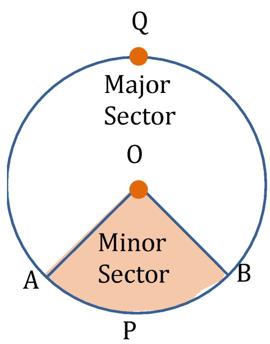
8 Areas Related to Circles

Do You Remember ?

Sector: The portion (or part) of the circular region enclosed by two radii and the corresponding arc of a circle is called a sector of the circle.

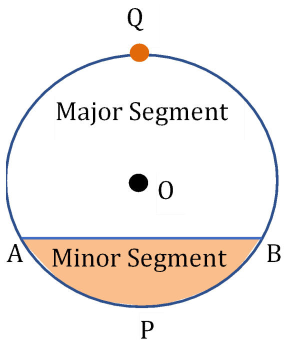


Here region OAPB is a sector of the circle with centre O. ∠AOB is called the angle of the sector. Here unshaded region OAPB is called minor sector and shaded region OAQB is called major sector.

Angle of the major sector = 360° – ∠AOB.

Do You Remember ?

- **Segment:** The portion (or part) of the circular region enclosed between a chord and the corresponding arc of a circle is called a segment of the circle.
  Shaded region APB is a segment of the circle. The unshaded region AQB is another segment of the circle formed by the chord AB.



When we write 'segment' and 'sector', we mean the 'minor segment' and the 'minor sector' respectively, unless stated otherwise.

SPOT LIGHT

APB is called the minor segment and AQB is called the major segment.

Method to calculate area of sector and segment

Let OAPB be a sector of a circle with centre O and radius r. Let ∠AOB = θ. Now area of the circle (in fact a circular region of disc) = πr^2. We can regard this circular region on disc as a sector forming an angle of 360° (i.e. of degree measure 360) at the centre O. By unitary method, area of the sector OAPB can be calculated as follows

When the degree measure of the angle at the centre is 360, area of the sector = πr^2

When the degree measure is 1, area of sector = $\frac{1}{360} \times \pi r^2$

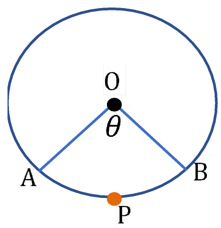
When the degree measure is θ, area of sector = $\frac{\theta}{360} \times \pi r^2$

Thus, area of the sector OAPB (i.e., sector of angle θ) = $\frac{\theta}{360} \times \pi r^2$

where r is the radius of the circle and θ is the angle of the sector.

Next, we shall find the length of the arc APB corresponding to this sector by unitary method.

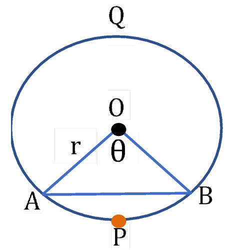
If angle subtended at the centre is 360°, then length of the arc = $2\pi r$



∴ If the angle at the centre is θ, then length of the arc of sector APB $= \frac{\theta}{360} \times 2\pi r$

Thus, the length of the arc of segment OAPB

i.e., length of arc of sector of angle θ = $\frac{\theta}{360} \times 2\pi r$



Now, area of segment APB.

= area of sector OAPB – area of ΔOAB

$\frac{\theta}{360} \times \pi r^2$ – area of ΔOAB

Cor. 1. Area of major sector AQB = πr^2 – area of the minor sector OAPB = $\frac{360-\theta}{360} \times \pi r^2$

Cor. 2. Area of major segment AQB = πr^2 – area of the minor segment APB.

Note: Area of ΔOAB with ∠AOB = θ is $\frac{1}{2} r^2 \sin\theta$ **or** $r^2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right)$

Quick Tips

- If θ = 90°, then ΔAOB is right triangle. So, area of ΔAOB = $\frac{1}{2}\times AO\times BO$ = $\frac{1}{2}\times r^2$

- If θ = 60°, then ΔAOB is equilateral triangles. So, of ΔAOB = $\frac{\sqrt{3}}{4}\times r^2$

- If θ = 120°, then ΔAOB is isosceles triangle. So, area of ΔAOB = $\frac{1}{2}r^2\sin 120°$

$= \frac{1}{2}r^2\times\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4}r^2$

Numerical Ability 1

The perimeter of a semi-circular protractor is 32.4 cm. Calculate:

(i) The radius of the protractor in cm

(ii) The area of the protractor in cm$_2$.

Solution

(i) Let the radius of the protractor be r cm.

Then, its perimeter = (πr + 2r) cm.

∴ πr + 2r = 32. 4

⇒ (π + 2)r = 32.4

⇒ $\left(\frac{22}{7}+2\right)$ r = 32.4

⇒ $\frac{36}{7}$ r = 32.4

⇒ r = $\left(32.4\times\frac{7}{36}\right)$ cm

= 6.3 cm.

Radius of the protractor = 6.3 cm.

(ii) Area of the protractor

$= \frac{1}{2}\pi r_2 = \left(\frac{1}{2}\times\frac{22}{7}\times6.3\times6.3\right) cm_2$

$= 62.37 cm_2$.

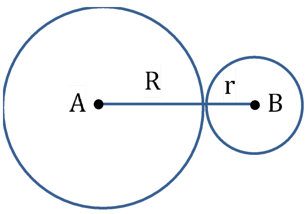
∴ Area of the protractor = 62.37 cm_2.

Numerical Ability 2

Two circles touch externally. The sum of their areas is 130π sq. cm and the distance between their centres is 14 cm. Determine the radii of the circles.

Solution

Let the radii of the given circles be R cm and r cm respectively as shown in figure. As the circles touch externally, distance between their centres = (R + r) cm.



$R + r = 14$ ... (i)

Sum of their areas = $(\pi R_2 + \pi r_2)$ cm_2 = $\pi(R_2 + r_2)$ cm_2.

$\pi(R_2 + r_2) = 130\pi$

⇒ $R_2 + r_2 = 130$ ...(ii)

We have the identity, $(R + r)_2 + (R - r)_2 = 2(R_2 + r_2)$

⇒ $(14)_2 + (R - r)_2 = 2 \times 130$ [From (i) and (ii)]

⇒ $(R - r)_2 = 64$

⇒ $R - r = 8$ ... (iii)

On solving (i) and (iii), we get R = 11 cm and r = 3 cm.

Hence, the radii of the given circles are 11 cm and 3 cm.

Numerical Ability 3

A chord of a circle of radius 14 cm makes a right angle at the centre. Calculate:

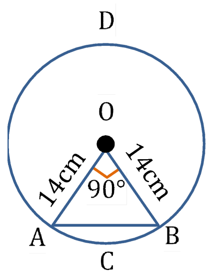
(i) The area of the minor segment of the circle.

(ii) The area of the major segment of the circle.

Solution

Let AB be the chord of a circle with centre O and radius 14 cm such that ∠AOB = 90°.

Thus, r = 14 cm and θ = 90°.



(i) Area of sector OACB = $\frac{\pi r^2 \theta}{360} = \left(\frac{22}{7} \times 14 \times 14 \times \frac{90}{360}\right) cm_2 = 154\ cm_2$.

Area of ΔOAB = $\frac{1}{2} r^2 \sin\theta = \left(\frac{1}{2} \times 14 \times 14 \times \sin 90^\circ\right) cm_2 = 98\ cm_2$.

∴ Area of minor segment ACBA = (Area of sector OACB) – (Area of ΔOAB)

$= (154 - 98)\ cm_2 = 56\ cm_2$.

(ii) Area of major segment BDAB = (Area of the circle) – (Area of minor segment ACBA)

$= \left[\left(\frac{22}{7} \times 14 \times 14\right) - 56\right] cm_2 = (616 - 56)\ cm_2 = 560\ cm_2$.

Numerical Ability 4

The minute hand of a clock is 10.5 cm long. Find the area swept by it in 15 minutes.

Solution

Angle described by minute hand in 60 minutes = 360°.

Angle described by minute hand in 15 minutes = $\left(\frac{360}{60}\times 15\right)^{\circ}$ = 90°.

Thus, the required area is the area of a sector of a circle with central angle, θ = 90° and radius, r = 10.5 cm.

∴ Required area = $\left(\frac{\pi r^2 \theta}{360}\right)$

= $\left(\frac{22}{7}\times 10.5\times 10.5\times \frac{90}{360}\right)$ cm_2 = 86.63 cm_2.

Area of combinations of plane figures

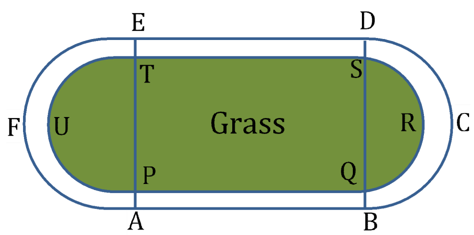
In this section, we shall calculate the areas of some combinations of plane figures. In our daily life, we see figures like flowerbeds, drain covers, window designs, designs on table covers etc. These are combinations of plane figures.

Numerical Ability 5

The figure shows a running track surrounding a grass enclosure PQRSTU. The enclosure consists of a rectangle PQST with a semi-circular region at each end. Given, PQ = 200 m and PT = 70 m.

(i) Calculate the area of the grassed enclosure in m_2.

(ii) Given that the track is of constant width 7 m, calculate the outer perimeter ABCDEF of the track.



Solution

(i) Diameter of each semi-circular region of grassed enclosure = PT = 70 m,

∴ Radius of each one of them = 35 m.

Area of grassed enclosure

⇒ (Area of rect. PQST) + 2 (Area of semi-circular region with radius 35 m)

⇒ (PQ × PT) + 2 × $\frac{1}{2}\pi r_2$ = $\left[(200\times70)+\frac{22}{7}\times35\times35\right]$ m_2 = 17850 m_2.

(ii) Diameter of each outer semi-circle of the track

⇒ AE = (PT + 7 + 7) m = 84 m.

∴ Radius of each one of them = 42 m.

Outer perimeter ABCDEF = (AB + DE + semi-circle BCD + semi-circle EFA)

⇒ (2PQ + 2 × circumference of semi-circle with radius 42 m)

⇒ (2 × 200 + 2 × π × 42) m = $\left[2\times200+2\times\frac{22}{7}\times42\right]$ m = 664 m.

Do You Remember ?

- In an equilateral Δ of side a units, Height = $\frac{\sqrt{3}}{2}$ a, Inradius = $\frac{a}{2\sqrt{3}}$, Circumradius = $\frac{a}{\sqrt{3}}$

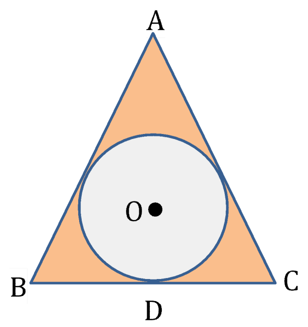
Numerical Ability 6

In an equilateral triangle of side 24 cm, a circle is inscribed, touching its sides. Find the area of the remaining portion of the triangle. Take $\sqrt{3}$ = 1.73 and π = 3.14.

Solution

Let ΔABC be the given equilateral triangle in which a circle is inscribed.

Side of the triangle, a = 24 cm.



Height of the triangle, h = $\left(\frac{\sqrt{3}}{2}\times a\right)$ cm = $\left(\frac{\sqrt{3}}{2}\times 24\right)$ cm = 12 $\sqrt{3}$ cm.

Radius of the incircle, r = $\frac{1}{3}$ h = $\left(\frac{1}{3}\times 12\sqrt{3}\right)$ cm = 4 $\sqrt{3}$ cm.

∴ Required Area = Area of the shaded region

⇒ (Area of ΔABC) – (Area of incircle)

⇒ $\left(\frac{\sqrt{3}}{4}\times 24\times 24-\pi\times 4\sqrt{3}\times 4\sqrt{3}\right)cm^2$

⇒ (144 $\sqrt{3}$ – 3 .14 × 48) cm_2 = (144 × 1.73 – 3.14 × 48) cm_2

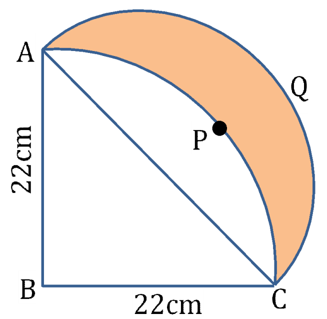
⇒ [48 × (3 × 1.73 – 3.14)] cm_2 = (48 × 2.05) cm_2 = 98.4 cm_2

Numerical Ability 7

In the given figure, ABCPA is a quadrant of a circle of radius 22 cm. With AC as diameter, a semi-circle is drawn. Find the area of the shaded region.

Solution

Since, ABCPA is a quadrant of a circle of radius 22 cm.



we have,

AB = BC = 22 cm and ∠ABC = 90°.

AC = $\sqrt{AB^2+BC^2}$ [by Pythagoras theorem]

= $\sqrt{(22)^2+(22)^2}$ cm = $\sqrt{2\times 484}$ cm = $22\sqrt{2}\,cm$

Radius of the semicircle = $\frac{1}{2}$ AC = $11\sqrt{2}\,cm$

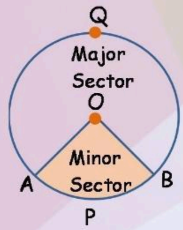
∴ Required area = (area of the semicircle with AC as diameter) + (area of ΔABC) – (area of the quadrant with r = 22 cm)

⇒ $\left[\frac{1}{2}\times\frac{22}{7}\times(11\sqrt{2})^2+\frac{1}{2}\times22\times22-\frac{1}{4}\times\frac{22}{7}\times22\times22\right]$ cm²

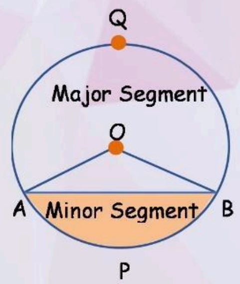
⇒ $\frac{2662}{7} + 242 - \frac{2662}{7} = 242 \text{ cm}^2$

Memory map

Sector → The portion (or part) of the circular region enclosed by two radii and the corresponding arc of a circle is called a sector of the circle.



Segment → The portion (or part) of the circular region enclosed between a chord and the corresponding arc of a circle is called a segment of the circle.

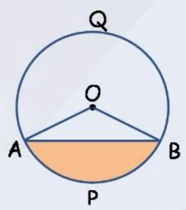


Length of the arc

Length of arc of sector of angle θ = $\frac{\theta}{360} \times 2\pi r$ sq. units

Area of sector

Area of the sector of angle θ = $\frac{\theta}{360} \times \pi r^2$ sq. units



Area of segment

Area of segment APB = area of sector OAPB – area of ΔOAB = $\frac{\theta}{360} \times \pi r^2$ – area of ΔOAB

1. Area of major sector AQB = πr^2 – area of the minor sector OAPB = $\frac{360-\theta}{360} \times \pi r^2$
2. Area of major segment AQB = πr^2 – area of the minor segment APB.

Note: Area of ΔOAB with $\angle$AOB = θ is $\frac{1}{2} r^2 \sin\theta$ **or** $r^2 \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$

where, r is the radius of the circle and θ is the angle of the sector.