D....)
• Elements of a set always denoted by small letters (a, b, c, d....)

▶ **Onto OR Surjective** :- A function $f: X \to Y$ is said to be onto if every element of Y is the image of some element of X under f ; i.e. for every $y \in Y$, element of Y there exists an element x in X such that $f(x) = y$.

▶ **One-One and onto or Bijective** :- A function $f: X \to Y$ is said to be one-one and onto, if f is both one-one and onto.

note:- $f: X \to Y$ is onto if and only if Range of $f = Y$

➡ **Composition Of Function** :- let $f: A \to B$ and $g: B \to C$ be two function then the composition of f and g denoted by gof and defined as the function $gof: A \to C$

$$\boxed{gof(x) = g[f(x)], \forall \ x \in A}$$

note:- ∴ dom (gof) = dom(f)

➡ **Identity Function**

let R be the set of real numbers, a function $I: R \to R$ such that $I(x) = x \ \forall x \in R$ is called identity function.

Invertible Function :-

A function $f: X \to Y$ is defined to be invertible, if there exists a function $g: Y \to X$ such that $gof = I_X$ and $fog = I_Y$. The function g is called the inverse of f and is denoted by f^{-1}.

Binary Operation :-

A binary operation $*$ on a set A is a function $*: A \times A \to A$. We denote $*(a,b)$ by $a * b$.

- A binary operation $*$ on a set A is called commutative, if $a*b = b*a$, for every $a, b \in A$.
- A binary operation $*: A \times A \to A$ is said to be associative if $(a*b)*c = a*(b*c)$, $\forall\, a, b, c \in A$.
- A binary operation $*: A \times A \to A$, an element $e \in A$ (identity element), if it exists, is called identity for the operation $*$, if $a*e = e*a$, $\forall\, a \in A$.
- A binary operation $*: A \times A \to A$ with the identity element e in A, an element $a \in A$ is said to be invertible with respect to the operation $*$, if there exists an element b in A such that $a*b = e = b*a$ and b is called the inverse of a and is denoted by a^{-1}.

No. of Function :-

Let $f: A \to B$ be any mapping and $|A| = n$ and $|B| = m$

where, $|A|$ represent no. of elements in set A

$|B|$ represent no. of elements in set B

Then, Total no. of function from A to $B = m^n$

- **case (i)** If $n = m$; then

  Total no. of mapping $= n^n$
  Total no. of one-one mapping $= n!$
  Total no. of onto mapping $= n!$

- **case (ii)** If $n < m$; then

  Total no. of mapping $= m^n$
  Total no. of one-one mapping $= {}^mC_n \, n!$
  Total no. of onto mapping $= 0$

- **case (iii)** If $n > m$; then

  Total no. of mapping $= m^n$
  Total no. of one-one mapping $= 0$
  Total no. of onto mapping $= \sum_{r=0}^{m-1} (-1)^r \, {}^mC_r \, (m-r)^n$

UNIT-I (RELATIONS AND FUNCTIONS)

CHAPTER-2

Inverse Trigonometric Functions

Functions	Domain (x)	Range
$y=\sin^{-1}x$	$[-1,1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$y=\cos^{-1}x$	$[-1,1]$	$[0,\pi]$
$y=\tan^{-1}x$	R	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
$y=\operatorname{cosec}^{-1}x$	$R-(-1,1)$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$
$y=\sec^{-1}x$	$R-(-1,1)$	$[0,\pi]-\left\{\frac{\pi}{2}\right\}$
$y=\cot^{-1}x$	R	$(0,\pi)$

note:- $\in \rightarrow$ belongs to
$\forall \rightarrow$ for all

$R=$ Real Numbers

	Principal Value	General Value
$\sin\theta=\sin\alpha$	$\theta=\alpha$ if $-\frac{\pi}{2}<\alpha<\frac{\pi}{2}$	$\theta=n\pi+(-1)^n\alpha$; $n\in Z$
$\cos\theta=\cos\alpha$	$\theta=\alpha$ if $0<\alpha<\pi$	$\theta=2n\pi\pm\alpha$; $n\in Z$
$\tan\theta=\tan\alpha$	$\theta=\alpha$ if $-\frac{\pi}{2}<\alpha<\frac{\pi}{2}$	$\theta=n\pi+\alpha$; $n\in Z$

➡ Properties of Inverse trigonometric Functions :-

1. $\sin^{-1}(\sin x)=x$, $\forall x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
2. $\cos^{-1}(\cos x)=x$, $\forall x\in[0,\pi]$
3. $\tan^{-1}(\tan x)=x$, $\forall x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
4. $\cot^{-1}(\cot x)=x$, $\forall x\in(0,\pi)$
5. $\sec^{-1}(\sec x)=x$, $\forall x\in[0,\pi]-\left\{\frac{\pi}{2}\right\}$
6. $\operatorname{cosec}^{-1}(\operatorname{cosec} x)=x$, $\forall x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$

$\sin(\sin^{-1}x) = x \;\forall\; x \in [-1,1]$

$\cos(\cos^{-1}x) = x \;\forall\; x \in [-1,1]$

$\tan(\tan^{-1}x) = x \;\forall\; x \in R$

$\cot(\cot^{-1}x) = x \;\forall\; x \in R$

$\sec(\sec^{-1}x) = x \;\forall\; x \in R-(-1,1)$

$\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x \;\forall\; x \in R-(-1,1)$

$\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1}x \;\forall\; x \in R-(-1,1)$

$\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}x \;\forall\; x \in R-(-1,1)$

$\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1}x \;\forall\; x > 0$

$\tan^{-1}\left(\frac{1}{x}\right) = -\pi + \cot^{-1}x \;\forall\; x < 0$

$\sin^{-1}(-x) = -\sin^{-1}x \;\forall\; x \in [-1,1]$

$\cos^{-1}(-x) = \pi - \cos^{-1}x \;\forall\; x \in [-1,1]$

$\tan^{-1}(-x) = -\tan^{-1}x \;\forall\; x \in R$

$\cot^{-1}(-x) = \pi - \cot^{-1}x \;\forall\; x \in R$

$\sec^{-1}(-x) = \pi - \sec^{-1}x \;\forall\; x \in R-(-1,1)$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x \;\forall\; x \in R-(-1,1)$

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \;\forall\; x \in [-1,1]$

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \;\forall\; x \in R$

$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2} \;\forall\; x \in R-(-1,1)$

$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}$ if $-1 \le x, y \le 1$ & $x^2+y^2 \le 1$ OR $xy < 0$ & $x^2+y^2 > 1$

$\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\}$ If $-1 \le x, y \le 1$ & $x^2+y^2 \le 1$ OR $xy > 0$ & $x^2+y^2 > 1$

$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\}$ If $-1 \le x, y \le 1$ & $x+y \ge 0$

$\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\}$ If $-1 \le x, y \le 1$ & $x \le y$

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ If $xy < 1$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ If $xy > -1$

$$3\sin^{-1}x = \sin^{-1}(3x-4x^3) \quad \text{if } -\frac{1}{2} \le x \le \frac{1}{2}$$

$$3\cos^{-1}x = \cos^{-1}(4x^3-3x) \quad \text{if } \frac{1}{2} \le x \le 1$$

$$3\tan^{-1}x = \tan^{-1}\left(\frac{3x-x^3}{1-3x}\right) \quad \text{if } -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

$$\sin^{-1}x = \cos^{-1}\left(\sqrt{1-x^2}\right) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$= \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$$

$$\cos^{-1}x = \sin^{-1}\left(\sqrt{1-x^2}\right) = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

$$= \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \sec^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$\tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$= \cot^{-1}\left(\frac{1}{x}\right) = \sec^{-1}\left(\sqrt{1+x^2}\right) = \operatorname{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$$

$$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right) \quad \text{if } -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$$

$$2\cos^{-1}x = \cos^{-1}(2x^2-1) \quad \text{if } 0 \le x \le 1$$

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) \quad \text{if } -1 < x < 1$$

$$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right) \quad \text{if } -1 \le x \le 1$$

$$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \quad \text{if } 0 \le x < \infty$$

$$2\tan^{-1}x = \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right) \quad \text{if } x > 1$$

Expression	Substitution
a^2+x^2 OR $\sqrt{a^2+x^2}$	$x=a\tan\theta$ OR $x=a\cot\theta$
a^2-x^2 OR $\sqrt{a^2-x^2}$	$x=a\sin\theta$ OR $x=a\cos\theta$
x^2-a^2 OR $\sqrt{x^2-a^2}$	$x=a\sec\theta$ OR $x=a\,\mathrm{cosec}\,\theta$
$\sqrt{\frac{a-x}{a+x}}$ OR $\sqrt{\frac{a+x}{a-x}}$	$x=a\cos 2\theta$
$\sqrt{\frac{a^2-x^2}{a^2+x^2}}$ OR $\sqrt{\frac{a^2+x^2}{a^2-x^2}}$	$x^2=a^2\cos 2\theta$
$\sqrt{\frac{x}{a-x}}$ OR $\sqrt{\frac{a-x}{x}}$	$x=a\sin^2\theta$ OR $x=a\cos^2\theta$
$\sqrt{\frac{x}{a+x}}$ $\sqrt{\frac{a+x}{x}}$	$x=a\tan^2\theta$ OR $x=a\cot^2\theta$

CHAPTER-3

Matrices

➡ **Matrix** :- A matrix is a rectangular arrangement of numbers or functions arranged into a fixed number of rows and columns. A matrix is written inside brackets [ ]. Each entry in a matrix is called an element of the matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

→ First row (R_1)

→ Second row (R_2)

(C_1) First column Second column (C_2)

$m \times n$ → order of Matrix (No. of rows X No. of Columns)

We shall write $A = [a_{ij}]_{m \times n}$ → order of Matrix

i-th row j-th column

➡ Types Of Matrices ~

★ **Column Matrix** :-

A matrix is said to be a column matrix if it has only one column. Ex:- $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ 4×1 column

★ **Row Matrix** :-

A matrix is said to be a row matrix if it has only one row.

Ex- $[a\ b\ c\ d]$ 1×4 row

★ **Square Matrix** : No. of rows (m) = No. of columns (n)

$$Ex:- \begin{array}{c|c} \begin{bmatrix} 3 & 5 & 9 \\ 4 & 7 & 4 \\ 1 & 0 & 2 \end{bmatrix} & \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \\ \begin{matrix} C_1 & C_2 & C_3 \end{matrix} & \end{array} \quad (3 \times 3)$$

m n

★ **Diagonal Matrix** :- A square matrix is said to be a diagonal matrix if all its non diagonal elements are zero.

$$Ex \begin{bmatrix} -1 & \boxed{0} \\ \boxed{0} & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

non-diagonal elements zero.

★ **Scalar Matrix** :- A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal.

$$Ex- \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$$

diagonal elements equal

★ **Identity Matrix** :- A square matrix in which all diagonal elements are 1 and rest are all zero. Ex- $[1]$ $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ diagonal elements (1)

★ **Null OR Zero Matrix** :- If all its elements are zero. We donate zero matrix by O. Ex- $[0]$, $\begin{bmatrix}0&0\\0&0\end{bmatrix}$

➡ **Equal Matrix** :- Two matrices $A=[a_{ij}]$ and $B=[b_{ij}]$ are said to be equal if–

(i) They are of the same order.

(ii) Each elements of A is equal to the corresponding element of B i.e. $a_{ij}=b_{ij}$ for all i and j.

➡ **Upper triangular Matrix** :- An upper triangular matrix, if $a_{ij}=0\ \forall\ i>j$, i.e. all entries below principal diagonal are zero.

Example – $\begin{bmatrix}1&1\\0&1\end{bmatrix}$ $\begin{bmatrix}1&4&5\\0&2&6\\0&0&3\end{bmatrix}$ upper triangular

➡ **lower triangular Matrix** :- A lower triangular matrix, if $a_{ij}=0\ \forall\ i<j$, i.e. all entries above principal diagonal are zero. Example – $\begin{bmatrix}1&0\\1&1\end{bmatrix}$ $\begin{bmatrix}1&0&0\\4&2&0\\5&6&3\end{bmatrix}$ lower triangular

➡ **Transpose OF A Matrix** :- Matrix obtained by interchanging rows and columns of A and denoted by A' OR A^T.

example :- if $A=\begin{bmatrix}3&5\\\sqrt{3}&1\\0&\frac{-1}{5}\end{bmatrix}_{3\times2}$, then $A'=\begin{bmatrix}3&\sqrt{3}&0\\5&1&\frac{-1}{5}\end{bmatrix}_{2\times3}$

★ **Properties Of Transport of the matrices** :-

(i) $\boxed{(A^T)^T=A}$ (ii) $\boxed{(kA)^T=kA^T}$ k is any constant

(iii) $\boxed{(A+B)^T=A^T+B^T}$ (iv) $\boxed{(AB)^T=B^TA^T}$ (v) $\boxed{(ABC)^T=C^TB^TA^T}$

➡ **Symmetric Matrices** :- $\boxed{A' = A}$ For example $A = \begin{bmatrix} \sqrt{3} & 2 & 3 \\ 2 & -1.5 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ is a symmetric matrix as $A' = A$

➡ **Skew-Symmetric Matrices** :- $\boxed{B' = -B}$ For example $B = \begin{bmatrix} 0 & e & f \\ -e & 0 & g \\ -f & -g & 0 \end{bmatrix}$ is a skew symmetric matrix as $B' = -B$

Important Points :-

▲ Diagonal Elements Of a skew symmetric matrice are zero.

▲ For any square matrix A with real entries than $(A+A^T)$ is a symmetric and $(A-A^T)$ is skew symmetric.

▲ Any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix. $\boxed{A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')}$

➡ **Operation On Matrices** :-

1. **Addition Of Matrices** :- If $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ two matrices Of the same order $m \times n$, then their sum $A+B$ is $m \times n$ matrix such that, $(A+B)_{ij} = a_{ij} + b_{ij}$ $\forall\ i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots n$

★ **Properties Of Matrix addition** :-

(i) Commutativity $\boxed{A+B = B+A}$

(ii) Associativity $\boxed{(A+B)+C = A+(B+C)}$

(iii) Existence of identity $\boxed{A+O = A+O = A}$ zero matrix

(iv) Existence of inverse $\boxed{A+(-A) = O = (-A)+A}$

(v) Cancellation laws $\boxed{A+B = A+C \Rightarrow B = C \quad \text{and} \quad B+A = C+A \Rightarrow B = C}$

2. **Scalar Multiplication Of a matrix** :- let $A = [a_{ij}]_{m\times n}$ be a matrix and k is a scalar. Then the matrix obtained by multiplying each element Of matrix A by k and is denoted by kA or Ak

Properties Of Scalar Multiplication Of a Matrix :-

If $A=[a_{ij}]$ and $B=[b_{ij}]$ be two matrices of the same order, say $m \times n$, and k and l are scalars, then (i) $\boxed{k(A+B)=kA+kB}$ (ii) $\boxed{(k+l)A = kA+lA}$

3. **Multiplication Of Matrices** :- Two Matrices A and B are said to be defined for multiplication, if the number of columns of A (pre-multiplier) is equal to the number of rows of B (post multiplier)

$$\boxed{A_{m \times n} \times B_{n \times p} = AB_{m \times p}}$$

equal

Properties Of Multiplication Of Matrices :-

1. Associative Law $\boxed{(AB)C = A(BC)}$

2. Distributive law (a) $\boxed{A(B+C)=AB+AC}$ (b) $\boxed{(A+B)C=AC+BC}$

3. Existence Of Multiplicative identity $\boxed{IA=AI=A}$ Identity matrix

Elementary Operation (transformation) Of a Matrix :-

There are six operations (transformation) on a matrix 3 of which due to row and 3 of due to column, called Elementary Transformation.

- The interchange of any two rows or two column.
- The multiplication of the elements of any row or column by a non-zero number.
- The addition to the elements of any row or column, the corresponding elements of any other row or column multiplied by any non-zero number.

Invertible Matrix :- If A is square matrix of order $m \times n$ and if there exist another square matrix B of the same order such that $AB = BA = I_m$

The A is invertible and B is called inverse of A.

- Inverse Of a square matrix, if it exists, is unique.

- If A and B are invertible matrices of the same order, then $(AB)^{-1}=B^{-1}A^{-1}$

★ Properties Of Invertible Matrix :-

If A is an invertible Matrix, then

1. $\boxed{(A^{-1})^{-1} = A}$
2. $\boxed{A^{-1} = \frac{1}{|A|}(\text{adj}A)}$
3. $\boxed{(A^{T})^{-1} = (A^{-1})^{T}}$
4. $\boxed{|A^{-1}| = \frac{1}{|A|}}$
5. Square Matrix is invertible ; If it is non - singular.
6. The inverse of an invertible symmetric matrix is a symmetric matrix.

SUCCESS is a journey not a Destination.....

UNIT-2 (ALGEBRA)

CHAPTER-4

Determinants

Determinant :- To every square matrix $A=[a_{ij}]$ of order n, we can associate a no. (real or complex) called determinant of the square matrix A, where $a_{ij} = (i,j)^{th}$ element of A. Donated as: $\det A$ or $|A|$.

Determinant of Matrix of order one :-

let $A = [a]$ be the matrix of order 1, then determinant of A is defined to be equal to a.

Determinant of Matrix of order 2X2 :- let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2} \Rightarrow \det(A) = |A| = \Delta$

$$= \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}_{2\times2} = a_{11}a_{22} - a_{21}a_{22}$$

Determinant of matrix of order 3X3 :- let $A = \begin{array}{c} \begin{matrix} \oplus & \ominus & \oplus \end{matrix} \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \end{array}$

Expansion along First row (R_1)

$$\det(A) = |A| = \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} - \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

ऐसे ही 2^{nd}, 3^{rd} row and 1^{st}, 2^{nd}, 3^{rd} column के through भी expansion कर सकते हैं। और answer हर बार Same आयेगा चाहे expansion कैसे भी करो।

note:- • For matrix A, $|A|$ is read as determinant of A and not modulus of A.

■ Only square matrices have determinants.

Properties Of Determinants :-

▲ The value of the determinant remains unchanged if its rows and columns interchanged.

▲ If any two rows (or columns) of a determinant are interchanged, then value of determinant is zero.

▲ If any two rows (or columns) of a determinant are identical, then value of determinant is zero.

▲ If each element of a row (or a column) of a determinant is multiplied by a constant k, then its value gets multiplied by k.

▲ If some or all elements of a row or column of a determinant, is expressed sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.

▲ If, to each element of any row or column of a determinant, the equimultiple of corresponding elements of other row (or column) are added, then the value of determinant remains the same i.e. the value of determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$

Area Of Triangle :-

Area of triangle with vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\boxed{\Delta = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]}$$

note:-

(i) Area is a positive quantity, we always take the absolute value of Δ.

(ii) If area is given, use both positive and negative values of the determinant for calculation.

(iii) The area of the triangle formed by three collinear points is zero.

➡ **Minors** :- Minor of an element a_{ij} of the $|A|$ is determinant obtained by deleting i^{th} row and j^{th} column and is denoted by M_{ij}.

➡ **Cofactors** :- Cofactor of an element a_{ij}, denoted by A_{ij} is defined by $A_{ij} = (-1)^{i+j} M_{ij}$ mirror of a_{ij}

➡ **Adjoint of a matrix** :- The adjoint of a square matrix $A = [a_{ij}]$ is defined as the transpose of the matrix $[A_{ij}]_{m\times n}$. Adjoint of the matrix A denoted by adj A. (the cofactor of the element a_{ij})

➡ **Singular Matrix** :- A square matrix A is said to be singular if $|A| = 0$

➡ **Non-Singular Matrix** :-

A square matrix A is said to be non-singular if $|A| \neq 0$.

➡ **Theorem 1** ~ If A be any given square matrix of order n, then

$$A(\text{adj}A) = (\text{adj}A)A = |A|\,I.$$

➡ **Theorem 2** ~ If A and B are non-singular matrices of the same order, then AB and BA are also non-singular matrices of the same order.

➡ **Theorem 3** ~ $|AB| = |A|\,|B|$ where A and B are square matrices of same order.

➡ **Theorem 4** ~ A square matrix A is invertible if and only if A is non-singular matrix.

➡ **Consistent System** :- If system of equation have solution (one or more) exists.

➡ **Inconsistent System** :- If system has no solution or solution does not exist.

➡ **System of linear equation using inverse of a matrix** :-

consider the equations,

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Here, $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B \Rightarrow \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

Then the system of equations can be written as,

$$AX = B \Rightarrow \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Case I :- If A is a non-singular matrix, then its inverse exists. Now

$X = A^{-1}B$ (consistent & unique sol^n)

Case II :- If A is a singular matrix,

then $|A| = 0$ $(Adj\,A)\,B \neq 0$ sol^n does not exist (incosistent)

$(Adj\,A)\,B = 0$ infinitely many sol^n or no sol^n (consistent or inconsistent)

➡ **System Of Equation** : $\boxed{AX = B}$

- if $\boxed{B = 0}$ (Homogenous system of Equation)
- if $\boxed{B \neq 0}$ (Non-Homogenous system of Equation)

➡ **Homogenous System Of Equation** :- $\boxed{AX = 0}$

- if $\boxed{|A| = 0}$ then unique solution (Trivial Solution)
- if $\boxed{|A| \neq 0}$ then infinitely solutions.

- If A is skew symmetric matrix of odd order, then $|A| = 0$
- The determinant of a skey-symmetric Matrix of even order is a perfect square.

UNIT-3 (CALCULUS)

Continuity And Differentiability

Continuity :- Suppose f is a real function on a subset of the real numbers and let c be a point in the domain f. Then f is continuous at c if $\boxed{\lim_{x \to c} f(x) = f(c)}$

Discontinuity :- A function said to be discontinuous at point $x = a$, if it is not continuous at this point. This point $x = a$ where the function is not continuous is called the point of discontinuity.

Theorem :- 1 Suppose f and g be two real functions continuous at a real no. then,

1. $f + g$ is continuous at $x = c$
2. $f - g$ is continuous at $x = c$
3. $f \cdot g$ is continuous at $x = c$
4. $\frac{f}{g}$ is continuous at $x = c$, {provided $g(c) \neq 0$}
5. $a \cdot f$ is continuous at $x = c$, where a is constant.
6. $|f|$ is continuous at $x = c$

↗ **Every constant** function is continuous function.

↗ **Every Polynomial** is continuous function.

↗ **Identity function** is continuous function.

↗ **Every Exponential & logarithmic function** is continuous function.

Theorem :- 2

Suppose f and g are real valued functions such that (fog) is defined at c. If g is continuous at c and if f is continuous at g(c), then (fog) is continuous at c.

Differentiability :- Suppose f is a real function and c is a point in its domain. The derivative of f at c defined by $\lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$ provided this limit exists. Derivative of at c is denoted by $f'(c)$ or $\frac{d}{dx}[f(x)]_c$.

The function defined by $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ wherever the limit exists is defined to be the derivative of f. The derivative denoted by $f'(x)$ or $\frac{d}{dx}[f(x)]$ or if $y = f(x)$ by $\frac{dy}{dx}$ or y'.

Algebra Of Derivaties :-

(Quotient Rule)

3. $\boxed{\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}, \text{ whenever } v \neq 0}$

ALGEBRA OF DERIVATIES

1. $\boxed{(u \pm v)' = u' \pm v'}$

2. $\boxed{(uv)' = u'v + uv'}$

(Leibnitz or product rule)

Theorem :- 3

If a function f is differentiable at a point c, then it is also continuous at that point.

note :- Every differentiable function is continuous.

Chain Rule :- let f be a real valued function which is a composite of two functions u and v i.e. $f = vou$; suppose $t = u(x)$ and if $\frac{dt}{dx}$ and $\frac{dv}{dt}$ exist, we have $\frac{df}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx}$

Suppose f is real valued function which is a composite of three functions u, v and w ; i.e. $f = (w o u) o v$ and if $t = v(x)$ and $s = u(t)$ then $\frac{df}{dx} = \frac{d(wou)}{dt} \cdot \frac{dt}{dx} = \frac{dw}{ds} \cdot \frac{ds}{dt} \cdot \frac{dt}{dx}$

Some Properties Of Logorithmic Function :-

$$\log_a p = \frac{\log_b p}{\log_b a}$$

$$\log_b pq = \log_b p + \log_b q$$

$$\log_b p^2 = \log_b p + \log_b p = 2\log_b p$$

$$\log_b p^n = n\log_b p$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b x = \frac{1}{\log_x b}$$

Learn it Important for limit Questions :-

$$e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots$$

$$\log|1-x| = -x - \frac{x^2}{\lfloor 2} - \frac{x^3}{\lfloor 3} - \cdots$$

$$e^{-x} = 1 - \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} - \frac{x^3}{\lfloor 3} + \cdots$$

$$\sin x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \frac{x^7}{\lfloor 7} + \cdots$$

$$a^x = 1 + \frac{x\log_e a}{\lfloor 1} + \frac{x^2(\log_e a)^2}{\lfloor 2} + \cdots$$

$$\cos x = 1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} - \frac{x^6}{\lfloor 6} + \cdots$$

$$\log|1+x| = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^5 + \cdots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2}x^2 + \frac{n(n-1)(n-2)x^3}{\lfloor 3} + \cdots$$

Rolle's Theorem :-

If $f:[a,b] \to R$ is continuous on $[a,b]$ and differentiable on (a,b) such that $f(a) = f(b)$ then there exists some C in (a,b) such that $f'(c) = 0$.

Langrage Theorem Or Mean Value Theorem:

If $f:[a,b]\rightarrow R$ is continuous on $[a,b]$ and differentiable on (a,b). Then there exists some c in (a,b) such that $\boxed{f'(c)=\frac{f(b)-f(a)}{b-a}}$

Some Standard derivative :-

$\frac{d}{dx}(c)=0$ c=constant

$\frac{d}{dx}(x^n)=nx^{n-1}$

$\frac{d}{dx}(e^x)=e^x$

$\frac{d}{dx}(\log_e x)=\frac{1}{x}$

$\frac{d}{dx}(a^x)=a^x\log_e a$

$\frac{d}{dx}(\log_a x)=\frac{1}{x}\log_a e$

$\frac{d}{dx}(\sin x)=\cos x$

$\frac{d}{dx}(\cos x)=-\sin x$

$\frac{d}{dx}(\tan x)=\sec^2 x$

$\frac{d}{dx}(\cot x)=-\operatorname{cosec}^2 x$

$\frac{d}{dx}(\sec x)=\sec x\cdot\tan x$

$\frac{d}{dx}(\operatorname{cosec} x)=-\operatorname{cosec} x\cdot\cot x$

$\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\cos^{-1}x)=\frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$

$\frac{d}{dx}(\cot^{-1}x)=-\frac{1}{1+x^2}$

$\frac{d}{dx}(\sec^{-1}x)=\frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}(\operatorname{cosec}^{-1}x)=\frac{-1}{x\sqrt{x^2-1}}$

Derivative of functions in parametric forms :-

$x = f(t) = g(t)$ parametric form with t as a parameter.

$$\frac{dy}{dx}=\frac{dy}{dx}\cdot\frac{dx}{dt}$$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$ whenever $\frac{dx}{dt}\neq 0$

$$\frac{dy}{dx}=\frac{g'(t)}{f'(t)}$$ $\left[\text{as } \frac{dy}{dt}=g'(t) \text{ and } \frac{dx}{dt}=f'(t)\right]$ $\{\text{provided } f'(t)\neq 0\}$

note:-

Exponential form	logarithim form
$2^3 = 8$	$\log_2 8 = 3$
$b^1 = b$	$\log_b b = 1$
$b^0 = 1$	$\log_b 1 = 0$

note:- Higher order derivative may be defined similarly.

Logarithmic Differentiation :-

$y = f(x) = [u(x)]^{v(x)}$

$\log y = v(x) \log[u(x)]$ taking log both sides,

$\frac{1}{y}\frac{dy}{dx} = v(x) \cdot \frac{1}{u(x)} \cdot u'(x) + v'(x) \cdot \log[u(x)]$ using chain rule to differentiate

$$\boxed{\frac{dy}{dx} = y\left[\frac{v(x)}{u(x)} \cdot u'(x) + v'(x) \cdot \log[u(x)]\right]}$$

Second order derivative :-

Let $y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)$ —— (i)

differentiate (i) again w.r.t to x,

$\frac{d}{dx}\frac{dy}{dx} = \frac{d}{dx}[f'(x)] \Rightarrow \boxed{\frac{d^2y}{dx^2} = f''(x)}$ denoted by D^2y or D''

BELIEVE you can AND you WILL.......

UNIT-3 (CALCULUS)

CHAPTER-6

Application Of Derivatives

Rate Of Change :-

If a quantity y varies with another quantity x, satisfying some rule $y=f(x)$, then $\left.\frac{dy}{dx}\right]_{x=x_0}$ (or $f'(x_0)$) represents the rate of change of y with respect to x at $x=x_0$.

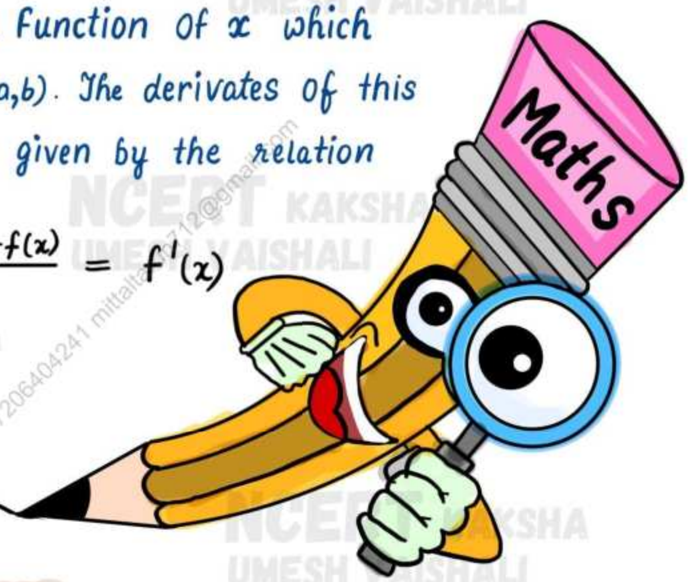
Differentials :-

Let $y=f(x)$ be any function of x which is differentiable in (a,b). The derivates of this function at some point x of (a,b) is given by the relation

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = f'(x)$$

$$\Rightarrow \frac{dy}{dx} = f'(x) \Rightarrow dy = f'(x)\,dx$$

differential of the function



Increasing And decreasing Functions :-

Increasing	Decreasing
(a) Increasing on an interval (a,b) if $x_1 < x_2$ in $(a,b) \Rightarrow f(x_1) \le f(x_2)$ for all $x_1, x_2 \in (a,b)$.	(b) decreasing on an interval (a,b) if $x_1 < x_2$ in $(a,b) \Rightarrow f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in (a,b)$

Theorem 1 ~

(a) f is increasing in $[a,b]$ if $f'(x)>0$ for each $x \in (a,b)$.

(b) f is decreasing in $[a,b]$ if $f'(x)<0$ for each $x \in (a,b)$

(c) f is a constant function in $[a,b]$ if $f'(x)=0$ for each $x \in (a,b)$

Tangent to a curve :- The equation of the tangent at (x_0, y_0) to the curve $y = f(x)$ is given by-

$\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$ OR $f'(x_0) = m$ = slope of tangent at (x_0, y_0)

$$\boxed{y - y_0 = \left.\frac{dy}{dx}\right]_{(x_0, y_0)} (x - x_0)}$$

⇒ If $\frac{dy}{dx}$ does not exist at the point (x_0, y_0), then the tangent at this point is parallel to the Y-axis and its equation is $x = x_0$.

⇒ If tangent to a curve $y = f(x)$ at $x = x_0$ is parallel to x-axis, then $\left.\frac{dy}{dx}\right]_{x=x_0} = 0$

Normal to the curve :-

Equation of the normal to the curve $y = f(x)$ at a point (x_0, y_0) is given by,

$$y - y_0 = \frac{-1}{\left.\frac{dy}{dx}\right]_{(x_0, y_0)}} (x - x_0)$$

$\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$ OR $f'(x_0) = m$ = slope of tangent at (x_0, y_0)

⇒ If $\frac{dy}{dx}$ at the point (x_0, y_0) is zero, then equation of the normal is $x = x_0$

⇒ If $\frac{dy}{dx}$ at the point (x_0, y_0) does not exist, then the normal is parallel to x-axis and its eq. $y = y_0$

$$\boxed{\text{Slope of the Normal} = \frac{-1}{\text{slope of the tangent}}}$$

Approximation :- Let $y = f(x)$, Δx be a small increament in x and Δy be the increment in y corresponding to the increment in x, i.e. $\Delta y = f(x + \Delta x) - f(x)$. Then approximate value of $\Delta y = \left(\frac{dy}{dx}\right) \Delta x$.

Maximum or Minimum value of a Function :-

⇒ A function f is said to attain maximum value at a point $a \in D_f$, if $f(a) \geq f(x)$ $\forall$ $x \in D_f$ then $f(a)$ is called absolute maximum value of f.

⇒ A function f is said to attain minimum value at a point $b \in D_f$, if $f(b) \leq f(x)$ $\forall\ x \in D_f$ then $f(b)$ is called absolute minimum value of f.

➡ Local Maxima And Local Minima (Relative Extrema) :-

Local Maxima ~ A function $f(x)$ is said to attain a local maxima at $x = a$, if there exists a neighbourhood $(a-\delta, a+\delta)$ of 'a' such that $f(x) < f(a)\ \forall\ x \in (a-\delta, a+\delta)$, $x \neq a$, then $f(a)$ is the local maximum value of $f(x)$ at $x = a$.

Local Minima :- A function $f(x)$ is said to attain a local minima at $x = a$, if there exists a neighbourhood $(a-\delta, a+\delta)$ of 'a' such that $f(x) > f(a)\ \forall\ x \in (a-\delta, a+\delta)$, $x \neq a$, then $f(a)$ is the local minimum value of $f(x)$ at $x = a$.

(a) **First derivative test :-**

- If $f'(x)$ changes sign from positive to negative as x increases through c, then 'c' is a point of local maxima and $f(c)$ is local maximum value.
- If $f'(x)$ changes sign from negative to positive as x increases through c, then 'c' is a point of local minima and $f(c)$ is local minimum value.
- If $f'(x)$ does not changes sign as x increases through c, then c is neither a point of local minima nor a point of local maxima. Such a point is called point of inflection.

(b) **Second Derivative Test :-** Let f be a function defined on an interval I and $c \in I$. Let f be twice differentiable at c. Then,

- $x = c$ is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$. In this case $f(c)$ is called local maximum value.
- $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$. In this case $f(c)$ is called local minimum value.

- The test fails of $f'(c)=0$ and $f''(c)=0$. In this case, we go back to first derivative test.

Working Rule for finding absolute maximum or absolute minimum values :-

Step I :- Find all the critical points of f in the given interval, i.e, find points x where either $f'(x)=0$ or f is not differentiable.

Step II :- Take the end points of the interval.

Step III :- At all these points, Calculate the value of f.

Step IV :- Identify the maximum and minimum value of f out the values calculated in step III. The maximum value will be the absolute maximum value of f and the minimum value will be the absolute minimum value of f.

Critical Point :-

A point C in the domain of a function f at which either $f'(c)=0$ or f is not differentiable is called a critical point of f.

Useful for Questions :-

Area Of Square $= x^2$ & Perimeter $= 4x$

Side of Square

Area Of Rectangle $= x \cdot y$ & Perimeter $= 2(x+y)$

length

Breadth

Area Of Circle $= \pi r^2$

circumference $= 2\pi r$

radius

Area Of Trapezium

$= \frac{1}{2}$ Sum of Parallel Side × Perpendicular distance between them

CUBE

= Surface Area = $6x^2$

Volume = x^3

Side of Cube

Right Circular Cylinder

Height

Total surface Area = $2\pi r h + 2\pi r^2$

Curved surface Area = $2\pi r h$

Volume = $\pi r^2 h$

Area Of Equilateral Triangle

$= \frac{\sqrt{3}}{4} (\text{Side})^2$

Right circular cone

Slant height

Total surface Area = $\pi r^2 + \pi r l$

Curved surface Area = $\pi r l$

Volume = $\frac{1}{3} \pi r^2 h$

Sphere

Volume of Sphere = $\frac{4}{3} \pi r^3$

Surface Area = $4 \pi r^2$

UNIT-3 (CALCULUS)

CHAPTER-7

Integrals

➡ **Integration** (Anti differentiation) :- Integration is the inverse process of differentiation. Instead of differentiating a function. We are given the derivative of a function and asked to find its primitive, i.e., the original function. Such a process is called integration or anti differentiation.

Example :- $\boxed{y = \int f(x)\,dx}$

Derivatives	Integrals (Antideratives)
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$
$\frac{d}{dx}(x) = 1$	$\int dx = x + C$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\, dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x\, dx = -\cos x + c$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + c$
$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x\, dx = -\cot x + C$
$\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$	$\int \sec x \tan x\, dx = \sec x + C$
$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$	$\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$

$\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1}x + C$
$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$	$\int \frac{dx}{1+x^2} = \tan^{-1}x + c$
$\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$	$\int \frac{dx}{1+x^2} = -\cot^{-1}x + C$
$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x + c$
$\frac{d}{dx}(\text{cosec}^{-1}x) = \frac{-1}{x\sqrt{x^2-1}}$	$\int \frac{dx}{x\sqrt{x^2-1}} = -\text{Cosec}^{-1}x + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}(\log x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log\lvert x\rvert + c$
$\frac{d}{dx}\left(\frac{a^x}{\log a}\right) = a^x$	$\int a^x dx = \frac{a^x}{\log a} + C$

Integration By Substitution Method :-

$\int \tan x \, dx = \log\lvert\sec x\rvert + c$

$\int \cot x \, dx = \log\lvert\sin x\rvert + c$

$\int \sec x \, dx = \log\lvert\sec x + \tan x\rvert + c$

$\int \text{cosec} x \, dx = \log\lvert\text{cosec} x - \cot x\rvert$

Integrals Of Some Particular Functions :-

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2-a^2}\right| + C$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right| + c$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$$

➡ To find the integral $\int \frac{dx}{ax^2+bx+c}$

we write, $ax^2+bx+c = a\left[x^2+\frac{bx}{a}+\frac{c}{a}\right] = a\left[\left(x+\frac{b}{2a}\right)^2+\left(\frac{c}{a}-\frac{b^2}{4a^2}\right)\right]$

Now; put $x+\frac{b}{2a} = t \Rightarrow dx = dt$ and $\frac{c}{a}-\frac{b^2}{4a^2} = \pm K^2$

The integral becomes $\frac{1}{a}\int \frac{dt}{t^2 \pm K^2}$

➡ To find the integrated of the type:

$$\int \frac{px+q}{ax^2+bx+c}dx \Rightarrow$$

where p, q, a, b, c are constants.

To find the real numbers A, B such that,

$px+q = A\frac{d}{dx}(ax^2+bx+c) + B = A(2ax+b) + B$

note:-

Integral of the form $\int \sin^m x \cdot \cos^n x \, dx$

1. if (m) exponent of $\sin x$ is an odd positive integer, then put $\cos x = t$
2. if (n) exponent of $\cos x$ is an odd integer, then put $\sin x = t$

Integration by Partial fraction :-

Form of Rational function	Form of partial function
$\frac{px+q}{(x-a)(x-b)}$; $a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{(x-a)} + \frac{Bx+C}{(x^2+bx+c)}$

where x^2+bx+c cannot be factorised further.

Integration By Parts :-

$$\int f(x)\, g(x)\, dx = f(x)\int g(x)\, dx - \int \left[f'(x)\int g(x)\, dx\right] dx$$

Integral of the type :-

$$\int e^x[f(x)+f'(x)]\, dx = \int e^x f(x)\, dx$$

$$\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\left|\log x + \sqrt{x^2-a^2}\right| + c$$

$$\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\left|\log x + \sqrt{x^2+a^2}\right| + c$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

Fundamental Theorem Of Calculus :-

Area Function : $A(x) = \int_a^x f(x)\,dx$

First Fundamental Theorem Of Integral calculus:

Theorem 1 ~ let f be a continuous function on the closed interval [a,b] and let A(x) be the area function. then $\boxed{A'(x) = f(x)}$, for all $x \in [a,b]$

Second fundamental theorem of integral calculus:

Theorem 2 ~ f be continuous function defined on the closed interval [a,b] and F be an antiderivative of f.

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$$

Definite Integral :-

If F(x) is the integral of f(x) over the interval [a,b], i.e. $\int f(x)\,dx = F(x)$ then the definite integral of f(x) over the interval [a,b] is denoted by $\int_a^b f(x)$ is defined as

upper limit
lower limit

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Define integral as the limit of the sum :-

$$\int_a^b f(x)\,dx = \lim_{h\to\infty} h[f(a) + f(a+h) + \ldots f(a+(n-1)h)]$$

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}[f(a) + f(a+h) + \ldots f(a+(n-1)h)]$$

where $h = \frac{b-a}{n} \to 0$ as $n \to \infty$

Some Properties Of Define Integrals :-

P_0 : $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

P_8 : $\int_0^{na} f(x)\,dx = n\int_0^a f(x)\,dx$; if $f(x) = f(a+x)$

P_1 : $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

P_2 : $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ where $(a<c<b)$

P_3 : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

P_4 : $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

P_5 : $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

P_6 : $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & : \text{if } f(2a-x) = f(x) \\ 0 & : f(2a-x) = -f(x) \end{cases}$

P_7 : (i) $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$: $f(x)$ is even function, i.e. $f(-x) = f(x)$

(ii) $\int_{-a}^a f(x)\,dx = 0$ if $f(x)$ is odd function. i.e. $f(-x) = -f(x)$

P_8 : $\int_0^{na} f(x)\,dx = n\int_0^a f(x)\,dx$; if $f(x) = f(a+x)$

your only limit is your Mind

UNIT-3 (CALCULUS)

Applications Of Integrals

- The area of region bounded by the curve $y=f(x)$, x-axis and the lines $x=a$ and $x=b$ $(b>a)$ is given by $\boxed{\text{Area} = \int_a^b f(x)\,dx = \int_a^b y\,dx}$

- The area of region bounded by the curve $x=\phi(y)$, y-axis and the lines $y=c$ and $y=d$ is $\boxed{\text{Area} = \int_c^d x\,dy = \int_c^d \phi(y)\,dy}$

- Area enclosed between; $y=f(x)$ and $y=g(x)$ and the lines; $x=a$, $x=b$

$$\boxed{\text{Area} = \int_a^b [f(x)-g(x)]\,dx \; ; \; f(x) \geq g(x) \text{ in } [a,b]}$$

- If $f(x) \geq g(x)$ in $[a,c]$ and $f(x) \leq g(x)$ in $[c,b]$, $a<c<b$ then,

$$\boxed{\text{Area} = \int_a^c [f(x)-g(x)]\,dx + \int_c^b [g(x)-f(x)]\,dx}$$

➡ Important For Questions :-

$$\boxed{\Sigma(n-1) = 1+2+3+\dots+(n-1) = \frac{n(n-1)}{2}}$$

$$\boxed{\Sigma(n-1)^2 = 1^2+2^2+3^2+\dots+(n-1)^2 = \frac{n(n-1)(2n-1)}{6}}$$

$$\boxed{\Sigma(n-1)^3 = 1^3+2^3+3^3+\dots+(n-1)^3 = \left[\frac{n(n-1)}{2}\right]^2}$$

$$\boxed{a + ar + ar^2 + \dots + ar^{n-1} = \begin{cases} a\left(\dfrac{r^n-1}{r-1}\right) & \text{if } r>1 \\ a\left(\dfrac{1-r^n}{1-r}\right) & \text{if } r<1 \end{cases}}$$

UNIT-3 (CALCULUS)

CHAPTER-9

Differential Equations

➡ Differentiation :-

An equation involving the independent variable x say, dependent variable y say and the differential coefficients of dependent variable with respect to independent variable i.e. $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}, \ldots\ldots,$ etc.

Example :- $\frac{dy}{dx} + 8y = 3x$, $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 7y = x^2$ are differential equations.

➡ Order And Degree Of a differential equation :-

The order of a differential equation is the highest order derivative occurring in the differential equation.

The degree of a differential equation is the degree of the highest order derivative occurring in the equation, when the differential coefficients are made free from radicals, fractions and it is written as a polynomial in differential co-efficient.

Example ~ $\boxed{\frac{d^3y}{dx^3}} + \frac{8d^2y}{dx^2} - \frac{3dy}{dx} + y = 0$

Highest order derivative = 3 ⇒ order :- 3

The degree of the highest order derivative occurring in the equation ⇒ degree = 1

$\left(\frac{d^2y}{dx^2}\right)^3 + \sin\frac{dy}{dx} = 0$

order :- 2

degree = not defined (because this differential eq^n cannot be written in the form of polynomial in diff^n co-efficient.)

note :-

order and degree of a differential eq^n are always positive integers

Classification Of Differential Equation :-

A According to D.E. Order
- 1st order D.E. (in which only 1^{st} order derivative of the dependent variable occurs)
- Higher order D.E. (in which two or more order derivative of the dependent variable occurs)

B According to D.E. linearity
- linear D.E.
- Non-linear D.E.

linear And Non-linear D.E. :-

A D.E. is which the dependent variable and its derivative occurs only in the 1^{st} degree & are not multiplied together, is called a linear D.E. other it is Non-linear D.E.

note:- Every linear D.E. is always of the 1^{st} degree but D.E. of 1^{st} degree need not to be the linear D.E.

note:- If the homogenous differential equation is in the form $\frac{dx}{dy} = F(x,y)$ then we substitute $x = vy$ and so $\frac{dx}{dy} = v + y\frac{dv}{dy}$ and proceed as above.

General Solution :-

The solution which contains arbitrary constants is called the general solution (primitive) of the differential equation.

Particular Solution :

The solution obtained from the general solution by giving particular values to the arbitrary constants is called a particular solution of the differential equation.

Equations in Variable separable form:-

Consider the Equation $\frac{dy}{dx} = X \cdot Y$

where X is a function of x only and Y is a function of y only.

[i] Put the equation in the form $\frac{1}{Y}dy = X \cdot dx$

[ii] Integrating both the sides, we get $\int \frac{dy}{Y} = \int X dx + C$ where c is an arbitrary constant.
Thus the required solⁿ is obtained.

➡ Equations Reducible to variables seperable form :-

1. Write the given equation in form $\frac{dy}{dx} = f(ax + by + c)$

2. Put $ax + by + c = z$, so that $\frac{dy}{dx} = \frac{1}{b}\left(\frac{dz}{dx} - a\right)$

3. Putting this $\frac{dy}{dx}$ in the given equation, we get $\frac{1}{b}\left(\frac{dz}{dx} - a\right) = f(z)$. This eq^n is reduced in the form : $\frac{dz}{a + bf(z)} = dx$. After integrating, we get the required result.

➡ Homogenous differential Equation :-

A differential equation of the form $\frac{dy}{dx} = F(x,y)$ is said to be homogenous differential equation if $F(x,y)$ is a homogenous function of degree zero.

1. Suppose $y = vx$ and so $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

2. The value $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ is substituted in given eq^n. The eq^n reduces to variable separable form, which can be solved by integrating both sides.

4. Finally v is replaced by $\frac{y}{x}$ to get the required solution.

➡ First order linear differential equation :-

$$\frac{dy}{dx} + Py = Q \quad \text{---(i)}$$

where P and Q are constants or function of x only

$I.F. = e^{\int P dx}$ (I.F.= Integrating factor)

solution of (i) is,

$$\boxed{y.(I.F.) = \int Q \times (I.F.)\, dx + c}$$

$$\frac{dx}{dy} + Px = Q \quad \text{---(i)}$$

where P and Q are constants or function of y only

$I.F. = e^{\int P dy}$ (I.F.= Integrating factor)

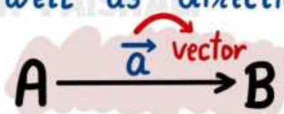
solution of (i) is,

$$\boxed{x.(I.F.) = \int Q \times (I.F.)\, dy + c}$$

UNIT- 4 (VECTORS AND THREE - DIMENSIONAL GEOMETRY)

Vectors

➡ **Vector** :- A quantity that has magnitude as well as direction is called a vector denoted by $\overrightarrow{AB}$ or $\vec{a}$.



➡ **Initial Point**

The point A where from the vector $\overrightarrow{AB}$ starts is known as initial point.

➡ **Terminal Point**

The point B, where it ends is said to be the terminal point.

➡ **Magnitude** :-

The distance between initial and terminal points of a vector is called the magnitude (or length) of the vector.

➡ **Scalar** :-

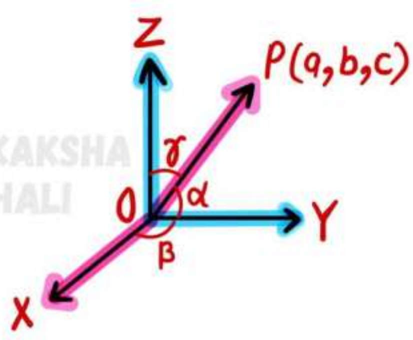
Those physical quantities which have only magnitude are called scalar.

Example - area, volume, mass etc.

➡ **Direction Cosines** :- If $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ makes angle α, β, γ with +ve direction of x-axis, y-axis and z-axis respectively, than $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the direction cosines of $\vec{r}$ and are denoted by l, m and n where,

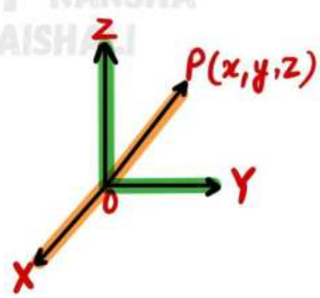
$$l = \cos\alpha = \frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \cos\beta = \frac{b}{\sqrt{a^2+b^2+c^2}},$$

$$n = \cos\gamma = \frac{c}{\sqrt{a^2+b^2+c^2}}$$



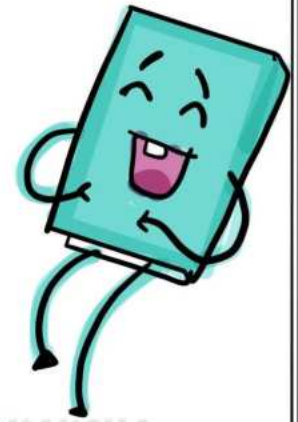
➡ **Direction Ratios** :- If numbers a, b, c are proportional to direction cosines l, m and n respectively of $\vec{r}$, then a, b, c are called direction ratios of $\vec{r}$.

➡ **Position Vector** :- Consider a point (x, y, z) in space. The vector $\overrightarrow{OP}$ with initial point, origin O and terminal point P, is called the position vector of P.



Zero Vector :- A vector whose initial and terminal points coincide is known as zero vector.

Unit Vector :- A vector whose magnitude is unity is said to be unit vector denoted by $\hat{a}$. $\boxed{\hat{a} = \frac{1}{|\vec{a}|}\vec{a}}$



Co-initial Vectors :-

Two or more vectors having the same initial point are called coinitial vectors.

Collinear Vectors :- Two or more vectors are said to be collinear if they are parallel to the same line, irrespective of their magnitudes and directions.

Equal Vectors :- Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal, if they have the same magnitude and direction regardless of the positions of their initial points, and written as $\vec{a} = \vec{b}$.

Negative Of a vector :- A vector whose magnitude is the same as that of a given vector, but direction is opposite to that of it, is called negative of the given vector. $\overrightarrow{BA} = -\overrightarrow{AB}$

Addition Of Vectors :- $\boxed{\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}}$ (Triangle law Of Vector addition)

Properties Of A Vector addition :-

(i) $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

(ii) $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

(iii) $\vec{a} + \vec{0} = \vec{0} + \vec{a} = \vec{a}$

Multiplication Of A Vector by a scalar :-

$\boxed{|\lambda\vec{a}| = |\lambda||\vec{a}|}$ Scalar

Midpoint :-

$$\boxed{\vec{r} = \frac{\vec{a}+\vec{b}}{2}}$$

note :- for any scalar k, $\boxed{k\vec{0} = \vec{0}}$

Component Form :-

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ vector component

$|\vec{r}| = \sqrt{x^2+y^2+z^2}$

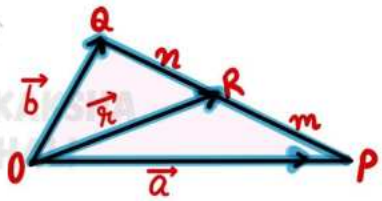
x, y, z = scalar components of $\vec{r}$

Vector Joining Two Points :

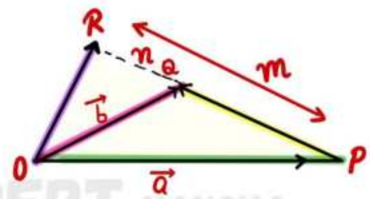
$$|\vec{P_1P_2}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Section Formula :-

case I When R divides PQ internally $\boxed{\vec{r} = \dfrac{m\vec{b}+n\vec{a}}{m+n}}$



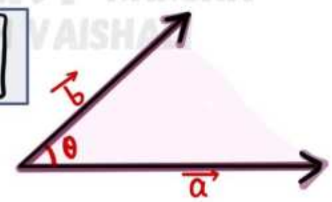
Case II When R divides PQ externally $\boxed{\vec{r} = \dfrac{m\vec{b}-n\vec{a}}{m-n}}$

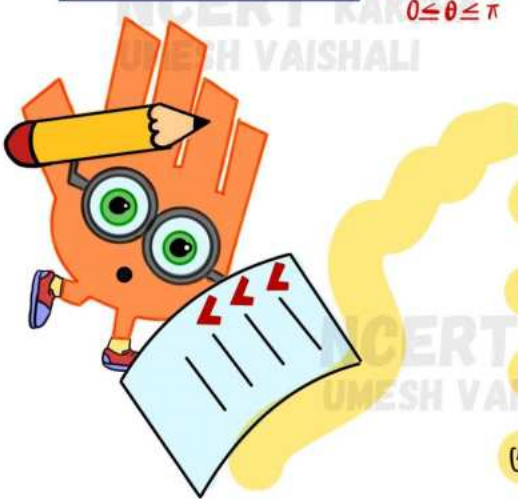


Scalar (or dot) product Of Two vectors :-

$\boxed{\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta}$ θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$

$\boxed{\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}}$ $\boxed{\theta = \cos^{-1}\left[\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right]}$





Properties :-

(i) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

(2) $\vec{a}\cdot(\vec{b}+c) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$

(3) $(\lambda\vec{a})\cdot\vec{b} = \lambda(\vec{a}\cdot\vec{b}) = \vec{a}\cdot(\lambda\vec{b})$

(4) $\vec{a}\cdot\vec{b} = 0 \Leftrightarrow \vec{a} = 0, \vec{b} = 0$ OR $\vec{a} \perp \vec{b}$

5. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

6. Projection Of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ and Projection vector of $\vec{a}$ on $\vec{b} = \left|\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right| \cdot \vec{b}$

7. Projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$ and Projection vector of $\vec{b}$ on $\vec{a} = \left|\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right|\cdot\vec{a}$

note:-

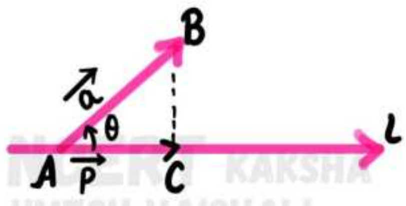
- If two vectors $\vec{a}$ and $\vec{b}$ are given in component form as $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ $\quad \vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$
- If $\vec{a}$ & $\vec{b}$ are the position vectors of two points A & B, then $\quad \overrightarrow{AB} = \vec{b} - \vec{a}$
- Two vectors are said to be arthogonal if they are perpendicular to each other.

➡ Observations:-

- ► $\vec{a}\cdot\vec{b}$ is a real number.
- ► Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors, then $\vec{a}\cdot\vec{b} = 0$ if and only if $\vec{a}$ and $\vec{b}$ are perpendicular to each other i.e. $\vec{a}\cdot\vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$
- ► If $\theta = 0$ then $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$. In Particular $\vec{a}\cdot\vec{a} = |\vec{a}|^2$, as θ in this case is 0.
- ► If $\theta = \pi$ then $\vec{a}\cdot\vec{b} = -|\vec{a}||\vec{b}|$. In Particular $\vec{a}\cdot(-\vec{a}) = -|\vec{a}|^2$, as θ in this case is π.
- ► In view of the observations 2 and 3, for mutually perpendicular unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$, we have $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$ $\qquad \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
- ► The angle between two non-zero vectors $\vec{a}$ and $\vec{b}$ is given by $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$
- ► The scalar product is commutative i.e. $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

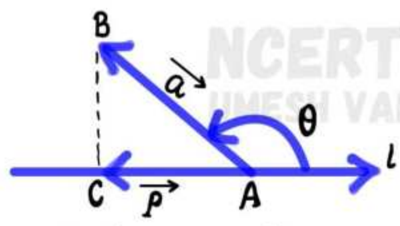
➡ Projection of a vector on a line :-

The $\vec{P}$ is called the projection vector and its magnitude $|\vec{P}|$ is simply called as the projection of the vector $\overrightarrow{AB}$ on the directed line l.



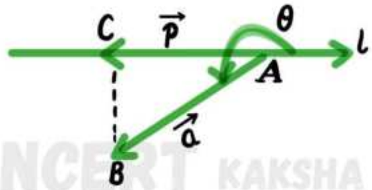
$(0° < \theta < 90°)$

(i)

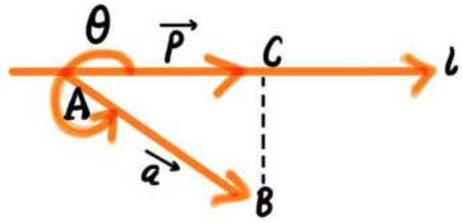


$(90° < \theta < 180°)$

(ii)



(180° < θ < 270°)
(iii)



(270° < θ < 360°) (iv)

Observations :-

* If $\vec{P}$ is the unit vector along a line l, then the projection of a vector $\vec{a}$ on the line l is given by $\vec{a}\cdot\hat{p}$.
* Projection of a vector $\vec{a}$ on other vector $\vec{b}$, is given by

$$\vec{a}\cdot\hat{b} \text{ OR } \vec{a}\cdot\left[\frac{\vec{b}}{|\vec{b}|}\right] \text{ OR } \frac{1}{|\vec{b}|}(\vec{a}\cdot\vec{b})$$

* If θ = 0, then the projection vector of $\vec{AB}$ will be $\vec{AB}$ itself and if θ = π, then the projection vector of $\vec{AB}$ will be $\vec{BA}$.
* If $\theta = \frac{\pi}{2}$ OR $\theta = \frac{3\pi}{2}$, then the projection vector of $\vec{AB}$ will be zero vector.

note :-

If α, β and γ are the direction angles of vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, then its direction cosines may be given as -

$$\cos\alpha = \frac{\vec{a}\cdot\hat{i}}{|\vec{a}||\hat{i}|} = \frac{a_1}{|\vec{a}|}, \quad \cos\beta = \frac{a_2}{|\vec{a}|} \quad \text{and} \quad \cos\gamma = \frac{a_3}{|\vec{a}|}$$

$$\vec{a} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$$

$\vec{a}$ = unit vector
a_1, a_2, a_3 = scalar components

Vector (or cross) Product Of Two Vectors :-

$$\boxed{\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}} \quad \text{OR} \quad \boxed{\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|}}$$

θ is the angle between $\vec{a}$ & $\vec{b}$, $0 \le \theta \le \pi$ n = unit vector ⊥ to the plane $\vec{a}$ and $\vec{b}$

Properties :-

1. $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$
2. If $\vec{a}\times\vec{b} = 0 \Leftrightarrow \vec{a} = 0, \vec{b} = 0$ or $\vec{a} \parallel \vec{b}$

3. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Observations :-

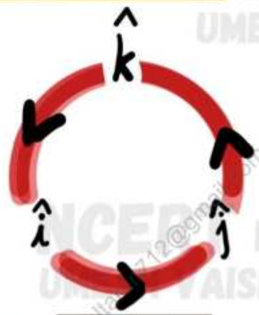
[1] $\vec{a} \times \vec{b}$ is a vector.

[2] If $\theta = \frac{\pi}{2}$ then $\boxed{\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|}$

[3] Angle between two vectors $\vec{a}$ and $\vec{b}$ $\boxed{\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}}$

[4] $\boxed{\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}}$

$\boxed{\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{k} = \hat{i},\ \hat{k} \times \hat{i} = \hat{j}}$



(5) $\boxed{\hat{j} \times \hat{i} = -\hat{k}}$, $\boxed{\hat{k} \times \hat{j} = -\hat{i}}$ and $\boxed{\hat{i} \times \hat{k} = -\hat{j}}$

Area of triangle ABC $= \frac{1}{2}|\vec{b}||\vec{a}|\sin\theta = \boxed{\frac{1}{2}|\vec{a} \times \vec{b}|}$

Area of Parallelogram ABCD $= |\vec{b}||\vec{a}|\sin\theta = \boxed{|\vec{a} \times \vec{b}|}$

Projection Formulae :-

- $a = b\cos C + c\cos B$
- $b = c\cos A + a\cos C$
- $c = a\cos B + b\cos A$

Lagrange's Identity :-

$$\boxed{|\vec{a} \times \vec{b}|^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} \end{vmatrix}} \quad \text{OR} \quad \boxed{(\vec{a}\cdot\vec{b})^2 + (\vec{a} \times \vec{b})^2 = |\vec{a}|^2 + |\vec{b}|^2}$$

note :-

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$$

$$|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2\left[|\vec{a}|^2 + |\vec{b}|^2\right]$$

$$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$$

Scalar Triple Product :- $[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a}.(\vec{b}\times\vec{c}) = \vec{b}.(\vec{c}\times\vec{a}) = \vec{c}\cdot(\vec{a}\times\vec{b})$

note :-

$[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\,\vec{c}\,\vec{a}] = [\vec{c}\,\vec{a}\,\vec{b}]$

$[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c}) = 0 \Leftrightarrow \vec{a}.\vec{b}.\vec{c}$ are coplanar

Find a way not an Excuse.

UNIT- 4 (VECTORS AND THREE - DIMENSIONAL GEOMETRY)

CHAPTER - 11

Three Dimensional Geometry

➡ Relation between the direction cosines of a line $\boxed{l^2 + m^2 + n^2 = 1}$ direction cosines

➡ Direction cosines of a line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

$$\boxed{\frac{x_2 - x_1}{PQ} = \frac{y_2 - y_1}{PQ} = \frac{z_2 - z_1}{PQ}}$$

where $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

➡ If l, m, n are the direction cosines and a, b, c are the direction ratios of a line

$$\boxed{l = \frac{a}{\sqrt{a^2+b^2+c^2}}} \; ; \; \boxed{m = \frac{b}{\sqrt{a^2+b^2+c^2}}} \; ; \; \boxed{n = \frac{c}{\sqrt{a^2+b^2+c^2}}}$$

➡ Vector Equation of a line that passes through the given point whose position vectors $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\boxed{\vec{r} = \vec{a} + \lambda \vec{b}}$ (vector form)

Cartesian Equation,

$$\boxed{\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}} \quad \text{OR} \quad \boxed{\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}}$$

direction Ratios

➡ Vector Equation of a line that passes through two points

$$\boxed{\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}, \; \lambda \in R$$

Position vectors

Cartesian Equation points :- (x_1, y_1, z_1) (x_2, y_2, z_2)

$$\boxed{\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}}$$

➡ If a_1, b_1, c_1 and a_2, b_2, c_2 are the direction ratios of two lines and θ is the acute angle between two lines then;

$$\boxed{\cos\theta = \left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|}$$

Two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are

(i) Perpendicular $\theta = 90^\circ$ $\boxed{a_1 a_2 + b_1 b_2 + c_1 c_2 = 0}$

(ii) parallel $\theta = 0^\circ$ $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$

If θ is the acute angle between the line $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda \vec{b_2}$; then θ is given by: $\boxed{\cos\theta = \left|\frac{|\vec{b_1} \cdot \vec{b_2}|}{|\vec{b_1}||\vec{b_2}|}\right|}$ OR $\boxed{\theta = \cos^{-1} \frac{|\vec{b_1} \cdot \vec{b_2}|}{|\vec{b_1}||\vec{b_2}|}}$

cartesian form, lines $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ is

$$\boxed{\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1 c_2 - b_2 c_1)^2 + (c_1 a_2 - c_2 a_1)^2 + (a_1 b_2 - a_2 b_1)^2}}}$$

➡ The shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ is $\boxed{\left|\frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|}\right|}$ (vector form)

➡ Shortest Distance between Parallel Lines $\vec{r} = \vec{a_1} + \lambda \vec{b}$ and $\vec{r} = \vec{a_2} + \mu \vec{b}$ is $\boxed{\left|\frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|}\right|}$

➡ Equation of a plane in a normal form $\boxed{\vec{r} \cdot \hat{n} = d}$ (vector Form)

Position vector · unit normal vector · distance from origin

cartesian form, $\boxed{lx + my + nz = d}$

➡ The equation of a plane through a point whose Position vector is $\vec{a}$ and perpendicular to the vector $\vec{N}$ is $\boxed{(\vec{r} - \vec{a}) \cdot \vec{N} = 0}$ (vector Form)

cartesian form, $\boxed{A(x - x_1) + B(y - y_1) + C(z - z_1) = 0}$

➡ Equation of a Plane Passing through three non collinear points

$\boxed{(\vec{r} - \vec{a})[\vec{b} - \vec{a}] \times (\vec{c} - \vec{a})] = 0}$ (vector Form) cartesian form, $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

➡ Plane Passing through the intersection of Two given Planes $\boxed{\vec{r}\cdot(\vec{n_1}+\lambda\vec{n_2}) = d_1+\lambda d_2}$ (vector form)

cartesian form,

$$\boxed{(A_1x+B_1y+C_1z-d_1)+\lambda(A_2x+B_2y+C_2z-d_2)=0}$$

➡ Angle between Two Planes $\boxed{\cos\theta = \left|\dfrac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right|}$

cartesian form, $\boxed{\cos\theta = \left|\dfrac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|}$

➡ Angle between a line and a plane $\boxed{\cos\theta = \left|\dfrac{\vec{b}\cdot\vec{n}}{|\vec{b}|\cdot|\vec{n}|}\right|}$

the angle ϕ between the line and the plane is given by $90^\circ-\theta$, i.e.

$\sin(90^\circ-\theta) = \cos\theta$ $\boxed{\sin\phi = \left|\dfrac{\vec{b}\cdot\vec{n}}{|\vec{b}|\cdot|\vec{n}|}\right|}$ **OR** $\boxed{\phi = \sin^{-1}\left|\dfrac{\vec{b}\cdot\vec{n}}{|\vec{b}|\cdot|\vec{n}|}\right|}$

note:- If $\vec{b}\cdot\vec{n} = 0 \Rightarrow \sin\phi = 0 \Rightarrow \phi = 0^\circ \Rightarrow$ line is parallel to the plane

➡ Equation of a Plane in intercept form: $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c} = 1$ where a, b, c are intercepts made by the Plane on the x-axis, y-axis & z-axis respectively

note:- (i) If $\vec{n_1}\cdot\vec{n_2} = 0 \Rightarrow$ planes are perpendicular

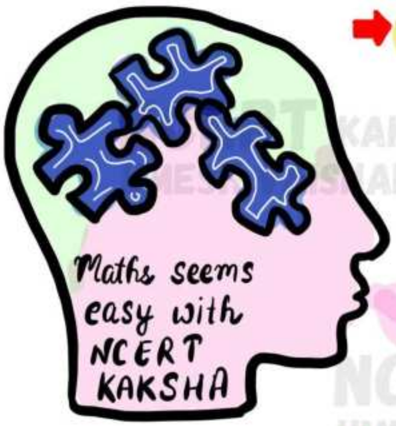
(ii) If $\vec{n_1} = \lambda\vec{n_2} \Rightarrow$ both planes are parallel

(iii) angle between two planes is always taken as acute angle.

CHAPTER-12

Linear Programming

➡ **Linear Programming** :- Linear Programming (LP) is an optimisation technique in which a linear function is optimised (i.e. minimised or maximised) subject to certain constraints which are in the form of linear inequalities and equations. The function to be optimised is called objective function.



➡ **Applications Of Linear Programming** :- Linear Programming optimum combination of several variables subject to certain constraints or restrictions.

➡ **Formation Of Linear Programming Problem (LPP)** :-

The basic problem in the formulation of a linear programming problem is to set-up some mathematical model. This can be done by asking the following questions:

(a) What are the unknown (variables)?

(b) What is the objective?

(c) What are the restrictions?

For this, let $x_1, x_2, x_3 \ldots\ldots x_n$ be the variables. Let the objective function to be optimized (i.e. minimised or maximised) be given Z.

(i) $Z = C_1 x_1 + C_2 x_2 + \ldots + C_n x_n$ where $c_i x_i$ $(i = 1, 2, \ldots, n)$ are constraints.

(ii) Let there be mn constants and let a be a set of constants such that

$$a_{11} x_1 + a_{12} x_2 + \ldots\ldots + a_n x_n \ (\le, = \text{ or } \ge)\ b_1$$

$$a_{21} x_1 + a_{22} x_2 + \ldots\ldots + a_n x_n \ (\le, = \text{ or } \ge)\ b$$

$$\ldots\ldots \qquad \ldots\ldots \qquad \ldots\ldots$$

$$\ldots\ldots \qquad \ldots\ldots \qquad \ldots\ldots$$

$$a_{m_1} x_1 + a_{m_2} x_2 + \ldots\ldots + a_{mn} x_n \ (\le, = \text{ or } \ge)\ b_m$$

(iii) Finally, let $x_1 > 0, x_2 > 0, \ldots x_n \ge 0$ called non-negative constraints.

The problem of determinating values of $x_1, x_2, \ldots x_n$ which makes z, a minimum or maximum and which satisfies (ii) and (iii) is called the general linear programming problem.

General LPP :-

(a) Decision Variables ~ The variables $x_1, x_2, x_3, \ldots, x_n$ whose values are to be decided, are called decision variables.

(b) Objective function :- The linear function $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ which is to be optimized (maximised or minimised) is called the objective function or preference function of the general linear programming problem.

(c) Structural Constraints ~ The inequalities given in (ii) are called the structural constraints of the general linear programming problem. The structural constraints are generally in the form of inequalities of $\geq$ type or $\leq$ type, but occasionally, a structural constraint may be in the form of an equation.

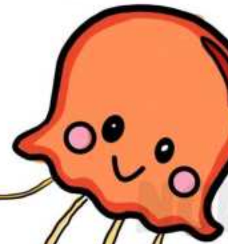
(d) Non-Negative Constraints :- The set of inequalities (iii) is usually known as the set of non-negative constraints of the general LPP. These constraints imply that the variables $x_1, x_2, \ldots x_n$ cannot take negative values.

(e) Feasible Solution :- Any solution of a general LPP which satisfies all the constraints, structural and non negative, of the Problem, is called a feasible solution of general LPP.

(f) Optimum Solution :- Any feasible solution which optimizes (i.e. minimize or maximises) the objective function of the LPP is called optimum solution.

Requirements for Mathematical Formulation of LPP :-

Before getting the mathematical form of a linear Programming problem, it is important to recognize the problem which can be handled by linear programming problem. For the formulation of a linear programming problem, the problem must satisfy the following requirements.



1. There must be an objective to minimise or maximise something. The objective must be capable of being clearly defined mathematically as a linear function.

2. There must be alternative sources of action so that the problem of selecting the best course of actions may arise.

3. The resources must be in economically quantifiable limited supply. They gives the constraints to LPP.

4. The constraints (restrictions) must be capable of being expressed in the form of linear equations or inequalities.

➡ Solving linear Programming Problem :-

To solve linear Programming problems, corner point method is adopted under this method following steps are performed:

▶ **Step I** ~ At first, feasible region is obtained by plotting the graph of given linear constraints and its corner points are obtained by solving the two equations of the lines intersecting at that point.

▶ **Step II** ~ The value of objective function $Z = ax + by$ is obtained for each corner point by putting its x and y coordinate in place of x and y in $Z = ax + by$. Let M and m be largest and smallest value of Z respectively.

case I : If the feasible is bounded, then M and m are the maximum and minimum values of z.

case II : If the feasible is unbounded, then we proceed as follow :-

▶ **Step III** : The open half plane determined by $ax + by > M$ and $ax + by < m$ are obtained.

case I : If there is no common point in the half plane determined by $ax+by>M$ and feasible region, there M is maximum value of z, otherwise z has no maximum value.

case II : If there is no common point in the half plane determined by $ax+by<m$ and feasible region, then m is minimum value of z, otherwise z has no minimum value.

UNIT-6 (PROBABILITY)

CHAPTER-13

Probability

Conditional Probability :- If E and F are two events associated with the same sample space of a random experiment, then the conditional probability of the event E under the condition that the event F has occurred written as $P\left(\frac{E}{F}\right)$ is given by,

$$\boxed{P\left(\frac{E}{F}\right)=\frac{P(E\cap F)}{P(F)}}\ ;\ P(F)\neq 0$$

note :- $\boxed{0\leq P\left(\frac{E}{F}\right)\leq 1}$

Properties of Conditional Probability :- let E and F be events associated with the sample space S of an experiment. Then,

- $P\left(\frac{S}{F}\right)=P\left(\frac{F}{F}\right)=1$
- $P\left[\frac{(A\cup B)}{F}\right]=P\left(\frac{A}{F}\right)+P\left(\frac{B}{F}\right)-P\left[\frac{(A\cap B)}{F}\right]$
- $P\left(\frac{E'}{F}\right)=1-P\left(\frac{E}{F}\right)$

Multiplication Theorem on Probability :- let E and F be Two events associated with a sample space of an experiment. Then,

$$P(E\cap F)=P(E)P\left(\frac{F}{E}\right);\ P(E)\neq 0$$
$$=P(F)\,P\left(\frac{E}{F}\right);\ P(F)\neq 0$$

IF E, F and G are 3 events associated with a sample space. then,

$$P(E\cap F\cap G)=P(E)\ P\left(\frac{F}{E}\right)\ P\left[\frac{G}{E\cap F}\right]$$

Independent Events :- let E and F be two events associated with the same random experiment, then E and F are said to be independent if,

$$\boxed{P(E\cap F)=P(E).\,P(F)}$$

note :- $\boxed{P\left(\frac{E}{F}\right)=P(E),\ P(F)\neq 0}$ $\boxed{P\left(\frac{F}{E}\right)=P(F),\ P(E)\neq 0}$

Dependent Events: Two events E and F are said to be dependent if they are not independent, i.e. if $\boxed{P(E \cap F) \neq P(E).P(F)}$

Three events A, B and C are said to be independent of all the following conditions hold:

$$P(A \cap B) = P(A)\,P(B)$$
$$P(A \cap C) = P(A)\,P(C)$$
$$P(B \cap C) = P(B)\,P(C)$$
$$\text{and } P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$$

Bayes' Theorem:- If $E_1, E_2 \ldots E_n$ are mutually exclusive and exhaustive events associated with a sample space and A as any event of non-zero probability then;

$$\boxed{P\left(\frac{E_i}{A}\right) = \frac{P(E_i)\,P\left(\frac{A}{E_i}\right)}{\sum_{j=1}^{n} P(E_j)\,P\left(\frac{A}{E_j}\right)}}$$

Theorem Of Total Probability:-

let $\{E_1, E_2, \ldots E_n\}$ be a partition of the sample space S. let A be any event associated with S, then:

$$\boxed{P(A) = \sum_{j=1}^{n} P(E_j)\,P\left(\frac{A}{E_j}\right)}$$

Random variable and its probability Distribution:-

A random variable is a real valued whose domain is the sample space of a random experiment, The Probability distribution of a random variable X is the system of numbers.

$$\begin{cases} X & : x_1 \quad x_2 \ldots\ldots x_n \\ P(X) & : P_1 \quad P_2 \ldots\ldots P_n \end{cases} \quad \text{where } P_i > 0;\ \sum_{i=1}^{n} P_i = 1 \quad i = 1, 2 \ldots\ldots n$$

Mean Of a Random variable:- let X be a random variable assume $x_1, x_2, \ldots x_n$ [The expectation of X or E(X)] with probabilities $P_1, P_2, \ldots P_n$ respectively. Mean of X, denoted by μ is the number $\sum_{i=1}^{n} x_i P_i$

$$E(X)=\mu=\sum_{i=1}^{n} x_i P_i = x_1 p_1 + x_2 p_2 + \dots + x_n p_n$$

Variance Of A Random Variable :- $$\sigma^2=\sum_{i=1}^{n}(x_i-\mu)^2 P_i=\sum_{i=1}^{n} x_i^2 P_i-\mu^2$$

or equivalently $$\sigma^2=E(X-\mu)^2$$

Standard deviation of the random variable X is defined as :

$$\sigma=\sqrt{\text{Variance}(X)}=\sqrt{\sum_{i=1}^{n}(x_i-\mu)^2 P_i}$$

Bernoulli Trials :- Trials of a random experiment are called Bernoulli trivals, if they satisfy the following conditions :-

1. There should be fine no. of trials.
2. The trials should be independent.
3. Each trial has exactly two outcomes : success or failure.
4. The Probability of success (or failure) remains the same in each trial.

Binomial Distribution :- A random variable X taking values 0, 1, 2 ..., n is said to have binomial distribution with parameters n and P of its probability distribution is given by :

$$P(X=r)={}^{n}C_r\, P^r q^{n-r}$$ where $q=1-p$ and $r=0,1,2 \dots n$

note :- $$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

In the case of three events ~

$$P(A\cap B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$$

$P(A\cup B)=P(A)+P(B)$, If two events A and B are mutually exclusive.

$P(\bar{A}\cap B)=P(B)-P(A\cap B)$, where $\bar{A}$ and B are independent events.

$\boxed{P(A\cap\bar{B}) = P(A)-P(A\cap B)}$, where A and $\bar{B}$ are independent events.

$\boxed{P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B}) = 1-P(A\cup B) = P(\bar{A})\times P(\bar{B})}$, where A and B are mutually exclusive events

$\boxed{P\left(\frac{\bar{A}}{B}\right) = \frac{P(\bar{A}\cap B)}{P(B)} = \frac{P(B)-P(A\cap B)}{P(B)}}$, where $\bar{A}$ and B are independent events and $P(B)\neq 0$.

$\boxed{P\left(\frac{\bar{B}}{\bar{A}}\right) = \frac{P(\bar{A}\cap\bar{B})}{P(\bar{A})} = \frac{1-P(A\cap B)}{1-P(A)}}$, where $\bar{A}$ and $\bar{B}$ are independent events and $P(A)\neq 0$.

Focus on the Goal ♥